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Modular Cosmological Parameter Estimation for Modified Gravity with CosmoSIS

Tesis

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A mis padres y mis hermanas. Los amo.

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Introduction

Modern science (or Physics in most cases) has to keep up with the current technological revolution. Most of the more successful companies, industries and investigation groups in the world are taking advantage of the new programming languages, frameworks, software methodologies, hardware improvements to change and adapt to the actual tendencies of the information era.

This work is about Cosmology, how and why we as cosmologists can use this new technologies and implement them in a way that our jobs and analysis become more accurate, precise and easy.

The main focus of this work is how we can use the framework CosmoSIS (fully described in the content), to run some calculations and analysis in modified gravity. But why? There are so many codes and programs that do this calculations, and been doing them for years. The difference here is that CosmoSIS embraces a software methodology construction that is similar to the ones that computer scientists do in their own jobs. Is an open source code, in the full meaning of the words, it is well documented, extremely detailed about the algorithms it implements and very easy to install and use. One subject that we will discuss in this work, is the modularity that CosmoSIS exposes to do its calculations, how is this important and why we should be taking this example to create most of our codes in physics.

We chose modified gravity for this work because is a well explain topic in these days, but not that well explored. So in the first chapter we start by setting the basis for modern Cosmology, namely, General Relativity (GR), Friedmann equations and cosmological perturbation theory. This concepts and definitions we'll give are in accordance with our Standard Model of Cosmology (SMC), that we call nowadays ΛCDM. So in order to understand the meaning of ΛCDM we give some insights about the *dark* side of the Universe (Dark Energy and Dark Matter), to obtain a full picture of our current view of the history, formation and evolution of the Universe. In the end of the first chapter we go into some statistical details that are very useful when comparing ΛCDM theory with observations, and that constrain the parameters, or *cosmological parameters*, for this particular model of the Universe.

In the first chapter we point out some of the problems and details of ΛCDM that are not fully understand, or aren't accepted by the whole cosmological community, and in the second chapter we introduce a theory that aims to fix one of them. The problem we are talking about is the accelerated expansion of the Universe, and that maybe ΛCDM and the standard theory of gravity, GR, is not correct for all scales in the Universe. This theory has been around for a few decades and we call it Modified Gravity (MG). The name is fairly obvious, but we have to be careful when we talk about modifying gravity, because GR explains very well some astronomical scenarios, like the way our Solar System behaves, so this modifications of gravity, have to become meaningless in the well explained region of GR or even Newtonian gravity. We present such a theory beginning with a short explanation of $f(R)$ models, and then a parametrization for modified gravity that makes easy its study and analysis. The parametrized framework we chose for this work is the so called Parametrized Post-Friedmann (PPF) Framework, and we go into details about it, its origin, use and perks. Finally we describe the specific PPF model we use for our calculations, the Bertschinger and Zukin (BZ) parametrization.

The third chapter is devoted to CosmoSIS, its structure, use and how to interact with the framework. But first we do two important things, (1) first we present Bayesian Cosmology, starting from the basis of Bayesian Statistics, Bayesian parameter inference and Bayesian model selection, stating why this paradigm of Statistics is very useful for Astronomy, Astrophysics and Cosmology, why it is becoming the main statistical theory to use when analyzing results of physical experiments and that it will become more important in the near future, then (2) we explain the need for a framework like CosmoSIS. The need for a truly open source code for Cosmology, why it is important that it be open source, and how to face the problems that other communities like Astropy are dealing with, so CosmoSIS can become the new and improved standard computational cosmological framework, and an example of coding and software methodologies for Computational Cosmology and even Computational Physics.

In the fourth and final chapter we show our results. Even though most of the chapters present some results and explanations that are original, we dedicate this last chapter to the specific results of running MG modules in CosmoSIS. We use MGCAMB as our principal module for computing MG models, and in here we give a full description of the code and its implementation in CosmoSIS. The plots we obtained, the statistical calculations and more were all obtained with CosmoSIS, and in this chapter we also explain how to use this modules for our advantage and how we can change them for our convenience.

Finally the appendices present two possible installations for CosmoSIS. Both of them uses mostly the code created by the authors, but we developed and change some of the files for an easier installation, and to provide full reproducibility of the codes we used. All of the code, files, and configurations will be located in an free access on-line repository, with some more explanations about the specific code we used for our calculations.

Chapter 1

Standard Model of Cosmology

Since the advances in computation, astronomical technology and theoretical Physics, we have developed a Standard Model for Cosmology (SMC). This model has been around for several decades now, but each year we increase the precision of the constraints, parameters and predictions for it. It is said that cosmology has entered the *precision era*, where we can make accurate measurements for most of the features and subtleties of our models. Although there are some parts of the model we don't fully understand, there is a lot of confidence in the way it explains the beginning, evolution and current state of the observable Universe. It has to be pointed out in this moment, that even though the SMC is very successful in describing the Universe, still exists some features of it that the cosmological community has not agreed to, and there are several other proposes that may explain this problems. In the next chapters we will focus on some of these problems and propose a theoretical and phenomenological framework that can solve them.

This model is called ΛCDM (Λ Cold Dark Matter), a term we will explain in this chapter. We start by setting the theoretical background, covering the basics of Einstein's General Theory of Relativity, then using the conventional assumptions taken by modern Cosmology, discuss briefly the Friedmann Equations and define some important cosmological parameters. Then we'll begin a description of the "Dark side" of the Universe, namely Dark Matter and Dark Energy, a subject that will be expanded in the rest of the thesis; and finally we present the anticipated ΛCDM model of Cosmology and some of its predictions.

1.1 Basics of Einstein's General Theory of Relativity

The SMC is based on two main assumptions, namely that the Universe is statistically homogeneous and isotropic at large scales (about 150 Mpc), and that gravity is described by Einstein's General Theory of Relativity (Koyama, 2016; Barreira, 2016). General Relativity has proven successful over many years of experimental tests (Will, 2014); in his theory Einstein stated that gravity is interpreted as the manifestation of the location-dependance of the metric, and his field equations relates the distribution of matter and the geometry of the Universe, in other words the field equations of General Relativity provides a specific way in which the metric is determined from the content of the spacetime (Pearson, 2014).

Einstein's field equations have a tensorial form and they can be written as (Coles and Lucchin, 2002; Charles Misner and Wheeler, 1973):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{1.1}$$

where we can define he Einstein tensor $G_{\mu\nu}$, as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{1.2}$$

and where $R_{\mu\nu}$ the Ricci tensor, which is a contraction of the Riemann tensor $R^{\mu}_{\mu\alpha\beta}$, $g_{\mu\nu}$ is the metric tensor, or metric, which describes the geometry of spacetime and R is the Ricci Scalar, which is the contraction of the Ricci tensor. In 1.1, G is the Newton's gravitational constant, an the entire proportionality factor $8\pi G$ is determined by comparing the limit of the field equations in the low gravitational field (Newtonian limit) with the Newtonian field equations and assures that the standard Poisson equation maintains its form in this limit (Pearson, 2014; Carmeli, 2002). Einstein tensor describes the curvature of spacetime in the field equations of General Relativity (Charles Misner and Wheeler, 1973). Finally, $T_{\mu\nu}$ is the energy-momentum tensor, which describes the distribution of matter in the Universe.

1.2 Friedmann equations

Now that we've set the field equations for the gravitational theory in the SMC, we have to construct the dynamic equations that describes it, using its main assumptions. Before doing that, there is one extra ingredient that we have not discussed yet, and that is that in the SMC, the Universe is expanding. This great discovery by Edwin Hubble (Hubble, 1929) and others, amaze Einstein that was one of the believers that the Universe was static (see section 1.5). It can be demonstrated (Trodden and Carroll, 2004) that the the simplest metric that describes the more general spacetime that follows the assumptions of the SMC is the so called Friedmann-Lamaitre-Robertson-Walker (FLRW) metric (Trodden and Carroll, 2004; Coles and Lucchin, 2002):

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right], \tag{1.3}$$

where the set of coordinates (r, θ, ϕ) are called the comoving coordinates, t is the cosmological time and the expansion of the Universe is introduced via $a(t)$ which is called the scale factor or expansion parameter, finally K is the curvature parameter, a constant that can only take the values of -1, 0 or 1, and it gives us three possibilities for the geometry of the Universe: open, flat or closed, respectively (Trodden and Carroll, 2004).

To find this dynamical equations we need the basic assumptions presented above and three more (Rich, 2010):

- The Universe is constituted by a homogeneous fluid with energy density ρ and pressure p.
- The energy-momentum tensor is a linear function of the coefficients of cuadratic terms of the locally Lorentzian metric, derived from the FLRW metric, and has the form Trodden and Carroll

(2004):

$$T^{\nu}_{\mu} = (\rho + p)u^{\mu}u_{\nu} + p\delta^{\mu}_{\nu}, \tag{1.4}$$

with u^{μ} the 4-velocity.

- The energy is conserved locally.

Now, the solutions to Einstein equations, for the type of Universe assumed and with the FLRW metric, are the dynamic equations we are looking for (Trodden and Carroll, 2004; Rich, 2010):

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right) = \frac{8\pi G}{3} - \frac{K}{a^2}, \tag{1.5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{1.6}$$

$$\dot{\rho} = -3H(\rho + p). \tag{1.7}$$

Where H is the Hubble parameter, defined as $H = \frac{\dot{a}}{a}$. Only the first two equations are linearly independent, and the third one can be obtained from the first two. The first two equations are known as the Friedmann equations (Trodden and Carroll, 2004; Barreira, 2016) and they govern the evolution of the scale factor, meanwhile the third equation is known as the conservation equation and it governs the energy density of the various energy species in the Universe.

1.3 Cosmological Perturbations

If we recall, one of the main assumptions of the SMC is that the Universe is statistically isotropic and homogeneous at large scales, and its metric has the form of a FLRW metric. But such a simplistic model cannot describe the complexity of the distribution of matter and energy in the actual Universe, where stars forms galaxies, galaxies forms groups of galaxies and even clusters of galaxies (Coles and Lucchin, 2002; Rich, 2010). We need to find a way for describing inhomogeneity and anisotropy. One of the easiest ways of doing this is recurring to a perturbative method that can increase the complexity of the *background* cosmology described by the FLRW metric (Houjun Mo, 2010; Malik and Wands, 2009).

We will not fully derive or describe the theory of cosmological perturbations, rather we will find some important results that will guide our work in following sections and chapters, so to the interested reader we suggest the careful reading of the books of (Luca Amendola, 2010) and (Houjun Mo, 2010). Also we will work for this introductory section only with first order perturbations.

Following (Luca Amendola, 2010) and (Pearson, 2014) in order to perturb General Relativity we first perturb the metric, and that at first order is

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}, \tag{1.8}$$

where the quantities with superscript (0) will denote the background cosmology, and the ones preceded with a δ will be the perturbed part. Now, a metric perturbed about a conformally flat FLRW background can be written as

$$ds^2 = a^2(t)(\eta_{\mu\nu} + \gamma_{\mu\nu})dx^{\mu}dx^{\nu}, \tag{1.9}$$

where $\gamma_{\mu\nu}$ can be space and time dependent, and can be parametrized as (Pearson, 2014; Barreira, 2016)

$$ds^2 = a^2(t)\left[-(1 + 2\Phi)dt^2 + 2N_i dx^i dt + (\delta_{ij} + \gamma_{ij})dx^i dx^j\right], \tag{1.10}$$

where $\Phi = \Phi(t, x)$ is the Newtonian gravitational potential, $N_i = N_i(t, x)$ the lapse function and $\gamma_{ij} = h_{ij}(t, x)$ are the spacial metric perturbations.

In General Relativity the split between a background metric and a perturbed one is not unique (Luca Amendola, 2010). And if we want to leave the FLRW metric as the background, we need to select an infinitesimal transformation that leaves $g_{\mu\nu}^{(0)}$ as it is, while allowing $\delta g_{\mu\nu}$ to change, this class of transformations are called gauge transformations. There are several interesting gauge choices that have been used in the last decades, some of them are listed in (Malik and Wands, 2009), but for the purpose of this work, we will use the so called *conformal Newtonian gauge* in which we set $N_i = 0$ and $h_{ij} = -2\Psi\gamma_{ij}$.

So, the full metric for a perturbed Universe with a FLRW metric in the background can be written, in the conformal Newtonian gauge as (Bertschinger and Zukin, 2008)

$$ds^2 = a^2(t)\left[-(1+2\Phi)dt^2 + (1-2\Psi)\gamma_{ij}dx^i dx^j\right]. \tag{1.11}$$

From this point, normally we would derive the Einstein Equations for a given energy-momentum tensor in the perturbed regime, we won't do that, but only use the results, if the reader wants to see a demonstration we recommend (Houjun Mo, 2010) and (Ma and Bertschinger, 1995).

If we consider a single-fluid model with energy-momentum tensor $T_{\mu\nu}$, and limit ourselves to perfect fluids [1] ($T_{\mu\nu} = 0(i \neq j)$), we can write it in its perturbed form and with equation of state $\omega = p/\rho$ as

$$\delta T_{\mu\nu} = \rho[\delta(1+c_s^2)u_\mu u_\nu + (1+\omega)(\delta u_\nu u_\mu + u_\nu \delta u_\mu) + c_s^2\delta\delta_{\mu\nu}], \tag{1.12}$$

where we have already defined ρ, p and u_μ in 1.4, and we've defined

$$\delta \equiv \frac{\delta\rho}{\rho}, \tag{1.13}$$

as the density contrast, and

$$c_s^2 \equiv \frac{\delta p}{\delta\rho}, \tag{1.14}$$

as the sound velocity. Now the first-order Einstein's equations can be written as

$$\delta G_{\mu\nu} = 8\pi G\delta T_{\mu\nu}, \tag{1.15}$$

and resolving this equations using the energy-momentum tensor introduced above, leads to (Houjun Mo, 2010):

$$3H(H\Psi - \dot{\Phi}) + \nabla^2\Phi = -4\pi G a^2\delta\rho, \tag{1.16}$$

$$\nabla^2(\dot{\Phi} - H\Psi) = 4\pi G a^2(1+\omega)\rho\theta, \tag{1.17}$$

$$\Psi = -\Phi, \tag{1.18}$$

$$\ddot{\Phi} + 2H\dot{\Phi} - H\dot{\Psi} - (H^2 + 2\dot{H})\Psi = -4\pi G a^2 c_s^2\delta\rho. \tag{1.19}$$

Where $\theta \equiv \nabla_i u^i$ is the velocity divergence. In this regime we can find that the continuity equation can be written as Ma and Bertschinger (1995)

$$\dot{\delta} + 3H(c_s^2 - \omega)\delta = -(1+\omega)(\theta + 3\dot{\Phi}). \tag{1.20}$$

Because the continuity equation holds, we know that $\delta T_{\mu\nu;\mu} = 0$ (where ; stands for the covariant derivative), so for $\nu = i$ we get

$$\dot{\theta} + \left[H(1-3\omega) + \frac{\dot{\omega}}{1+\omega}\right]\theta = \nabla^2\left(\frac{c_s^2}{1+\omega} + \Psi\right), \tag{1.21}$$

that for non-relativistic matter is known as the Euler equation Houjun Mo (2010). Now, if we Fourier expand all the perturbation quantities we get:

[1] We can assume that the perturbed fluid will still be a perfect fluid (Houjun Mo, 2010).

$$\Phi = \int \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \Phi_k d^3k, \tag{1.22}$$

$$\Psi = \int \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \Psi_k d^3k, \tag{1.23}$$

$$\delta = \int \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \delta_k d^3k, \tag{1.24}$$

$$\theta = \int \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \theta_k d^3k, \tag{1.25}$$

where k represents a Fourier mode for each wavenumber k. Now rewriting 1.21 in the Fourier space

$$\dot{\theta} + \left[H(1-3\omega) + \frac{\dot{\omega}}{1+\omega}\right]\theta = k^2\left(\frac{c_s^2}{1+\omega} + \Psi\right), \tag{1.26}$$

and combining it with 1.20 we obtain the relativistic Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho[\delta + 3H(\omega+1)\theta/k^2] = 4\pi G a^2 \rho\Delta, \tag{1.27}$$

where $\Delta = \delta + 3H(\omega+1)\theta/k^2$ is the *total-matter* variable (or the comoving density contrast).

1.4 Cosmological Parameters

It is convenient to express the Friedmann equations in a way that they contain the current expansion rate, and some parameters that can be compared with observations. We have define one of this parameters, the Hubble constant H which is related with the age of the Universe, but there are other interesting parameters that can be defined and used in cosmology. One of them is the density parameter, which is very helpful to write down in a simple fashion the density of the Universe and its content (Rich, 2010).

If we recall the form of the first Friedmann equation, we see that for a given H, there is a special value of H, that gives us a spatially flat Universe, i.e. $K = 0$, and solving that equation for ρ we can define the critical density of the Universe (ρ_c):

$$\rho_c(t) = \frac{3H^2}{8\pi G}. \tag{1.28}$$

So now we can express the density of the Universe in terms of its relative value to the critical density. This adimensional quantity is known as the density parameter Ω,

$$\Omega(t) = \frac{\rho}{\rho_c}, \tag{1.29}$$

and using Ω we can write the first Friedmann equation as

$$\Omega(t) - 1 = \frac{K}{a^2H^2} \tag{1.30}$$

This Ω is a mix, for an arbitrary K, of the distinct components of the Universe. These components will be explain in the next section. We need a few more terms to fully describe the properties of the SMC; in order to describe the perturbations in curvature we need two more parameters, that should describe the amplitude of the initial spectrum and the spectral index n_s. For the amplitude we use σ_8 which refers to amplitude of the power spectrum (see section 1.6) as measured today from a distance of 8 Mpc. The spectral index n_s refers to the initial condition of the perturbations and it's related to the power spectrum with

$$P(k) = A\, k^{n_s-1}, \tag{1.31}$$

where A is a constant amplitude. We need another parameter to specify the the reionization of the cosmic medium. This parameter is known as the optical depth τ, and it represents the probability of a photon to be scattered. This parameter is important in cosmology because the optical depth of the Universe to low-energy photons after decoupling, determines the fraction of CMB photons that travel to us uninterrupted since that epoch. Finally we need a parameter to measure the present expansion rate of the Universe, and this is known as the Hubble constant H_0. This parameter relates the redshift of a distant object to its distance, also it sets the age of the Universe and an estimate for the size of the observable Universe. The critical density of the Universe is also dependent on H_0, $\rho_0 = 3H_0^2/8\pi G$. The values for all of this parameters for the SMC will be presented in section 1.6.

Another important parameter that we can define is the deceleration parameter; as we stated the Universe is expanding, an a way of measuring this expansion is via the Hubble parameter. This parameter changes with time, and the deceleration parameter allow us to quantify that change. It can be written as (Rich, 2010; Coles and Lucchin, 2002)

$$q_0 = -H_0^{-2}\left[\frac{\ddot{a}}{a}\right]_{t_0}, \tag{1.32}$$

where the subscript 0 means the evaluation at the present time. The name of this parameter, the "deceleration" parameter, was given thinking that the expansion should be slowing down with time due to the gravitational attraction between its components. So it was a big surprise when several teams across the world, see for example the paper of the Supernova Cosmology Project collaboration (S. Pelmutter, 1998), find out that q_0 was negative, it meant that the expansion was accelerating!. This astonishing discovery started a new conflict in Cosmology, the *dark energy*, and together with the problem of *dark matter*, they form the *dark* sector of the Universe, and the topic for next section.

1.5 Dark matter and Dark energy

In the last section we introduced the "dark side" of the Universe. This sector of the Universe was discovered without being expected or theorized by observing different cosmological and astrophysical systems and then analyzing the obtained data within the SMC. These observations came from different probes, observatories, telescopes, collaborations and these studies came from distances from supernovae, gravitational lensing, galaxy rotation curves, the cosmic microwave background and structure formation (Pearson, 2014; Joyce et al., 2016).

What is important here is that the Universe expansion, that observations reveal and data implies, are in accordance with Einstein's General Relativity only if we introduce the ingredients of the *dark side*, commonly referred as "dark matter" and "dark energy". In the historical context, dark matter was introduced to fix galaxy rotation curves and for structure formation, and dark energy as an explanation for the origin of the observed accelerating expansion of the Universe. They are now part of the standard picture of modern cosmology, but it has to be pointed out that neither is completely accepted by all the experts in the field (Penrose, 2010).

Dark matter shares some of it properties with regular (baryonic) matter, having the same gravitational effects and negligible pressure, but it does not interact with electromagnetic fields, that is why it's "dark". We can only know is present, for now, from its gravitational effects. In the other hand, dark energy has a somewhat ghostly nature, but is has to be included as the current dominant gravitating species in the content of the Universe Koyama (2016); Joyce et al. (2016). We will not discuss more about dark matter, and we'll focus in the different explanations for dark energy, because it is more close to the objectives of this work.

In order to include dark energy into the SMC we need to take the modified form of Einstein's equations that he derived in 1916(7) (and later ruled out). These modifications were added to create and non-expanding Universe, as Einstein picture it, but are really useful to incorporate one of the possibilities for the theoretical explanations of dark energy, that it can be described by a cosmological constant (in this case it has to be tiny and positive). In this case the Einstein's equations 1.1 has the form (Coles and Lucchin, 2002):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1.33}$$

This modification doesn't change the covariant character of the equations, and doesn't the form of the continuity equation. As explained by (Bertschinger and Zukin, 2008), when we impose these form of

Einstein's equations on the FLRW metric, we encounter a set of Friedmann equations that imply that stress-energy-momentum component with negative pressure is needed to explain cosmic acceleration, and we also can deduce Pearson (2014) that it must be a substance with a equation of state $\omega \approx -1$. This substance may be vacuum energy (a cosmological constant), a scalar field (Bertschinger and Zukin, 2008) or more there may be another explanation, but we will explore this scenario in the next chapter.

According to the results of the (Planck Collaboration et al., 2016b) the current known values for the densities of dark matter and dark energy are

$$\Omega_c = 0.2589 \pm 0.0057 \tag{1.34}$$

$$\Omega_\Lambda = 0.6911 \pm 0.0062 \tag{1.35}$$

We have added the last ingredients for our SMC, and now we can start calling it in its usual name ΛCDM, where Λ stands for the before introduced cosmological constant, and CDM stands for *Cold Dark Matter*, which is the more plausible form of dark matter we have to explain this side of the dark sector; it refers to particles without significant random movement, or in other words, its random movement is not relativistic Coles and Lucchin (2002). In the next section we will describe the main aspects and current values for the parameters of this cosmological model, in preparation for the next chapter in which we will find some alternatives to it, to better describe some parts of our SMC using another tools and theories.

1.6 ΛCDM Model of Cosmology

All the previous definitions, equations and discussion lead us to this section, and to the description of our SMC, ΛCDM. As we have stated, in this model, gravity is described by Einstein's General Theory of Relativity, in which the field equations are the Einstein Equations, with a FLRW metric for a statistically isotropic and homogeneous expanding (and accelerating) Universe, the matter and energy are described by a energy-momentum tensor for a perfect fluid with pressure p and density ρ, and the accelerated expansion is caused by a dark energy of a somewhat undiscovered nature plus the majority of matter in the Universe is compressed in a dark matter component. These model has been studied and analyzed with extreme precaution and detail by most cosmological collaborations around the globe and its predictions are in agreement with most observations performed by astronomers. In the next chapter we will see some of its problems, and the parts that we don't fully understand, leading us to propose some alternatives to one or more of its ingredients, but for now lets present some of the more important results for cosmological parameters we have found following the ΛCMD model.

In the last sections we have presented some of the results for parameters in the ΛCDM cosmology, but here we present a table that resumes them and adds some extra information following the (Planck Collaboration et al., 2016b) paper:

Parameter	Name	Value (today)
$\Omega_b h^2$	Physic Baryonic Density	0.02230 $\pm$ 0.00014
Ω_Λ	Dark Energy Density	0.6911 $\pm$ 0.0062
H_0	Hubble Parameter	67.74 $\pm$ 0.74 kms^{-1}Mpcs^{-1}
n_s	Spectral Index	0.9667 $\pm$ 0.0040
Ω_m	Matter Density	0.3089 $\pm$ 0.0062
σ_8	Power Spectrum Amplitude	0.815 $\pm$ 0.0086
τ	Optical Depth	0.066 $\pm$ 0.012
ω	Equation of State for Dark Energy	-1.006 $\pm$ 0.045

These parameters were explained in the section 1.4 and are very well measured and analyzed constantly by various cosmological collaborations. They describe several features of ΛCDM and because they agree so good between observations and theory, they impose constraints on new cosmological models. There are other descriptors that are really important for our actual understanding of the

ΛCDM model, they are based on statistical theory and we will give a short explanation for them below.

1.6.1 2-point correlation function

Galaxies, and many other astronomical sources, are not randomly distributed in space, but cluster together in high-density regions. This clustering may be quantified using the correlation function. If $dN_{ab} = \langle n_a n_b \rangle$ is the average number of pairs in the volumes dV_a and dV_b (i.e. the product of the number of particles in one volume times the number in another volume), separated by r_{ab} , then we define the 2-point correlation function $\xi(r_{ab})$, as

$$dN_{ab} = \langle n_a n_b \rangle = \rho_0^2 dV_a dV_b [1 + \xi(r_{ab})], \tag{1.36}$$

where ρ is the average numerical density for N particles in a volume V. If the distribution has been obtained by throwing the N particles at random (i.e. without any preference with respect to the place), then there is no reason for dN_{ab} to depend on the location. Therefore the average number of pairs is exactly equal to the product of the average number of particles in the two volumes, $\langle n_a n_b \rangle = \langle n_a \rangle \langle n_b \rangle = \rho_0^2 dV_a dV_b$, and the correlation ξ vanishes. Conversely, if ξ is non-zero, we say that the particles are correlated. Then the correlation function can be written as a spatial average of the product of the density contrast $\delta(r_a) = n_a/(\rho_0 dV_a) - 1$ at two different points:

$$\xi_{ab} = \frac{dN_{ab}}{\rho_0^2 dV_a dV_b} - 1 = \langle \delta(r_a)\delta(r_b) \rangle. \tag{1.37}$$

When $\xi(\mathbf{r})$ depends only on the separation r and not on r_a and r_b individually, the system is said to be statistically homogeneous. In practice the simplest way to measure xi is to compare the real catalog to an artificial random catalog with exactly the same boundaries and the same selection function. Then the estimator can be written as

$$\xi = \frac{DD}{DR} - 1, \tag{1.38}$$

where DD means the number of galaxies at distance r counted by an observer centered on a real galaxy (data D). This is divided by the number of galaxies DR at the same distance but in the random catalog (if to reduce the scatter the random catalog contains α times more galaxies than the real one then DR has to be divided by α), in this case when DD and DR are the same, the correlation function, will be zero, and the interpretation here is that, DR is random, so if DD is equal to DR, it means that it is random, so the galaxies are not correlated. In other words, instead of calculating the volume of the shell (which is a difficult task in realistic cases), we estimate ξ by counting the galaxies in the Monte Carlo realization (Luca Amendola, 2010).

1.6.2 Power Spectrum

The linear perturbation variables contain important physics, both for dark energy and cosmology in general. A convenient way to study perturbation variables is to decompose fluctuations into orthonormal modes because at the linear level they evolve independently. Since by definition the average of a perturbation variable is zero, the simplest non-trivial statistics corresponds to a quadratic function of the variables. In Fourier space, any real quadratic function of a perturbation variable is called a power spectrum. The power spectrum is the Fourier-space version of the correlation spectrum derived from the correlation function we described above. The power spectrum is defined as (with dimension of volume)

$$P(\mathbf{k}) = V|\delta_k|^2 = V\delta_k \delta_k^*, \tag{1.39}$$

and using the definition of the Dirac's delta in Fourier space it follows that

$$P(\mathbf{k}) = \frac{1}{V}\int \delta(\mathbf{x})\delta(\mathbf{y}) e^{i\mathbf{k}\cdot(x-y)} dV_x dV_y, \tag{1.40}$$

and now setting $\mathbf{r} = \mathbf{x} - \mathbf{y}$, this spectrum reduces to

$$P(\mathbf{k}) = \int \xi(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} dV, \tag{1.41}$$

where $\xi(\mathbf{r})$ is defined as above

$$\xi(\mathbf{r}) = \langle \delta(\mathbf{y}+\mathbf{r})\delta(\mathbf{y}) = \frac{1}{V}\int \delta(\mathbf{y}+\mathbf{r})\delta(\mathbf{y}) dV_y. \tag{1.42}$$

Therefore, the power spectrum is the Fourier transform of the correlation function (Barreira, 2016).

1.6.3 Cosmic Microwave Background (CMB)

In their 1965 landmark paper A. Penzias and R. Wilson (Penzias and Wilson, 1965), who were investigating the origin of radio interference at what at the time were considered high frequencies, reported a 3.5 K signal from the sky at 4 GHz that was "isotropic, unpolarized, and free from seasonal variations" within the limits of their observations. In a companion paper published in the same issue of the Astrophysical Journal, (Dicke et al., 1965) proposed the explanation that the isotropic sky signal seen by Penzias and Wilson was in fact emanating from a hot big bang, as had been suggested in the 1948 paper of (Alpher et al., 1948) suggesting the presence of the photon blackbody component having a temperature of approximately a few K. Their prediction was based on considering the conditions required for successful nucleosynthesis in an expanding universe—that is, to create an appreciable fraction of primordial helium from the neutrons that are decaying as the universe is expanding. The importance of this discovery became almost immediately apparent, and others set out to better characterize this excess emission, which later would become known as the Cosmic Microwave Background (CMB), or sometimes in the older literature the Cosmic Microwave Background Radiation (CMBR).

The observations of temperature anisotropies in the CMB provided another independent test for the existence of dark energy. The oldest sky we can see is the so-called last scattering surface at which electrons are trapped by hydrogen to form atoms (dubbed "decoupling" or "recombination"). All of the matter components in the Universe (dark matter, neutrinos, ...) are coupled to gravity through the Einstein equations. The scalar part of the perturbations is the main source for the CMB temperature anisotropies. When we confront the predicted temperature anisotropies with CMB observations, we expand the perturbation Θ in terms of spherical harmonics:

$$\Theta(\mathbf{x},\eta) = \sum_{l=1}^{\infty}\sum_{m=l}^{2} a_{lm}(\mathbf{x},\eta) Y_{lm}(\hat{n}), \tag{1.43}$$

where the subscripts l and m are conjugate to a real space unit vector $\hat{n}$ represent ing the direction of incoming photons. The coefficients a_{lm} in this equation are assumed to be statistically independent. This means that the mean value of a_{lm}'s is zero ($\langle a_{lm}\rangle = 0$) with a non-zero variance defined by

$$C_l = \langle |a_{lm}|^2 \rangle. \tag{1.44}$$

The variance C_l can be express in terms of the temperature field $\Theta_l(k)$ in Fourier space as (Luca Amendola, 2010)

$$C_l = \frac{2}{\pi}\int_0^{\infty} dk k^2 |\Theta_l(k)|^2. \tag{1.45}$$

In the figure below we can see the predicted CMB temperature anisotropies $D^{TT} = l(l+1)C_l/2\pi$ versus the multipole moment l together with the 2015 Planck observational data (Planck Collaboration et al., 2016b).

