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QUANTUM IN FORMATION: A FACTUAL APPROACH TO QUANTUM MECHANICS

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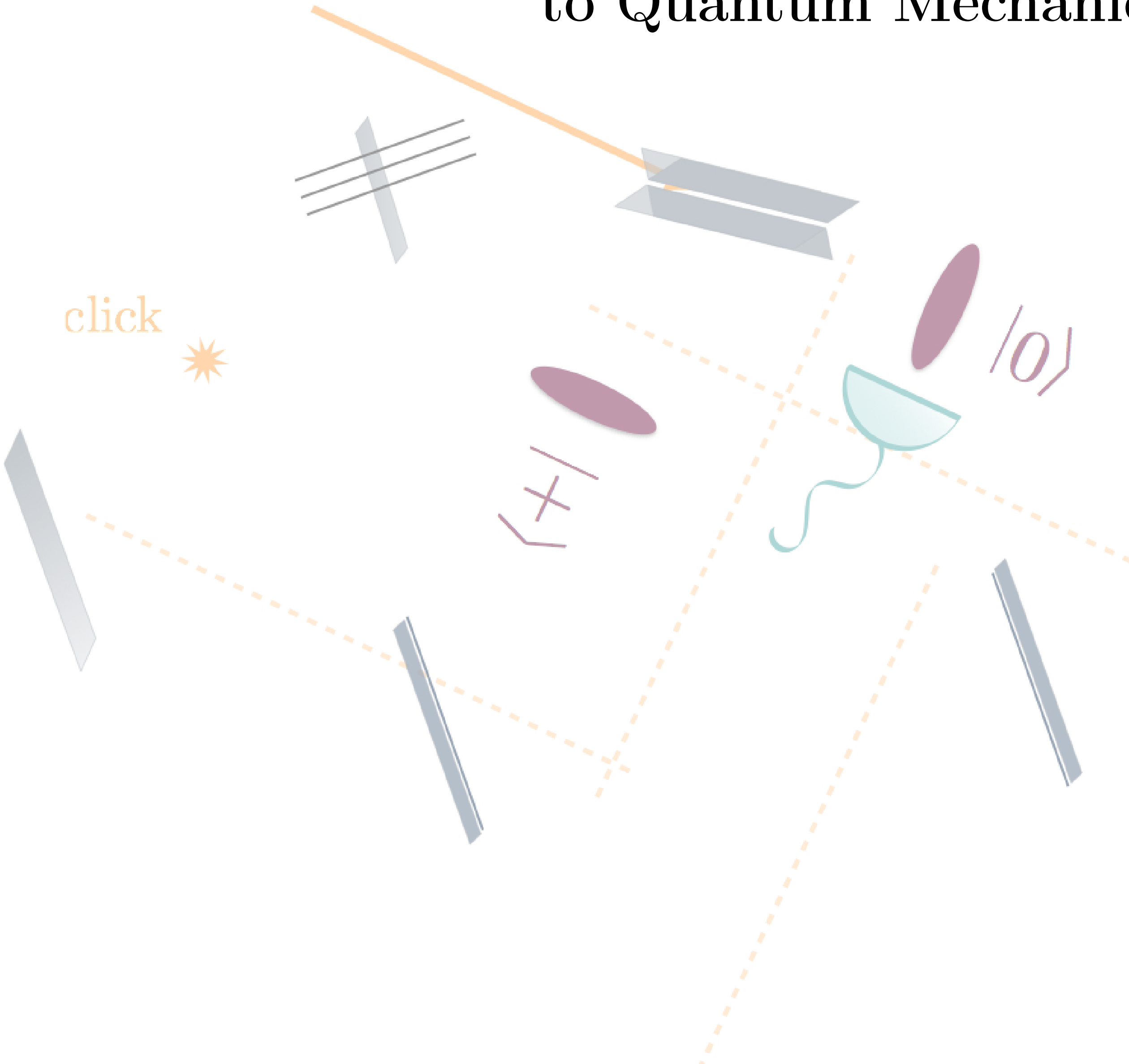

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A mi par enredado:
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Cuanto en formación:
Una aproximación factual a la Mecánica Cuántica

Resumen

En este trabajo se trata el debate sobre localidad, contextualidad y realismo en la Mecánica Cuántica y para ello se exponen y analizan los principales teoremas de imposibilidad de variables ocultas. Se muestra el realismo contextual de la teoría, mientras que se da una interpretación que permite recuperar localidad en la misma, *factualidad*. Se examina el teorema de PBR para demostrar que el realismo de un estado y la medición del mismo están inseparablemente ligados, es decir, que sin medición no se puede concluir realismo de un cierto estado cuántico. Se utiliza este hecho para construir el formalismo de factualidad.

El formalismo de factualidad clasifica a los estados entre estados cuánticos (que son mera información) y estados ontológicos (que tienen una contraparte en la realidad) y postula la existencia de una función de evolución determinista que dictará el estado ontológico de un sistema a todo tiempo t como función de ciertas variables ocultas λ y de t.

En la tesis se muestra que esta función determinista puede existir sin violación alguna de la desigualdad de Bell, siempre y cuando se mantenga la premisa de factualidad sobre la realidad. De este modo, se reconcilian la Mecánica Cuántica y la Relatividad Especial.

Quantum in formation:
A factual approach to Quantum Mechanics

Abstract

In this work the debate about locality, contextuality and realism in Quantum Mechanics is treated and, to this end, the major no-hidden-variables theorems are exposed and analyzed. The contextual realism of the theory is shown, while an interpretation that allows for the recovery of locality is given, *factuality*. The PBR theorem is examined to demonstrate that the realism of a state and its measurement are inseparably linked, that is, that without a measurement, realism of a certain quantum state cannot be concluded. This fact is used to construct the factuality formalism.

The factuality formalism classifies states between quantum states (which are mere information) and ontological states (which have a counterpart in reality) and postulates the existence of a deterministic evolution function that would dictate the ontological state of a system at any given time t, as a function of certain hidden variables λ and of t.

It is demonstrated that, as long as the premise of factuality over reality is maintained, this deterministic function can exist with no violation whatsoever of Bell's inequality. In this way, Quantum Mechanics and Special Relativity are reconciled.

Contents

Introduction

Here is yet another work on the paradoxical aspects of Quantum Mechanics. Work keeps coming out because no winning interpretation has been found; and we are not giving a final interpretation here, we are just giving out some tools that might help us arrive to one.

We can derive all the paradoxical aspects of Quantum Mechanics from the double-slit experiment. After all, the outcomes of this experiment are extremely non-local, they display two incompatible aspects of matter (particle and wave), and the result is linked to our knowledge of any one of the incompatible aspects: knowledge of position erases all momentum properties.

In order to get a clear view of what is behind these paradoxical aspects, we are going to make a revision of what has been said since the beginning of Quantum Theory regarding incompatible quantities and non-locality. We dive in this way into the EPR paradox, and the currents that responded to it. These currents take us to the no-go theorems that have been built in order to grasp what can be and cannot be said about this quantum ocean. Are there hidden entities in the unexplored deep waters that determine what we see from above? We will address these questions with a special tool: the factuality anchor. This anchor is going to let things that seem non-local from the surface have a local explanation from the bottom, in spite of the theorems that impose non-locality to the root.

For the factuality tool we will propose a deterministic formalism that takes into account only that which actually has a counterpart in reality, as can be shown by means of the PBR theorem. With this formalism we strive to single out what we see in the world because it is real, from what we might see because it is possible. This distinction being important considering how the view of the world as possibilities has puzzled our understanding of Nature.

Chapter 1

Theoretical framework

The entrance of Quantum Mechanics to the human understanding of Nature was not a glorious one, not even an easily accepted one. Much debate started when the uncommon predictions and aspects of the theory began to be realized. One of these aspects was put forward by Einstein, Podolsky and Rosen [1] , who showed that Quantum Mechanics was not a complete theory in the sense that there were elements of reality that were not comprised by the theory.

After the mentioned paper was published, a wave of interpretations entered the picture. Questions about the reality of the quantum state (or wave function), the ontology of the wave-particle duality, what lies behind the uncertainty principle, the non-local correlations that arise from entanglement and the measuring process, have found divergent answers. Nowadays, it seems that it is an individual task to pick the most convincing one when, as physicists, we would like to have an experimentally testable hypothesis that would do both things: overcome the present paradoxes and provide us with a stable and objective answer.

This first chapter is a revision of the main ideas that have been floating around since the beginning of the last century concerning these questions.

1.1 The Einstein, Podolsky and Rosen paper

In a paper entitled *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* [1], Einstein, Podolsky and Rosen (EPR) gave proof that Quantum Mechanics, as it was, was an incomplete theory, the condition of completeness being that:

> every element of the physical reality must have a counterpart in the physical theory.

Of course, to pose this condition one has to define an *element of physical reality*, and they did so through the following statement:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

And, since the wave function description of reality given by Quantum Mechanics does not provide the information necessary to predict with certainty two different quantities that are represented by operators that do not commute, they ascertained that: either the quantum-mechanical description is not complete, or these two quantities do not have simultaneous reality.

To demonstrate incompleteness, they proposed an experiment of two particles that are prepared together in the state

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{ik(x_1 - x_2 + x_0)} dp, \tag{1.1}$$

where x_0 is some constant and $k = p/\hbar$. They put forward the eigenfunctions of momentum of the two particles $u_p(x_1) = e^{ikx_1}$ and $\psi_p(x_2) = e^{-ik(x_2 - x_0)}$ (with eigenvalues p and $-p$ respectively) so that state (1.1) can be expanded as:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} u_p(x_1)\psi_p(x_2) dp. \tag{1.2}$$

They then showed that state (1.1) can also be expanded in eigenfunctions of position. This is done by using the fact that

$$\delta(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ips} dp,$$

setting $s = 2\pi(x - x_2 + x_0)/h$ so that

$$\delta\left(\frac{2\pi}{h}(x - x_2 + x_0)\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ip(2\pi/h)(x - x_2 + x_0)} dp.$$

But

$$\delta(ax) = \frac{1}{|a|}\delta(x),$$

so

$$h\delta(x - x_2 + x_0) = \int_{-\infty}^{\infty} e^{ik(x - x_2 + x_0)} dp,$$

and $\varphi_x(x_2) = h\delta(x - x_2 + x_0)$ is an eigenfunction of position of the second particle with eigenvalue $x + x_0$. Since $v_x(x_1) = \delta(x - x_1)$ is an eigenfunction of position of the first particle with eigenvalue x, the expansion of the state is straightforward:

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} v_x(x_1)\varphi_x(x_2) dx. \tag{1.3}$$

Now that these two expansions have been given, we can continue with EPR's argument.

If one were to measure the position of particle 1 and got a result of, say, q then through reduction of the wave packet (1.3) one would know that particle 2 is in the state $\varphi_q(x_2)$, with position $q + x_0$. Analogously, if one were to measure the momentum of particle 1 and got maybe r, through reduction of (1.2) one would be positive that particle 2 is in the state $\psi_r(x_2)$, with momentum $-r$.

Since just by measuring over system 1 *without in any way disturbing system 2* (locality assumption) *we can predict with certainty the value of position or momentum of system 2*, they concluded that the quantities of position and momentum must have a simultaneous reality, and therefore that Quantum Mechanics is not a complete theory, as it does not give an account of these two properties of the system.

1.2 Bohr and the complementarity principle

The first to respond to the logic of EPR in disagreement was Bohr [2]. He showed, by means of a thought experiment, that the measurement requirement of EPR was unrealizable. He argued that through measuring the position of one of the particles, the correlation of momentum between them is completely destroyed and, thus, that the requirement of a physical situation in which the momentum and position of particle 2 could be *both* predicted with certainty (from measurements performed over particle 1) is not a realistic one. He was therefore convinced of the fact that these two quantities need not have a simultaneous counterpart in the theory, from where they could be simultaneously predicted.

Bohr had to attend to the condition of physical reality imposed by EPR, "without in any way disturbing a system", and he did so by clarifying:

> Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.* Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. ... In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena, that the notion of complementarity aims at characterizing.

The key aspect of this affirmation is the link Bohr put forward between *conditions* and *physical reality*: he used the word inherent to show the inseparable nature of these two sides of any phenomenon. He went on talking about *mutual exclusion* and *complementary physical* *quantities* to specify how important it is to grasp these new features of the physical description of reality, so important that we should coin a new term: complementarity.

Complementarity meant for Bohr an understanding of physical reality in regards to reference frames, the defining objects of reference frames being the measuring apparatuses and the quantities coming into being within these reference frames as complementary, meaning that two or more complementary quantities cannot manifest in one and the same reference frame, and that each quantity must manifest in its corresponding reference frame.

Complementarity and reference frames was a main part of Bohr's interpretation of Quantum Mechanics but, as Faye [3] affirms, he

> flatly denied the ontological thesis that the subject has any direct impact on the outcome of a measurement. Hence, when he occasionally mentioned the subjective character of quantum phenomena and the difficulties of distinguishing the object from the subject in quantum mechanics, he did not think of it as a problem confined to the observation of atoms alone. For instance, he stated that already "the theory of relativity reminds us of the subjective character of all physical phenomena" [4]. Rather, by referring to the subjective character of quantum phenomena he was expressing the epistemological thesis that all observations in physics are in fact context-dependent. There exists, according to Bohr, no view from nowhere in virtue of which quantum objects can be described.

And as Dickson [5] puts it:

> Bohr believes that the 'uncontrollable exchange' of momentum and energy between a measured system and a measuring apparatus entails that experimental situations that allow the determination of a particle's position relative to a given reference frame forbid the determination of its (simultaneous) momentum relative to that frame[.]

So Bohr regarded Quantum Mechanics as a complete theory, in which properties of Nature are defined in terms of reference frames of the observer, viewing the reference frames as the specific context in which one can talk about the reality of anything.

1.3 Bohm's version

In his book *Quantum Theory* [6] Bohm talked about the EPR paradox giving a rather different example. He used the fact that the spin projection of a particle exhibits a complementary behavior in its non-commuting operators[1], instead of the complementary quantities of position and momentum, and gave as an example a pair of particles of spin $\hbar/2$ glued in a state of total spin equal to zero (*a molecule containing two atoms*). The proposed system is then in a singlet state, i.e.,

[1] $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$, where $S_\alpha = \frac{\hbar}{2}\hat{\sigma}_\alpha$.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big].$$

Taking this initial state of the system, Bohm went through the same arguments of EPR, with any two non-commuting operators of spin projection instead of the position and momentum operators. As Dickson notes, the response Bohr gave to the EPR example is not automatically extended to the example of Bohm. Bohm provided his own response to the EPR paradox he built: he had a concept of reality that differed from that given by EPR, in that he did not regard properties such as momentum and position (and spin) *as belonging to the electron alone*, but as properties that arise when a particular interaction with a measurement device takes place. Furthermore, he discarded the one-to-one correspondence between mathematical aspects of the theory and physical aspects of the systems as a necessary requirement for a complete theory. By doing this, Bohm could conclude that

> the wave function (in principle) can provide the most complete possible description of the system that is consistent with the actual structure of matter[,]

and thus, that Quantum Mechanics is a complete theory.

