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Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas

On the relationship between neural complexity and sociality: A research compilation on embodied dyadic interaction models

TESIS

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PRESENTA:
Georgina Montserrat Reséndiz Benhumea

DIRECTOR DE TESIS :
Dr. Tom Froese
Okinawa Institute of Science and Technology Graduate University

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Dedication

> “ *I am because we are, and since we are, therefore I am.* ”
>
> John Mbiti, *African Religions and Philosophy*

This work is dedicated to my beloved family and friends. Here, I want to share some of my deepest thoughts for each of them:

To my loving mother, Georgina Araceli Benhumea Sierra, thanks to her my life has been a vibrant and joyful explosion of colors and fulfillment. You are my muse, my greatest source of inspiration. You are one-of-a-kind, the strongest, bravest and most virtuous, brilliant, and hardworking woman in this world. You are my vital pillar, all that I am now is thanks to you. I will be forever grateful for your boundless patience, support, encouragement, caring, confidence, surprises, help, values, and huge efforts into everything that you have done for all of us. I will be always here for you, I will always protect your beautiful smile. Thank you for being my guidance, thank you for your pure feelings of unconditional love and tenderness, thank you for being the best mother to me! And, thank you for always being by my side to create and experience the most meaningful moments of life together, you are my most precious and valuable treasure!

To my caring father, Alberto Guadalupe Reséndiz Reséndiz, thanks to him my world has been illuminated with fireworks of happiness and positiveness. Thank you for nurturing my imagination and curiosity since I was a child, thank you for being the best storyteller, and thank you for playing with us! I feel very fortunate for having your words of wisdom and encouragement when I need them, thank you for listening to me, I will always be there to listen to you and take care of you! I am very grateful for all the trips we have shared together, thank you for showing us the most beautiful places in Mexico and igniting the spark of my wanderlust! I will always appreciate all your hard work, perseverance, warmth, kindness, support, motivation, confidence, bravery, creativity, and inventiveness. Thank you for guiding me on the right path of life and for your unconditional love, you are the best father to me!

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To my cousin, Herman Peralta Benhumea, for being like a brother to me. I will always cherish the memories we shared together on our journey from childhood to adulthood. Thank you for sharing with me your passion for computers!

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To my dear friends, Gabriel de los Santos and family, for their timeless friendship, great support and enjoyable conversations. Thank you for being our extended family for so long!

To my dear neighbors and great friends, Mrs. Juana, Lety, Lulú, Mrs. Norma, Raúl, and Anastacio, for being attentive, kind, and supportive. You are more than my friends, you are part of my family!

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To all my friends from the ECSU-Team, OIST, the 4E Cognition Group, IIMAS-UNAM, and FESC-UNAM, for being a great source of happiness, fellowship, togetherness, collaboration, and harmony. Thank you for enriching my life with your presence!

To my dear friends, Teresa Hernández, Tomoka Yoseyama, and Milena Menezes, for their great help, support, and friendship. Thank you for introducing and guiding me into the Japanese culture, and showing me the beauty of Okinawa, Japan!

In memory of Dr. Humberto Maturana, whose bright and eye-opening ideas have radically changed the way I see life. Thank you for your great legacy, I will keep working hard for the good of humanity as you did!

Finally, this work is dedicated as well to all my loyal pets, whose wholehearted company, love, and warmth, have been an essential part of my life!

To all of you, thank you (ありがとうございます)!

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"Por mi raza hablará el espíritu"

Abstract

Traditional cognitive science has assumed that cognition only occurs inside of our brains and has supported the idea that behavior is merely a product of internal neural activity. Recent approaches to cognitive science, namely, *"4E" cognition*, have called into question all these assumptions, by arguing that the basis of cognition spans over brain, body, and environment. In this sense, social interaction can have an effect in the neural and behavioral complexity of the interactive agents. In order to investigate these proposals as proof of concepts, researchers have made use of agent-based modeling to generate minimal cognitive models following an evolutionary robotics methodology and a dynamical systems approach to cognition. Following this line, recent studies demonstrated that social interaction increases the complexity of neural dynamics of interactive agents, which cannot be achieved in isolation. Inspired in these previous studies, here we present a scientific research compilation that in conjunction led to the development of the first minimal cognitive model that investigates the consequences of brain size reduction, in terms of neural complexity, in the evolutionary transitions from solitary to social groups, as is the case of Neolithic humans and eusocial insects. We proposed to investigate and compare neural complexity of *smaller-brained* (2-neuron model) and *bigger-brained* (3-neuron model) agents in different levels of coupling (dyadic interaction, isolation and in the presence of a *ghost* partner). To achieve this goal, we divided our studies into three main stages: (1) *smaller-brained agent couplings*, from the perspective of complex systems, by integrating agent-based modeling and social network analysis for a better visualization of the interactions between these simple components, and, from a dynamical systems approach to cognition, by analyzing the state spaces of their decoupled continuous-time recurrent neural networks to gain insights into their intrinsic basins of attraction; (2) *bigger-brained agent couplings*, from the perspective of evolutionary robotics, by creating a new condition of evolving agents with *ghost* agents as a source of non-social stimulation; and (3) *smaller-brained vs bigger-brained agent couplings*, from an evolutionary robotics methodology and a dynamical systems approach to cognition, by following standard practices of nonlinear time series analysis to determine and compare the embedding dimension of their evolved neural activity in solitary and social conditions. In short, we found that *smaller-brained* social agents can achieve comparable levels of neural complexity as *bigger-brained* solitary agents, which suggests that *smaller-brained* social agents take advantage of their capacity for interacting to enhance their intrinsic neural complexity, thus, allowing them to achieve similar behavioral performance with smaller brains. Our results open the possibility to test this hypothesis in novel experimental work with real social species, such as social insects.

Publications and Presentations

This thesis with title *On the relationship between neural complexity and sociality: A research compilation on embodied dyadic interaction models*, corresponds to a compilation of four scientific research papers previously submitted, presented, and/or published between 2019 and 2021. These works were developed at *IIMAS*, *UNAM*, Mexico City, Mexico, and as *Special Research Student (SRS)* (from January 13, 2020 to September 30, 2020) and as a *Research Intern (RI)* (from October 1, 2020 to March 31, 2021) in the *Embodied Cognitive Science Unit*, under the supervision of Dr. Tom Froese, at *Okinawa Institute of Science and Technology Graduate University (OIST)*, Okinawa, Japan. The details of each scientific research paper are presented as follows:

- **Main research papers:**

  1. **Conference paper**

     Authors: Georgina Montserrat Reséndiz-Benhumea, Tom Froese, Gabriel Ramos-Fernández, and Sandra E. Smith-Aguilar.

     Title: *Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior*

     Presented at: *The Artificial Life Conference 2019 (ALIFE 2019).*

     Date: August 2, 2019.

     Place: Newcastle upon Tyne, UK.

Published in: *Proceedings of the Artificial Life Conference 2019 (ALIFE 2019). MIT Press*, Cambridge, MA.

2. **Workshop paper**

   Authors: Georgina Montserrat Reséndiz-Benhumea and Tom Froese.

   Title: *Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis*

   Presented at: *The Second International Workshop on Agent-Based Modelling of Human Behaviour (2020 ABMHuB Workshop)* held in conjunction with *the 2020 Conference on Artificial Life (ALIFE 2020).*

   Date: July 17, 2020.

   Place: Montreal, Canada (virtual workshop).

   Published in: Online *2020 ABMHuB Workshop proceedings.*

3. **Conference paper**

   Authors: Georgina Montserrat Reséndiz-Benhumea, Ekaterina Sangati, and Tom Froese.

   Title: *Levels of Coupling in Dyadic Interaction: An Analysis of Neural and Behavioral Complexity*

   Presented at: *IEEE Symposium on Artificial Life (IEEE ALIFE)* held as part of the *2020 IEEE Symposium Series on Computational Intelligence (IEEE SSCI 2020).*

   Date: December 4, 2020.

   Place: Canberra, Australia (virtual symposium).

   Published in: *Proceedings of the 2020 IEEE Symposium Series on Computational Intelligence (SSCI), IEEE.*

4. **Journal article**

   Authors: Georgina Montserrat Reséndiz-Benhumea, Ekaterina Sangati, Federico Sangati, Soheil Keshmiri, and Tom Froese.

   Title: *Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity*

   Submitted to: *Frontiers in Neurorobotics* in the Research Topic on *Emergent Behavior in Animal-Inspired Robotics.*

   Status: Accepted for publication in 2021.

   Manuscript presentation at: *Dyadic Interactions in a Comparative Perspective workshop* organized by *OIST (Okinawa Institute of Science and Technology* and *IRCN (International Research Center for Neurointelligence, The University of Tokyo)*, at *OIST Seaside House* on December 10, 2020.

- **Other related papers, posters, and presentations:**

  1. **Talk**

     Authors: Georgina Montserrat Reséndiz-Benhumea, Tom Froese, Gabriel Ramos-Fernández, and Sandra E. Smith-Aguilar.

     Presentation title: *Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior*

     Presented at: *Joint Workshop on Cognitive Robotics and Cognitive Science.*

     Date: June 26, 2019.

     Place: *IIMAS*, *UNAM*, Mexico City, Mexico.

2. **Workshop paper**

   Authors: Georgina Montserrat Reséndiz-Benhumea and Tom Froese.

   Title: *Social Network Analysis Applied to Agent-Based Modeling to Understand Social Withdrawal as an Emergent Property*

   Presented at: *2019 International Workshop on Agent-Based Modelling of Human Behaviour (ABMHuB)* held in conjunction with the *2019 Conference on Artificial Life*

   Date: July 31, 2019.

   Place: Newcastle upon Tyne, UK.

   Published in: Online *2019 ABMHuB Workshop proceedings.*

3. **Poster presentation**

   Authors: Georgina Montserrat Reséndiz-Benhumea and Tom Froese.

   Poster title: *Understanding Social Withdrawal as an Emergent Property Using SNA Applied to ABM Approach*

   Presented at: *MobiCom 2019*, as part of the *America's Student Symposium on Emerging Technologies (ASSET).*

   Date: October 22, 2019.

   Place: Los Cabos, Baja California Sur, Mexico.

4. **Lightning talk**

   Authors: Georgina Montserrat Reséndiz-Benhumea and Tom Froese.

   Presentation title: *Modeling the interdependence of neural and social complexity*

   Presented at: *2020 Emerging Researchers in Artificial Life Workshop (2020 ERA Workshop).*

Date: July 16, 2020.

Place: Montreal, Canada (virtual workshop held as part of the *2020 Conference on Artificial Life (ALIFE 2020)*).

5. **Poster presentation**

   Authors: Ekaterina Sangati, Georgina Montserrat Reséndiz-Benhumea, Federico Sangati, Alexey Yudin, and Tom Froese.

   Poster title: *Maximizing Transfer Entropy in Agent-Based Models*

   Presented at: *International Symposium on Artificial Intelligence and Brain Science 2020 (AIBS 2020).*

   Date: October 10 - 12, 2020.

   Place: Tokyo, Japan (virtual symposium).

Contents

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Introduction

> “*Science has explored the microcosmos and the macrocosmos; we have a good sense of the lay of the land. The great unexplored frontier is complexity.*”
>
> Heinz Pagels, *The Dreams of Reason*

1.1 Background and Problem Statement

Social interaction is inherently rooted in our human nature. We, as social beings, are constantly involved in interaction with others, either in dyads (pairs), smaller, or larger groups. For instance, when having a conversation, collaborating in different activities at school or work, dancing, among many others (De Jaegher et al., 2010). Undoubtedly, relying in others has brought us several benefits in different scales, from satisfying our most basic survival needs to achieving collective goals. Similarly, other social species have taken advantage of social interaction, such as social insects. Social insects, also known as *eusocial* insects, such as ants, bees, termites, and wasps, are *self-organized* systems that commonly live in densely populated groups, also known as *colonies*. They display complex collective behavior, e.g. building nests, foraging, task allocation, feeding the brood, among other collective tasks, that arise from local interactions between their simple individuals, and between these individuals and their environment (Bonabeau et al., 1999). As we can see from these examples, social interaction results crucial to profoundly understand these complex phenomena, however, it has remained scientifically poorly understood (Candadai et al., 2019).

Traditional cognitive science has supported the idea that cognition only relies on the brain and that behavior is just a product of internal neural activity. However, alternative paradigms to cognitive science, namely embodied, embedded, extended, and enactive cognition (*"4E" cognition*), also known in conjunction as *embodied cognitive science*, has challenged those assumptions by claiming that the basis of cognition spans over brain, body, and environment. In this sense, social interaction can play a role as an enabler of cognition, thus, enhancing our individual capacities (De Jaegher et al., 2010). Recently, researchers from this community demonstrated that social interaction increases the complexity of the dynamics of neural activity of embodied agents and revealed that this cannot be achieved on their own (Candadai et al., 2019).

Inspired in these previous studies, here we provide a scientific research compilation that in conjunction aims to gain novel insights into *the relationship between neural complexity and sociality*, more specifically, we focused on the development of the first minimal cognitive model that investigates the consequences of brain size reduction, in terms of *neural complexity*, in the evolutionary transitions from solitary to social groups, as evidence has shown for the case of Neolithic humans (Brown, 1987; Brown and Maeda, 2004; Henneberg, 1988; Henneberg and Steyn, 1993) and some eusocial insects, such as wasps (O'Donnell et al., 2015).

1.2 Hypothesis

In general, we focused on the role of brain size in generating neural complexity in solitary and social conditions. Specifically, our hypothesis is the following:

> *"It is possible for social agents with a smaller neural network to exhibit at least the same complexity of neural activity as a solitary agent with a larger neural network"*.

1.3 Objectives

1.3.1 General objective

Investigate and compare neural complexity of *smaller-brained* (2-neuron model) and *bigger-brained* (3-neuron model) agents in different levels of coupling (dyadic interaction, isolation and in the presence of a *ghost* partner).

1.3.2 Specific objectives

- Integrate agent-based modeling and social network analysis for a better visualization of the interactions and a better understanding of the emergent properties of complex systems consisted of *smaller-brained* components.

- Replicate and extend the Candadai et al. (2019) model to analyze the state spaces of the decoupled continuous-time recurrent neural networks of interactive *smaller-brained* agents to get clarity on their intrinsic complexity.

- Address two of the main limitations of the Candadai et al. (2019) model by: (1) including one more neuron in the internal layer of the neural architecture to build *bigger-brained* agents, and (2) creating a new condition in which we evolve agents with a *ghost* agent as a source of non-social stimulation to see the effects of a richer environment.

- Create the first minimal cognitive model that analyzes and compares the levels of neural complexity of *smaller-brained* and *bigger-brained* agents evolved in solitary and social conditions to gain novel insights into the consequences of brain size reduction in the evolutionary transitions from solitary to social species, particularly, the cases of Neolithic humans and some eusocial insects, such as wasps.

1.4 Thesis organization

In order to achieve our general and specific objectives, we divided our studies into three main stages:

1. *Smaller-brained agent couplings* **(Chapter 2):** In this first stage, we investigated them from two main perspectives:

   (1) From the perspective of complex systems, we present our conference research paper entitled: *Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior*. Here we integrated agent-based modeling (ABM) and social network analysis (SNA) for a better visualization of the interactions between simple components.

This was applied to a minimalist case study in swarm robotics loosely inspired by ant foraging behavior.

(2) From a dynamical systems approach to cognition, we present our workshop research paper entitled: *Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis*. Here we analyzed the state spaces of the decoupled continuous-time recurrent neural network of interactive embodied agents, based on the Candadai et al. (2019) model, to gain insights into their intrinsic basins of attraction.

2. *Bigger-brained agent couplings* **(Chapter 3):** In this second stage, we investigated them from the following perspective:

   (1) From an evolutionary robotics approach, we present our conference research paper entitled: *Levels of Coupling in Dyadic Interaction: An Analysis of Neural and Behavioral Complexity*. Here we addressed two main limitations of the Candadai et al. (2019) model. First, we added one more neuron to the original configuration of the neural architecture of the embodied agents, thereby, building *bigger-brained* agents (3-neuron model). And, second, we created a new condition in which we evolved agents with *ghost* agents, as a source of non-interactive stimulation.

3. *Smaller-brained vs bigger-brained agent couplings* **(Chapter 4):** In this third stage, we investigated them from the following perspectives:

   (1) From an evolutionary robotics methodology and a dynamical systems approach to cognition, we present our journal article (*accepted for publication*) entitled: *Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity*. Here we show through a minimal cognitive model that the neural complexity of *smaller-brained* agents evolved in interaction is comparable to the neural complexity of *bigger-brained* agents evolved in solitary conditions.

> We followed standard practices of nonlinear time series analysis to determine and compare the embedding dimension of the corresponding agent's evolved neural activity in solitary and social conditions.

Finally, in **Chapter 5** we provide the conclusions and future directions for this work.

2

Smaller-brained agent couplings

In this chapter, we present our following conference and workshop papers:

- *Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior* by Georgina Montserrat Reséndiz-Benhumea, Tom Froese, Gabriel Ramos-Fernández, and Sandra E. Smith-Aguilar. Presented at *The Artificial Life Conference 2019 (ALIFE 2019).*

- *Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis* by Georgina Montserrat Reséndiz-Benhumea and Tom Froese. Presented at *The Second International Workshop on Agent-Based Modelling of Human Behaviour (2020 ABMHuB Workshop)* held in conjunction with *the 2020 Conference on Artificial Life (ALIFE 2020).*

2.1 *Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior*

Abstract

Social network analysis and agent-based modeling are two approaches used to study biological and artificial multi-agent systems. However, so far there is little work integrating these two approaches. Here we present a first step toward integration. We developed a novel approach that allows the creation of a social network on the basis of measures of interactions in an agent-based model for purposes of social network analysis. We illustrate this approach by applying it to a minimalist case study in swarm robotics loosely inspired by ant foraging behavior. For simplicity, we measured a network's inter-agent connection weights as the total number of interactions between mobile agents. This measure allowed us to construct weighted directed networks from the simulation results. We then applied standard methods from social network analysis, specifically focusing on node centralities, to find out which are the most influential nodes in the network. This revealed that task allocation emerges and induces two classes of agents, namely foragers and loafers, and that their relative frequency depends on food availability. This finding is consistent with the behavioral analysis, thereby showing the compatibility of these two approaches.

Applying Social Network Analysis to Agent-Based Models: A Case Study of Task Allocation in Swarm Robotics Inspired by Ant Foraging Behavior

Georgina Montserrat Reséndiz-Benhumea[1], Tom Froese[1, 2], Gabriel Ramos-Fernández[1, 3] and
Sandra E. Smith-Aguilar[4]

[1]Institute for Applied Mathematics and Systems Research, National Autonomous University of Mexico, Mexico City, Mexico
[2]Center for the Sciences of Complexity, National Autonomous University of Mexico, Mexico City, Mexico
[3]Unidad Profesional Interdisciplinaria en Ingeniería y Tecnologías Avanzadas, Instituto Politécnico Nacional, Mexico City, Mexico
[4]Conservación Biológica y Desarrollo Social A.C., Mexico City, Mexico
gmontserb@comunidad.unam.mx

Abstract

Social network analysis and agent-based modeling are two approaches used to study biological and artificial multi-agent systems. However, so far there is little work integrating these two approaches. Here we present a first step toward integration. We developed a novel approach that allows the creation of a social network on the basis of measures of interactions in an agent-based model for purposes of social network analysis. We illustrate this approach by applying it to a minimalist case study in swarm robotics loosely inspired by ant foraging behavior. For simplicity, we measured a network's inter-agent connection weights as the total number of interactions between mobile agents. This measure allowed us to construct weighted directed networks from the simulation results. We then applied standard methods from social network analysis, specifically focusing on node centralities, to find out which are the most influential nodes in the network. This revealed that task allocation emerges and induces two classes of agents, namely foragers and loafers, and that their relative frequency depends on food availability. This finding is consistent with the behavioral analysis, thereby showing the compatibility of these two approaches.