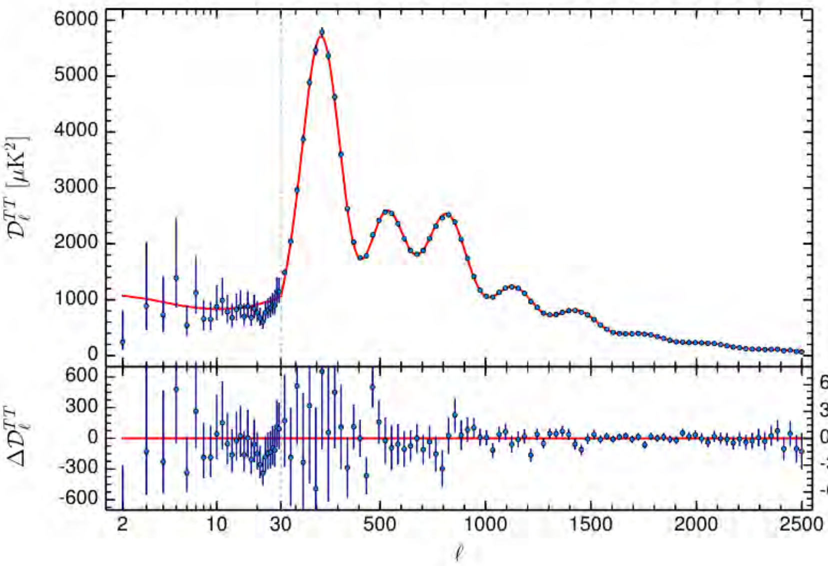


Figure 1.1: Planck 2015 temperature power spectrum. The best-fit base ΛCDM theoretical spectrum fitted to the Planck TT+lowP likelihood is plotted in the upper panel. Residuals with respect to this model are shown in the lower panel. Took from (Planck Collaboration et al., 2016b).

According to the paper the χ^2 of the coadded TT spectrum plotted in this figure relative to the best-fit base ΛCDM model is 2547 for 2479 degrees of freedom ($30 \leq 1 \leq 2500$), which is a 0.96 σ fluctuation. These numbers confirm the extremely good fit of the base ΛCDM cosmology to the Planck TT data at high multipoles.

1.6.4 Baryon Acoustic Oscillations (BAO)

One of the most important observational tools we have in Astronomy and Cosmology are the Standard Candles, that with new knowledge of the extra-galactic distance ladder, helped us, among other things, probe the cosmic expansion history. Nowadays that we can do this accurately beyond the local universe, we needed more sophisticated tools to measure, we called them *standard rulers*, and using the statistical tools we define above and other, one can define a Statistical Standard Ruler (SSR). According to (Bassett and Hlozek, 2010) a standard ruler is an object of a known size at a single redshift, z, or a population of objects at different redshifts whose size changes in a well-known way (or is actually constant) with redshift. Now a SSR comes with the idea that the clustering of galaxies may have a preferred scale in it which, when observed at different redshifts, can be used to constrain the angular diameter distance.

Baryon acoustic oscillations (BAO) are the imprint of oscillations in the baryon-photon plasma on the matter power spectrum, they are frozen relics left over from the pre-decoupling universe. The pressure waves that propagated at this time imprinted a characteristic scale in the clustering of matter and galaxies, which appears in the galaxy correlation function as a localized peak at the sound horizon scale[2] r_s, or in the power spectrum as a series of oscillations (Mortonson et al., 2014; Planck Collaboration et al., 2016c). The main advantage that BAO provides as a SSR is that it is primarily a linear physics phenomenon, which means we can ignore nonlinear effects to good approximation (Bassett and Hlozek, 2010), but they propose a problem, because this is a weak signal at a large scale, one needs to map enormous volumes of the universe to detect the BAO and obtain a precise distance measurement (Weinberg et al., 2013).

The BAO scale can be measured in the power spectrum of cosmic microwave background (CMB) and in the maps of large-scale structure at lower redshifts (Wang et al., 2017). This scale in a galaxy redshift survey determines the angular diameter distance $D_A(z)$ and the expansion rate $H(z)$. Errors

[2] $r_s = \int_0^{t^*} \frac{c_s(t)}{a(t)} dt$, where t^* is the time of recombination and c_s is the speed of sound.

on the two quantities are correlated, and its value is approximately (Mortonson et al., 2014):

$$D_V(z) = [czD_A^2(z)/H(z)]^{1/3}. \tag{1.46}$$

As we said before, the first direct evidence for cosmic acceleration came from surveys of Type Ia supernovae (SNe) (S. Pelmutter, 1998), BAO measurements complement the SN measurements by providing an absolute distance scale, direct measurement of the expansion rate $H(z)$, and robustness to systematic errors (Alam et al., 2016). BAO leads to small oscillations in the matter power spectrum $P(k)$,(Cole et al., 2005) and (Eisenstein et al., 2005) found a peak of baryon acoustic oscillations in the large-scale correlation function at $107h^{-1}$ Mpc separation measured from a spectroscopic sample of 46,748 luminous red galaxies from the Sloan Digital Sky Survey (SDSS) (Luca Amendola, 2010), in the figure below we can see one of the first accurate detections of this peak.

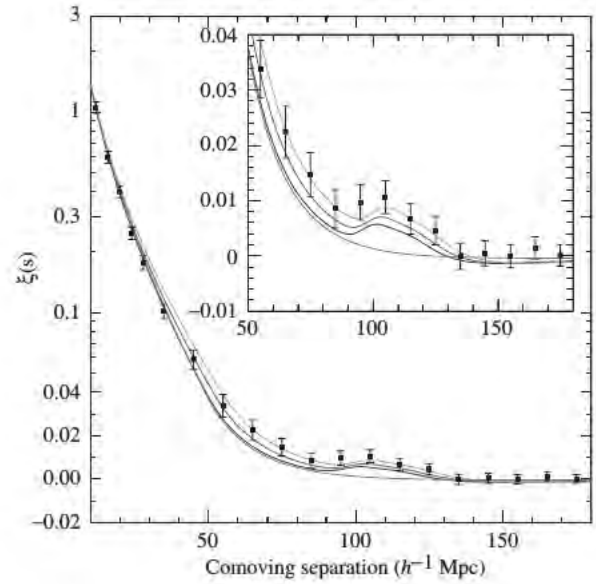


Figure 1.2: The large-scale redshift-space correlation function of the SDSS sample. The inset shows an expanded view with a linear vertical axis. From top to bottom each curve corresponds to $\Omega_m^{(0)}h^2 = 0.12, 0.13, 0.14$ with $\Omega_b^{(0)}h^2 = 0.024$ pure CDM (no baryons) model with $\Omega_m^{(0)}h^2 = 0.15$. The observational data clearly show the existence of an acoustic peak around the comoving separation scale $107h^{-1}$ Mpc, in agreement with the predictions except for the pure CDM model. Taken from (Luca Amendola, 2010)

Several experiments in the last years have improved these detections, e.g. BOSS (Alam et al., 2016). They as expected measured the angular diameter distance DM and Hubble parameter H from the baryon acoustic oscillation (BAO) method after applying reconstruction to reduce non-linear effects on the BAO feature. BOSS is part of the project that was set up to to develop dark energy experiments, that were classified in stages with higher numbered stages reflecting more sophisticated and powerful experiments. BOSS and DES (The Dark Energy Survey Collaboration, 2005), were part of the Stage-III, and they are finishing their tasks and publishing last results. In the next years, the Stage-IV experiments will start, some of them are under construction, experiments as EUCLID, DESI, WFIRST, CHIME and BAOBAB, promise to improve precision on the measured values of the parameters describing the expansion of the Universe and its time evolution by a factor of 10 Levi et al. (2013). DESI will be the first Stage-IV dark energy experiment, surveying 30 M luminous red galaxies (LRGs), emission line galaxies (ELGs) and quasars (QSOs), and one of the authors of this thesis is part of the Mexican Collaboration for DESI. DESI provides at least an order of magnitude improvement over BOSS both in the comoving volume it probes and the number of galaxies it will map. This will significantly advance our understanding of the expansion history of the Universe, providing more than thirty sub-percent accuracy distance measurements (DESI Collaboration et al., 2016).

1.6.5 Redshift-Space Distortions (RSD)

Different cosmological observations measures different geometrical quantities. As we have seen The Cosmic Microwave Background (CMB), Supernovae (SNe) and the Baryon Acoustic Oscillations (BAO) measure the distances in the background universe, hence the expansion history of the Universe H(z). Lastly we have to define and set the theory, for an important observable that help us constraint our cosmological models. Redshift-Space Distortions (RSD) of galaxy clustering, induced by peculiar motions, probe structure growth by constraining the parameter combination $f(z)\sigma_8(z)$, where $f(z)$ is the growth function, that is just the logarithmic derivate of the growth rate ($G(z)$) defined by (Mortonson et al., 2014)

$$f(z) = -\frac{d\ln G}{d\ln(1+z)}, \tag{1.47}$$

Transverse versus line-of-sight anisotropies in the redshift-space clustering of galaxies can, potentially, provide a powerful way of constraining the growth rate of structure (Planck Collaboration et al., 2016b), also, fits which constrain RSD frequently also measure the BAO scale, $D_V(z)/r_s$ at the drag epoch (Planck Collaboration et al., 2016c), and finally RSD can be used to measure the velocity of dark matter (Koyama, 2016).

In GR the behavior of $f(z)$, is given, to a good approximation by (DESI Collaboration et al., 2016)

$$f(z) \approx \Omega_m(z)^\gamma, \tag{1.48}$$

where γ is the growth index. It is expected that DESI could constrain γ to 0.04 (7 %) (DESI Collaboration et al., 2016). These measurements can improve constraints on dark energy models assuming GR to be correct, and they can be used to constrain (or reveal) departures from GR by testing consistency of the growth and expansion histories (Weinberg et al., 2013).

The linear theory for RSD only depends on the growth parameters $f(z)\sigma_8(z)$ as stated before, but testing non-GR models such as $f(R)$ gravity[3] with RSD will probably require full numerical simulations to capture the non-linear effects in these models. The major challenge in modeling RSD is taken into account non-linear effects, including non-linear or scale-dependent bias between galaxies and matter, at the level of accuracy demanded by the measurement precision (DESI Collaboration et al., 2016; Weinberg et al., 2013). Finally to put these words into equations, we can specify the relation between the real-space matter power spectrum $P(k)$ and the redshift-space galaxy power spectrum $P_g(k,\mu)$ at redshift z as (Weinberg et al., 2013; Mortonson et al., 2014):

$$P_g(k,\mu) = [b_g + \mu^2 f(z)]^2 P(k), \tag{1.49}$$

where $b_g(z)$ is the galaxy bias factor and μ is the cosine of the angle between the wavevector $\mathbf{k}$ and the line of sight. So, modeling the full redshift-space galaxy power spectrum one can extract the parameter combination $f(z)\sigma_8(z)$, and then have a review of the physics of RSD.

On large-scales the growth of structure is the dominant source of RSD. If we consider a galaxy on the near-edge of a strong over-density: this galaxy will tend to be falling in to the over-density, away from us, increasing its redshift and moving its apparent position closer to the center of the over-density. A galaxy at the far-edge of an over-density will appear closer, and we therefore see that clusters will appear "squashed" along the line-of-sight in redshift-space. By a similar argument, under-dense regions appear "stretched" along the line-of-sight. This is shown in figure below.

[3] See next chapter.

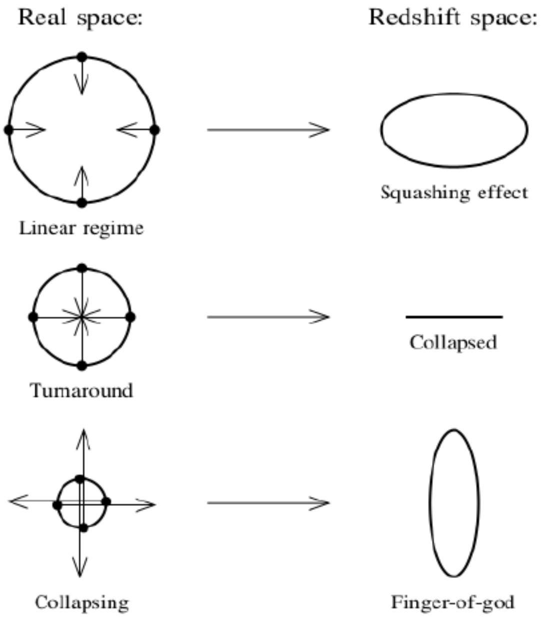


Figure 1.3: The points are galaxies experiencing a drop to a spherical overdensity, Tte arrows represent the peculiar velocities. On a large scale, the peculiar velocities of a falling layer are smaller compared to its radius and the layer appears crushed. At smaller scales (near the center), not only is the radius of a smaller layer, but also its peculiar speed is larger but it is no longer due to gravitational collapse but to the movement around its center. Source (Hamilton, 1998)

RSD analyses of the same redshift surveys conducted for BAO could provide powerful constraints on dark energy and stringent tests of GR growth predictions, but exploiting this potential will require development of theoretical modeling methods that are accurate at the sub-percent level in the moderately non-linear regime.

Chapter 2

Modified Gravity Theories

In the last chapter we have described the current Standard Model of Cosmology, know as ΛCDM. We state again that in this model, gravity is described by Einstein's General Theory of Relativity, with a FLRW metric for a statistically isotropic and homogeneous expanding (and accelerating) Universe, the matter and energy are described by a energy-momentum tensor for a perfect fluid with pressure p and density ρ, and the accelerated expansion is caused by a dark energy of a somewhat undiscovered nature plus the majority of matter in the Universe is compressed in a dark matter component. In the last sections we presented a short description of the statistical tools and observations that cosmology have used in the last decades to constrict the cosmological parameters towards a standard cosmology. Most of these tests, however, only probe length scales that are much smaller than those relevant for cosmology (Barreira, 2016). This therefore motivates research on the observational signatures that alternative gravity models can leave on cosmological observables.

Although most (or all) current observations are consistent with a cosmological constant (Hu and Sawicki, 2007; Luca Amendola, 2010; Planck Collaboration et al., 2016b), there are some theoretical and naturalness problems with it (Joyce et al., 2016), and since the mid 2000's, a plethora of alternatives have arise trying to solve these. It can be said that there are problems concerning the nature of the Dark Side of the Universe, and both Dark Matter and Dark Energy are two of the fundamental problems in modern cosmology to be solved. In this chapter and the rest of the thesis, we will focus on some alternatives to Dark Energy (DE), following the classification of (Joyce et al., 2016) that defines a boundary between DE and modifications to the general relativistic gravitational law or Modified Gravity (MG); this classification invokes the Strong Equivalence Principle[1] (SEP): We will call anything which obeys the SEP, DE, and anything which does not, MG.

We start this chapter with a short description of Modified Gravity in general, followed by a section for $f(R)$ (where R is the Ricci scalar) gravity, a class of theories can be specified by giving a Lagrangian with free parameters; then we will discuss Parametrized Modified Gravity and specifically we present the parametrized post-Friedmann (PPF) framework that describes all three regimes of modified gravity[2] models that accelerate the expansion without dark energy; finally we show a specific parametrization developed by Edmund Bertschinger and Phillip Zukin (Bertschinger and Zukin, 2008), setting the necessary basis to do the calculations in the next chapters.

2.1 Modified Gravity

The main assumption of the theories will present below is that perhaps General Relativity is not the gravitational theory that should be applied on cosmological scales, and maybe a new theory will produce better predictions about our universe which are compatible with experimental observations without the need for introducing dark energy (or something else). It is important to mention, as stated by (Bertschinger and Zukin, 2008) that "modified gravity theories must pass tests within the solar

[1]The SEP extends the universality of free fall to massive bodies, i.e. to be completely independent of a body's composition, and extends the Weak Equivalence Principle that states there exists some metric to which all matter species couple universally (Joyce et al., 2016).

[2]These regimes will be discussed soon.

system and in relativistic binaries. They are expected to show significant departures from general relativity only on cosmological distance scales". And more general it has to pass three regimes in order to accelerate the Universe without Dark Energy. In the paper of (Hu and Sawicki, 2007) they define this three regimes as[3]:

1. **First Regime**: On large scales, the evolution of scalar metric and density perturbations must be compatible with the expansion history defined by distance measures.

2. **Second Regime**: On intermediate scales in the linear regime, they form a scalar-tensor theory with a modified Poisson equation.

3. **Third Regime**: On small scales in dark matter halos such as our own galaxy, modifications must be suppressed in order to satisfy stringent local tests of general relativity.

We will present such a theory of MG that fully describe these three regimes in the sections below, but first, let's introduce a few more the formalities behind MG. In the last chapter we presented the Einstein field equations, and how they describe gravity in our SMC. These equations can be derived from the Einstein-Hilbert action (Hilbert, 1935),

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} R + S_m[g_{\mu\nu}, \phi_m], \tag{2.1}$$

where $\kappa^2 = 8\pi G$, $g \equiv \det(g_{\mu\nu})$, R is the Ricci Scalar (see equation 1.2), and S_m is the action of the matter fields ϕ_m. Now a popular way for constructing theories of modified gravity is to add extra terms or extra fields to the gravitational Lagrangian density. So in a more general manner we can write the action as

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \mathcal{L}, \tag{2.2}$$

where $\mathcal{L}$ is the gravitational Lagrangian density (we see that in GR $\mathcal{L} = R$). Following the work of (Pearson, 2014) we will classify MG theories into to broad categories: (a) only the metric is used to mediate gravity and (b) extra mediators of gravity are introduced. Below we present a table for the subcategories of our broad classification. We recommend the reader to see the cited papers of this table.

[3]For a full review of this regimes, and its equations, view (Hu and Sawicki, 2007).

Name	**Category**	**Lagrangian**	**Examples**
Tensor Theories	(a)	$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \partial_\alpha \partial_\beta g_{\mu\nu})$	• Cosmological Constant • $f(R)$ gravity • Gauss-Bonnet gravities.
Scalar-Tensor Theories	(b)	$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \partial_\alpha \partial_\beta g_{\mu\nu}, \dots, \phi, \delta_\mu \phi, \dots)$	• Brans-Dicke Theory • Kinetic gravity braiding theory • Horndeski's theory • Covariant galileon theory
Tensor-Vector Theories	(b)	$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \partial_\alpha \partial_\beta g_{\mu\nu}, \dots, A^\mu, \delta_\nu A^\mu, \dots)$	Einstein-_ther theory
Tensor-Vector-Scalar Theories	(b)	$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \partial_\alpha \partial_\beta g_{\mu\nu}, \dots, \phi, \delta_\mu \phi, \dots A^\mu, \delta_\nu A^\mu, \dots)$	TeVeS

Table 2.1: Classification of Modified Gravity models based on the types of mediator of gravity it uses and examples for each.

For more examples and Lagrangians for MG models we recommend the review of (Koyama, 2016) and the work of (Barreira, 2016). If we go to this and other articles or reviews, we'll see names as *quintessence*, *Cubic Galileon*, *K-mouflage*, *Dvali–Gabadadze–Porrati (DGP) model*, *Braneworld models*, and more. We will not explain in detail any of these models, and the focus will be on presenting a theory of MG that fully describe the three regimes described above, in a parametrized fashion. One of the ways of accommodating some models to these regimes, i.e. quintessence models, are the *screening mechanisms*, which dynamically suppress the modifications to gravity in regions like the Solar System. There is a variety of screening mechanisms out there which include the chameleon, symmetron, dilaton, Vainshtein and K-mouflage type screenings (Barreira, 2016). In all of these, the implementation of the screening effects relies on the presence of nonlinear terms in the equations of motion, which act to suppress the size of the fifth force in regions where some criterion is met, satisfying the third regime explained above. In the next sections we derive other ways a model can accommodate to these and other tests, parametrizing the modifications to gravity at the level of the equations of motion.

2.2 $f(R)$ Gravity

One of the most-studied theories of MG is the $f(R)$ gravity. This model consists of higher-curvature corrections to the Einstein–Hilbert action (Joyce et al., 2016)

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,(R + f(R)) + S_m[g_{\mu\nu}, \phi_m], \tag{2.3}$$

where $f(R)$ is a function only of the Ricci scalar. This function is chosen to become significant in the low-curvature (infrared) regime $R \to 0$ and such that the resulting equations for the metric admit self-accelerating solutions (Hojjati et al., 2012). We can design $f(R)$ functions to match any expansion history of the universe, this is due to the fourth-order nature of the $f(R)$ equations of motion; in these theories there is an additional scalar degree of freedom coupled to all matter with gravitational strength, and they can be written as a scalar-tensor gravity theory if we transform to a conformally related frame $f(R)$. This scalar field is usually called the *scalaron* (Silvestri et al., 2013), this field can constraint fifth-force interactions via the Chamaleon mechanisms (Khoury and Weltman, 2004), can mediate an attractive force and can evade solar system tests (Will, 2014; Li et al., 2011).

Although viable $f(R)$ models are forced to have an expansion history that is indistinguishable from that of the ΛCDM model with $w = -1$, there can be detectable differences between them and ΛCDM when we study the kinematics of clustering of matter, both on linear and non-linear scales (Hu and Sawicki, 2007).

Now, if we vary the action 2.3 with respect to $g_{\mu\nu}$ we find that the cosmological evolution at the background level is described by the Friedmann equation,

$$(1 + f_R)H^2 + \frac{a^2}{6}f - \frac{\ddot{a}}{a}f_R + H\dot{f}_R = \frac{a^2}{3\kappa^2}\rho, \tag{2.4}$$

and the trace equation (Hojjati et al., 2012),

$$\Box f_R = \frac{1}{3}(R + 2f - Rf_R) - \frac{\rho - 3P}{3\kappa^2}, \tag{2.5}$$

where $f_R \equiv df/dR$. Equation 2.4 can be rewritten as a second-order equation for $f(a)$, and then given an expansion history $H(a)$, and thus $R(a)$, it can be used to solve for $f(a)$ and then determine $f(R)$; in the other hand, equation 2.5 can be interpreted as an equation of motion for a scalar field[4] f_R, with an effective potential that depends on the density of matter. Two important properties of a $f(R)$ model are the f_R we defined and the inverse-mass or Compton scale associated; the square of the latter in units of the Hubble length is proportional to

$$B = \frac{f_{RR}}{1 + f_R}\dot{R}\frac{\dot{H}}{H}, \tag{2.6}$$

where $f_{RR} \equiv d^2f/dR^2$. Constraints on $f(R)$ are often presented in terms of a parameter B_0, which is related to the mass of the scalaron today (Song et al., 2007). The growth of structure and evolution of the CMB anisotropies that we discussed in the last chapter, are described expanding the $f(R)$ field equations to first order in perturbations. These results are derived in a similar fashion that the ones we found in section 1.3, so in here we'll just present them. First the anisotropy equation that is spatial off-diagonal component of the linearized Einstein equations, in GR it's

$$k^2(\Phi - \Psi) = \frac{a^2}{3\kappa^2}(\rho + P)\sigma, \tag{2.7}$$

where σ is the anisotropic stress of matter. In $f(R)$ this equation becomes

$$k^2(\Phi - \Psi) - k^2\frac{\delta f_R}{1 + f_R} = \frac{3a^2}{2\kappa^2}\frac{(\rho + P)}{\mathcal{F}}\sigma, \tag{2.8}$$

[4]This is the one we call scalaron.

where $\mathcal{F} \equiv 1 + f_R$. Now, the Poisson equation that we found in equation 1.27 for $f(R)$ becomes

$$k^2\Psi - k^2\frac{\delta f_R}{2\mathcal{F}} + \frac{3}{2}\left[(\dot{H} - H^2)\frac{\delta f_R}{\mathcal{F}} + (\dot{\Phi} + H\Psi)\frac{\dot{\mathcal{F}}}{\mathcal{F}}\right] = -\frac{a^2}{2\kappa^2}\frac{\rho}{\mathcal{F}}\Delta, \tag{2.9}$$

where we have defined $\delta f_R \equiv f_{RR}\delta R$ and again $\Delta = \delta + 3H(\omega + 1)\theta/k^2$. $f(R)$ gravity has been explored in a variety of scenarios, and the interested reader can go to (De Felice and Tsujikawa, 2010; Joyce et al., 2016; Koyama, 2016) for more of the specifics of the theory.

2.3 Parametrized Modified Gravity

As we said before, this chapter is aimed to present a theory of MG that fully describes the three regimes of MG and accelerate the expansion without dark energy. In the last section we presented one of the more general and studied types of model for MG, but now, we want to focus on more general parametrized frameworks that cover many theories at once and minimize the risk of missing potential hints of modified gravity in the data (Silvestri et al., 2013). In the last decade, several papers have presented parametrized frameworks, their implication, the growth of structure that they allow and their departures from ΛCDM; we will use some of the principal works on this subject to present their results and understand the parametrized model they proposed, and them we will use an specific parametrization to infer cosmological parameters and test its cosmological implications. We recommend the reader, that for a more complete description and derivation of this models, to study in detail the papers of (Hu and Sawicki, 2007), (Bertschinger and Zukin, 2008), (Hojjati et al., 2012), (Silvestri et al., 2013) and (Joyce et al., 2016).

In this models, the departures from ΛCDM are quantified in terms of arbitrary functions of time and, sometimes, scale; the constraining approach they use is to parametrize the modifications to gravity at the level of the equations of motion. It is important to notice that the models that we have presented and will present not only deal with modified gravity, but also, with modified growth of structures. The modification to the dynamics of scalar perturbations on linear scales can be encoded into two time- and scale-dependent functions, $\mu(a,k)$ $\gamma(a,k)$, that generalize the Poisson 1.27 and anisotropy 2.7 equations of GR to:

$$k^2\Psi = -4\pi\mu(a,k)Ga^2\rho\Delta, \tag{2.10}$$

$$\Phi = \gamma(a,k)\Psi, \tag{2.11}$$

with $\Delta = \delta + 3H(\omega + 1)\theta/k^2$. These functions are defined in such a way that we recover ΛCDM and GR with $\mu = \gamma = 1$. In the beginnings of these theories the authors tried to find a way of reducing the arbitrariness in the explicit form of the functions μ and γ, so they could be easy to analyze and constrain, and finally (Silvestri et al., 2013) showed that in local theories of gravity, in the Quasi-Static Approximation (QSA), these functions must be ratios of even polynomials in k, with the numerator of one function being equal to the denominator of the other, and also that these polynomials where of second degree in practically all viable models considered to that day.

Without the adoption of QSA μ and γ do not necessarily have a simple form in specific models of MG (Hojjati et al., 2012), that is because if you don't make the assumptions of QSA one needs to solve the differential equations in order to determine μ and γ, making them dependent on the initial conditions. So one may wonder what are the assumptions that are involved in QSA. According to (Silvestri et al., 2013) they are two:

1. The relative smallness of the time derivatives of metric perturbations compared to their space derivatives,

2. The subhorizon approximation $k/aH \gg 1$.

So if we consider QSA, then the expressions for μ and γ introduce five functions of the background $p_i(a)$ and they can be written as[5]

$$\gamma = \frac{p_1(a) + p_2(a)k^2}{1 + p_3(a)k^2}, \tag{2.12}$$

$$\mu = \frac{1 + p_3(a)k^2}{p_4(a) + p_5(a)k^2}. \tag{2.13}$$

They also showed that even though their ansatz was derived using QSA, it allows for near- and superhorizon modifications of gravity as $\gamma(a, k \to 0) = p_1(a) \neq 1$. It turns out that the QSA is correct when considering the structure formation at distances bellow the horizon scale ($k/aH \gg 1$) (Sawicki and Bellini, 2015).

2.3.1 Parametrized Post-Friedmann Framework

In their amazing review, (Will, 2014) explained and extended the theories for several tests of General Relativity, and one of the most important part of their work is their description of the parametrized post-Newtonian (PPN) formalism, pioneered by (Nordtvedt, 1968). Post-Newtonian formalism is sufficiently accurate to encompass most solar-system tests, and PPN formalism inserts parameters in front of the metric potentials whose values depend on the theory under study; the values for the parameters in Will (2014) PPN formalism are summarized in a table in the page 31 of the article in a way that they measure or indicate general properties of metric theories of gravity.

Several cosmological parametrizations approaches to gravity existed in the last decade, but none of them described the three regimes of MG that we presented above, so in (Hu and Sawicki, 2007) developed such a parametrization that that described all three regimes of MG models that accelerate the expansion without dark energy. This phenomenological parametrized approach inspired by PPN formalism, for testing the predictions of a cosmological constant and phrasing constraints in a model-independent language was called by the authors parametrized post-Friedmann (PPF) framework. As explicitly explained by the authors[6]: "This framework features compatibility in the evolution of structure with a background expansion history on large scales, a modification of the Poisson equation on intermediate scales, and a return to general relativity within collapsed dark matter halos". They also showed that with an appropriate choice of parameters this framework can also described linear perturbations in the $f(R)$ modified action and DGP braneworld gravity models (Dvali et al., 2000). It may be used in place of the more complicated 4th order and higher dimensional dynamics exhibited in these models, respectively, when studying phenomena such as the integrated Sachs-Wolfe effect in the CMB, large-scale gravitational lensing, and galaxy clustering.

2.4 Bertschinger and Zukin (BZ) parametrization

One of the most used parametrizations that follows the PPF framework we have just described, and also the Silvestri type of μ and γ, is the Bertschinger and Zukin (BZ) parametrization (Bertschinger and Zukin, 2008). It is important to clarify here that (1) BZ did their work some years before that Silvestri showed the general form for μ and γ, (2) In the BZ parametrization μ and γ tend to 1 as k goes to 0 reducing the theory to GR in that limit, (3) in the original article for their parametrization BZ does not set the numerator of μ equal to the denominator of γ and other subtleties.