Later on, in a paper published with Aharonov [7], Bohm took *the essence of the paradox of* EPR to be that, while it is understandable that a measurement over particle 1 would imply the realization of a certain potentiality[2], it is not clear how this potentiality could be realized by a system (particle 2) on which no measurement or interaction has taken place.

This moved the focus of the paradox from the question of completeness to the question of locality, for if Quantum Mechanics is complete, and no new physical interpretation enters the picture, then Quantum Mechanics must be a non-local theory. Bohm and Aharonov explicitly gave two ways out: either there is a hidden non-local interaction *(quantum potential)* between the spatially separated particles, or one can develop a *further new explanation of the quantum theory in terms of a deeper subquantum-mechanical level.* This new explanation would be what later became known as Bohm's pilot wave interpretation and Bohmian Mechanics.

1.4 No-go theorems

Soon enough, a handful of interpretations started to wander throughout the realm of Quantum Physics and, as a way to confirm or discard such different interpretations, no-go theorems appeared. In this section we will see the most relevant no-go theorems for the present work.

1.4.1 Contextuality

It is convenient to begin with the no-go theorems that address contextuality due to their strong link with the principle of complementarity (Bohr) and with the view of Quantum Mechanics

[2] In Bohm [6]:*properties of matter do not, in general, exist separately in a given object in a precisely defined form. They are, instead, incompletely defined potentialities realized in more definite form only in interaction with other systems, such as a measuring apparatus[.]*

as a description of potentialities (Bohm).

For Bohr complementarity meant the existence of mutually exclusive quantities which arise in mutually exclusive reference frames (each reference frame defined by a measurement apparatus). For Bohr a measurement of position defined one reference frame and a measurement of momentum defined another (both mutually exclusive), but he was never explicit about spin measurements for non-compatible observables (non-commuting operators). What he did expose was the nature of context-dependent quantities and the explicit link between a context and a reference frame. Hence, if we regard spin as a context-dependent quantity, we can make a leap and regard non-compatible projections of spin as complementary quantities which arise in specific contexts, each context defined by a different measuring condition.

On a parallel side, Bohm interpreted the wave function description of a system as a set of potentialities the system *has* and *realizes* with a certain probability when analyzed under certain conditions, i.e., measurement settings.

For both, the physical properties of a system are not pre-existing properties, independent of the context, instead they only come into being when the particular system interacts with a certain context, that is, with any kind of environment that would force a given property to arise.

Along the same lines, contextuality no-go theorems prove that non-contextual hidden variables cannot be assigned to a quantum system, i.e., that it is impossible to specify the value of every possible property of the system without specifying the context in which each of those properties might attain such a value. The work on this theme was developed by several theorists[3], and the most successful proof at the time was given by Kochen and Specker [12]: they showed that it was impossible to assign to a spin-1 system simultaneous values of S_α^2 for 117 different directions (denoted by α); their demonstration required a very elaborate construction.

Since then, simpler versions of the Kochen-Specker theorem have been constructed, one of the simplest one was proposed by Peres in 1990 [13] and complemented by Mermin [14] that same year. Peres took as an example two particles in a singlet state, and demonstrated that if one were to assign values α_i to observables $\hat{\sigma}_\alpha^i$ ($i = 1, 2$; $\alpha = x, y, z$), then a simultaneous assignment of values to the six observables

$$\hat{\sigma}_x^1,\ \hat{\sigma}_x^2,\ \hat{\sigma}_y^1,\ \hat{\sigma}_y^2,\ \hat{\sigma}_x^1\hat{\sigma}_y^2,\ \text{and}\ \hat{\sigma}_y^1\hat{\sigma}_x^2,$$

was unrealizable, since it led to a contradiction, as we will see. Peres knows that the product of the values α_1 and α_2 assigned to $\hat{\sigma}_\alpha^1$ and $\hat{\sigma}_\alpha^2$ ($\alpha = x, y, z$) must satisfy

$$x_1 x_2 = y_1 y_2 = z_1 z_2 = -1, \tag{1.4}$$

considering that the system is in a singlet state. Furthermore, values $x_1 y_2$ and $y_1 x_2$ can be simultaneously assigned to the observables $\hat{\sigma}_x^1\hat{\sigma}_y^2$ and $\hat{\sigma}_y^1\hat{\sigma}_x^2$, given that these two observables commute (the commutation relations are shown in Appendix A.1). Now, as is also shown in Appendix A.1:

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^1\hat{\sigma}_x^2 = \hat{\sigma}_z^1\hat{\sigma}_z^2\,;$$

[3] Von Neumann, Gleason, Jauch, Piron, Bell, to mention a few [8–11].

consequently, the values assigned to the observables must satisfy the relation

$$(x_1 y_2)(y_1 x_2) = z_1 z_2, \tag{1.5}$$

and the contradiction arrises:

$$1 = (-1)(-1) \overset{(1.4)}{=} (x_1 x_2)(y_1 y_2) = (x_1 y_2)(y_1 x_2) \overset{(1.5)}{=} z_1 z_2 \overset{(1.4)}{=} -1.$$

The equality in the middle is true since values are real numbers, so they trivially commute.

Mermin showed that by adding three observables to the set given by Peres one could arrive to a contradiction when arranging them in sets of three mutually commuting operators, regardless of the state of the system; the arrangement is given in Table 1.1. The proof is done by demonstrating that the product of the three operators in each row gives I_2 just as the product of the three operators in each of the first two columns, while the product of the operators in the last column gives $-I_2$ (cf. Appendix A.2). This means that if one were to multiply all the values assigned to a state related to all the observables in the table by multiplying the rows the result would be 1, while if one were to do it by multiplying the columns the result would be -1, which is contradictory.

Table 1.1: Set of observables proposed by Mermin to derive a contradiction if one were to assign a pre-existing value to each observable.

$\hat{\sigma}_x^1$	$\hat{\sigma}_x^2$	$\hat{\sigma}_x^1 \hat{\sigma}_x^2$
$\hat{\sigma}_y^2$	$\hat{\sigma}_y^1$	$\hat{\sigma}_y^1 \hat{\sigma}_y^2$
$\hat{\sigma}_x^1 \hat{\sigma}_y^2$	$\hat{\sigma}_y^1 \hat{\sigma}_x^2$	$\hat{\sigma}_z^1 \hat{\sigma}_z^2$

This is a special case in which hidden variables that determine the values of all the possible properties of one system cannot exist independent of the context. Many other no-go theorems of contextual hidden variables have been built, leading us to the certainty that a Quantum Theory with non-contextual hidden variables cannot be built, but, can hidden variables exist in a local way? I.e., can the value of a property of one system be independent of that of a spatially separated system? We turn to the next section to examine this question.

1.4.2 Locality

The widely renowned proof of the non-local nature of Quantum Mechanics is given by the violation of Bell's inequality. Hereby we present his demonstration.

Bell works with the thought experiment of Bohm and Aharonov: suppose a pair of entangled electrons in a singlet state is split into two electrons at time $t = t_0$. If the spin projection of electron A is measured at a later time along direction $\vec{z}$ and we get, for example, $|\uparrow\rangle_A^z$ then we can be sure that the spin projection of electron B is $|\downarrow\rangle_B^z$.

Through locality, he associates with the value of spin projection of each electron a hidden variable quality, that is, the existence of a function A of the direction of the detector $\vec{a}$ and of a hidden variable λ such that $A(\vec{a}, \lambda) = \pm 1$ results in the value of the spin projection of particle A along direction $\vec{a}$, and same for B along any direction $\vec{b}$. The expectation value of the correlation between A (measured in the direction $\vec{a}$) and B (measured in the direction $\vec{b}$) naturally arises,

$$E(\vec{a}, \vec{b}) = \int_\Lambda A(\vec{a}, \lambda) B(\vec{b}, \lambda) \rho(\lambda) d\lambda,$$

and, as Bell shows [15], this expectation value must satisfy:

$$\left| E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c}) \right| \leq 1 + E(\vec{b}, \vec{c}), \tag{1.6}$$

where $\vec{a}$, $\vec{b}$, and $\vec{c}$ are three alternative directions of the detectors used to measure the spin projection of the electrons—inequality (1.6) is Bell's inequality and its derivation is given in Appendix A.3. Therefore, if the non-local correlation between distant systems could be explained by means of local hidden variables, inequality (1.6) should be satisfied.

The expectation value of the correlation between measurements of spin projection, over two particles in a singlet state, is predicted by Quantum Mechanics to be:

$$E(\vec{a}, \vec{b}) = \langle \hat{\sigma}_a \hat{\sigma}_b \rangle = -\cos\theta_{ab} = -\vec{a} \cdot \vec{b}, \tag{1.7}$$

as shown in Appendix A.4. So, if one were to choose three directions such that $\theta_{ab} = \pi/2$, $\theta_{ac} = \pi/4$, and $\theta_{bc} = \pi/4$, and to substitute in equation (1.6) with the quantum mechanical predictions, one would arrive to the contradiction:

$$\begin{aligned} \left| -\cos\frac{\pi}{2} + \cos\frac{\pi}{4} \right| &\leq 1 - \cos\frac{\pi}{4} \\ \longrightarrow \quad \frac{1}{\sqrt{2}} &\leq 1 - \frac{1}{\sqrt{2}} \\ \longrightarrow \quad \sqrt{2} &\leq 1, \end{aligned}$$

which has been experimentally tested[4] [17] [18]. This fact allowed Bell to conclude that Quantum Mechanics cannot be completed by a local hidden variables account.

[4]The inequality that is tested for in experiments is the CHSH inequality [16], which uses the same hypotheses as Bell's inequality and is built on the same grounds. For the purpose of our theoretical work, it is enough to deal with the original version of the inequality.

In 1989, 25 years after Bell derived his inequality, Greenberger, Horne and Zeilinger (GHZ) [19] came up with another way to arrive at the same conclusion. Following the logic of GHZ, in 1990 Mermin [20] wrote the version of the GHZ theorem that is used nowadays. For his version, he put forward the three commuting operators[5]

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3, \;\; \hat{\sigma}_y^1\hat{\sigma}_x^2\hat{\sigma}_y^3, \;\; \text{and} \;\; \hat{\sigma}_y^1\hat{\sigma}_y^2\hat{\sigma}_x^3, \tag{1.8}$$

and the GHZ state of three spin-1/2 particles

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\Big[|{\uparrow\uparrow\uparrow}\rangle - |{\downarrow\downarrow\downarrow}\rangle\Big], \tag{1.9}$$

which is an eigenstate of all three observables in (1.8) with eigenvalue 1—as shown in Appendix A.5.

Since (1.9) is an eigenstate of all three observables in (1.8) with eigenvalue 1, the result of measuring any of the observables in (1.8) over the three-particle system must yield unity. As a consequence, if $\hat{\sigma}_y$ is measured over two of the particles, one can immediately conclude the value x_i related to a measurement of $\hat{\sigma}_x^i$ over the third; analogously, if $\hat{\sigma}_x$ is measured over one particle, and $\hat{\sigma}_y$ is measured over another, one can derive the value y_i that a measurement of $\hat{\sigma}_y^i$ over the last would yield[6].

Mermin went on assuming that the three particles of the system could be separated and subsequently measured, and he noted that measuring over two particles would determine either the value x_i, or the value y_i, of the third. In order for such values to be local, one has to impose that x_i and y_i are pre-existing values in every one of the particles, and so, an assignment of values to the observables $\hat{\sigma}_x^i$ and $\hat{\sigma}_y^i$ must be made. Given the eigenvalue equation, the values assigned must be such that:

$$x_1y_2y_3 = 1, \;\; y_1x_2y_3 = 1, \;\; \text{and} \;\; y_1y_2x_3 = 1.$$

Now, the multiplication of all three equalities gives unity, y_j appears twice for each particle, and $y_j = \pm 1$; so the multiplication of all y_j's is 1 and can be factorized so as to leave:

$$x_1x_2x_3 = 1. \tag{1.10}$$

Mermin finished by showing that $\hat{\sigma}_x^1\hat{\sigma}_x^2\hat{\sigma}_x^3|GHZ\rangle = -1$, which is contradictory with the result in (1.10), and thus, demonstrated that a local assignment of values is at odds with the predictions of Quantum Mechanics.

The experimental realization of a GHZ state was achieved in 2000 [21]. There, the predicted correlations between the three particles were ascertained, and the eigenvalue equations confirmed, ruling out local interpretations of Quantum Mechanics.

[5] That they pairwise commute arises from the fact that any two of these three operators share one observable on the same Hilbert space and what is left out is always the commutator $\left[\hat{\sigma}_x^i\hat{\sigma}_y^j, \hat{\sigma}_y^i\hat{\sigma}_x^j\right]$ which equals zero, as it is shown in Appendix A.1.

[6] Here we use the notation of Section 1.4.1 in which x_1 is the value assigned to the observable $\hat{\sigma}_x^1$ and so forth.

In this way, the theorems of Bell and GHZ left us with no possibility to assign the necessary values to properties of the particles that would allow for Quantum Mechanics to be reconstructed locally. We will revisit these theorems in Chapter 3, where we will deal with them from our *factuality* point of view in order to show that these results can be reinterpreted, and that locality can be recovered.

1.4.3 Reality

In the previous sections we talked about aspects of the theory. We reviewed the strange behavior of the realm described by Quantum Mechanics implicit in the existence of complementary quantities, in the essence of contextual dependent values, and in the non-local behavior of the entities this theory characterizes. It is this framework that raises the interest in addressing a particular question: to what extent do those and other aspects mimic the nature of *reality*?

Embedded in our scientific labour is the premise that there exists an objective universe suitable for description. Our job is to construct theories which faithfully describe this universe. From labels, definitions, experimental observations, postulates, laws, and an interplay between them, we strive to make predictions and to answer fundamental questions about the physical surroundings. Our present interest is directed towards answering the question of how is it that quantum mechanical concepts and phenomena have a counterpart in reality. Is it as physical entities or is it as mere information?

We have a definition of an electron, and properties such as its mass and charge have been studied and measured; we view the electron, its mass and its charge as real. We have a classical understanding of these properties. We also have a definition of a wave and a definition of a particle. And, although they are opposite in nature, both describe an ensemble of electrons in its behavior, so the question arises: is the wave function a real property of the electron or just a statistical description (information) of the ensemble of electrons?

On a recently published paper [22] Pusey, Barrett and Rudolph (PBR) addressed this matter. They demonstrated that

> [if] a system has a ‘real physical state’—not necessarily completely described by quantum theory, but objective and independent of the observer... [and if] systems that are prepared independently have independent physical states[,]

then a pure state cannot be regarded as mere information, or else one would arrive to a contradiction with the quantum theoretical framework.

