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QUE PARA OPTAR POR EL GRADO DE:
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PRESENTA:
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Firma

Capítulo 1
Generalidades

1.1 Introducción.
En este trabajo se presenta primero un marco teórico acerca de los robots paralelos industriales. Posteriormente se da una descripción del proyecto y se comienza con el análisis cinemático del mismo.

1.2 Objetivo.
Analizar la cinemática y dinámica de un robot paralelo espacial de 6 grados de libertad para la obtención de un modelo.

1.3 Fundamentos de Robots.

1.3.1 Definición de robot industrial.
Existen diversas definiciones acerca de lo que es un robot industrial según la institución de la cual provenga dicha definición. Algunas de estas son:

Según la **Asociación de Industrias Robóticas** (RIA):

- Un robot industrial es un manipulador multifuncional reprogramable, capaz de mover materias, piezas, herramientas o dispositivos especiales, según trayectorias variables, programadas para realizar tareas diversas (Barrientos et al., 2007).

Según la **Organización Internacional de Estándares** (ISO):

- Manipulador funcional reprogramable con varios grados de libertad, capaz de manipular materias, piezas, herramientas o dispositivos especiales según trayectorias variables programadas para realizar tareas diversas (Barrientos et al., 2007).

Al leer las dos definiciones anteriores se observa que son muy parecidas aunque en la de la ISO se especifica que debe tener varios grados de libertad. Las dos hacen referencia a que es un manipulador aunque no se da la definición de éste. Esto no ocurre en la definición de la **Asociación Francesa de Normalización** (AFNOR) que primero define lo que es un manipulador:

- Manipulador: mecanismo formado generalmente por elementos en serie, articulados entre sí, destinado al agarre y desplazamiento de objetos. Es multifuncional y puede ser gobernado directamente por un operador humano o mediante un dispositivo lógico (Barrientos et al., 2007).

- Robot: manipulador automático servo-controlado, reprogramable, polivalente, capaz de orientar y posicionar piezas útiles o dispositivos especiales, siguiendo trayectorias variables reprogramables, para la ejecución de tareas variadas. Normalmente tiene la forma de uno o varios brazos terminados en una muñeca. Su unidad de control incluye un dispositivo de memoria y ocasionalmente de percepción del entorno. Normalmente su uso es el de realizar una tarea de manera cíclica, pudiéndose adaptar a otra sin cambios permanentes en su material (Barrientos et al., 2007).

1.3.2 Estructura mecánica del robot.

Mecánicamente un robot está formado por una serie de elementos o eslabones interconectados entre sí por medio de articulaciones o juntas, que permiten el movimiento y establecen el número de grados de libertad entre dos elementos consecutivos. Los grados de libertad se pueden definir como el número de movimientos independientes que tiene un eslabón con respecto a otro.

El movimiento de las articulaciones puede ser de rotación o de traslación o una combinación de ambas, por lo que existen seis diferentes tipos de articulación (Barrientos et al., 2007):

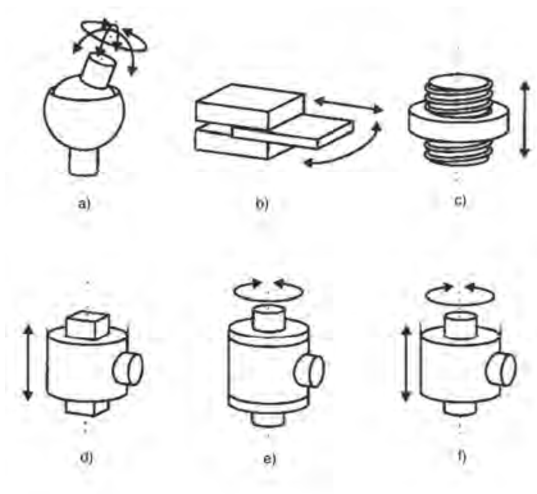


Figura 1-1 Tipos de articulación.

a) **Esférica (3 GDL):** Permite tres giros (θ_x θ_y θ_z) entre eslabones.
b) **Plana (3 GDL):** Permite dos traslaciones (x, y) y un giro (θ) entre eslabones.
c) **Tornillo (1 GDL):** Permite un traslación (x) y un giro (θ) pero están relacionados.
d) **Prismática (1 GDL):** Permite una traslación (x) entre eslabones.
e) **Rotacional (1 GDL):** Permite un giro (θ) entre eslabones.
f) **Cilíndrica (2 GDL):** Permite un giro (θ) y una traslación (x) entre sus eslabones.

El empleo de diferentes combinaciones de articulaciones, da lugar a diferentes configuraciones, con características a tener en cuenta tanto en el diseño y construcción del robot como en su aplicación. A partir de estas combinaciones se puede establecer una clasificación que se muestra en la imagen siguiente (Barrientos et al., 2007):

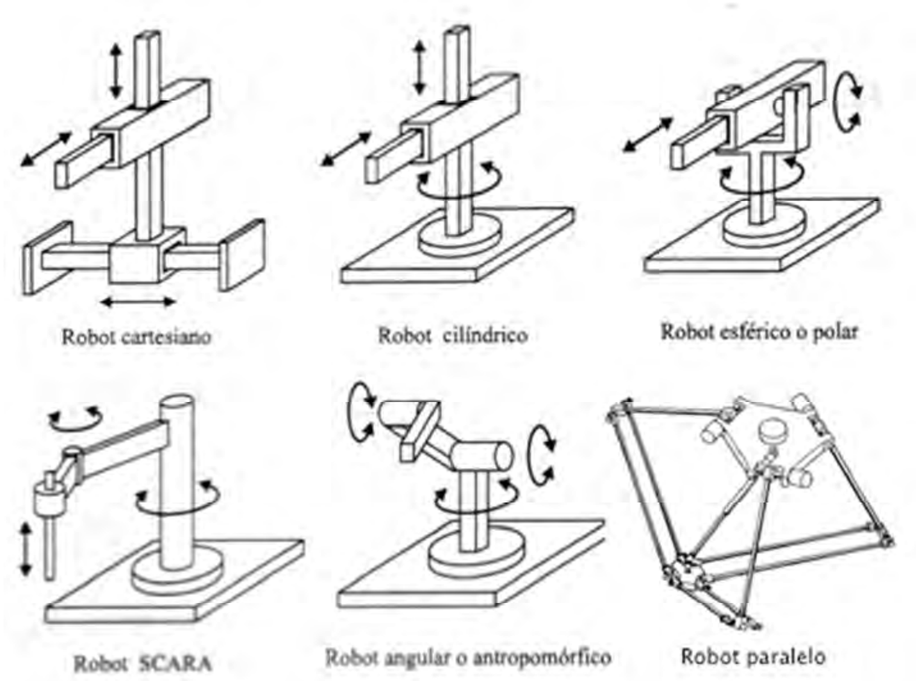


Figura 1-2 Clasificación de robots industriales.

De esta manera un robot paralelo está formado por un efector y una base fija unidos por al menos dos cadenas cinemáticas independientes en donde el número de actuadores es igual número de grados de libertad (Merlet, 2000).

1.4 Estado del arte.

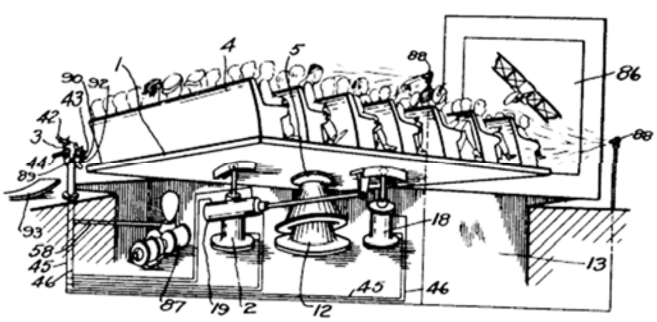


Figura 1-3 Plataforma diseñada por James Gwinnett.

Los robots paralelos fueron originalmente propuestos para lidiar con las desventajas de los robots seriales siendo estas su limitada capacidad de carga, poca precisión y baja rigidez (Angeles, 1997).

El primer robot espacial paralelo fue una plataforma de movimiento dedicada a la industria del entretenimiento y diseñada por James E. Gwinnett (Gwinnett, 1931). Su dispositivo estaba basado en un robot esférico paralelo y fue patentado en 1931.

Una década después un nuevo robot paralelo fue inventado para el pintado automatizado con spray por Willard L.V. Pollard (Pollard, 1940) y es conocido como el primer robot paralelo industrial. Su invento tiene 5 grados de libertad, consta de tres cadenas cinemáticas y fue patentado en 1942 (US Patent No. 2,286,571).

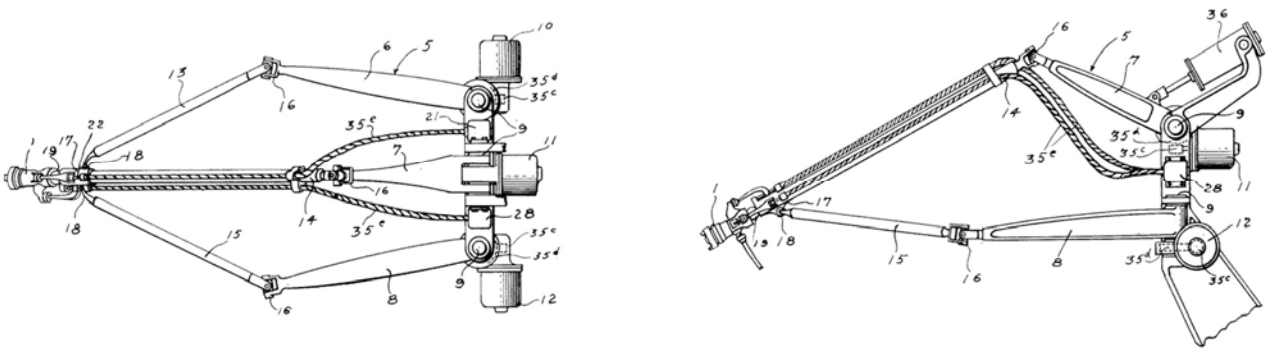


Figura 1-4 Robot patentado por Willard L.G. Pollard Jr.

En Octubre 29 de 1934 el hijo de Willard L.V. Pollard, Willard L.G. Pollard Jr. (Pollard, 1942), presentó una patente de una máquina para pintado con spray. La patente consistía en dos partes, una era el sistema de control el cual consistía en películas perforadas en las cuales la velocidad de rotación de los motores era proporcional a la densidad de los orificios. La otra parte era un manipulador mecánico el cual estaba basado en un pantógrafo con dos motores como actuadores en la base. La patente fue emitida el 27 de agosto de 1940.

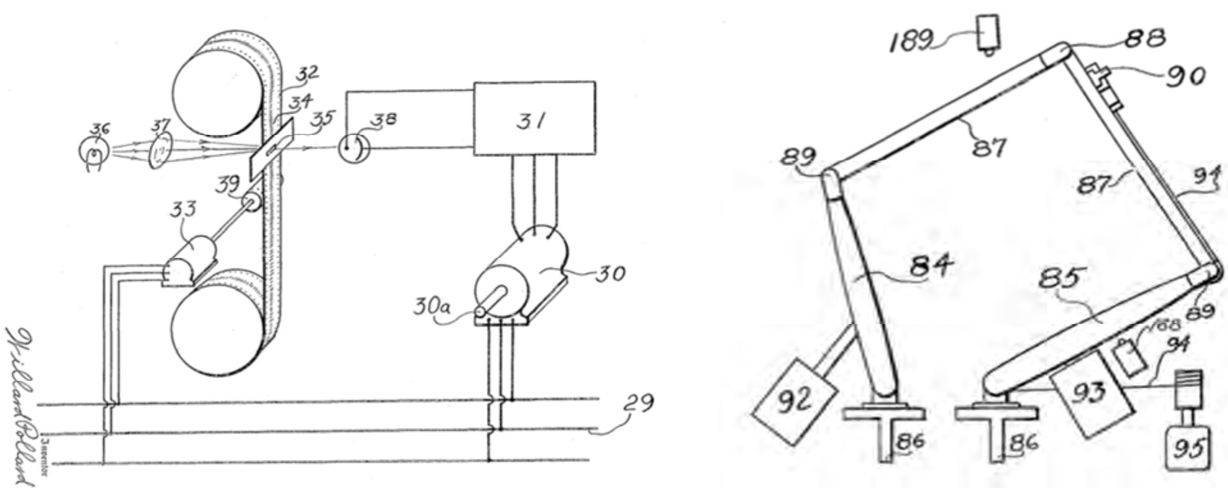


Figura 1-5 Ilustraciones presentes en la patente de DeVilbiss.

Mientras tanto en 1937 le fue concedida a una licencia a la compañía DeVilbiss, que más tarde en 1941 se convertiría en el primer proveedor de robots industriales, completando su primer prototipo bajo la dirección de Harold Roselund (Roselund, 1944). La patente de este robot fue emitida el 14 de Marzo de 1944 y utilizaba el sistema de control diseñado por Pollard Jr. Es por esto que el crédito del primer robot industrial construido se le atribuye tanto a Ruselund como a Pollard Jr.

Existen otros robots paralelos presentes en la historia como el hexápodo de Gough diseñado a partir de la necesidad de medir las deflexiones causadas por fuerzas aplicadas en cualquier dirección de un neumático (Gough y Whitehall, 1962). Una configuración similar fue un mecanismo de 8 barras con propósitos diversos desde espaciales hasta simuladores de vuelo (Stewart, 1965). Debido a la similitud entre ambos mecanismos las plataformas basadas en octaedros hexápodos son conocidas como plataformas Gough-Stewart (Gallardo, 2016). A pesar de los trabajos previos la primer patente para un mecanismo octaedro hexápodo fue concedida a Klaus Cappel en 1967 (Cappel, 1967).

En 1980 Raymond Clavel presentó un mecanismo original que hoy se conoce como "Robot Delta" el cual hace uso de paralelogramos para restringir las rotaciones entre la plataforma fija y la móvil (Clavel 1988). La patente para dicho robot la obtuvo en 1990 (Clavel, 1990). El robot delta y variantes del mismo son de los más usados para tareas de tomar y colocar en la actualidad. Una extensión a seis grados de libertad del robot delta fue desarrollada por Pierrot François y compañía, el robot fue conocido como "Hexa" (Pierrot et al., 1991). Entre 1993 y 1996 Gosselin y compañía desarrollaron un robot esférico para la rápida orientación de una cámara llamado "Agile Eye" (Gosselin et al., 1994). En 2003 Kim y Tsai desarrollaron un robot cartesiano donde el movimiento en cada eje es independiente. A este tipo de robot se le conoce como robot paralelo lineal (Kim y Tsai, 2003).

Muchos otros robots con aplicaciones específicas han sido diseñados con base en los antes mencionados, algunos de estos son; un robot para la emulación de operaciones espaciales basado en una plataforma de Stewart (Nguyen et al., 1993), mecanismo de 6 GDL, robot para lelo dedicado a operaciones de excavación subterránea (Arai et al., 1991), un delta modificado para resucitación cardiopulmonar (Yangmin y Qingsong, 2005) o un robot de 6 grados de libertad para el aislamiento de vibraciones (Geng y Haynes, 1993).

1.5 Marco teórico.

El desarrollo de la cinemática de robots se puede dividir en dos enfoques, la cinemática directa y la inversa. La cinemática directa consiste en describir la posición y orientación del efector final dados los valores de las variables de las juntas (Spong, 1989). El enfoque de cinemática inversa consiste en descubrir el comportamiento de las juntas dada una trayectoria específica (Spong, 1989).