The BZ parametrization was inspired by works that investigated modified gravity effects for $f(R)$ theories. Specifically by the form of μ and γ that can reproduce equations 2.10 and 2.11 if one assumes the QSA, and they have the form (Bertschinger and Zukin, 2008):

$$\mu^Q = \frac{1}{f_R}\frac{1 + (4/3)Q}{1 + Q}, \tag{2.14}$$

$$\gamma^Q = \frac{1 + (2/3)Q}{1 + (4/3)Q}, \tag{2.15}$$

[5] The derivation of this equations can be found in the II section of (Silvestri et al., 2013).
[6] (Hu and Sawicki, 2007) page 8.

with

$$Q = 3\frac{k^2}{a^2}\frac{f_{RR}}{f_R} \approx \left(\frac{\lambda_C}{\lambda}\right)^2,$$

where $\lambda_C = 2\pi/m_{f_R}$ is the Compton wavelength of the scalaron, $m^2_{f_R} \equiv \frac{1}{3}\left(\frac{1+f_R}{f_{RR}} - R\right)$ is the effective scalaron mass, f_R and f_{RR} are the ones defined in section 2.2. There are two regimes in the dynamics imposed by λ_C, (1) for scales above λ_C modifications are negligible and GR is recovered and (2) in scales smaller than λ_C the growth is enhanced by a fifth force and in addition the metric potentials are no longer equal. So using these assumptions and results we can now present the BZ parametrization that was introduced in (Bertschinger and Zukin, 2008), but in an time- and scale-dependent fashion presented by (Zhao et al., 2009)[7]

$$\mu(a,k)^{BZ} = \frac{1+\frac{4}{3}\lambda_1^2 k^2 a^s}{1+\lambda_1^2 k^2 a^s}, \tag{2.16}$$

$$\gamma(a,k)^{BZ} = \frac{1+\frac{2}{3}\lambda_1^2 k^2 a^s}{1+\frac{4}{3}\lambda_1^2 k^2 a^s}, \tag{2.17}$$

where λ_1 represents the Compton wavelength and s encodes its time dependence, that comes from (Hojjati et al., 2012)

$$a^s = \frac{6H_0^2}{B_0}\frac{f_{RR}}{a^2\mathcal{F}}. \tag{2.18}$$

In (Zhao et al., 2009) it was shown that this form presented for BZ parametrization with $s \approx 4$ is quite accurate and can be safety used for deriving constraints on f (R) for a wide range of models.

[7] They showed that this expressions coincide with the scale-dependent parametrization introduced in the original article.

Chapter 3

CosmoSIS: modular cosmological parameter estimation

In the past chapters we've developed a theory that led us to a framework for parametrizing Modified Gravity (MG) in a way that it can describe the regimes for explaining the acceleration of the Universe without Dark Energy (DE). In this chapter we will focus on explaining the rest of the statistical theory for constraining this models of gravity, and the computational tools available for doing so. We will center on a relatively new framework for constraining cosmological parameters, that actually does much more than that. The name of this framework is Cosmology Survey Inference System (CosmoSIS) and it is described in (Zuntz et al., 2015); it sets a new way of developing and using software in the cosmological community. We will expend several pages explaining the need for this software, the current problems of computational cosmology, how it is structured and how to use it.

Before going to specifics of CosmoSIS we need to set the basis of Bayesian Cosmology, so in the beginning of the chapter we will briefly explain the concepts of Bayesian Statistics and its link to Cosmology. Then in the next section we will expose some of the current problems in computational cosmology (that can be extrapolated in some way to all of computational physics), and how CosmoSIS solve them, then we will go to the specifics of CosmoSIS, its structure and how it works. In this last section we will talk about some of the *modules* we used to do our calculations that will be analyzed in the last chapter oh this thesis. Most of this chapter is inspired by the main article of Cosmosis (Zuntz et al., 2015), the problems proposed for open source software in the astronomical community by some of the Astropy developers (Muna et al., 2016), and two online webinars that the creators of ComoSIS did this year[1].

[1]https://bitbucket.org/joezuntz/cosmosis/wiki/Home

3.1 Bayesian Cosmology

Statistics is not new in Physics, nor in Astronomy or Cosmology. But since the 1980's a great number of articles in this fields have been using the methods proposed by Bayesian Statistics (BS). In the next few sections we will give some notions of Bayesian tools, biased towards Cosmology. This is not a comprehensive review of the statistical theory behind this tools or concepts, but merely a resume of the most important definitions and the theoretical framework for BS we will use to analyze and discuss our results. This Bayesian revolution has been accompanied by the era of high-performance computing, allowing astronomers and cosmologists for the first time to deploy the power of Bayesian statistics thanks to numerical implementations. Paraphrasing the introduction of (Hobson et al., 2009)[2], three distinct elements have to be taken into account to understand the Bayesian revolution: (1) the ability we have now to to extract accurate predictions from extremely sophisticated cosmological models, (2) the huge and precise observational datasets we acquired in the last decades and the need to use, analyze and extract valuable information from them to constrain those models, (3) the advanced statistical techniques that we have developed to extract the best possible constraints from those datasets.

Below we can see the increase in the articles in Astronomy and Cosmology with "Bayesian" in the title; a very interesting analysis made by (Trotta, 2017). We will explain some of the reasons for this exponential increase in the interest of Bayesian methods for Astronomy and Cosmology in the next sections.

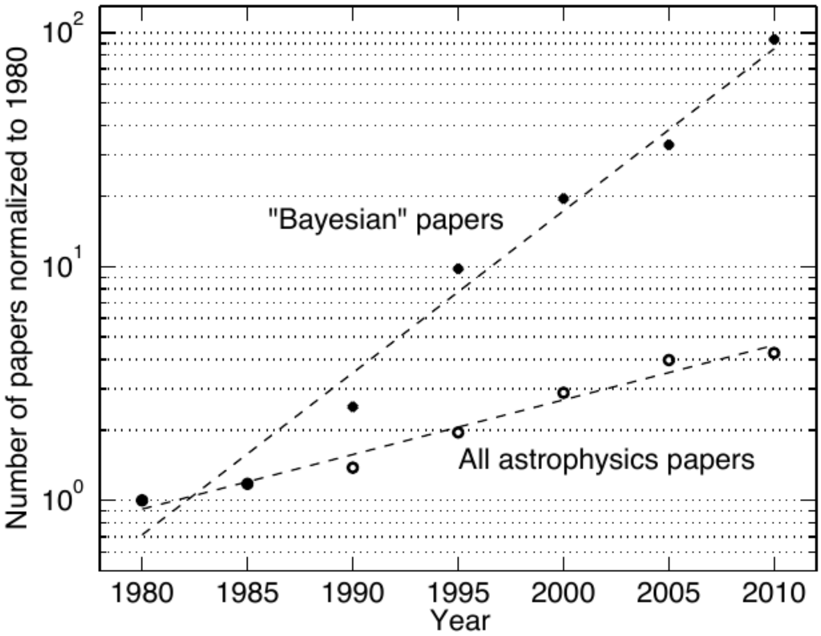


Figure 3.1: The rise of Bayesian methods in astrophysics. The number of Bayesian papers doubles every 4.3 years, while the total number of papers doubles "only" every 12.6 years. At the present rate, by 2060 all papers on the archive will be Bayesian. Source: (Trotta, 2017).

3.1.1 Bayesian Statistics

In statistical theory there are mainly two ways of thinking about probability, the classical and more common is the *frequentist* that establishes that probabilities are related to the frequency of outcomes over a long series of trials. The key in this way of thinking is repeatability of an experiment (Trotta, 2017). In the other hand, the Bayesian thinking is that probability expresses a degree of belief in a proposition, based on the available knowledge of the experimenter. Here the information is the key. The fundamental idea behind Bayesian statistics is the Bayes Theorem, first introduced by Rev. Thomas Bayes, in a posthumously published paper in 1763 (Bayes, 1763).

[2] A must read for Bayesian Cosmology.

Bayes theorem can be stated and understand in several ways, but for our purpose we will write it as (Downey, 2013; Gelman et al., 2014; Trotta, 2017):

$$P(\theta|d) = \frac{P(d|\theta)P(\theta)}{P(d)}, \tag{3.1}$$

where θ refers to the *parameters* and d to the *data*. Now we will use the "diachronic" interpretation to think about this equation, where diachronic means that something is happening over a period of time, and in this case is the probability of our hypothesis about the parameters of a model changes over time as we see new data. So in this interpretation each term of 3.1 has a name:

- $p(d)$ is the probability of the parameters before we see the data, is called the prior probability or "prior" for short. It represents our degree of belief in the value of θ before we see the data.
- $p(\theta|d)$ is what we want to compute, it represents our degree of belief about the value of θ after we have seen the data. It is called the posterior probability or "posterior".
- $p(d|\theta)$ is It is the probability of the data given a certain value of the parameters. It is usually called "Likelihood". In here the probability (density) of observing the data that have been obtained is considered as a function of the parameters θ. Two important remarks here is that (1) the likelihood is not a probability density function (pdf), and this is because (2) it is normalized over d and not θ.
- $p(d)$ is a normalizing constant, usually called "the evidence" or "the marginal likelihood", and it ensures that the posterior is normalized to unity. It is simply a marginalization:

$$P(d) = \int d\theta P(d|\theta)P(\theta).$$

An important remark is that the posterior and the likelihood are two different quantities with different interpretation, and in general $P(\theta|d) \neq P(d|\theta)$.

3.1.2 Bayesian Parameter Inference

Bayesian inference is the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model and on unobserved quantities such as predictions for new observations (Gelman et al., 2014). Bayesian inference works by updating our state of knowledge about a parameter (or hypothesis) as new data flow in. The posterior from a previous cycle of observations becomes the prior for the next (Trotta, 2017).

There are several advantages of using the Bayesian thinking, that the cosmology and astrophysics communities have been embracing since the last decades as we stated before. They are:

1. Bayesian thinking facilitates a common-sense interpretation of statistical conclusions. For example, a Bayesian (probability) interval for an unknown quantity of interest can be directly regarded as having a high probability of containing the unknown quantity, in contrast to a frequentist (confidence) interval, which may strictly be interpreted only in relation to a sequence of similar inferences that might be made in repeated practice (Gelman et al., 2014).
2. It can be shown that application of Bayes' Theorem recovers frequentist results (in the long run) for cases simple enough where such results exist, while remaining applicable to questions that cannot even be asked in a frequentist context (Trotta, 2017).
3. The probabilities are constantly updated in response to new data, and at any given instant provide a snapshot of the best current understanding (Hobson et al., 2009).
4. Bayesian inference deals effortlessly with *nuisance parameters*: those are parameters that have an influence on the data but are of no interest for our particular study. Frequentist methods offer no simple way of dealing with nuisance parameters. This is important because neglecting nuisance parameters or fixing them to their best–fit value can result in a very serious underestimation of the uncertainty on the parameters of interest Trotta (2017).

5. In many situations prior information is highly relevant and omitting it would result in seriously wrong inferences. The use of Bayes' Theorem ensures that relevant prior information is accounted for in the final inference and that physically meaningless results are weeded out from the beginning (Trotta, 2017).

6. Bayesian statistics only deals with the data that were actually observed, while frequentist methods focus on the distribution of possible data that have not been obtained. As a consequence, frequentist results can depend on what the experimenter thinks about the probability of data that have not been observed (Trotta, 2017).

In the beginning of this section we stated the cycle for Bayesian inference. But there is something that we have not discussed. If the prior is the degree of belief we have in the value of θ before we see the data, we have to start somewhere by specifying an initial prior. This is not an easy task because the prior is not determined by the theory or model, but it needs to be given by the user. The important thing here is that the prior should represent fairly the state of knowledge of the user about the quantity of interest, so we have to be very careful in the selection of this prior. The good thing here is that in most of the cases the posterior will converge to a unique (objective) result even if different scientists start from different priors.

We will not discuss here how to select a prior, and the interested reader can go to (Gelman et al., 2014) and (Trotta, 2017) for more information. Even though, there are some aspects that are obvious in the selection of our prior, for example if our parameter θ describes a quantity that has to be strictly negative, our prior will have to be 0 for values $\theta > 0$. In the literature will find that one of the standard choice for a prior is the "uniform" or "flat" prior, where the posterior becomes functionally identical to the likelihood up to a proportionality constant. Other common priors used are *reference priors*, *ignorance priors* and *conjugate priors*.

In the paper inside (Hobson et al., 2009) titled "Bayesian experimental design and model selection forecastin", Roberto Trotta proposed a recipe for optimizing the Bayesian inference process for the WFMOS survey (Bassett et al., 2005), and in (Trotta, 2017) he generalized the process and created the general Bayesian recipe to inferential problems. The process is next:

1. Choose a model containing a set of hypotheses in the form of a vector of parameters θ.

2. Specify the priors for the parameters.

3. Construct the likelihood function for the measurement, which usually reflects the way the data are obtained. Nuisance parameters related to the measurement process might be present in the likelihood. If external measurements are available for the nuisance parameters, they can be incorporated either as an informative prior on them, or else as additional likelihood terms.

4. Obtain the posterior distribution (usually, up to an overall normalisation constant) either by analytical means or, more often, by numerical methods (We will discussed some of this methods in the sections below)

Because in Bayesian statistics conclusions about a parameter θ are made in terms of probability statements, we need to understand what do we mean by probability in this framework. Probability in the "Bayesian way" is used as the fundamental measure or yardstick of uncertainty (Gelman et al., 2014). Bayesian methods enable statements to be made about the partial knowledge available (based on data) concerning some situation or "state of nature" (unobservable or as yet unobserved) in a systematic way, using probability as the yardstick.

3.1.3 Bayesian Model Selection

Following the work of (Trotta, 2017) it is convenient to divide Bayesian inference in three different levels:

1. Level 1: We assume a model $\mathcal{M}_0$ to be true, and we want to learn about its parameters θ_0. This is very common in cosmology, for example if we take ΛCDM to be the true model for the Universe, and then try to constrain its parameters. Normally this is just the usual parameter inference.

2. Level 2: We consider a series of alternative models ($\mathcal{M}_1, \mathcal{M}_2, \ldots$) and then we want to see which of these models is in best agreement with the data we have. For example wanting to see if the best description for the Universe is a Lagrangian with a simple dependence of R like ΛCDM, or if it's better to consider a $f(R)$ model. This is a problem of model selection.

3. Level 3: We don't know which of the N models that we considered in Level 2 is the "best" model. So we want to report inferences on parameters that account for this model uncertainty. For example we want to determine Ω_m independently of the assumed dark energy model. And finally this is a problem of Bayesian model averaging.

The main difference between the Frequentist approach and Bayesian approach takes is that the latter doesn't reject a model unless there are specific alternatives available: it takes therefore the form of model comparison. The key quantity for model comparison is the Bayesian evidence. That we define as the normalization integral on the right–hand–side of Bayes' theorem 3.1. It is the average of the likelihood under the prior for a specific model choice. Thus in the Bayesian framework we want to compare two models, e.g. $\mathcal{M}_a$ and $\mathcal{M}_b$, the thing we want to measure is the ratio of the posterior probabilities, given by (Barber, 2012):

$$\frac{p(\mathcal{M}_a|d)}{p(\mathcal{M}_b|d)} = B_{ab}\frac{p(\mathcal{M}_a)}{p\mathcal{M}_b}, \tag{3.2}$$

where B_{ab} is the Bayes Factor[3], and it's the ratio of the model's evidences,

$$B_{ab} = \frac{p(d|\mathcal{M}_a)}{p(d|\mathcal{M}_b)}. \tag{3.3}$$

A value $B_{ab} > 1$ represents an increase of the support in favor of model a versus model b given the observed data. Two important things here to mention is that (1) Bayesian model comparison is usually conservative when it comes to admitting a new quantity in our model, even in the case when the prior width is chosen "incorrectly", and (2) Bayesian model selection does not penalize parameters which are unconstrained by the data, so normally we have to take into consideration the Bayesian model complexity that may help us in judging model performance (Trotta, 2017). For the interested reader that want to dig in more about model selection we recommend the 4th section of (Trotta, 2017), the 12th chapter of (Barber, 2012) and the 6th chapter of (Gelman et al., 2014).

The description of Bayesian Statistics might seem obscure now, but it will be of great importance in analyzing our final results, and the structure of the CosmoSIS framework.

3.2 The need for CosmoSIS

The birth of CosmoSIS was inspired by some of the current challenges we face in Cosmology, specifically in this era of precision and computationally driven Cosmology. Some of this challenges are[4]:

1. Many, sometimes correlated, observables, e.g., CMB, lensing, galaxy clustering, supernovae, clusters, among others.

2. Different theoretical models, e.g., Supernovae light curve fitters, bias models for galaxy clustering.

3. Different parameters, systematics in each model, that leads us to the question: how to sample over each in an MCMC chain? (or some other sampling method).

4. Complicated, possibly multimodal, Posterior/ Likelihoods: sampling choice may impact results, estimate/model covariances.

5. Large collaborations (hundreds of people e.g. DES, DESI & Planck), that leads to two problems:
    - how to track contributions, ensure reproducibility & consistency?
    - how to use wealth of existing code/data without wasting PhD deciphering it all, learning new coding language, or interacting with several (dying) communities?

[3] Which does not require integration/summation over all possible models.

[4] https://bytebucket.org/joezuntz/cosmosis/wiki/oct16_webinars/CosmoSIS_Webinar_2.pdf

CosmoSIS was designed to address each of these issues. We will discuss its structure with detail in the next sections, but for now we can say, it is an open source code which community actively contributes to multi-language modules (Python, C++, C, Fortran), that allows the choice of physics and likelihood modules, it has a large collection of samplers - mostly in Python, and nice Python plotting functions.

One of the key words in the last paragraph is "Open Source"[5]. For the unfamiliar reader, open source software is software developed by and for the user community. Open source projects provide tremendous opportunities for developers to share and learn through collaboration. This software is freely accessed, used, changed, and shared (in modified or unmodified form) by anyone. It's made by many people, and distributed under licenses that comply with the Open Source Definition.

CosmoSIS as an open source project has all of this advantages. It is not the unique open source project for computational cosmology, but its documentation, code and user interactions resembles the traditional software development approaches, something very important for creating solid, testable and reliable software.

Most of what we do in CosmoSIS is in Python, a much easier language to interact than C or Fortran. The shift to Python in the Astronomical and Cosmological communities is recent, but since the beginning of the 2010's, serious packages and frameworks were developed (or translated from older languages) in pure Python. One of the most important was Astropy[6] (Astropy Collaboration et al., 2013), that emerged mostly because of the widespread dissatisfaction with IDL[7] (Muna et al., 2016). Astropy had to break two barriers for the astronomical community to adopt Python: (1) astronomers would be required to learn a new language, and (2) the substantial IDL libraries that astronomers rely on daily needed to be replaced with Python versions. Right now, Astropy did break this barriers, but with several drawbacks.

Sadly, the physical community haven't fully recognize the importance and need for open source projects, and its founding is practically null. In the Astropy Problem paper (Muna et al., 2016), some of the authors of the package, with detail, explain the problems they have faced in the developing of the framework. Summarized the problem is: Non-official funding. Employees of NASA institutions say that each dollar spent must be allocated to a specific project or mission; money cannot be spent on general purpose, community efforts. And others at academic institutions say their primary mission is education, not software development. Individual surveys received the money they did (either through government or private funding) to deliver the science products promised, not to develop community software. Even though all NASA mission *pipelines*, nearly all national laboratories, surveys, and telescopes depend on Astropy, there is still no solution to the problem[8].

Open source projects need funding, it is impossible that a complete and robust software can be built for free, because hundreds or thousands of people need to be involved in the stages of software development, some coding, some testing, some solving issues, some updating versions and creating new features, and a lot more. Because we don't want to see a "The CosmoSIS Problem" paper, we, as a community need to be prepared and come up with solutions for these problems, right now, otherwise they will exist in the near future. Soon these problems will be discussed with the CosmoSIS community for finding optimal solution to them, and we think these conversations need to happen in universities, institutions (both educational and government), and between the software creators for all the open source projects in physics.

The need for CosmoSIS was to solve the problems or challenges we have now in computational cosmology, but it's important that this new framework establishes a solid foundation for sustainable development, so we don't need to create in the near future another framework that solves the problems created by CosmoSIS.

[5]https://opensource.org/

[6]Actually, the foundations for Astropy began in 1998 at the Space Telescope Science Institute but difficulties in working with IRAF (both technical and managerial) motivated a move to Python in the early 2000's (Muna et al., 2016).

[7]IDL, short for Interactive Data Language, is a programming language used for data analysis. It was popular in particular areas of science, such as astronomy, atmospheric physics and medical imaging.

[8]These authors present a subscription based model for Astropy and creating the role of full time developers for Astropy, but none of them have been accepted by the whole community.

3.3 CosmoSIS structure

The key concept in CosmoSIS is modularity. This means, breaking large and complicated code into small bits that we call module. Each module has a specific task to complete and its functionality does not depend directly on what other modules are used in the *pipeline*. Modules in CosmoSIS don't have direct read and write access to the data each other hold, alternatively the inputs they take and the outputs they make are passed on only through a specific set of functions designed for this purpose, these mechanisms of communication is guided by the *datablock* (that we discuss later). In CosmoSIS a module in the sequence can be replaced at runtime by another calculation of the same step without affecting the others (Zuntz et al., 2015). The concept of modularity and independence made possible the creation of this flexible cosmological parameter estimation (CPE) framework, where systematic models and alternative cosmologies can be fully explored.

In (Zuntz et al., 2015) and (Bridle et al., 2015) it's stated that there are benefits and costs for the modularity of CosmoSIS. In here we present a short review of both:

Benefits:

- Replacement: It's easy to run different models without re-writing and recompiling code each time is easy in a modular architecture. In CosmoSIS all you have to do is change a configuration file or even in console, to switch between models.
- Verifiability: Because each module have their limitations and assumptions clearly specified, it's easier and faster to test these individual parts of the *pipeline*, than to try to regression-test the whole calculation.
- Debugging[9]: With the CosmoSIS architecture the inputs to a module are absolutely explicit and the connection between modules is clear. So it is easy to find errors in the inputs, calculations, and outputs, module by module without so much effort.
- Consistency: It is essential a correct treatment of shared physics and systematics that is consistent across probes in order to obtain accurate constraints on cosmological parameters. The shared CosmoSIS *datablock* assures this consistency as we will see later.
- Languages: The CosmoSIS plug-in approach to adding modules makes it easy to switch between languages for different parts of the code. Right now computational intensive calculations can be coded in compiled languages like Fortran, C or C++, and fast portions that doesn't need the velocity of compiled languages can be coded in Python. This is an essential part of CosmoSIS, it is not a secret that most of the physics code we have available are poorly coded, with outdated methodologies and without following the standards of software engineering; that's why we consider that the open source, serious and structure way that the CosmoSIS community is building this framework has to be a starting point or inspirations for future collaborations in computational physics.
- Publishing: When you have a modular framework to build your calculations, and plug them to create a *pipeline* for solving a problem, it is way easier to share code, and in a consistent way. CosmoSIS allows different communities to communicate their code and results easily making the field of computational cosmology to advance faster than before.
- Samplers: Because of modularity we can create a *pipeline* decoupled somewhat from the sampler, so switching between samplers becomes a far easier proposition. This is a really important consideration as some samplers are ineffective at fully exploring multimodal distributions or parameter degeneracies.
- Legacy: A wide body of disorganized but powerful code already exists in cosmology. With CosmoSIS is simple for the user to add and implement this existing software as modules for their calculations, and do it in a way that they don't have to create a complete system for

[9] The terms "bug" and "debugging" are popularly attributed to Admiral Grace Hopper in the 1940s. While she was working on a Mark II Computer at Harvard University, her associates discovered a moth stuck in a relay and thereby impeding operation, whereupon she remarked that they were "debugging" the system. http://foldoc.org/Grace%20Hopper

analyzing or calculating hard parts, just add your calculations to existing modules in CosmoSIS and you are good to go.

But of course we have some drawbacks from the modularity that CosmoSIS imposes. Now we present the costs that this modularity has in the design and execution of calculations. We will find that surprisingly part of the benefits we presented become costs in some cases.

Costs:

- Overheads: For complex modules with a lot of different options we can have overhead of code, or, additional (or excess) object code generated by a compiler to provide machine code which will be executed by a specific CPU. And also, this being another layer of separation between parts of the pipeline, it is also another place bugs can enter.

- Interpolation: In a modular structure data must be explicitly saved by one module to be usable by another. This can mean that data is not sampled at the points that a module is expecting, and therefore require interpolation. This can be a source of inaccuracy.

- Speed: Short-cuts and efficiencies available in a tightly-coupled code may not be available in a modular context. Even though, the connections between modules in CosmoSIS is fast enough that they do not slow down cosmological likelihoods significantly.

- Consistency: A modular approach is more vulnerable to misuse compared to a rigid monolithic code that is designed from the start with consistency in mind. This can be particularly true in complex cases such as those where errors are cosmology-dependent.

- Temptation: As it becomes easier to specify and design a pipeline the temptation to over-complicate and build large and complex pipeline grows. The authors of the framework doesn't recommend doing this because the more parameters and steps a process has the more prone to error it is, and the more difficult the associated sampling problem becomes. One of the worst part of creating weird and complicated pipelines is that it makes harder to diagnose convergence. Likelihoods has to be simplified as much as possible.

- Legacy: Most existing code is not written with modularity in mind. Much of it needs to be modified to fit into the CosmoSIS framework. This is not that hard, and we give a short explanation of how to convert an external code or calculation into a CosmoSIS module (see section 3.3.2).

Now that we have presented the pros and cons of CosmoSIS' modularity, we present its structure.

3.3.1 Pipeline

The concept of pipelining is used in most of the machine learning and Bayesian computation approaches. This is because it is far more easy to have small bits or code, like functions or methods, that do specific part of your calculations that can be later liked together, than a big monolithic application that does everything without separation. This is taken the lemma "Divide and Conquer" to an extreme that makes easy the code to be run, debug, share, and analyze.

In simple words a pipeline in computer science is to divide a calculation into portions, or modules, where each of them are independent but have a way of communicating to one another, the results of each module is passed through the pipeline through stages in a sequence. In CosmoSIS a pipeline is then the sequence of modules to be run in your analysis for each likelihood; these calculations compute a joint likelihood from a series of parameters. Below there is a graphical depiction of a pipeline, each of the "pipes" are actually the modules, and the operation of passing the data through the modules is done by the *datablock*.

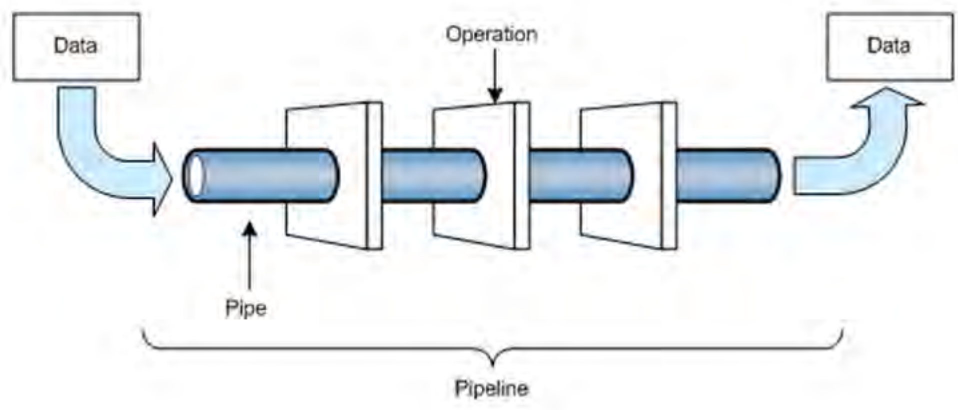


Figure 3.2: Pipeline.

3.3.2 Modules

Modules are the the individual "pipes" in the pipeline, each of which performs a separate step in the calculation. They are run as a sequence, each taking inputs from previous modules and providing new outputs for later ones. As we said before modules do not communicate directly with each other, nor do they directly communicate with the *sampler*. They communicate indirectly through the *datablock* (Bridle et al., 2015). Some of them do physics calculations, others interpolation, and at the end some generate likelihoods.

Lots of modules are included in the ComoSIS standard library[10] and the user can create new ones. There is a complete section in the CosmoSIS repository[11] that describes the steps necessary for creating the code and structure of a new module.

Before going into detail about the existing CosmoSIS modules, lets see how can an user create a new module. A module has two parts - the bit that does the actual calculation, and the bit that plugs in that calculation to CosmoSIS. For the calculation part the user can (1) convert external code into a module, (2) write a module with new code from scratch or (3) making changes to an existing module.

If you have a code that does some cosmological calculation, all you need to do is need to turn your program into one more functions, by moving code around so that the main program is a just a handful of very simple calls to those functions. You need to put these functions in separate files from the main program, and change your makefile to compile them separately. When you have these separate functions we can connect them to CosmoSIS. One of the good things about the open source of this framework is that the community is happy to help anyone to transform their code into a CosmoSIS module.

You can create ad hod calculation modules for some specific task, but, if you want to share your code and make it widely usable module you can alter your program so that cosmological inputs to the program are loaded from the *datablock* instead of having a functional form assumed. Then you need to create a connecting interface, writing the three functions described on the modules page[12] - setup, execute, and cleanup. This can be done in Python, Fortran, C or C++.

This three special functions have the next meaning:

- setup: called once at the start; reads parameters from input *param file*.
- execute: called for each sample; reads inputs from *datablock*.
- cleanup: called once at the end; frees memory/resources (optional).

[10] https://bitbucket.org/joezuntz/cosmosis-standard-library
[11] https://bitbucket.org/joezuntz/cosmosis/wiki/creating_modules
[12] https://bitbucket.org/joezuntz/cosmosis/wiki/modules

The easiest language for writing a module is Python. In here we reproduce the example given in the CosmoSIS wiki[13], that assumes there is a python code called `my_calculation.py` that does some calculation.