They carry out their demonstration by adopting the notion given by Harrigan and Spekkens [23] of what ‘mere information’ looks like. This notion goes as follows: if λ is a label for the physical state of the system, and two different, but non-orthogonal preparations, $|\psi_0\rangle$ and $|\psi_1\rangle$, are mere information about some physical state, then the distribution $\mu_0(\lambda)$, which assigns the probability of $|\psi_0\rangle$ resulting in a physical state λ, *and* the distribution $\mu_1(\lambda)$ of $|\psi_1\rangle$, overlap; so that $|\psi_0\rangle$ and $|\psi_1\rangle$ can both result in a physical state λ_p from the overlap region with non-zero probability. The scheme of this notion is given in Figure 1.1.

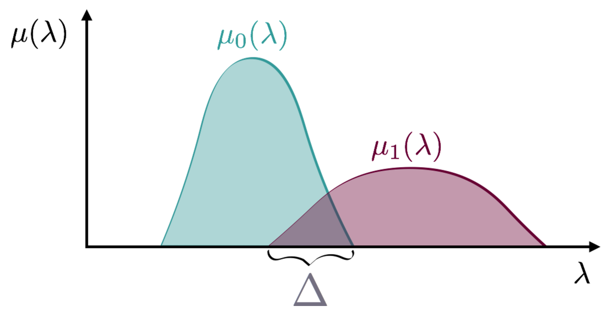


Figure 1.1: The notion of $|\psi_0\rangle$ and $|\psi_1\rangle$ representing mere information is illustrated. $|\psi_0\rangle$ and $|\psi_1\rangle$ are two non-orthogonal states. $\mu_0(\lambda)$ is the probability distribution of $|\psi_0\rangle$ over the space of physical states, and $\mu_1(\lambda)$ is that of $|\psi_1\rangle$. The defining quality is that these two distributions overlap, and there is a non-zero probability that the physical state of both preparations,—$|\psi_0\rangle$ and $|\psi_1\rangle$,—might be in the overlap region Δ.

PBR go on by considering two identical and independent preparation devices, each device prepares a system in either the quantum state $|\psi_0\rangle = |0\rangle$ or the quantum state $|\psi_1\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. They assume that if these two states represent mere information, there is a probability $q^2 > 0$ that both systems result in physical states, λ_1 and λ_2, from the overlap region, Δ. So if this happens, when the two states are brought together the complete system is compatible with any of the four quantum states:

$$|0\rangle \otimes |0\rangle, \;\; |0\rangle \otimes |+\rangle, \;\; |+\rangle \otimes |0\rangle, \;\; \text{and} \;\; |+\rangle \otimes |+\rangle.$$

The complete system can be subsequently measured, and for this they propose an entangled measurement with the four possible outcomes:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |1\rangle \;+\; |1\rangle \otimes |0\rangle\Big],$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |-\rangle \;+\; |1\rangle \otimes |+\rangle\Big],$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |1\rangle \;+\; |-\rangle \otimes |0\rangle\Big],$$

$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |-\rangle \;+\; |-\rangle \otimes |+\rangle\Big];$$

but the probability that the quantum state $|0\rangle \otimes |0\rangle$ results in $|\xi_1\rangle$ is zero, same for $|0\rangle \otimes |+\rangle$ resulting in $|\xi_2\rangle$, for $|+\rangle \otimes |0\rangle$ resulting in $|\xi_3\rangle$, and for $|+\rangle \otimes |+\rangle$ resulting in $|\xi_4\rangle$. This takes them to the conclusion that if the state $\lambda_1\lambda_2$ that arrives to the detector is compatible with the four quantum states, then the measuring device could give a result that should occur with zero probability. This contradiction arises simply by assuming that the distributions of $|0\rangle$ and

$|+\rangle$ overlap, so their distributions cannot overlap, and therefore $|0\rangle$ and $|+\rangle$ cannot represent mere information of an underlying physical system.

They extend their demonstration to any pair of quantum states,

$$|\psi_0\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle,$$
$$|\psi_1\rangle = \cos\frac{\theta}{2}|0\rangle - \sin\frac{\theta}{2}|1\rangle,$$

and thereby show that pure states must have a direct counterpart in reality: that $|\psi\rangle$ must represent a physical property of the physical system.

Chapter 2

Factuality

In this chapter we will explore the results given before, and elaborate on a possible way to solve the problems that come by when one is confronted with the strange aspects of Quantum Mechanics and the no-go theorems that act as a barrier we cannot cross.

As a starting point we shall define one concept and its negation, **factuality** and **counterfactuality**.

We begin with the **factuality** concept by enunciating that the physical world we live in is factual iff solely what actually happens has a counterpart in reality.

On the other side, **counterfactuality** implies that something that could have happened is as real as that which actually happened.

2.1 EPR, Bohr, Bohm and contextuality

If we go back to the work of EPR we see that the hypotheses their logic is built on are that:

- Reality is local.
- Quantum Mechanics is complete.
- Quantum Mechanics is counterfactual.

And with these hypotheses they conclude that:

- Non-commuting quantities have a simultaneous reality.

But this conclusion is equivalent to the premise that Quantum Mechanics is non-contextual, and we know this cannot be true from the wide variety of no-go theorems that dismiss non-contextuality, so one of the three hypotheses must be incorrect.

It is clear that Bohr denied counterfactual reasoning, by showing that a counterfactual circumstance was physically impossible within the example EPR put forward. Bohm went further with his personal example, disregarding the conclusion EPR arrived at from his potentialities point of view, noticing the non-local correlations that arise instead, and dismissing non-local interactions as a possible explanation for these correlations. Therefore, both would regard the third hypothesis as the incorrect one. In our analysis we do so too.

We move on by postulating that *Quantum Mechanics is factual.* However, before going further, we shall note that EPR responded to our previous reasoning from the very beginning, given that in their 1935 paper they wrote:

> Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted.* [...] This makes the reality of P and Q [momentum and position of the second system] depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

Yet, by now, we know that whatever definition of reality we arrive at must permit this "dependence". Such definition of reality does not necessarily have to imply a straightforward dependence of distant systems between them, but must at least encompass the nature and mechanism of entanglement, that is, we must have a definition of reality in which non-local correlations arise in a natural way.

We will show that by restricting Quantum Mechanics to be factual, we have a definition of reality that permits quantum correlations to arise, while remaining local and contextual.

2.2 The Pusey, Barrett and Rudolph paper

Before getting into the factuality formalism, we will revisit the PBR paper, to show that their proof of the reality of the quantum state necessarily implies a measurement over the quantum system.

It is important to notice that the experimental setup PBR propose entails a preparation method that makes the state $|0\rangle$ distinguishable from the state $|+\rangle$ (cf. Subsection 1.4.3) and thus gives the observer knowledge of the state of each system that is being prepared. This is due to the fact that, if there were no distinguishability between the two preparation methods, then, as shown in Appendix B.1, the quantum state that would arrive at the measuring device is predicted by Quantum Theory to be:

$$\begin{aligned} |\Psi\rangle &= N\Big[|0\rangle + |+\rangle\Big] \otimes N\Big[|0\rangle + |+\rangle\Big] \\ &= N^2\Big[|0\rangle \otimes |0\rangle \;+\; |0\rangle \otimes |+\rangle \;+\; |+\rangle \otimes |0\rangle \;+\; |+\rangle \otimes |+\rangle\Big] \end{aligned}$$

with normalization constant

$$N^2 = \frac{\sqrt{2}}{2\sqrt{2}+2}\,.$$

The state that would arrive at the detector would then be compatible with the four possible measurement outcomes they propose, namely,

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |1\rangle \;+\; |1\rangle \otimes |0\rangle\Big],$$

$$|\xi_2\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes |-\rangle \;+\; |1\rangle \otimes |+\rangle\Big],$$

$$|\xi_3\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |1\rangle \;+\; |-\rangle \otimes |0\rangle\Big],$$

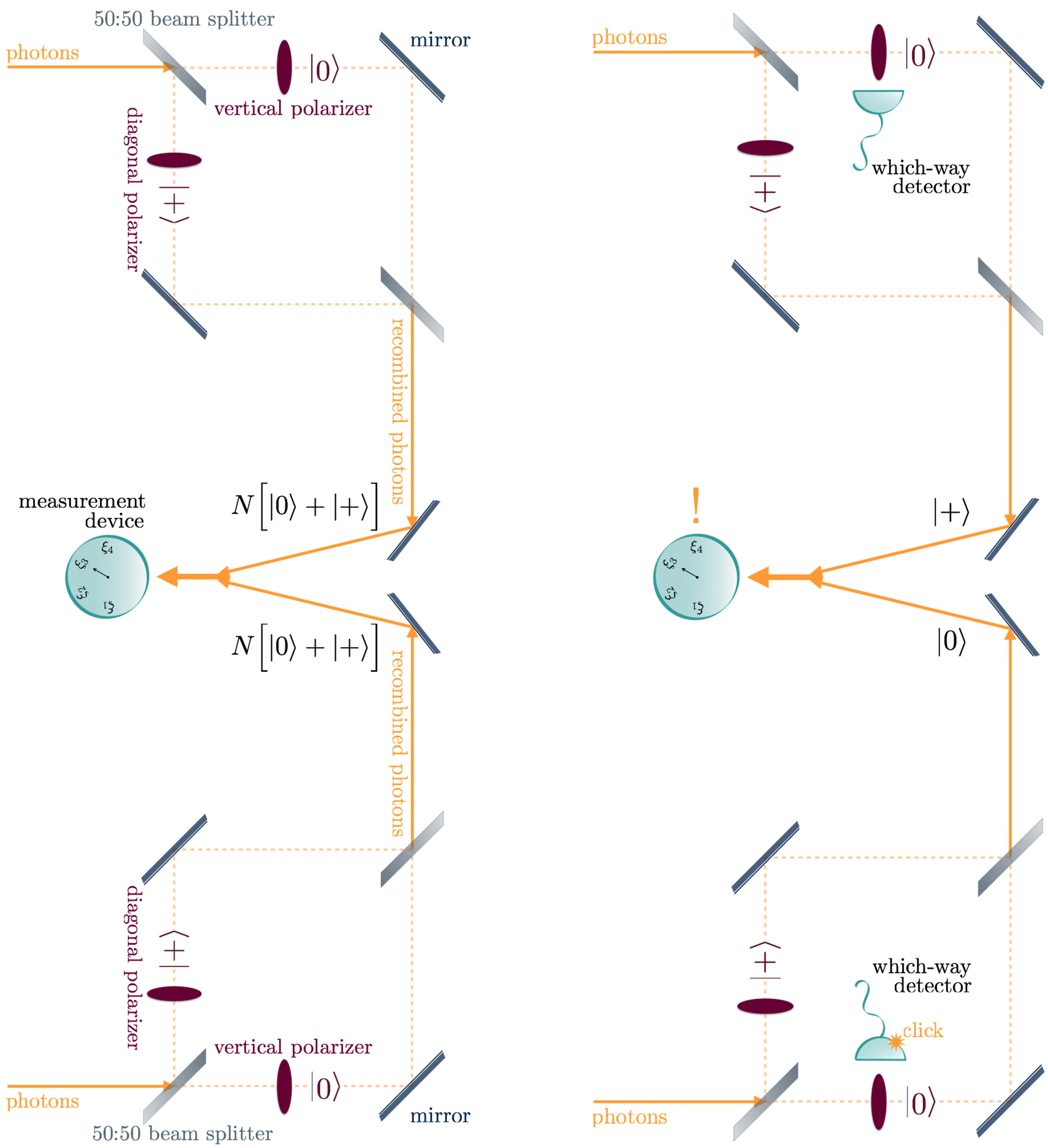
$$|\xi_4\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle \otimes |-\rangle \;+\; |-\rangle \otimes |+\rangle\Big],$$

meaning it could result in any of them with non-zero probability, and no contradiction arises. This leads us to conclude that, if there is no distinguishability in the preparation method they propose, the states $|0\rangle$ and $|+\rangle$ can be regarded as mere information, i.e., they can both result in physical states in the overlapping region Δ of phase space and the information that would arrive to the detector, $|\Psi\rangle$, would be compatible with any of the four quantum states $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |0\rangle$, and $|+\rangle \otimes |+\rangle$ *and* with any of the four measurement results, $|\xi_1\rangle$, $|\xi_2\rangle$, $|\xi_3\rangle$, and $|\xi_4\rangle$.

On the other hand, if we have a preparation procedure which distinguishes between the two methods, then the state that arrives at the detector is indeed any of the four states $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |0\rangle$, and $|+\rangle \otimes |+\rangle$, and in this case the states $|0\rangle$ and $|+\rangle$ cannot be regarded as mere information, they must arise from disjoint regions of phase space (as PBR demonstrate), and thus represent a physical property of the system.

We give Figure 2.1 to show that the only difference between the first experimental setup and the second one is that in the second experimental setup there is a certain knowledge of the state of the system that is being prepared, and this knowledge entails a measurement of some kind, while in the first experimental setup there is no measurement whatsoever between the preparation procedure and the detector. In the first experimental setup (2.1a) we can regard states $|0\rangle$ and $|+\rangle$ as mere information and no contradiction arises, while in the second experimental setup (2.1b) if we regard $|0\rangle$ and $|+\rangle$ as mere information, the contradiction derived by PBR arises.

We can conclude form this that the degree of “physicality” of a property depends directly on the measurement such property has undergone. A property that has not been measured can be regarded as pure information, while a property that has been measured has to be regarded as a physical property, having a counterpart in reality.



(a) No contradiction arises. (b) The contradiction derived by PBR arises.

Figure 2.1: Each experimental setup is a combination of two identical preparation devices. In **(a)** the preparation devices consist of a source of photons which undergoes a splitting of half and half in optical path, each optical path is polarized in a certain direction and the two paths are recombined, the photon coming out of the two paths is therefore in the state $N[|0\rangle + |+\rangle]$. At the end, the photons of the two preparation devices are brought together and measured. In **(b)** the preparation devices are as that of (a), except now two which-way detectors are added, one for each preparation device: if a detector clicks, the photon that has been prepared is in the state $|0\rangle$, while if a detector does not click, the photon comes out in the state $|+\rangle$.

2.3 Factuality and reality

The conclusion we arrived at in the previous section strongly favours our view of reality as factual: a property that has been measured (that what actually happens) has to be regarded as a physical property which has a counterpart in reality, while a property that has not been measured (does not actually happen) can be regarded as pure information and therefore such a property does not have a direct counterpart in reality.

If we were to put it in Bohr's terms, we would say that measurements define the reference frame in which a certain reality is actual (and all the other possibilities remain as fantasies). Now, to the factuality formalism.