Introduction

Social network analysis (SNA) has been widely used in the study of biological multi-agent systems (Krause et al., 2015). In recent years, there has been an increasing interest in analyzing animal social networks (Scott and Carrington, 2014). For example, there are studies in social networks of spider monkeys (Ramos-Fernández et al., 2009), crows (Rutz et al., 2012) and social insects (Charbonneau et al., 2013). Similarly, agent-based modeling (ABM) has been applied to the same area. Ramos-Fernández et al. (2006) studied the emergence of animal social structure using agent-based models. Guo and Wilensky (2016), researchers in Alife, have demonstrated the utility of agent-based models of social insects as powerful tools to understand complex system principles. Moreover, Wang et al. (2019) studied collective behavior of bacteria, which use signaling systems known as quorum-sensing (QS) to communicate and cooperate. They used an agent-based modeling approach to understand the emergence of complex QS architectures and functions.

On the other hand, there are few studies using these two approaches (SNA and ABM) in combination in artificial multi-agent systems (MAS), particularly, in swarm robotics. Swarm robotics is a recent approach in the field of artificial swarm intelligence to study the coordination of multi-robot systems (MRS) without central control inspired on swarms observed in nature, such as those of social insects. Collective behavior emerges from robot-robot and robot-environment interactions (Tan and Zheng, 2013). There is a strong potential found in mimicking social insect behavior because this is highly convenient for solving complex coordination tasks (Alers et al., 2014). For example, ant foraging behavior induces task allocation as an emergent property, which is suitable for swarm robotics (Labella et al., 2006).

In this study, we are interested in applying social network analysis to agent-based modeling. There are previous studies that successfully combined SNA and ABM (Fontana and Terna, 2015) or SNA and MAS (Ma et al., 2009; Grant, 2009). For a better understanding, we have developed a taxonomy of social interaction models based on the approach of Powers et al. (2018), as shown in Figure 1.

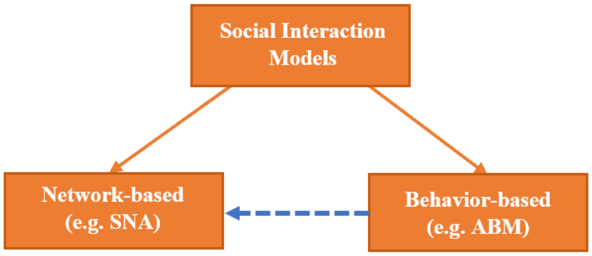


Figure 1: A taxonomy of social interaction models. We have two classes of social interaction models: network-based and behavior-based. Social network analysis (SNA) is an instance of network-based model and agent-based modeling (ABM) is an instance of behavior-based model. We propose there should be a bridge (dashed blue arrow) from behavior-based to network-based models to have a complete perspective of the network dynamics in a complex system in order to get new insights on their emerging properties. That is, moving from agent-based modeling to social network analysis.

Figure 1 shows our proposed taxonomy where we consider there should be a bridge from behavior-based (e.g. ABM) to network-based (e.g. SNA) models to have a complete perspective of the network dynamics in a complex system in order to get new insights on its emerging properties. Thus, our representation of moving from ABM to SNA.

We found that this approach (from ABM to SNA) has not been exploited in foraging and task allocation in swarm robotics. However, there are previous papers using either one of these two approaches (ABM or SNA). Iba (2013), developed agent-based modeling and simulations with swarms; Palestra et al. (2017), modeled and simulated rescue robots using the swarm robotics approach; Koval et al. (2009), introduced a social network to a swarm robotics system in order to improve accuracy in automatic target recognition.

The main goal of this study is to apply our proposed approach, from agent-based modeling to social network analysis, to a case study in swarm robotics inspired by ant foraging behavior to show task allocation as an emergent property of the complex system.

The case of ant foraging behavior in swarm robotics

Task allocation, in social insects, refers to the processes by which a task is carried out by each member of the colony. As examples, we have foraging and brood care. Additionally, these processes adapt to changing conditions (Gordon, 2016). In this paper, we are interested in task allocation as an emergent property of ant foraging behavior.

The main features of ant foraging behavior can be summarized as follows (Labella et al., 2006):

- The ant explores the environment in random displacements until it finds food. There are three cases of how to take it to the nest: (i) the ant pulls it, if it is not too heavy, (ii) the ant cuts it, (iii) the ant uses long or short recruitment (as a result of spreading pheromone trail).
- In individual or collective retrieval, food is directly pulled to the nest.
- When a forager returns to the nest, it unloads food by mouth-to-mouth contact into the *crops* (a pouch located just upstream of their stomachs) of other ants (Greenwald et al., 2018).
- After retrieving food, the ant goes straight back to the location where it found food.

Deneubourg et al. (1987) modeled an ant of the species *Pachycondyla apicalis* as an agent. Each agent has a probability P_l of leaving the nest, that varies depending on prior successes or failures. That is, when an ant retrieves food, its P_l increases by a constant Δ. Conversely, when an ant spends a lot of time without retrieving food, its P_l decreases by a constant Δ. P_l is bounded in the range $[P_{min}, P_{max}]$. They showed, by means of numerical simulations, that this model can explain task allocation and adaptation to the environment in ants (Labella et al., 2006).

The Variable Delta Rule algorithm (VDR) was based on Deneubourg et al.'s model. The main change was to multiply Δ by the number of consecutive successes or failures when increasing or decreasing the probability of leaving the nest, P_l, to carry out experiments in less time (Labella, 2003; Labella et al., 2006). This simple algorithm might be well suited for use in the context of swarm robotics.

Foraging, in test application for multi-robot systems (MRS), refers to searching for objects and taking them to a place called "nest" (Labella, 2003).

A swarm of interacting robots produces emergent behaviors. We can analyze the local interactions that allow the process of self-organization in these robots using social network analysis. Social network analysis studies the structural properties of groups or individuals in a network. It considers the effect of the interconnections on each other (Srivastava et al., 2014).

We developed an agent-based model based on the Variable Delta Rule algorithm to simulate a swarm of robots inspired by ant foraging behavior. Furthermore, for simplicity we focused on one of the main traits of *Pachycondyla apicalis* ants, that is hunting alone, consequently, there is no need of pheromone trails (Monmarché et al., 2000). Therefore, we modeled the case in which each forager takes only a unit of food when having a successful food retrieval without using pheromone trails. Then, we applied social network analysis to show task allocation as an emergent property of this model.

Methods

In this section, we present the methodology and tools that we used to implement, simulate and analyze the agent-based model of swarm robotics.

Variable Delta Rule Algorithm

We implemented the Variable Delta Rule algorithm (Labella, 2003; Labella et al., 2006). It consists in the following rules: each time the mobile agent has a success in food retrieval, the number of successes is increased and multiplied by Δ, then it is added to its P_l. Conversely, if the mobile agent has a failure in food retrieval, the number of failures is increased and multiplied by Δ, then it is subtracted from its P_l. Therefore, each mobile agent's probability of leaving the nest, P_l, is determined by the number of consecutive successful or failed food retrieval events. Note that P_l is bounded in the range $[P_{min}, P_{max}]$. This is shown in Algorithm 1.

Algorithm 1 Variable Delta Rule

```
Initialization:
    successes ← 0
    failures ← 0
    $P_l$ ← Initial value

if food is retrieved then
    successes ← successes + 1
    failures ← 0
    $P_l \leftarrow P_l$ + (successes * $\Delta$)
    if $P_l > P_{max}$ then
        $P_l \leftarrow P_{max}$
    end if
else if timeout then
    failures ← failures + 1
    successes ← 0
    $P_l \leftarrow P_l$ - (failures * $\Delta$)
    if $P_l < P_{min}$ then
        $P_l \leftarrow P_{min}$
```

Agent-based model (ABM) of swarm robotics

Environment. The simulated environment is a bounded two-dimensional grid (when a mobile agent reaches an edge it rotates 180 degrees and continues moving) and has a size of 91 x 91 units, with a unique nest located at the center (cluster of brown patches). A unit of the grid is represented by a patch of 5 x 5 pixels. A unit of food is represented by a unit of the grid located in a food source.

A fixed value in the range [0, 200] is assigned to each unit of the grid as follows: the distance between the focal unit of the grid and the center of the nest is calculated, then it is subtracted from 200 to obtain its “nest scent” value. This value is greater as the focal unit of the grid is closer to the nest. This approach is used by mobile agents to find their way back to the nest, it is known as following “nest scent” and it is described as follows: before each step forward when coming back directly to the nest, the mobile agent is going to head toward the greatest value of “nest scent” that is ahead of it and between the angles -45, 0 or 45. This is repeated until reaching the nest (Wilensky, 1997).

On the grid, food sources are clusters of units of food that are established in a fixed position and have a variable size between small (9 units of food), medium (45 units of food) or large (109 units of food). We have three food sources identified with the following colors, from the closest to the furthest from the nest: magenta, lime and turquoise. Figure 2 shows the distribution and different sizes for food sources in the environment. The environment is dynamic. A food source decreases by one unit of food each time a mobile agent has a successful food retrieval.

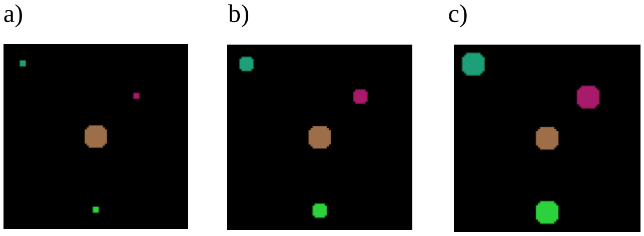


Figure 2: The simulated environments with different sized food clusters: a) Small (9 units of food for each food source), b) Medium (45 units of food for each food source), c) Large (109 units of food for each food source). The nest is in the center of the environment (cluster of brown patches). There are three available food sources, the color of each one indicates the distance to the nest, from the closest to the furthest we have: magenta, lime and turquoise.

Mobile Agents. We consider six mobile agents with initial positions in the center of the nest. Each mobile agent represents a robot. Movements, behaviors and interactions of mobile agents are described as follows:

Movements. Mobile agents have two classes of movements, these are described as follows:

- **Foraging movement:** When a mobile agent is out of the nest, it moves around the environment by random displacements to right and left each time-step, while considering not to take an occupied unit of the grid where another mobile agent is, as an obstacle avoidance mechanism. A displacement has a maximum turning angle of ± 40 degrees (Wilensky, 1997).
- **Nest seeking movement:** When a mobile agent is returning to the nest, it moves by displacements following the *“*nest scent*”* in each time-step. That is, it moves towards the next unit of the grid that has the greatest value of “nest scent” until reaching the nest, while considering not to take an occupied unit of the grid where another mobile agent is, to avoid obstacles.

Behaviors. Each mobile agent assumes one of the following behaviors per time-step depending on its own parameters and environment conditions (Labella et al., 2006):

- **Rest:** Stays in the nest.
- **Search for food:** Explores the environment while checking if there is a unit of food in the path. If there is one, the mobile agent takes it and returns to the nest with food (its number of successes is increased). If there is not one, the mobile agent keeps randomly moving around until a timeout occurs and it returns to the nest without food (its number of failures is increased).
- **Return to nest:** Finds the way back to the nest following the “nest scent” (Wilensky, 1997). It returns to the nest if a unit of food was successfully retrieved or a timeout occurs.
- **Feed:** Transfers food to all the mobile agents in the nest, when arriving to it after a successful food retrieval. Its number of successes is increased by one, therefore its probability of leaving the nest is going to be higher when updating it.

Furthermore, the mobile agents change their color to identify the performed behavior, as shown in Table 1.

Behavior	Color
Rest	Blue
Search for food	Red
Return to nest (with food)	Yellow
Return to nest (without food)	Violet
Feed	Orange

Table 1: Colors representing the behavior of each agent.

Interactions.

- **Agent - Agent (among mobile agents):** When a mobile agent arrives to the nest after retrieving a unit of food, there is an interaction between that mobile agent (emitter) and all the mobile agents in the nest (receivers), which represents food transfer. When a mobile agent is the emitter, its corresponding interaction variables (each one corresponds to an emitter-receiver interaction) increase by one. This is prompted by the forager ant’s interactions with the rest of the colony to feed them. Figure 4 shows an example of interaction among mobile agents.

Figure 4: Interaction between mobile agents. The orange-colored mobile agent (emitter) returned to the nest after retrieving a unit of food, when it arrives to the nest it interacts with all the blue-colored mobile agents (receivers) that are on the cluster of brown patches. This interaction represents food transfer (white arrow) from emitter to receivers and is loosely inspired by a forager ant feeding the rest of the colony in the nest.

- **Agent - Food Source (among mobile agents and food source clusters):** When a mobile agent finds and retrieves a unit of food, there is an interaction between that mobile agent and the retrieved unit of food from a food source, this is inspired by the forager ant's interactions with a food source. Each time a unit of food is retrieved from a food source, the number of units of food of that food source is decreased and the retrieved unit of food changes to color black to represent it was taken. Figure 5 shows an example of interaction among a mobile agent and a food source.

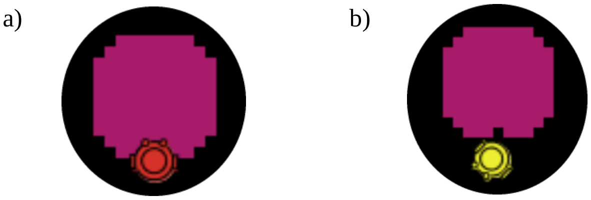


Figure 5: Interaction between a mobile agent and a food source. a) When a red-colored mobile agent finds out a unit of food, it interacts with the food source and b) it changes its color to yellow. The retrieved unit of food changes to color black to represent it was taken.

Experiments

The simulation-based experiments consisted in introducing a swarm of six mobile agents and three food sources (clusters of magenta, lime and turquoise patches), which we varied from small sizes (9 units of food for each food source cluster), medium sizes (45 units of food for each food source cluster) and large sizes (109 units of food for each food source cluster) to show task allocation under changing conditions of the environment. We created 30 instances per food sources size, i.e. 90 simulations in total. Each simulation lasted 2400 time-steps. The model was initialized with the following parameters (Labella, 2003): The search timeout was fixed to 228 units of time, Δ was set to 0.005, P_{min} to 0.0015, P_{max} to 0.05 and P_{init} to 0.033. Figure 6 shows a representative simulation of the agent-based model of swarm robotics and its components.

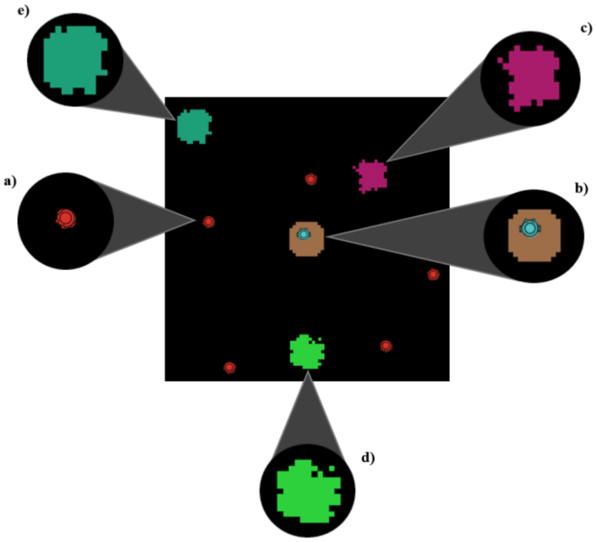


Figure 6: Screenshot of the agent-based model of swarm robotics after 2400 time-steps. a) Red-colored mobile agents are searching for food and b) blue-colored mobile agents are resting in the nest (cluster of brown patches). There are three food sources, from closest to furthest to the nest: c) magenta, d) lime and e) turquoise.

Social network analysis (SNA)

We followed the proposal by Wasserman and Faust (1994), who used network graphs to represent agent structures and network measures such as strength and centrality, to determine the particular role of individuals in the network's structure. We propose to represent mobile agents and their interactions in each simulation as a weighted directed network and focus on outdegree and weighted outdegree centralities to identify the induced classes, as a result of task allocation: foragers and loafers.

We constructed ninety weighted directed networks, from the 90 simulations, i.e. 30 simulations per food sources size (small, medium and large) as described in the Methods. We added a directed edge between two nodes (source and target) to represent whenever one of the two mobile agents (emitter) interacted with another one in the nest (receiver) to represent food transfer, this is inspired by the forager ants' interactions with the rest of the colony to feed them. Weights were assigned according to the number of interactions between the two mobile agents. Nodes were labeled with the six mobile agents' identifiers, from 0 to 5.

Measures were computed for each weighted directed network. We focused on outdegree and weighted outdegree centralities. Degree centrality shows the quality of a network node's interconnectedness by the number of direct contacts (Landherr et al., 2010). The outdegree is the number of ties that a node directs to others, it is interpreted as a quantity of information that is spread from one node to other (by outgoing edge). A high value is interpreted as sociability (Mansur et al., 2016). The centrality of nodes allows us to identify the most important or central nodes in a network.

Thus, outdegree centrality is a measure of the importance of a node, based on its number of ties. It is interpreted as the involvement of a node in the network. Weighted outdegree centrality is a measure of the importance of a node, based on its strength in terms of the total weight of their connections. It is interpreted as strength of collaborative ties (Opsahl et al., 2010). To calculate node strength, we have the following equation:

$$s_i = \sum_{j}^{N} w_{ij} \tag{1}$$

where w is the weighted adjacency matrix and w_{ij} represents the weight of the tie, it is greater than 0 if the node i is connected to node j (Opsahl et al., 2010).

Outdegree centrality can lead us to identify the mobile agents who are the most interconnected to others (i.e. more ties), whereas weighted outdegree centrality can lead us to identify the mobile agents who have the greatest number of interactions (i.e. wider edges) with the rest. Hence, we need both centrality measures to identify the expected induced classes, as a result of task allocation: foragers and loafers. Foragers' task consists in searching and retrieving food to feed the rest of the mobile agents; loafers' task consists in staying in the nest. Thus, foragers must be the most interconnected to the rest (i.e. having more ties) and with the greatest number of interactions (i.e. having wider edges). In the shown networks, node size refers to the value of outdegree or weighted outdegree centralities.

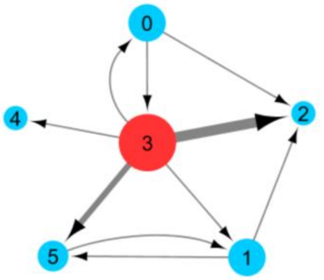
According to Labella's (2003) experimental results with *MindS-bots* (swarm of robots), he found that the distribution of probability of leaving the nest had two peaks and the boundary between the two groups was around 0.025, therefore there were two groups of *MindS-bots*: foragers ($P_l \geq 0.025$) and loafers ($P_l < 0.025$). As described in the Experiments, our model was initialized under Labella's (2003) experimental parameters, thus, we compared the results with the second parameter to identify foragers and loafers: mean probability of leaving the nest (mean P_l). Therefore, those mobile agents with mean $P_l \geq 0.025$ are likely to be foragers (red-colored nodes) and those with mean $P_l < 0.025$ are likely to be loafers (blue-colored nodes).