Let's assume is need one fixed parameter telling it what to do, called "mode", and it uses a single cosmological parameter input, `omega_m` and has one cosmological parameter output, `cluster_mass`.

```python
from cosmosis.datablock import names, option_section
import my_calculation

# We have a collection of commonly used pre-defined block section names.
# If none of the names here is relevant for your calculation you can use any
# string you want instead.
cosmo = names.cosmological_parameters

def setup(options):
    #This function is called once per processor per chain.
    #It is a chance to read any fixed options from the configuration file,
    #load any data, or do any calculations that are fixed once.

    #Use this syntax to get a single parameter from the ini file section
    #for this module.  There is no type checking here - you get whatever the user
    #put in.

    mode = options[option_section, "mode"]

    #The call above will crash if "mode" is not found in the ini file.
    #Sometimes you want a default if nothing is found:
    high_accuracy = options.get(option_section, "high_accuracy", default=False)

    #Now you have the input options you can do any useful preparation
    #you want.  Maybe load some data, or do a one-off calculation.
    loaded_data = my_calculation.prepare_something(mode)

    #Whatever you return here will be saved by the system and the function below
    #will get it back.  You could return 0 if you won't need anything.
    return loaded_data


def execute(block, config):
    #This function is called every time you have a new sample of cosmological and other parameters.
    #It is the main workhorse of the code. The block contains the parameters and results of any
    #earlier modules, and the config is what we loaded earlier.

    # Just a simple rename for clarity.
    loaded_data = config

    #This loads a value from the section "cosmological_parameters" that we read above.
    omega_m = block[cosmo, "omega_m"]

    # Do the main calculation that is the purpose of this module.
    # It is good to make this execute function as simple as possible
    cluster_mass = my_calculation.compute_something(omega_m, loaded_data)

    # Now we have got a result we save it back to the block like this.
    block[cosmo, "cluster_mass"] = cluster_mass

    #We tell CosmoSIS that everything went fine by returning zero
    return 0

def cleanup(config):
    # Usually python modules do not need to do anything here.
    # We just leave it in out of pedantic completeness.
    pass
```

Is not necessary to understand all the code right now, and for more information the reader can visit CosmoSIS repository. Still the easiest way of creating a custom calculation is see if an existing module

[13] https://bitbucket.org/joezuntz/cosmosis/wiki/modules_python

can be modified, in here you might make add modified gravity or some other piece of new physics to a calculation module. The authors of the framework recommend creating a new repository for your project, to this you can use the next commands[14]:

```
$ ./cosmosis/tools/create-repository.py NAME_OF_PROJECT
```

Now follow the instructions that appear:

1. Copy the existing module into a subdirectory your new repository:

```
$ cp -r cosmosis-standard-library/EXISTING_MODULE  modules/NAME_OF_PROJECT/
  NAME_OF_MODULE

```

2. Add the copied files to the repository:

```
$ cd modules/NAME_OF_PROJECT/NAME_OF_MODULE
$ make clean  # unless module is in python
$ git add .
$ git commit -m 'Initial commit of files for NAME_OF_PROJECT'

```

3. Make any modifications you want to the code. Then save the results:

```
#  ... lots of work here ...
$ git commit -am 'Made my changes'
$ git push

```

Now you can use your modified module by changing the path in the `ini` file to point to it (we will explain what the `ini` file is later).

In the CosmoSIS standard library are about 60 modules already programmed and explain for the user. They are divided by the calculation they make, and their code and implementation is in the CosmoSIS standard library repository https://bitbucket.org/joezuntz/cosmosis-standard-library. The list below, is presented in this way[15], updated to June, 2017:

Type of calculation

- [Name and Web page] (version) Short description of module

Background:

- [Distances] (2015) Output cosmological distance measures for dynamical dark energy.

Bias

- [Clerkin (1) Compute galaxy bias as function of k, z for 3-parameter Clerkin et al. model (Clerkin et al., 2015).
- [Constant_bias] (1) Apply a galaxy bias constant with k and z.
- [No_bias] (1) Generate galaxy $P(k)$ as though galaxies were unbiased DM tracers.

Boltzmann

- [Camb] (Jan15) Boltzmann and background integrator for BG, CMB, and matter power.
- [Camb] (Nov13) Boltzmann and background integrator for BG, CMB, and matter power (Note some settings in this module may not be consistent with other modules e.g. the Planck likelihood. Camb 2015 is recommended in this case).
- [Class] (2.4.1) Boltzmann and background integrator for BG, CMB, matter power, and more.
- [Extrapolate] (1.0) Simple log-linear extrapolation of $P(k)$ to high k.
- [Halofit] (Camb-Oct-09) Compute non-linear matter power spectrum.
- [Halofit_Takahashi] (Camb-Nov-13) Compute non-linear matter power spectrum.

[14] Make sure you have git installed https://git-scm.com/

[15] Source https://bitbucket.org/joezuntz/cosmosis/wiki/default_modules

- [Isitgr-camb] (1.1) Modified version of CAMB to implement phenomenological modified gravity models.
- [Mgcamb] (Feb14) Modified Gravity Boltzmann and background integrator for BG, CMB, and matter power.
- [Sigma_r] (1.0) Compute anisotropy dispersion $\sigma(R, z)$.

Intrinsic Alignments

- [Linear_alignments] (1.0) Compute the terms P_{II} and P_{GI} which go into intrinsic alignment calculations.
- [Ia_z_powerlap] (1.0) Takes an existing model of the IA power spectra and modifies them by giving them additional evolution in z.

Likelihood

- [2pt] (1) Generic 2-point measurement Gaussian likelihood.
- [WiggleZBao] (1401.0358v2) Compute the likelihood of the supplied expansion history against WiggleZ BAO data.
- [BBN] (PDG13) Simple prior on $Omega_b h^2$ from light element abundances.
- [BICEP2] (20140314) Compute the likelihood of the supplied CMB power spectra.
- [BOSS] (1303.4486) Compute the likelihood of supplied f_{σ_8} $(z = 0.57)$, $H(z = 0.57)$, $D_a(z = 0.57)$, $\omega_m h^2$, $b_{\sigma_8}(z = 0.57)$.
- [Extreme_Value_Statistics] (1.0) PDF of the maximum cluster mass given cosmological parameters.
- [Cluster_mass] (1.0) Likelihood of z=1.59 Cluster mass from (Santos et al., 2011).
- [Fgas] (2014) Likelihood of galaxy cluster gas-mass fractions.
- [JulloLikelihood] (2012) Likelihood of (Jullo et al., 2012) measurements of a galaxy bias sample.
- [planck] (1.0) Likelihood function of CMB from Planck.
- [planck2015] (2) Likelihood function of CMB from Planck 2015 data.
- [planck2015_simple] (2) Simplified Likelihood function of CMB from Planck 2015 TT TE EE data.
- [Riess11] (2011) Likelihood of hubble parameter H0 from Riess et al supernova sample.
- [shear_xi] (1.0) Compute the likelihood of a tomographic shear correlation function dataset.
- [planck_sz] (1.0) Prior on (σ_8, Ω_m) from Planck SZ cluster counts.
- [wmap] (4.1) Likelihood function of CMB from WMAP.
- [wmap] (5) Likelihood function of CMB from WMAP.
- [wmap_shift] (1.0) Massively simplified WMAP9 likelihood reduced to just shift parameter.

Luminosity function

- [Joachimi_Bridle_alpha] (1.0) Calculate the gradient of the galaxy luminosity function at the limiting magnitude of the survey.

Mass function

- [Press_Schechter_MF] (1) Code to compute the PressSchechter mass function given Pk from CAMB, based on Komatsu's CRL.
- [Sheth-Tormen MF] (1) Code to compute the Sheth-Tormen mass function given Pk from CAMB, based on Komatsu's CRL.
- [Tinker_MF] (1) Code to compute the Tinker et al. mass function given Pk from CAMB, based on Komatsu's CRL.

Number density

- [Gaussian_window] (1) Compute Gaussian $n(z)$ window functions for weak lensing bins.
- [load_nz] (1) Load a number density $n(z)$ for weak lensing from a file.
- [Load_nz_fits] (1) Load a number density $n(z)$ from a FITS file.
- [Photoz_bias] (1) Modify a set of loaded $n(z)$ distributions with a multiplicative or additive bias.
- [Smail] (1) Compute window functions for photometric $n(z)$.

Shear

- [Add_intrinsic] (1.0) Sum together intrinsic aligments with shear signal.
- [Apply_astrophysical_biases] (1.0) Apply various astrophysical biases to the matter power spectrum $P(k, z)$.
- [Cl_to_xi_nicaea] (1.0) Compute WL correlation functions $\xi+$, $\xi-$ from C_{ell}.
- [Shear_bias] (1) Modify a set of calculated shear C_{ell} with a multiplicative bias.
- [Wl_spectra] (1.0) Compute various weak lensing C_{ell} from $P(k, z)$ with the Limber integral.
- [Wl_spectra_ppf] (1.0) Compute weak lensing C_{ell} from $P(k, z)$ and MG $D(k, z)$ with the Limber integral.

Strong lensing

- [balmes] (1) Balmes & Corasaniti measured H_0 using strong lensing systems.
- [suyu_time_delay] (1) The likelihood of a strong lensing time-delay system as modelled in http://arxiv.org/pdf/1306.47 and http://arxiv.org/pdf/0910.2773v2.pdf.

structure

- [CRL_Eisenstein_Hu] (1) Komatsu's CRL code to compute the power spectrum using EH fitting formula.
- [FrankenEmu] (2.0) Emulate N-body simulations to compute nonlinear matter power.
- [Growth_factor] (1) Returns linear growth factor and growth rate for flat cosmology with either const w or variable DE eos $w(a) = w + (1 - a) * wa$.
- [Mead] (1.0) Uses an extended Halo model to compute non-linear and baryonic power.
- [project_2d] (1.0) Project 3D power spectra to 2D tomographic bins using the Limber approximation.

supernovae

- [jla] (3) Supernova likelihood for SDSS-II/SNLS3.

utility - [BBN-Consistency] (0705.0290) Compute consistent Helium fraction from baryon density

given BBN.
- [consistent_parameters] (1.0) Deduce missing cosmological parameters and check consistency.
- [sigma8_rescale] (1.0) Rescale structure measures to use a specified σ_8.

For a full explanation for every model go to the CosmoSIS wiki[16]. Some of these models were used for our calculations, and we will explain them with detail in the next chapter.

3.3.3 DataBlock

Before ComoSIS brute force approach was to use files and bash scripts to manage input/output, which is very prone to error/bugs. In this chapter we have used the word *DataBlock* a lot to explain some of the functionality of CosmoSIS. But what is this datablock? and why was it created?

The creators of CosmoSIS faced a problem in the beginning of the architecture solutions planned for the framework: How can different packages, written in a variety of languages, communicate with each other? So they created the datablock, a cross-language key-value store, an object that can be passed down the pipeline. For a given set of parameters all module inputs are read from the datablock and all module outputs are written to it.

Storing all the cosmology information in one places makes it easier to serialize blocks. It also makes debugging easier because all the inputs that a given module receive are explicitly clear. The datablock is a CosmoSIS object that stores scalar, vector, and n-dimensional integer, double, or complex data, as well as strings (Zuntz et al., 2015).

[16] https://bitbucket.org/joezuntz/cosmosis/wiki/default_modules

Figure 3.3: Pictorical representation of a DataBlock. Source: https://bytebucket.org/joezuntz/cosmosis/wiki/oct16_webinars/CosmoSIS_Webinar_2.pdf

Datablocks are organized into sections, named categories of information. A number of common sections are predefined in CosmoSIS, but they are simple strings and new ones can be arbitrarily created[17]. A datablock may be thought of as a dictionary mapping from a pair of strings (a section name and a specific name for data in that section) to a generic value (Zuntz et al., 2015).

Native (Application Programming Interface) APIs that act on datablocks exist for C, Fortran, C++, and Python to read or write the data stored in the block. The interfaces to modules call this API, as do the samplers when they create the block in the first place. In all the languages available in CosmoSIS the API defines a way for a module to save and load data from a block designed to collect together all the theoretical predictions about a cosmology. For example in Python, the values for a specific section name (predefined) can be saved with

```
block["section_name", "value_name"] = value
```

All the python functions are methods on a DataBlock object. For more information about the API for the other languages check the Appendix E of (Zuntz et al., 2015).

3.3.4 Sampler

A sampler is anything that produces one or more sets of input parameters, in any way, runs the pipeline on them, and then does something with the results (Zuntz et al., 2015). They are in charge for the exploration of the parameter space according to some specified algorithm (e.g. Metropolis-Hastings). The user specifies the configuration of the sampler defines the parameter space to be explored: the number and names of parameters, as well as the ranges of variation of the parameters. The sampler is influenced by the likelihood modules through the likelihood values they return. The likelihood modules are influenced by the sampler through the values of the parameters being varied in the problem, communicated through the datablock (Bridle et al., 2015).

Samplers are connected to CosmoSIS by sub-classing from a base class which provides access to the pipeline and to configuration file input, and to output files. Subclasses implement methods to

[17] In this file can be found the code names for sections. Other names can be used; these are just the predefined ones to help avoid typo errors

configure, execute and test for convergence to see if the process should stop (Zuntz et al., 2015).

Default CosmoSIS installation comes with lots of samplers that are suitable for different likelihoods spaces. Bellow we present all of them (existing by June 2017), in the following way:

Type of sampler:
- [Name and Web page] Short description of module. "Large description (if existing) taken from (Zuntz et al., 2015)"

Simple:

- [test sampler] Evaluate a single parameter set. "Evaluates the CosmoSIS pipeline at a single point in parameter space and is useful for ensuring that the pipeline has been properly configured. The test sampler is particularly useful for generating predictions for theoretical models, outside the context of parameter estimation."
- [list sampler] Re-run existing chain samples.

Classic:

- [metropolis sampler] Classic Metropolis-Hastings sampling. "Implements a straightforward Metropolis–Hastings algorithm with a proposal similar to the one in cosmomc, using a multivariate Gaussian. Multiple chains can be run with MPI."
- [importance sampler] Importance sampling.
- [fisher sampler] Fisher Matrices.

Max-Like:

- [maxlike sampler] Find the maximum likelihood using various methods in scipy. "wrapper around the SciPy minimize optimization routine, which is by default an imple mentation of the Nelder–Mead downhill simplex algorithm".
- [gridmax sampler] Naive grid maximum-posterior.
- [minuit sampler] MPI-aware maxlike sampler from the ROOT package.

Ensemble:

- [emcee sampler] Ensemble walker sampling. "Emcee (Foreman-Mackey et al., 2013) sampler is a python implementation of an affine invariant MCMC ensemble sampler (Goodman and Weare, 2010). The emcee sampler simultaneously evolves an ensemble of walkers where the proposal distribution of one walker is updated based on the position of all other walkers in a complementary ensemble. The number of walkers specified in the CosmoSIS `ini` file must be even to allow a parallel stretch move where the ensemble is split into two sets. The output will be (walkers × samples) number of steps for each parameter".
- [kombine sampler] Clustered KDE.
- [multinest sampler] Nested sampling. Proposed by (Feroz et al., 2009). "Is a multi-modal nested sampler that integrates the likelihood throughout the prior range of the space using a collection of live points and a sophisticated proposal to sample in an ellipsoid containing them. It produces the Bayesian evidence in addition to samples from the posterior".
- [pmc sampler] Adaptive Importance Sampling

Grid:

- [grid sampler] Regular posterior grid. "Is used to sample the CosmoSIS parameters in a regularly spaced set of points, or grid. This is an efficient way to explore the likelihood functions and gather basic statistics, particularly when only a few parameters are varied. When the number of parameters is large, the number of sampled points in each dimension must necessarily be kept small. This can be mitigated somewhat if the grid is restricted to parameter ranges of interest".
- [snake sampler] Intelligent Grid exploration

3.3.5 Runtime

The runtime is the code layer that connects the above components together, coordinates execution, and provides an output system that saves relevant results and configuration (Zuntz et al., 2015). It is written in Python and can be found here. The sampler and modules are both invoked by the runtime, which is also responsible for writing the relevant output files (Bridle et al., 2015). It also makes available, at the command line, the correct version of the C, C++ and Fortran compilers, of the Python interpreter, and of widely-used packages like LAPACK, NumPy, matplotlib. The sampler or samplers to be used in the pipeline are defined in the [runtime] section of the `ini` file.

3.3.6 User Interface (UI)

The CosmoSIS User Interface (UI) is the space where interactions between humans and the CosmoSIS framework occur. The primary UI is configuration files in the "ini" format, extended slightly beyond the standard to allow the inclusion of other files. The `ini` file is converted into a DataBlock object to initialize modules.

To run CosmoSIS the user needs to (Zuntz et al., 2015):

1. Choose a sequence of modules to form the pipeline.
2. Create a parameter file describing that pipeline.
3. Create a values file describing the numerical inputs for the parameters or their sampling ranges.
4. Check that your pipeline can run using CosmoSIS with the test sampler.
5. Choose and configure a sampler, such as grid, maxlike, multinest, or emcee.
6. Run CosmoSIS with that sampler
7. Run the postprocess command on the output to generate constraint plots and statistics.

The main file parameter file, e.g. `name_of_your_project.ini`. In here the user defines the sampler, selection of modules to be executed and the order of the modules. Bellow we reproduce the `ini` file for the Demo 5[18].

[18]This demo runs a full MCMC analysis on data from the SDSS-II/SNLS3 Joint Light-curve Analysis (JLA). Can be found at https://bitbucket.org/joezuntz/cosmosis/wiki/Demo5

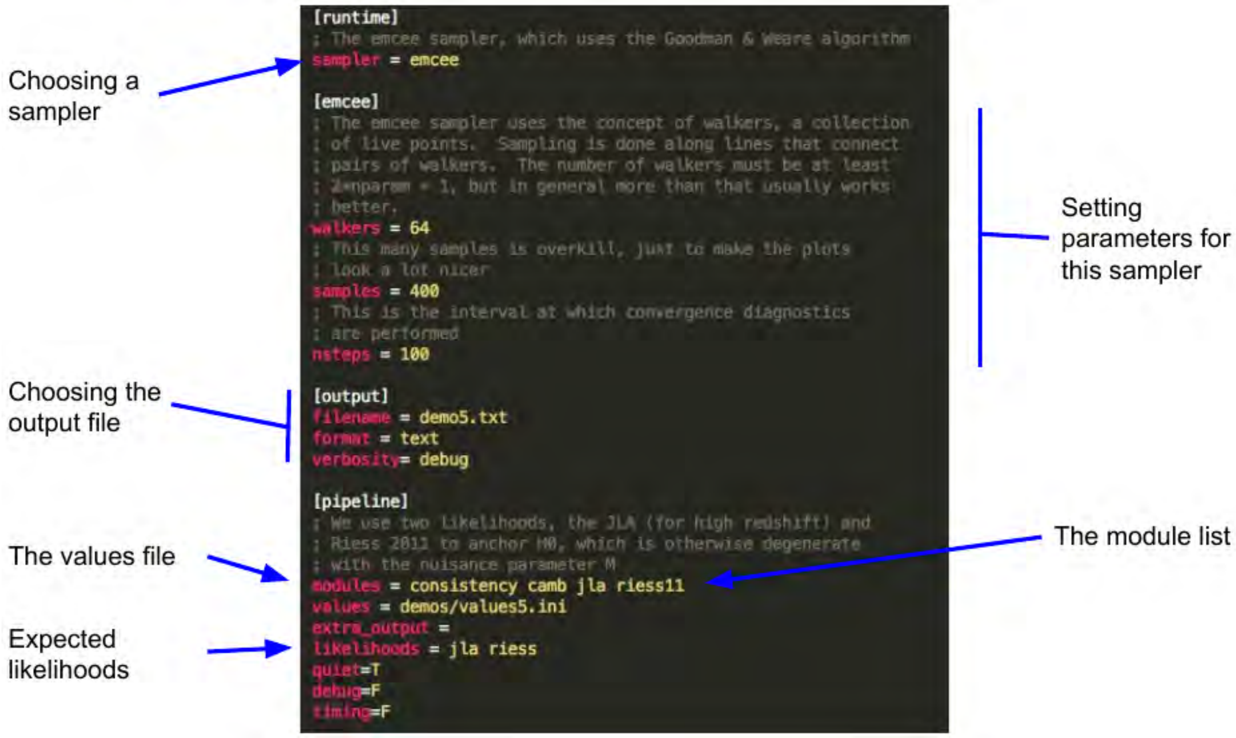


Figure 3.4: Part 1 of `ini` file from Demo 5.
Source: https://bytebucket.org/joezuntz/cosmosis/wiki/oct16_webinars/CosmoSIS_Webinar_2.pdf

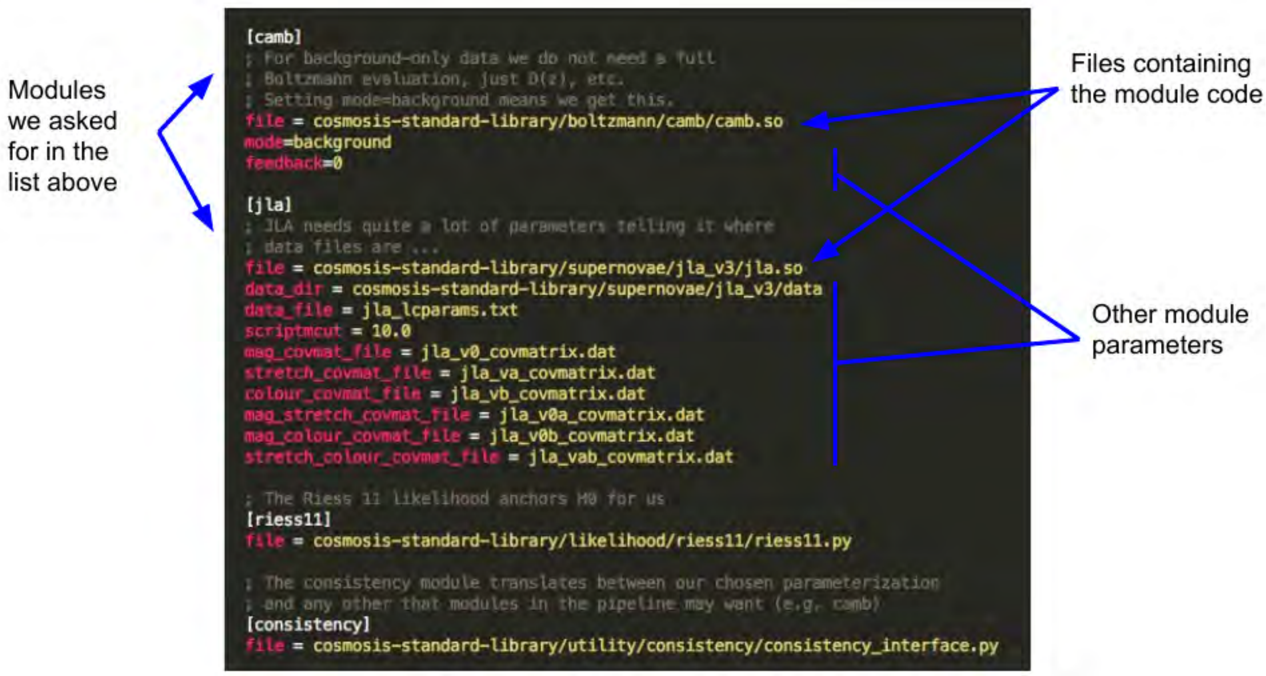


Figure 3.5: Part 2`ini` file from Demo 5.
Source: https://bytebucket.org/joezuntz/cosmosis/wiki/oct16_webinars/CosmoSIS_Webinar_2.pdf

Then the user has to specify the values describing the numerical inputs for the parameters or their sampling ranges. This file is also a `ini` file. Below we reproduce the values file for the Demo 5

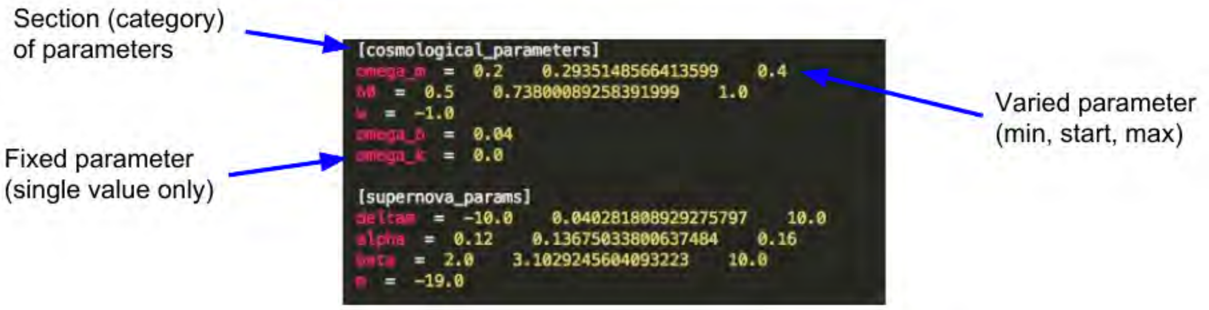


Figure 3.6: Values file from Demo 5.
Source: https://bytebucket.org/joezuntz/cosmosis/wiki/oct16_webinars/CosmoSIS_Webinar_2.pdf

3.3.7 Processing outputs

One of CosmoSIS best functionalities is the `postprocess` command. This program generates a collection of plots and statistics automatically for all the samplers that CosmoSIS supports. You can also make your own plots, or use the CosmoSIS `postprocess` command to generate plots from other codes. You call it on the same `ini` file that was used to generate the chain in the first place, so that any type of chain (grid, mcmc, or any others that we add) are analyzed with the same executable.

In the Postprocessing web page, there is a full description of the different things that the `postprocess` command can do, and all the (amazing) plots in the Demos[19] are created using this command.

[19] https://bitbucket.org/joezuntz/cosmosis/wiki/Home#markdown-header-try-some-short-demos

Chapter 4

Modified Gravity with CosmoSIS

This is our result and analysis chapter. We have presented the Standard Model of Cosmology (SMC), ΛCDM, its main equations, pros and cons, also some of the main statistical tools we have nowadays to constrain this and other cosmological models. Then we described Modified Gravity (MG), as a solution to some of ΛCDM open problems, and went into details for a parametrized model for MG, that fully describes the three regimes for MG, specifying its main equations and assumptions. Finally we presented CosmoSIS as a framework for computational cosmology, specifically for constraining cosmological parameters. In this discussion we also set the basis of Bayesian Statistics, its importance for modern cosmology, and then how and why CosmoSIS was created, its structure and interface.

Now we show our results for using CosmoSIS as an parameter inference framework for MG. CosmoSIS currently includes two of the most used codes for MG, they have been implemented in versions of the Boltzmann code CAMB. The first one is ISiTGR (Dossett et al., 2011): Integrated Software in Testing General Relativity, and MGCAMB version 2 (Hojjati et al., 2011). These two codes use a phenomenological parameterization of modifications to the perturbed Einstein equations on large scales. For our analysis we used the MGCAMB implementation existing in CosmoSIS, mostly because it already had the Bertschinger and Zukin (BZ) coded into it, we remind that this is a model that complies with the main physical properties of MG, with only two parameters, and it is easy to understand and modify.

4.1 MGCAMB in CosmoSIS

MGCAMB is a modified version of CAMB in which the linearized Einstein equations of General Relativity (GR) are modified. Specifically this patch to CAMB made possible to evaluate cosmological observables using a parametrized modification of linear Einstein equations. The motivation for this parametrization in terms of functions $\mu(a, k)$ and $\gamma(a, k)$, its consistency, and the relation to other models in the literature was fully discussed in (Pogosian et al., 2010), summarizing, their choice was guided by the fact that on super-horizon scales μ naturally becomes irrelevant and we are left with only one function, γ, as expected on those scales, they also showed that the single system of equations that results out of this parametrization can be used consistently to evolve perturbations across all linear scales, and that they make easy the derivation of other parameters which may be more suitable for interpreting observational constraints.

MGCAMB has various versions, the most recent one (as of June, 2017) is from February, 2016. Even though, the version implemented in CosmoSIS is from February, 2014. It would be a complete thesis to update the version of the code in CosmoSIS, so we used this older version. The important thing here is that, for the parametrization we used these versions are exactly the same, so it was not of much importance to use this version of MGCAMB. MGCAMB implements several parametrizations, that are fully described in (Zhao et al., 2009; Hojjati et al., 2011) and in its web page[1]. In the code they are referenced as:

- Model 0: default GR.
- Model 1: BZ (μ, γ) [introduced in (Bertschinger and Zukin, 2008)].
- Model 2: (Q,R) [introduced in (Bean and Tangmatitham, 2010)].
- Model 3 : (Q0,R0,s) [introduced in (Bean and Tangmatitham, 2010)].
- Model 4 : f(R) [introduced in (Giannantonio et al., 2009)].
- Model 5 : Chameleon [introduced in (Giannantonio et al., 2009)].
- Model 6 : Linder's gamma [introduced in (Linder, 2005)].

[1] http://aliojjati.github.io/MGCAMB/mgcambexamples.html

4.1.1 Parameters

These parameters can be set in the module's section in the `ini` parameter file. If no default is specified then the parameter is required.

Parameter	Description
mode	String, choose from Background, thermal, cmb, or all. In background mode only the expansion history is calculated. In thermal mode the recombination history is computed and rs_zdrag and related quantities also. In cmb mode the CMB power spectra are also calculated. In all mode the matter power spectrum at low redshift and sigma8 are also calculated.
mg_model	Integer, from 0-6, choice of MG model to use
lmax	Integer, only if mode!=background, default 1200 - the max ell to use for cmb calculation
feedback	Integer, amount of output to print. 0 for no feedback. 1 for basic, 2 for extended.
use_tabulated_w	Logical, set to true to load $w(z)$ from previous module (default F)
k_eta_max_scalar	Integer, maximum value of (k_η) to evolve for scalars. (default 2*lmax)
do_tensors	Include tensor modes (default F)
zmin	Min value to save $P(k,z)$ (default 0)
zmax	Max value to save $P(k,z)$ (default 4)
nz	Number of z values to save $P(k,z)$ (default 401, so that $d_z = 0.01$)
do_nonlinear	Apply non-linear halofit corrections to matter-power. Relevant only for lensing right now (default F)
do_lensing	Include lensing of CMB, and save C_ell $\phi - \phi$ (default F)
high_ell_template	Required for lensing - set to the file included in the camb dir (no default)

Table 4.1: Parameters for MGCAMB

4.1.2 Inputs

These parameters and data are inputs to the module, either supplied as parameters by the sampler or computed by some previous module. They are loaded from the data block.