In Quantum Mechanics we represent measurements by hermitian operators (observables) and states by vectors (wave functions). An operator acts on the Hilbert space of states, action which results in an element of the Hilbert space. Furthermore, any operator has an orthogonal set of eigenstates, which can be used as the basis of the Hilbert space the operator acts on. This set of eigenstates can always be normalized, so they satisfy the condition[1]:

$$\langle\psi_i|\psi_j\rangle = \delta_{ij},$$

There exists a complete set of compatible measurements for a state to be completely specified; it exists for every quantum system and it is known as the complete set of commuting operators, all the operators in the set share a common basis of eigenstates.

A linear combination of eigenstates of an operator, $\hat{O}$, is an eigenstate of the operator iff all the eigenstates in the combination have the same eigenvalue, i.e., the eigenstate equation:

$$\hat{O}|\phi\rangle = o|\phi\rangle \quad \text{with} \quad |\phi\rangle = \sum_i a_i|\psi_i\rangle$$

is satisfied iff

$$\hat{O}|\psi_i\rangle = o|\psi_i\rangle \quad \forall i.$$

Having gone through the previous reminder, we define two types of states:

Quantum states as the states generated from linear combinations of different eigenstates of an observable (with different eigenvalues), and denote them by $|\psi\rangle$.

Ontological states as the eigenstates of a complete set of mutually commuting observables, and denote them by $|\Omega\rangle$.

It is important to notice that, if we have a quantum state description of a system, we can always perform a basis transformation so that this description becomes an ontological state description. For example: $\frac{1}{\sqrt{2}}\left[|\uparrow\rangle^z + |\downarrow\rangle^z\right]$ is a quantum state description of the observable

[1] $$\int \psi^*(x)\psi(x')dx = \delta(x-x') \quad \text{in the position representation.}$$

$\hat{\sigma}_z$, but acquires an ontological meaning when we switch to the $\hat{\sigma}_x$-diagonal basis[2], resulting in $|\uparrow\rangle^x$.

We can also use the density matrix formalism to turn quantum states into ontological states (mathematically speaking). For example, a quantum state $|\psi\rangle^\alpha$ that belongs to a Hilbert space of 5 dimensions (let us say, spin $S = 2$) might look like:

$$\rho_\alpha = \begin{pmatrix} 1/3 & 1/3 & 0 & 0 & 1/3 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \end{pmatrix}$$
$$= \frac{1}{\sqrt{3}}\Big[|\,2\rangle^\alpha + |1\rangle^\alpha + |-2\rangle^\alpha\Big]\Big[{}^\alpha\langle 2| + {}^\alpha\langle 1| + {}^\alpha\langle -2|\Big]\frac{1}{\sqrt{3}} = |\psi\rangle^{\alpha\alpha}\langle\psi|$$

when represented in a certain basis (denoted by α) in which a certain set of operators $\{\hat{O}_\alpha\}$ are diagonal. But this density matrix is diagonal in another basis (that will be denoted by β) in which some other operators $\{\hat{O}_\beta\}$ are diagonal and, since ρ_α is a pure state, when diagonalized will take the form:

$$|\psi\rangle^{\alpha\alpha}\langle\psi| = |\Omega\rangle^{\beta\beta}\langle\Omega| = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \rho_\beta.$$

So $|\Omega\rangle^\beta$ is an eigenstate of the complete set of commuting operators $\{\hat{O}_\beta\}$ and hence, we can assert that $|\Omega\rangle^\beta$ is an ontological state, that can be described as the quantum state $|\psi\rangle^\alpha$. By this account we arrive at two important conclusions:

1. Every quantum state can be changed into an ontological state by performing a basis transformation.

2. Pure states are ontological states in their diagonal representation.

One would ask, then, what is the use of separating states into quantum or ontological if in the end they can all take on an ontological state form? Well, what is important is to distinguish what is ontological and what is quantum in each basis. States cannot be all ontological in one and the same basis. The election of basis is directly done by the measurement or preparation procedure a particular state goes through. To every measurement there is an associated operator, the choice of basis is that in which this operator (in fact, the complete set of commuting operators, associated to a complete set of measurements) is diagonal. This

[2] That is, $\frac{1}{\sqrt{2}}\left[|\uparrow\rangle^z + |\downarrow\rangle^z\right]$ is ontological when the chosen set of commuting observables is $\{S^2, S_x\}$, rather than $\{S^2, S_z\}$.

automatically tells us which states are ontological and which are quantum, in other words, what has a counterpart in reality and what does not.

Furthermore, ensembles of individual particles might be described either as pure states or as mixed states. Of course, each description characterizes different ensembles. In a pure state description we regard the ensemble as if every one of its components were in precisely that pure state, while in a mixed state description we regard the ensemble as one where different components of the ensemble are in different pure states, with a certain probability distribution. We will denote these two descriptions as ρ and $\tilde{\rho}$ respectively, i.e.,

$$\rho = |\Omega\rangle\langle\Omega|,$$
$$\tilde{\rho} = \sum_i c_i|\Omega_i\rangle\langle\Omega_i|.$$

Along the same line in which quantum states emerge only as a mathematical description of a system, mixed states only represent a statistical description of an ensemble that is comprised of many entities, each one in a pure state.

2.4 Evolution and determinism

In physics we talk about evolution, cause and effect, condition and response, all in a time related narrative. When we talk about evolution we mean evolution of a physical quantity, any real given property of a certain system. In Quantum Mechanics we have the usual hamiltonian (unitary) evolution, put forward in Schrödinger's equation. But this evolution has two downsides, the first one is widely known and the second one is related to the perspective of the present work:

1. The unitary evolution of the state is broken when a measurement takes place and there is no account in the theory regarding how the state evolves when measured.

2. Schrödinger's equation is solely about the evolution of information (quantum states), not about the evolution of real states onto real states.

Then, the real and complete evolution of the system has not been described by Quantum Mechanics thus far. Given this situation, we will propose a schematic description of what happens when a system is measured and of what happens in between measurements.

We have until now seen that:

- Measurement instruments set the reference frame for reality to appear, compatible observables can be simultaneously real, non-compatible observables are not all real in one and the same reference frame. (Bohr-like view and contextuality demonstration).

- Measured states have a counterpart in reality, a state that has not been measured can be regarded as mere information. (PBR demonstration).

- In a given reference frame solely ontological states have a counterpart in reality. (Factuality formalism and natural consequence of the two previous statements).

What we propose is that, since every ontological state has a counterpart in reality and is thus a physical state, then every ontological state must arise as an effect of physical causes. Even more, we postulate that different physical causes give rise to different ontological states, while identical physical causes give rise to one and the same ontological state, always. The physical causes that result in a certain ontological state are not accessible to the observer, so they appear as hidden variables that determine the state at a given time. The state of the system is then a function $\mathcal{F}$ of a hidden variable $\lambda \in \Lambda$ and time $t \in \mathbf{R}$, onto the Hilbert space of states $\mathcal{H}$.

$$\mathcal{F} : \Lambda \times \mathbf{R} \longrightarrow \mathcal{H}$$
$$(\lambda, t) \xrightarrow{\mathcal{F}} |\Omega\rangle.$$

We call this function the **deterministic evolution function** of a system.

So, what happens when a system is measured? An ontological state is realized. What was prone to happen given the initial conditions of a certain system, happens[3]. And what happens in between measurements? Well, the system evolves with probabilities governed by Schrödinger's equation with initial conditions defined by the last ontological state realized by the system, that is:

$$i\hbar \frac{\partial}{\partial t}|\Omega\rangle = \hat{H}|\Omega\rangle.$$

[3]The reader might object free will, the free will to choose the measurement setting in which instead of, say, the ontological state $|\uparrow\rangle^x$ that actually happened, the ontological state $|\uparrow\rangle^z$ could have happened if one had chosen to measure S_z instead of S_x. But the factuality assumption impedes one to go through this reasoning. The factuality assumption is a way to look at the system, from effect to cause, from future results to past conditions, in which reality is described only by what actually took place and not by a superposition of possibilities. It is the only accessible description by Quantum Mechanics of a physical system.

Chapter 3

Locality

We will now turn our eyes to locality. Can locality be salvaged in the quantum domain? Can phenomena remain only dependent on the causes Special Relativity taught us were accessible to them, or has Special Relativity no saying and no truth in the quantum world? In this chapter we will address the two no-go theorems of locality from a factuality point of view, that is:

- Solely what actually happens has a counterpart in reality.
- Ontological states are determined by a function $\mathcal{F}(\lambda, t)$.

3.1 Examination of Bell's inequality

Let us remember that we have a system whose initial state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big]$$

and the functions associated by Bell to recover locality, $A(\vec{j}, \lambda)$ and $B(\vec{k}, \lambda)$. First we need to find the relation between the functions $A(\vec{j}, \lambda)$ and $B(\vec{k}, \lambda)$ and our deterministic evolution function $\mathcal{F}(\lambda, t)$.

Functions A and B are both results of a measurement, so they must be related to the function $\mathcal{F}$ when the latter is evaluated at the time of measurement, say $t = t_1$. Now if $\mathcal{F}(\lambda, t_1)$ results in the ontological state of the system of the two particles, then it must yield a pair of orientations, $(\vec{o}_A, \vec{o}_B)$. These two orientations are those of the spin projection for particles A and B respectively at the time of measurement; it is important to recall that the orientation of the two detectors is also encoded in the value of the hidden variable λ.

Now, function $A(\vec{j}, \lambda)$ raises the question, "given a detector device with orientation $\vec{j}$ and a hidden variable λ, is the electron's spin orientation $\vec{j}$ or $-\vec{j}$?". So for this question to be posed, the electron's spin orientation *must* be $\vec{j}$ or $-\vec{j}$. Analogously for function $B(\vec{k}, \lambda)$. Then, these two questions can be posed iff $\mathcal{F}(\lambda, t_1) = (\pm\vec{j}, \pm\vec{k})$.

Fact 1 functions $A(\vec{j}, \lambda)$ and $B(\vec{k}, \lambda)$ are simultaneously well defined iff $\mathcal{F}(\lambda, t_1) = (\pm\vec{j}, \pm\vec{k})$.

Let us take **Fact 1** to examine the three experimental scenarios in Figure 3.1. In the left hand side scenario of Figure 3.1, the measurement outcome can be any of four different possibilities, $(\vec{a}, \vec{b})$, $(\vec{a}, -\vec{b})$, $(-\vec{a}, \vec{b})$ and $(-\vec{a}, -\vec{b})$, that is $\mathcal{F}(\lambda, t_1) = (\pm\vec{a}, \pm\vec{b})$. In the second scenario, $\mathcal{F}(\lambda, t_1) = (\pm\vec{a}, \pm\vec{c})$ and in the right hand scenario $\mathcal{F}(\lambda, t_1) = (\pm\vec{b}, \pm\vec{c})$.

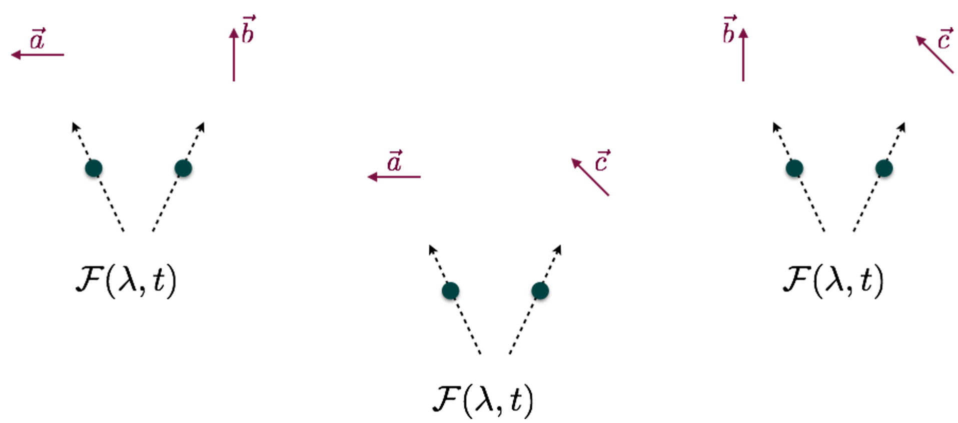


Figure 3.1: Three ontological states whose spin projection degree of freedom is described by the function $\mathcal{F}(\lambda, t)$, each state related to a different set of measurements at time $t = t_1$.

Under the factuality condition, each of these sets of outcomes must come from a different set of hidden variables, that is:

$$\mathcal{F}(\lambda, t_1) = (\pm\vec{a}, \pm\vec{b}) \leftrightarrow \lambda \in \Lambda_1, \tag{3.1}$$

$$\mathcal{F}(\lambda, t_1) = (\pm\vec{a}, \pm\vec{c}) \leftrightarrow \lambda \in \Lambda_2, \tag{3.2}$$

$$\mathcal{F}(\lambda, t_1) = (\pm\vec{b}, \pm\vec{c}) \leftrightarrow \lambda \in \Lambda_3. \tag{3.3}$$

Furthermore, $\Lambda_1 \cap \Lambda_2 = \Lambda_1 \cap \Lambda_3 = \Lambda_2 \cap \Lambda_3 = \emptyset$, which can be seen by the simple reasoning:

$$\text{If } \lambda \in \Lambda_1, \text{ then } \mathcal{F}(\lambda, t_1) = (\pm\vec{a}, \pm\vec{b}) \neq (\pm\vec{a}, \pm\vec{c}), \text{ then } \lambda \notin \Lambda_2; \text{ etc.}$$

So, given **Fact 1** and equations (3.1), (3.2) and (3.3):

Fact 2.1 functions $A(\vec{a}, \lambda)$ and $B(\vec{b}, \lambda)$ are simultaneously well defined iff $\lambda \in \Lambda_1$.

Fact 2.2 functions $A(\vec{a}, \lambda)$ and $B(\vec{c}, \lambda)$ are simultaneously well defined iff $\lambda \in \Lambda_2$.

Fact 2.3 functions $A(\vec{b}, \lambda)$ and $B(\vec{c}, \lambda)$ are simultaneously well defined iff $\lambda \in \Lambda_3$.

To follow Bell's steps to derive his inequality, we start by comparing the expectation values,

$$E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) = \int_{\Lambda_1} A(\vec{a},\lambda)B(\vec{b},\lambda)\rho(\lambda)d\lambda - \int_{\Lambda_2} A(\vec{a},\lambda)B(\vec{c},\lambda)\rho(\lambda)d\lambda, \tag{3.4}$$

where we have explicitly written the integration domains imposed by **Fact 2.1** and **Fact 2.2**. Since $\Lambda_1 \cap \Lambda_2 = \emptyset$ we cannot carry out Bell's first step in order to derive his inequality (cf. Appendix A.3), so:

> **Fact 3** In a local deterministic scenario, governed by factuality, Bell's inequality cannot be derived, therefore the violation of his inequality by experiments does not show that the assumption of locality in this scenario is incorrect.