Existen algunos métodos para calcular el comportamiento cinemático del robot, como son; el método algebraico, el método iterativo, el método matricial que puede ser abordado por medio de transformaciones homogéneas o por la convención de Denavit-Hatenberg. El enfoque de transformaciones homogéneas consiste en expresar los movimientos del robot en términos de matrices de rotación y traslación (Tsai, 1999). Para el caso de un robot espacial se definen 6 trasformaciones, tres para las rotaciones y tres para las traslaciones. La convención de Denavit-Hatenberg consiste en usar una matriz de transformación compuesta por dos rotaciones y dos traslaciones (Spong, 1989). La teoría de tornillos también puede ser usada para el cálculo cinemático. Esta teoría permite expresar desplazamientos, velocidades, fuerzas y torques en un espacio tridimensional, usando la idea de que el movimiento de un cuerpo rígido puede ser representado por la unión de una rotación alrededor de un eje y una traslación sobre el mismo, donde el eje es coincidente con el objeto (Gallardo, 2016).

El cálculo dinámico consiste en calcular las fuerzas y torques necesarios para que el mecanismo alcance una posición deseada en un tiempo dado (Spong, 1989). El desarrollo de un modelo dinámico es importante para la simulación del robot ya que de esta manera es posible predecir el comportamiento del mismo antes de ser construido. Otro aspecto de importancia es que el modelo obtenido es usado para el diseño de una estrategia de control. Un controlador eficiente requiere de un modelo dinámico realista para tener un desempeño óptimo. Así mismo el análisis dinámico ayuda al diseño de los actuadores, eslabones y rodamientos, mediante la obtención de las fuerzas y momentos de reacción en las juntas (Tsai, 1999).

Al igual que con la cinemática, el problema de la dinámica puede ser abordado de dos maneras. Una es la dinámica directa, la cual consiste encontrar la respuesta del robot ante una serie de fuerzas y torques aplicados en función del tiempo. La otra es la dinámica inversa que se centra en calcular las fuerzas y torques necesarios para que el robot siga una trayectoria deseada (Tsai, 1999).

Las ecuaciones dinámicas de movimiento pueden ser formuladas de distintas maneras. Un enfoque es la aplicación del método de Newton-Euler, lo cual deriva en la obtención de ecuaciones que contemplan tanto las fuerzas aplicadas como las fuerzas de restricción. Otro método comúnmente es el de Euler-Lagrange el cual elimina las fuerzas de restricción y se enfoca únicamente en las fuerzas o torques encargados del movimiento (Tsai, 1999). Alternativamente un tercer método puede ser usado el cual se conoce como método de Kane, el cual al igual que el de Euler-Lagrange, prescinde de las fuerzas de restricción y se enfoca en el uso de fuerzas generalizadas para su formulación (Kane y Lenvinson, 1985).

1.6 Justificación.

El robot diseñado por Pollard fue el primer mecanismo paralelo en ser propuesto para propósitos industriales, desafortunadamente nunca llegó a ser construido. Por su configuración mecánica se puede considerar como el ancestro del robot delta sin embargo su análisis resulta ser más complicado debido a la presencia de juntas esféricas en el órgano terminal. Debido a esta mayor dificultad, la mayoría de los estudios se han centrado en analizar y mejorar configuraciones basadas en el delta. A causa de esta falta de documentación referente a su análisis, resulta ser un buen caso de estudio para evaluar el comportamiento que tendría el robot en procesos industriales.

1.7 Metodología.

En el desarrollo de la tesis se seguirá la siguiente metodología:

Análisis Cinemático

1. Análisis de Posición
2. Análisis de Velocidad
3. Análisis de Aceleración

Análisis Dinámico

Formulación Euler-Lagrange

1. Energía Cinética
2. Energía Potencial
3. Función Lagrangiana
4. Fuerzas Generalizadas

1.8 Arquitectura del robot.

El modelo de robot a analizar está basado en el diseño de Willard L.G. Pollard.

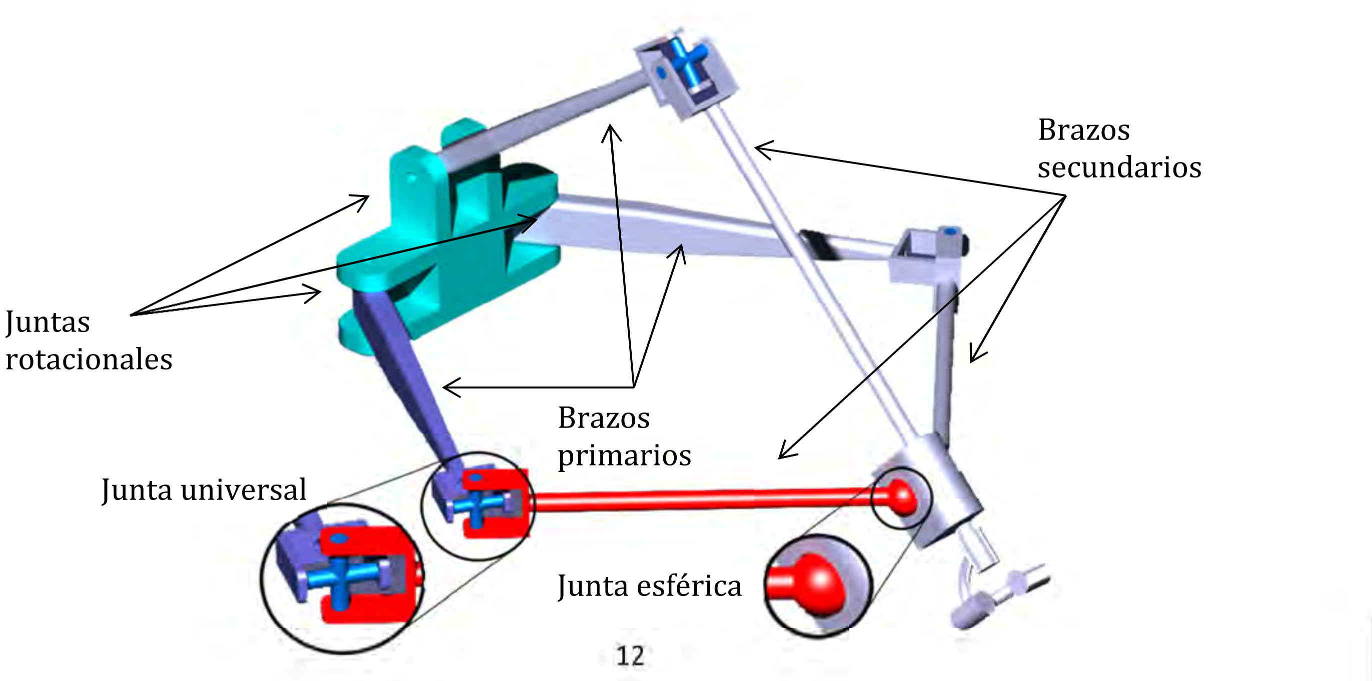


Figura 1-6 Modelo del robot basado en el diseño de Willard L.G. Pollard Jr.

El robot cuenta con tres brazos primarios unidos a una base mediante juntas rotacionales. Estos a su vez están unidos cada uno a tres brazos secundarios mediante juntas universales. Dos de los tres brazos convergen en un punto mediante juntas esféricas y el tercer brazo secundario está unido rígidamente a este mismo punto. En el punto donde convergen los tres brazos va montado el órgano terminal mediante una junta esférica. En la base, donde van montados los brazos primarios irán acoplados a estos los actuadores encargados de dar la posición al órgano terminal y en el lugar en el cual convergen los brazos secundarios irán los otros tres actuadores encargados de dar la orientación a la herramienta.

Capítulo 2
Análisis cinemático

2.1 Introducción

Se usaron Transformaciones Homogéneas para el análisis cinemático (Stejskal y Valáek, 1996). Este método consiste en crear sistemas de referencia locales a lo largo de cadenas cinemáticas con el fin de aplicar una serie de rotaciones y traslaciones por medio de matrices de transformación.

Una matriz homogénea de transformación es una matriz de 4x4 cuyo propósito es proyectar un vector de posición de un sistema de coordenadas a otro (Tsai, 1999). En general se puede dividir en 4 submatrices de manera siguiente:

$$\boldsymbol{T}_B^A = \begin{bmatrix} \boldsymbol{R}_B^A(3x3) & \boldsymbol{q}(3x1) \\ \boldsymbol{\gamma}(1x3) & \rho(1x1) \end{bmatrix}$$

Donde la submatriz superior izquierda de 3x3 denota la orientación de un marco móvil con respecto a un marco de referencia. La submatriz superior derecha de 3x1 denota la posición del origen del marco móvil con respecto al marco de referencia. La submatriz inferior izquierda denota una transformación de perspectiva y el elemento inferior derecho es un factor de escalamiento (Tsai, 1999), (Nevatia et al., 1941).

De esta manera se definen 6 transformaciones:

- Rotación alrededor en el eje X ($\boldsymbol{T}_{z1}$).
- Rotación alrededor en el eje Y($\boldsymbol{T}_{z2}$).
- Rotación alrededor en el eje Z($\boldsymbol{T}_{z3}$).
- Traslación sobre el eje X ($\boldsymbol{T}_{z4}$).
- Traslación sobre el eje Y ($\boldsymbol{T}_{z5}$).
- Traslación sobre el eje Z ($\boldsymbol{T}_{z6}$).

Por convención se nombra a cada una de ellas con $\boldsymbol{T}$ y se les da un subíndice para distinguirlas.

$$\boldsymbol{T}_{z1}(x) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{z2}(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{z3}(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{z4}(\theta_x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\,(\theta) & -sen(\theta) & 0 \\ 0 & sen(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{z5}(\theta_y) = \begin{bmatrix} cos(\theta) & 0 & sen(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sen(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{T}_{z6}(\theta_z) = \begin{bmatrix} cos(\theta) & -sen(\theta) & 0 & 0 \\ sen(\theta) & cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2 Grados de libertad.

Los grados de libertad es el número de entradas independientes necesarias para definir la configuración de un mecanismo. Para un robot tridimensional el cálculo de los grados de libertad puede ser realizado por medio de la fórmula de Chebyshev-Grübler-Kutzbach de manera siguiente:

$$L = 6(b - g - 1) + \sum_k f_k$$

Donde b es el número de cuerpos, g es el número de juntas y f_k es el número de grados de libertad de la junta k. Para este caso particular los valores son los siguientes:

$$b = 8$$
$$g = 9$$
$$\sum_k f_k = 18$$

Al sustituir los valores anteriores se tiene:

$$L = 6(8 - 9 - 1) + 18$$
$$L = 6$$

Por lo tanto el robot tiene 6 grados de libertad

2.3 Solución algebraica.

Para la solución algebraica se usó el enfoque de cinemática inversa, es decir que se calcularon la posición, velocidad y aceleración de los ángulos de cada una de las juntas del robot en función de una trayectoria dada. De esta manera las variables de entrada son la posición y orientación del efector definidas con la notación $x, y, z, \psi, \theta, \phi$.

2.4 Cálculo de posición para la cadena 0.

Para el cálculo de la posición de la cadena 0 se tiene como variables a calcular los ángulos $\theta_{21}, \theta_{43}, \theta_{54}, \theta_{1817}, \theta_{1716}, \theta_{1615}$. Para comenzar con el análisis se estableció un punto de origen sobre a base del robot a partir del cual se comenzaran a escribir las matrices de transformación. Además se definieron tres cadenas cinemáticas que se muestran en la figura siguiente.

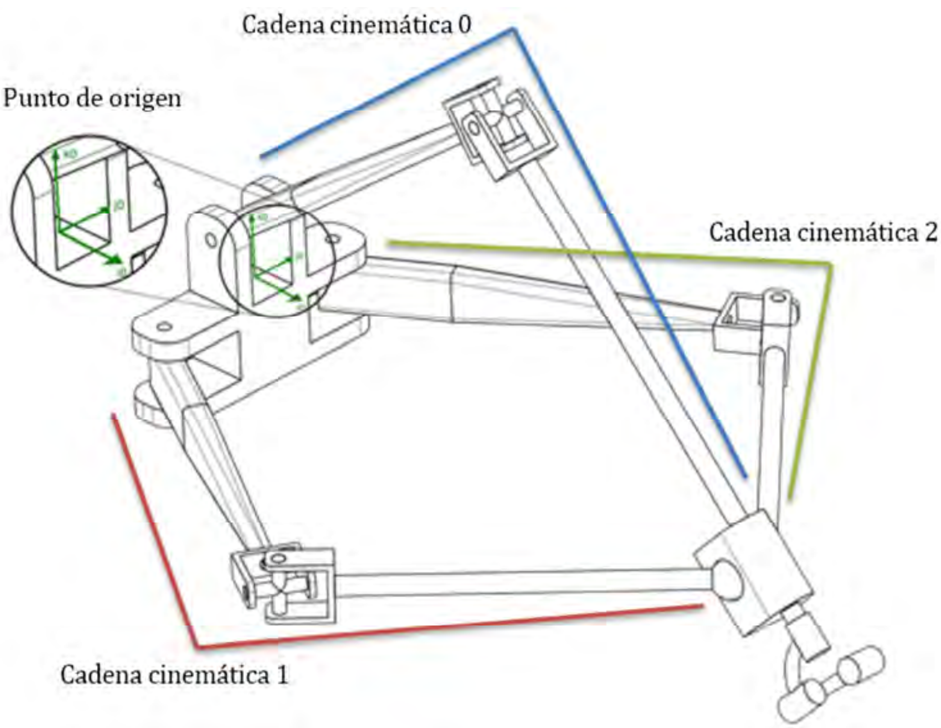


Figura 2-1 Cadenas cinemáticas.

2.4.1 Solución del ángulo θ_{21}.

Para la solución del ángulo se busca que éste quede expresado en términos de longitudes constantes y ángulos constantes o conocidos. De ésta manera se forman lazos vectoriales a lo largo de la cadena cinemática 0.

Se generan las bases locales a partir de la base inercial por medio de transformaciones homogéneas. En las figuras 2-2 a 2-5 se muestra la secuencia en la cual se fueron generando las bases locales.

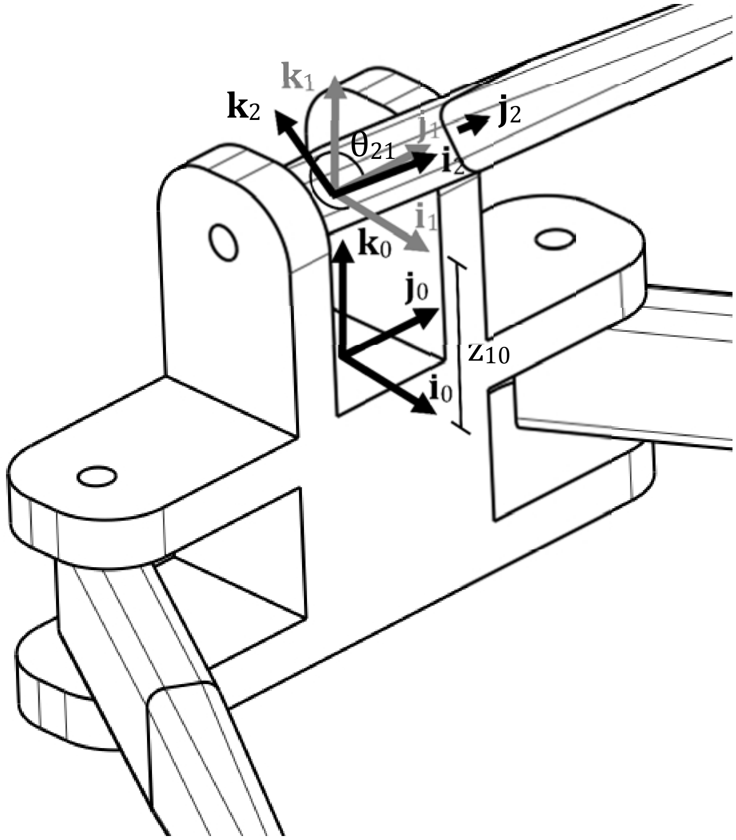


Figura 2-2 Bases de la 0 a la 2.