Section	Parameter	Description
modified_gravity	grtrans	real, scale factor of transition from GR
	b1	real, model 1, β_1 in $\mu(a,k)$
	b2	real, model 1, β_2 in $\gamma(a,k)$
	lambda1_2	real, model 1, λ_1^2 in $\mu(a,k)$
	lambda2_2	real, model 1, λ_2^2 in $\gamma(a,k)$
	ss	real, model 1, scale factor power index in μ and γ
	MGQfix	real, model 2, Constant Q value
	MGRfix	real, model 2, Constant R value
	Qnot	real, model 3, Q_0 term in $Q(k,a)$
	Rnot	real, model 3, R_0 term in $R(k,a)$
	sss	real, model 3, scale factor power index for Q and R
	b0	real, models 4 & 5, B_0 term that goes into λ_1^2 in $\mu(a,k)$
	beta1	real, model 5, β_1 term that goes into λ_2^2 term in $\mu(a,k)$
	s	real, model 5 scale factor power index for mu
	linder_gamma	real, model 6, γ_L power law in Ω_M for growth rate
cosmological_parameters	omega_b	real, baryon density fraction today
	omega_c	real, cdm density fraction today
	omega_k	real, curvature density fraction today (default 0.0)
	omega_lambda	real, dark energy density fraction today
	hubble	real, hubble parameter H_0 (km/s/Mpc)
	tau	real, optical depth to last-scattering (ignored in background mode)
	n_s	real, scalar spectral index (ignored in background/thermal mode)
	A_s	real, scalar spectrum primordial amplitude (ignored in background/thermal mode)
	k_s	real, Power spectrum pivot scale (default 0.05/Mpc)
	r_t	real, tensor to scalar ratio (default 0.0)
	n_run	real, running of scalar spectrum d_{n_s}/d_{log_k} (default 0.0)
	n_t	real, tensor spectral index (default 0.0)
	omega_nu	real, neutrino density fraction today (default 0.0)
	massless_nu	real, effective number of massless neutrinos (default 3.046)
	massive_nu	integer, number of massive neutrinos (default 0)
	sterile_neutrino	integer, number of sterile neutrinos (default 0)
	delta_neff	real, contribution to N_{eff} by sterile neutrino (default 0)
	sterile_mass _fraction	real, fraction of omega_nu in sterile neutrino
	yhe	real, helium fraction (default 0.24)
	w	real, w(z=0) equation of state of dark energy (default -1.0)
	wa	real, equation of state parameter $w(z) = w_0 + w_a z/(1+z)$ (default 0.0)
	cs2_de	real, dark energy sound speed/c (default 1.0)

Table 4.2: Inputs for MGCAMB

4.1.3 Outputs

These parameters and data are computed as outputs from the module.

Section	Parameter	Description
post_friedmann_parameters	z	1D real array, redshifts of samples
	k_h	1D real array, k wavenumbers of samples in Mpc/h.
	D	2D real array, $D(k,z)$ modification to first perturbed Einstein equation
	Q	2D real array, $Q(k,z)$ modification to first perturbed Einstein equation
cosmological_parameters	sigma_8	real, amplitude of linear matter power at 8/h Mpc at $z = 0$. Only calculated if mode=all
distances	nz	integer, number of z samples
	z	1D real array, redshifts of samples
	d_a	1D real array, angular diameter distance in Mpc
	d_m	1D real array, co-moving distance in Mpc
	d_l	1D real array, luminosity distance in Mpc
	mu	1D real array, distance modulus
	h	1D real array, hubble parameter with in units of Mpc
	rho	1D real array, matter density, in kg/m^3 Only if mode=all
	age	real, age of universe in GYr
	zdrag	real, redshift where baryons no longer dragged by photons. Only if mode!=background
	rs_zdrag	real, sound horizon size at zdrag. Only if mode!=background
	zstar	real, redshift of unity optical depth. Only if mode!=background
	theta	real, angular size of sound horizon at zstar. Only if mode!=background
	chistar	real, comoving distance to zstar. Only if mode!=background
	z	1D real array, redshifts of samples. Only if mode=all
matter_power_lin	k_h	1D real array, k wavenumbers of samples in Mpc/h. Only if mode=all
	p_k	2D real array, matter power spectrum at samples in (Mpc/h)$^{-3}$. Only if mode=all
linear_cdm_transfer	z	1D real array, redshifts of samples. Only if mode=all
	k_h	1D real array, k wavenumbers of samples in Mpc/h. Only if mode=all
	delta_cdm	2D real array, Linear CDM transfer function at samples. Only if mode=all
cmb_cl	ell	1D integer array, angular frequencies. Only if mode=cmb or all
	tt	1D real array, ell * (ell+1) $C_{ell}^{TT}/2\pi$ in mu K^2. Only if mode=cmb or all
	ee	1D real array, ell * (ell+1) $C_{ell}^{EE}/2\pi$ in mu K^2. Only if mode=cmb or all
	bb	1D real array, ell * (ell+1) $C_{ell}^{BB}/2\pi$ in mu K^2. Only if mode=cmb or all
	te	1D real array, ell * (ell+1) $C_{ell}^{TE}/2\pi$ in mu K^2. Only if mode=cmb or all
	PhiPhi	1D real array, Lensing spectrum; note ell scaling: ell * (ell+1) $C_{ell}^{\Phi\Phi}$. Only if mode=cmb or all

Table 4.3: Outputs for MGCAMB

We didn't set all of these parameters, inputs or outputs, and in the next section we will point out which ones we used for our calculations. The MGCAMB implementation in CosmoSIS is (here).

In CosmoSIS the MGCAMB module has all the code of the second version of MGCAMB, and four more files:

- `Makefile`: A special format file that together with the `make` utility will automatically build and manage the compilation of the MGCAMB module.
- `camb_module.F90`: It has the implementations of the setup, execute, and cleanup functions. It is written in Fortran, and it connects MGCAMB code with CosmoSIS.
- `camb_interface`.F90: It has the initial setups for the parameters of basic CAMB and MGCAMB, the implementation of the datablock for MGCAMB, and all of calculations for the different models coded.
- `module.yaml`: YAML[2] is a human friendly data serialization standard for all programming languages. It contains all of MGCAMB documentation in CosmoSIS.

4.1.4 Limitations of MGCAMB and its implementation in CosmoSIS

The first obvious limitation of MGCAMB in CosmoSIS is that its version is from Feb. 2014, and the more recent version of the code is from Feb. 2016. Although this version is a little outdated, for our purposes is the same because it doesn't affect, changes or improves the models we used for our calculations.

On the other hand, the core implementation of MGCAMB has some disadvantages when compare to its father CAMB. According to the respond of an issue in the CosmoSIS repository[3], some of the modules that can be included in the pipeline for computations are not compatible with some models that MGCAMB implements. For example the `halofit` and `growthfunction` modules that we presented above are not compatible, the first is specific to ΛCDM or wCDM cosmologies, and its implementation for MG (MGHalofit), written by Gong Bo Zhao (Zhao, 2014) which is publicly available and can be used for the models in MGCAMB is not implemented in CosmoSIS; the second one can be only used for ΛCDM or wCDM cosmologies where the growth is scale independent (neglecting any clustering of the scalar field in the case of a quintessence type model), and in the BZ parametrization the growth rate is scale dependent so the `growthfunction` module is not compatible with a pipeline that uses that model for MG. We leave as future work for someone to create these modules for CosmoSIS in a way that they are compatible with most MG models.

In the official documentation of the MGCAMB module in CosmoSIS this is not stated or fully explained, but we hope that in the near future, this and other subtleties of the module can be further explored and discussed by the creator of the code.

4.2 Results and Analysis

In the last few sections we have described the MGCAMB module and its implementation in CosmoSIS, we also discussed some of its limitations. But the great advantage of using MGCAMB in this framework, is that is really easy to try different configurations, explore different modules, different samplers, and then use the great plotting and analysis functions that the framework provides to study our results. We used and modified some of the existing examples in CosmoSIS to test the BZ parametrization as a model in MGCAMB, and this section provides an explanation of the different calculations we did, and its analysis. The BZ parametrization in MGCAMB is the first model (aside from ΛCDM) and it can be set with the `mg_model` $= 1$ in the `ini` file.

[2]Originally YAML was said to mean Yet Another Markup Language, referencing its purpose as a markup language with the yet another construct, but it was then repurposed as YAML Ain't Markup Language, a recursive acronym, to distinguish its purpose as data-oriented, rather than document markup http://yaml.org/.

[3]https://bitbucket.org/joezuntz/cosmosis/issues/205/rsd-in-mgcamb

4.2.1 Basic `ini` files for MGCAMB

The main `ini` file contains the modules that we want to run, the sampler, and the order of the pipeline. To set MGCAMB as a module in the pipeline we just need to add it in the `[pipeline]` section[4]:

```
[pipeline]
mgcamb ; Add more modules here

```

Then we need to define the `[mgcamb]` section so we can specify all the different parameters for MGCAMB and its path:

```
[mgcamb]
file = cosmosis-standard-library/boltzmann/mgcamb/camb.so
mode=cmb
lmax=2650
feedback=0
mg_model=1
do_tensors=F
do_lensing=true
do_nonlinear=F
; And more parameters...
```

And remember to include the consistency parameter if you are going to use different modules that can have different definitions for its cosmological parameters. Add:

```
; The consistency module translates between our chosen parameterization
; and any other that modules in the pipeline may want (e.g. mgcamb)
[consistency]
file = cosmosis-standard-library/utility/consistency/consistency_interface.py
```

4.2.2 Finding the best fit cosmological parameters for a Planck likelihood

This section is based on the Demo 4 in the CosmoSIS webpage[5]. In this example we will find the maximum likelihood cosmological parameters for Planck 2015 (Planck Collaboration et al., 2016b). We kept the Planck nuisance parameters fixed, so we have six varying cosmological parameters, and first fixed the modified gravity parameters for the BZ parametrization so the code will not behave in unpredictable ways.

This module was run first because the samplers we used are sensitive to the starting point, so the closer this point is to the point of maximum likelihood, the faster it is for the samples to converge. This first calculations used `maxlike`, a sampler which aims at finding the points of maximum likelihood of the parameters, then used the outputs from `maxlike` as priors for the `metropolis`, `emcee`, `multinest` samplers or other we wanted to use.

As a usual and recommended step we first run the pipeline with the `test` sampler and everything ran without any issue. The priors used for the `maxlike` sampler come from (Planck Collaboration et al., 2016b), and they are[6]

[4] The comments in the `ini` files start with ;

[5] https://bitbucket.org/joezuntz/cosmosis/wiki/Demo4

[6] This data is obtained from the third column of the Table 4 in (Planck Collaboration et al., 2016b). According to the authors when using ΛCDM as a background cosmology, a conservative choice would be to use the parameter values listed in this column. This column refers to the $TT + lowP$+lensing power spectra. Where the TT refers to the full Planck temperature-only C_l^{TT} likelihood and $lowP$ denotes the polarization part of the likelihood including both temperature and polarization for multipoles $l \leq 29$; for more see (Planck Collaboration et al., 2016d). Over the multipole range $l = 2 - 29$, the power spectrum is derived from the "Commander" component-separation algorithm applied to the combination of Planck 2015 temperature data between 30 and 857 GHz, the 9-year WMAP sky maps, and the 408-MHz (Haslam et al., 1982) survey, including 93% of the sky; for more see (Planck Collaboration et al., 2016a). For multipoles equal or greater than $l = 30$, instead, the spectrum is derived from the "Plik" likelihood (Planck Collaboration et al., 2016d) by optimally combining the spectra in the frequency range 100-217 GHz, and correcting them for unresolved foregrounds using the best-fit foreground solution from a Planck $TT + lowP$ ΛCDM run.

Cosmological Parameter	Planck Value	Sampling Range
$\Omega_b h^2$	0.2227	[0.015, 0.030]
$\Omega_c h^2$	0.1184	[0.100, 0.120]
h_0	0.67	[0.600, 0.800]
n_s	0.9681	[0.940, 1.000]
A_s	2.143e-9	[2.0e-9, 2.3e-9]
τ	0.067	[0.060, 0,070]

Table 4.4: Priors for cosmological parameters used by the `maxlike` sampler

The `maxlike` sampler will take a little while to run, since we have to do several hundred different cosmologies. Once done it will create two files, `mgcamb_maxlike.txt` that contains the log for the run, and `mgcamb_maxlike_output.ini` that contains the best fit for the cosmological parameters we asked for. We used a maximum iteration number of 1000 and a tolerance of 0.01.

On the other hand, the modified gravity parameters we used as priors as we said were fixed. We used the values proposed by (Bertschinger and Zukin, 2008) and (Zhao et al., 2009). Specifically in (Zhao et al., 2009) they derive this next values for the MG parameters in the BZ parametrization, proposing an action and then deriving its corresponding Poisson equation, next they find a similar, but more complicated, parametrization for μ and γ and when they fix some of its constants they found that for a BZ model, as (Bertschinger and Zukin, 2008) found before, a good option for β_1 is 4/3 and for β_2 is 1/2, also that s parameter should run between 1 and 4, and finally that for cosmological tests λ_1^2 is $\lesssim 10^6$ Mpc^2 and that because $\lambda_2^2 = \beta_1 \lambda_1^2 \rightarrow \lambda_2^2 \approx 10^3\text{Mpc}^2$.

The best fit values we found for our cosmological parameters using the `maxlike` sampler, and the next MG parameters as priors

MG parameter	Value
β_1	4/3
β_2	1/2
λ_1^2	10^6 Mpc^2
λ_2^2	10^3 Mpc^2
s	4.0

Table 4.5: Priors for MG parameters.

were

Cosmological Parameter	Best Fit (`maxlike`)
$\Omega_b h^2$	0.02292
$\Omega_c h^2$	0.1142
h_0	0.7063
n_s	0.9794
A_s	2.1195e-9
τ	0.0651

Table 4.6: Best fit for cosmological parameters obtained by the `maxlike` sampler.

Although some of the values are not fully consistent with the Planck values, we continue our analysis to test the other samplers and find a better fit for them varying the MG parameters. One of the reasons some these parameters are not that close to the Planck values is maybe because we are not using the best values for our MG parameters, and that is what we are going to find out.

As a note here, it should be said that we tried to use the `minuit` sampler[7], it attempts to find

[7] https://seal.web.cern.ch/seal/MathLibs/Minuit2/html/

the single point in parameter space with the highest posterior probability. It is a wrapper around the powerful MINUIT2 library that is widely used in particle physics. We failed to obtain best fit values with this sampler, using two different combinations of values and algorithms, first with the MIGRAD algorithm, which is pretty robust unless there are sharp edges in the parameter space. It also re-parameterizes so that the formal parameter edges (the limits in your values file) are shifted to +- infinity; and them with the SIMPLEX algorithm. We even tried the medium, and fast way of convergence with no good results. In CosmoSIS this sampler is not paralleized so it took around 6 hours for each run, and after 1000 iterations it still didn't work. The files used and more explanation about the sampler can be found in the thesis repository.

4.2.3 Getting standard cosmological theory functions for our cosmology

After getting the maximum likelihood cosmological parameters and modified gravity parameters, we will calculate standard cosmological distances, and matter and CMB power spectra using the `test`, `metropolis` and `emcee` samplers.

- Metropolis sampler

Metropolis-Hastings (MH) is the classic Monte-Carlo Markov Chain method for sampling from distributions (Metropolis et al., 1953). MH as a Markov process where from each point in chain there is a process for choosing the next point in such a way that the distribution of chain points tends to the underlying distribution. In MH a proposal function is defined that suggests a possible next point in the chain. The posterior of that point is evaluated and if: $P_{new}/P_{old} > U[0,1]$ where $U[0,1]$ is a random number from 0-1, then the new point is 'accepted' and becomes the next chain element. Otherwise the current point is repeated.

To test if our pipeline is correct we start by using the test sampler and everything ran perfectly. We then set the `ini` file to use the metropolis sampler. The input file for the values of cosmological parameters were the ones obtained in the last calculation, using the `maxlike` sampler, and they were on the file `mgcamb_maxlike_output.ini`.

The results obtained with this sampler were:

Cosmological Parameter	Marginalized mean ± std-dv
$\Omega_b h^2$	0.0295 ± 0.00021
$\Omega_c h^2$	0.1144 ± 0.0015
H_0	0.7060 ± 0.0072
n_s	0.9796 ± 0.0045
a_s	$2.121 \times 10^{-9} \pm 1.33 \times 10^{-11}$
τ	0.0654 ± 0.0024
λ_1^2	89851.96 ± 45639.4
λ_2^2	886.83 ± 69.90
s	3.76353 ± 0.2086

Table 4.7: Constraints for marginalized means and its standard deviation as a result of the `metropolis` sampler.

Cosmological Parameter	Marginalized median ± std-dv
$\Omega_b h^2$	0.0229 ± 0.00021
$\Omega_c h^2$	0.1144 ± 0.0015
H_0	0.7059 ± 0.0072
n_s	0.9796 ± 0.0045
a_s	$2.121 \times 10^{-9} \pm 1.33 \times 10^{-11}$
τ	0.0655 ± 0.0024
λ_1^2	90011.6 ± 4563.94
λ_2^2	896.33 ± 69.99
s	3.8230 ± 0.2086

Table 4.8: Constraints for marginalized medians and its standard deviation as a result of the `metropolis` sampler.

Cosmological Parameter	Best likelihood
$\Omega_b h^2$	0.0229
$\Omega_c h^2$	0.1145
H_0	0.7057
n_s	0.9782
a_s	2.1013×10^{-9}
τ	0.0604
λ_1^2	80980.7
λ_2^2	835.823
s	3.9363

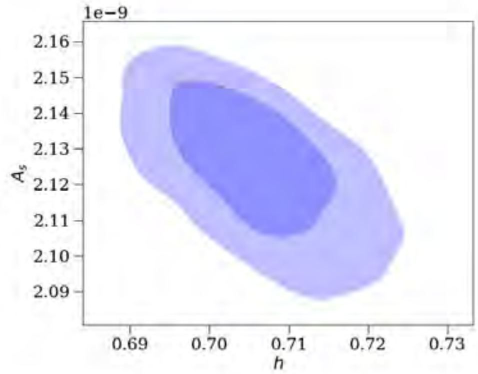
Table 4.9: Best likelihood for the `metropolis` sampler.

Cosmological Parameter	95% lower limits
$\Omega_b h^2$	> 0.0226
$\Omega_c h^2$	> 0.1119
H_0	> 0.6945
n_s	> 0.9720
a_s	$> 2.0985 \times 10^{-9}$
τ	> 0.0610
λ_1^2	> 81955.9
λ_2^2	> 759.50
s	> 3.3290

Table 4.10: 95% lower limits for the `metropolis` sampler.

Cosmological Parameter	95% upper limits
$\Omega_b h^2$	< 0.0233
$\Omega_c h^2$	< 0.1169
H_0	< 0.7184
n_s	< 0.9874
a_s	$< 2.1436 \times 10^{-9}$
τ	< 0.0693
λ_1^2	< 97865.5
λ_2^2	< 986.79
s	< 3.9851

Table 4.11: 95% upper limits for the `metropolis` sampler.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2.15
2.14
2.13
2.12
2.11
2.10
2.09
A_s
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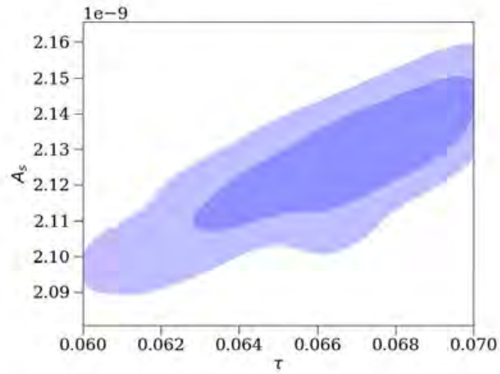
0.70
0.71
0.72
0.73
h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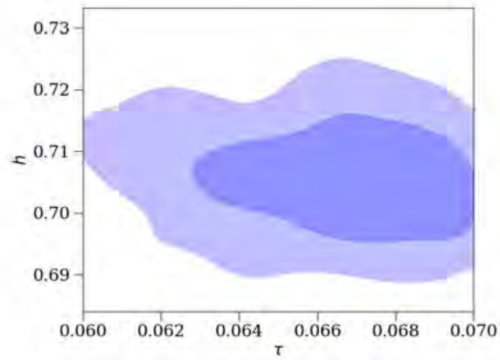
2.16
2.15
2.14
2.13
2.12
2.11
2.10
2.09
A_s
0.060
0.062
0.064
0.066
0.068
0.070
τ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0.72
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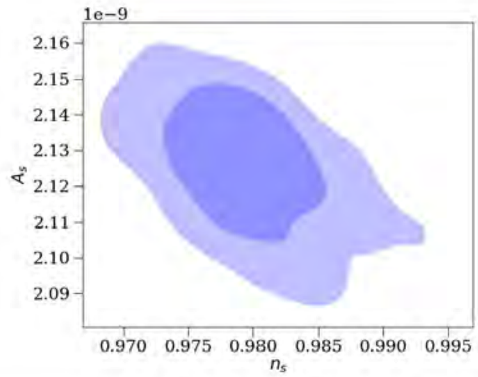
0.70
0.69
h
0.060
0.062
0.064
0.066
0.068
0.070
τ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2.15
2.14
2.13
2.12
2.11
2.10
2.09
A_s
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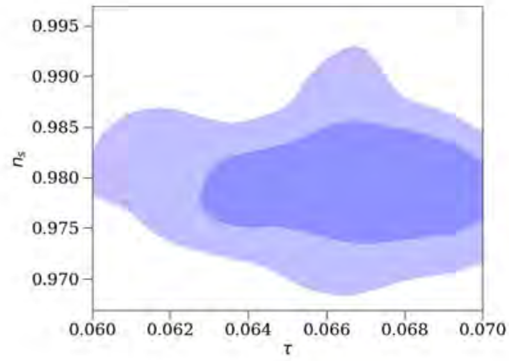
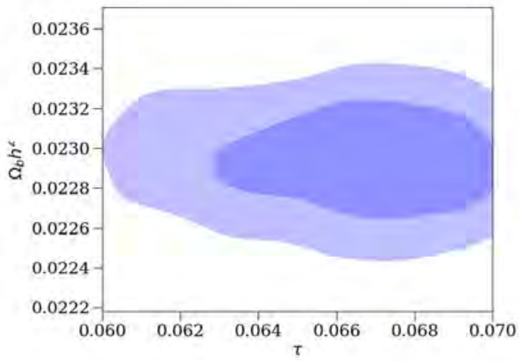
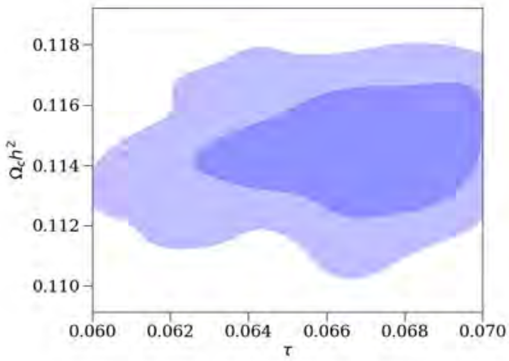
0.980
0.985
0.990
0.995
n_s



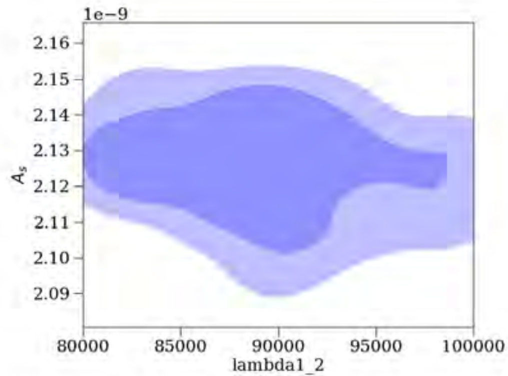
0.995
0.990
0.985
n_s
0.980
0.975
0.970
0.060
0.062
0.064
0.066
0.068
0.070
τ



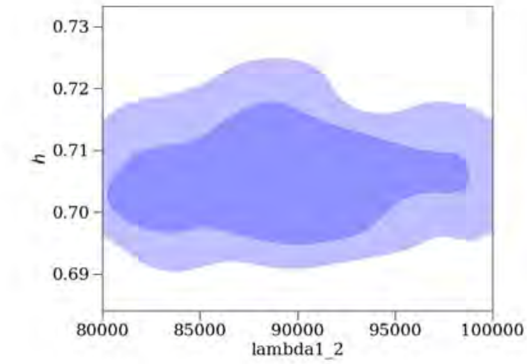
0.0236
0.0234
0.0232
$\Omega_b h^2$
0.0230
0.0228
0.0226
0.0224
0.0222
0.060
0.062
0.064
0.066
0.068
0.070
τ

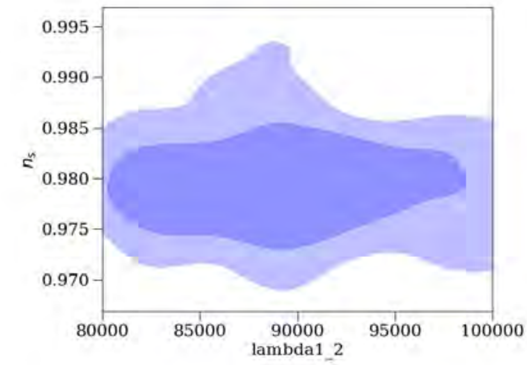


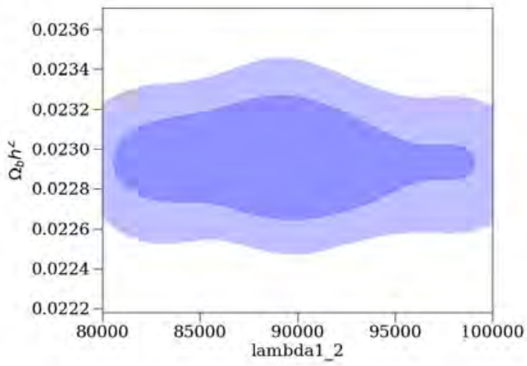
0.118
0.116
$\Omega_c h^2$
0.114
0.112
0.110
0.060
0.062
0.064
0.066
0.068
0.070
τ

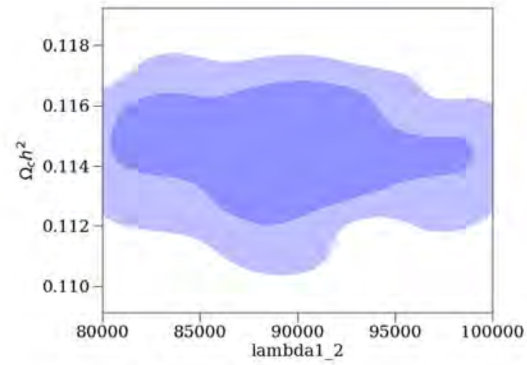


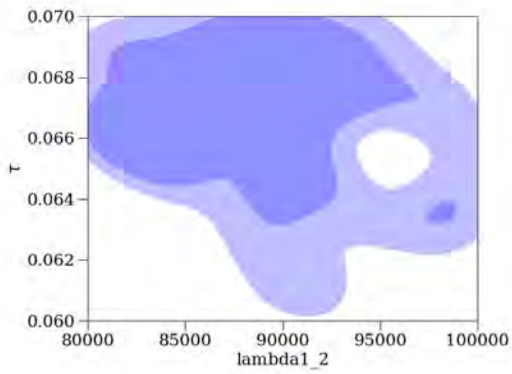
1e−9
2.16
2.15
2.14
2.13
A_s
2.12
2.11
2.10
2.09
80000
85000
90000
95000
100000
lambda1_2

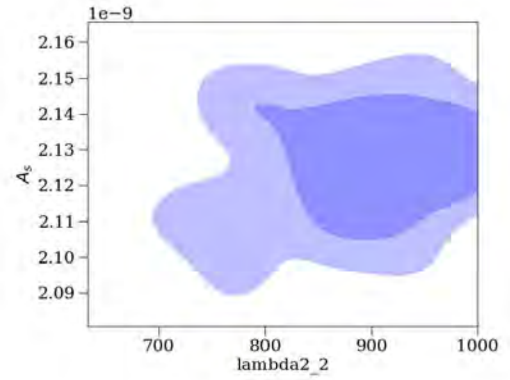

0.73
0.72
0.71
0.70
0.69
h
80000
85000
90000
95000
100000
lambda1_2

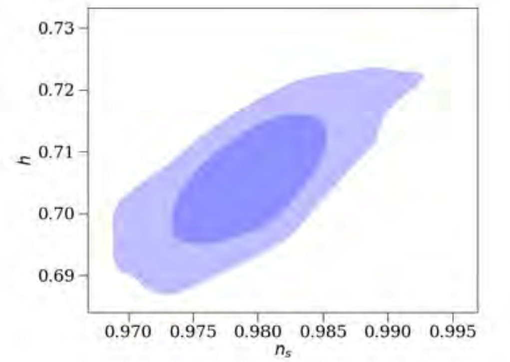

0.995
0.990
0.985
0.980
0.975
0.970
n_s
80000
85000
90000
95000
100000
lambda1_2


0.0236
0.0234
0.0232
0.0230
0.0228
0.0226
0.0224
0.0222
$\Omega_b h^2$
80000
85000
90000
95000
100000
lambda1_2


0.118
0.116
0.114
0.112
0.110
$\Omega_c h^2$
80000
85000
90000
95000
100000
lambda1_2