Results

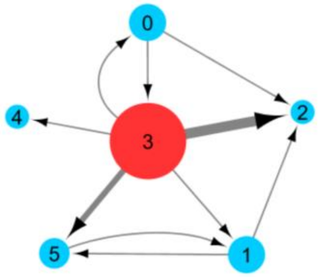
First, we show the results for three representative simulations (each one with a different food sources size). Then, we show in summary the results for the 90 simulations.

Simulation 1 - Small food sources size

Figure 7 shows the weighted directed network obtained with the results of simulation 1. The mean outdegree centrality of this network was 1.83, that indicates there were few nodes that were the most interconnected to others, in this case, only node 3 had ties to all the other nodes. The mean weighted outdegree centrality was 2.17, that indicates there were few interactions between mobile agents. There were two edges with high weight values, those were (3,2) and (3,5), which represented the greatest number of interactions between the mobile agents. Node 3 had the greatest number of ties and wider edges, moreover, it has a $P_l > 0.025$, therefore we interpreted it as a forager. The mean probability of leaving the nest of all nodes was 0.021, which was less than 0.025, so we expected more loafers than foragers. Likely agents to be foragers by P_l were represented by red-colored nodes and likely agents to be loafers by P_l were represented by blue-colored nodes. After analyzing the results, we got 1 forager (node 3) and 5 loafers (nodes 0, 1, 2, 4, 5).



(a) Social network 1 with nodes sized by their outdegree centrality



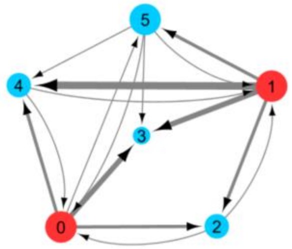
(b) Social network 1 with nodes sized by their weighted outdegree centrality

Figure 7: Graphs of social network 1 (the size of food sources is small) between six mobile agents where node sizes are reflecting: (a) Outdegree centrality, (b) Weighted outdegree centrality. Edge widths are reflecting the number of interactions between mobile agents. Node colors represent the probability of leaving the nest: if $P_l \geq 0.025$ the node is red, therefore, it is likely to be a forager and if $P_l < 0.025$ the node is blue, therefore, it is likely to be a loafer. As it can be seen, node color and size are consistent with each other, that means bigger nodes and probability to be a forager coincide; similarly, smaller nodes and probability to be a loafer also coincide. Therefore, both approaches obtain same results (in this case, 1 forager and 5 loafers).

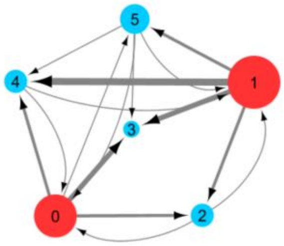
Simulation 2 - Medium food sources size

Figure 8 shows the weighted directed network obtained with the results of simulation 2. The mean outdegree centrality of this network was 2.67, that indicates there was a moderate number of nodes that were the most interconnected to others, more than in Simulation 1. The mean weighted outdegree centrality was 4.5, that indicates there was a greater number of interactions between mobile agents than in Simulation 1.

There were seven edges with high weight values, those were (1,5), (1,4), (1,3), (1,2), (0,4), (0,3) and (0,2) which represented the greatest number of interactions between agents. Nodes 0 and 1 had the greatest number of ties and wider edges, moreover, their $P_l > 0.025$, therefore we interpreted them as foragers. The mean probability of leaving the nest of all nodes was 0.022, which was less than 0.025, so we expected more loafers than foragers. After analyzing the results, we got 2 foragers (nodes 0, 1) and 4 loafers (nodes 2, 3, 4, 5).



(a) Social network 2 with nodes sized by their outdegree centrality

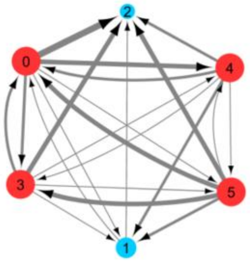


(b) Social network 2 with nodes sized by their weighted outdegree centrality

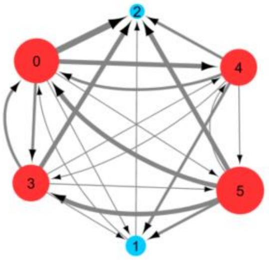
Figure 8: Graphs of social network 2 (the size of food sources is medium) between six mobile agents where node sizes are reflecting: (a) Outdegree centrality, (b) Weighted outdegree centrality. Edge widths are reflecting the number of interactions between mobile agents. Node colors represent the probability of leaving the nest: if $P_l \geq 0.025$ the node is red; therefore, it is likely to be a forager and if $P_l < 0.025$ the node is blue; therefore, it is likely to be a loafer. As it can be seen, node color and size are consistent with each other, that means bigger nodes and probability to be a forager coincide; similarly, smaller nodes and probability to be a loafer also coincide. Therefore, both approaches obtain same results (in this case, 2 foragers and 4 loafers).

Simulation 3 - Large food sources size

Figure 9 shows the weighted directed network obtained with the results of simulation 3. The mean outdegree centrality of this network was 3.6, that indicates there were many nodes that were the most interconnected to others, more than in Simulations 1 and 2. The mean weighted outdegree centrality was 6.83, that indicates there was a greater number of interactions between mobile agents than in Simulations 1 and 2. There were many edges with high weight values, due to high food availability. Nodes 0, 3, 4 and 5 had the greatest number of ties and wider edges, moreover, their $P_l > 0.025$, therefore we interpreted them as foragers. The mean probability of leaving the nest of all nodes was 0.029, which was greater than 0.025, so we expected more foragers than loafers. After analyzing the results, we got 4 foragers (nodes 0, 3, 4, 5) and 2 loafers (nodes 1, 2).



(a) Social network 3 with nodes sized by their outdegree centrality



(b) Social network 3 with nodes sized by their weighted outdegree centrality

Figure 9: Graphs of social network 3 (the size of food sources is large) between six mobile agents where node sizes are reflecting: (a) Outdegree centrality, (b) Weighted outdegree centrality. Edge widths are reflecting the number of interactions between mobile agents. Node colors represent the probability of leaving the nest: if $P_l \geq 0.025$ the node is red; therefore, it is likely to be a forager and if $P_l < 0.025$ the node is blue; therefore, it is likely to be a loafer. As it can be seen, node color and size are consistent with each other, that means bigger nodes and probability to be a forager coincide; similarly, smaller nodes and probability to be a loafer also coincide. Therefore, both approaches obtain same results (in this case, 4 foragers and 2 loafers).

Summary of results

The results of the social network analysis applied to the 90 weighted directed networks obtained from the simulation experiments are summarized in Table 2. It reports the mean and standard deviation of number of foragers and loafers.

Figure 10 shows the results of mean and standard deviation of probability of leaving the nest of the six mobile agents in the 30 experiments per food sources size (i.e. 90 experiments in total).

Contrasting the results of Table 2 and Figure 10, we can see that the social network analysis results confirmed the expectations of number of loafers and foragers obtained by the mean probability of leaving the nest varying the food sources size. Hence, the results proved task allocation among mobile agents as an emergent property of this model, inducing two classes: foragers and loafers. The number of foragers and loafers was adapted to the environment conditions (in this case, food availability).

Food Sources Size	**Food availability**	**Number of Foragers**	**Number of Loafers**
Small	Low	1.1 ± 0.3051	4.9 ± 0.3051
Medium	Medium	3.07 ± 0.7397	2.93 ± 0.7397
Large	High	4.77 ± 0.4302	1.23 ± 0.4302

Table 2: Mean and standard deviation of number of foragers and loafers calculated over 30 simulations per food sources size (i.e. 90 simulations in total) by applying social network analysis to the obtained weighted directed networks. The low values of standard deviation indicate that the behavior of the model was consistent across simulations.

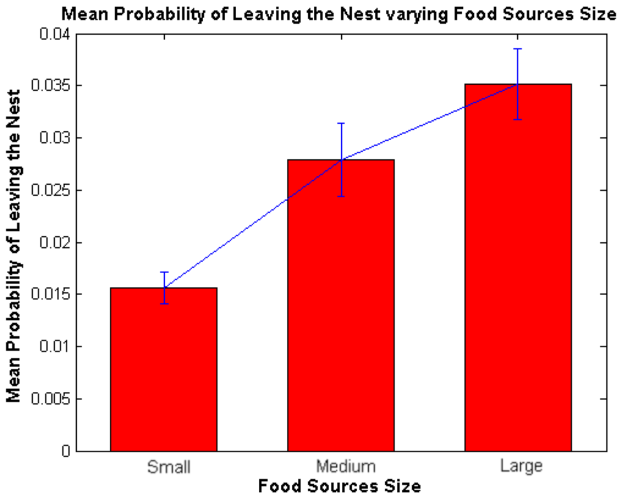


Figure 10: Mean and standard deviation of probability of leaving the nest while varying food sources size. The low standard deviation indicates that the behavior of the model was consistent across simulation experiments. These results show that with a low availability of food (small sized food sources) the mean $P_l < 0.025$, therefore, we expected more loafers than foragers; with a medium availability of food (medium sized food sources) the mean P_l is a little above 0.025, therefore, we expected similar number of loafers and foragers; with a high availability of food (large sized food sources) the mean $P_l > 0.025$, therefore, we expected more foragers than loafers.

Emergent property - Task allocation

In all simulations we observed that task allocation emerged and induced two classes: foragers and loafers. There were more loafers than foragers with low food availability (i.e. small sized food sources); there was similar number of loafers and foragers with medium food availability (i.e. medium sized food sources); and there were more foragers than loafers with high food availability (i.e. large sized food sources).

Discussion

As we have seen, moving from agent-based modeling (ABM) to social network analysis (SNA) lead us to a better understanding of the complex system by studying its emergent properties. In our agent-based model of swarm robotics we have shown that task allocation emerges and induces the creation of two classes: foragers and loafers. Furthermore, one of our main results was that the number of foragers and loafers changed with the conditions of the environment, as in real ant colonies. It means, task allocation changes as conditions vary (Gordon, 1999). Our model highlights that when more food is available, more foragers appear, and vice versa, as we observed in the weighted directed networks that we created for each simulation results. Thus, we conclude task allocation implies an adaptive and self-organized process (Labella, 2003).

A distinctive property revealed by the social network analysis was that the nodes with the greatest outdegree centralities were the most interconnected with the others (i.e. more ties) and those with the greatest weighted outdegree centralities had wider edges, therefore those nodes which were bigger in both graphs were the most interconnected mobile agents (i.e. having more ties) with the greatest number of interactions (i.e. having wider edges), hence we can call them, the “influentials” in the colony. These are the foragers.

Conclusions and Future Work

To summarize, we presented and analyzed our agent-based model of swarm robotics using social network analysis to show that it exhibits task allocation as an emergent property due to the Variable Delta Rule algorithm (Labella, 2003; Labella et al., 2006), which was inspired by ant foraging behavior. In future work, we are going to explore more complicated scenarios, for example, considering cheaters, those social insects that exploit the benefits of biological cooperation without contributing to them (Dobata and Tsuji, 2009). Moreover, this can be extended by using social network analysis to develop agent-based models, that is, moving from social networks to multi-agent systems in order to establish the measures of those networks and then design agent’s behaviors that will reach those measures. This could potentially be used in order to run game theoretic (network) models in an agent-based modeling framework.

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2.2 *Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis*

Abstract

Although cognitive complexity has been usually related to brain size or number of neurons, they are neither necessary nor sufficient conditions, since there are many species in nature showing that even with simple brains, they can exhibit unexpected levels of behavioral complexity. More recent approaches to cognitive science, such as enactive cognition, have been investigating social interaction in itself as part of an individual's cognition. Recently, Candadai et al. (2019) have demonstrated through a minimal model that social interaction increases agents' neural complexity and revealed that this cannot be achieved in isolation. In this paper, we first replicate the Candadai et al. (2019) model to analyze the state-space of the autonomous continuous-time recurrent neural networks of the interacting agents. Our results show that in terms of complexity, it is as simple as it can be, a single fixed-point attractor. Then, we proceed to ask whether, after loosening up the parameter constraints of this model, we will find more complexity in the state-space as there will be a broader variety of values in the parameters of neural controllers encoded in the genotype of each agent. Surprisingly, the state-space of this second approach leads to the same results, a single fixed-point attractor. Our findings, then, support the idea that cognitive complexity is mainly driven by the dynamics of social interaction rather than internal complexity.

Enhanced Neural Complexity is Achieved by Mutually Coordinated Embodied Social Interaction: A State-Space Analysis

Georgina Montserrat Reséndiz-Benhumea[1,2] and Tom Froese[1]

[1]Embodied Cognitive Science Unit, Okinawa Institute of Science and Technology Graduate University, 1919-1 Tancha, Onna-son,
Okinawa 904-0495, Japan
[2]Institute for Applied Mathematics and Systems Research, National Autonomous University of Mexico, 3000 Circuito Escolar,
Ciudad Universitaria, Mexico City 04510, Mexico
georginamontserrat.resendizbenhumea@oist.jp

Abstract

Although cognitive complexity has been usually related to brain size or number of neurons, they are neither necessary nor sufficient conditions, since there are many species in nature showing that even with simple brains, they can exhibit unexpected levels of behavioral complexity. More recent approaches to cognitive science, such as enactive cognition, have been investigating social interaction in itself as part of an individual's cognition. Recently, Candadai et al. (2019) have demonstrated through a minimal model that social interaction increases agents' neural complexity and revealed that this cannot be achieved in isolation. In this paper, we first replicate the Candadai et al. (2019) model to analyze the state-space of the autonomous continuous-time recurrent neural networks of the interacting agents. Our results show that in terms of complexity, it is as simple as it can be, a single fixed-point attractor. Then, we proceed to ask whether, after loosening up the parameter constraints of this model, we will find more complexity in the state-space as there will be a broader variety of values in the parameters of neural controllers encoded in the genotype of each agent. Surprisingly, the state-space of this second approach leads to the same results, a single fixed-point attractor. Our findings, then, support the idea that cognitive complexity is mainly driven by the dynamics of social interaction rather than internal complexity.

Introduction

Traditionally, cognitive complexity has often been associated with brain size or number of neurons. The social brain hypothesis supports these ideas by assuming that socially living species, such as human and nonhuman primates, should have enlarged brain sizes as a result of the demands of social life, in comparison to the more isolated ones. Briefly, this hypothesis is based on the correlation between social complexity and brain/group sizes (Dunbar, 1998; Barret et al., 2007). In contrast to these ideas, Barret et al. (2007) suggested that cognitive complexity might emerge from the interaction of brain, body and environment, and it is not simply attributable to the level of internal complexity itself. Furthermore, Barret (2011) argued that there are many of the so-called simpler animals with small brains that exhibit striking levels of behavioral complexity, such as paper wasps, which have the ability to recognize facial traits and use it to preserve social order in their hives and ants, which are able to find the shortest path to food sources through self-organizing processes. Therefore, these examples reinforce the idea that the complexity of behavioral activity is not merely a consequence of internal complexity and they highlight the importance of social interaction.

From an enactive cognition approach, social interaction is defined as a complex phenomenon involving engagement of at least two agents in a complex co-regulated pattern that enables social cognition (De Jaegher et al., 2010). In the last decades, there have been increasing efforts in investigating social interaction by using agent-based models to provide proof of concepts in order to make conceptual advances. Some of these models known as "minimal models" have been inspired by William Grey Walter's "turtle" mobile robots (Walter, 1950). These robots, which had only touch and light sensors and a very simple control architecture, performed surprising complex patterns of behaviors when interacting. On the basis of these findings, Di Paolo (2000) developed a simulation model of acoustically coupled embodied agents to study social coordination. In his model, these agents showed interesting behaviors, such as turn-taking and organized movement that emerged through their interaction via an acoustic medium. On the other hand, Froese et al. (2013a) created a minimal model of two acoustically coupled agents and demonstrated that in interactive scenario the neural dynamics of these agents has formal properties that could not be generated in isolation. Similar minimal approaches have been shown also to be very productive in studies of real social interaction in humans, e.g. perceptual crossing experiments (Froese, 2018).

Based on those previous works and following an evolutionary robotics methodology, Candadai et al. (2019) demonstrated that social interaction increases the complexity of an agent's neural activity and revealed that this cannot be achieved in isolation. They performed experiments on agent-based modeling using pairs of acoustically coupled embodied agents in an empty 2-dimensional environment, as shown in Figure 1.

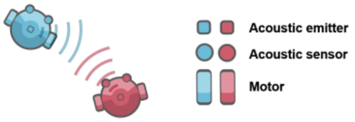

Figure 1: Illustration of a pair of acoustically coupled embodied agents in Candadai et al. (2019) model.

These experiments consisted of measuring neural entropy as an index of internal complexity. This was motivated by its interpretability, computational tractability and because of previous studies which have related higher levels of neural entropy with enhanced cognitive performance, e.g. enhanced generalization in motor learning tasks (Dotov and Froese, 2018). There were two different scenarios: by artificially evolving interacting pairs of embodied agents (maximizing interaction entropy) and by artificially evolving isolated agents in the environment (maximizing isolation entropy). Moreover, they measured the interaction entropy of an agent in the presence of a "ghost" partner, which was playing back pre-recorded behavior of previous trials, thus, they were not able to mutually interact with each other. As a result, this led to a loss of internal complexity of the "live" agent and proved that active interdependent interaction increases their neural complexity.

We found the Candadai et al. (2019) model very insightful, however, they did not provide a dynamical systems analysis to get clarity on the internal complexity, consequently, we got the next open questions: How complex are the internal state spaces of these agents? And how is that internal complexity related to their underlying genetic complexity, and how does it evolve over generations? Therefore, in this study we aim to achieve three novel contributions:

1. **Loosening up genetic constraints:** By proposing a novel approach that we call "layer-based unconstrained", in order to see the effects of the loosened parameter constraints and allow a broader diversity in the genotypes of the agents.

2. **State-space analysis:** By performing the dynamical systems analysis of the autonomous continuous-time recurrent neural network (CTRNN) of each best pair of agents for both approaches (layer-based constrained (original configuration) and layer-based unconstrained (proposed configuration)) in order to analyze their internal complexity.

3. **Evolutionary analysis of neural entropy:** By observing how the normalized neural entropy of the best pair of agents is evolving through time in two different scenarios: interactive and under "ghost" condition.

Thus, these goals share the following underlying reason for being studied: To demonstrate that social interaction matters and makes a difference in the complexity of neural dynamics.

Methods

We started working on our model as a replication of the Candadai et al. (2019) model. We choose to maximize only interaction entropy. The fitness function for the evolutionary search (i.e. neural entropy, which has been used as an indication of cognitive complexity) does not explicitly optimize social interaction nor does it optimize any specific task. The implementation details of this model and our modifications are presented as follows:

A. Body

The body of each embodied agent is designed as circular with a radius, R, of 4 units, which has two acoustic sensors symmetrically positioned at an angle of 45 degrees to the central axis (i.e. positioned at 45 and 315 degrees, respectively); an acoustic emitter placed on the center of the body, therefore, equidistant to its own sensors; and two motors driving wheels placed on opposite sides of the agent that enable movement in a 2-dimensional environment. This design was initially inspired by Di Paolo's acoustically coupled agents (Di Paolo, 2000). The strength of the emitted signal experiences linear loss with distance. It will be maximum and equal to that of the emitted strength at a distance $D = 2R$, between the center of the agents and linearly decays with increasing distance. Furthermore, the "self-shadowing" mechanism, i.e. experienced attenuation when the signal travels within the embodied agent, is modeled as a scaling factor over the sensory inputs in a range from 0.1 to 1. The equations for calculating the shielded distance, D_{sh}, that the signal passes through the body can be found in the Supplementary Material of Candadai et al. (2019). Then, the process of obtaining the sensory input for each sensor consists in first calculating it by applying the inverse square law without any "self-shadowing" attenuation, based on the distance between the sensor and the source, and then multiplying by the "self-shadowing" attenuating factor linearly mapped from 1 (when $D_{sh} = 0$) to 0.1 (when $D_{sh} = 2R$).