The statement above begs the questions: under which conditions can Bell's inequality be derived? And, do experiments which follow such conditions violate this inequality? We will address these two questions in the next subsection.

3.1.1 Building Bell's inequality

In order to build Bell's inequality it is required that the set of hidden variables that lie behind the three different scenarios in Figure 3.1 be one and the same ($\lambda \in \Lambda$). If we want this requirement to be satisfied, we can take on two possible paths:

- Each different scenario can be governed by a different function, $\mathcal{F}_i(\lambda, t)$, $i = 1, 2, 3$, for $\lambda \in \Lambda$.
- On each different scenario the measurement can take place at a different time, so the final state would be described by $\mathcal{F}(\lambda, t_i)$, $i = 1, 2, 3$.

When taking any of these two paths, Bell's steps can be followed all the way, so we will take on the first path here and show the second path procedure in Appendix C.1. As before, we will start by identifying the functions A and B used by Bell with our function $\mathcal{F}$.

We can directly see that functions $A(\vec{a},\lambda)$ and $B(\vec{b},\lambda)$ can only be simultaneously identified with $\mathcal{F}_1(\lambda, t_1)$ and we know that

$$\mathcal{F}_1(\lambda, t_1) = (\vec{o}_{A_1}(\lambda, t_1), \vec{o}_{B_1}(\lambda, t_1)) = (\pm\vec{a}, \pm\vec{b});$$

then we can define

$$A_1(\vec{a},\lambda) \equiv \text{sign}(\vec{o}_{A_1}(\lambda, t_1))$$

and

$$B_1(\vec{b},\lambda) \equiv \text{sign}(\vec{o}_{B_1}(\lambda, t_1)).$$

The subscript 1 distinguishes these functions from the ones defined by $\mathcal{F}_2(\lambda, t_1)$. In this second case we have:

$$\mathcal{F}_2(\lambda, t_1) = (\vec{o}_{A_2}(\lambda, t_1), \vec{o}_{B_2}(\lambda, t_1)) = (\pm\vec{a}, \pm\vec{c}),$$

and we can simultaneously define

$$A_2(\vec{a}, \lambda) \equiv \text{sign}(\vec{o}_{A_2}(\lambda, t_1))$$

and

$$B_2(\vec{c}, \lambda) \equiv \text{sign}(\vec{o}_{B_2}(\lambda, t_1)).$$

And in the third case:

$$\mathcal{F}_3(\lambda, t_1) = (\vec{o}_{A_3}(\lambda, t_1), \vec{o}_{B_3}(\lambda, t_1)) = (\pm\vec{b}, \pm\vec{c}),$$

so

$$A_3(\vec{b}, \lambda) \equiv \text{sign}(\vec{o}_{A_3}(\lambda, t_1))$$

and

$$B_3(\vec{c}, \lambda) \equiv \text{sign}(\vec{o}_{B_3}(\lambda, t_1)).$$

Now that each scenario is governed by a different function $\mathcal{F}_i$ we can go back to Bell's first step,

$$E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c}) = \int_\Lambda A_1(\vec{a}, \lambda) B_1(\vec{b}, \lambda) \rho(\lambda) d\lambda - \int_\Lambda A_2(\vec{a}, \lambda) B_2(\vec{c}, \lambda) \rho(\lambda) d\lambda,$$

where we have implicitly written the subscripts that define each function A_i, B_i in terms of the deterministic evolution function of each different experiment.

And to his second step,

$$\left| E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c}) \right| = \left| \int_\Lambda [\boldsymbol{A_1(\vec{a}, \lambda)} B_1(\vec{b}, \lambda) - \boldsymbol{A_2(\vec{a}, \lambda)} B_2(\vec{c}, \lambda)] \rho(\lambda) d\lambda \right|, \tag{3.5}$$

where we have highlighted $A_1(\vec{a}, \lambda)$ and $A_2(\vec{a}, \lambda)$ to stress the fact that for his third step, Bell takes these two functions to be identical. This is his first assumption (out of three). We will analyze what can be said about the quantity $\left| E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c}) \right|$ in two cases: while taking Bell's three assumptions, and while taking none of them.

Within Bell's assumptions

Bell's three assumptions are (shown in Appendix C.2):

$$A_1(\vec{a}, \lambda) = A_2(\vec{a}, \lambda),$$

$$B_1(\vec{b}, \lambda) = -A_3(\vec{b}, \lambda),$$

$$B_2(\vec{c},\lambda) = B_3(\vec{c},\lambda).$$

These are constraints on the functions $\mathcal{F}_i(\lambda,t)$ that have to be met in order for Bell's inequality to be derived. So, the applicable domain of his inequality is the one that behaves according to these constraints, that is, the deterministic functions $\mathcal{F}_i(\lambda,t)$ that govern the three experiments built to test Bell's inequality have to be so that these constraints are satisfied.

This has an implication on the expectation values of the correlation between measurements. If these three constraints are satisfied, the predicted expectation values result in:

$$E(\vec{a},\vec{b}) = -\cos\theta_{ab},$$
$$E(\vec{a},\vec{c}) = -\cos\theta_{ac},$$
$$E(\vec{b},\vec{c}) = -\cos\theta_{ab}\cos\theta_{ac},$$

which is caused by the fact that the given constraints tamper with the probabilities of getting $(\pm\vec{a})$, $(\pm\vec{b})$ or $(\pm\vec{c})$ in the measurements performed. The derivation of these results is given in Appendix C.3.

Now, this result drives us into two conclusions:

- If the experiments were to satisfy the constraints necessary to build Bell's inequality, then the expectation values would be such that when plugged into the inequality one would get:

  $$|-\cos\theta_{ab} + \cos\theta_{ac}| \le 1 - \cos\theta_{ab}\cos\theta_{ac} \tag{3.6}$$

  and, as shown in Appendix C.4, this inequality is always satisfied.

- **The experiments** used to test Bell's inequality do not result in an expectation value given by a product of cosines $(-\cos\theta_{ab}\cos\theta_{ac})$, so they **do not behave according to the constraints necessary to build Bell's inequality**, so they do not have to satisfy such an inequality and the violation of the inequality by the experiments does not show that reality cannot behave in a local deterministic way.

Without Bell's assumptions

We will now go back to his second step and build a Bell-like inequality without his assumptions.

$$\begin{aligned}\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| &= \left|\int_\Lambda [A_1(\vec{a},\lambda)B_1(\vec{b},\lambda) - A_2(\vec{a},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda\right| \\ &= \left|\sum_{i=1}^{8}\int_{\tilde{\Lambda}_i} [A_1(\vec{a},\lambda)B_1(\vec{b},\lambda) - A_2(\vec{a},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda\right|,\end{aligned}$$

where we build the sets $\tilde{\Lambda}_i$, $i = 1,\ldots,8$, in terms of the different relations that the functions A_1, A_2, A_3, B_1, B_2 and B_3 may hold between them. These 8 sets $\tilde{\Lambda}_i$ are defined as:

$$\tilde{\Lambda}_1 = \{\lambda \,|\, A_1 = A_2\,,\, B_1 = -A_3\,,\, B_2 = B_3\},$$

$$\tilde{\Lambda}_2 = \{\lambda \,|\, A_1 = A_2\,,\, B_1 = -A_3\,,\, B_2 = -B_3\},$$

$$\tilde{\Lambda}_3 = \{\lambda \,|\, A_1 = A_2\,,\, B_1 = A_3\,,\, B_2 = B_3\},$$

$$\tilde{\Lambda}_4 = \{\lambda \,|\, A_1 = A_2\,,\, B_1 = A_3\,,\, B_2 = -B_3\},$$

$$\tilde{\Lambda}_5 = \{\lambda \,|\, A_1 = -A_2\,,\, B_1 = -A_3\,,\, B_2 = B_3\},$$

$$\tilde{\Lambda}_6 = \{\lambda \,|\, A_1 = -A_2\,,\, B_1 = -A_3\,,\, B_2 = -B_3\},$$

$$\tilde{\Lambda}_7 = \{\lambda \,|\, A_1 = -A_2\,,\, B_1 = A_3\,,\, B_2 = B_3\},$$

$$\tilde{\Lambda}_8 = \{\lambda \,|\, A_1 = -A_2\,,\, B_1 = A_3\,,\, B_2 = -B_3\}.$$

So

$$\left| \int_{\tilde{\Lambda}_1} [A_1(\vec{a},\lambda)B_1(\vec{b},\lambda) - A_2(\vec{a},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda \right| \leq \int_{\tilde{\Lambda}_1} [1 + A_3(\vec{b},\lambda)B_3(\vec{c},\lambda)]\rho(\lambda)d\lambda$$

$$= Z(\tilde{\Lambda}_1) - Z(\tilde{\Lambda}_1)\cos\theta_{ab}\cos\theta_{ac}\,,$$

where $Z(\tilde{\Lambda}_1)$ is the measure of the set $\tilde{\Lambda}_1$. The integral of the product $A_3B_3\rho(\lambda)$ is calculated by a sum of values ($A_3B_3 = 1$ and $A_3B_3 = -1$) each with a certain probability distribution and results in $-Z(\tilde{\Lambda}_1)\cos\theta_{ab}\cos\theta_{ac}$ given that, when λ belongs to $\tilde{\Lambda}_1$, the functions A_3 and B_3 are correlated precisely by the constraints used to build Tables C.1 - C.4.

Since we are not as familiar with the constraints in $\tilde{\Lambda}_2$ as those in $\tilde{\Lambda}_1$ we will perform the next integral in more detail.

$$\left| \int_{\tilde{\Lambda}_2} [A_1(\vec{a},\lambda)B_1(\vec{b},\lambda) - A_2(\vec{a},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda \right|$$

$$= \left| \int_{\tilde{\Lambda}_2} A_1(\vec{a},\lambda)B_1(\vec{b},\lambda)[1 + A_3(\vec{b},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda \right|,$$

given the first two constraints, which is then

$$\leq \int_{\tilde{\Lambda}_2} [1 - A_3(\vec{b},\lambda)B_3(\vec{c},\lambda)]\rho(\lambda)d\lambda$$

by use of the last constraint. We can determine this integral by means of the probability distributions of the product $A_3B_3 = 1$ and the product $A_3B_3 = -1$ that span from Table 3.1.

Table 3.1: Joint probabilities of $A_3(\vec{b},\lambda)$ and $B_3(\vec{c},\lambda)$, when λ belongs to $\tilde{\Lambda}_2$. The **LHS** table results from substituting with $A_1 = A_2$ and $B_1 = -A_3$ in Table C.1 and the **RHS** table results from substituting with $B_2 = -B_3$ in the LHS of Table C.3.

$-A_3(\vec{b},\lambda)$ / $A_2(\vec{a},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$	$\cos^2\frac{\theta_{ab}}{2}$
-1	$\frac{1}{2}$	$\cos^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$	$\sin^2\frac{\theta_{ab}}{2}$

$-B_3(\vec{c},\lambda)$ / $A_2(\vec{a},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2\frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\cos^2\frac{\theta_{ac}}{2}$
-1	$\frac{1}{2}$	$\cos^2\frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\sin^2\frac{\theta_{ac}}{2}$

Such probability distributions are derived under the following logic:

- λ can belong to two different sets, if $A_2 = 1$ then $\lambda \in \tilde{\Lambda}_{2+}$, and if $A_2 = -1$ then $\lambda \in \tilde{\Lambda}_{2-}$.
- If $\lambda \in \tilde{\Lambda}_{2+}$, then $A_3 = 1$ with probability $\cos^2(\theta_{ab}/2)$ and $A_3 = -1$ with probability $\sin^2(\theta_{ab}/2)$; while $B_3 = 1$ with probability $\cos^2(\theta_{ab}/2)$ and $B_3 = -1$ with probability $\sin^2(\theta_{ab}/2)$.
- If $\lambda \in \tilde{\Lambda}_{2-}$ then $A_3 = 1$ with probability $\sin^2(\theta_{ab}/2)$ and $A_3 = -1$ with probability $\cos^2(\theta_{ab}/2)$; while $B_3 = 1$ with probability $\sin^2(\theta_{ab}/2)$ and $B_3 = -1$ with probability $\cos^2(\theta_{ab}/2)$.
- With this information one can calculate the probability of $A_3B_3 = 1$ and $A_3B_3 = -1$, taking into account that $\lambda \in \tilde{\Lambda}_{2+}$ with probability 1/2 and that $\lambda \in \tilde{\Lambda}_{2-}$ with probability 1/2.

So we get

$$\mathcal{P}(A_3 \cdot B_3 = 1) = \sin^2\frac{\theta_{ab}}{2}\sin^2\frac{\theta_{ac}}{2} + \cos^2\frac{\theta_{ab}}{2}\cos^2\frac{\theta_{ac}}{2} \tag{3.7}$$

and

$$\mathcal{P}(A_3 \cdot B_3 = -1) = \cos^2\frac{\theta_{ab}}{2}\sin^2\frac{\theta_{ac}}{2} + \sin^2\frac{\theta_{ab}}{2}\cos^2\frac{\theta_{ac}}{2}. \tag{3.8}$$

The result of the integral is obtained by subtracting (3.8) from (3.7) which gives:

$$\int_{\tilde{\Lambda}_2} A_3(\vec{b},\lambda)B_3(\vec{c},\lambda)]\rho(\lambda)d\lambda = Z(\tilde{\Lambda}_2)\cos\theta_{ab}\cos\theta_{ac}\,,$$

so that

$$\int_{\tilde{\Lambda}_2} [1 - A_3(\vec{b},\lambda)B_3(\vec{c},\lambda)]\rho(\lambda)d\lambda = Z(\tilde{\Lambda}_2) - Z(\tilde{\Lambda}_2)\cos\theta_{ab}\cos\theta_{ac}\,.$$

Following the same procedure, one can verify that

$$\left|\int_{\tilde{\Lambda}_i} [A_1(\vec{a},\lambda)B_1(\vec{b},\lambda) - A_2(\vec{a},\lambda)B_2(\vec{c},\lambda)]\rho(\lambda)d\lambda\right| \leq Z(\tilde{\Lambda}_i) - Z(\tilde{\Lambda}_i)\cos\theta_{ab}\cos\theta_{ac}$$

$\forall i$. Adding all these integrals over i, and normalising to the volume of Λ, i.e.

$$\sum_{i=1}^{8} Z(\tilde{\Lambda}_i) = 1,$$

yields the value $1 - \cos\theta_{ab}\cos\theta_{ac}$.

This shows that the inequality the two expectation values must satisfy, when assuming no specific relation between functions A and B, is:

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| \leq 1 - \cos\theta_{ab}\cos\theta_{ac}\,, \tag{3.9}$$

where $\cos\theta_{ab}\cos\theta_{ac}$ is just a quantity, not an expectation value of a specific scenario. We have already shown that Quantum Mechanics' predictions and experimental results always satisfy inequality (3.9).