0.070
0.068
0.066
0.064
0.062
0.060
τ
80000
85000
90000
95000
100000
lambda1_2


1e−9
2.16
2.15
2.14
2.13
2.12
2.11
2.10
2.09
A_s
700
800
900
1000
lambda2_2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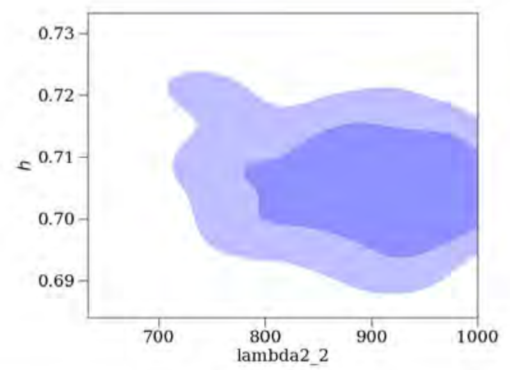
0.72
0.71
0.70
0.69
h
0.970
0.975
0.980
0.985
0.990
0.995
n_s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h
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800
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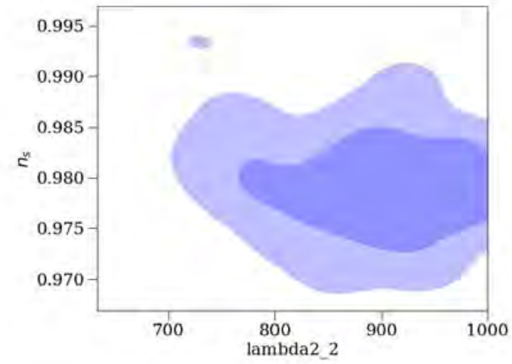
1000
lambda2_2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n_s
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700
800
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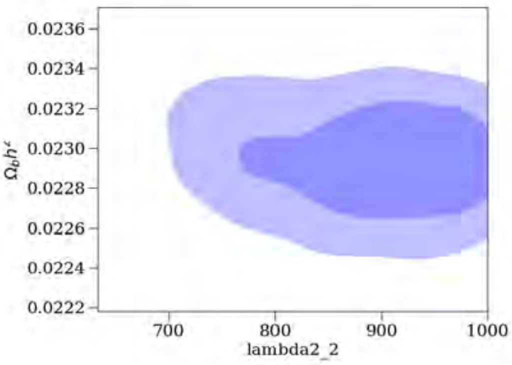
lambda2_2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0.0234
0.0232
$\Omega_b h^2$
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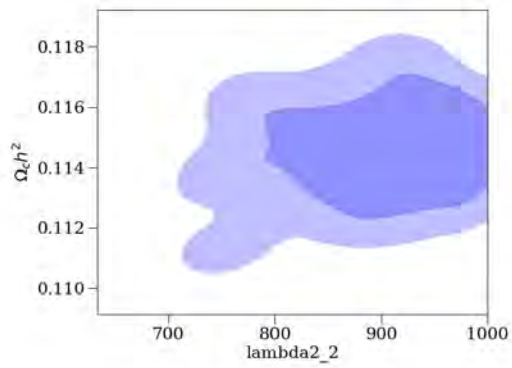
0.0228
0.0226
0.0224
0.0222
700
800
900
1000
lambda2_2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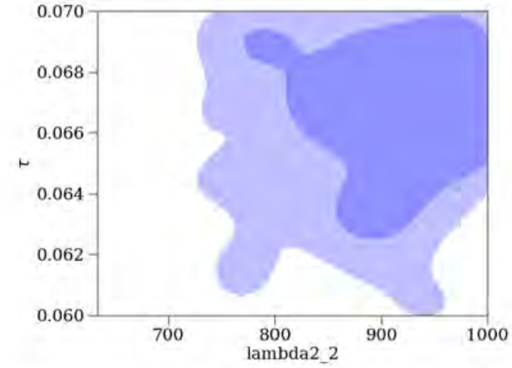
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0.066
τ
0.064
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0.060
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800
900
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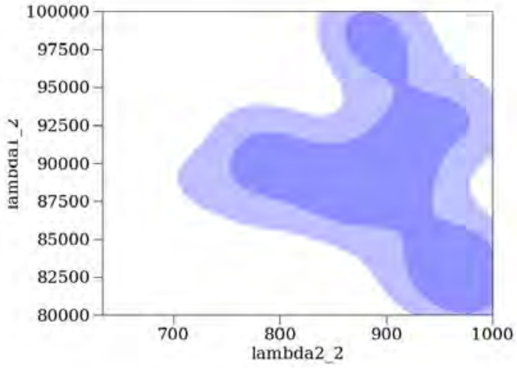
lambda2_2


lambda1_2
lambda2_2
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92500
90000
87500
85000
82500
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800
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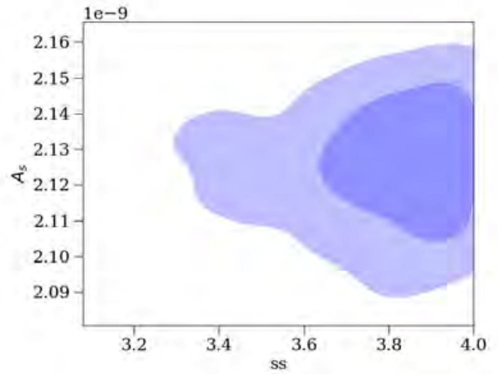
1000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2.10
2.09
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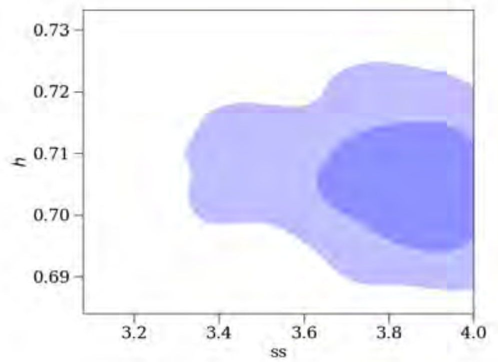
3.6
3.8
4.0


h
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3.2
3.4
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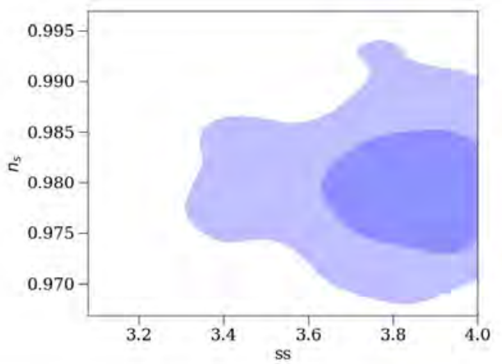
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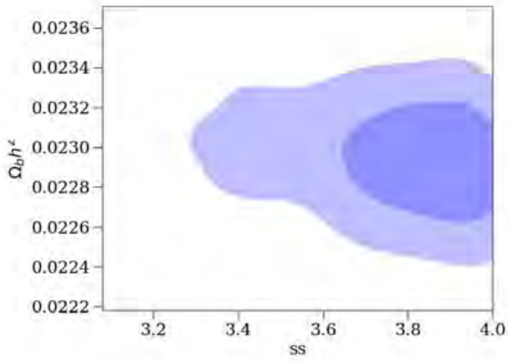

0.0236
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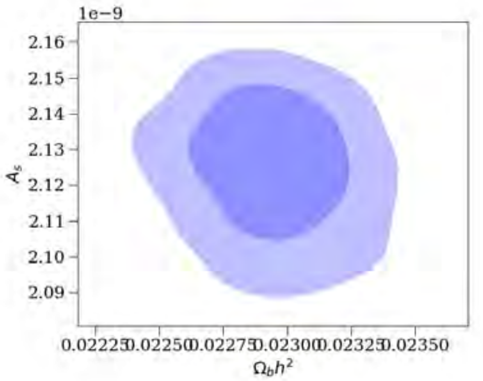
0.0224
0.0222
$\Omega_b h^2$
3.2
3.4
3.6
3.8
4.0
ss



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2.11
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2.09
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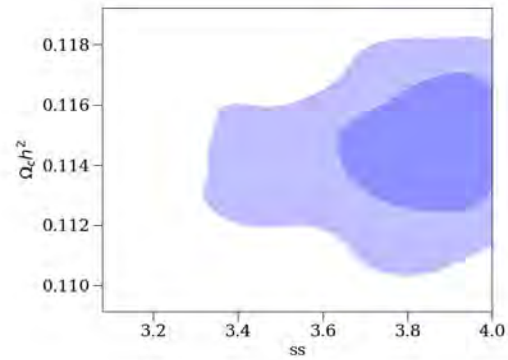
0.02325
0.02350
$\Omega_b h^2$



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0.116
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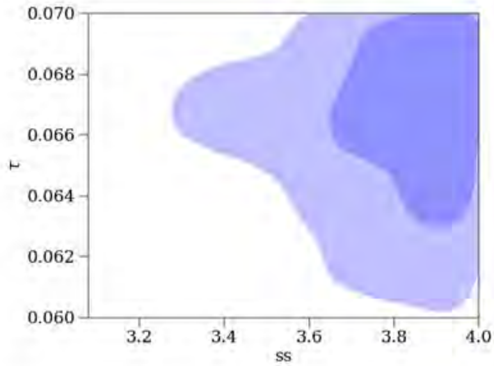
0.112
0.110
$\Omega_c h^2$
3.2
3.4
3.6
3.8
4.0
ss



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0.066
0.064
0.062
0.060
τ
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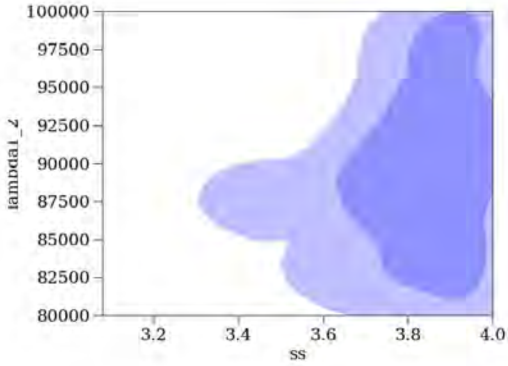
3.4
3.6
3.8
4.0
ss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92500
90000
87500
85000
82500
80000
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3.4
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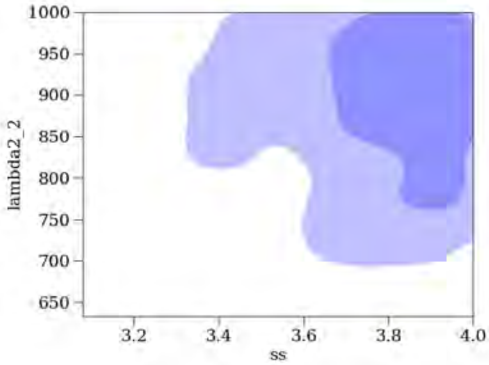
3.8
4.0
ss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950
900
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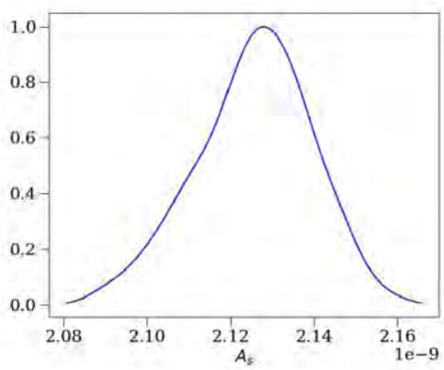
750
700
650
3.2
3.4
3.6
3.8
4.0
ss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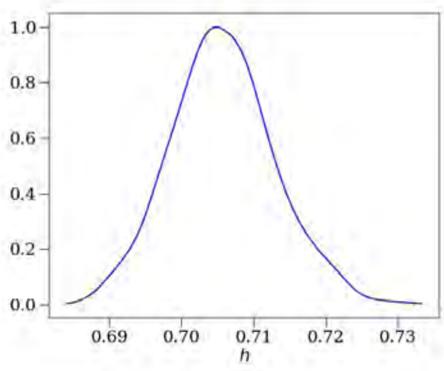
2.10
2.12
2.14
2.16
A_s
1e−9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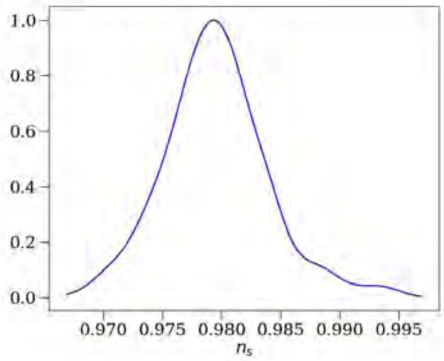
0.70
0.71
0.72
0.73
h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0.0
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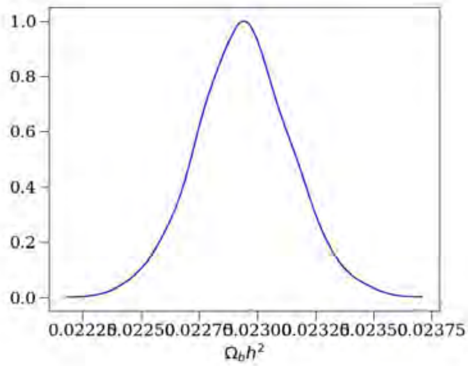
n_s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0.0
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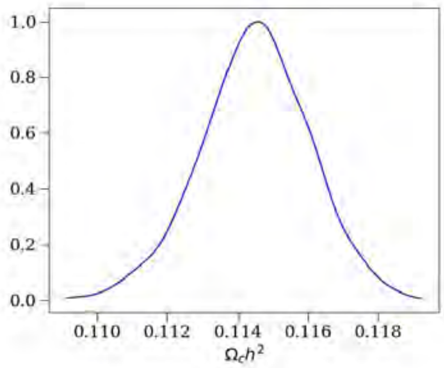
$\Omega_b h^2$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0.4
0.2
0.0
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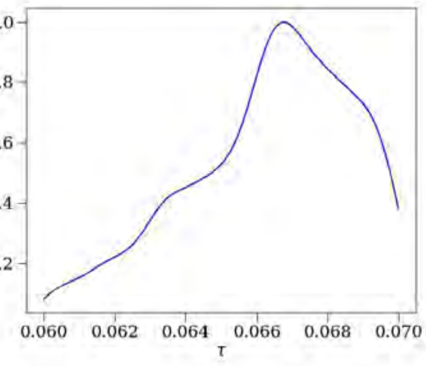
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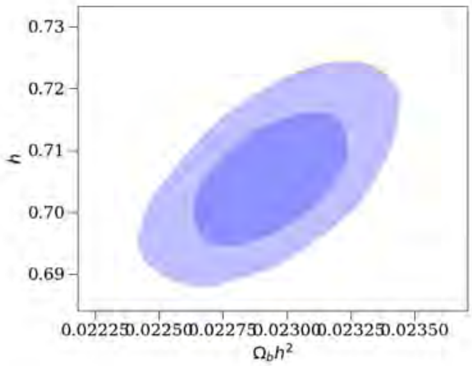
0.4
0.2
0.060 0.062 0.064 0.066 0.068 0.070
τ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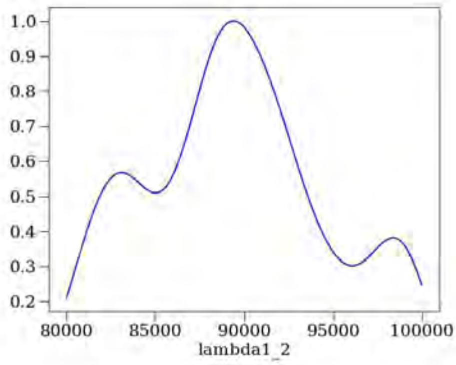
0.72
0.71
0.70
0.69
h
0.02225
0.02250
0.02275
0.02300
0.02325
0.02350
$\Omega_b h^2$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0.9
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0.6
0.5
0.4
0.3
0.2
80000
85000
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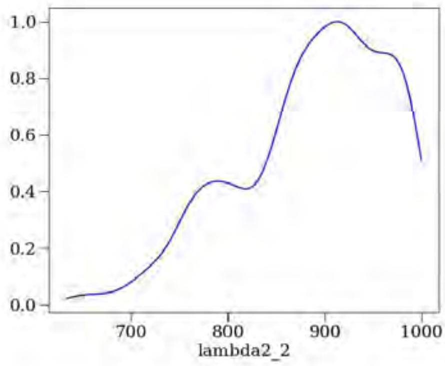
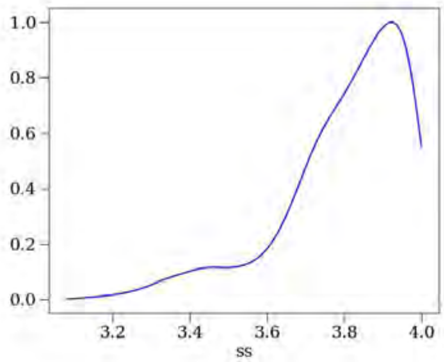
95000
100000
lambda1_2


1.0
0.8
0.6
0.4
0.2
0.0
700
800
900
1000
lambda2_2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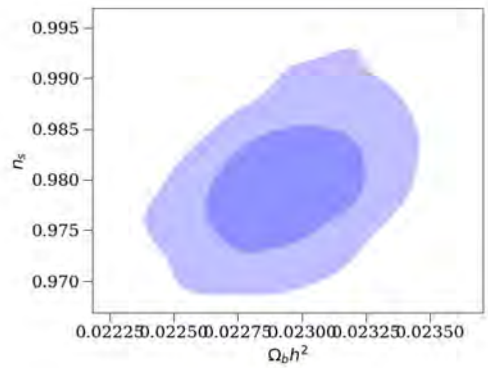
0.6
0.4
0.2
0.0
3.2
3.4
3.6
3.8
4.0
ss


0.995
0.990
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0.980
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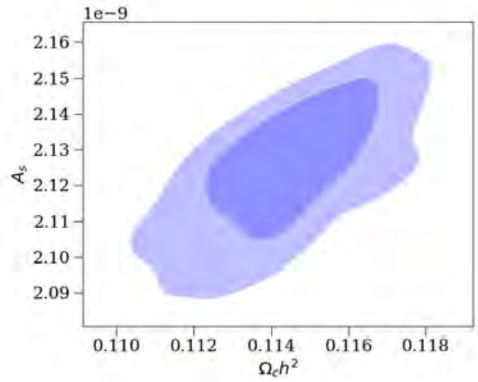
0.970
n_s
0.02225 0.02250 0.02275 0.02300 0.02325 0.02350
$\Omega_b h^2$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2.16
2.15
2.14
2.13
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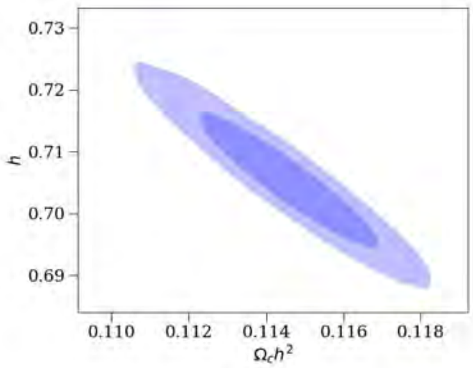
2.11
2.10
2.09
A_s
0.110 0.112 0.114 0.116 0.118
$\Omega_c h^2$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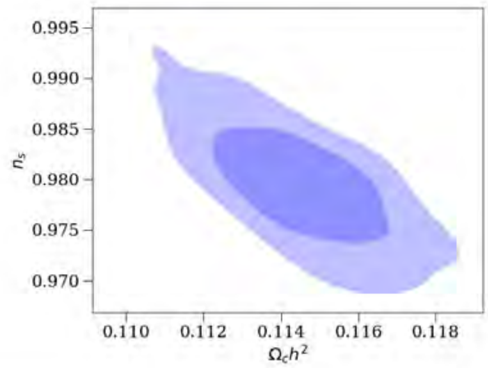
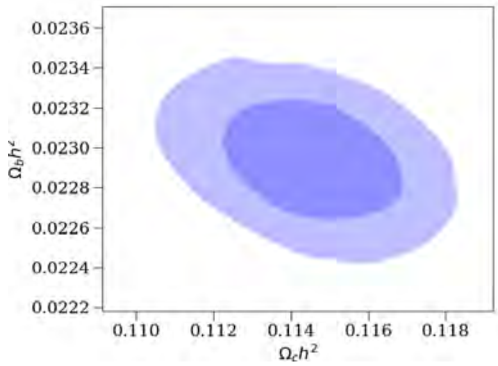
0.71
0.70
0.69
h
0.110 0.112 0.114 0.116 0.118
$\Omega_c h^2$


0.995
0.990
0.985
0.980
0.975
0.970
n_s
0.110 0.112 0.114 0.116 0.118
$\Omega_c h^2$



From the last figures we can see that some of the parameters for our parametrization and the standard cosmology are poorly correlated like the λ_1^2 and λ_2^2 and almost every parameter, being the poorest correlated (and making weird contours) with the λ_2^2. Some other parameters are fully uncorrelated as we can see in the figures above. The cosmological parameters of the BZ model have some weird contours when compared with the ΛCDM or even with the same BZ parameters. In general these plots and results were the worst when comparing with the `emcee` and `multinest` samplers as we will discuss bellow.

- Emcee sampler

The `emcee` sampler is a form of Monte-Carlo Markov Chain that uses an ensemble of 'walkers' that explore the parameter space (Foreman-Mackey et al., 2013). Each walker chooses another walker at random and proposes along the line connecting the two of them using the Metropolis acceptance rule. The proposal scale is given by the separation of the two walkers. It is parallel, so multiple processes can be used to speed up the running. It is also affine invariant, so that no covariance matrix or other tuning is required for the proposal.

The burn-in behavior of emcee can sometimes be poor; it is much better to start the chain as near to the maximum posterior as possible. This is why we used the `maxlike` to find this. The total number of samples taken is walkers*samples.

The results obtained with this sampler were:

Cosmological Parameter	Marginalized mean ± std-dv
$\Omega_b h^2$	0.0229 ± 0.00022
$\Omega_c h^2$	0.1142 ± 0.0019
H_0	0.7066 ± 0.0091
n_s	0.9801 ± 0.0057
a_s	$2.117 \times 10^{-9} \pm 1.58 \times 10^{-11}$
τ	0.0647 ± 0.0028
λ_1^2	90181.9 ± 5341.87
λ_2^2	856.07 ± 103.16
s	3.7267 ± 0.2395

Table 4.12: Constraints for marginalized means and its standard deviation as a result of the `emcee` sampler.

Cosmological Parameter	Marginalized median ± std-dv
$\Omega_b h^2$	0.0229 ± 0.00022
$\Omega_c h^2$	0.1142 ± 0.0019
H_0	0.7061 ± 0.0091
n_s	0.9803 ± 0.0057
a_s	$2.117 \times 10^{-9} \pm 1.58 \times 10^{-11}$
τ	0.0647 ± 0.0028
λ_1^2	90090.2 ± 5341.87
λ_2^2	881.56 ± 103.16
s	3.7923 ± 0.2395

Table 4.13: Constraints for marginalized medians and its standard deviation as a result of the `emcee` sampler.

Cosmological Parameter	Best likelihood
$\Omega_b h^2$	0.0227
$\Omega_c h^2$	0.1146
H_0	0.7034
n_s	0.9785
a_s	2.1013×10^{-9}
τ	0.0609
λ_1^2	86462.8
λ_2^2	851.74
s	3.9504

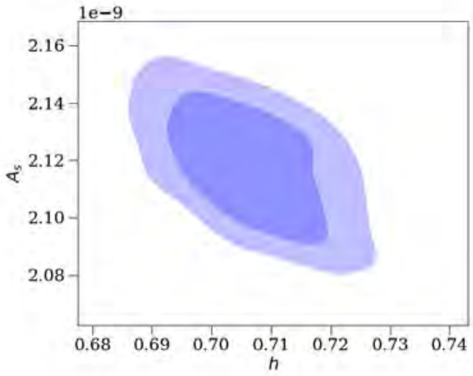
Table 4.14: Best likelihood for the `emcee` sampler.

Cosmological Parameter	95% lower limits
$\Omega_b h^2$	> 0.0225
$\Omega_c h^2$	> 0.1111
H_0	> 0.6918
n_s	> 0.9703
a_s	$> 2.0913 \times 10^{-9}$
τ	> 0.0604
λ_1^2	> 81287.1
λ_2^2	> 646.98
s	> 3.2271

Table 4.15: 95% lower limits for the `emcee` sampler.

Cosmological Parameter	95% upper limits
$\Omega_b h^2$	< 0.0233
$\Omega_c h^2$	< 0.1174
H_0	< 0.7220
n_s	< 0.9886
a_s	$< 2.1435 \times 10^{-9}$
τ	< 0.0692
λ_1^2	< 98670.6
λ_2^2	< 987.72
s	< 3.9870

Table 4.16: 95% upper limits for the `emcee` sampler.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2.16
2.14
2.12
2.10
2.08
A_s
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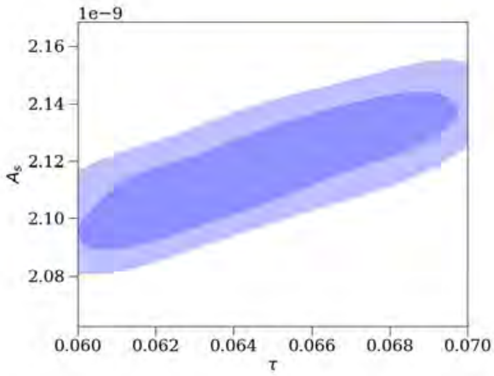
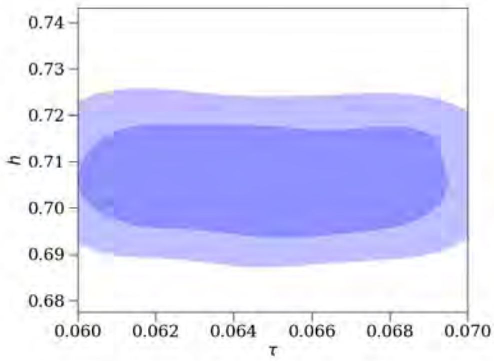
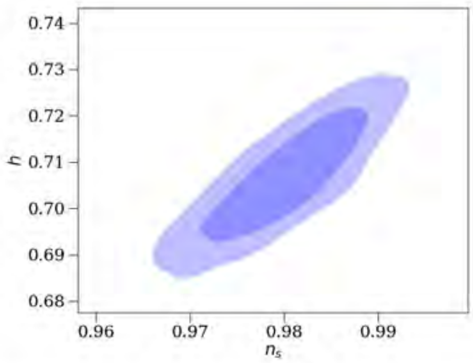
h


1e−9
2.16
2.14
2.12
2.10
2.08
A_s
0.060 0.062 0.064 0.066 0.068 0.070
τ


0.74
0.73
0.72
0.71
0.70
0.69
0.68
h
0.060 0.062 0.064 0.066 0.068 0.070
τ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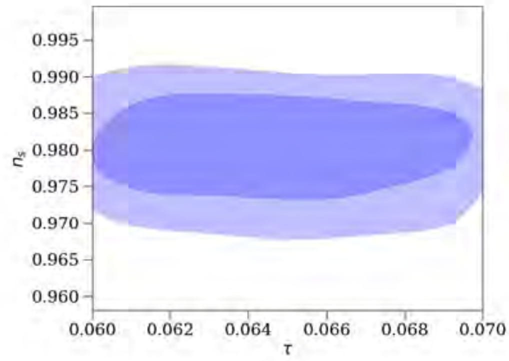
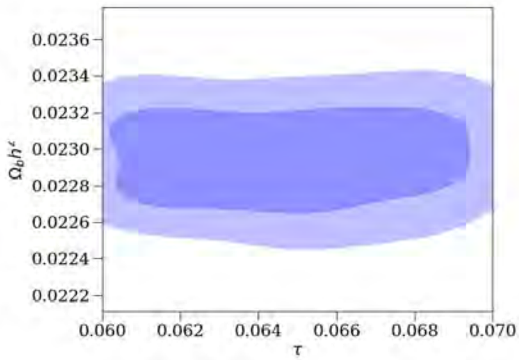
0.71
0.70
0.69
0.68
h
0.96 0.97 0.98 0.99
n_s



0.995
0.990
0.985
0.980
0.975
0.970
0.965
0.960
n_s
0.060
0.062
0.064
0.066
0.068
0.070
τ



0.0236
0.0234
0.0232
0.0230
0.0228
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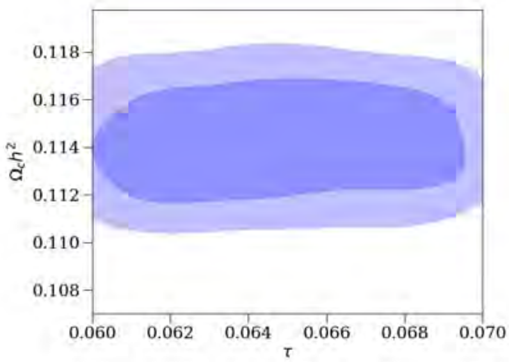
0.0224
0.0222
$\Omega_b h^2$
0.060
0.062
0.064
0.066
0.068
0.070
τ



0.118
0.116
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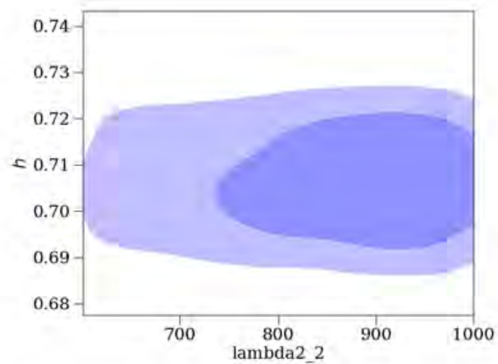
0.108
$\Omega_c h^2$
0.060
0.062
0.064
0.066
0.068
0.070
τ