B. Environment

The simulated environment is a 2-dimensional unlimited arena. Collisions are modeled as point elastic, i.e. no changes in their angular velocity (no friction between bodies) and conserving the momentum of the whole system by having zero net effect on their velocity vectors. This is achieved by exchanging the velocity vectors of the embodied agents, so they simply bounce off each other without loss of energy. Modeling energy transfer is considered for future work.

C. Neural architecture

The neural architecture of each of the embodied agents is composed of three layers, we called them as follows: sensor layer, neuron layer and actuator layer.

1. Sensor layer: The sensor layer consists in two sensor nodes with a sigmoidal activation function. Their output is given by:

$$o_s = g_s \sigma(I_s + \theta_s) \tag{3}$$

where $\sigma(x) = 1/(1 + e^{-x})$ is the sigmoidal activation function, g_s is the sensory gain, I_s is the sensory input and θ_s is the bias.

2. Neuron layer: The neuron layer is modeled as a continuous-time recurrent neural network (CTRNN) (Beer, 1995), consisting in two fully recurrently connected neurons, this corresponds to a 2-dimensional dynamical system. The activity in each neuron is governed by the following state equation:

$$\tau_i \frac{dy_i}{dt} = -y_i + \sum_{j=1}^{N} w_{ij}\, \sigma(y_j + \theta_j) + \sum_{s=1}^{2} w_{is} o_s \tag{4}$$

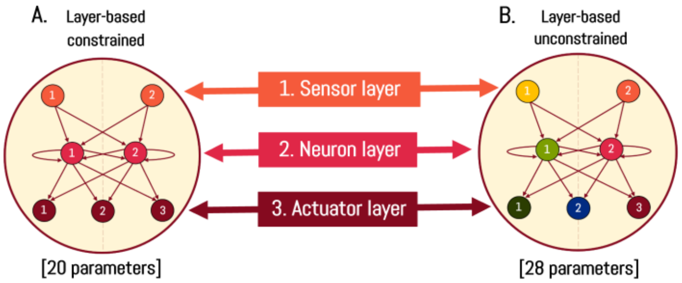

Figure 2: Two approaches based on layer parameter constraints of the neural controllers for the evolutionary search. **(A)** Layer-based constrained. This is the original configuration in the Candadai et al. (2019) model, where for N agents, the genotype contained 20N parameters. In this approach, the two acoustic sensor nodes have common gain and bias, the two neurons have common time-constant and bias, and the three actuator nodes (two motors and an acoustic emitter) have common gain and bias. **(B)** Layer-based unconstrained. This is the proposed configuration in order to loosen up the original parameter constraints and allow more diversity in the genotype, where for N agents, the genotype contained 28N parameters. In this approach, sensor nodes, neurons and actuator nodes do not have common parameters.

where dy_i/dt refers to the rate of change of internal state y_i of neuron i based on a time constant τ_i. This rate of change depends on three values: the current state, the weighted sum of outputs from the two neurons ($N = 2$) in the network and the total external input. The input from other neuron is calculated by weighting their output with weights from j to i, i.e. w_{ij}. The output of each neuron based on its internal state is given by $\sigma(y_j + \theta_j)$ where θ_j refers to a bias term for that neuron. Lastly, the state is influenced as well by the total external input received by the neuron, given by the weighted sum of the sensory input with weights w_{is} from sensor node s to neuron i and o_s being the sensory output from two sensors.

3. Actuator layer: The neurons feed into the actuator layer, where the input to each actuator node is a weighted sum of the outputs of the neuron. The actuator layer contains three actuator nodes, two corresponding to the left and right motors and one corresponding to the acoustic signal emitter. All of them are sigmoidal units with a gain and bias such that the output of actuator node i, m_i, is given by:

$$m_i = g_m \sigma\left(\sum_{n=1}^{N} w_{ni} * o_n + \theta_i\right) \tag{5}$$

where o_n is the output of the neuron, that is weighted by w_{ni} and θ_i is the bias term, and g_m is their gain.

Note that locomotion is managed by the effective control of the two motors. Net linear velocity is given by the average of their corresponding outputs and angular velocity which rotates the agent is given by their difference divided by the radius of the agent.

D. Neural entropy

During the entire course of behavior, i.e. 4 trials, the neural activity of each of the agents is recorded. Then, neural entropy in the 2-dimensional time series from the outputs of the two neurons (neuron layer) is measured as the neural complexity. The outputs of the two neurons are bounded in the range [0,1], as they are obtained from a sigmoid function. The output space is binned with 100 bins along each dimension, i.e. each axis corresponds to the outputs of the first and the second neuron, respectively, each axis goes from 0 to 1 and is divided into 100 bins, totaling 10,000 bins. Thus, a 2-dimensional histogram is created at the end of the 4 trials with all the recorded binning data points. The entropy H of the neural time series is given by:

$$H = \sum_{i=1}^{100}\sum_{j=1}^{100} -p_{ij}\log(p_{ij}) \tag{6}$$

where the probability of the neural activity in a particular bin [i,j], p_{ij}, is given by the number of data points in that bin divided by the total number of data points. The neural entropy then is normalized in the range [0,1] through dividing by the maximum possible entropy, $log(100 * 100)$, which is obtained when all bins are uniformly populated, therefore, a uniform distribution over the 2-dimensional histogram is achieved. Hence, normalized neural entropy is given by:

$$\widehat{H} = H/\log(100 * 100) \tag{7}$$

E. Evolutionary algorithm

A real-valued genetic algorithm was used as an optimization technique for the parameters of the neural controllers in order to maximize the agents' neural entropy. In contrast to the Candadai et al. (2019) model, here we followed two different approaches for the evolutionary optimization, namely, layer-based constrained and layer-based unconstrained, as shown in Figure 2. Each of these approaches is described as follows:

1. Layer-based constrained: This approach followed the original configuration in the Candadai et al. (2019) model, where each agent had 20 parameters, i.e. for N agents, the genotype contained 20N parameters. Here, both sensor nodes shared same gain and bias, both neurons shared same time-constant and bias, and the three actuator nodes shared same gain and bias. Therefore, in each of the three layers (sensor layer, neuron layer and actuator layer) the parameters were limited to have common values.

2. Layer-based unconstrained: This is the proposed approach in order to loosen up the identified parameter constraints to allow more diversity in the genotypes and see the possible effects in the subsequently analysis. Here, in each of the three layers (sensor layer, neuron layer and actuator layer) there were no common parameter values. Thus, each agent had 28 parameters, i.e. for N agents, the genotype contained 28N parameters.

In both approaches, each of the parameters were initially encoded in the range [-1,1]. When performing the trials to evaluate the performance, these parameters were scaled in different ranges in order to build the agents. For the sensor layer and actuator layer, their gains were scaled in the range [1,5] and their biases were scaled in the range [-3,3]. For the neuron layer, its time-constants were set in the range [1,2] and their biases were set in the range [-3,3]. Additionally, all weights from the three layers, were scaled in the range [-8,8]. All these parameter ranges were the same as those in the Candadai et al. (2019) model.

We performed 10 independent runs for each of the approaches (layer-based constrained and layer-based unconstrained), totaling 20 runs. Each of the runs started with a random population of 96 solutions, where each of them encoded the parameters for two agents in the interactive scenario and were evolved for up to 2000 generations.

In each generation, the agents built from the genotype were evaluated over 4 independent trials. Each trial lasted 200 units of time at a step size of 0.1 At the beginning of the trials, the agents were placed at 20 units from each other but varying their relative angle as $[0, \pi/2, \pi, 3\pi/2]$ for each trial. During the 4 trials, the neural activity of each agent was recorded, and at the end, the normalized neural entropy was calculated, and fitness was set as the average normalized neural entropy of the two agents.

After the performance evaluation, an elite population of the top 4% solutions was kept as is, and the remainder of the solutions for a new population was created by mutating and, then, crossing over this elite fraction. Mutation was applied by adding a zero-mean Gaussian mutation noise with variance 0.1 to the solutions, while, crossover involved that each parameter between a pair of solutions was swapped with a probability of 0.1.

F. Analysis under “ghost” condition

In order to delineate the role played by interdependent interaction on internal complexity, the best pair of agents in each of the selected generations (0, 1, 2, 3, 4, 5, 10, 50, 100, 500, 1000, 2000) were tested under a “ghost” condition. Blue agent was referred as the “ghost” agent and red agent was referred as the “live” agent. The “ghost” agent was replaying pre-recorded behavior from previous trials and the “live” agent was allowed to interact with it. The active agent started at a different random initial angle from “ghost” agent, in order to not repeat its behavior from those trials, while keeping the initial distance the same (20 units). As the evolutionary fitness evaluation, 4 trials were conducted, and the normalized neural entropy of the active agent was measured based on its behavior in the presence of a “ghost” partner.

G. Dynamical Systems Analysis

In order to analyze the complexity of the state-spaces for the best pair of agents in each approach, we performed the dynamical systems analysis of the autonomous continuous-time recurrent neural network (CTRNN) using *Dynamica* (version 1.0.9), a *Mathematica* package for the analysis of smooth dynamical systems, developed by Randall D. Beer.

Results

This section presents the results obtained from the best pair of agents in each of the approaches, layer-based constrained and layer-based unconstrained. These results are analyzed in detail in the Discussion and Conclusions section.

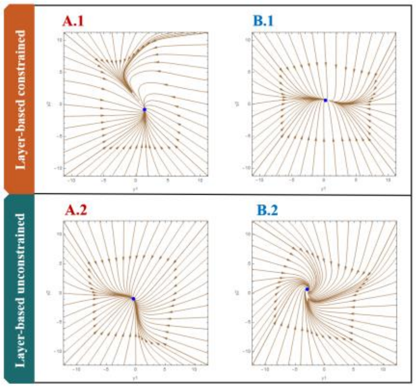

Figure 3: Dynamical systems analysis of the autonomous continuous-time recurrent neural network for the best pair of agents in the layer-based constrained (**A.1 and B.1)** and layer-based unconstrained **(A.2 and B.2)** approaches. The state-spaces of the dynamical systems are showing representative flow structure of a region of the activation space of the CTRNN. It can be seen that for red **(A.1 and A.2)** and blue **(B.1 and B.2)** agents there is a single stable fixed-point attractor (blue dot), the position of this point and its structure of attraction depend on the input values. In this case, the input values were those obtained at the end of the corresponding simulation (Generation 2000, Trial 1, best run in each case). The coordinates of the attractors are: (1.30753, -0.768629) in **A.1**, (0.2131, 0.6247) in **B.1**, (−0.42956,−0.96966) in **A.2** and (−2.9214, 0.6546) in **B.2**, where y1-axis represents the states of neuron 1 and y2-axis represents the states of neuron 2.

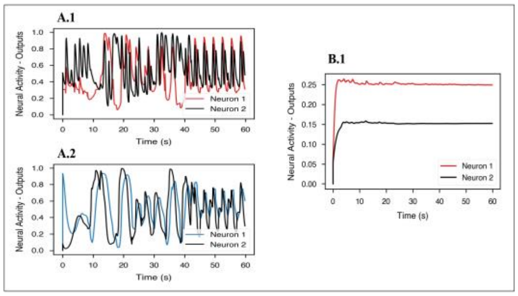

Figure 4: (A.1 and A.2) Neural activity of the two neurons of red agent **(A.1)** and blue agent **(A.2)** in interactive scenario (representative trial from the best pair of agents in layer-based unconstrained approach). As it can be seen, when both agents are interacting the neural activity shows chaotic aperiodic activity that cannot be produced by 2-dimensional decoupled CTRNNs, as demonstrated in Figure 4. **(B.1)** Neural activity of the two neurons of red agent under “ghost” condition. It can be observed that when red agent is in the presence of a “ghost” partner, the neural activity demonstrates remarkably lower complexity than the neural activity of the same agent in interactive scenario **(A.1)**.

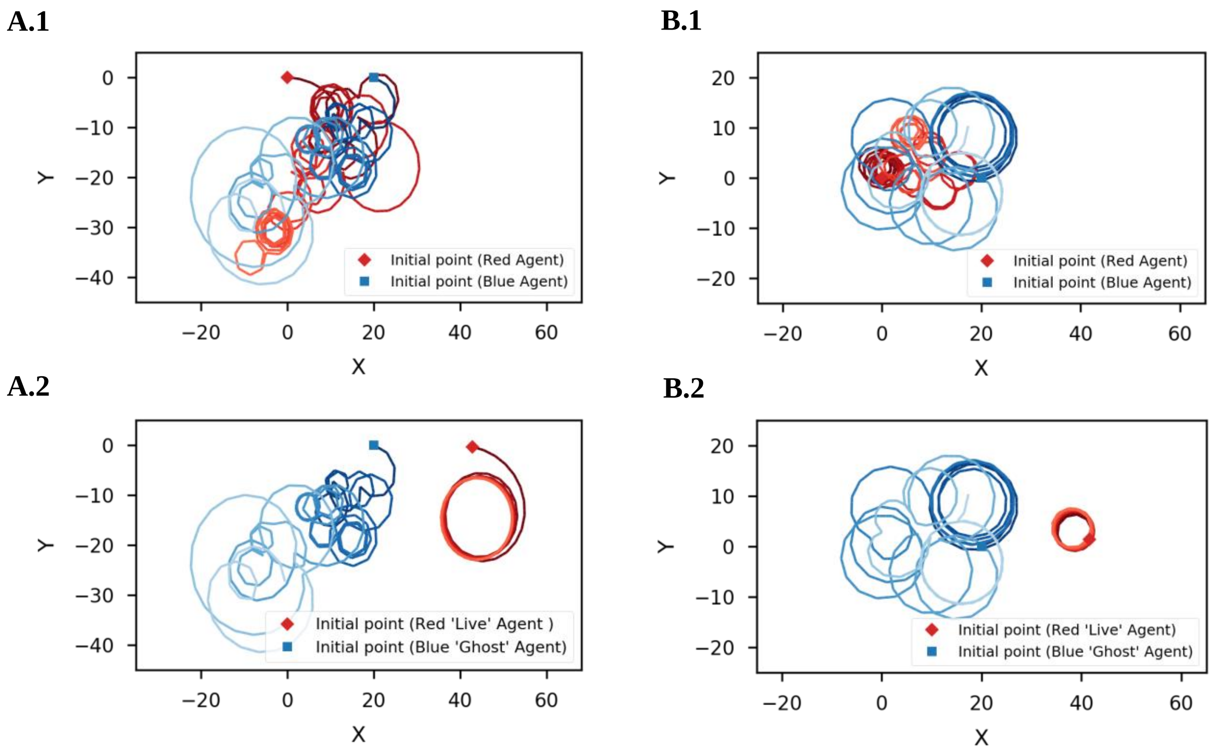

Figure 5: Results of behavioral patterns in different interaction conditions (in all images, earlier motion is darker than later motion to show directionality). **(A.1 and A.2)** Behavioral patterns for layer-based constrained approach. (**A.1)** Red and blue agents interacting in spiraling pairwise movement. (**A.2)** Red agent under "ghost" condition. In the presence of a "ghost" partner, the red agent moves in simple loops, therefore, its behavioral complexity is significantly reduced. **(B.1 and B.2)** Behavioral patterns for layer-based unconstrained approach. **(B.1)** Red and blue agents interacting in spiraling nested loops movement, where the red agent moves in smaller loops and the blue agent moves in bigger loops. **(B.2)** Red agent under "ghost" condition. In the presence of a "ghost" partner, the red agent moves in simple loops, therefore, its behavioral complexity is significantly reduced.

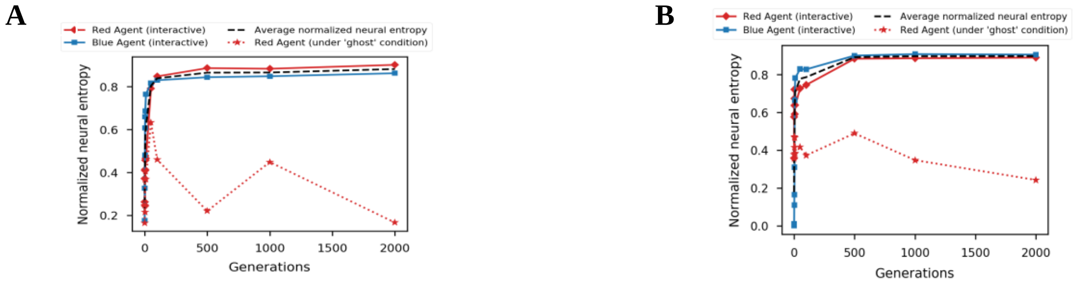

Figure 6: Normalized neural entropy of best pair of agents in each selected generation - best run (interactive vs under "ghost" condition scenarios) **(A)** Layer-based constrained approach. **(B)** Layer-based unconstrained approach. For both approaches it can be observed that when agents are able to mutually interact with each other they exhibit higher levels of normalized neural entropy, while under "ghost" condition, the "live" agent's normalized neural entropy drops, therefore, it suffers a loss in internal complexity. The highest entropy scores of red agent under "ghost" condition are achieved during the initial generations until generation 50 (for **A**) and generation 500 (for **B**), after that they start to drop until generation 2000.

Discussion and Conclusions

Our results in Figure 3 have shown that either in the layer-based constrained or the layer-based unconstrained approaches, we got a single fixed-point attractor in the state-space analysis of the autonomous continuous-time recurrent neural networks of each agent, it follows that, what insights can we get from these findings?

Previous minimal models of adaptive behavior in different task domains that have followed evolutionary robotics methodology have gotten similar results, i.e. only single fixed-point attractors. For instance, Campos and Froese (2017) developed a minimal model on referential communication based on the waggle dance of the bee to set the task for a receiver and a sender, then, by performing dynamical system analysis, they found only one fixed-point attractor that changed to different positions for each role, instead of having two different attractors, therefore, being an example of action switching models (Agmon and Beer, 2014). Furthermore, in social interaction minimal models, these results have been consistent as well, even when increasing the number of neurons (Froese and Fuchs, 2012) or having structurally identical pair of agents interacting, i.e. clones (Froese et al., 2013a). What is interesting to remark here is that in our findings the state-spaces also showed complementary roles of the CTRNN for red and blue agents, however, there was never specified a task to achieve, as described before for other previous models. This is one of the most insightful ideas that we got by going deeper in the Candadai et al. (2019) model: by maximizing the neural entropy in the evolutionary optimization, we were not expecting a particular behavior from the agents, however, in both approaches the strategy that the agents found was the same: by mutually interacting. At this point, it is worth recalling that elevated levels of neural entropy have been associated with improved cognitive performance. Therefore, if those elevated levels of neural entropy in the agents were achieved by mutually interacting, as shown in Figure 6, this suggests that social interaction might play a relevant role for cognition. Thus, these ideas might reject the classical view of cognitive science where cognitive complexity relies only on internal complexity.