Para llegar al sistema de referencia 2 se emplea una traslación sobre el eje Z y una rotación sobre el eje Y, debido a que la primera junta es de tipo rotacional.

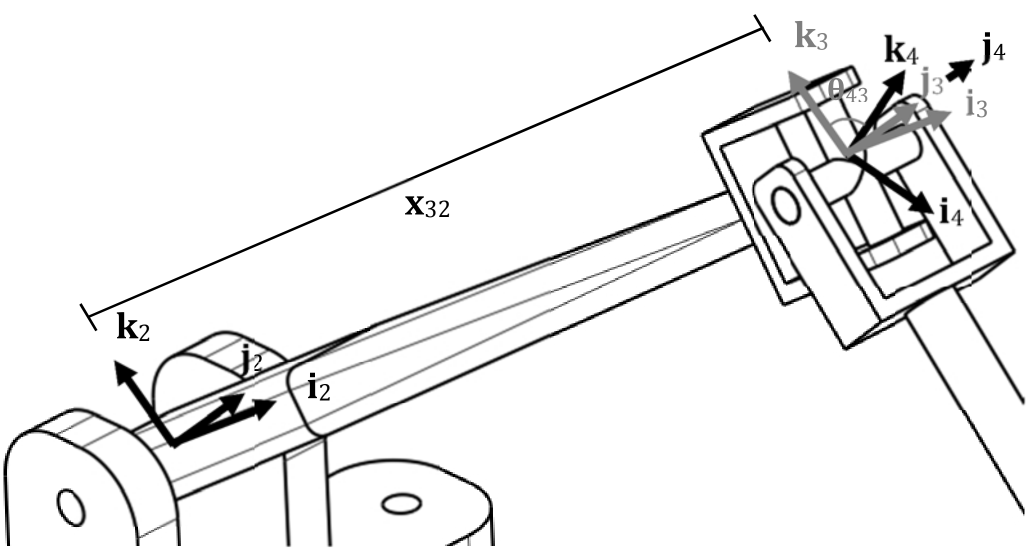


Figura 2-3 Bases de la 2 a la 4.

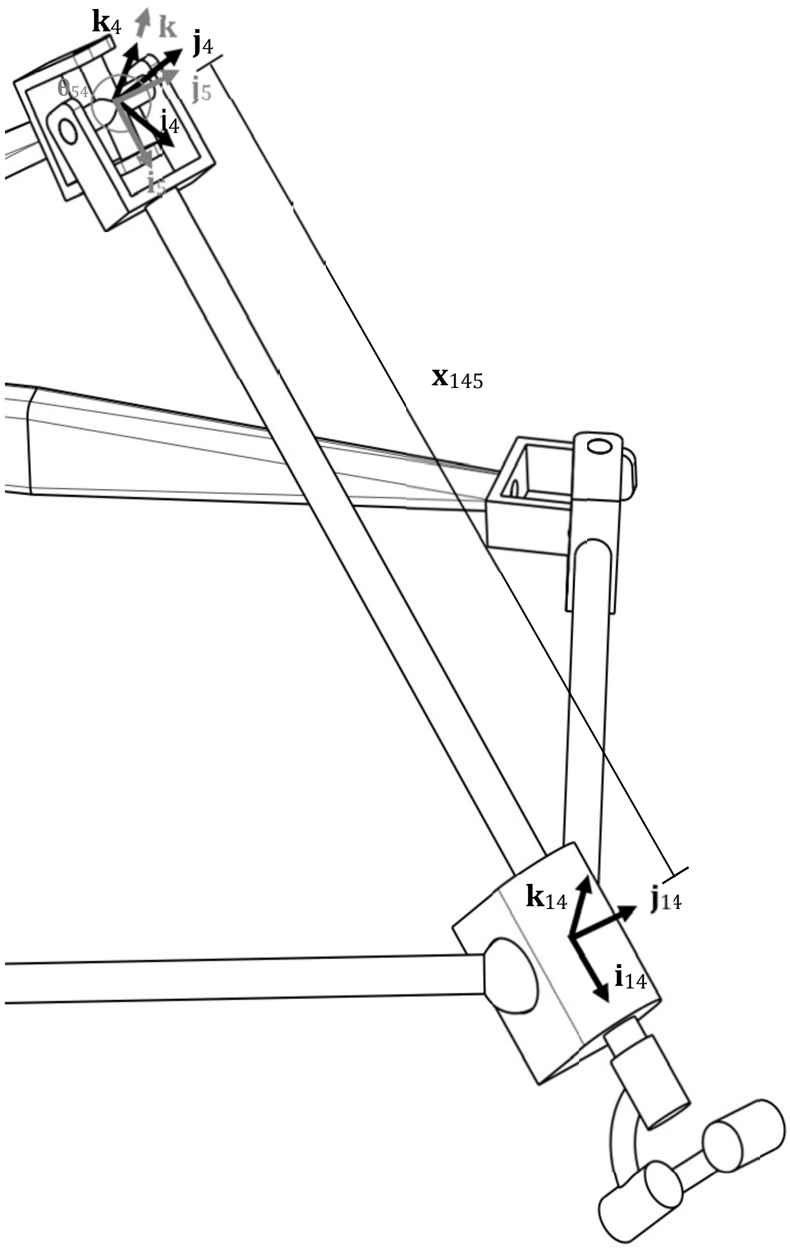


Figura 2-4 Bases de la 4 a la 14

Después se emplea una traslación sobre el eje X y dos rotaciones, por tratarse de una junta universal, sobre el eje Y y sobre el eje Z. De esta manera se llega a la base 5.
Para llegar a la base 14 se emplea una traslación sobre el eje X.

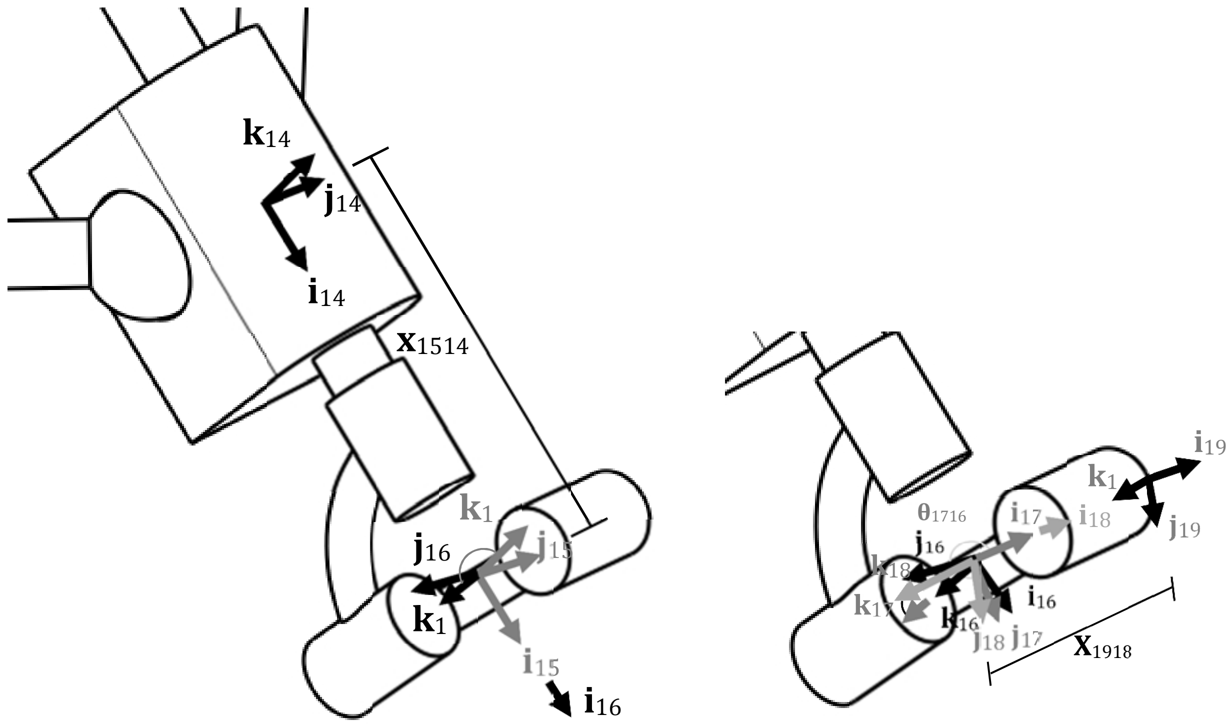


Figura 2-5 Bases de la14 a la 19.

Una traslación sobre el eje X se usa para llegar a la base 15. Una junta esférica es empleada para dar dirección al rociador por lo que es necesario definir tres traslaciones, que en este caso se eligió que fueran, sobre X, Z y X consecutivamente. Por último se usa una traslación sobre X para llegar a la base 19 donde se encuentra la punta del rociador.

De esta manera se forma la ecuación de lazo:

$$\boldsymbol{r_{10}} + \boldsymbol{r_{32}} + \boldsymbol{r_{14,5}} + \boldsymbol{r_{15,14}} = \boldsymbol{r_{19,18}} + \boldsymbol{r_5} \tag{2.1}$$

Cuyos vectores se ilustran en la figura 2-6.

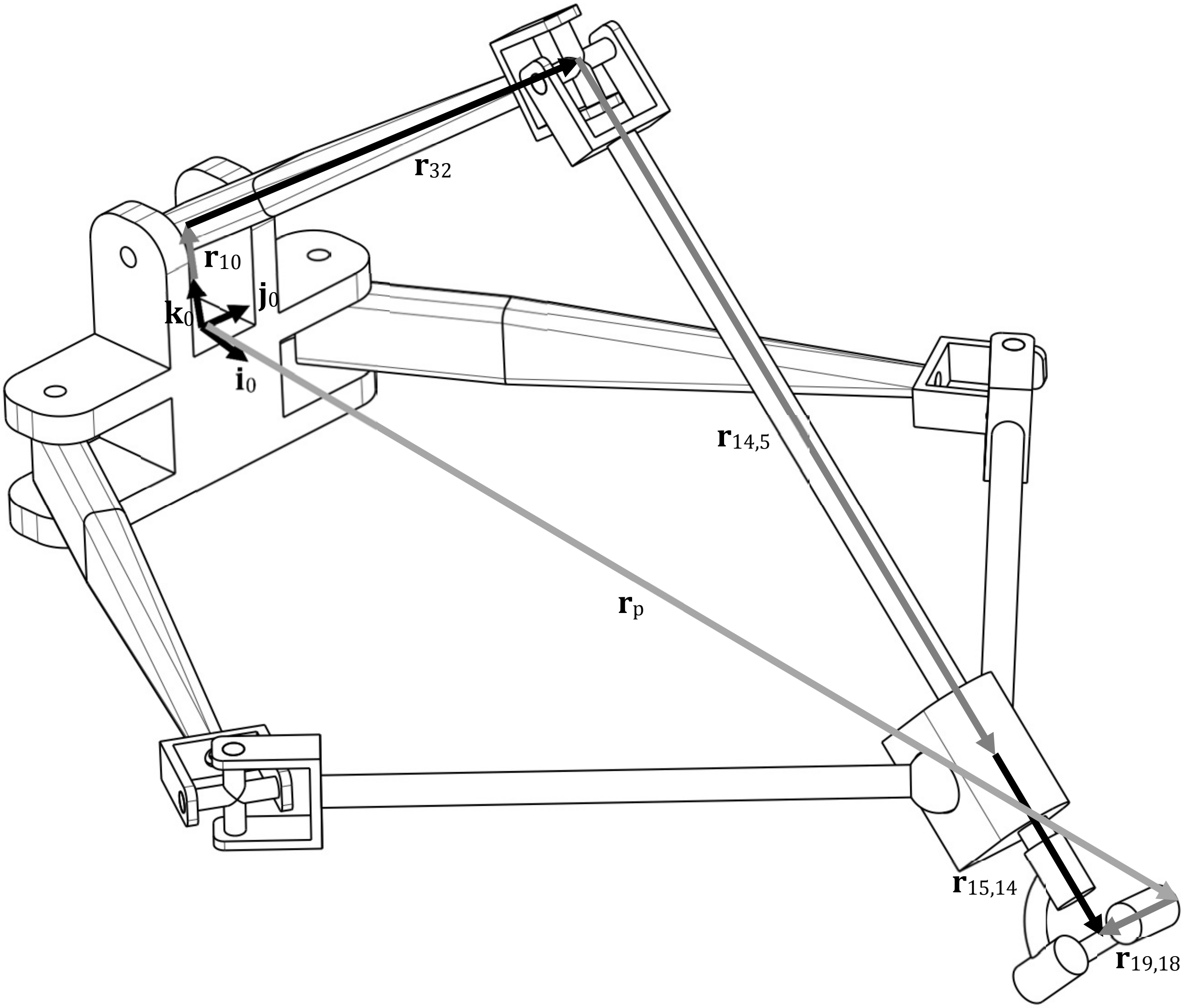


Figura 2-6 Lazo vectorial para la cadena 0.

Se renombran ambos lados de la ecuación (2.1) de la manera siguiente:

$$\boldsymbol{T}_{03}\boldsymbol{T}_{315} = \boldsymbol{T}_{LD0} \quad (2.2)$$

Dónde:

$$\boldsymbol{T}_{03} = \boldsymbol{T}_{z3}(z_{10})\boldsymbol{T}_{z5}(\theta_{21})\boldsymbol{T}_{z1}(x_{32})$$
$$\boldsymbol{T}_{315} = \boldsymbol{T}_{z5}(\theta_{43})\boldsymbol{T}_{z6}(\theta_{54})\boldsymbol{T}_{z1}(x_{65})\boldsymbol{T}_{z1}(x_{1514})$$
$$\boldsymbol{T}_{LD0} = \boldsymbol{T}_{z1}(x)\boldsymbol{T}_{z2}(y)\boldsymbol{T}_{z3}(z)\boldsymbol{T}_{z4}(\theta)\boldsymbol{T}_{z5}(\psi)\boldsymbol{T}_{z6}(\phi)\boldsymbol{T}_{z1}(-x_{1918})$$

Ahora pasamos el término $\boldsymbol{T}_{03}$, el cual contiene a θ_{21}, del lado derecho de la ecuación, además se posmultiplica ambos lados por un vector $\boldsymbol{n}$ con el fin de obtener la última columna de la matriz de transformación, con lo cual tenemos:

$$\boldsymbol{T}_{315}\boldsymbol{n} = \boldsymbol{T}_{03}^{-1}\boldsymbol{T}_{LD0}\boldsymbol{n} \tag{2.3}$$

Dónde:

$$n = [0\ ,\quad 0\ ,\quad 0\ ,\quad 1]^T$$

Si renombramos el lado izquierdo de la ecuación (2.3) como $\boldsymbol{Ec}_1$ y el lado derecho como $\boldsymbol{Ec}_2$ se obtiene:

$$\boldsymbol{Ec}_1 = \boldsymbol{Ec}_2 \tag{2.4}$$

En esta ecuación se tienen como incógnitas a θ_{21}, θ_{43} y θ_{54}. Como nos interesa despejar a θ_{21} hay que eliminar las otras dos, lo cual se logra al obtener la magnitud de ambos lados de la ecuación:

$$\|\boldsymbol{Ec}_1\| = \|\boldsymbol{Ec}_2\| \tag{2.5}$$

$$\sqrt{\boldsymbol{Ec}_1{}^{\boldsymbol{T}}\boldsymbol{Ec}_1} = \sqrt{\boldsymbol{Ec}_2{}^{\boldsymbol{T}}\boldsymbol{Ec}_2} \tag{2.6}$$