This last examination has given us ground to show that Quantum Mechanics' predictions and facts do not necessarily imply the extinction of locality. One just needs to add factuality as a condition of physical realism. In order to see how the factuality formalism is applied to another case, we will revisit the second proof of non-locality given in the first chapter, the GHZ theorem.

3.2 On the Greenberger, Horne and Zeilinger theorem

The GHZ theorem is based on a system in the state

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\Big]$$

and the observables

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3,\;\; \hat{\sigma}_y^1\hat{\sigma}_x^2\hat{\sigma}_y^3,\;\; \text{and}\;\; \hat{\sigma}_y^1\hat{\sigma}_y^2\hat{\sigma}_x^3. \tag{3.10}$$

We all know that these observables commute only when each one is *completely* measured. By completely we mean *the three measurements done simultaneously over all of the three components of the system*, so measuring only over two of the components will inevitably induce a change in frame of reference, that is, a new ontological state that was not ontological under the complete set in (3.10).

Now, when the system is separated into its three components, and no measurement has been done, there is no change in frame of reference and the system remains in the ontological state $|GHZ\rangle$, so if no measurement is done everything goes smoothly for GHZ. But, as soon as there is a measurement, the ontological state changes to an eigenstate of say $\hat{\sigma}_x^1$, $\hat{\sigma}_y^2$ and $\hat{\sigma}_y^3$ *separately*. And yes, given the ontological state the system was in prior to measurement, y_3 could be predicted by just measuring over particle 1 and 2, but this prediction cannot be taken further, one can no longer predict as to what x_i of a third would be, just by measuring $\hat{\sigma}_y^j$ over the other two. So there is no need for a simultaneous assignment of values x_i, y_i to the system, given that such a simultaneous reality never existed and never will exist.

Again, the argument is that *one could have chosen to measure over* ***any two*** *of the particles and predicted* ***any two*** *of the values x_i, y_i of the third.* However, this reasoning is counterfactual, and we have discarded counterfactual premises when looking into Nature from a realistic point of view. So there is no proof of non-locality in the theorem of GHZ if one keeps the hypothesis of factuality true.

3.3 Final interpretation

The framework we have constructed is based on the following tools:

- Complete sets of mutually commuting operators, which are used to specify the frame of reference in which a certain state appears.
- Eigenstates of those sets, which are the ontological states $|\Omega\rangle$ in the frame of reference the complete sets define.
- A deterministic evolution function $\mathcal{F}(\lambda, t)$ which is defined by the particular Hilbert space of a system and that univocally determines the ontological state of the given system.

These tools taken over to the no-go theorems of locality allowed us to conclude that quantum phenomena could be interpreted in a local deterministic way.

The quantum correlations that arise in experiments are a consequence of the ability one has in Quantum Mechanics to change the frame of reference of the system. In this way, **ontological states display quantum features when the frame of reference is changed**. For example, in the thought experiment of Bohm and Aharonov, the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big]$$

is ontological in the initial frame of reference $\{(S_A + S_B)^2, S_{Az}S_{Bz}, S_{Ax}S_{Bx}\}$ but displays a quantum behaviour[1] in the subsequent frame of reference, $\{{S_A}^2, S_{Aa}, {S_B}^2, S_{Bb}\}$. The state, in the end, is ontological in the last frame of reference and evolved from the former to the latter governed by $\mathcal{F}(\lambda, t)$.

[1] Comprised in the correlation between the spin projection of the two particles.

It is useful to underline what a change of frame of reference entails, and for this we will propose an experiment.

Imagine an electron is prepared in the state $|\uparrow\rangle^z$ by means of a Stern-Gerlach magnet. Now, along the path of this electron place a second Stern-Gerlach, this time oriented towards $\vec{x}$. The electron might project to $|\uparrow\rangle^x$ or $|\downarrow\rangle^x$. Recombine these two paths and place a third Stern-Gerlach along the recombined path oriented towards $\vec{z}$, (cf. Figure 3.2)

You can either add or not a detector to the experimental setting before the electrons are recombined. Build a setting without a detector and a setting with a detector. In the first case (Figure 3.2a) no change in frame of reference occurs, just a perturbation, while in the second case (Figure 3.2b) the frame of reference changes twice.

The state of the electron on the recombined path is predicted by Quantum Mechanics through the action of $\hat{\sigma}_x$ over the state $|\uparrow\rangle^z$ as precisely $|\downarrow\rangle^z$ so, when the interaction is that of setting 3.2a, the state just evolves in one and the same frame of reference from the ontological state $|\uparrow\rangle^z$ to the ontological state $|\downarrow\rangle^z$,

$$|\uparrow\rangle^z \xrightarrow{\mathcal{F}} |\downarrow\rangle^z.$$

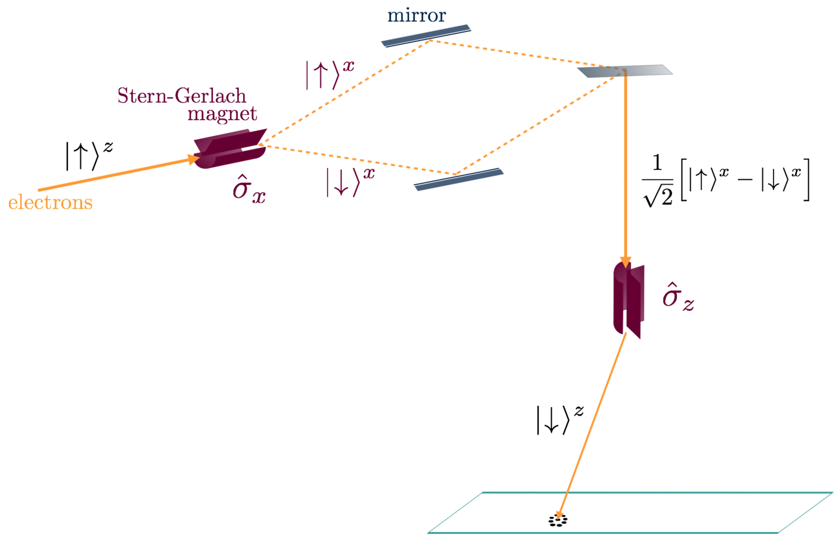
But if there is a detector before the two paths are recombined, one either gets $|\uparrow\rangle^x$ or $|\downarrow\rangle^x$ on the recombined path and not a superposition, so, if the interaction is that of setting 3.2b, there are two changes in frame of reference and the state of the electron evolves in four possible ways:

$$\begin{aligned}
|\uparrow\rangle^z &\xrightarrow{\mathcal{F}} |\uparrow\rangle^x \xrightarrow{\mathcal{F}} |\uparrow\rangle^z,\\
|\uparrow\rangle^z &\xrightarrow{\mathcal{F}} |\uparrow\rangle^x \xrightarrow{\mathcal{F}} |\downarrow\rangle^z,\\
|\uparrow\rangle^z &\xrightarrow{\mathcal{F}} |\downarrow\rangle^x \xrightarrow{\mathcal{F}} |\uparrow\rangle^z,\\
|\uparrow\rangle^z &\xrightarrow{\mathcal{F}} |\downarrow\rangle^x \xrightarrow{\mathcal{F}} |\downarrow\rangle^z.
\end{aligned}$$

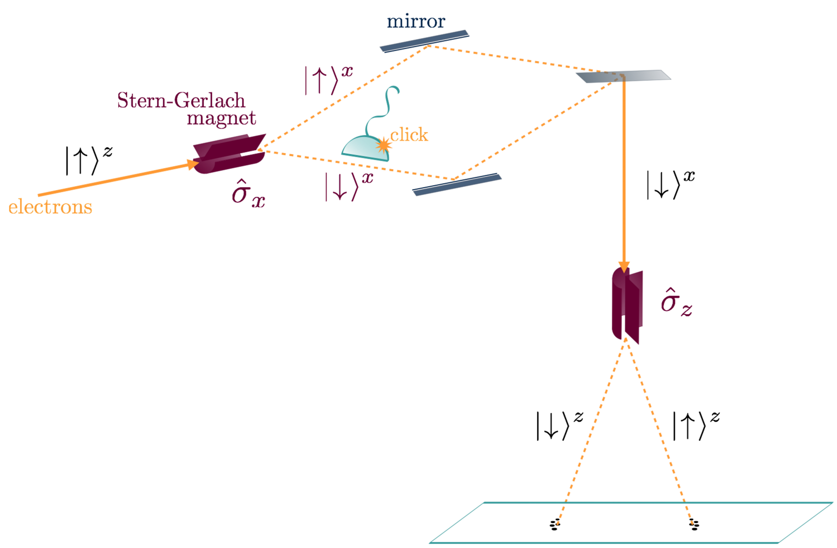
This example is extended to show that a change in the frame of reference necessarily entails knowledge about some quantity, incompatible with the known quantities of the previous frame of reference. If there is no subsequent knowledge of incompatible quantities, there is no change in frame of reference.

Every time there first is knowledge of the result of a measurement of one observable and then knowledge of the result of a measurement of another, incompatible observable, there is a transition of reference frame. When there is a transition of reference frame, the (previously) ontological state will change into an ontological state in the new reference frame. And certainly, if there is no transition of reference frame ontological states will only evolve into ontological states within the same reference frame. In any case, for an ontological state to change there has to be some kind of interaction of the quantum system with the environment.

When a state is measured we set the reference frame in which the ontological state emerges. Of course, any change in frame of reference has to be encoded in $\mathcal{F}(\lambda, t)$, hence, decisions of what to measure and when to measure it are governed by $\mathcal{F}(\lambda, t)$. $\mathcal{F}(\lambda, t)$ is just the accessible



(a) No change in frame of reference, just an interaction which changes the ontological state from $|\uparrow\rangle^z$ to $|\downarrow\rangle^z$.



(b) Two subsequent changes in frame of reference.

Figure 3.2: Two experimental devices that consist in a couple of Stern-Gerlach magnets which one after another measure the spin projection of an incoming electron. In **(a)** there is no knowledge of the state after its spin projection along $\vec{x}$ is measured, while in **(b)** one knows which one of the two possible states comes out of the Stern-Gerlach magnet. From this slight difference between experimental settings (a) and (b) a whole different state emerges when $\hat{\sigma}_z$ is measured.

way we have to describe reality, however reality might behave.

Moreover, we have also to encode what lies behind function $\mathcal{F}(\lambda, t)$, what defines it and which are the necessary hidden variables for its domain. These questions parallel the questions one has to answer when looking into entanglement. The hidden mechanism a system undergoes when it is entangled is precisely that which defines the shape of $\mathcal{F}(\lambda, t)$ in that system.

While we have given the general structure that leads to the recovery of locality, we are not faced with a final interpretation. Rather, we are faced with more questions, questions that can take us to look deeper into the structure of reality and that can open the door for local explanations of quantum phenomena.

Chapter 4

Conclusions

The use of the factuality assumption to recover locality is not a novel idea. Superdeterminism has long been known to respond to theorems that prove non-locality, given that the existence of free will is a main hypothesis used to construct the non-locality demonstrations. The main focus of this work has been to give the fundamental tools, scope, and predictions of a factual interpretation of reality, in regards to the questions of contextuality, non-locality and the paradoxical nature of measurement.

Probably the most controversial or uncomfortable aspect of this work is the proposal of a deterministic function $\mathcal{F}(\lambda, t)$ that seems to govern all happenings, including the decisions of the observer. It is left to the will of the reader and to future observations to analyze how such a function might work in Nature: be it that observers just follow the same physical laws all other objects are subjected to, be it retrocausality[1], be it an impossibility in our human understanding of Nature to see it another way, be it a direct consequence of the building blocks of our natural philosophy, physics. Whatever the nature of this function $\mathcal{F}$ might be, real, physical, in the external world, or epistemic, informational, in our line of thought, we find the quest for the essence of this function not only a compelling matter, but also a promising path.

If we were to find in our experiments the physical counterpart of $\mathcal{F}$, we would be confronted with the local mechanism behind Quantum Mechanics. But even if function $\mathcal{F}$ were just an epistemic tool that we needed to be able to describe reality, it might nevertheless open the path for more epistemic tools, until we find the counterparts in physical entities that derivate from these tools.

[1]Retrocausality, also known as backwards causation, is a notion proposed by Huw Price in 1994 [24] based on the statement that future events might give rise to present effects. Work on this notion has been developed further in [25–28]. That $\mathcal{F}(\lambda, t)$ can mimic retrocausality is given, since the determinacy of a future event from past causes is symmetric to the notion of a past effect coming from a future cause.

4.1 Main results

In the present work we saw that Quantum Mechanics has to be understood as a contextual theory, which talks about physical quantities always in regards to measurements being performed. To apprehend this matter we inherited Bohr's concept of **frame of reference** and carried it all the way into our interpretation. We also took the contextual view a little further by showing that, in order to guarantee the physical reality of any state by means of the PBR theorem, the particular state must result from an actual measurement.

With this view in mind we defined two types of states, **quantum states** and **ontological states**, the former being mere information and the latter having a counterpart in reality. We saw that the information carried by quantum states may exhibit a quantum behavior in any frame of reference in which they are not ontological. The evolution that a state undergoes when there is a change in the frame of reference is not yet understood, but, as we postulated, it is comprised in a **deterministic evolution function** $\mathcal{F}(\lambda, t)$ and it is determined by hidden variables.

We termed this formalism of frame of references, ontological states, hidden variables, and a deterministic evolution function as the **factuality formalism** and saw that when factuality is taken into account, Bell's inequality cannot be derived, so that there are no existing grounds to conclude non-locality by the violation of this inequality.

Furthermore, we put forward the necessary conditions for deriving Bell's inequality and showed that if the experiments were to satisfy such conditions, they would also satisfy his inequality. Moreover, we demonstrated that the experiments built to test his inequality do not behave in accordance with the necessary conditions for its construction, and therefore, that the violation of Bell's inequality is again no solid ground for concluding non-locality. We did all this from a factual approach.

We revisited Bell's theorem as well as the GHZ example to show that their proofs of non-locality do not "pass the factuality test" and thereby concluded that in a factual scenario, determinism and locality can be recovered for Quantum Mechanics.

4.2 A word on the double-slit experiment

Maybe we haven't stressed enough the importance of the 'frame of reference' concept in Quantum Mechanics. This section is devoted to show how this concept is embedded in every quantum situation and that embracing the 'frame of reference' concept might help to simplify the way we interpret any quantum phenomena.

When a myriad of particles encounters a double-slit, they form an interference pattern on a screen on the other side of the double-slit. This interference pattern is an indirect measurement of momentum, given that by the interference pattern we can figure out the wavelength of the incoming particles, and by their wavelength we can conclude their momentum:

$$p = \frac{h}{\lambda} \,.$$

The frame of reference in which this phenomenon occurs is incompatible with the frame of reference in which a measurement of position (which slit did the particle go through?) is carried out.