Following the previous points, now we can understand the reason why our results showed only single fixed-point attractors. According to Zarco and Froese (2018), in “world-involving” scenarios obtained by evolutionary robotics methodology, the evolutionary search leads to a CTRNN structure that makes the agent to be interactively guided by the world. When an agent is evolved to display adaptive behavior, its CTRNN controllers usually exhibit a single attractor, however, still able of fruitful dynamics. Then, from the dynamical perspective, we can conclude that our agents in interaction became the whole brain-body-environment-body-brain system (Froese et al., 2013b) demonstrating chaotic aperiodic neural activity as shown in Figure 4 (A.1 and A.2), which in principle should require 3-dimensional decoupled CTRNNs. Thus, when an agent was in the presence of a “ghost” partner, this system was incomplete, generating consequently, a loss in neural and behavioral complexity of the “live” agent as shown in Figures 4 (B.1), 5 (A.2 and B.2) and 6. Finally, we propose thinking about how human cognition is enriched in real “world-involving” scenarios given that our daily life is full of social interactions.

In future work we will extend these results by implementing 3-neurons model and maximizing transfer entropy.

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3

Bigger-brained agent couplings

In this chapter, we present our conference paper:

- *Levels of Coupling in Dyadic Interaction: An Analysis of Neural and Behavioral Complexity* by Georgina Montserrat Reséndiz-Benhumea, Ekaterina Sangati, and Tom Froese. Presented at the *IEEE Symposium on Artificial Life (IEEE ALIFE)* held as part of the *2020 IEEE Symposium Series on Computational Intelligence (IEEE SSCI 2020).*

3.1 *Levels of Coupling in Dyadic Interaction: An Analysis of Neural and Behavioral Complexity*

Abstract

From an enactive approach, some previous studies have demonstrated that social interaction plays a fundamental role in the dynamics of neural and behavioral complexity of embodied agents. In particular, it has been shown that agents with a limited internal structure (2-neuron brains) that evolve in interaction can overcome this limitation and exhibit chaotic neural activity, typically associated with more complex dynamical systems (at least 3-dimensional). In the present paper we make two contributions to this line of work. First, we propose a conceptual distinction in levels of coupling between agents that could have an effect on neural and behavioral complexity. Second, we test the generalizability of previous results by testing agents with richer internal structure and evolving them in a richer, yet non-social, environment. We demonstrate that such agents can achieve levels of complexity comparable to agents that evolve in interactive settings. We discuss the significance of this result for the study of interaction.

Levels of Coupling in Dyadic Interaction: An Analysis of Neural and Behavioral Complexity

Georgina Montserrat Reséndiz-Benhumea
Embodied Cognitive Science Unit
Okinawa Institute of Science and
Technology Graduate University
Okinawa, Japan
Institute for Applied Mathematics
and Systems Research
National Autonomous University of Mexico
Mexico City, Mexico
georginamontserrat.resendizbenhumea@oist.jp

Ekaterina Sangati
Embodied Cognitive Science Unit
Okinawa Institute of Science and
Technology Graduate University
Okinawa, Japan
ekaterina.sangati@oist.jp

Tom Froese
Embodied Cognitive Science Unit
Okinawa Institute of Science and
Technology Graduate University
Okinawa, Japan
tom.froese@oist.jp

***Abstract*—From an enactive approach, some previous studies have demonstrated that social interaction plays a fundamental role in the dynamics of neural and behavioral complexity of embodied agents. In particular, it has been shown that agents with a limited internal structure (2-neuron brains) that evolve in interaction can overcome this limitation and exhibit chaotic neural activity, typically associated with more complex dynamical systems (at least 3-dimensional). In the present paper we make two contributions to this line of work. First, we propose a conceptual distinction in levels of coupling between agents that could have an effect on neural and behavioral complexity. Second, we test the generalizability of previous results by testing agents with richer internal structure and evolving them in a richer, yet non-social, environment. We demonstrate that such agents can achieve levels of complexity comparable to agents that evolve in interactive settings. We discuss the significance of this result for the study of interaction.**

***Index Terms*—agent-based modeling, social interaction, coupling, neural entropy, evolutionary robotics, continuous-time recurrent neural network, minimal cognition, dyad.**

I. INTRODUCTION

Recent years have seen an increase in efforts to understand the role of social interaction in social cognition from an embodied perspective [1]–[7]. It has been argued that rather than being merely an outcome of the dynamics of individual cognitive agents, social interaction can itself constitute cognition and have an effect on the individuals that partake in it[1]. According to one specific embodied cognition account – enactivism – social interaction is defined as an active co-regulated coupling between two or more autonomous agents, where their role of interactors co-emerges with the interaction itself and their individual cognitive capacities can be reduced or augmented [1]–[3]. In this paper we focus on how different levels of coupling can influence the agent's neural and behavioral complexity.

The types of couplings distinguished in this and previous work are inspired by a well-known experiment from developmental psychology: the "double TV monitor" paradigm [9]. In this experiment, 2-month-old infants interact with their mothers through a live video link. When the live video is replaced with a recorded replay of the previous actions of the mother, the infants become distressed, distracted and upset, suggesting that the reciprocity of the interaction makes a difference. That is, passive social input that is not sensitive to one's own response is not sufficient for a positive social experience. Simulation studies described here show that it might also not be sufficient for individual cognition.

Based on the previous approaches, we propose to distinguish the following levels of coupling in dyadic interaction:

- 2-way or bidirectional coupling: Both agents are mutually interacting, e.g. normal interaction as a mother playing with her infant. Other examples are conversations, dancing, collaborative work, etc. [3].
- 1-way or unidirectional coupling: Active agent is in the presence of a non-interactive agent, which is showing pre-recorded behavior, e.g. "double TV monitor" experiment.
- 0-way or no-way coupling: Active agent is not in the presence of a social partner, i.e. is alone.

This distinction is not meant to be exhaustive and future adjustments might be required. For instance, a case in which one agent is fully interactive while the other is present but staying still might fall somewhere between a 0-way and a 1-way coupling. Additionally, even within the isolated condition, one could distinguish different ways in which the agent can couple to its physical environment – bidirectionally in a full sensorimotor loop [10] or one-directionally, whereby the agent is passively receiving stimulation from the environment. However, this work focuses on the social interaction scenario and we think it is still useful to think of levels of coupling within this domain and how they might have distinct effects on individual cognitive capacities.

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[1] This is admittedly a contentious claim, cf. [8].

A. *Previous work*

Evolutionary robotics (ER) has been used as a scientific tool to study minimal models of cognition. It is a methodology especially suited for embodied cognition because it allows for developing integrated sensorimotor systems which act in close coupling with their environments, i.e. ER takes into account both embodiment and situatedness in how they contribute to the solution of particular cognitive tasks [11]. The Candadai et al. (2019) model [7] is a minimal model of cognition in interaction based on ER. This model demonstrates as a proof of concept that social interaction transforms the neural and behavioral dynamics of embodied agents more than what is achievable by agents operating in isolation. In this model, pairs of agents were evolved in a 2-dimensional environment to maximize their individual neural entropy[2]. When the agents were evolved together (interaction condition) they exhibited mutually coordinated behavior and higher individual neural entropy compared to when they were evolved alone (isolation condition). Furthermore, when agents that evolved in the interaction condition were tested in the presence of a ”ghost” non-interactive partner (ghost condition), they exhibited a loss in neural and behavioral complexity. This condition is analogous to a “double TV monitor experiment” in which infants used to normal 2-way interaction mode are suddenly placed in a 1-way interaction mode.

In our previous work [12], we have replicated the Candadai et al. (2019) model [7] with a less constrained parameter space of the agents, leading to a broader diversity in the agent genotypes but achieving similar results in terms of neural complexity. We also provided a state-space analysis of the evolved agents' neural network activity, which showed a single fixed-point attractor. This is consistent with previous ER work that shows that CTRNN controllers are often able to display rich dynamics despite having a single attractor because the attractor landscape is constantly shifting in the agent's interaction with the environment [13], [14].

B. *Current work*

The Candadai et al. (2019) model [7] demonstrated that 2-way coupling leads to higher neural complexity than 1-way coupling and 0-way coupling. However, the specific implementation adopted is open to two central criticisms.

First, it is known that continuous dynamical systems can only exhibit chaotic behavior when they have more than 2 dimensions [15]. This is indeed an assumption of the Candadai et al. (2019) model [7]: the fact that the agents' 2D brains exhibit chaotic activity is remarkable because it means that interaction can overcome the inherent limitations of their brains. However, a corollary of this is that if agents had brains with more dimensions, they could generate neural and behavioral complexity without interaction. If so, this would limit the result to very constrained systems and tell us nothing about cognition in multi-dimensional actual brains.

Second, it could be argued that a comparison between agents evolved in isolation and agents evolved in interaction is not a strong comparison that would justify far-reaching claims about the role of interaction in cognitive complexity. The isolated agents might be less complex not only because they don't interact but because they evolve in an overall poorer environment from which they receive no input. It could be that if the environment contained some stimulation, even if not social, the agents could leverage it to create more neural complexity.

In this study, we address these two limitations and investigate whether the Candadai et al. (2019) model [7] results still hold when we 1) include one more neuron in the internal layer of the neural architecture of the agents, i.e. we construct a 3-neuron model instead of 2-neuron model and 2) we create a new condition in which we evolve agents with a ghost agent, thereby providing them with a source of non-interactive stimulation. As a result, we implement 4 conditions based on 3 levels of coupling distinguished above, as illustrated in Fig. 1, 2, 3:

- **Interactive condition (2-way coupling):** pairs of agents are evolved in bidirectional interaction.
- **Ghost-test condition (1-way coupling):** pairs of agents are evolved in bidirectional interaction but tested with a ghost at the end.
- **Ghost-evolution condition (1-way coupling):** an active agent is evolved in the presence of a sufficiently complex ghost.
- **Isolated condition (0-way coupling):** agents are evolved in isolation.

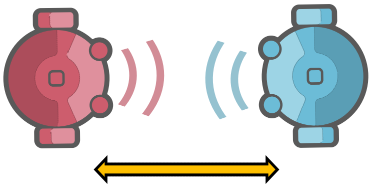

Fig. 1. Two-way coupling: Fully interactive condition, agents interact with each other by sending and receiving acoustic signals.

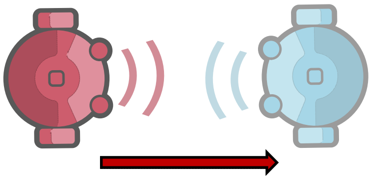

Fig. 2. One-way coupling: “Live” agent (red) tested or evolved with a “ghost” agent (blue). Live agent is sending and receiving acoustic signals. Ghost agent is playing back pre-recorded behavior of previous trials and so sending pre-recorded signals but not receiving anything.

In all conditions agents are evolved to maximize their neural complexity operationalized, for simplicity, as Shannon

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[2] It is important to emphasize here that maximizing this measure was not meant to achieve any particular adaptive outcomes. Rather, the point is to establish whether 2D brains can be evolved to become more complex and whether interaction is an enabling condition for this.

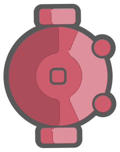

Fig. 3. Zero-way coupling. The "isolated" agent (red agent) is evolved on its own, therefore, the agent neither receives any input from the environment nor interacts with other agents.

entropy of neural outputs. However, the maximization of predictive information (PI, [16]), which has been successfully applied in previous self-organizing robotic systems where they exhibited a wider behavioral spectrum [17], is considered as an alternative approach for future work.

We then compare the level of neural and behavioral complexity in the last generation across all conditions. We find that despite having more powerful brains, agents that evolve in isolation exhibit lower neural and behavioral complexity. However, agents that evolve in the presence of the ghost exhibit an interesting divergence between the complexity of their neural output and behavior: their neural complexity is comparable to that of agents that evolve in interaction while their behavioral complexity is lower. We also investigate the quality of interaction between social conditions and conclude by placing our results in a broader theoretical context of embodied cognition.

II. METHODS

A. Model Design

Our model is a replication of the Candadai et al. (2019) model [7], except for the increased number of neurons in the internal layer of the neural architecture: three neurons instead of two.

1) Agents and environment: Agent bodies are circular, with a radius of 4 units. Each of them has two acoustic sensors symmetrically placed in the front of the agent at a 45° angle to the central axis; an acoustic emitter positioned in the center of the body; and two motors located in the right and left sides of the agent to allow movement in a 2-dimensional empty environment, which is an unlimited arena. Collisions are modeled as point elastic, which means, no change in the agents' angular velocity (i.e. no friction between them) and zero net effect on their velocity vectors (i.e. energy of the complete system is conserved). This is achieved by exchanging the agents' velocity vectors, which causes them to bounce off each other without loss of energy [4]. Each agent can emit and sense acoustic signals. The strength of the acoustic signal experiences two kinds of attenuation:

a) Attenuation due to distance: The maximum strength of the emitted signal is exhibited from the source to a distance equal to 2R between the center of the bodies of the agents. The intensity of the signal obtained by each sensor is then calculated by applying the inverse square law using the distance between the sensor and the source.

b) Attenuation due to "self-shadowing" mechanism: This attenuation occurs when the emitted signal passes within the body of the sensing agent (as the intensity of the acoustic signal is weakened due to the agent's own embodiment). It is modeled as a scaling factor over the sensory inputs in a range from 0.1 (when the sensors of the sensing agent are diametrically opposite from the source) to 1 (when the sensing agent is facing the source). The equations to calculate the distance that the emitted signal travels within the body of the sensing agent, i.e. the shielded distance (D_{sh}), are available in the Supplementary Material of the Candadai et al. (2019) model [7].

According to the previous points, the sensory input for each sensor of an agent is calculated using the distance between the sensor and the source (applying the inverse square law), and it is then multiplied by the "self-shadowing" attenuating factor which goes linearly from 1 (when $D_{sh} = 0$) to 0.1 (when $D_{sh} = 2R$).

2) Neural architecture: The neural architecture of each agent consists of three layers, in our previous work we called them: sensor layer, neuron layer and actuator layer. The main difference from our current model and the Candadai et al. (2019) model [7] is presented in the neuron layer, where instead of two neurons, we use three neurons, as shown in Fig. 4

a) Sensor layer: In this layer, there are two sensor nodes with a sigmoidal activation function, whose output is given by:

$$o_s = g_s \sigma(I_s + \theta_s) \tag{1}$$

where gs is the sensory gain, $\sigma(x) = 1/(1 + e^{-x})$ is the sigmoidal activation function, I_s is the sensory input and θ_s is the bias. In this layer, both sensor nodes share common gain and bias.

b) Neuron layer: This layer is modeled as a continuous-time recurrent neural network (CTRNN) [18]. In contrast to the Candadai et al. (2019) model [7], this layer now consists of three fully recurrently connected neurons, which corresponds to a 3-dimensional dynamical system. Each neuron's activity is governed by the following state equation:

$$\tau_i \frac{dy_i}{dt} = -y_i + \sum_{j=1}^{N} w_{ij} \sigma(y_j + \theta_j) + \sum_{s=1}^{2} w_{is} o_s \tag{2}$$

where dy_i/d_t refers to the rate of change of internal state y_i of neuron i based on a time constant τ_i. The rate of change dy_i/d_t depends on the current state of the neuron, the weighted sum of outputs from other internal neurons and the total external input from the sensors. The output of each neuron based on its internal state is given by a sigmoid activation function $\sigma(y_i + \theta_j)$ where θ_j refers to the neuron's bias. In this implementation, the three neurons share same time-constant and bias.

c) Actuator layer: The three internal neurons feed into the actuator layer, where the input to each actuator node is a weighted sum of the outputs of the neuron. The actuator

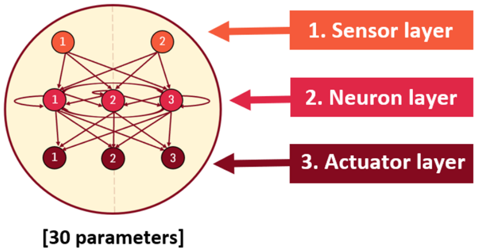

Fig. 4. Neural architecture of the 3-neuron model based on the Candadai et al. (2019) [7] model, where the number of neurons in increased to three. In this approach, the two sensor nodes share common gain and bias; the three neurons share common time-constant and bias; and, the three actuator nodes share common gain and bias.

layer contains three actuator nodes, two corresponding to the motors and one corresponding to the acoustic signal emitter. All of them are sigmoidal units with a gain and bias (but no internal state) such that the output of the actuator node i, m_i, is given by:

$$m_i = g_m \sigma \left(\sum_{n=1}^{N} w_{ni} * o_n + \theta_i \right) \tag{3}$$

where o_n is the output of the neuron, that are weighted by w_{ni}, θ_i is the bias and g_m is the gain. In this layer, the three actuator nodes share common gain and bias.

Locomotion is managed by the effective control of the two motors. Net linear velocity is given by the average of their corresponding outputs and angular velocity which rotates the agent is given by their difference divided by the radius of the agent.

3) *Evolutionary optimization:*

a) *Fitness function: Neural entropy:* The fitness function for the evolutionary algorithm is the Shannon entropy of neural outputs, i.e. neural entropy, which has been used as a proxy of cognitive complexity. This function does not optimize any particular task. The agents are initialized in a random position in the environment (see below) and allowed to move around over 4 trials, during which their neural activity is recorded. The neural complexity is measured as the Shannon entropy in the three-dimensional time series from the outputs of the three neurons, which are bounded in the range from 0 to 1. The output space is binned with 100 bins along each of the three dimensions, i.e. one million bins in total. Then, a 3-dimensional histogram is created using all the binning data points acquired during the 4 trials. Thus, the Shannon entropy H of the neural time series is given by:

$$H = \sum_{i=1}^{100} \sum_{j=1}^{100} \sum_{k=1}^{100} -p_{ijk} log(p_{ijk}) \tag{4}$$

where the probability of the neural activity in a specific bin $[i, j]$, p_{ij}, is given by the number of data points in that bin divided by the total number of data points. The neural entropy is then normalized to be in the range from 0 to 1 by dividing by the maximum neural entropy, i.e. $log(100 * 100 * 100)$, when all bins are uniformly populated. Therefore, the normalized neural entropy is given by:

$$\hat{H} = H / log(100 * 100 * 100) \tag{5}$$

b) *Genetic algorithm:* We used a real-valued genetic algorithm to optimize the parameters of the neural controllers, such as weights, gains, biases and time-constants in order to maximize the neural entropy of the agents. Each agent had 30 parameters, i.e. for N agents, the genotype consisted of 30N parameters that were initially encoded in the range $[-1, 1]$. When building the agents from each genotype to perform the 4 trials, these parameters were scaled appropriately, following the same parameter ranges as in the Candadai et al. (2019) model [7] such that: for sensor and actuator nodes, the gains were scaled in the range $[1, 5]$ and the biases were scaled in the range $[-3, 3]$; for neuron nodes, the time-constants were scaled in the range $[1, 2]$ and the biases were scaled in the range $[-3, 3]$; all weights were scaled in the range $[-8, 8]$.

The performance evaluation of the agents was obtained according to each of the 4 conditions: interactive condition, ghost-test condition, ghost-evolution condition and isolated condition. The experimental setup for each of the 4 conditions is described in detail in the Experiments section.

For all conditions, after the performance evaluation, we generated a new population by, first, keeping an elite population of the top 4% solutions as it is and, second, by mutating and crossing over this elite fraction to get the remainder of the solutions. Mutation was obtained by adding a zero-mean Gaussian mutation noise with variance 0.1 to the solutions and, then, crossover was obtained by swapping each parameter between a pair of solutions with a probability of 0.1.