Simplificado:

$$\boldsymbol{Ec}_1{}^{\boldsymbol{T}}\boldsymbol{Ec}_1 - \boldsymbol{Ec}_2{}^{\boldsymbol{T}}\boldsymbol{Ec}_2 = \boldsymbol{0} \tag{2.7}$$

Al factorizar esta última ecuación en términos de $s\theta_{21}$ y $c\theta_{21}$ se obtiene una expresión de la forma:

$$C_{10} + A_{10}c\theta_{21} + B_{10}s\theta_{21} = 0 \tag{2.8}$$

Dónde:

$$\begin{aligned} A_{10} &= 2\,x_{32}\,(x - x_{19181}\ c\phi\ c\psi) \\ B_{10} &= -2x_{32}(z - z_{10} - x_{1918}s\theta\ s\phi + x_{1918}c\theta\ c\phi\ s\psi) \\ C_{10} &= -x_2 + x_{1514}^2 - x_{1918}^2 - x_{32}^2 + 2x_{1514}x_{65} + x_{65}^2 - y^2 - z^2 + 2zz_{10} - z_{10}^2 \\ &\quad + 2x_{1918}\ y\ c\theta\ s\phi + 2x_{1918}\ z\ s\theta\ s\phi - 2x_{1918}\ z_{10}\ s\theta\ s\phi \\ &\quad + 2x_{1918}\ c\phi(x\ c\psi + \big((-z + z_{10}) * c\theta + y\ s\theta\big) * s\psi) \end{aligned} \tag{2.9}$$

De esta manera en la forma en la que se encuentra la ecuación (2.8) se puede despejar a θ_{21} como:

$$\theta_{21} = 2\ arcTan\left(\frac{B_{10} \pm \sqrt{{A_{10}}^2 + {B_{10}}^2 - {C_{10}}^2}}{A_{10} - C_{10}}\right) \tag{2.10}$$

2.4.2 Solución del ángulo θ43.

Del desarrollo anterior se puede obtener la forma matricial de ambos lados de la ecuación (2.4) como sigue:

$$\boldsymbol{Ec_1} = \begin{bmatrix} (x_{1514} + x_{65})\ c\theta_{43}\ c\theta_{54} \\ (x_{1514} + x_{65})\ s\theta_{54} \\ -(x_{1514} + x_{65})\ c\theta_{54}\ c\theta_{43} \\ 1 \end{bmatrix} \tag{2.11}$$

$$\boldsymbol{Ec_2} = \begin{bmatrix} x - x_{32}c\theta_{21} - x_{1918}c\phi c\psi \\ y - x_{1918}\ (c\theta s\theta +\ c\phi s\phi\ s\psi]) \\ z - z10 + x_{32}s\theta_{21} - x_{1918}s\theta s\phi + x_{1918}c\theta c\phi s\psi \\ 1 \end{bmatrix} \tag{2.12}$$

Se observa que $\boldsymbol{Ec_2}$ está formada por elementos constantes y por θ_{21} que es el ángulo que se ya se despejó, por lo que para un manejo más cómodo se renombraran sus elementos como X, Y, Z es decir:

$$\boldsymbol{Ec}_2 = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{2.13}$$

De esta manera se manipulará más fácilmente el lado izquierdo de la ecuación, es decir $\boldsymbol{Ec}_1$, que contiene el ángulo que nos interesa despejar.
Entonces se tiene el sistema de ecuaciones siguiente:

$$(x_{1514} + x_{65})\ c\theta_{43}\ c\theta_{54} = X \tag{2.14}$$
$$(x_{1514} + x_{65}) s\theta_{54} = Y \tag{2.15}$$
$$-(x_{1514} + x_{65})\ c\theta_{54}\ s\theta_{43} = Z \tag{2.16}$$

Ahora de (2.14) se despeja $c\theta_{54}$ y se sustituye en (2.16) con lo que se obtiene:

$$-\frac{X\ x_{1514}\ tan\theta_{43}}{x_{1514} + x_{65}} - \frac{X\ x_{65}\ tan\theta_{43}}{x_{1514} + x_{65}} = Z \tag{2.17}$$

De (2.17) se despeja $tan(\theta_{43})$:

$$tan\theta_{43} = \frac{-Z}{X} \tag{2.18}$$

De esta manera:

$$\theta_{43} = arctan\left(\frac{-Z}{X}\right) \tag{2.19}$$

Dónde:

$$\begin{aligned} X &= x - x_{32}c\theta_{21} - x_{1918}c\phi\ c\psi \\ Y &= y - x_{1918}(c\theta\ s\phi +\ c\phi\ s\theta\ s\psi) \\ Z &= z - z_{10} + x_{32}s\theta_{21} - x_{1918}s\theta\ s\phi + x_{1918}c\theta\ c\phi\ s\psi \end{aligned} \tag{2.20}$$

2.4.3 Solución del ángulo θ_{54}.

Para el despeje de este ángulo se utilizan las ecuaciones del desarrollo anterior, entonces de la ecuación (2.14) se despeja $c\theta_{54}$ y de la ecuación (2.15) se despeja $s\theta_{54}$ con lo cual se obtiene que:

$$\begin{aligned} c\theta_{54} &= \frac{X\ sec\theta_{43}}{x_{1514}\ +\ x_{65}} \\ s\theta_{54} &= \frac{Y}{x_{1514}\ +\ x_{65}} \end{aligned} \tag{2.21}$$

De las ecuaciones (2.21) se llega a que:

$$tan\ \theta_{54} = \frac{Yc\theta_{43}}{X} \tag{2.22}$$

Y por lo tanto:

$$\theta_{54} = arctan\left(\frac{Yc\theta_{43}}{X}\right) \tag{2.23}$$

2.4.4 Solución del ángulo θ_{1817}.

Para la solución de los ángulos anteriores se formaron las ecuaciones de lazo sin considerar los giros de la junta esférica, lo cual se compensó al extraer la última columna de las transformaciones homogéneas. Es decir solo se expresó a los ángulos en términos de distancias.

Para despejar los ángulos de la junta esférica del órgano terminal se formarán las ecuaciones de lazo pero esta vez considerando tanto distancias como ángulos.

Al expresarse en transformaciones homogéneas:

$$\boldsymbol{T}_{03}\boldsymbol{T}_{315}\boldsymbol{T}_{1519} = \boldsymbol{T}_{z1}(x)\boldsymbol{T}_{z2}(y)\boldsymbol{T}_{z3}(z)\boldsymbol{T}_{z4}(\theta)\boldsymbol{T}_{z5}(\psi)\boldsymbol{T}_{z6}(\phi) \tag{2.24}$$

Dónde:

$$\begin{aligned} \boldsymbol{T}_{03} &= \boldsymbol{T}_{z3}(z_{10})\boldsymbol{T}_{z5}(\theta_{21})\boldsymbol{T}_{z1}(x_{32}) \\ \boldsymbol{T}_{315} &= \boldsymbol{T}_{z5}(\theta_{43})\boldsymbol{T}_{z6}(\theta_{54})\boldsymbol{T}_{z1}(x_{65})\boldsymbol{T}_{z1}(x_{1514}) \\ \boldsymbol{T}_{1519} &= \boldsymbol{T}_{z4}(\theta_{1615})\boldsymbol{T}_{z6}(\theta_{1716})\boldsymbol{T}_{z4}(\theta_{1817})\boldsymbol{T}_{z1}(x_{1918}) \end{aligned}$$

Renombrando el lado derecho de la ecuación (2.24):

$$\boldsymbol{T}_{03}\boldsymbol{T}_{315}\boldsymbol{T}_{1519} = \boldsymbol{T}_{LDn} \tag{2.25}$$

Dónde:

$$\boldsymbol{T}_{LDn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{2.26}$$

Se observa que los ángulos de interés se encuentran contenidos en $\boldsymbol{T}_{1519}$, es decir están en el lado izquierdo de la ecuación (2.25). Se observa que además de los tres ángulos de la junta esférica (θ_{1615}, θ_{1716} y θ_{1817}) lo demás es conocido debido a que es dato o ya se calculó, por lo que se procederá a dejar las transformaciones asociadas a los ángulos desconocidos del lado izquierdo de la ecuación y todo lo demás del lado derecho.

$$\boldsymbol{T}_{z4}(\theta_{1615})\boldsymbol{T}_{z6}(\theta_{1716})\boldsymbol{T}_{z4}(\theta_{1817}) = \boldsymbol{T}_{315}^{-1}\boldsymbol{T}_{03}^{-1}\boldsymbol{T}_{LDn}\boldsymbol{T}_{z1}(-x_{1918}) \tag{2.27}$$

Renombrando ambos lados de la ecuación (2.27):

$$\boldsymbol{T}_{LI} = \boldsymbol{T}_{LDn} \tag{2.28}$$

Dónde:

$$\boldsymbol{T}_{LI} = \begin{bmatrix} c\theta_{1716} & -c\theta_{1817}s\theta_{1716} & s\theta_{1716}s\theta_{1817} & 0 \\ c\theta_{1615}s\theta_{1716} & c\theta_{1615}c\theta_{17161}c\theta_{1817} - s\theta_{1615}s\theta_{1817} & -c\theta_{1817}s\theta_{1615} - c\theta_{1615}c\theta_{1716}s\theta_{1817} & 0 \\ s\theta_{1615}s\theta_{1716} & c\theta_{1716}c\theta_{1817}s\theta_{1615} + c\theta_{1615}s\theta_{1817} & c\theta_{1615}c\theta_{1817} - c\theta_{1716}s\theta_{1615}s\theta_{1817} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.29}$$

$$\boldsymbol{T}_{LDn2} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \tag{2.30}$$

Igualando los elementos (1,2) y (1,3) de las ecuaciones (2.29) y (2.30):

$$\begin{aligned} -c\theta_{18171}s\theta_{17161} &= b_{12} \\ s\theta_{17161}s\theta_{18171} &= b_{13} \end{aligned} \tag{2.31}$$

Despejando $c\theta_{1817}$ y $s\theta_{1817}$:

$$\begin{aligned} c\theta_{1817} &= -b_{12}\ csc\theta_{1716} \\ s\theta_{1817} &= b_{13}\ csc\theta_{1716} \end{aligned} \tag{2.32}$$

Formando la función tangente:

$$tan\theta_{1817} = \frac{b_{13}}{-b_{12}} \tag{2.33}$$

Por lo tanto:

$$\theta_{1817} = arctan\left(\frac{b_{13}}{-b_{12}}\right) \tag{2.34}$$

Dónde:

$$\begin{aligned} b_{12} &= s\theta_{54}(c\theta c\phi - s\theta s\phi s\psi) - c\theta_{54}(c\phi s\theta s(\theta_{21} + \theta_{43}) + s\phi(c(\theta_{21} + \theta_{43})c\psi \\ &\quad + c\theta s(\theta_{21} + \theta_{43})s\psi)) \\ b_{13} &= -c\theta c\theta_{54}c\psi s(\theta_{21} + \theta_{43}) - c\psi s\theta s\theta_{54} + c(\theta_{21} + \theta_{43})c\theta_{54}s\psi \end{aligned} \tag{2.35}$$

2.4.5 Solución del ángulo θ_{1716}.

De las ecuaciones (2.29) y (2.30) se igualan los elementos (1,1) y (1,3):

$$\begin{aligned} c\theta_{1716} &= b_{11} \\ s\theta_{1716}s\theta_{1817} &= b_{13} \end{aligned} \tag{2.36}$$

Despejando $c\theta_{1716}$ y $s\theta_{17161}$:

$$\begin{aligned} c\theta_{1716} &= b_{11} \\ s\theta_{1716} &= b_{13}\ csc\theta_{1817} \end{aligned} \tag{2.37}$$

Formando la función tangente:

$$tan\theta_{1716} = \frac{b_{13}csc\theta_{18171}}{-b_{11}} \tag{2.38}$$

Por lo tanto:

$$\theta_{1716} = arctan\left(\frac{b_{13}csc\theta_{18171}}{-b_{11}}\right) \tag{2.39}$$

Dónde:

$$\begin{aligned} b_{11} = {} & c(\theta_{21} + \theta_{43})c\theta_{54}c\phi c\psi + c\theta_{54}s(\theta_{21} + \theta_{43})(-s\theta s\phi + c\theta c\phi s\psi) \\ & + s\theta_{54}(c\theta s\phi + c\phi s\theta s\psi) \end{aligned} \tag{2.40}$$

2.4.6 Solución del ángulo θ_{1615}.

De la ecuación (2.27) se despeja $\boldsymbol{T}_{z4}(\theta_{1615})$:

$$\begin{aligned} \boldsymbol{T}_{z4}(\theta_{1615}) &= \boldsymbol{T}_{315}^{-1}\boldsymbol{T}_{03}^{-1}\boldsymbol{T}_{LDn}\boldsymbol{T}_{z1}(-x_{1918})\boldsymbol{T}_{z4}(-\theta_{1817})\boldsymbol{T}_{z6}(-\theta_{1716}) \\ \boldsymbol{T}_{z4}(\theta_{1615}) &= \boldsymbol{T}_{LDn2}\boldsymbol{T}_{z4}(-\theta_{1817})\boldsymbol{T}_{z6}(-\theta_{1716}) \end{aligned} \tag{2.41}$$

Renombrando el lado derecho de la ecuación (2.41):

$$\boldsymbol{T}_{z4}(\theta_{1615}) = \boldsymbol{T}_{LDn3} \tag{2.42}$$

Dónde*:*

$$\boldsymbol{T}_{z4}(\theta_{1615}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta_{1615} & -s\theta_{16151} & 0 \\ 0 & s\theta_{16151} & c\theta_{16151} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.43}$$

$$\boldsymbol{T}_{LDn3} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \tag{2.44}$$

De las ecuaciones (2.43) y (2.44) se igualan los elementos (2,2) y (3,2)

$$\begin{aligned} c\theta_{1615} &= c_{22} \\ s\theta_{1615} &= c_{32} \end{aligned} \tag{2.45}$$

Formando la función tangente:

$$tan\theta_{1615} = \frac{c_{32}}{c_{22}} \tag{2.46}$$

Por lo tanto:

$$\theta_{1615} = arctan\left(\frac{c_{32}}{c_{22}}\right) \tag{2.47}$$

Dónde:

$$\begin{aligned} c_{22} = s\theta_{1716}(&-c(\theta_{21}+\theta_{43})c\phi c\psi s\theta_{54} + s(\theta_{21}+\theta_{43})s\theta_{54}(s\theta s\phi - c\theta c\phi s\psi) \\ &+ c\theta_{54}(c\theta s\phi + c\phi s\theta s\psi)) + c\theta_{1716}(s\theta_{1817}(c\theta_{54}c\psi s\theta \\ &+ s\theta_{54}(-c\theta c\psi s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})s\psi)) \\ &+ c\theta_{1817}(c(\theta_{21}+\theta_{43})c\psi s\theta_{54}s\phi + s(\theta_{21}+\theta_{43})s\theta_{54}(c\phi s\theta \\ &+ c\theta s\phi s\psi) + c\theta_{54}(c\theta c\phi - s\theta s\phi s\psi))) \\ c_{32} = s\theta_{1716}(&c\phi c\psi s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})(s\theta s\phi - c\theta c\phi s\psi)) \\ &+ c\theta_{1716}(-s\theta_{1817}(c\theta c(\theta_{21}+\theta_{43})c\psi + s(\theta_{21}+\theta_{43})s\psi) \\ &+ c\theta_{1817}(-c\psi s(\theta_{21}+\theta_{43})s\phi + c(\theta_{21}+\theta_{43})(c\phi s\theta \\ &+ c\theta s\phi s\psi))) \end{aligned} \tag{2.48}$$

2.5 Cálculo de velocidad para la cadena 0.

Para el cálculo de la velocidad se tiene como variables de entrada la velocidad del efector $(\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi})$ y las variables a calcular son las velocidades de los ángulos de la cadena 0 es decir $\dot{\theta}_{21}, \dot{\theta}_{43}, \dot{\theta}_{54}, \dot{\theta}_{1817}, \dot{\theta}_{1716}, \dot{\theta}_{1615}$. Para esta sección se asume que la posición de los ángulos es conocida debido a que se calculó en la sección anterior.