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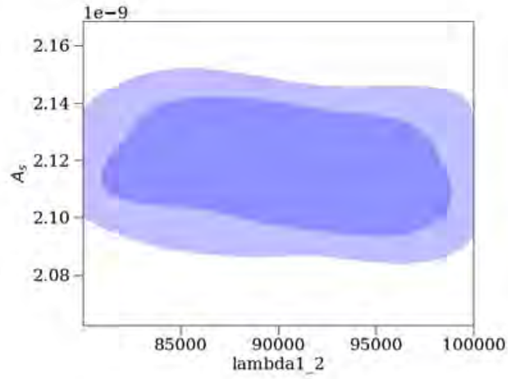
0.69
0.68
h
700
800
900
1000
lambda2_2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2.08
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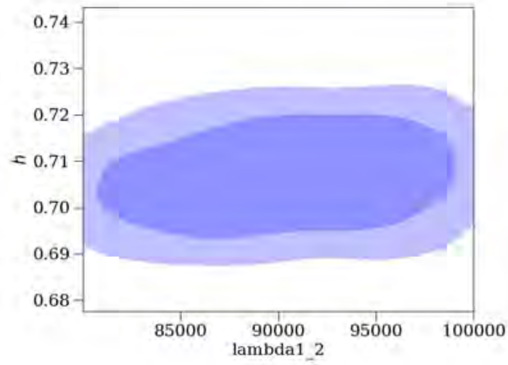
90000
95000
100000
lambda1_2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0.70
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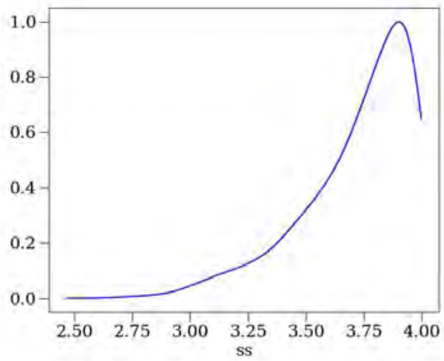
lambda1_2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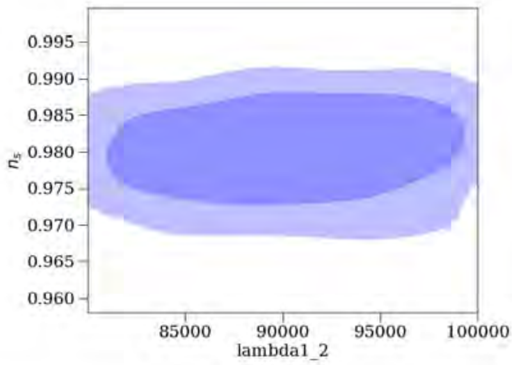
0.2
0.0
2.50
2.75
3.00
3.25
3.50
3.75
4.00
ss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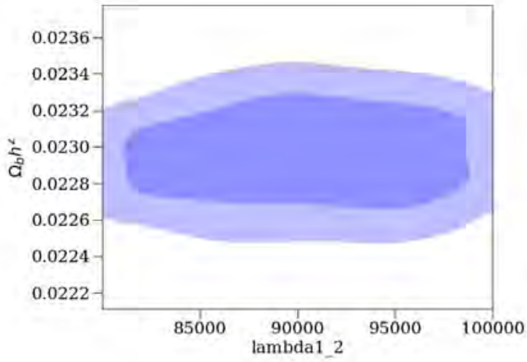
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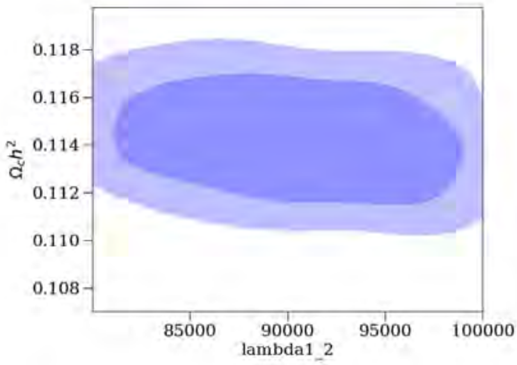
lambda1_2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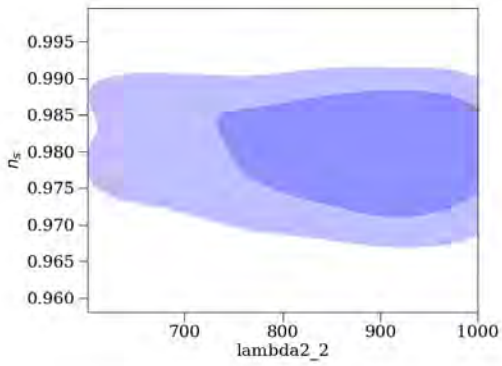
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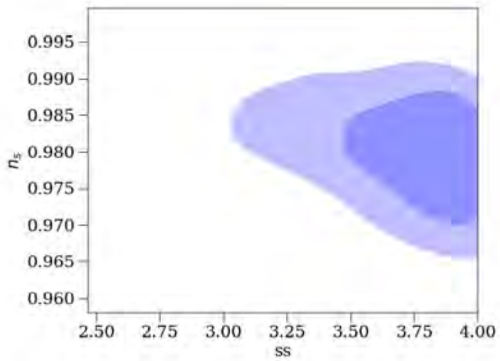
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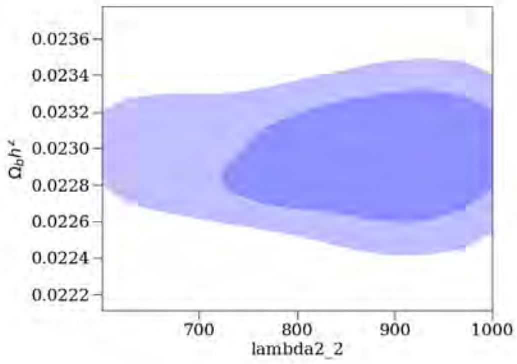
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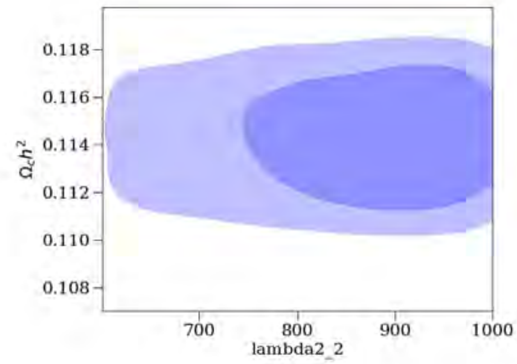
lambda2_2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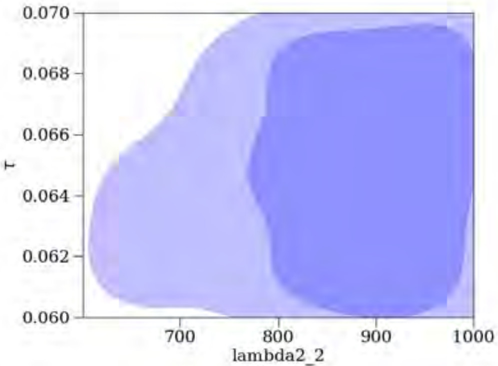
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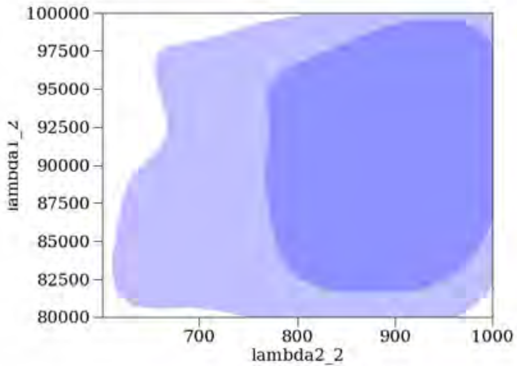
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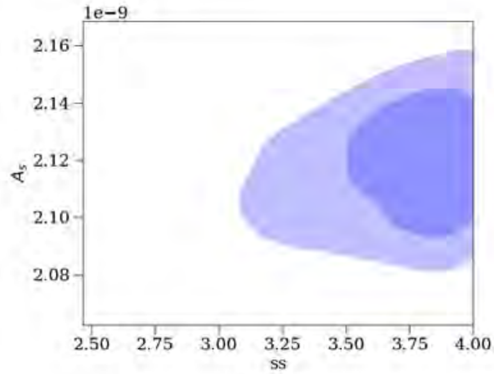
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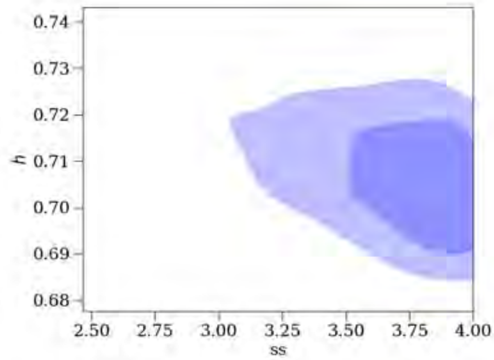
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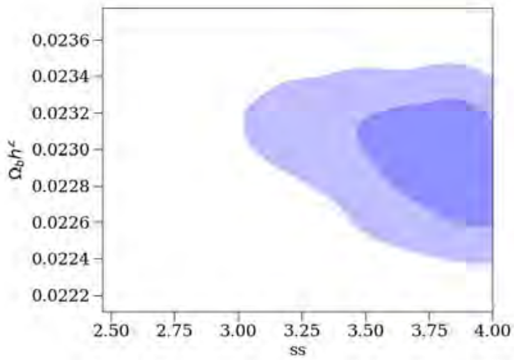
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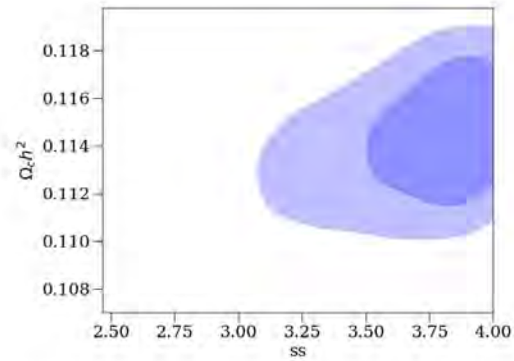
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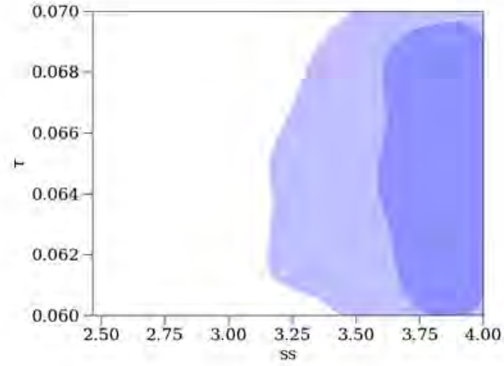
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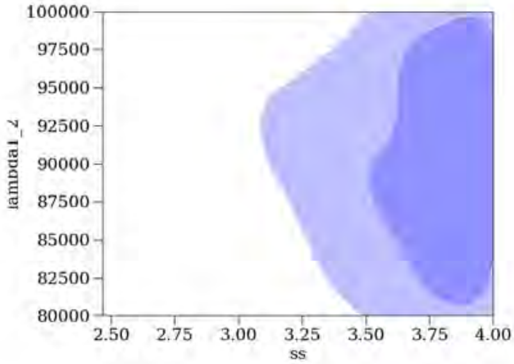
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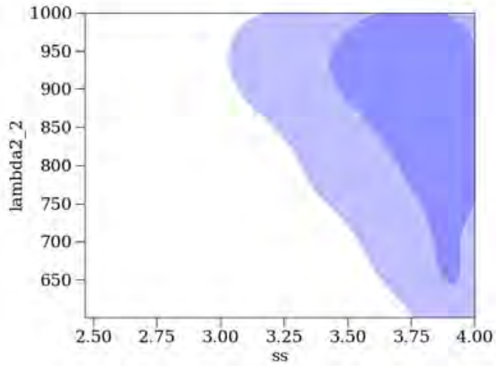
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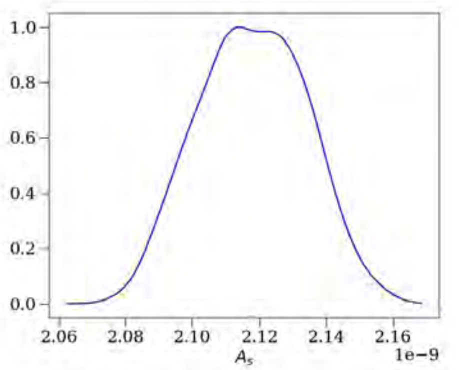
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A_s
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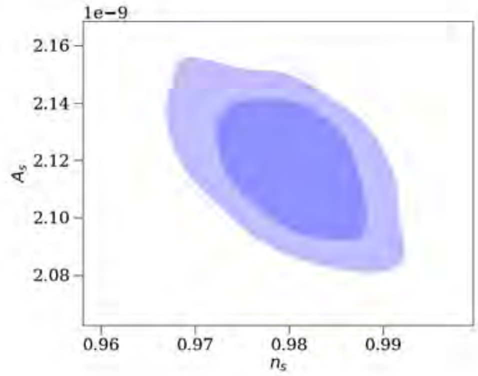
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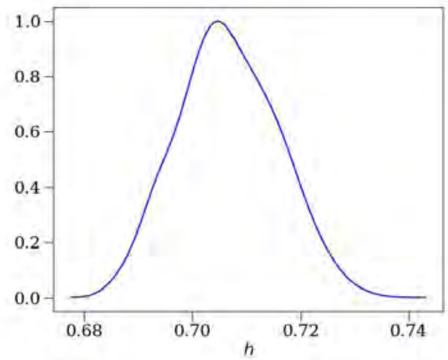
n_s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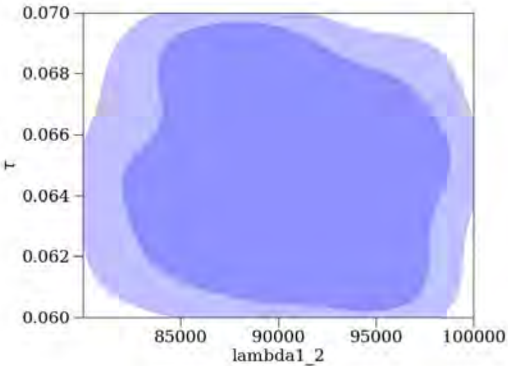
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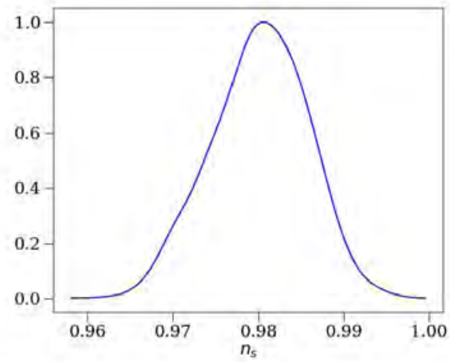
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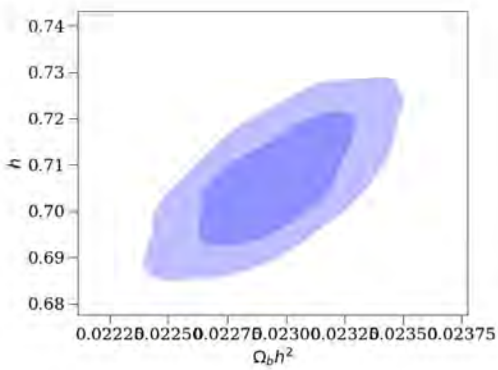
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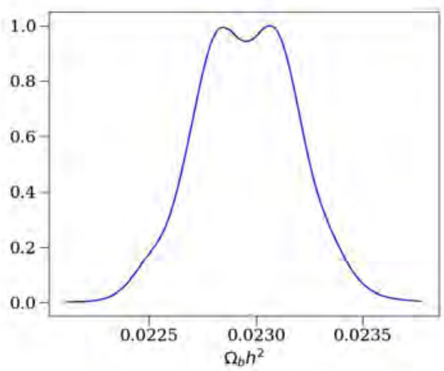
0.02350
0.02375
$\Omega_b h^2$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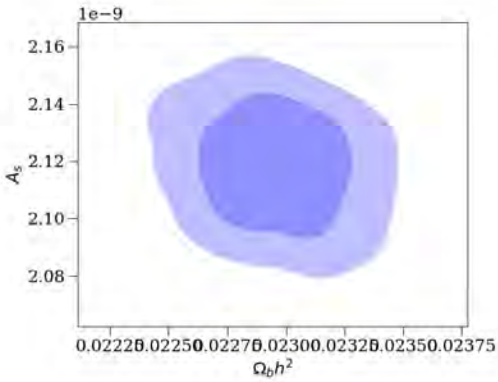
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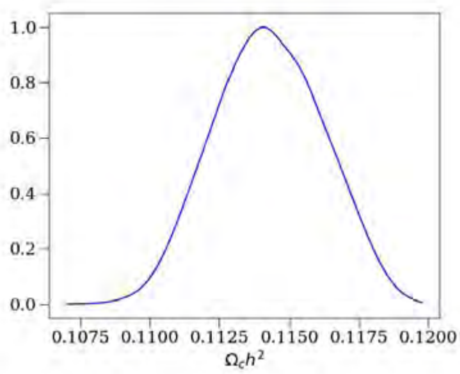
$\Omega_b h^2$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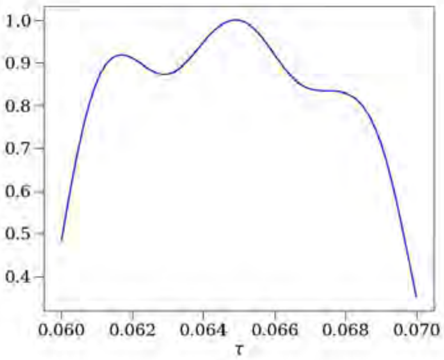
$\Omega_c h^2$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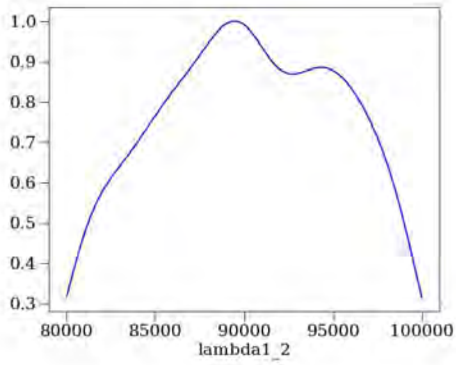
τ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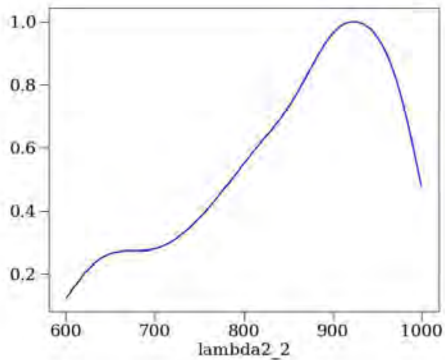
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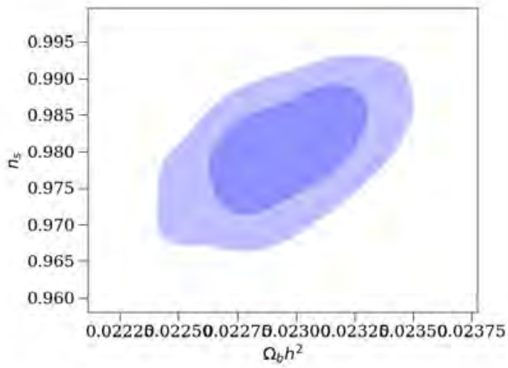
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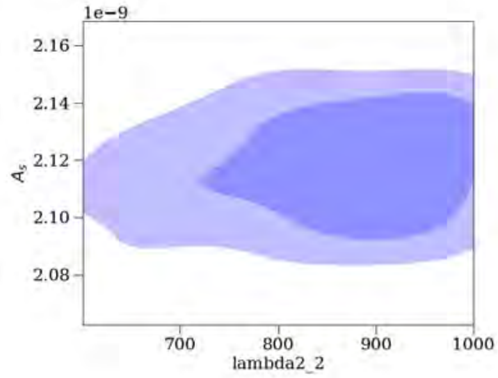
$\Omega_b h^2$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A_s
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800
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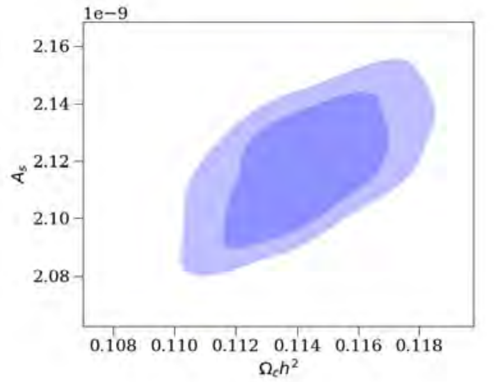
lambda2_2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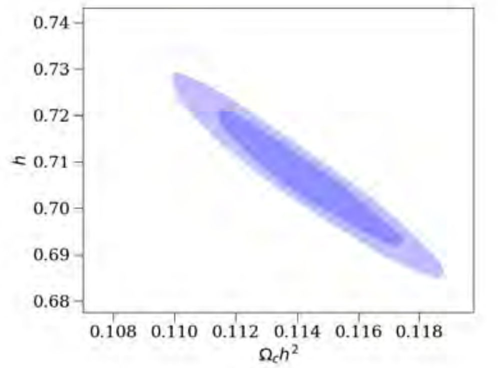
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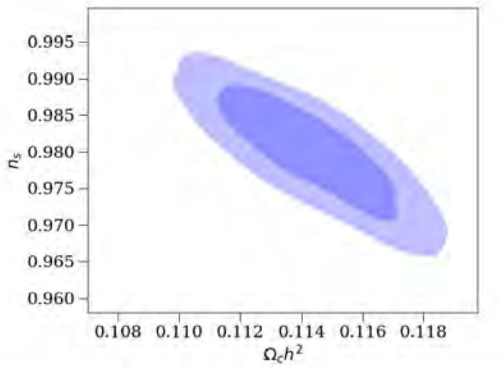
0.118
$\Omega_c h^2$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0.114
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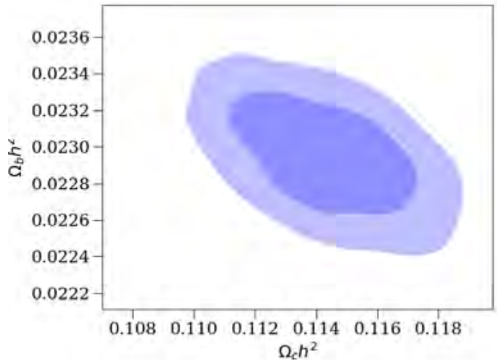
0.118
$\Omega_c h^2$



With the `emcee` we see much clear correlation between parameters, and more similarity with the standard plots for ΛCDM. Still, we can see that the relationships between the parameters of our model and the ones for the standard cosmology are not clear, and some differ a lot from the ones obtained from Metropolis. Something very important, as we will see, is that in this two samplers the s parameter has converged to 4.

4.2.4 Multinest

As we discussed in the last chapter, Bayesian evidence is a method for model selection - it tells us how likely an entire cosmological model is, rather than just a particular choice of parameters in that model. In this calculation we used the sampler Multinest to calculate the Bayesian evidence. As a by-product we can also construct all the usual parameter constraint plots, or get a set of posterior samples. The `multinest` code and algorithm is described in (Feroz et al., 2009).

Nested sampling is a method designed to calculate the Bayesian Evidence of a distribution, for use in comparing multiple models to see which fit the data better. Remember that the evidence is the integral of the likelihood over the prior; it is equivalent to the probability of the model given the data. Nested sampling is an efficient method for evaluating this integral using members of an ensemble of live points and steadily replacing the lowest likelihood point with a new one from a gradually shrinking proposal so and evaluating the integral in horizontal slices. Multinest is a particularly sophisticated implementation of this which can cope with multi-modal distributions using a k-means clustering algorithm and a proposal made from a collection of ellipsoids.

The output from `multinest` is not a set of posterior samples, but rather a set of weighted samples - when making histograms or parameter estimates these must be included. The primary `multinest` parameter is the number of live points in the ensemble. If this number is too small you will get too few posterior samples in the result, and if it is too large the sampling will take a long time. A few hundred seems to be reasonable for typical cosmology problems.
The results obtained with this sampler were:

Cosmological Parameter	Marginalized mean ± std-dv
$\Omega_b h^2$	0.0229 ± 0.00021
$\Omega_c h^2$	0.1140 ± 0.0016
H_0	0.7080 ± 0.0080
n_s	0.9804 ± 0.0050
a_s	$2.116 \times 10^{-9} \pm 1.48 \times 10^{-11}$
τ	0.0646 ± 0.0028
λ_1^2	88591.7 ± 5568.09
λ_2^2	818.203 ± 111.33
s	3.7834 ± 0.1840

Table 4.17: Constraints for marginalized means and its standard deviation as a result of the `multinest` sampler.

Cosmological Parameter	Marginalized median ± std-dv
$\Omega_b h^2$	0.0229 ± 0.00021
$\Omega_c h^2$	0.1139 ± 0.0016
H_0	0.7080 ± 0.0080
n_s	0.9805 ± 0.0050
a_s	$2.115 \times 10^{-9} \pm 1.48 \times 10^{-11}$
τ	0.0644 ± 0.0028
λ_1^2	87983.5 ± 5568.09
λ_2^2	826.62 ± 111.36
s	3.834 ± 0.1840

Table 4.18: Constraints for marginalized medians and its standard deviation as a result of the `multinest` sampler.

Cosmological Parameter	Best likelihood
$\Omega_b h^2$	0.0229
$\Omega_c h^2$	0.1148
H_0	0.7035
n_s	0.9780
a_s	2.112×10^{-9}
τ	0.0630
λ_1^2	83682.5
λ_2^2	988.637
s	3.9556

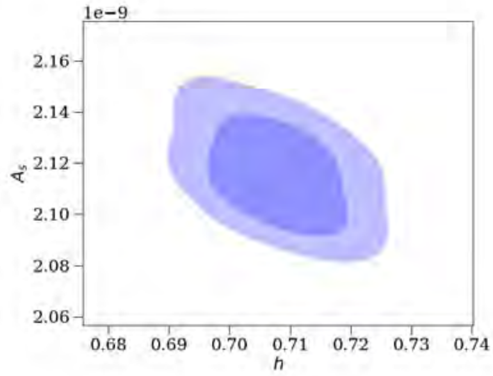
Table 4.19: Best likelihood for the `multinest` sampler.

Cosmological Parameter	95% lower limits
$\Omega_b h^2$	> 0.0226
$\Omega_c h^2$	> 0.1111
H_0	> 0.6950
n_s	> 0.9720
a_s	$> 2.0923 \times 10^{-9}$
τ	> 0.0604
λ_1^2	> 80781.2
λ_2^2	> 629.30
s	> 3.416

Table 4.20: 95% lower limits for the `multinest` sampler.

Cosmological Parameter	95% upper limits
$\Omega_b h^2$	< 0.0233
$\Omega_c h^2$	< 0.1167
H_0	< 0.7212
n_s	< 0.9887
a_s	$< 2.1415 \times 10^{-9}$
τ	< 0.0694
λ_1^2	< 98366.6
λ_2^2	< 982.20
s	< 3.9853