B. *Experiments*

Here, we describe the implementation of the 4 conditions introduced in the Introduction:

1) *Condition 1: Interactive condition (2-way coupling):* In this condition, we performed 100 independent runs using 96 pairs of agents that were able to interact with each other. Each pair's agent parameters were encoded in a single genotype subjected to evolutionary search. Initially, the agents were placed at 20 units from each other. For each trial, their relative angle was modified as $[0, \pi/2, \pi, 3\pi/2]$, respectively, where both agents' heading direction was set to the right. The population was evolved for up to 2000 generations to maximize the neural entropy of both agents.

2) *Condition 2: Ghost-test condition (1-way coupling):* In this condition, we selected the best pair of agents of the best 10 runs previously obtained in the fully interactive scenario and, then, tested them under a "ghost" condition. Red agent was the "live" agent and blue agent was the "ghost" agent. The "live" agent was able to interact with the "ghost" partner, while, the "ghost" agent was just playing back pre-recorded behavior from the previous trials in a fully interactive scenario. The "live" agent was initially positioned at a different

angle (randomly selected from $[0, \pi/2, \pi, 3\pi/2]$ but different to the one it was chosen when the "live" agent was in 2-way interaction) from the "ghost" agent to avoid repeating the behavior of those previous trials. The initial distance between both agents was 20 units. We conducted 4 trials for each pair of agents, as in the fully interactive condition, and measured the normalized neural entropy of the "live" agent.

3) Condition 3: Ghost-evolution condition (1-way coupling): In this condition, we selected the best blue agent of the best run in fully interactive scenario and used it as the non-interactive or "ghost" agent. We performed 10 independent runs, where only the interactive or "live" agent was evolved in the presence of the "ghost" partner, which was just playing back the pre-recorded behavior in fully interactive scenario (different for the 4 trials). The same playback was used for all runs, all agents and all generations, in order to evolve the agents to respond to a specific set of conditions and see the effects on neural and behavioral complexity through each generation. The population in each run consisted of 96 individuals, where each individual was encoding the parameters of only the "live" agent. For each trial, the "live" agent was placed 20 units from the "ghost" agent and their relative angle was modified as $[0, \pi/2, \pi, 3\pi/2]$, respectively. The population was evolved for up to 2000 generations to maximize the interaction entropy of "live" agents. Evaluation was performed the same way as for Condition 1 with fitness derived only from the "live" agent's neural entropy.

4) Condition 4: Isolated condition (0-way coupling): In this condition, we performed 10 independent runs using isolated agents that were not receiving any input and were evolved on their own to maximize their isolation entropy. Red agent was referred as the "isolated" agent. The population in each run consisted of 96 individuals, where each individual was encoding the parameters for only one agent ("isolated" agent). The population was evolved up to 2000 generations.

In all conditions, the agents' initial heading direction was set to the right.

III. RESULTS

Fig. 5 shows example trajectories and neural activation of the best agents in 4 conditions from the best runs, respectively. Complex behavior and neural activity can be clearly seen on the fully interactive condition. This complexity seems to be lost in the ghost-test condition, in line with Candadai et al. (2019) model [7] results. The agent that exhibits complex movement trajectory when interacting with a live partner, starts to literally run in circles when the partner is non-responsive. This is the case even though the agents in the current experiment have a more complex 3-neuron brain that could in principle exhibit chaotic activity. In the isolated condition, the agent shows highly regular behavior and an oscillatory pattern of neural activity, again, in line with the results of the original model and despite a more complex brain. The most interesting for our purposes ghost-evolution condition, in which the agents are evolved in the presence of a ghost partner displays something that seems as an intermediate level of behavioral and neural complexity between the fully interactive and the isolated case.

In order to go beyond intuitions and understand the general pattern of differences between conditions, we run 4 statistical tests comparing the best agents' neural activity and movement trajectories.

Specifically, in the first test we compared the average of the means of neural entropy of 100 agent pairs in interactive condition against the means of the best live agents in other conditions. Given unequal sample size and significantly different variances between conditions, $F(3, 127) = 21.24$, $p < .001$, we performed a non-parametric Kruskal-Wallis test, which showed a significant difference in neural entropy between conditions, $H(3) = 41.7, p < .001$. Focused comparisons of the mean ranks between groups showed that agents in isolated condition had significantly lower neural entropy compared to interactive and ghost-evolution conditions, as expected. However, somewhat surprisingly, the entropy in ghost-evolution condition was significantly higher than in the interactive or ghost-test conditions. This trend can also be seen in Fig. 6A.

Next, we obtained a measure of behavioral complexity of the live agents in all conditions. We recorded heading direction angles at each time point of agent trajectories, which resulted in 1D time series. We then computed sample entropy for each such time series. Since there does not seem to be a universally agreed upon measure of behavioral trajectory complexity, we used sample entropy as a measure that has been shown to be an appropriate index of complexity for biological time series more broadly [19]. We have informally validated this measure by checking that it reliably distinguishes between fully regular circular and more complex movement patterns[3]. The data subjected to statistical analysis were time series sample entropy for all trials of the 10 best runs in each condition. In the interactive condition, only one of live agents was considered. Levene's test for equality of variances was not significant and there was a significant effect of condition on the level of behavioral entropy, $F(3, 153) = 20.98$, $p < .001$. Post-hoc Bonferroni-corrected tests showed that isolated agents' entropy was significantly lower than entropy in all other conditions ($p < .001$) but also ghost-evolution condition entropy was significantly lower than entropy in the interactive condition (see Fig. 6B).

As the difference in neural and behavioral complexity in the ghost-evolution condition compared to interactive condition was in opposite directions (higher and lower respectively), we run two further tests to investigate the type of interaction between the agents in 3 conditions with coupling (thus, excluding the isolated condition). In the first test we have computed the entropy of the distance between the agents while in the second their synchrony. Distance entropy was measured with a binned

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[3] We originally tried to estimate behavioral complexity using image entropy calculated on generated plots of trajectories. However, this measure delivered inconsistent results, most likely because it does not consider the temporally ordered nature of movement coordinates and instead takes into account only their spatial dispersion.

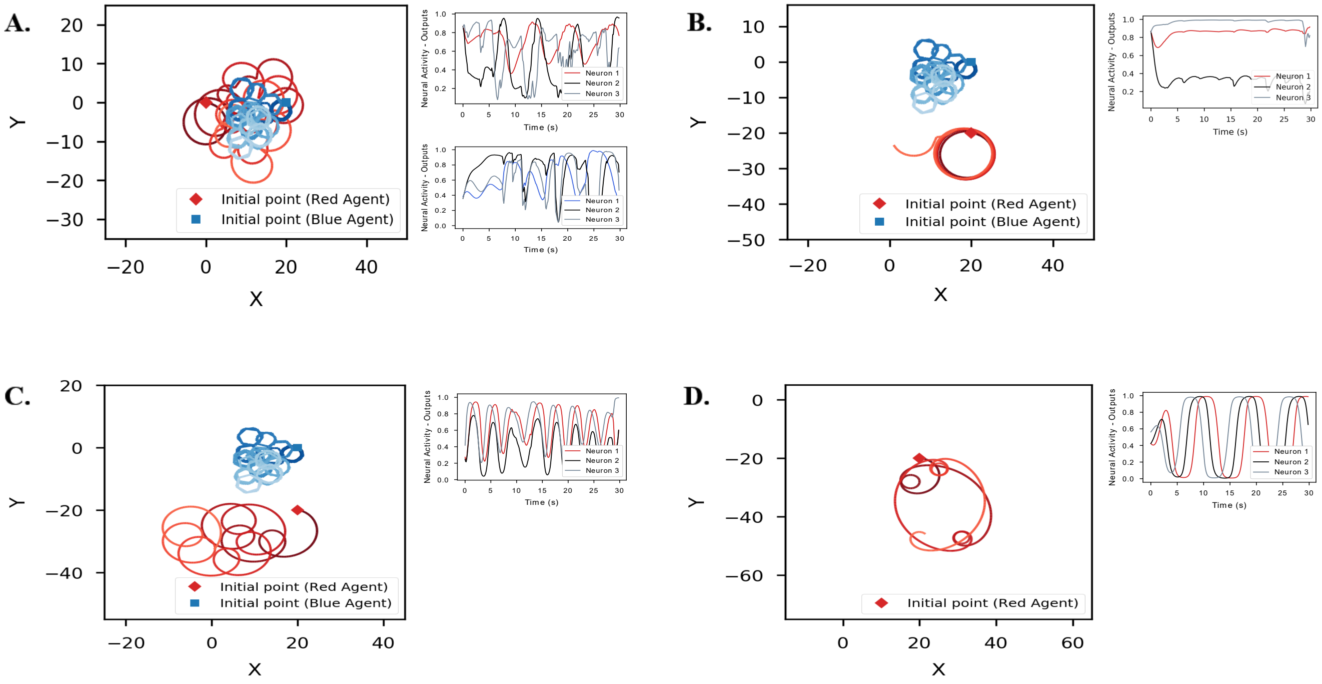
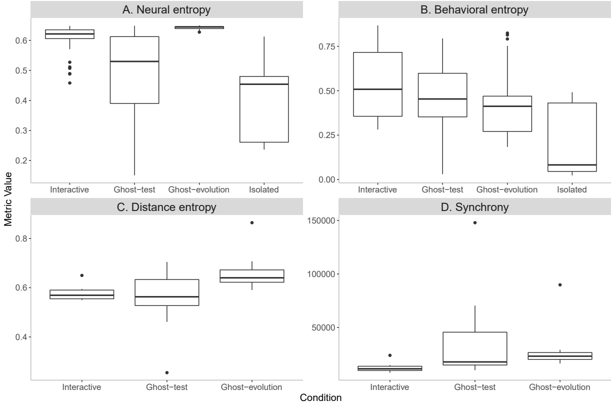


Fig. 5. Example plots of movement trajectories and neural activity across 4 conditions. Movement trajectories in red are of a live agent, movement trajectories in blue are of a live agent in Figure A and of ghost replay in figures B and C. Neural activity plots show output of 3 internal neurons of the live agents.

Shannon entropy approach adopted in the original Candadai et al. (2019) paper [7] . Synchrony was estimated with a Dynamic Time Warping metric applied to 2-dimensional time series representing the xy-coordinates of the two agents in all trials. Only the first measure showed a significant overall effect of condition, $F(2, 27) = 4.27$, $p < .05$ and a significant post-hoc pairwise comparison between Ghost-evolution and Ghost-test conditions. Fig. 6C and Fig. 6D show the overall trends for these measures.


A. Neural entropy

B. Behavioral entropy

C. Distance entropy

D. Synchrony

Fig. 6. Individual and interaction measures across experimental conditions. Figures A and B show neural and behavioral entropy of live agents. C: distance entropy between agents. D: DTW-estimated distance between agents (lower distance means more synchrony).

IV. DISCUSSION AND CONCLUSIONS

In this work, we have extended the Candadai et al. (2019) model [7] in order to address some of its limitations. By including one more neuron in the neuron layer, i.e. three neurons instead of two, we have found that the results in terms of neural and behavioral complexity are similar to those in the original configuration. In particular, the agents' neural entropy is still higher in the interactive condition than in the ghost-test or isolated conditions. Additionally, we have quantitatively assessed behavioral complexity in all conditions and this measure was found to be also lower in the isolated condition. This means, perhaps unsurprisingly, that a powerful brain operating in isolation, without any input is not able to achieve high levels of neural or behavioral complexity.

Our second generalization test delivered mixed insights. On the one hand, by observing example plots of neural activation (Fig. 5), it can be suggested that in the interactive condition, the neural activity of the three neurons exhibits more chaotic activity than in the other conditions, including the more stringent ghost-evolution condition. On the other hand, this suggestion is not borne out by the statistical test that shows that neural complexity in this condition is higher than in the interactive condition. This would mean that richer environment that provides constant but complex input to the agent can compensate the relative poverty of the 1-way coupling and lack of contingent response of the interaction partner. At the same time, despite a higher neural complexity, behavioral complexity in this setting is lower. Thus, agents that are evolved to maximize their neural complexity in a rich environment end up behaving in more predictable and regular manner than agents that receive specifically social stimulation.

Briefly, our findings suggests that the richness of the environment may compensate 1-way interaction in terms of neural complexity, however, this does not apply in terms of behavioral complexity. This suggests that the social world (i.e. real-time interaction between agents becoming the whole brain-body-environment-body-brain system [20]) allows for a greater repertoire of behaviors transforming our individual capacities [3].

Some possible limitations may be encountered in our model based on the specific metrics that we used as a measure of neural and behavioral complexity. Further work is needed to compare the results by implementing alternative methods, e.g. evolving agents to maximize predictive information (PI), using permutation entropy and applied it to raw xy-coordinates, etc.

V. Future Work

In this paper, we have explored the neural and behavioral complexity of embodied agents using different levels of coupling in dyadic interaction. Future work will investigate how different modes of coupling can affect individual and interactive capacities of evolved agents. Specifically, it could be argued that an opportunity to interact with multiple partners could further enhance individual complexity. Alternatively, allowing agents to use different interactive modalities, such as distal and proximal coupling analogous to pheromone and saliva-based interactions in ants, could enrich their cognition. This will allow us to further understand how individual complexity can be generated by interaction.

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4

Smaller-brained vs bigger-brained agent couplings

In this chapter, we present our journal article:

- *Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity* by Georgina Montserrat Reséndiz-Benhumea, Ekaterina Sangati, Federico Sangati, Soheil Keshmiri, and Tom Froese. Accepted for publication in *Frontiers in Neurorobotics* in the Research Topic on *Emergent Behavior in Animal-Inspired Robotics.*

4.1 *Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity*

Abstract

The social brain hypothesis proposes that enlarged brains have evolved in response to the increasing cognitive demands that complex social life in larger groups places on primates and other mammals. However, this reasoning can be challenged by evidence that brain size has decreased in the evolutionary transitions from solitary to social larger groups in the case of Neolithic humans and some eusocial insects. Different hypotheses can be identified in the literature to explain this reduction in brain size. We evaluate some of them from the perspective of recent approaches to cognitive science, which support the idea that the basis of cognition can span over brain, body and environment. Here we show through a minimal cognitive model using an evolutionary robotics methodology that the neural complexity, in terms of neural entropy and degrees of freedom of neural activity, of smaller-brained agents evolved in social interaction is comparable to the neural complexity of larger-brained agents evolved in solitary conditions. The nonlinear time series analysis of agents' neural activity reveals that the decoupled smaller neural network is intrinsically lower dimensional than the decoupled larger neural network. However when smaller-brained agents are interacting, their actual neural complexity goes beyond its intrinsic limits achieving results comparable to those obtained by larger-brained solitary agents. This suggests that the smaller-brained agents are able to enhance their neural complexity through social interaction, thereby offsetting the reduced brain size.

Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity

Georgina Montserrat Reséndiz-Benhumea[1, 2*], Ekaterina Sangati[1], Federico Sangati[1], Soheil Keshmiri[3], Tom Froese[1]

[1]Okinawa Institute of Science and Technology Graduate University, Japan, [2]IIMAS Academic Unit, Institute of Applied Mathematics and Systems Research, National Autonomous University of Mexico, Mexico, [3]Okinawa Institute of Science and Technology Graduate University (OIST), Embodied Cognitive Science Unit (ECSU), Japan

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Author contribution statement

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

Keywords

Agent-based modeling, social interaction, Complexity, entropy, social brains, Evolutionary Robotics, Continuous-time recurrent neural network, nonlinear time series analysis

Abstract

Word count: 230

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Contribution to the field

This is the first agent-based model that investigates the consequences of brain size reduction in social species. The modeling results suggest that reduced brain size does not necessarily imply reduced cognitive capacity, as the complexity of neural activity that can be realized in social interaction is greater than in isolation.

Ethics statements

Studies involving animal subjects

Generated Statement: No animal studies are presented in this manuscript.

Studies involving human subjects

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Inclusion of identifiable human data

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Shrunken Social Brains? A Minimal Model of the Role of Social Interaction in Neural Complexity

Georgina Montserrat Reséndiz-Benhumea [1,2,*], **Ekaterina Sangati** [1], **Federico Sangati** [1], **Soheil Keshmiri** [1] **and Tom Froese** [1]

[1] *Embodied Cognitive Science Unit, Okinawa Institute of Science and Technology Graduate University, Okinawa, Japan*
[2] *Computer Science and Engineering Postgraduate Program, Institute for Applied Mathematics and Systems Research, National Autonomous University of Mexico, Mexico City, Mexico*

Correspondence*:
Georgina Montserrat Reséndiz-Benhumea
georginamontserrat.resendizbenhumea@oist.jp

ABSTRACT

The social brain hypothesis proposes that enlarged brains have evolved in response to the increasing cognitive demands that complex social life in larger groups places on primates and other mammals. However, this reasoning can be challenged by evidence that brain size has decreased in the evolutionary transitions from solitary to social larger groups in the case of Neolithic humans and some eusocial insects. Different hypotheses can be identified in the literature to explain this reduction in brain size. We evaluate some of them from the perspective of recent approaches to cognitive science, which support the idea that the basis of cognition can span over brain, body and environment. Here we show through a minimal cognitive model using an evolutionary robotics methodology that the neural complexity, in terms of neural entropy and degrees of freedom of neural activity, of smaller-brained agents evolved in social interaction is comparable to the neural complexity of larger-brained agents evolved in solitary conditions. The nonlinear time series analysis of agents' neural activity reveals that the decoupled smaller neural network is intrinsically lower dimensional than the decoupled larger neural network. However when smaller-brained agents are interacting, their actual neural complexity goes beyond its intrinsic limits achieving results comparable to those obtained by larger-brained solitary agents. This suggests that the smaller-brained agents are able to enhance their neural complexity through social interaction, thereby offsetting the reduced brain size.

Keywords: agent-based modeling, social interaction, complexity, entropy, social brains, evolutionary robotics, continuous-time recurrent neural network, nonlinear time series analysis

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[1] Manuscript length: 3938 words, Figures: 4

1 INTRODUCTION

It is widely accepted that increased social group size is an important factor for explaining the evolution of increased brain size in primates, because of a concomitant increase in the cognitive demands posed by more complex forms of social competition and/or social bonding (Byrne, 1996; Dunbar and Shultz, 2007). However, this social brain hypothesis has come under pressure from various directions. Brain tissue is energetically expensive, and so primate brain size can only increase as an adaptive response to increased cognitive demands if such growth is enabled by a sufficiently high-quality diet. In fact, diet seems to be the primary factor that predicts brain size in primates (DeCasien et al., 2017). In this context the evolution of human brain size during the Paleolithic period can then be seen as fueled by the development of cooking, which significantly enhanced our energy intake and brought along with it an increase in cognitive and social complexity (Herculano-Houzel, 2016; Wrangham, 2009).

It might therefore be expected that during the subsequent Neolithic period, with the advent of domestication, agriculture, and settled living in ever larger groups, human brain size will have continued to increase or at least remained stable. However, the opposite tendency has been observed: during this time period around the world human brain size underwent a notable reduction beyond that expected from an overall reduction in body size (Brown, 1987; Brown and Maeda, 2004; Henneberg, 1988; Henneberg and Steyn, 1993). In particular, reduction in cranial capacity is associated with the highest levels of population density, coinciding with the emergence of larger, socially and economically organized ways of life that mark the start of the Neolithic period (Bailey and Geary, 2009). The overall extent of the reduction is controversial (Leach, 2003), but according to some estimates it is comparable in extent to the increase in brain size associated with previous speciation events in human evolution (Henneberg, 2004).

Although this curious fact about the most recent period of human brain evolution has so far received less attention, several hypotheses have been developed to account for it. These hypotheses can be grouped into two broad categories depending on whether they appeal to an increase in selection pressures favoring smaller brains or to a decrease in selection pressures favoring bigger brains. The former category includes selection pressure on improved brain efficiency, and more prominently the self-domestication hypothesis, which has proposed that there was an increase in selection pressure for reducing in-group competition (Hare, 2017; Hare et al., 2012), which has led to the prediction of reduced cranial capacity in Holocene humans (Cieri et al., 2014), as this is also observed in other domesticated animals (Leach, 2003).