Of course, Bohr noticed this from the very beginning and always referred to the double-slit experiment in this way. What we are doing here is bringing back his interpretation of complementary quantities and his notion of reference frame as a main element of Quantum Theory.

We can only talk about two different aspects of reality if they are simultaneously ontological in one and the same reference frame. All other aspects have no counterpart in reality. Position has no counterpart in reality if we are standing in the momentum reference frame.

Furthermore, how does the interference pattern appear? Well, when each one of the particles hits the screen we have a measurement of position and know nothing about its momentum, but when all the particles have hit the screen, we know the momentum the particles had before they hit the screen. So, particle i starts in an ontological state of momentum[2] $|\Omega\rangle^p$, goes through the double-slit, and is then in an ontological state of position $|\Omega_i\rangle^x$ when it hits the screen, displaying the quantum behavior of the state $|\Omega\rangle^p$, which is a quantum state in the new frame of reference.

Our proposed description is that the particles in the ensemble share hidden variables, and that the evolution of the state of each particle is simply:

$$|\Omega\rangle^p \xrightarrow{\mathcal{F}} |\Omega_i\rangle^x;$$

but when the states of position $|\Omega_i\rangle^x$ of all the particles are seen together we look straight into the interference pattern. So, we entertain the view that the interference pattern arises from hidden variables that the particles share, and a function $\mathcal{F}(\lambda, t)$ that indicates in which place of the screen every one of the particles of the ensemble is going to hit.

Now, why does the interference pattern disappear when we measure position? Well, the hidden variables in this case have to be in a different set, these hidden variables are such that when $\mathcal{F}(\lambda', t)$ is taken over them, the story of the particles goes a different way, namely:

$$|\Omega\rangle^p \xrightarrow{\mathcal{F}} |A\rangle^x \xrightarrow{\mathcal{F}} |\Omega_i\rangle^x,$$

where $|A\rangle^x$ is the state of a particle that has gone through slit A and $|\Omega_i\rangle^x$ indicates the final position of the particle in the screen. Since the change in frame of reference was already done in the slits, the screen "sees" an ontological state arrive and the particles do not display a quantum behavior. In this case, from the hidden variables λ' shared between the particles of the ensemble and the function $\mathcal{F}(\lambda', t)$ that governs the ontological state of everyone of the particles, no interference pattern arises.

4.3 A Universe of information

Ours is a Universe of information, which we have managed to arrange by physical laws and labels. This information is the most fundamental aspect of any real object. Deep in the

[2] All particles in the ensemble have the same momentum.

structure of matter is the information on how to appear, how to behave, how to interact. Nature simply acts as a decoding mechanism of this information. Such is the *Quantum in Formation*, Nature assuming its shape, governed by physical laws, all of them comprised in $\mathcal{F}$. Yet, how is this information stored and how is it interchanged between any two objects standing in the Universe? This is our main question, left open for future scrutiny.

4.4 Symmetry breaking

There is an idea, that may have been floating around for some time now, that the measurement process is a physical process analogous to the symmetry breaking process of a statistical system when several ground states become available to its initial conditions.

In both cases this symmetry breaking has to do with one particular action: choosing. The statistical system "chooses" one of several possible ground states, just as an observer chooses one detector orientation over an infinite set of possibilities.

The physical mechanism by which one system chooses one particular ground state over the others has not been understood and Statistical Mechanics does not seem to be able to go as far as to understand this process. So we might have to construct a theory which is able to answer this question. And when we do so, when we solve the symmetry breaking process that Nature does to our theory, when Nature no more breaks any symmetry because there are no more symmetric conditions embedded in our theory, maybe then we can begin to understand the measurement problem, because then we could find an answer to the question: what lies behind the choice of an observer? Why is this choice the one selected amongst the other possibilities? And, therefore, how is it that this choosing process determines the reality of a quantum system?

Appendix A

Theoretical Framework

A.1 Commutation and group relations of spin operators

First we will show that the observables $\hat{\sigma}_x^1\hat{\sigma}_y^2$ and $\hat{\sigma}_y^1\hat{\sigma}_x^2$ commute. We know by properties of the commutator that

$$\begin{aligned}\left[\hat{\sigma}_x^1\hat{\sigma}_y^2, \hat{\sigma}_y^1\hat{\sigma}_x^2\right] &= \hat{\sigma}_x^1\left[\hat{\sigma}_y^2, \hat{\sigma}_y^1\hat{\sigma}_x^2\right] + \left[\hat{\sigma}_x^1, \hat{\sigma}_y^1\hat{\sigma}_x^2\right]\hat{\sigma}_y^2\\ &= \hat{\sigma}_x^1\hat{\sigma}_y^1\left[\hat{\sigma}_y^2, \hat{\sigma}_x^2\right] + \left[\hat{\sigma}_x^1, \hat{\sigma}_y^1\right]\hat{\sigma}_x^2\hat{\sigma}_y^2,\end{aligned}$$

and from the group structure of momentum operators $\hat{\sigma}_i\hat{\sigma}_j = i\epsilon_{ijk}\hat{\sigma}_k$, so

$$\left[\hat{\sigma}_x^1\hat{\sigma}_y^2, \hat{\sigma}_y^1\hat{\sigma}_x^2\right] = i\hat{\sigma}_z^1\left[\hat{\sigma}_y^2, \hat{\sigma}_x^2\right] + \left[\hat{\sigma}_x^1, \hat{\sigma}_y^1\right]i\hat{\sigma}_z^2\,;$$

but $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$, then

$$\left[\hat{\sigma}_x^1\hat{\sigma}_y^2, \hat{\sigma}_y^1\hat{\sigma}_x^2\right] = i\hat{\sigma}_z^1\left(-2i\hat{\sigma}_z^2\right) + \left(2i\hat{\sigma}_z^1\right)i\hat{\sigma}_z^2 = 0,$$

which gives proof of our claim.

Now, we will see that $\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^1\hat{\sigma}_x^2 = \hat{\sigma}_z^1\hat{\sigma}_z^2$. Each operator acts on a Hilbert space denoted by the superindex $i = 1, 2$, given this we can rearrange the observables in the following way:

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^1\hat{\sigma}_x^2 = \hat{\sigma}_x^1\hat{\sigma}_y^1\hat{\sigma}_y^2\hat{\sigma}_x^2,$$

preserving the order inside each Hilbert space. But $\hat{\sigma}_x^1\hat{\sigma}_y^1 = i\hat{\sigma}_z^1$ and $\hat{\sigma}_y^2\hat{\sigma}_x^2 = -i\hat{\sigma}_z^2$, so

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^1\hat{\sigma}_x^2 = (i\hat{\sigma}_z^1)(-i\hat{\sigma}_z^2) = \hat{\sigma}_z^1\hat{\sigma}_z^2.$$

A.2 Identities in Mermin's table

In the upper two rows and in the first two columns of Table 1.1 each observable $\hat{\sigma}_\alpha^i$ appears twice. We know that $\hat{\sigma}_\alpha^i \hat{\sigma}_\alpha^i = I_2$, where

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

so the product of the observables of these sets is trivially I_2.

In the bottom row we have the observables $\hat{\sigma}_x^1 \hat{\sigma}_y^2$, $\hat{\sigma}_y^1 \hat{\sigma}_x^2$ and $\hat{\sigma}_z^1 \hat{\sigma}_z^2$. We saw in the previous appendix that the product of the first two observables is

$$\hat{\sigma}_x^1 \hat{\sigma}_y^2 \hat{\sigma}_y^1 \hat{\sigma}_x^2 = \hat{\sigma}_z^1 \hat{\sigma}_z^2,$$

so if we take the product of these two and multiply it with the last observable $\hat{\sigma}_z^1 \hat{\sigma}_z^2$, we again arrive to I_2.

Finally, in the last column we find the observables $\hat{\sigma}_x^1 \hat{\sigma}_x^2$, $\hat{\sigma}_y^1 \hat{\sigma}_y^2$ and $\hat{\sigma}_z^1 \hat{\sigma}_z^2$. Following the same steps of the last appendix we see that:

$$\begin{aligned} \hat{\sigma}_x^1 \hat{\sigma}_x^2 \hat{\sigma}_y^1 \hat{\sigma}_y^2 &= \hat{\sigma}_x^1 \hat{\sigma}_y^1 \hat{\sigma}_x^2 \hat{\sigma}_y^2 \\ &= (i\hat{\sigma}_z^1)(i\hat{\sigma}_z^2) = -\hat{\sigma}_z^1 \hat{\sigma}_z^2\,; \end{aligned}$$

hence, the product of these two observables with $\hat{\sigma}_z^1 \hat{\sigma}_z^2$ gives $-I_2$.

A.3 Bell's inequality

Bell starts by comparing the expectation values of measurements along directions $\vec{a}$ and $\vec{b}$ and measurements along directions $\vec{a}$ and $\vec{c}$,

$$\left| E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \right| = \left| \int_\Lambda \left[A(\vec{a},\lambda)B(\vec{b},\lambda) - A(\vec{a},\lambda)B(\vec{c},\lambda) \right] \rho(\lambda) d\lambda \right|;$$

then he factorizes the function $A(\vec{a},\lambda)$ and uses the fact that $B(\vec{b},\lambda)B(\vec{b},\lambda) = 1$ to get:

$$\left| E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \right| = \left| \int_\Lambda A(\vec{a},\lambda)B(\vec{b},\lambda) \left[1 - B(\vec{b},\lambda)B(\vec{c},\lambda) \right] \rho(\lambda) d\lambda \right|.$$

Afterwards he takes the absolute value function under the integral and, since $\left| A(\vec{a},\lambda)B(\vec{b},\lambda) \right| = 1$, he arrives to:

$$\left| E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \right| \leq \int_\Lambda \left| \left[1 - B(\vec{b},\lambda)B(\vec{c},\lambda) \right] \rho(\lambda) \right| d\lambda,$$

but what is inside the absolute value function is always positive, so he just discards the bars. Now he changes $B(\vec{b},\lambda)$ for $-A(\vec{b},\lambda)$ so that

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| \leq \int_{\Lambda} \left[1 + A(\vec{b},\lambda)B(\vec{c},\lambda)\right] \rho(\lambda) d\lambda,$$

which takes him to his final step,

$$\int_{\Lambda} \left[1 + A(\vec{b},\lambda)B(\vec{c},\lambda)\right] \rho(\lambda) d\lambda = 1 + E(\vec{b},\vec{c}),$$

concluding,

$$\left|E(\vec{a},\vec{b}) - E(\vec{a},\vec{c})\right| \leq 1 + E(\vec{b},\vec{c}).$$

A.4 Expectation value of two spin measurements

We have two particles in a singlet state, expressed in the $\hat{\sigma}_z$-diagonal basis,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big].$$

The first step is to express $\hat{\sigma}_a$ and $\hat{\sigma}_b$ in that basis:

$$\hat{\sigma}_a = \sin\theta_{az}\Big[|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\Big] + \cos\theta_{az}\Big[|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|\Big],$$

where θ_{az} is the angle between the detector orientation $\vec{a}$ and the chosen direction $\vec{z}$, and

$$\hat{\sigma}_b = \sin\theta_{bz}\Big[|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\Big] + \cos\theta_{bz}\Big[|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|\Big],$$

where θ_{bz} is the angle between the detector orientation $\vec{b}$ and the chosen direction $\vec{z}$.

For the calculation of $\langle\psi|\hat{\sigma}_a\hat{\sigma}_b|\psi\rangle$ it is understood that $\hat{\sigma}_a$ acts on the first of the two Hilbert spaces in $|\psi\rangle$, and that $\hat{\sigma}_b$ acts on the second Hilbert space. This calculation goes as follows:

$$\begin{aligned}\langle\psi|\hat{\sigma}_a\hat{\sigma}_b|\psi\rangle = &\frac{1}{2}\sin\theta_{az}\sin\theta_{bz}\Big[-\langle\uparrow\downarrow|\uparrow\downarrow\rangle\langle\downarrow\uparrow|\downarrow\uparrow\rangle - \langle\downarrow\uparrow|\downarrow\uparrow\rangle\langle\uparrow\downarrow|\uparrow\downarrow\rangle\Big]\\ &+\frac{1}{2}\cos\theta_{az}\cos\theta_{bz}\Big[-\langle\uparrow\downarrow|\uparrow\downarrow\rangle\langle\uparrow\downarrow|\uparrow\downarrow\rangle - \langle\downarrow\uparrow|\downarrow\uparrow\rangle\langle\downarrow\uparrow|\downarrow\uparrow\rangle\Big],\end{aligned}$$

where we have only kept the terms that are non-zero, given that $\langle\uparrow|\downarrow\rangle = 0$. Furthermore, $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1$, so

$$\begin{aligned}\langle\psi|\hat{\sigma}_a\hat{\sigma}_b|\psi\rangle &= \frac{1}{2}\sin\theta_{az}\sin\theta_{bz}[-2] + \frac{1}{2}\cos\theta_{az}\cos\theta_{bz}[-2]\\ &= -\sin\theta_{az}\sin\theta_{bz} - \cos\theta_{az}\cos\theta_{bz}\\ &= -\cos(\theta_{az}-\theta_{bz});\end{aligned}$$

but the difference between the two angles $(\theta_{az}-\theta_{bz})$ is simply $(\pm)$ the angle between $\vec{a}$ and $\vec{b}$, $\pm\theta_{ab}$, and $\cos(\pm\theta_{ab}) = \cos(\theta_{ab})$, thereby

$$\langle\hat{\sigma}_a\hat{\sigma}_b\rangle = -\cos\theta_{ab}\,,$$

which takes us to equation (1.7).

A.5 GHZ state

The $|GHZ\rangle$ state is given in the $\hat{\sigma}_z$-diagonal basis, so it is useful to remember that:

$$\begin{aligned}\hat{\sigma}_x|\uparrow\rangle^z &= |\downarrow\rangle^z,\\ \hat{\sigma}_x|\downarrow\rangle^z &= |\uparrow\rangle^z,\\ \hat{\sigma}_y|\uparrow\rangle^z &= i|\downarrow\rangle^z,\\ \hat{\sigma}_y|\downarrow\rangle^z &= -i|\uparrow\rangle^z;\end{aligned}$$

therefore,

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3|\uparrow\uparrow\uparrow\rangle = (1)(i)(i)|\downarrow\downarrow\downarrow\rangle = -|\downarrow\downarrow\downarrow\rangle$$

and

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3|\downarrow\downarrow\downarrow\rangle = (1)(-i)(-i)|\uparrow\uparrow\uparrow\rangle = -|\uparrow\uparrow\uparrow\rangle.$$

Hence, by linear combination of these two results,

$$\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3\frac{1}{\sqrt{2}}\Big[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\Big] = \frac{1}{\sqrt{2}}\Big[|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle\Big],$$

which is the main part of what we wanted to prove.