2.5.1 Solución de la velocidad para ángulo θ_{21}.

Derivando la ecuación (2.8) respecto al tiempo:

$$\dot{\theta}_{21} = \frac{-\dot{C}_{10} - \dot{A}_{10}c\theta_{21} - \dot{B}_{10}s\theta_{21}}{B_{10}c\theta_{21} - A_{10}s\theta_{21}} \tag{2.49}$$

Donde $A_{10}, B_{10}\ y\ C_{10}$ se encuentran en las ecuaciones (2.9), además:

$$\begin{aligned} \dot{A}_{10} &= 2x_{32}(\dot{x} + \dot{\phi}x_{1918}c\psi s\phi + \dot{\psi}x_{1918}c\phi s\psi) \\ \dot{B}_{10} &= -2x_{32}(\dot{z} - x_{1918}c\phi s\theta(\dot{\phi} + \dot{\theta}s\psi) + x_{1918}c\theta(\dot{\psi}c\phi c\psi - s\phi(\dot{\theta} + \dot{\phi}s\psi))) \end{aligned} \tag{2.50}$$

$$\dot{C}_{10} = 2(-\dot{x}x - \dot{y}y - \dot{z}z + \dot{z}z_{10} - \dot{\phi}xx_{1918}c\psi s\phi + \dot{z}x_{1918}s\theta s\phi - \dot{\theta}x_{1918}ys\theta s\phi - \dot{\phi}x_{1918}ys\theta s\phi s\psi + x_{1918}c\theta(c\phi(\dot{\phi}y - \dot{\psi}(z - z_{10})c\psi - \dot{z}s\psi + \dot{\theta}ys\psi) + s\phi(\dot{y} + \dot{\theta}z - \dot{\theta}z_{10} + \dot{\phi}(z - z_{10})s\psi)) + x_{1918}c\phi(c\psi(\dot{x} + \dot{\psi}ys\theta) - \dot{\psi}xs\psi + s\theta(\dot{\phi}(z - z_{10}) + (\dot{y} + \dot{\theta}z - \dot{\theta}z_{10})s\psi)))$$

Sustituyendo las ecuaciones $A_{10}, B_{10}, C_{10}, \dot{A}_{10}, \dot{B}_{10}\ y\ \dot{C}_{10}$ en (2.49) y al agrupar en $\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}$ se tiene:

$$\dot{\theta}_{21} = \frac{1}{V_1}\left(\dot{x}V_2 + \dot{y}V_3 + \dot{z}V_4 + \dot{\psi}V_5 + \dot{\theta}V_6 + \dot{\phi}V_7\right) \tag{2.51}$$

Donde los coeficientes se encuentran en el apéndice D.

2.5.2 Solución de la velocidad para ángulo θ_{43}.

Derivando respecto del tiempo la ecuación (2.19):

$$\dot{\theta}_{43} = \frac{-\dot{Z}c\theta_{43} - \dot{X}s\theta_{43}}{Xc\theta_{43} - Zs\theta_{43}} \tag{2.52}$$

Dónde:

$$\begin{aligned} \dot{X} &= \dot{x} + \dot{\theta}_{21}x_{32}s\theta_{21} + \dot{\phi}x_{1918}c\psi s\phi + \dot{\psi}x_{1918}c\phi s\psi \\ \dot{Y} &= \dot{y} - x_{1918}(c\theta c\phi(\dot{\phi} + \dot{\theta}s\psi) + s\theta(\dot{\psi}c\phi c\psi - s\phi(\dot{\theta} + \dot{\phi}s\psi))) \\ \dot{Z} &= \dot{z} + \dot{\theta}_{21}x_{32}c\theta_{21} - \dot{\phi}x_{1918}c\phi s\theta - \dot{\theta}x_{1918}c\phi s\theta s\psi + x_{1918}c\theta(\dot{\psi}c\phi c\psi - s\phi(\dot{\theta} + \dot{\phi}s\psi)) \end{aligned} \tag{2.53}$$

Sustituyendo X, Z que se encuentran en (2.20) así como $\dot{X}, \dot{Z}$ en (2.52), simplificando y agrupando:

$$\dot{\theta}_{43} = \frac{1}{V_8}\left(\dot{x}V_9 + \dot{z}V_{10} + \dot{\psi}V_{11} + \dot{\theta}V_{12} + \dot{\phi}V_{13} + \dot{\theta}_{21}V_{14}\right) \tag{2.54}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo la ecuación (2.51) en (2.54) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{43} = \frac{1}{V_8}\left(\dot{x}E_1 + \dot{y}E_2 + \dot{z}E_3 + \dot{\psi}E_4 + \dot{\theta}E_5 + \dot{\phi}E_6\right) \tag{2.55}$$

Dónde:

$$
\begin{aligned}
E_1 &= \frac{V_{14}V_2}{V_2} + V_9 & E_4 &= V_{11} + \frac{V_{14}V_5}{V_1} \\
E_2 &= \frac{V_{14}V_3}{V_1} & E_5 &= V_{12} + \frac{V_{14}V_6}{V_1} \\
E_3 &= V_{10} + \frac{V_{14}V_4}{V_1} & E_6 &= V_{13} + \frac{V_{14}V_7}{V_1}
\end{aligned} \tag{2.56}
$$

2.5.3 Solución de la velocidad, ángulo θ_{54}.

Derivando respecto del tiempo la ecuación (2.20):

$$
\dot{\theta}_{54} = \frac{c\theta_{54}{}^2((\dot{Y}X - \dot{X}Y)c\theta_{43} - \dot{\theta}_{43}XYs\theta_{43})}{X^2} \tag{2.57}
$$

Sustituyendo X, Y que se encuentran en (2.20) así como $\dot{X}, \dot{Y}$ en (2.57), simplificando y agrupando:

$$
\dot{\theta}_{54} = \frac{1}{V_{15}}\left(\dot{x}V_{16} + \dot{y}V_{17} + \dot{\psi}V_{18} + \dot{\theta}V_{19} + \dot{\phi}V_{20} + \dot{\theta}_{21}V_{21} + \dot{\theta}_{43}V_{22}\right) \tag{2.58}
$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55) en (2.58) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$
\dot{\theta}_{54} = \frac{1}{V_{15}}\left(\dot{x}E_7 + \dot{y}E_8 + \dot{z}E_9 + \dot{\psi}E_{10} + \dot{\theta}E_{11} + \dot{\phi}E_{12}\right) \tag{2.59}
$$

Dónde:

$$
\begin{aligned}
E_7 &= V_{15} - \frac{V_2V_{20}}{V_1} + \frac{E_1V_{21}}{V_8} & E_{10} &= V_{17} - \frac{V_{20}V_5}{V_1} + \frac{E_4V_{21}}{V_8} \\
E_8 &= V_{16} - \frac{V_{20}V_3}{V_1} + \frac{E_2V_{21}}{V_8} & E_{11} &= V_{18} - \frac{V_{20}V_6}{V_1} + \frac{E_5V_{21}}{V_8} \\
E_9 &= -\frac{V_{20}V_4}{V_1} + \frac{E_3V_{21}}{V_8} & E_{12} &= V_{19} - \frac{V_{20}V_7}{V_1} + \frac{E_6V_{21}}{V_8}
\end{aligned} \tag{2.60}
$$

2.5.4 Solución de la velocidad, ángulo θ_{1817}.

Derivando respecto del tiempo la ecuación (2.34):

$$
\dot{\theta}_{1817} = \frac{(b_{13}\dot{b}_{12} - b_{12}\dot{b}_{13})c\theta_{1817}{}^2}{b_{12}{}^2} \tag{2.61}
$$

Donde $b_{12}\ y\ b_{13}$ se encuentran en (2.35), además:

$$\begin{aligned}\dot{b}_{12} = {} & \dot{\theta}_{54} c\theta_{54}(c\theta c\phi - s\theta s\phi s\psi) + \dot{\theta}_{54} s\theta_{54}(c\phi s\theta s(\theta_{21} + \theta_{43}) + s\phi(c(\theta_{21} \\ & + \theta_{43})c\psi + c\theta s(\theta_{21} + \theta_{43})s\psi)) - s\theta_{54}(c\phi s\theta(\dot{\theta} + \dot{\phi} s\psi) \\ & + s\phi(\dot{\psi} c\psi s\theta + c\theta(\dot{\phi} + \dot{\theta} s\psi))) - c\theta_{54}(c(\theta_{21} + \theta_{43})(c\phi(\dot{\phi} c\psi \\ & + (\dot{\theta}_{21} + \dot{\theta}_{43})s\theta) + (-\dot{\psi} + (\dot{\theta}_{21} + \dot{\theta}_{43})c\theta)s\phi s\psi) + s(\theta_{21} \\ & + \theta_{43})(-s\phi((\dot{\theta}_{21} + \dot{\theta}_{43})c\psi + s\theta(\dot{\phi} + \dot{\theta} s\psi)) + c\theta(\dot{\psi} c\psi s\phi \\ & + c\phi(\dot{\theta} + \dot{\phi} s\psi)))) \\ \dot{b}_{13} = {} & s\theta_{54}(-c\theta c\psi(\dot{\theta} - \dot{\theta}_{54} s(\theta_{21} + \theta_{43})) + \dot{\psi} s\theta s\psi) - c\theta_{54}(c\psi s\theta(\dot{\theta}_{54} \\ & - \dot{\theta} s(\theta_{21} + \theta_{43})) + (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi} c\theta)s(\theta_{21} + \theta_{43})s\psi) \\ & + c(\theta_{21} + \theta_{43})((\dot{\psi} - (\dot{\theta}_{21} + \dot{\theta}_{43})c\theta)c\theta_{54} c\psi - \dot{\theta}_{54} s\theta_{54} s\psi)\end{aligned} \tag{2.62}$$

Sustituyendo las ecuaciones $b_{12}, b_{13}, \dot{b}_{12}, \dot{b}_{13}$ en (2.61), simplificando y agrupando:

$$\dot{\theta}_{1817} = \frac{1}{V_{23}}\left(\dot{\psi} V_{24} + \dot{\theta} V_{25} + \dot{\phi} V_{26} + \dot{\theta}_{21} V_{27} + \dot{\theta}_{43} V_{27} + \dot{\theta}_{54} V_{28}\right) \tag{2.63}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59) en (2.63) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{1817} = \frac{1}{V_{23}}\left(\dot{x} E_{13} + \dot{y} E_{14} + \dot{z} E_{15} + \dot{\psi} E_{16} + \dot{\theta} E_{17} + \dot{\phi} E_{18}\right) \tag{2.64}$$

Dónde:

$$\begin{aligned} E_{13} &= \frac{V_2 V_{27}}{V_1} + \frac{E_7 V_{28}}{V_{15}} + \frac{E_1 V_{27}}{V_8} & E_{16} &= V_{24} + \frac{E_{10} V_{28}}{V_{15}} + \frac{V_{27} V_5}{V_1} + \frac{E_4 V_{27}}{V_8} \\ E_{14} &= \frac{E_8 V_{28}}{V_{15}} + \frac{V_{27} V_3}{V_1} + \frac{E_2 V_{27}}{V_8} & E_{17} &= V_{25} + \frac{E_{11} V_{28}}{V_{15}} + \frac{V_{27} V_6}{V_1} + \frac{E_5 V_{27}}{V_8} \\ E_{15} &= \frac{E_9 V_{28}}{V_{15}} + \frac{V_{27} V_4}{V_1} + \frac{E_3 V_{27}}{V_8} & E_{18} &= V_{26} + \frac{E_{12} V_{28}}{V_{15}} + \frac{V_{27} V_7}{V_1} + \frac{E_6 V_{27}}{V_8} \end{aligned} \tag{2.65}$$

2.5.5 Solución de la velocidad, ángulo θ_{1716}.

Derivando respecto del tiempo la ecuación (2.39):

$$\dot{\theta}_{1716} = -\frac{c\theta_{1716}{}^2(b_{13}\dot{b}_{11}\csc\theta_{1817} - b_{11}\dot{b}_{13}\csc\theta_{1817} + b_{11}b_{13}\dot{\theta}_{1817}\cot\theta_{1817}\csc\theta_{1817})}{b_{11}{}^2} \tag{2.66}$$

Dónde:

$$\begin{aligned}\dot{b}_{11} = {} & s\theta_{54}(-\dot{\theta}_{54}c(\theta_{21}+\theta_{43})c\phi c\psi + c\theta c\phi(\dot{\phi} + (\dot{\theta} - \dot{\theta}_{54}s(\theta_{21}+\theta_{43}))s\psi) \\ & + s\theta(\dot{\psi}c\phi c\psi - s\phi(\dot{\theta} - \dot{\theta}_{54}s(\theta_{21}+\theta_{43}) + \dot{\phi}s\psi))) \\ & - c\theta_{54}(c\phi((\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}c\theta)c\psi s(\theta_{21}+\theta_{43}) + (\dot{\psi} - (\dot{\theta}_{21} \\ & + \dot{\theta}_{43})c\theta)c(\theta_{21}+\theta_{43})s\psi + s\theta(-\dot{\theta}_{54}s\psi + s(\theta_{21}+\theta_{43})(\dot{\phi} \\ & + \dot{\theta}s\psi))) + s\phi(c(\theta_{21}+\theta_{43})(\dot{\phi}c\psi + (\dot{\theta}_{21}+\dot{\theta}_{43})s\theta) \\ & + c\theta(-\dot{\theta}_{54} + s(\theta_{21}+\theta_{43})(\dot{\theta} + \dot{\phi}s\psi))))\end{aligned} \tag{2.67}$$

Sustituyendo las ecuaciones $b_{11}, b_{13}, \dot{b}_{11}, \dot{b}_{13}$ en (2.66), simplificando y agrupando:

$$\dot{\theta}_{1716} = \frac{1}{V_{29}}\left(\dot{\psi}V_{30} + \dot{\theta}V_{31} + \dot{\phi}V_{32} + \dot{\theta}_{21}V_{33} + \dot{\theta}_{43}V_{33} + \dot{\theta}_{54}V_{34} + \dot{\theta}_{1817}V_{35}\right) \tag{2.68}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.64) en (2.68) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{1716} = \frac{1}{V_{29}}\left(\dot{x}E_{19} + \dot{y}E_{20} + \dot{z}E_{21} + \dot{\psi}E_{22} + \dot{\theta}E_{23} + \dot{\phi}E_{24}\right) \tag{2.69}$$

Dónde:

$$\begin{aligned} E_{19} &= \frac{V_2V_{33}}{V_1} + \frac{E_7V_{34}}{V_{15}} + \frac{E_{13}V_{35}}{V_{23}} + \frac{E_1V_{33}}{V_8} \\ E_{20} &= \frac{V_3V_{33}}{V_1} + \frac{E_8V_{34}}{V_{15}} + \frac{E_{14}V_{35}}{V_{23}} + \frac{E_2V_{33}}{V_8} \\ E_{21} &= \frac{E_9V_{34}}{V_{15}} + \frac{E_{15}V_{35}}{V_{23}} + \frac{V_{33}V_4}{V_1} + \frac{E_3V_{33}}{V_8} \\ E_{22} &= V_{30} + \frac{E_{10}V_{34}}{V_{15}} + \frac{E_{16}V_{35}}{V_{23}} + \frac{V_{33}V_5}{V_1} + \frac{E_4V_{33}}{V_8} \\ E_{23} &= V_{31} + \frac{E_{11}V_{34}}{V_{15}} + \frac{E_{17}V_{35}}{V_{23}} + \frac{V_{33}V_6}{V_1} + \frac{E_5V_{33}}{V_8} \\ E_{24} &= V_{32} + \frac{E_{12}V_{34}}{V_{15}} + \frac{E_{18}V_{35}}{V_{23}} + \frac{V_{33}V_7}{V_1} + \frac{E_6V_{33}}{V_8} \end{aligned} \tag{2.70}$$