Alternatively, it could be that domesticated ways of life placed fewer cognitive demands on individuals' brains, which would then put this hypothesis in the latter category. Indeed, rather than just reducing cognitive demands, the human sociocultural environment makes cognition more efficient and more complex (Clark, 2006; Sterelny, 2017). Human evolution is characterized by an increase in sociocultural scaffolding of learning and apprenticeship (Sterelny, 2012), and so it makes sense that the increase in social institutions in the Neolithic period permitted the human brain of "doing more with less" size (Bednarik, 2014). Hodder (2020) jokingly refers to this possibility as the "smart phones, dumb people" syndrome. To be more precise, the latest theoretical developments in cognitive science promote an embodied, embedded, extended, and enactive ("4E") approach to the mind (Newen et al., 2018), which argues that the basis of cognition is not limited to the brain but can spread out over brain, body and environment. On this view, cognitive processes are underdetermined by brain structure, which undermines the underlying assumption of the social brain hypothesis (Barona, 2020; Barrett et al., 2007). Living in a world of enhanced sociocultural scaffolding of cognition would permit brains to become smaller but, importantly, without a reduction in overall cognitive capacity of the appropriately scaffolded person. We could call this the social scaffolding hypothesis for decreased brain size.

Interestingly, a similar distributed cognition hypothesis has been proposed for the case of eusocial insects: in wasps the strongest changes in brain investment, namely a reduction in central processing brain regions, accompanied the evolutionary transition from solitary to social species but did not correlate specifically with the degree of social complexity (O'Donnell et al., 2015). More generally, the evolution of insect societies appears to have occurred without the evolution of any new dedicated neural structures (Farris, 2016). Reasons for this are unclear, but a computational model suggests that there may be energetic advantages to optimizing total colony brain mass over individual brain mass (Feinerman and Traniello, 2016).

This suggests a more general hypothesis for decreased brain size, in which the key explanatory factor is not the complexity of the social world per se, but rather a more general principle of decreased in-group conflict and increased possibilities of relying on responsive others. In the case of insect colonies, this is achieved by the colony serving as an extended phenotype of the reproductive individuals, with most individuals being closely related to each other. In the Neolithic period human groups could no longer rely on such genetic relatedness for in-group support, but they were able to expand in size from extended family groups to large villages by developing new symbolic forms of group identification. This allowed unfamiliar individuals with a shared symbolic identity to rely on each other for support, based on early developing mutual socio-moral expectations (Jin and Baillargeon, 2017).

However, in general it remains unclear whether the brain size reduction associated with the evolution of such large, yet tightly integrated and highly coordinated in-groups is better characterized as a case of "smart phone, dumb user" or of "doing more with less". In other words, is it the case that brain size could decrease because it was possible for brain activity to become *less complex* ("dumb") due to a reduction of cognitive demands in the social milieu (highly dependable and organized "smart" society)? Or could it also be the case that brain size could decrease because brain activity became *more complex* due to the cognitive scaffolding provided by that reliable social milieu? That is, a smaller brain could be producing activity equal in complexity to a larger brain ("doing more processing with less resources") because it became supported by extended social structures. It is difficult to empirically arbitrate between these two possibilities. One piece of evidence in support of the latter possibility is that during human evolution the brain's blood flow rate, which is indicative of levels of neural activity, has increased faster than brain size (Seymour et al., 2016). But also, more theoretical work is needed to deepen our conceptual understanding of how brains could do more with less during social interaction.

2 METHODS

In the following we will explore these questions by employing the synthetic approach to studying adaptive behavior based on agent-based modeling, evolutionary algorithms, and dynamical systems analysis (Beer, 1997; Cliff et al., 1993; Harvey et al., 2005). We will use this approach to create a simulated "thought experiment" (Di Paolo et al., 2000) that will permit us to investigate, in the most simplified manner possible, the potential roles of brain size and sociality in the generation of an individual's neural complexity, where we will operationalize it in two ways.

First, we will consider neural complexity to be captured by Shannon entropy calculated over neural output values. While it might seem that focusing on Shannon entropy, which is maximal for uniform distributions, will bring about cognitive or behavioral randomness, there are actually good reasons to consider it as a possible measure of complexity. For one, "the principle of maximum entropy", which states that the distribution that maximizes Shannon entropy is to be preferred (Jaynes, 1957), is found to be operational in biological systems. For instance, maximizing the neural response entropy amplifies mutual information between the brain activation and incoming stimuli (Laughlin, 1981). Furthermore, entropy has been suggested as a possible correlate of consciousness by the "entropic brain hypothesis" (Carhart-Harris, 2018).[2] Additionally, as stated in more recent work (Candadai et al., 2019), some previous studies have associated high levels of neural entropy with enhanced cognitive performance, e.g. improved generalization in motor learning tasks (Dotov and Froese, 2018).

Second, for analysis of the results, we will complement the measure derived from information theory with a notion more closely associated with system dynamics by looking at the dimension of the attractor in the state spaces of evolved neural activity. Here, we employ the embedding dimension as a measure of the effective degrees of freedom or the complexity of the dynamics (Stam, 2005) of the corresponding agent's neural activity (neural states). In order to calculate it, we follow standard practices of nonlinear time series analysis (Perc, 2006; Kodba et al., 2005; Froese et al., 2013), by: (1) using mutual information (MI) to estimate a proper embedding delay τ (Fraser and Swinney, 1986), and, (2) using the false nearest neighbor (FNN) method to determine a proper embedding dimension m (Kennel et al., 1992).

Research (Froese and Di Paolo, 2010; Froese et al., 2013; Campos and Froese, 2017; Candadai et al., 2019; Reséndiz-Benhumea et al., 2020) suggests that in the absence of in-group competition[3], it is easier to evolve increased complexity of neural activity in close interaction with other agents, than to do so alone by increasing the intrinsic complexity of the neural network architecture. In the present study, we extend these findings by focusing on the role of brain size in generating complexity of neural activity in solitary and social conditions. Our aim is to show that it is possible for social agents with a smaller neural network to exhibit at least the same complexity of neural activity as a solitary agent with a larger neural network.

2.1 Model

The implementation of the proposed model is based on the Candadai et al. (2019) model. Here, we evolved agents to maximize their entropy in individual and social scenarios using a smaller (2-neuron model) and a larger neural network (3-neuron model).

2.1.1 Simulated agents and environment

Agents have circular bodies, with a radius of 4 units. Each of them is provided with: two acoustic sensors, which are symmetrically positioned in its frontal side at $\pm 45°$ with respect to its central axis; an acoustic emitter, which is located in its body's center; and, two motors, which are driving two wheels in its left and right sides, respectively, that enable displacement in a 2-dimensional environment. The environment consists of an empty open-ended arena.

Each agent emits and senses the strength of the acoustic signal from another agent. The strength of an agent's acoustic signal experiences attenuation due to distance and "self-shadowing" (see **Supplementary Material** for details).

2.1.2 Neural architecture

The agent's neural architecture consists of three fully connected layers: sensor layer, neuron layer and actuator layer. In this work, we study the comparison between a 2-neuron and a 3-neuron model, which reflects the number of neurons in the inner layer. Sensor and actuator layers are kept the same between the two models, except for the number of connections to the inner layer (see **Figure 1**).

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[2] We admit that generalizability of our results needs to be checked with other possible measures of neural complexity. However, given that a choice of a particular measure is in itself a challenging endeavor and each of the measures has advantages and disadvantages for trying to capture any particular phenomenon (cf. a comparison of 12 different entropy measures for just anesthesia (Liang et al., 2015)), we believe using the most straightforward approach for understanding the fundamental conceptual issues we tackle here is a good starting point.

[3] In the studies mentioned above and the present study this feature is built-in given that selection pressure acts on pairs of agents together, rather than on individuals.

The sensor layer consists of two sensor nodes with a sigmoidal activation function and no inner state. The inner neuron layer is modeled as a continuous-time recurrent neural network (CTRNN) (Beer, 1995). In this layer, we are implementing 2 brain architectures: the use of two fully recurrently connected neurons for the 2-neuron model as shown in the **Figure 1A**, which corresponds to a 2-dimensional dynamical system; and the use of three fully recurrently connected neurons for the 3-neuron model as shown in the **Figure 1B**, which corresponds to a 3-dimensional dynamical system. In both architectures, each neuron's activity is governed by the standard CTRNN state equation. The actuator layer consists of three actuator nodes, two correspond to the left and right motors for agent locomotion and one corresponds to the acoustic signal emitter for modulating the strength of the emitted signal. All actuator nodes are sigmoidal units with no internal state. Agent locomotion is the result of the effective control of the two motors, where net linear velocity is given by the average of the two actuator nodes corresponding to the right and left motors; and, the angular velocity, which refers to how fast an agent rotates, is given by their difference divided by the radius of the agent. The equations for each layer are provided in the **Supplementary Material**.

2.1.3 Performance measure

The measure that we use for evaluating agent's performance is its entropy. In particular, we select simple multi-dimensional Shannon entropy calculated on neural output values, as in Candadai et al. (2019) model (see **Supplementary Material** for details).

2.1.4 Genetic algorithm

A real-valued genetic algorithm was used to optimize the parameters of the agents' neural controllers (i.e. the connection weights, the time-constants, the biases and the gains) to maximize their neural entropy. It is important to highlight that no particular task was explicitly optimized. For the 2-neuron model, each agent had 20 parameters, and, for the 3-neuron model, each agent had 30 parameters. These parameters were encoded as the real-valued genotype, where each value was contained in the interval [-1,1] and scaled during simulation to specific parameter ranges.

The agents were evaluated in each of the 4 independent trials and their neural outputs recorded in order to calculate their normalized neural entropy. Each trial lasted 200 simulation seconds at a step size of 0.1.

Upon ranking the population according to fitness values, the new population was generated by keeping an elite population of the top 4% of the existing solutions and by mutating and crossing over these elite individuals to get the rest of the new solutions. Mutation consisted in adding zero-mean Gaussian mutation noise with a variance of 0.1 to the solutions and crossover consisted in swapping each parameter between a pair of solutions with a probability of 0.1.

2.1.5 Experimental setup

For both brain sizes (2-neuron and 3-neuron models) we evolved agents in two different evolutionary conditions: individual and social. Here we describe the implementation details for both scenarios.

1. **Individual Evolution (IE):** We performed 10 independent runs with an initial random population of 96 solitary agents, i.e. without an agent partner as shown in **Figure 1C**. The parameters for each agent were encoded as a single genotype (one solution). The solitary agents were not sensing any input, neither from another agent nor from the environment. For each trial, the agent's initial position was set at coordinates $(0, 0)$. The agent's heading direction was initialized to the right. The population was evolved for 2000 generations to maximize the neural entropy of each solitary agent.
2. **Social Evolution (SE):** We performed 10 independent runs with an initial random population of 96 pairs of agents that were able to interact with each other as shown in **Figure 1D**. The parameters for each pair of agents were encoded as a single genotype (one solution) and were evolved together during

all generations. For each trial, there was a fixed initial position for each pair of agents: the first agent was always positioned at coordinates $(0, 0)$, while the second agent was placed 20 units of distance from the first one, just varying their relative angle as $[0, \pi/2, \pi, 3\pi/2]$, respectively. Furthermore, both agents' heading direction was initialized to the right. The population was evolved for 2000 generations to maximize the neural entropy of each pair of agents.

3 RESULTS

In this section, we present the results for the best agents in 2-neuron and 3-neuron models from the best runs. We focus on the comparison between 3-neuron agents evolved in solitary environment (Individual Evolution (IE)) and 2-neuron agents evolved in social environment (Social Evolution (SE)).

3.1 Agent Behavior

Figure 2A shows the trajectory of the best agent evolved in the IE condition in 3-neuron model. **Figure 2B** shows the trajectory of the best pair of agents evolved in the SE condition in 2-neuron model. In the comparison of both images, it can be observed that the solitary agent exhibits less complex behavior than the pair of agents in interaction. In the former case, the agent is moving in simple loops, while in the latter, the agents are enhancing each other behavior by displaying spiralling nested pairwise loops movement.

3.2 Statistical Analysis

In order to capture the statistical differences between the conditions tested, we compared the means of neural entropy of evolved pairs of agents in the SE conditions against the neural entropy of evolved agents in the IE conditions, between 2-neuron and 3-neuron models. Specifically, we took the neural entropy values from the best agent pair or the best agent in the last generation of all 10 independent runs in each condition. There was a significant main effect of condition $F(1, 36) = 6.55$, $p < .05$ and a significant main effect of the number of neurons $F(1, 36) = 4.62$, $p < .05$ but no significant interaction effects. That is, neural entropy was higher in SE condition than in IE condition, and higher in 3-neuron than in 2-neuron model but these factors did not affect each other (see **Figure 3**). Social interaction between the larger-brained agents did not lead to a larger (or smaller) neural entropy gain compared to the smaller-brained agents.

Since we were specifically interested in comparing neural entropy between agents evolved in social interaction (SE) in a 2-neuron model and agents evolved in isolation (IE) in a 3-neuron model, we also conducted a Bonferroni-corrected post-hoc t-test between these two conditions. The difference was not significant, $p = 0.79$.

3.3 Nonlinear Time Series Analysis

We performed the nonlinear time series analysis of the evolved agents' neural activity (neural states) to determine their proper embedding delay τ using MI method and, then, their proper embedding dimension m using FNN method (see **Supplementary Material** for details).

We distinguished two different testing modes for obtaining the time series of the evolved agents' (in IE or SE, respectively) neural activity: (1) decoupled, when the evolved agent is tested in isolation (Input = 0); and (2) coupled, when the evolved agent is tested in the presence of an interactive partner. Furthermore, we only considered the neural states of neuron 1, trial 1, of the best agents from each run (10 runs), from 2-neuron and 3-neuron models, correspondingly, to obtain their embedding dimension.

Our results (see **Figure 4**) confirmed our hypothesis:

1. By comparing the mean embedding dimension of the neural states of the coupled 2-neuron agent evolved in SE condition and the decoupled 3-neuron agent evolved in IE condition, we found that

the former was higher dimensional than the latter, thereby demonstrating that social smaller-brained agents have at least equivalent (i.e. equal or more) degrees of freedom of neural activity than isolated bigger-brained agents.

2. Importantly, we found that this increase in degrees of freedom of neural activity in coupled socially evolved agents does not depend in their intrinsic complexity, as when being decoupled, their embedding dimension is lower than what can be achieved. This suggests, in line with previous work (Candadai et al., 2019; Reséndiz-Benhumea et al., 2020), that agents in interaction are enhancing each other's neural complexity.

4 DISCUSSION

In this paper, we have investigated the idea that brain size could decrease because agents could reliably take advantage of social interaction. A key open question in this regard is whether this reduction in brain size is made possible because of decreased cognitive demands ("smart phone, dumb user"), or because of increased cognitive scaffolding ("doing more with less").

By performing a statistical analysis, we found that smaller-brained social agents are able to exhibit comparable levels of neural complexity, as larger-brained solitary agents. This is in line with the idea that the brain of an agent in a reliable social setting can do more with less, and is consistent with our previous findings (Candadai et al., 2019; Reséndiz-Benhumea et al., 2020). Second, by performing a nonlinear time series analysis, we found that the embedding dimension of the neural states of the decoupled neural network is lower when agents evolved under social condition, which seems to be more in line with the idea of the brain as a "dumb user". Therefore, in a way, both possibilities can co-exist depending on whether we focus on the topological structure (i.e. number of neurons) or the state dynamics (i.e. degrees of freedom) of the brain: during evolution of sociality an individual's brain topological structure can become simplified, while its state dynamics can become more complex at the same time.

An important implication of this computational proof of concept for the science of brain evolution, whether in humans or social insects, is that care should be taken when inferring cognitive capacities from brain size. The coupled brain, as part of a whole body and environment system, will exhibit neural dynamics that are underdetermined by the structure of the brain.

There are several limitations to our model that need to be taken into consideration. First, our simulation involved separate evolutionary runs for different brain sizes and different conditions rather than integrating these factors within the same evolution. An alternative setup would be evolving a population of solitary large-brained agents and then transitioning them to a social smaller-brained population. It would be interesting to examine whether in such a scenario the same types of neural state spaces would be observed. Conceivably, the process of first evolving individual agents with a larger, more complex brain would be in tension with its later simplification once a social dimension is introduced and brain size is reduced.

Second, an important characteristic of the brain neural complexity lies in its modular architecture in which the interplay between specialized (i.e., segregated) and integrated neuronal units results in variety and flexibility of cognition. In this regard, the use of Shannon entropy for measuring complexity is limited as it does not allow for quantification of such functional integration among differential neural activities. This can be addressed by exploring the use of more comprehensive measures of complexity (Tononi et al., 1994), thereby capturing the utility of such potential interplay between agent's neural units on their evolved behaviour.

Third, the lack of input to the agents in isolated condition was an unrealistic impoverishment in that one could attribute the higher neural complexity in social than individual evolution to a richer input provided to the former. This is consistent with our previous result (Reséndiz-Benhumea et al., 2020) that evolving an isolated agent in the presence of a non-interactive "partner" can also lead to high neural complexity. The point here, however, is not to exclude other factors that might enable neural complexity.

Fourth, social interaction simulated in our model is of a relatively simple kind: the agents were evolved always in the same pairs. It might be argued that our results are due to the availability of the same reliable partner and that a more realistic scenario in which they would have to interact with multiple, more unpredictable partners would not be possible with a smaller brain. Furthermore, brain size undoubtedly did increase over most of human evolution (Herculano-Houzel, 2016). We would like to point out that our work is not intended to deny this phenomenon but rather to put a more nuanced question into the spotlight. Namely, that it needs to be investigated what kind of social (and environmental) conditions lead to what type of consequences for brain structure and activity. Future work could explore how our findings scale up to larger groups and different kinds of social couplings.

Finally, the agents in our simulation did not have any specific task they were required to solve as we optimized neural complexity directly. This might make conclusions about *task-related* neural complexity as opposed to *task-independent* complexity not entirely justified. However, related work by Nagar et al. (2019) that did include a behavioral task found results similar to ours.

In addition to raising new modeling questions, our work leads to novel hypotheses that could be tested in experimental work. For instance, it is possible that even though social insects tend to have smaller brains than solitary insects, the individuals in a colony may nevertheless exhibit more complex neural activity.

CONFLICT OF INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

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FIGURE CAPTIONS

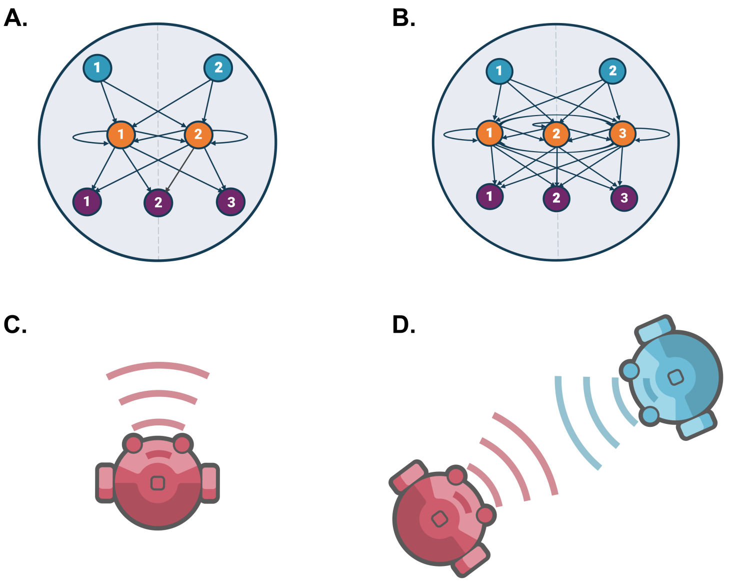

Figure 1. Agent neural architecture and evolutionary conditions, **(A)** 2-neuron model neural architecture, **(B)** 3-neuron model neural architecture. In both neural architectures, blue nodes refers to sensor nodes, orange nodes refers to neurons (fully recurrently connected neurons) and purple nodes refers to actuator nodes. **(C)** Evolving solitary agents (Individual Evolution (IE)), **(D)** Evolving interacting pair of agents (Social Evolution (SE)).