Since the state $|GHZ\rangle$ is indistinguishable under the exchange of particles 1,2,3 and, as we have shown, is an eigenstate of the observable $\hat{\sigma}_x^1\hat{\sigma}_y^2\hat{\sigma}_y^3$ with eigenvalue 1, we can conclude that it is an eigenstate of the other two observables in (1.8) with eigenvalue 1.

Appendix B

Factuality

B.1 PBR state under non-distinguishability

To demonstrate that if the preparation method does not "announce" a distinction between the states $|0\rangle$ and $|+\rangle$, then the state that arrives to the detector is

$$|\Psi\rangle = N\Big[|0\rangle + |+\rangle\Big] \otimes N\Big[|0\rangle + |+\rangle\Big],$$

we must first construct a preparation device that behaves according to the PBR conditions, *but* with no distinguishability between the prepared systems.

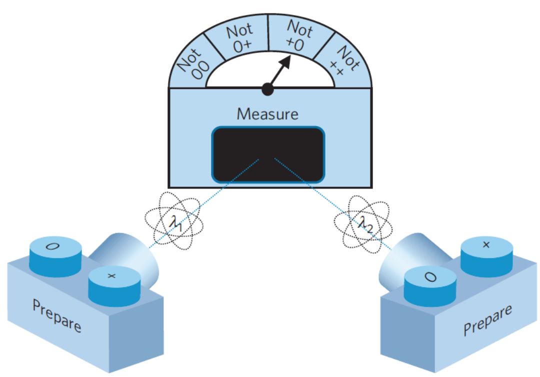


Figure B.1: Figure given by PBR in [22]

> Consider two methods of preparing a quantum system, corresponding to quantum states $|\psi_0\rangle$ and $|\psi_1\rangle$, with $|\langle\psi_0|\psi_1\rangle| = 1/\sqrt{2}$. Choose a basis of the Hilbert space so that $|\psi_0\rangle = |0\rangle$ and $|\psi_1\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. ... Now consider two systems whose physical states are uncorrelated. This can be achieved, for example, by constructing and operating two copies of a preparation device independently.

> Each system can be prepared such that its quantum state is either $|\psi_0\rangle$ or $|\psi_1\rangle$, as illustrated in [Figure B.1].

So, we have two identical preparation devices each of which prepares either the state $|0\rangle$ or the state $|+\rangle$ *without making an announcement of which of the two states was prepared.*

Each device can be built by putting together a photon source and a Mach-Zehnder interferometer, with a polarizer set in the vertical direction ($\equiv |0\rangle$) on one path of the laser, and a polarizer set to 45° ($\equiv |+\rangle$) on the other path of the laser. At the end, the two paths are recombined.

Under these conditions, the state that comes out of the preparation device is

$$|\psi\rangle = N\Big[|0\rangle + |+\rangle\Big],$$

where one can easily calculate the normalization constant N to obtain:

$$N^2 = \frac{\sqrt{2}}{2\sqrt{2}+2}\,.$$

Finally, when the two preparation devices are put together, one gets the state

$$|\Psi\rangle = N^2\Big[|0\rangle + |+\rangle\Big] \otimes \Big[|0\rangle + |+\rangle\Big],$$

which is what we wanted to show.

Appendix C

Locality

C.1 Second path for identical Λ's

We start from:

$$\mathcal{F}(\lambda, t_1) = (\vec{o}_A(\lambda, t_1), \vec{o}_B(\lambda, t_1)) = (\pm\vec{a}, \pm\vec{b}),$$

and we define:

$$A_1(\vec{a}, \lambda) \equiv \mathrm{sign}(\vec{o}_A(\lambda, t_1))$$

and

$$B_1(\vec{b}, \lambda) \equiv \mathrm{sign}(\vec{o}_B(\lambda, t_1)),$$

where we carried the subscript 1 to distinguish these functions from the ones defined by $\mathcal{F}(\lambda, t_2)$. Now:

$$\mathcal{F}(\lambda, t_2) = (\vec{o}_A(\lambda, t_2), \vec{o}_B(\lambda, t_2)) = (\pm\vec{a}, \pm\vec{c}),$$

so we can simultaneously define:

$$A_2(\vec{a}, \lambda) \equiv \mathrm{sign}(\vec{o}_A(\lambda, t_2))$$

and

$$B_2(\vec{c}, \lambda) \equiv \mathrm{sign}(\vec{o}_B(\lambda, t_2)).$$

And finally:

$$A_3(\vec{b}, \lambda) \equiv \mathrm{sign}(\vec{o}_A(\lambda, t_3))$$

and

$$B_3(\vec{c}, \lambda) \equiv \mathrm{sign}(\vec{o}_B(\lambda, t_3)).$$

Now, of course functions A_i and B_i defined this way are not necessarily identical to those defined by the the first path, just because $\vec{o}_A(\lambda, t_3)$ is not necessarily the same as $\vec{o}_{A_3}(\lambda, t_1)$, etc. The thing is that, once one defines a set of functions $\{A_1, B_1, A_2, B_2, A_3, B_3\}$, function $A_1(\vec{a}, \lambda)$ *can* be different from $A_2(\vec{a}, \lambda)$ (and so forth) and this is the argument we use in the rest of our development.

C.2 Bell's assumptions

Bell starts from equation (3.5):

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| = \left|\int_\Lambda \left[A_1(\vec{a}, \lambda)B_1(\vec{b}, \lambda) - A_2(\vec{a}, \lambda)B_2(\vec{c}, \lambda)\right] \rho(\lambda)d\lambda\right|$$

and makes his first assumption:

$$A_1(\vec{a}, \lambda) = A_2(\vec{a}, \lambda);$$

then equation (3.5) turns to:

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| = \left|\int_\Lambda A_1(\vec{a}, \lambda)B_1(\vec{b}, \lambda) \left[1 - B_1(\vec{b}, \lambda)B_2(\vec{c}, \lambda)\right] \rho(\lambda)d\lambda\right|,$$

where he uses the fact that $B_1 B_1 = 1$. Now, taking the absolute value function under the integral and using the fact that $|A_1 B_1| = 1$ his last equation turns to:

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| \leq \int_\Lambda \left|\left[1 - B_1(\vec{b}, \lambda)B_2(\vec{c}, \lambda)\right] \rho(\lambda)\right| d\lambda, \tag{C.1}$$

but what is inside the absolute value function is always positive, so he just discards the bars. Next comes his second assumption:

$$B_1(\vec{b}, \lambda) = -A_3(\vec{b}, \lambda),$$

so equation (C.1) becomes:

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| \leq \int_\Lambda \left[1 + A_3(\vec{b}, \lambda)B_2(\vec{c}, \lambda)\right] \rho(\lambda)d\lambda. \tag{C.2}$$

And finally, he takes a third assumption:

$$B_2(\vec{c}, \lambda) = B_3(\vec{c}, \lambda);$$

then equation (C.2) turns to:

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| \leq \int_\Lambda \left[1 + A_3(\vec{b}, \lambda)B_3(\vec{c}, \lambda)\right] \rho(\lambda)d\lambda,$$

which allows him to conclude,

$$\left|E(\vec{a}, \vec{b}) - E(\vec{a}, \vec{c})\right| \leq 1 + E(\vec{b}, \vec{c}).$$

C.3 Expectation values within Bell's assumptions

We begin by building a table of probabilities for the first scenario (detector settings $\vec{a}$ and $\vec{b}$), under the following knowledge: the probability of getting either $+1$ or -1 when measuring the spin projection of particle A is $1/2$, but once one of those is guaranteed, say $+1$, the probability of getting $+1$ when measuring the spin projection of particle B is $\sin^2(\theta_{ab}/2)$ and the probability of getting -1 is $\cos^2(\theta_{ab}/2)$. So we have the joint probabilities shown in Table C.1.

Table C.1: Joint probabilities for experiment 1. The probability for two of the outcomes in the edges is the product of the two numbers in the cell where the edges coincide.

$A_1(\vec{a},\lambda)$ \ $B_1(\vec{b},\lambda)$	1	-1
1	$\frac{1}{2}$ $\sin^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$ $\cos^2\frac{\theta_{ab}}{2}$
-1	$\frac{1}{2}$ $\cos^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$ $\sin^2\frac{\theta_{ab}}{2}$

Now, the assumption $A_1(\vec{a},\lambda) = A_2(\vec{a},\lambda)$ invites us to substitute A_1 for A_2 and the assumption $B_1(\vec{b},\lambda) = -A_3(\vec{b},\lambda)$, to substitute B_1 for $-A_3$, turning Table C.1 into Table C.2.

Table C.2: Joint probabilities under the assumptions $A_1(\vec{a},\lambda) = A_2(\vec{a},\lambda)$ and $B_1(\vec{b},\lambda) = -A_3(\vec{b},\lambda)$.

$A_2(\vec{a},\lambda)$ \ $-A_3(\vec{b},\lambda)$	1	-1
1	$\frac{1}{2}$ $\sin^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$ $\cos^2\frac{\theta_{ab}}{2}$
-1	$\frac{1}{2}$ $\cos^2\frac{\theta_{ab}}{2}$	$\frac{1}{2}$ $\sin^2\frac{\theta_{ab}}{2}$

The joint probabilities for experiment 2 are built accordingly and result in the left hand side of Table C.3. Taking into account the assumption $B_2(\vec{c},\lambda) = B_3(\vec{c},\lambda)$ one gets the right hand side of Table C.3.

Table C.3: **LHS** joint probabilities for experiment 2. **RHS** same, under the assumption $B_2(\vec{c},\lambda) = B_3(\vec{c},\lambda)$.

$A_2(\vec{a},\lambda)$ \ $B_2(\vec{c},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$
-1	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$

$A_2(\vec{a},\lambda)$ \ $B_3(\vec{c},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$
-1	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$

Table C.4 just brings together Table C.2 and the right hand side of Table C.3. We will use it to compute the joint probabilities of $A_3(\vec{b},\lambda)$ and $B_3(\vec{c},\lambda)$.

Table C.4: Joint probabilities of $A_3(\vec{b},\lambda)$ and $B_3(\vec{c},\lambda)$.

$A_2(\vec{a},\lambda)$ \ $-A_3(\vec{b},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ab}}{2}$	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ab}}{2}$
-1	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ab}}{2}$	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ab}}{2}$

$A_2(\vec{a},\lambda)$ \ $B_3(\vec{c},\lambda)$	1		-1	
1	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$
-1	$\frac{1}{2}$	$\cos^2 \frac{\theta_{ac}}{2}$	$\frac{1}{2}$	$\sin^2 \frac{\theta_{ac}}{2}$

The procedure is as follows:

$A_2(\vec{a},\lambda) = +1$ for λ in a certain set, say Λ_+, and from Table C.4 if $\lambda \in \Lambda_+$, then the probability that $A_3(\vec{b},\lambda) = 1$ is $\cos^2(\theta_{ab}/2)$ and the probability that $A_3(\vec{b},\lambda) = -1$ is $\sin^2(\theta_{ab}/2)$, while the probability that $B_3(\vec{b},\lambda) = 1$ is $\sin^2(\theta_{ac}/2)$ and the probability that $B_3(\vec{b},\lambda) = -1$ is $\cos^2(\theta_{ac}/2)$. So, for $\lambda \in \Lambda_+$ the probability of getting the same sign in both functions A_3 and B_3 is:

$$\mathcal{P}(A_3 \cdot B_3 = 1) = \cos^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} + \sin^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2},$$

and the probability of getting opposite signs is:

$$\mathcal{P}(A_3 \cdot B_3 = -1) = \cos^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2} + \sin^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2}.$$

Similarly, if $\lambda \in \Lambda_-$,

$$\mathcal{P}(A_3 \cdot B_3 = 1) = \sin^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2} + \cos^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2}$$

and

$$\mathcal{P}(A_3 \cdot B_3 = -1) = \sin^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} + \cos^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2}.$$

But at the same time, the probability that $\lambda \in \Lambda_+$ is $1/2$ as is the probability that $\lambda \in \Lambda_-$, so we must multiply all the four last equations by $1/2$ and then add them to obtain:

$$\mathcal{P}(A_3 \cdot B_3 = 1) = \cos^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} + \sin^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2} \tag{C.3}$$

and

$$\mathcal{P}(A_3 \cdot B_3 = -1) = \sin^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} + \cos^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2}. \tag{C.4}$$

Functions with the probability distributions given by equations (C.3) and (C.4) describe an experiment in which the expectation value of the correlation between these two functions would be:

$$\begin{aligned}
E(\vec{b}, \vec{c}) &= \mathcal{P}(A_3 \cdot B_3 = 1) - \mathcal{P}(A_3 \cdot B_3 = -1) \\
&= \cos^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} + \sin^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2} \\
&\quad - \sin^2 \frac{\theta_{ab}}{2} \sin^2 \frac{\theta_{ac}}{2} - \cos^2 \frac{\theta_{ab}}{2} \cos^2 \frac{\theta_{ac}}{2} \\
&= (\cos^2 \frac{\theta_{ab}}{2} - \sin^2 \frac{\theta_{ab}}{2})(\sin^2 \frac{\theta_{ac}}{2} - \cos^2 \frac{\theta_{ac}}{2}) \\
&= -\cos\theta_{ab} \cos\theta_{ac}\,.
\end{aligned}$$

C.4 Demonstration of the derived inequality

The inequality to be analyzed is:

$$|-\cos\theta_{ab} + \cos\theta_{ac}| \leq 1 - \cos\theta_{ab}\cos\theta_{ac}\,,$$

which turns to

$$\cos\theta_{ab}\cos\theta_{ac} - 1 \leq -\cos\theta_{ab} + \cos\theta_{ac} \leq 1 - \cos\theta_{ab}\cos\theta_{ac}\,.$$

The first inequality is satisfied iff

$$\cos\theta_{ab}\cos\theta_{ac} + \cos\theta_{ab} \leq \cos\theta_{ac} + 1,$$

or, equivalently,

$$\cos\theta_{ab}(\cos\theta_{ac} + 1) \leq \cos\theta_{ac} + 1, \tag{C.5}$$

and the second inequality is satisfied iff

$$\cos\theta_{ac} + \cos\theta_{ab}\cos\theta_{ac} \leq 1 + \cos\theta_{ab}\,,$$

or, equivalently,

$$\cos\theta_{ac}(1 + \cos\theta_{ab}) \leq 1 + \cos\theta_{ab}\,. \tag{C.6}$$

Finally, equations (C.5) and (C.6) are both true iff

$$\cos\theta_{ab} \leq 1 \quad \text{and} \quad \cos\theta_{ac} \leq 1,$$

which always holds. So inequality (3.6) is always satisfied.

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