2.5.6 Solución de la velocidad, ángulo θ_{1615}.

Derivando respecto del tiempo la ecuación (2.47):

$$\dot{\theta}_{1615} = -\frac{(c_{32}\dot{c}_{22} - c_{22}\dot{c}_{32})c\theta_{1615}{}^2}{c_{22}{}^2} \tag{2.71}$$

Al derivar la ecuación (2.48) se obtiene $\dot{c}_{22}\ y\ \dot{c}_{32}$, que se encuentran en el apéndice C. Sustituyendo las ecuaciones (2.48) así como $\dot{c}_{22}\ y\ \dot{c}_{32}$ en la ecuación (2.71), simplificando y agrupando:

$$\dot{\theta}_{1615} = \frac{1}{V_{36}}\left(\dot{\psi}V_{37} + \dot{\theta}V_{38} + \dot{\phi}V_{39} + \dot{\theta}_{21}V_{40} + \dot{\theta}_{43}V_{40} + \dot{\theta}_{54}V_{41} + \dot{\theta}_{1817}V_{42} + \dot{\theta}_{1716}V_{43}\right) \quad (2.72)$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.64), (2.69) en (2.72) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{1615} = \frac{1}{V_{36}}\left(\dot{x}E_{25} + \dot{y}E_{26} + \dot{z}E_{27} + \dot{\psi}E_{28} + \dot{\theta}E_{29} + \dot{\phi}E_{30}\right) \quad (2.73)$$

Dónde:

$$\begin{aligned}
E_{25} &= \frac{V_2V_{40}}{V_1} + \frac{E_7V_{41}}{V_{15}} + \frac{E_{13}V_{42}}{V_{23}} + \frac{E_{19}V_{43}}{V_{29}} + \frac{E_1V_{40}}{V_8}\\
E_{26} &= \frac{V_3V_{40}}{V_1} + \frac{E_8V_{41}}{V_{15}} + \frac{E_{14}V_{42}}{V_{23}} + \frac{E_{20}V_{43}}{V_{29}} + \frac{E_2V_{40}}{V_8}\\
E_{27} &= \frac{V_4V_{40}}{V_1} + \frac{E_9V_{41}}{V_{15}} + \frac{E_{15}V_{42}}{V_{23}} + \frac{E_{21}V_{43}}{V_{29}} + \frac{E_3V_{40}}{V_8}\\
E_{28} &= V_{37} + \frac{E_{10}V_{41}}{V_{15}} + \frac{E_{16}V_{42}}{V_{23}} + \frac{E_{22}V_{43}}{V_{29}} + \frac{V_{40}V_5}{V_1} + \frac{E_4V_{40}}{V_8}\\
E_{29} &= V_{38} + \frac{E_{11}V_{41}}{V_{15}} + \frac{E_{17}V_{42}}{V_{23}} + \frac{E_{23}V_{43}}{V_{29}} + \frac{V_{40}V_6}{V_1} + \frac{E_5V_{40}}{V_8}\\
E_{30} &= V_{39} + \frac{E_{12}V_{41}}{V_{15}} + \frac{E_{18}V_{42}}{V_{23}} + \frac{E_{24}V_{43}}{V_{29}} + \frac{V_{40}V_7}{V_1} + \frac{E_6V_{40}}{V_8}
\end{aligned} \quad (2.74)$$

2.6 Calculo de la aceleración para la cadena 0.

Para el cálculo de la aceleración se tiene como variables de entrada la aceleración del efector $\left(\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}\right)$ y las variables a calcular son las aceleraciones de los ángulos de la cadena 0 es decir $\ddot{\theta}_{21}, \ddot{\theta}_{43}, \ddot{\theta}_{54}, \ddot{\theta}_{1817}, \ddot{\theta}_{1716}, \ddot{\theta}_{1615}$. Para esta sección se asume que la posición y la velocidad de los ángulos son conocidas debido a que se calcularon en secciones anteriores.

2.6.1 Solución de la aceleración, ángulo θ_{21}.

Derivando la ecuación (2.49) respecto al tiempo:

$$\ddot{\theta}_{21} = \frac{-\ddot{C}_{10} - \ddot{A}_{10}c\theta_{21} - 2\dot{B}_{10}\dot{\theta}_{21}c\theta_{21} + A_{10}\dot{\theta}_{21}^{\,2}c\theta_{21} - \ddot{B}_{10}s\theta_{21} + 2\dot{A}_{10}\dot{\theta}_{21}s\theta_{21} + B_{10}\dot{\theta}_{21}^{\,2}s\theta_{21}}{B_{10}c\theta_{21} - A_{10}s\theta_{21}} \quad (2.75)$$

Donde A_{10}, B_{10}, C_{10} se encuentran en (2.9) y $\dot{A}_{10}, \dot{B}_{10}, \dot{C}_{10}$ en (2.50), además:

$$\begin{aligned}
\ddot{A}_{10} &= 2x_{32}(\ddot{x} + \ddot{\phi}x_{1918}c\psi s\phi - 2\dot{\phi}\dot{\psi}x_{1918}s\phi s\psi + x_{1918}c\phi((\dot{\phi}^2 + \dot{\psi}^2)c\psi \\
&\qquad + \ddot{\psi}s\psi)) \\
\ddot{B}_{10} &= -2x_{32}(\ddot{z} + \dot{\theta}^2 x_{1918}s\theta s\phi + \dot{\phi}^2 x_{1918}s\theta s\phi + 2\dot{\theta}\dot{\phi}x_{1918}s\theta s\phi s\psi \\
&\qquad - x_{1918}c\phi s\theta(\ddot{\phi} + 2\dot{\theta}\dot{\psi}c\psi + \ddot{\theta}s\psi) - x_{1918}c\theta(s\phi(\ddot{\theta} + 2\dot{\phi}\dot{\psi}c\psi \\
&\qquad + \ddot{\phi}s\psi) + c\phi(2\dot{\theta}\dot{\phi} - \ddot{\psi}c\psi + (\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2)s\psi))) \\
\ddot{C}_{10} &= 2(-\dot{x}^2 - \dot{y}^2 - \dot{z}^2 - \ddot{x}x - \ddot{y}y - \ddot{z}z + \ddot{z}z_{10} - x_{1918}y(\dot{\theta}^2 c\theta + \ddot{\theta}s\theta)s\phi \\
&\qquad + x_{1918}z(\ddot{\theta}c\theta - \dot{\theta}^2 s\theta)s\phi - 2\dot{\theta}x_{1918}s\theta(\dot{\phi}yc\phi + \dot{y}s\phi) \\
&\qquad + 2\dot{\theta}x_{1918}c\theta(\dot{\phi}zc\phi + \dot{z}s\phi) + x_{1918}c\theta((2\dot{y}\dot{\phi} + \ddot{\phi}y)c\phi + (\ddot{y} \\
&\qquad - \dot{\phi}^2 y)s\phi) + x_{1918}s\theta((2\dot{z}\dot{\phi} + \ddot{\phi}z)c\phi + (\ddot{z} - \dot{\phi}^2 z)s\phi) \\
&\qquad + x_{1918}z_{10}(-c\theta(2\dot{\theta}\dot{\phi}c\phi + \ddot{\theta}s\phi) + s\theta(-\ddot{\phi}c\phi + (\dot{\theta}^2 + \dot{\phi}^2)s\phi)) \\
&\qquad + x_{1918}((-\dot{\phi}^2 c\phi - \ddot{\phi}s\phi)(xc\psi + ((-z + z_{10})c\theta + ys\theta)s\psi) \\
&\qquad - 2\dot{\phi}s\phi(\dot{x}c\psi + \dot{\psi}c\psi((-z + z_{10})c\theta + ys\theta) - \dot{\psi}xs\psi + ((-\dot{z} \\
&\qquad + \dot{\theta}y)c\theta + (\dot{y} + \dot{\theta}(z - z_{10}))s\theta)s\psi) + c\phi(\ddot{x}c\psi + 2\dot{\psi}c\psi((-\dot{z} \\
&\qquad + \dot{\theta}y)c\theta + (\dot{y} + \dot{\theta}(z - z_{10}))s\theta) - 2\dot{x}\dot{\psi}s\psi + ((-\ddot{z} + 2\dot{y}\dot{\theta} + \ddot{\theta}y \\
&\qquad + \dot{\theta}^2 z - \dot{\theta}^2 z_{10})c\theta + (\ddot{y} + 2\dot{z}\dot{\theta} - \dot{\theta}^2 y + \ddot{\theta}z - \ddot{\theta}z_{10})s\theta)s\psi \\
&\qquad - x(\dot{\psi}^2 c\psi + \ddot{\psi}s\psi) + ((-z + z_{10})c\theta + ys\theta)(\ddot{\psi}c\psi - \dot{\psi}^2 s\psi))))
\end{aligned} \tag{2.76}$$

Sustituyendo $A_{10}, B_{10}, C_{10}, \dot{A}_{10}, \dot{B}_{10}, \dot{C}_{10}, \ddot{A}_{10}, \ddot{B}_{10}, \ddot{C}_{10}$ en la ecuación (2.75) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{21} = \frac{1}{V_1}\left(\ddot{x}G_1 + \ddot{y}G_2 + \ddot{z}G_3 + \ddot{\psi}G_4 + \ddot{\theta}G_5 + \ddot{\phi}G_6 + G_7\right) \tag{2.77}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.6.2 Solución de la aceleración, ángulo θ_{43}.

Derivando respecto del tiempo la ecuación (2.52):

$$\ddot{\theta}_{43} = \frac{-\ddot{Z}c\theta_{43} - 2\dot{X}\dot{\theta}_{43}c\theta_{43} + \dot{\theta}_{43}^{\ 2}Zc\theta_{43} - \ddot{X}s\theta_{43} + 2\dot{Z}\dot{\theta}_{43}s\theta_{43} + \dot{\theta}_{43}^{\ 2}Xs\theta_{43}}{Xc\theta_{43} - Zs\theta_{43}} \tag{2.78}$$

Donde X, Z se encuentran en (2.20) y $\dot{X}, \dot{Z}$ en 2.53, además:

$$\begin{aligned}
\ddot{X} &= \ddot{x} + \dot{\theta}_{21}{}^{2} x_{32} c\theta_{21} + \ddot{\theta}_{21} x_{32} s\theta_{21} + \ddot{\phi} x_{1918} c\psi s\phi - 2\dot{\phi}\dot{\psi} x_{1918} s\phi s\psi \\
&\quad + x_{1918} c\phi((\dot{\phi}^2 + \dot{\psi}^2)c\psi + \ddot{\psi} s\psi) \\
\ddot{Y} &= \ddot{y} + x_{1918}(c\theta(-c\phi(\ddot{\phi} + 2\dot{\theta}\dot{\psi}c\psi + \ddot{\theta}s\psi) + s\phi(\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}s\psi)) \\
&\quad + s\theta(s\phi(\ddot{\theta} + 2\dot{\phi}\dot{\psi}c\psi + \ddot{\phi}s\psi) + c\phi(2\dot{\theta}\dot{\phi} - \ddot{\psi}c\psi + (\dot{\theta}^2 + \dot{\phi}^2 \\
&\quad + \dot{\psi}^2)s\psi))) \\
\ddot{Z} &= \ddot{z} + \ddot{\theta}_{21} x_{32} c\theta_{21} - \ddot{\phi} x_{1918} c\phi s\theta - 2\dot{\theta}\dot{\psi} x_{1918} c\phi c\psi s\theta - \dot{\theta}_{21}{}^{2} x_{32} s\theta_{21} \\
&\quad + \dot{\theta}^2 x_{1918} s\theta s\phi + \dot{\phi}^2 x_{1918} s\theta s\phi - \ddot{\theta} x_{1918} c\phi s\theta s\psi \\
&\quad + 2\dot{\theta}\dot{\phi} x_{1918} s\theta s\phi s\psi - x_{1918} c\theta(s\phi(\ddot{\theta} + 2\dot{\phi}\dot{\psi}c\psi + \ddot{\phi}s\psi) \\
&\quad + c\phi(2\dot{\theta}\dot{\phi} - \ddot{\psi}c\psi + (\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2)s\psi))
\end{aligned} \tag{2.79}$$

Sustituyendo $X, Z, \dot{X}, \dot{Z}, \ddot{X}, \ddot{Z}$ en la ecuación (2.78) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{43} = \frac{1}{V_8}\left(\ddot{x}G_8 + \ddot{y}G_9 + \ddot{z}G_{10} + \ddot{\psi}G_{11} + \ddot{\theta}G_{12} + \ddot{\phi}G_{13} + G_{14}\right) \tag{2.80}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.6.3 Solución de la aceleración, ángulo θ_{54}.

Derivando respecto del tiempo la ecuación (2.57):

$$\begin{aligned}
\ddot{\theta}_{54} = \frac{1}{X^3} c\theta_{54}{}^{2}(&-2\dot{X}\dot{Y}Xc\theta_{43} + \ddot{Y}X^2c\theta_{43} + 2\dot{X}^2Yc\theta_{43} - \ddot{X}XYc\theta_{43} \\
&- \dot{\theta}_{43}{}^{2}X^2Yc\theta_{43} - 2\dot{Y}\dot{\theta}_{43}X^2s\theta_{43} + 2\dot{X}\dot{\theta}_{43}XYs\theta_{43} - \ddot{\theta}_{43}X^2Ys\theta_{43} \\
&- 2\dot{\theta}_{54}{}^{2}X^3\sec\theta_{54}{}^{2}\tan\theta_{54})
\end{aligned} \tag{2.81}$$

Sustituyendo $X, Y, \dot{X}, \dot{Y}, \ddot{X}, \ddot{Y}$ en la ecuación (2.81) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{54} = \frac{1}{G_{15}}\left(\ddot{x}G_{16} + \ddot{y}G_{17} + \ddot{z}G_{18} + \ddot{\psi}G_{19} + \ddot{\theta}G_{20} + \ddot{\phi}G_{21} + G_{22}\right) \tag{2.82}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.6.4 Solución de la aceleración, ángulo θ_{1817}.

Derivando respecto del tiempo la ecuación (2.61):

$$\ddot{\theta}_{1817} = -\frac{1}{b_{12}{}^{3}} c\theta_{1817}{}^{2}(2b_{13}\dot{b}_{12}{}^{2} - 2b_{12}\dot{b}_{12}\dot{b}_{13} - b_{12}b_{13}\ddot{b}_{12} + b_{12}{}^{2}\ddot{b}_{13} + 2b_{12}{}^{3}\dot{\theta}_{1817}{}^{2}\sec\theta_{1817}{}^{2}\tan\theta_{1817}) \quad (2.83)$$

Al derivar las ecuaciones (2.62) con respecto al tiempo se obtiene $\ddot{b}_{12}$ y $\ddot{b}_{13}$ que se encuentran en el apéndice C. De esta manera al sustituir $b_{12}, b_{13}, \dot{b}_{12}, \dot{b}_{13}, \ddot{b}_{12}, \ddot{b}_{13}$ en la ecuación (2.83) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{1817} = \frac{1}{G_{23}}\left(\ddot{x}G_{24} + \ddot{y}G_{25} + \ddot{z}G_{26} + \ddot{\psi}G_{27} + \ddot{\theta}G_{28} + \ddot{\phi}G_{29} + G_{30}\right) \quad (2.84)$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.6.5 Solución de la aceleración, ángulo θ_{1716}.