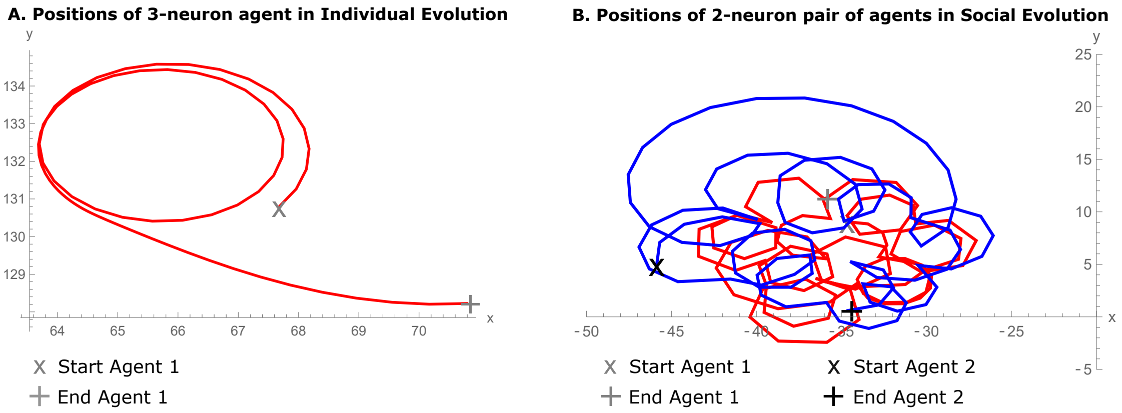

Figure 2. Behavior of agents during the last 10 simulation seconds of the trial. **(A)** Trajectory of the best 3-neuron agent evolved in Individual Evolution (IE) condition, showing simple loop movement. **(B)** Trajectories of the best pair of 2-neuron agents evolved in Social Evolution (SE) condition, showing spiralling nested pairwise loops movement.

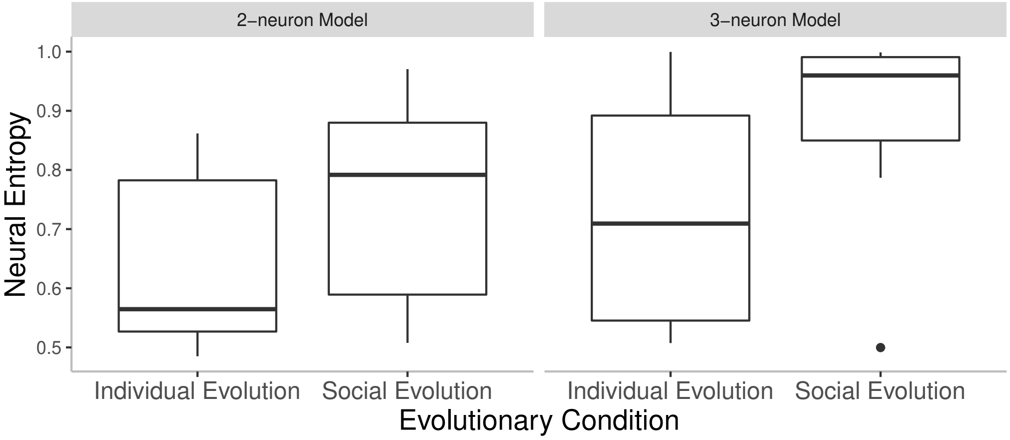

Figure 3. Neural entropy values in all conditions tested: values from the best agent (IE) or agents pair (SE) in the last generation in 10 runs of each condition. Note that the outlier in the SE, 3-neuron model was not removed from the analysis.

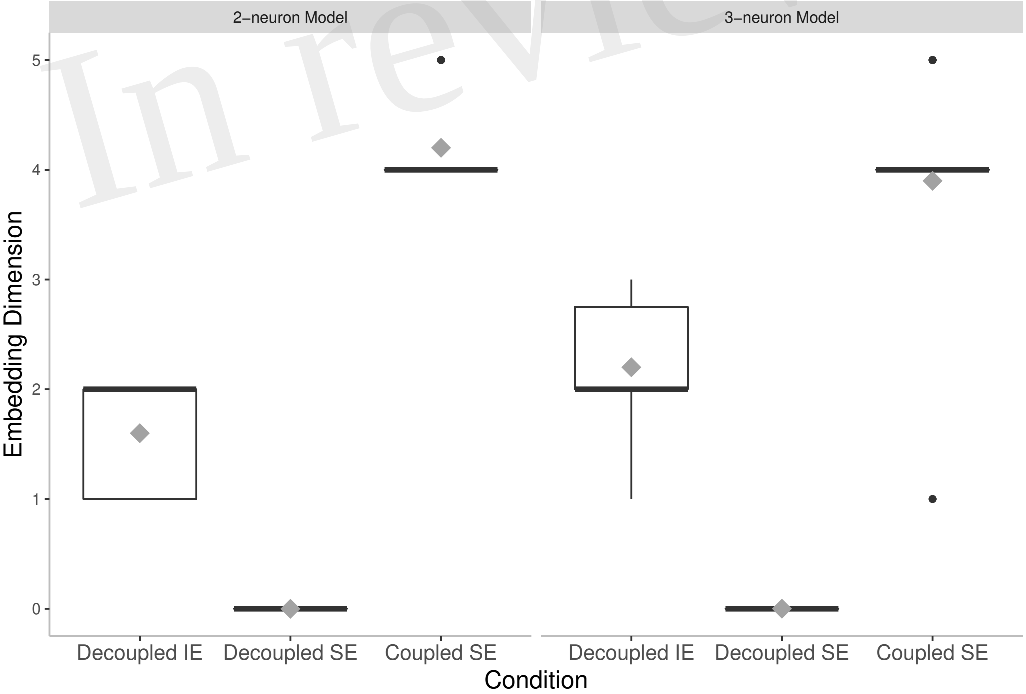

Figure 4. Embedding dimension in all conditions tested: The mean values are presented by gray diamonds. The mean value of the Coupled Social Evolution (SE) condition in 2-neuron model is higher than the mean value of the Decoupled Individual Evolution (IE) condition in 3-neuron model. Note that, for both models, in the Coupled SE condition, the mean values are higher than their intrinsic dimensional limitations (i.e. 2-dimensional system for the case of 2-neuron model and 3-dimensional system for the case of 3-neuron model). On the other hand, for both models, in the Decoupled SE condition, the mean values are lower than their intrinsic dimensional limitations.

Supplementary Material

This supplementary material document contains additional details of the methods section, including a description of the performed analysis.

1 METHODS

1.1 Simulated agents and environment

The strength of an agent's acoustic signal is attenuated in two different ways:

- **Attenuation due to distance:** The strength of the acoustic signal (or sound intensity) decays inversely proportional to the square of the distance from the sound source, i.e. it obeys the inverse square law. It will be maximum and equal to that of the emitted strength at a distance equal to the $2R$ between the center of the agents.
- **Attenuation due to "self-shadowing" mechanism:** The "self-shadowing" mechanism refers to the linear attenuation of the strength of the acoustic signal, which is proportional to the distance travelled by the signal within the agent's body, i.e. the shielded distance, D_{sh}. The range of D_{sh} goes from 0, when there is a direct line between the sound source and the sensor, to $2R$, when the sensor is directly opposed to the sound source. This attenuation is a natural consequence of the agent's embodiment (Di Paolo, 2000). The equations to calculate the shielded distance, D_{sh}, are available in the Supplementary Material of the Candadai et al. (2019) model.

Thus, the sensory input for each agent's acoustic sensor (i.e. the attenuated strength of acoustic signal) is obtained by first applying the inverse square law to the sound intensity at the position of the sensor and then multiplying it by an attenuating factor that goes linearly from 1, when $D_{sh} = 0$, to 0.1, when $D_{sh} = 2R$ (Di Paolo, 2000).

Agents are able to navigate freely in the environment except when they are involved in a collision. Collisions are modeled as point elastic, i.e. no effect in the agents' angular velocity (frictionless bodies) and the entire system's energy is conserved (no loss of energy). Agent bodies are considered as identical, so that when they are colliding, the result is the instantaneous exchange of their velocity vectors. Due to lack of inertia, agents take back control of their movement immediately after colliding (Di Paolo, 2000).

1.2 Neural architecture

The output of the **sensor layer** nodes is given by:

$$o_s = g_s \sigma(I_s + \theta_s) \tag{S1}$$

where $\sigma(x) = 1/(1 + e^{-x})$ is the sigmoidal activation function, I_s is the sensory input, θ_s is the bias and g_s is the gain. Both sensor nodes share common bias and gain.

In both 2-neuron and 3-neuron architectures, each **inner layer** neuron's activity is governed by the following state equation:

$$\tau_i \frac{dy_i}{dt} = -y_i + \sum_{j=1}^{N} w_{ij}\sigma(y_j + \theta_j) + \sum_{s=1}^{2} w_{is} o_s \tag{S2}$$

where dy_i/dt refers to the rate of change of internal state y_i of neuron i based on a time constant τ_i. The rate of change dy_i/dt depends on three values: the current state of the neuron, the weighted sum of outputs from all N neurons (for the 2-neuron model $N = 2$ and for the 3-neuron model $N = 3$) in the network and the total external input. The output of each neuron based on its internal state is given by a sigmoidal activation function $\sigma(y_j + \theta_j)$ where θ_j refers to the neuron's bias. The total external input received by the neuron is given by the weighted sum of the sensory input with weights w_is from sensor node s to neuron i and o_s is the sensory output from each sensor node. All neurons share common time-constant and bias.

The input to the **actuator layer** is a weighted sum of the neurons' outputs. The output of each of the actuator layer node i, m_i, is given by:

$$m_i = g_m\sigma\left(\sum_{n=1}^{N} w_{ni} * o_n + \theta_i\right) \tag{S3}$$

where o_n is the output of the corresponding neuron n, that are weighted by w_{ni}, θ_i is the bias and g_m is the gain. All actuator nodes share common bias and gain.

1.3 Performance measure

Neural complexity is calculated by creating a 2-dimensional or 3-dimensional (depending on the number of neurons in the inner layer) histogram of neural output values in a given trial. The neural outputs are obtained from a sigmoid function, therefore, they are bounded in the range $[0, 1]$. The output space is divided into 100 bins along each dimension, i.e. totaling ten thousand bins for the 2-neuron model (2-dimensional output space) and one million bins for the 3-neuron model (3-dimensional output space).

Then, the bins are filled with data points collected for each trial and the probability of the neural activity being in a given bin $[i, j]$, $p_{i,j}$ or $[i, j, k]$, $p_{i,j,k}$, respectively, is given by the number of points in that bin divided by the total number of points in the trial ($N = 2000$).

From these probabilities, the neural entropy H is given by:

$$H = \sum_{i=1}^{100}\sum_{j=1}^{100} -p_{ij}log_2(p_{ij}) \tag{S4}$$

$$H = \sum_{i=1}^{100}\sum_{j=1}^{100}\sum_{k=1}^{100} -p_{ijk}log_2(p_{ijk}) \tag{S5}$$

for the 2-neuron and 3-neuron model respectively.

Next, the neural entropy is normalized to be in the range $[0, 1]$ by dividing by the maximum neural entropy that can be achieved in the given dimension and with the given number of data points., i.e. $log_2(total\_number\_of\_fillable\_bins)$, where $total\_number\_of\_fillable\_bins = min(total\_number\_of\_bins, total\_number\_of\_data\_points)$. This is achieved when the data points for a specific trial are uniformly populated among the bins. Hence, the normalized neural entropy is given by:

$$\hat{H} = H/log_2(total\_number\_of\_fillable\_bins) \tag{S6}$$

For both 2-neuron model and 3-neuron model, the total number of data points is 2000 per trial (i.e. one data point per step size 0.1 for 200 simulation seconds). Since for both cases, the number of data points in each trial is less than the total number of bins, we have that $total\_number\_of\_fillable\_bins = 2000$. Thus, the normalized neural entropy per trial is given by:

$$\hat{H} = H/log_2(2000) \tag{S7}$$

In contrast to the Candadai et al. (2019) implementation, we calculate neural entropy measure for each experimental trial separately. The individual fitness value for an agent is calculated as the average normalized neural entropy of all trials. In the case of interacting pairs of agents, the fitness value for each pair is calculated as the average of their individual fitness values.

1.4 Genetic algorithm

Parameter ranges that were used to scale genotype values were as follows: for sensor and actuator nodes, the gains were scaled in the interval [1,20] and [1,5] respectively; for neurons, the time-constants were scaled in the interval [1,2]; all connection weights were scaled in the interval [-8,8] and all biases were scaled in the interval [-3,3].

1.5 Nonlinear Time Series Analysis

Nonlinear time series analysis is a useful approach to understanding the underlying dynamics of a system based solely on its realizations (e.g., time series observations) and without direct/explicit knowledge to its (unknown) properties. It is a *top-down* approach in which one utilizes the system's available observations to realize its potential state space and attractor(s) (Stam, 2005).

Let $\{x_0, x_1, x_2, ..., x_t, ..., x_n\}$ be a time series associated with a system in which x_t denotes the system's output at time t. Such time series along with *delay coordinate embedding* (Takens, 1981; Perc, 2006; Kodba et al., 2005) procedure can be utilized to reconstruct the system's attractor $\mathbf{p}(t) = (x_t, x_{t+\tau}, x_{t+2\tau}, ..., x_{t+(m-1)\tau})$, where τ and m are the embedding delay and the embedding dimension, respectively.

To estimate the optimal embedding delay τ, Fraser and Swinney (1986) used the position at which x_t and $x_{t+\tau}$ attained their first minimum mutual information (MI) i.e., the minimum amount of information that the state x_t provided about the state $x_{t+\tau}$ (Perc, 2006; Kodba et al., 2005).

For estimating a proper embedding dimension m, Kennel et al. (1992) introduced the false nearest neighbor method (FNN). The main idea of this method consists of minimizing the fraction of points having a false nearest neighbor through the choice of a sufficiently large embedding dimension m.

In the present study, we were interested in determining the above embedding dimension m for the time series of the evolved agents' neural activity (neural states) in different conditions. We achieved this objective as follows.

1. We obtained the time series of neural activity in both, 2-neuron and 3-neuron models, for following conditions:
    - **Decoupled Individual Evolution (IE):** From the best evolved agents (agent 1) of each run (10 runs) in IE condition, we tested them in isolation for 4 trials. Then, for determining the embedding dimension m, we used the time series of neural states of neuron 1, trial 1, from the isolated agent (agent 1).

- **Decoupled Social Evolution (SE):** From the best evolved pairs of agents (agent 1 and agent 2) of each run (10 runs) in SE condition, we tested agent 1 in isolation for 4 trials. Then, for determining the embedding dimension m, we used the time series of neural states of neuron 1, trial 1, from the isolated agent (agent 1).
- **Coupled Social Evolution (SE):** From the best evolved pairs of agents (agent 1 and agent 2) of each run (10 runs) in SE condition, we tested agent 1 and agent 2 in interaction for 4 trials. Then, for determining the embedding dimension m, we used the time series of neural states of neuron 1, trial 1, from agent 1.

2. In order to estimate the optimal embedding delay τ using MI, we employed the nonlinear time series analysis "*mutual.exe*" by Perc (2006). It requires the following parameters to be set by the user: number of data points, number of bins, and maximal embedding delay. We estimated the proper number of bins using *Freedman–Diaconis rule* (Freedman and Diaconis, 1981):

$$Bins = \frac{\|max(x) - min(x)\|}{2\frac{IQR(x)}{\sqrt[3]{n}}} \tag{S8}$$

   where $IQR(x)$ is the interquartile range of the data and n is the number of observations in the sample x.

3. In order to determine the proper embedding dimension m using the FNN with Euclidean metric (Kennel et al., 1992), we employed the *NoLiTSA Python module* by Manu Mannattil (Mannattil, 2018), where we specified the optimal embedding delay τ previously estimated.

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Conclusions and future work

This thesis has explored the relationship between neural complexity and sociality from the interdisciplinary perspective of embodied cognitive science using a wide variety of scientific methods to the study of cognition.

Overall, our results shed light on different insights that can be summarized as follows:

First, from a complex systems theory perspective, the antireductionist quote, *"the whole is more than the sum of its parts"*, which has been relevant for the new sciences such as chaos, systems biology, network theory, among others, to illustrate how complex behavior can emerge from large collections of simpler interactive components (Mitchell, 2009), can be extended according to our results of the *smaller-brained agent couplings* section. We propose the following extension:

> "The whole is ***more*** than the sum of its parts (in isolation) . . . since, its parts are also ***more*** when interacting, as they become integrated into 'the whole' of a higher dimensionality"

In other words, from a dynamical systems perspective, on the one hand, when our *smaller-brained* embodied agents are isolated, each of them is a 2-dimensional (2-neuron) autonomous system, that can exhibit only single fixed-point or limit-cycle attractors in their phase portraits (Beer, 1995). On the other hand, when our *smaller-brained* embodied agents are mutually interacting, they become integrated into a larger coupled system of a higher dimensionality, i.e. a brain-body-environment-body-brain system (Froese et al., 2013b), thereby, allowing an increase in their neural complexity beyond their intrinsic limits (i.e. degrees of freedom). This goes in line with the original results of the Candadai et al. (2019) model.

Second, from an evolutionary robotics approach, according to our results of the *bigger-brained agent couplings* section, by evolving embodied agents in the presence of a *ghost* partner as a source of non-social stimulation, we found that the richness of the environment may compensate unidirectional (or 1-way) interaction in terms of neural complexity, nonetheless, this is not the same in terms of behavioral complexity, thus, suggesting that a coupled system of embodied agents, that are responsive to each other when interacting, allows a broader repertoire of behaviors.

Third, from an evolutionary robotics perspective and a dynamical systems approach to cognition, our results in the *smaller-brained vs bigger-brained agent couplings* section demonstrated that *smaller-brained* social agents can exhibit comparable levels of neural complexity as *bigger-brained* solitary agents, importantly, without an increase in their intrinsic degrees of freedom of neural activity. This suggests that *smaller-brained* social agents take advantage of their capacity for interacting to enhance their intrinsic neural complexity, thus, allowing them to achieve similar behavioral performance with smaller brains.

Finally, we conclude that our results provide relevant implications for the science of brain evolution, as they suggest that a coupled brain, as part of a whole brain-body-environment-body-brain system (Froese et al., 2013b), will display neural dynamics that are underdetermined by the brain in itself.

In addition, our work opens the possibility to test this hypothesis in novel experimental work with solitary and social species of real social insects.

Future work will look at increasing the number of embodied agents, inspired in ant colonies, to get novel insights into the so-called *"liquid" brains* (Solé et al., 2019).

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