Table 4.21: 95% upper limits for the `multinest` sampler.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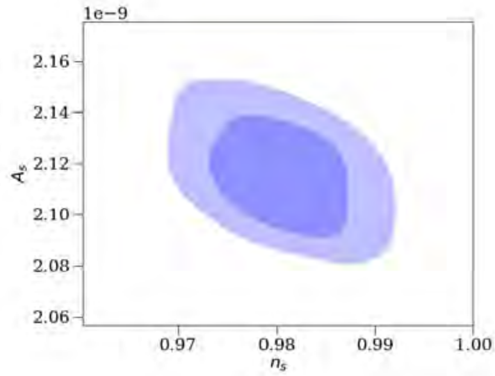
h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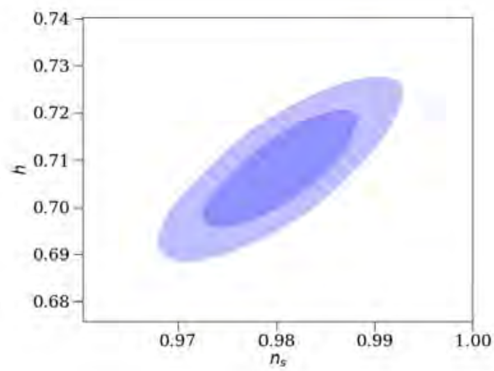
n_s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h
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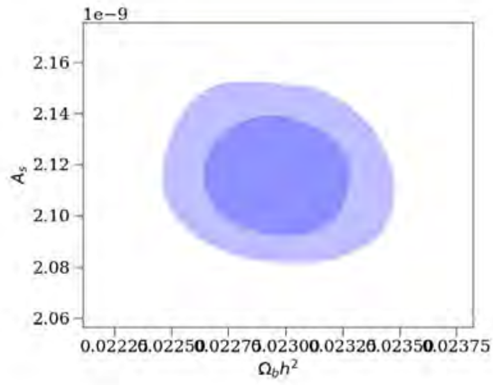
n_s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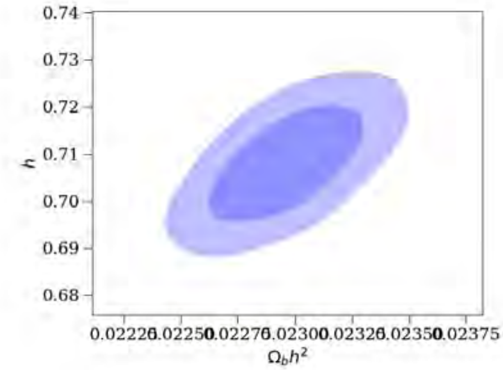
$\Omega_b h^2$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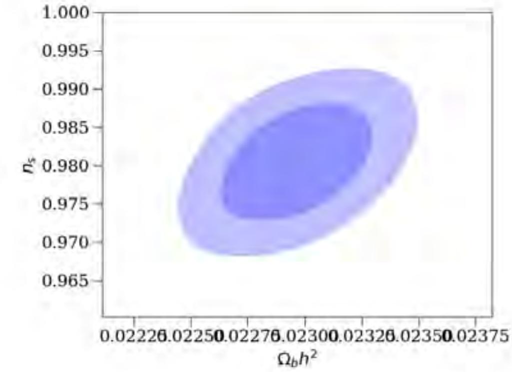
0.68
h
0.02225 0.02250 0.02275 0.02300 0.02325 0.02350 0.02375
$\Omega_b h^2$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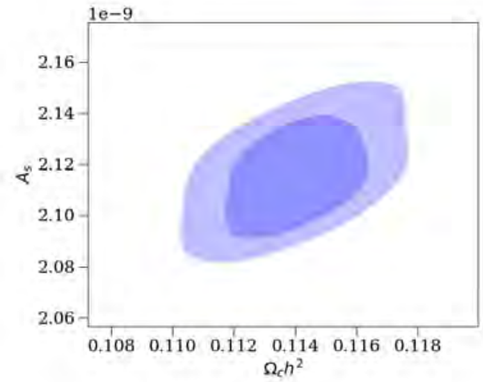
$\Omega_b h^2$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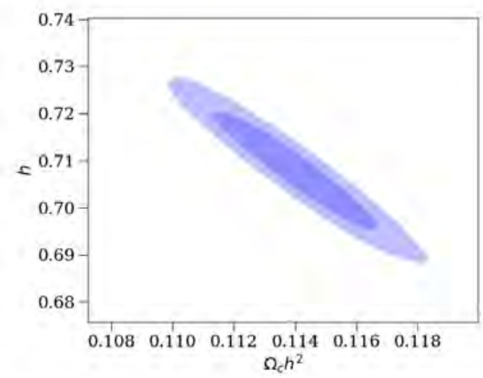
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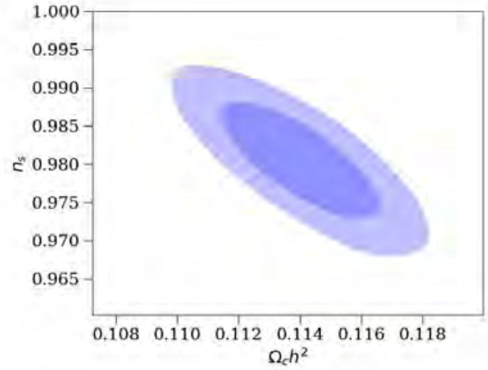
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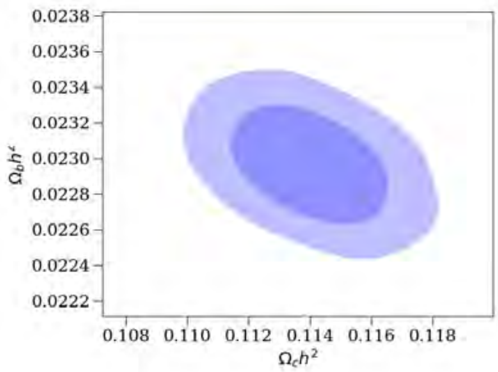
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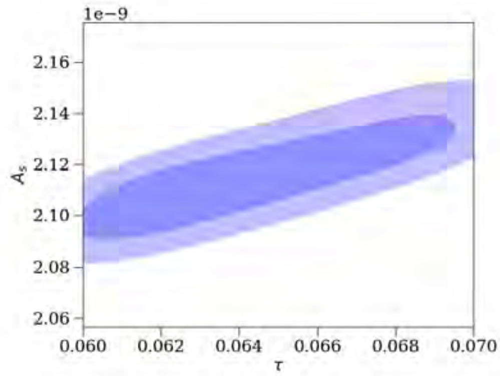
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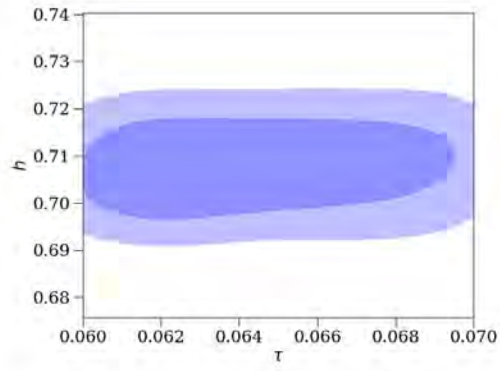
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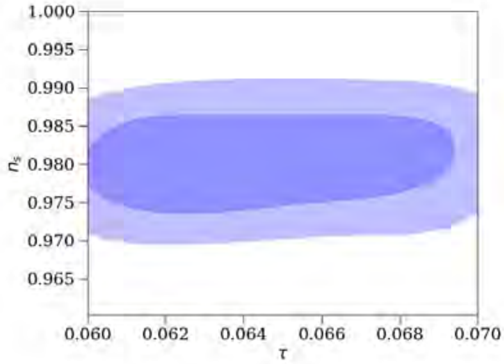
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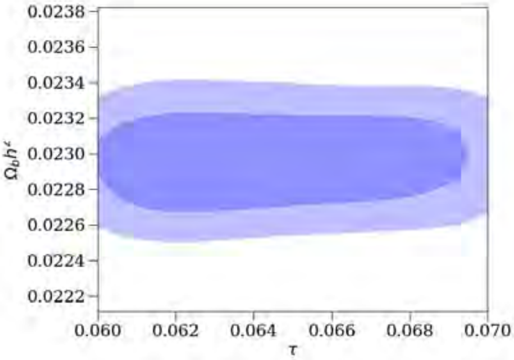
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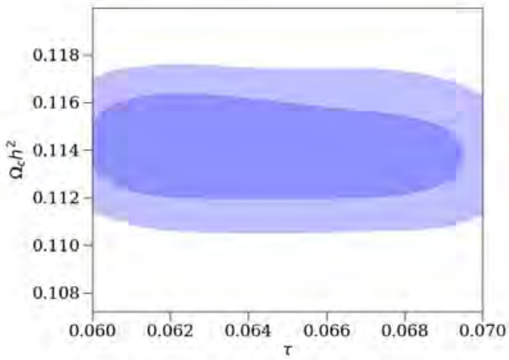
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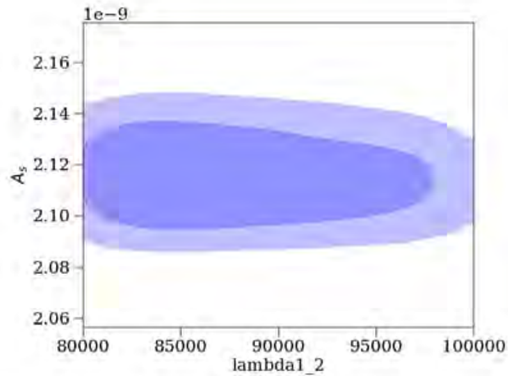
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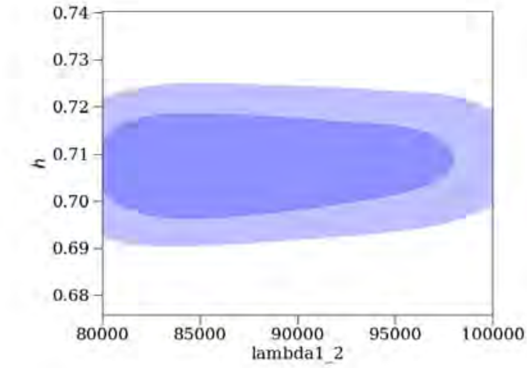
lambda1_2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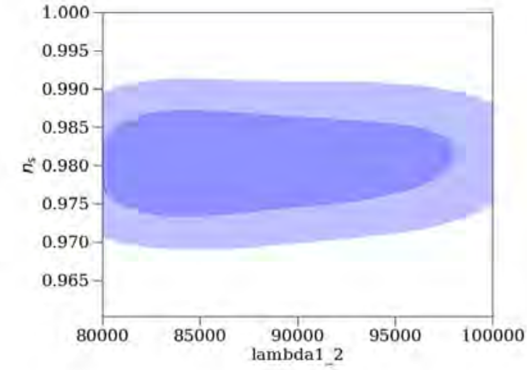
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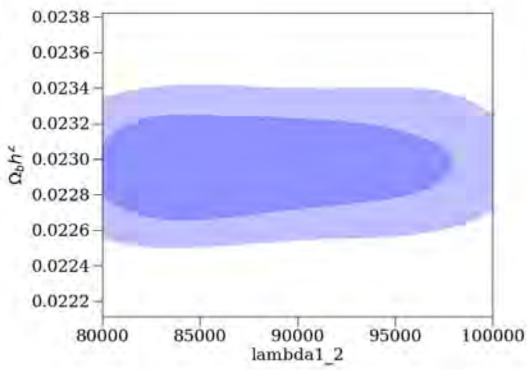
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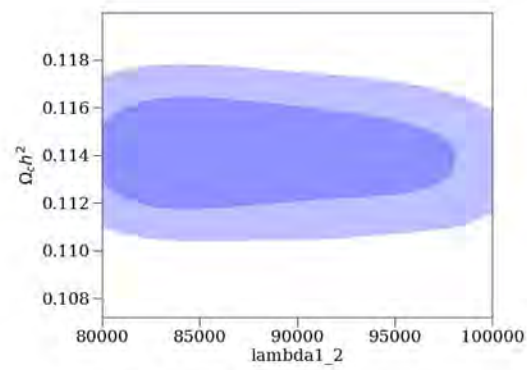
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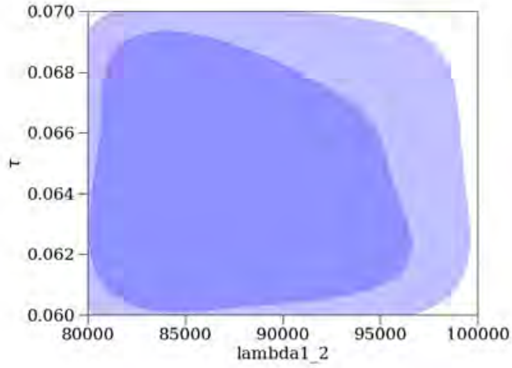
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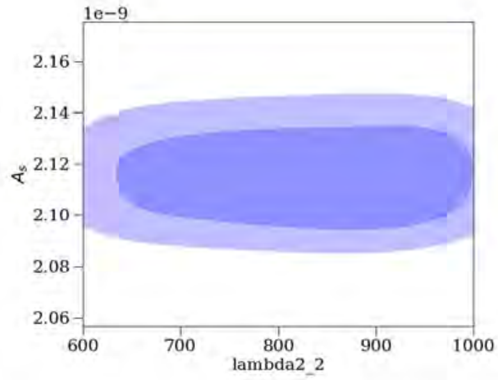
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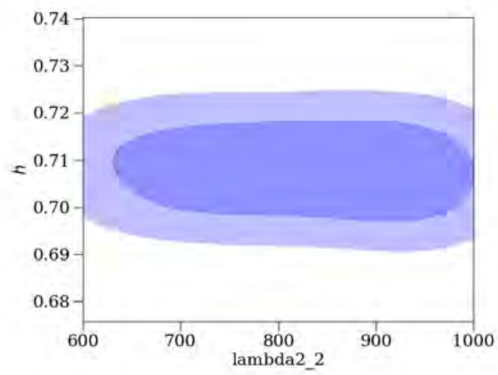
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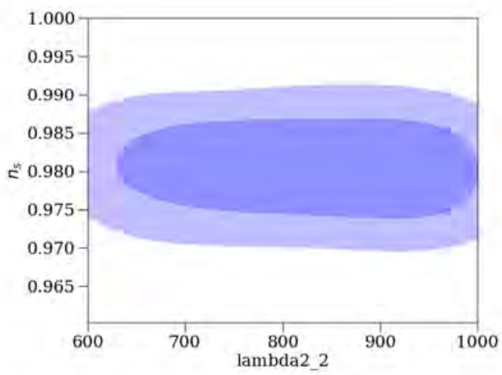
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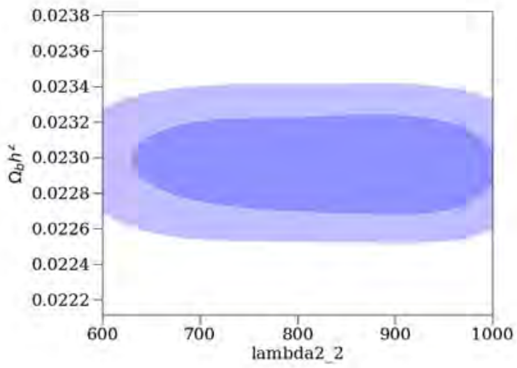
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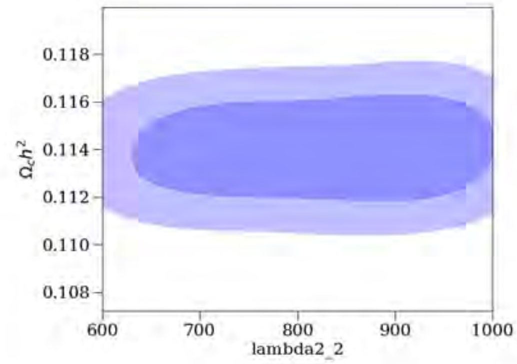
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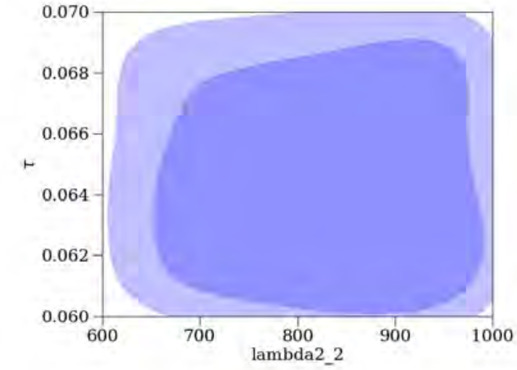
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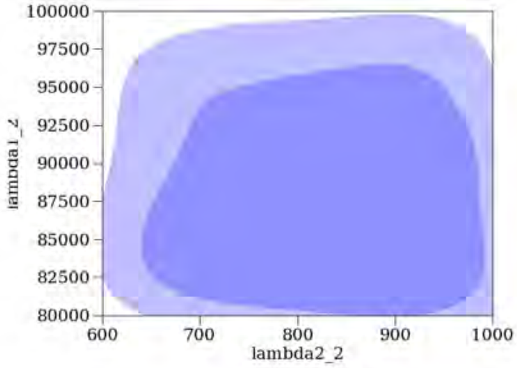
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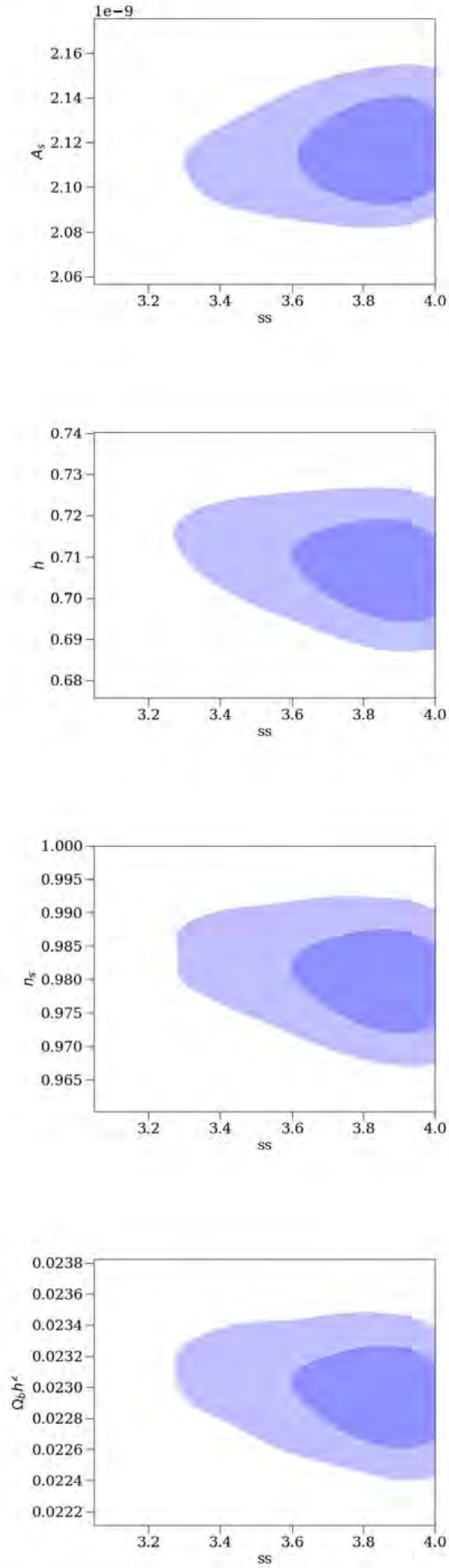
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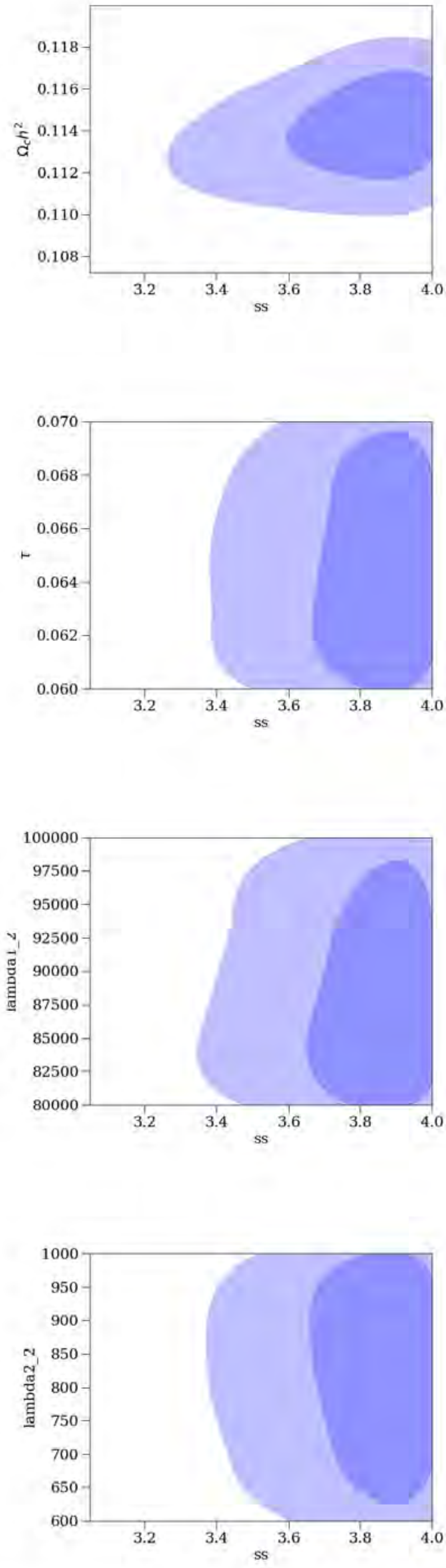
$\Omega_b h^2$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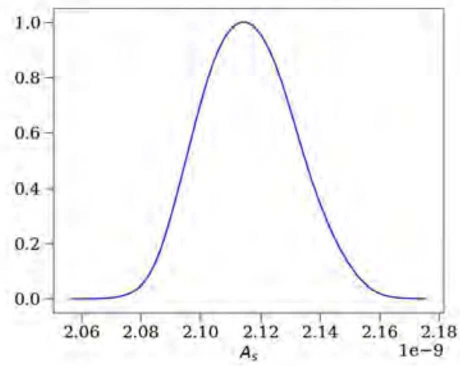
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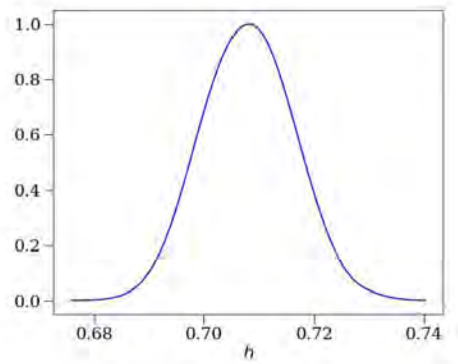
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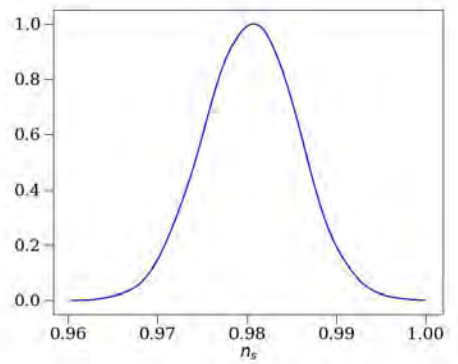
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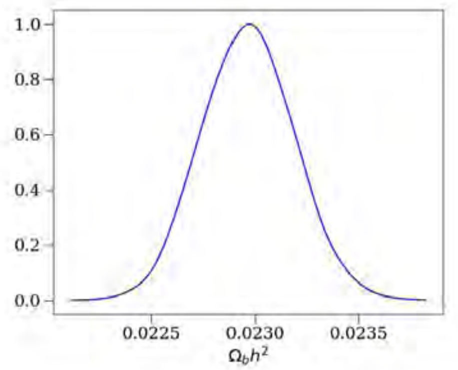
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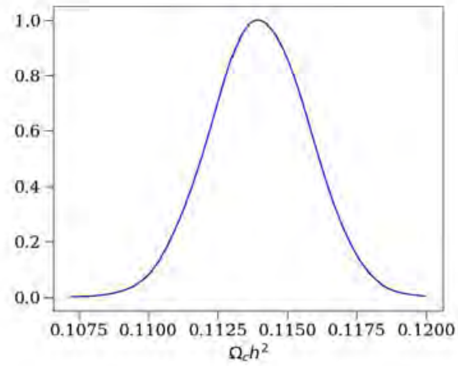
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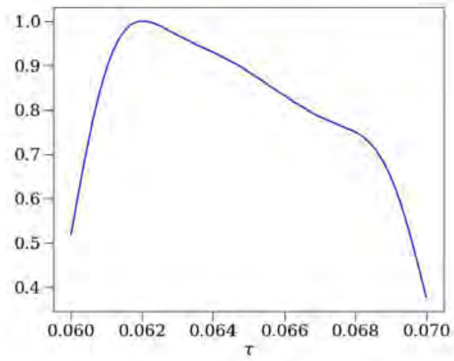
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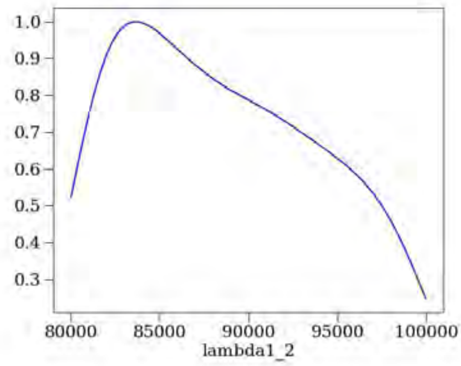
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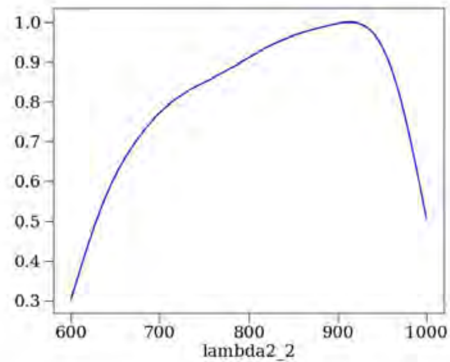
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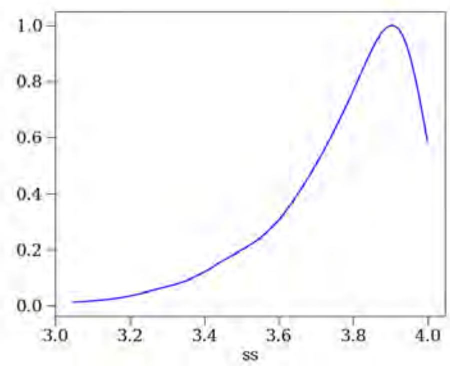
0.5
0.4
0.3
600 700 800 900 1000
lambda2_2



The `multinest` sampler converged much faster that the other methods. We can see also that the contours are much better in definition and the correlations are close to ones we expected from ΛCDM. The λ_1^2 and λ_2^2 parameters are close to the ones examined (Zhao et al., 2009), so we are reinforcing the comments explained in that article. Also the s parameter is close to 4, as expected and declared by (Zhao et al., 2009). In this paper they explain that cosmologically viable $f(R)$ models correspond to $s \approx 4$. And also from numerical simulations (Pogosian and Silvestri, 2008), we find that $f_{RR} \propto a^6$, corresponding to $s \approx 4$.

Conclusions

In this this thesis we built upon an existing Modified Gravity framework a complete experiment for analyzing the results of using the Bertschinger and Zukin (BZ) model and its comparison to the standard ΛCDM model of Cosmology.

We started by setting the theoretical background, covering the basics of Einstein's General Theory of Relativity, then using the conventional assumptions taken by modern Cosmology, we discussed briefly the Friedmann Equations and defined some important cosmological parameters. Then we described the "Dark side" of the Universe, Dark Matter and Dark Energy, focusing on Dark Energy; and finally we presented the ΛCDM model of Cosmology and some of its predictions.

We then gave a short description of Modified Gravity in general, $f(R)$ (where R is the Ricci scalar) gravity, Parametrized Modified Gravity and specifically we presented the parametrized post-Friedmann (PPF) framework that describes all three regimes of modified gravity models that accelerate the expansion without dark energy; finally we showed a specific parametrization developed by Edmund Bertschinger and Phillip Zukin (Bertschinger and Zukin, 2008), the Modified framework we used to build our experiments.

In the chapter about CosmoSIS we needed to set the basis of Bayesian Cosmology, so in the beginning of the chapter we briefly explained the concepts of Bayesian Statistics and its link to Cosmology. Then we exposed some of the current problems in computational cosmology, and how CosmoSIS solve them, then we went to the specifics of CosmoSIS, its structure and how it works. In this last section we talked about some of the modules we used to do our calculations in the last chapter oh this thesis.

Finally we showed our results for using CosmoSIS as an parameter inference framework for MG. We remind that for our analysis we used the MGCAMB implementation existing in CosmoSIS.

We divide the results obtained in this work in two, first the ones relating to computational physics, computational cosmology and the research in cosmology in general:

- We need to build serious open source framework to do computational physics. CosmoSIS is a representative and an example of this. The code has to be well documented, written using standard software engineering methods, have examples of both code and analysis, and finally have a good open source community to answer questions and improve the code.

- The integration of new advances in technology are fundamental for the advances in Cosmology. Here we refer to new languages, Big Data Frameworks (Like Spark), Software Engineering methodologies, programming best practices and Data Science environments should be used to fully describe the scientific process of analyzing data for all Physics.

- CosmoSIS solves lots of problems in the computational cosmology department, and even though if it's ending is not the new standard software for cosmological inference and bayesian cosmology, it should be a starting point for future frameworks.

- The implementation of MGCAMB in CosmoSIS is robust and easier to use than the original code. The BZ model in its implementation will give better results when used with the Multinest sampler.

- The CosmoSIS user has to understand several layers of complexity before successfully use the code, and it's recommended to understand the samplers and Boltzmann codes before using the modules. We don't want to create a black box of black boxes, and even though CosmoSIS is easier to use and interact to, it should be taken seriously by the community and the final user.

- If possible, the author of papers and thesis like this, should share the code created to obtain the results. This will improve reproducibility and it will make much easier the entrace for new cosmologists into the area. The code should be on an online platform like GitHub or BitBucket, not in a weird webpage with a download button.

And now we present the results relating to our results of using CosmoSIS to analyze the BZ framework:

- It is possible to use the CosmoSIS framework to study the properties of a Modified Gravity framework. It is also much easier than in other softwares.
- The results obtained using the MGCAMB module in CosmoSIS are consistent with the original MGCAMB code.
- The CosmoSIS process for cosmological inference should always start with running `maxlike`, a sampler which aims at finding the points of maximum likelihood, this will output priors for the other samplers.
- The `metropolis` sampler converges really slow for the BZ model, and it gives results that are not always correct in respect of correlation of cosmological parameters and its values.
- The `emcee` sampler converges faster than the `metropolis` sampler but the relationships between the parameters of our model and the ones for the standard cosmology were not clear, and some differ a lot from the ones obtained from `metropolis`.
- The `multinest` sampler was the best for our problem. It converged much faster (about 5x) that the other methods, the results of this sampler are the ones we used to explain the cosmological implications of our framework.
- The λ_1^2 and λ_2^2 parameters are close to the ones examined (Zhao et al., 2009), so we are reinforcing the comments explained in that article.
- The s parameter is close to 4, as expected and declared by (Zhao et al., 2009). In this paper they explain that cosmologically viable $f(R)$ models correspond to $s \approx 4$. And also from numerical simulations (Pogosian and Silvestri, 2008), we find that $f_{RR} \propto a^6$, corresponding to $s \approx 4$.

We end now with the same paragraph of the thesis applied to the case:

Cosmology, and computational cosmology, has to keep up with the current technological revolution. Most of the more successful companies, industries and investigation groups in the world are taking advantage of the new programming languages, frameworks, software methodologies, hardware improvements to change and adapt to the actual tendencies of the information era.

We propose that if one follows the process of this thesis one can achieve much better results, and interpret them more easily. This is not the ending of computational cosmology, this is only the beginning.

Appendices

Appendix A

CosmoSIS installation

A.1 Very easy (but limited) installation

This section is inspired by the Readme in the official web page of the tool[1]. This tool uses the "Docker"[2] system to set up the libraries that CosmoSIS depends on inside a "virtual machine"-like system. It is suitable for desktops and laptops through probably not for clusters and supercomputers without specific effort. It will work on Linux and OSX.

A.1.1 Prerequisites

- Update and upgrade Ubuntu repositories and libraries[3]:

```
$ sudo apt-get install update
$ sudo apt-get install upgrade
```

- Install git:

```
$ sudo apt-get install git
```

A.1.2 Instructions

- Install docker (step one of https://docs.docker.com/docker-for-mac/ on OSX) from www.docker.com
- Start Docker running by following the instructions on the site. You do not need to go through the tutorials.
- Download this tool by running:

```
$ git clone https://bitbucket.org/joezuntz/cosmosis-docker
```

- `cd` to the new cosmosis-docker directory and run:

```
$ sudo ./get-cosmosis-and-vm ./cosmosis
```

- Once the download and installation process are complete, run:

```
$ ./start-cosmosis-vm ./cosmosis
```

- First time only, run `update-cosmosis -develop` to get the development version which has some fixes in.
- The first time you do this (or to recompile later), run: `make`

[1] https://bitbucket.org/joezuntz/cosmosis-docker

[2] Docker technically provides "images" and "containers" not virtual machines. But the latter name is more familiar to most people and from the perspective of an end user there isn't that much difference between the two.

[3] The $ symbol represent the terminal and should not be copied when running the following commands

In future you just need to do step 5. Your terminal will now be in the CosmoSIS virtual machine, with a different operating system and files than your usual computer. The only overlap is the directory `cosmosis` where you start, which is mapped to the subdirectory cosmosis. You should use the "virtual" terminal just to run `make`, `cosmosis`, and `postprocess`, and use a separate terminal for everything else like editing files and viewing plots.

A.2 Easy (not that easy) installation

This installation is specific for Ubuntu systems (14.04 or 16.04). It is based on the official installation[4] but with some tweaks. It was the code used for the development of this work, and the git branch for the thesis will remain active for reproducibility.

Everything this script installs is put in a single directory which can be cleanly deleted and will not interfere with your other installed programs.

A.2.1 Prerequisites

- Update and upgrade Ubuntu repositories and libraries:

```
$ sudo apt-get install update
$ sudo apt-get install upgrade
```

- Install git:

```
$ sudo apt-get install git
```

- Install curl:

```
$ sudo apt-get install curl
```

A.2.2 Instructions

- Download and execute the bootstrap script[5] by running the following lines. Make sure you are running the bash shell.

```
$ curl -L --remote-name https://raw.githubusercontent.com/FavioVazquez/cosmosis-
    bootstrap/master/cosmosis-bootstrap
$ chmod u+x cosmosis-bootstrap
$ ./cosmosis-bootstrap -d <new desired target directory e.g. "cosmosis">
```

  The `-d` will download data from Planck and WMAP, so if you want a faster download omit it. You can always download them later.

- Go to your new cosmosis directory with

```
 $ cd <the new desired target directory that just got created>
```

- Set up the CosmoSIS environment (this must be done every time you use the program):

```
 $ source config/setup-cosmosis
```

- Build CosmoSIS libraries and included modules:

```
 $ make
```

For an installations with bootstrap for other systems visit https://bitbucket.org/joezuntz/cosmosis/wiki/bootstrap, or for a manual and advanced installation visit https://bitbucket.org/joezuntz/cosmosis/wiki/Manual%20Install.

[4] https://bitbucket.org/joezuntz/cosmosis/wiki/bootstrap
[5] https://github.com/FavioVazquez/cosmosis-bootstrap

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