Derivando respecto del tiempo la ecuación (2.66):

$$\ddot{\theta}_{1716} = \frac{1}{b_{11}{}^{3}} c\theta_{1716}{}^{2}((2b_{13}\dot{b}_{11}{}^{2} - 2b_{11}\dot{b}_{11}\dot{b}_{13} - b_{11}b_{13}\ddot{b}_{11} + b_{11}{}^{2}\ddot{b}_{13} - b_{11}(b_{11}b_{13}\ddot{\theta}_{1817} - 2b_{13}\dot{b}_{11}\dot{\theta}_{1817} + 2b_{11}\dot{b}_{13}\dot{\theta}_{1817})\cot\theta_{1817} + b_{11}{}^{2}b_{13}\dot{\theta}_{1817}{}^{2}\cot\theta_{1817}{}^{2})\csc\theta_{1817} + b_{11}{}^{2}b_{13}\dot{\theta}_{1817}{}^{2}\csc\theta_{1817}{}^{3} - 2b_{11}{}^{3}\dot{\theta}_{1716}{}^{2}\sec\theta_{1716}{}^{2}\tan\theta_{1716}) \quad (2.85)$$

Al derivar la ecuación (2.67) con respecto al tiempo se obtiene $\ddot{b}_{11}$ que se encuentran en el apéndice C. De esta manera al sustituir $b_{11}, b_{13}, \dot{b}_{11}, \dot{b}_{13}, \ddot{b}_{11}, \ddot{b}_{13}$ en la ecuación (2.85) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{1716} = \frac{1}{G_{31}}\left(\ddot{x}G_{32} + \ddot{y}G_{33} + \ddot{z}G_{34} + \ddot{\psi}G_{35} + \ddot{\theta}G_{36} + \ddot{\phi}G_{37} + G_{38}\right) \quad (2.86)$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.6.6 Solución de la aceleración, ángulo θ_{1615}.

Derivando respecto del tiempo la ecuación (2.71):

$$\ddot{\theta}_{1615} = \frac{1}{c_{22}{}^{3}} c\theta_{1615}{}^{2}(2c_{32}\dot{c}_{22}{}^{2} - 2c_{22}\dot{c}_{22}\dot{c}_{32} - c_{22}c_{32}\ddot{c}_{22} + c_{22}{}^{2}\ddot{c}_{32} - 2c_{22}{}^{3}\dot{\theta}_{1615}{}^{2}\sec\theta_{1615}{}^{2}\tan\theta_{1615}) \quad (2.87)$$

Al derivar las ecuaciones (2.62) con respecto al tiempo se obtiene $\ddot{c}_{22}\ y\ \ddot{c}_{32}$ que se encuentran en el apéndice C. De esta manera al sustituir $c_{22}, c_{32}, \dot{c}_{22}, \dot{c}_{32}, \ddot{c}_{22}, \ddot{c}_{32}$ en la ecuación (2.87) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{1615} = \frac{1}{G_{39}}\left(\ddot{x}G_{40} + \ddot{y}G_{41} + \ddot{z}G_{42} + \ddot{\psi}G_{43} + \ddot{\theta}G_{44} + \ddot{\phi}G_{45} + G_{46}\right) \tag{2.88}$$

Donde los coeficientes de la ecuación (2.88) se encuentran en el apéndice E.

2.7 Cálculo de posición para las cadenas i.

Para el cálculo de la posición de las cadenas i se tiene como variables a calcular los ángulos $\theta_{21i}, \theta_{65i}, \theta_{76i}, \theta_{1110i}, \theta_{109i}, \theta_{98i}$ y la posición x, y, z, ψ, θ, ϕ del efector está dada.

2.7.1 Solución del ángulo θ_{21i} de las cadenas i.

A continuación se muestran las bases y los vectores a partir de los cuales se construye la ecuación de lazo para las cadenas 1 y 2. Al ser simétricas las cadenas 1 y 2 se lleva a cabo el análisis sobre una de ellas y asigna un iterador i. Así mismo se hace uso de una variable auxiliar llamada "signo" para diferenciar algunas distancias iguales en magnitud pero contrarias en signo.

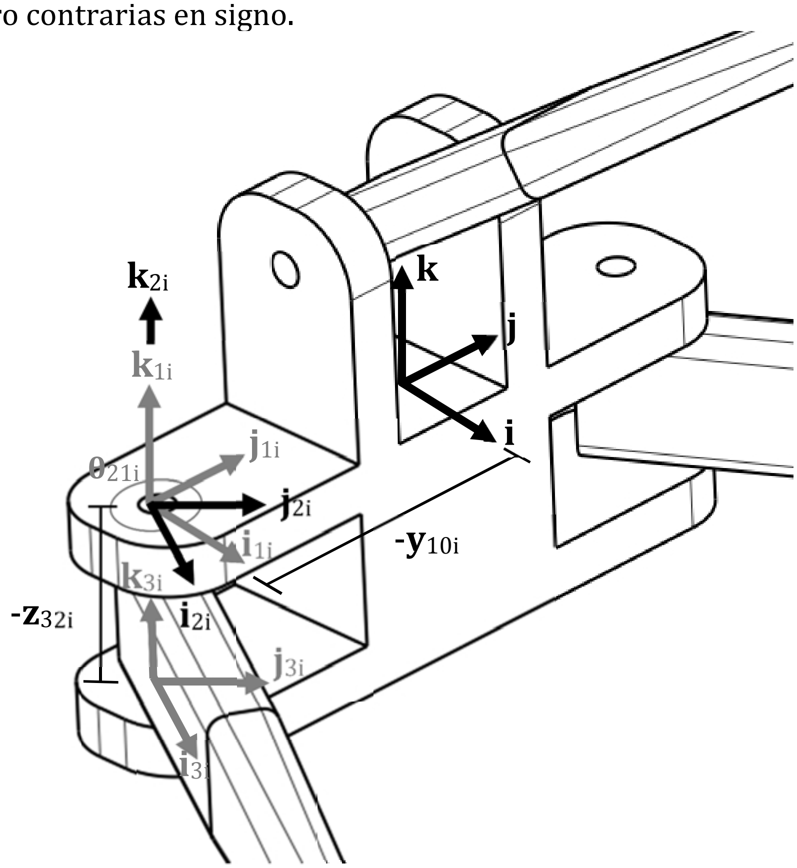


Figura 2-7 Bases locales de la 1i a la 3i para las cadenas i.

Partiendo de la base inercial se forma la base 1i por medio de una traslación sobre el eje Y. Posteriormente por tratarse de una junta rotacional se emplea una rotación sobre el eje Z para llegar a la base 2 y una traslación sobre Z para llegar a la base 3i como se observa en la figura (2-7).

En la figura (2-8) se aprecia que para formar la base 4i se traslada la base 3i sobre el eje X. Después para generar la base 5i se define un ángulo constante β_{54i} que le otorga una inclinación constante a la junta universal.

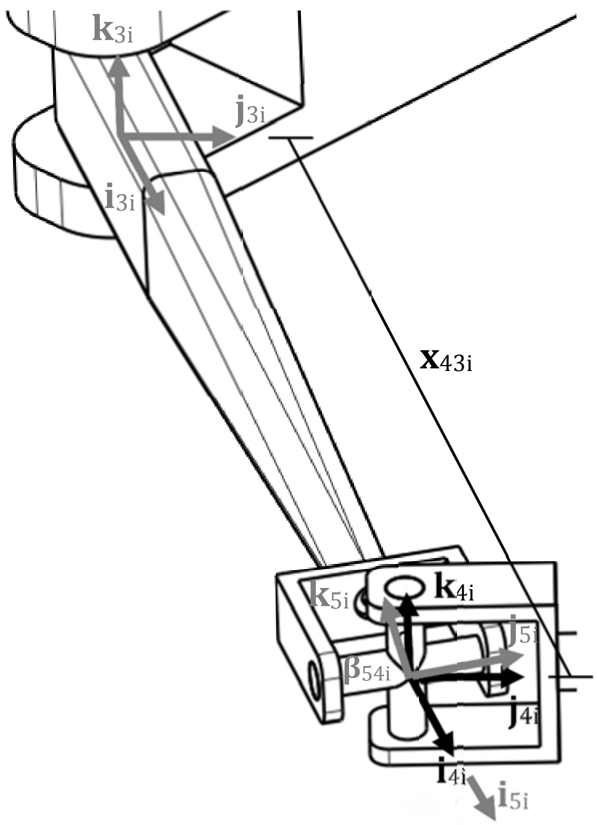


Figura 2-8 Bases locales 4i y 5i.

Por la naturaleza de la junta universal se necesita definir 2 rotaciones, lo cual se observa en la figura (2-9) generando así las bases 6i y 7i respectivamente. Posteriormente se crea la base 8i por medio de una traslación en el eje X, con lo cual se llega al centro de la junta esférica.

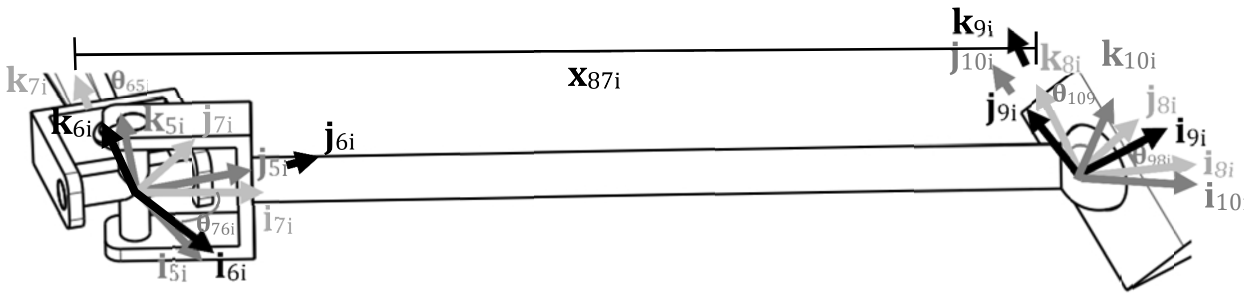


Figura 2-9 Bases locales de la 7i a la 10i.

Para la junta esférica es necesario definir tres rotaciones creando así las bases 9i y 10i, que se observan en la figura (2-9) y 11i que se observa en la figura (2-10)

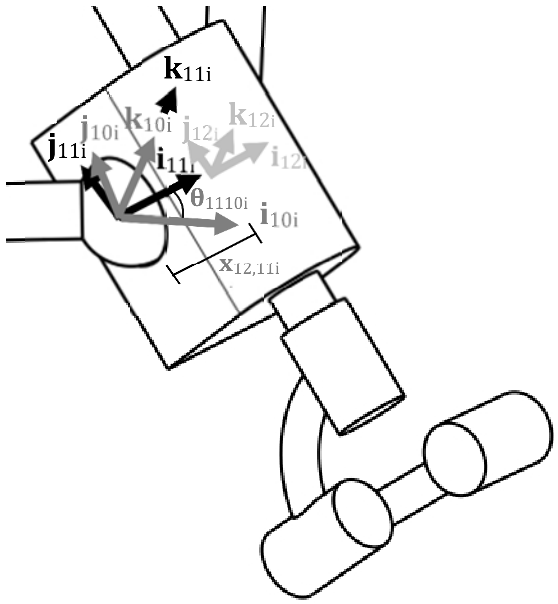


Figura 2-10 Bases locales de la 10i a la 12i.

Por último se busca cerrar el lazo vectorial con algunos de los vectores ya definidos para la cadena 0. Para esto se generan las bases 12i y 13i mediante dos traslaciones, como se aprecia en las figuras (2-10) y (2-11).

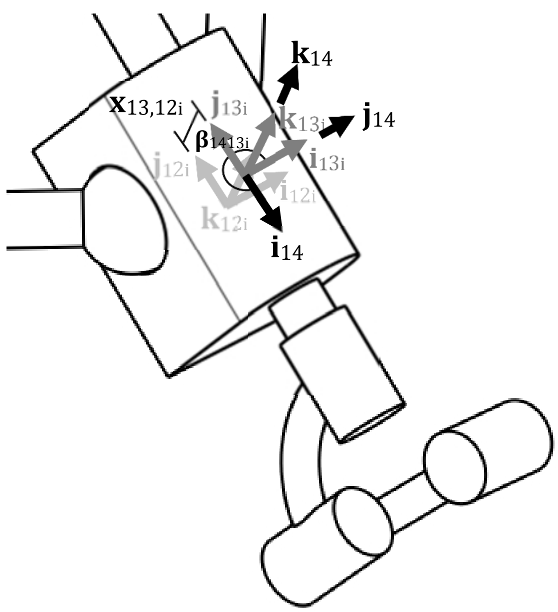


Figura 2-11 Bases locales de la 12i a la 14.

Finalmente se alinean las bases 13i y 14 mediante un ángulo β_{1413i} constante, cerrando de esta manera el lazo vectorial para las cadenas i.

En la figura (2-12) se muestra el lazo vectorial para las cadenas i.

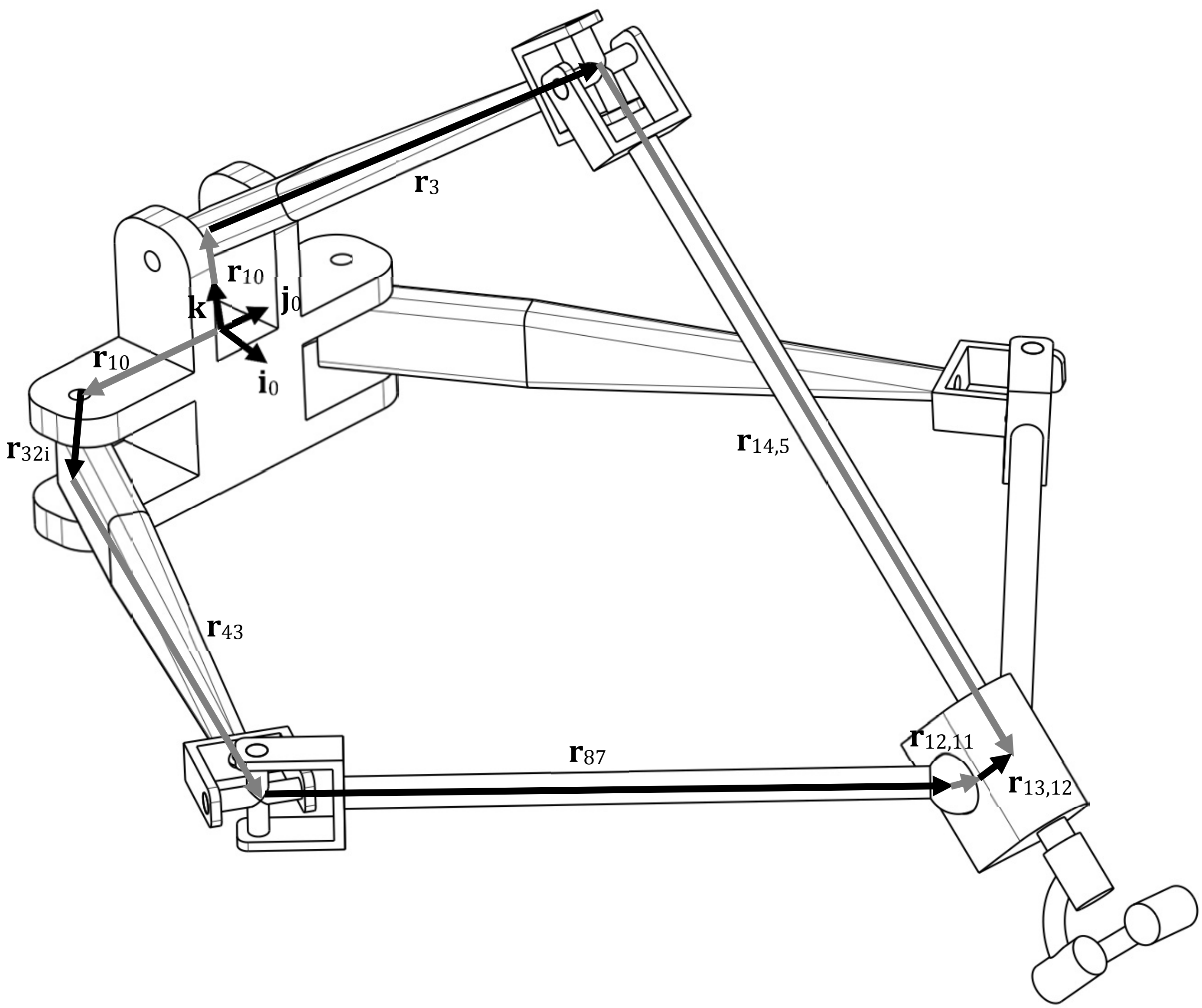


Figura 2-12 Lazo vectorial para las cadenas i.

A partir de la figura se forma la ecuación de lazo:

$$\boldsymbol{r}_{10i}^{0} + \boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{87i}^{0} = \boldsymbol{r}_{10}^{0} + \boldsymbol{r}_{32}^{0} + \boldsymbol{r}_{14,5i}^{0} - \boldsymbol{r}_{13,12i}^{0} - \boldsymbol{r}_{12,11i}^{0}$$

Al expresar la ecuación con transformaciones homogéneas se obtiene lo siguiente:

$$\boldsymbol{T}_{02i}\boldsymbol{T}_{24i}\boldsymbol{T}_{48i} = \boldsymbol{T}_{03}\boldsymbol{T}_{311i} \quad (2.89)$$

Dónde:

$$\boldsymbol{T}_{02i} = \boldsymbol{T}_{z2}(signo\ y_{10i})\boldsymbol{T}_{z6}(\theta_{21i})$$
$$\boldsymbol{T}_{24i} = \boldsymbol{T}_{z3}(-z_{32i})\boldsymbol{T}_{z1}(x_{43i})$$
$$\boldsymbol{T}_{48i} = \boldsymbol{T}_{z4}(\beta_{54i})\boldsymbol{T}_{z5}(\theta_{65i})\boldsymbol{T}_{z6}(\theta_{76i})\boldsymbol{T}_{z1}(x_{87i})$$
$$\boldsymbol{T}_{03} = \boldsymbol{T}_{z3}(z_{10})\boldsymbol{T}_{z5}(\theta_{21})\boldsymbol{T}_{z1}(x_{32})$$
$$\boldsymbol{T}_{311i} = \boldsymbol{T}_{z5}(\theta_{43})\boldsymbol{T}_{z6}(\theta_{54})\boldsymbol{T}_{z1}(x_{65})\boldsymbol{T}_{z6}(-\beta_{1413i})\boldsymbol{T}_{z3}(-z_{1312i})\boldsymbol{T}_{z1}(-x_{1211i})$$

Se despeja $\boldsymbol{T}_{48i}$ y se posmultiplica ambos lados de la ecuación por un vector $\boldsymbol{n}$ con el fin de obtener la última columna de la matriz de transformación, con lo cual tenemos:

$$\boldsymbol{T}_{48i}\boldsymbol{n} = \boldsymbol{T}_{24i}^{-1}\boldsymbol{T}_{02i}^{-1}\boldsymbol{T}_{03}\boldsymbol{T}_{311i}\ \boldsymbol{n} \tag{2.90}$$

Dónde:

$$n = [0\ ,\quad 0\ ,\quad 0\ ,\quad 1]^T$$

Si renombramos el lado izquierdo de la ecuación (2.90) como $\boldsymbol{Ec}_3$ y el lado derecho como $\boldsymbol{Ec}_4$ **,** se obtiene:

$$\boldsymbol{Ec}_3 = \boldsymbol{Ec}_4 \tag{2.91}$$

En esta ecuación se tienen como incógnitas a θ_{21i}, θ_{65i} y θ_{76i}. Como nos interesa despejar a θ_{21i} hay que eliminar las otras dos, lo cual se logra al obtener la magnitud de ambos lados de la ecuación:

$$\|\boldsymbol{Ec}_3\| = \|\boldsymbol{Ec}_4\| \tag{2.92}$$

$$\sqrt{\boldsymbol{Ec}_3{}^{\boldsymbol{T}}\boldsymbol{Ec}_3} = \sqrt{\boldsymbol{Ec}_4{}^{\boldsymbol{T}}\boldsymbol{Ec}_4} \tag{2.93}$$

Simplificado:

$$\boldsymbol{Ec}_3{}^{\boldsymbol{T}}\boldsymbol{Ec}_3 - \boldsymbol{Ec}_4{}^{\boldsymbol{T}}\boldsymbol{Ec}_4 = \boldsymbol{0} \tag{2.94}$$

Al factorizar esta última ecuación en términos de $s\theta_{21i}$ y $c\theta_{21i}$ se obtiene una expresión de la forma:

$$C_{1i} + A_{1i}c\theta_{21i} + B_{1i}s\theta_{21i} = 0 \tag{2.95}$$

Dónde:

$$\begin{aligned}
A_{1i} = x_{43i}(2\, x_{32}c\theta_{21} - x_{1211i}\, c(\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54}) \\
+ x_{65}\, c(\theta_{21} + \theta_{43} - \theta_{54}) - x_{1211i}\, c(\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}) \\
+ x_{65}\, c(\theta_{21} + \theta_{43} + \theta_{54}) - 2\, z_{1312i}\, s(\theta_{21} + \theta_{43})) \\
B_{1i} = 2x_{43i}(-signo\, y_{10i} + x_{1211i}s(\beta_{1413i} - \theta_{54}) + x_{65}s(\theta_{54})) \\
C_{1i} = -x_{1211i}{}^2 - x_{32}{}^2 - x_{43i}{}^2 - x_{65}{}^2 + x_{87i}{}^2 - signo^2 y_{10i}{}^2 - z_{10}{}^2 - z_{1312i}{}^2 \\
- 2z_{10}z_{32i} - z_{32i}{}^2 + 2x_{1211i}x_{65}c(\beta_{1413i}) + 2z_{1312i}(z_{10} \\
+ z_{32i})c(\theta_{21} + \theta_{43}) + x_{1211i}x_{32}c(\beta_{1413i} - \theta_{43} - \theta_{54}) \\
- x_{32}x_{65}c(\theta_{43} - \theta_{54}) + x_{1211i}x_{32}c(\beta_{1413i} + \theta_{43} - \theta_{54}) \\
- x_{32}x_{65}c(\theta_{43} + \theta_{54}) + 2x_{32}z_{10}s(\theta_{21}) + 2x_{32}z_{32i}s(\theta_{21}) \\
+ 2x_{32}z_{1312i}S(\theta_{43}) + 2signo x_{1211i}y_{10i}s(\beta_{1413i} - \theta_{54}) \\
+ x_{1211i}z_{10}s(\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54}) + x_{1211i}z_{32i}s(\beta_{1413i} \\
- \theta_{21} - \theta_{43} - \theta_{54}) + x_{65}z_{10}s(\theta_{21} + \theta_{43} - \theta_{54}) + x_{65}z_{32i}s(\theta_{21} \\
+ \theta_{43} - \theta_{54}) - x_{1211i}z_{10}s(\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}) \\
- x_{1211i}z_{32i}s(\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}) + 2signo x_{65}y_{10i}s(\theta_{54}) \\
+ x_{65}z_{10}s(\theta_{21} + \theta_{43} + \theta_{54}) + x_{65}z_{32i}s(\theta_{21} + \theta_{43} + \theta_{54})
\end{aligned} \tag{2.96}$$

De esta manera en la forma en la que se encuentra la ecuación (2.95) se puede despejar a θ_{21i} como:

$$\theta_{21i} = 2\, arcTan\left(\frac{B_{1i} \pm \sqrt{A_{1i}{}^2 + B_{1i}{}^2 - C_{1i}{}^2}}{A_{1i} - C_{1i}}\right) \tag{2.97}$$

2.7.2 Solución del ángulo θ_{65i} de las cadenas i.

Del desarrollo anterior se puede obtener la forma matricial de ambos lados de la ecuación (2.91) como sigue:

$$\boldsymbol{Ec_3} = \begin{bmatrix} x_{87i}c\theta_{65i}c\theta_{76i} \\ x_{87i}s\theta_{76i} \\ -x_{87i}c\theta_{76i}s\theta_{65i} \\ 1 \end{bmatrix} \tag{2.98}$$

Se observa que $\boldsymbol{Ec}_4$ esta formada por elementos constantes y por θ_{21i} que es el ángulo que se ya se despejó por lo que para un manejo más cómodo se renombraran sus elementos como X_i, Y_i, Z_i es decir:

$$
\boldsymbol{Ec}_4 = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix} \tag{2.99}
$$

Dónde:

$$
\begin{aligned}
X_i = {} & -x_{43i} - signoy_{10i} s\theta_{21i} + c\theta_{21} c\theta_{21i}(x_{32} + c\theta_{43}(-x_{1211i} c(\beta_{1413i} - \theta_{54}) \\
& + x_{65} c\theta_{54}) - z_{1312i} s\theta_{43}) - c\theta_{21i} s\theta_{21}(z_{1312i} c\theta_{43} \\
& - x_{1211i} c(\beta_{1413i} - \theta_{54}) s\theta_{43} + x_{65} c\theta_{54} s\theta_{43}) \\
& + x_{1211i} s\theta_{21i} s(\beta_{1413i} - \theta_{54}) + x_{65} s\theta_{21i} s\theta_{54} \\
Y_i = {} & -signoy_{10i} c\beta_{54i} c\theta_{21i} + z_{32i} s\beta_{54i} + s\beta_{54i}(z_{10} - x_{32} s\theta_{21}) \\
& - x_{32} c\beta_{54i} c\theta_{21} s\theta_{21i} + (s\beta_{54i} s\theta_{21} \\
& + c\beta_{54i} c\theta_{21} s\theta_{21i})(c\theta_{43}(x_{1211i} c(\beta_{1413i} - \theta_{54}) - x_{65} c\theta_{54}) \\
& + z_{1312i} s\theta_{43}) - (c\theta_{21} s\beta_{54i} - c\beta_{54i} s\theta_{21} s\theta_{21i})(z_{1312i} c\theta_{43} \\
& + (-x_{1211i} c(\beta_{1413i} - \theta_{54}) + x_{65} c\theta_{54}) s\theta_{43}) \\
& + c\beta_{54i} c\theta_{21i}(x_{1211i} s(\beta_{1413i} - \theta_{54}) + x_{65} s\theta_{54}) \\
Z_i = {} & z_{32i} c\beta_{54i} + signoy_{10i} c\theta_{21i} s\beta_{54i} + c\beta_{54i}(z_{10} - x_{32} s\theta_{21}) \\
& + x_{32} c\theta_{21} s\beta_{54i} s\theta_{21i} + (c\beta_{54i} s\theta_{21} \\
& - c\theta_{21} s\beta_{54i} s\theta_{21i})(c\theta_{43}(x_{1211i} c(\beta_{1413i} - \theta_{54}) - x_{65} c\theta_{54}) \\
& + z_{1312i} s\theta_{43}) - (c\beta_{54i} c\theta_{21} + s\beta_{54i} s\theta_{21} s\theta_{21i})(z_{1312i} c\theta_{43} \\
& + (-x_{1211i} c(\beta_{1413i} - \theta_{54}) + x_{65} c\theta_{54}) s\theta_{43}) \\
& - c\theta_{21i} s\beta_{54i}(x_{1211i} s(\beta_{1413i} - \theta_{54}) + x_{65} s\theta_{54})
\end{aligned} \tag{2.100}
$$

De esta manera se manipulará más fácilmente el lado izquierdo de la ecuación, es decir $\boldsymbol{Ec_3}$**,** la cual contiene el ángulo que nos interesa despejar.

Se tiene el sistema de ecuaciones siguiente:

$$
\begin{aligned}
x_{87i} c\theta_{65i} c\theta_{76i} &= X_i \\
x_{87i} s\theta_{76i} &= Y_i \\
-x_{87i} c\theta_{76i} s\theta_{65i} &= Z_i
\end{aligned} \tag{2.101}
$$

De las ecuaciones (2.101) se obtiene $tan\theta_{65i}$:

$$
tan\theta_{65i} = \frac{-Z_i}{X_i} \tag{2.102}
$$

De esta manera:

$$
\theta_{65i} = arctan\left(\frac{-Z_i}{X_i}\right) \tag{2.103}
$$

2.7.3 Solución del ángulo θ_{76i} las cadenas i.

De las ecuaciones (2.101) se tiene que:

$$\tan \theta_{76i} = \frac{Y_i c\theta_{65i}}{X_i} \tag{2.104}$$

Y por lo tanto:

$$\theta_{76i} = \arctan\left(\frac{Y_i c\theta_{65i}}{X_i}\right) \tag{2.105}$$

2.7.4 Solución del ángulo θ_{1110i} de las cadenas i.

Para el cálculo de los ángulos de la junta esférica que conecta la cadena i con el órgano terminal se formarán las ecuaciones de lazo pero esta vez considerando tanto distancias como ángulos.

Al expresarse en transformaciones homogéneas:

$$\boldsymbol{T}_{02i}\boldsymbol{T}_{24i}\boldsymbol{T}_{48i}\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i})\boldsymbol{T}_{z6}(\theta_{1110i}) = \boldsymbol{T}_{03}\boldsymbol{T}_{311i} \tag{2.106}$$

Dónde:

$$\begin{aligned}
\boldsymbol{T}_{02i} &= \boldsymbol{T}_{z2}(signo\ y_{10i})\boldsymbol{T}_{z6}(\theta_{21i})\\
\boldsymbol{T}_{24i} &= \boldsymbol{T}_{z3}(-z_{32i})\boldsymbol{T}_{z1}(x_{43i})\\
\boldsymbol{T}_{48i} &= \boldsymbol{T}_{z4}(\beta_{54i})\boldsymbol{T}_{z5}(\theta_{65i})\boldsymbol{T}_{z6}(\theta_{76i})\boldsymbol{T}_{z1}(x_{87i})\\
\boldsymbol{T}_{03} &= \boldsymbol{T}_{z3}(z_{10})\boldsymbol{T}_{z5}(\theta_{21})\boldsymbol{T}_{z1}(x_{32})\\
\boldsymbol{T}_{311i} &= \boldsymbol{T}_{z5}(\theta_{43})\boldsymbol{T}_{z6}(\theta_{54})\boldsymbol{T}_{z1}(x_{65})\boldsymbol{T}_{z6}(-\beta_{1413i})\boldsymbol{T}_{z3}(-z_{1312i})\boldsymbol{T}_{z1}(-x_{1211i})
\end{aligned}$$

Se observa que además de los tres ángulos de la junta esférica ($\theta_{98i}, \theta_{109i}$ y θ_{1110i}) lo demás es conocido debido a que es dato o ya se calculó, por lo que se procederá a dejar las transformaciones asociadas a los ángulos desconocidos del lado izquierdo de la ecuación y todo lo demás del lado derecho.

$$\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i})\boldsymbol{T}_{z6}(\theta_{1110i}) = \boldsymbol{T}_{48i}^{-1}\boldsymbol{T}_{24i}^{-1}\boldsymbol{T}_{02i}^{-1}\boldsymbol{T}_{03}\boldsymbol{T}_{311i} \tag{2.107}$$

Renombrando ambos lados de la ecuación (2.107):

$$\boldsymbol{T}_{LIi} = \boldsymbol{T}_{LDi} \tag{2.108}$$

Dónde:

$$\boldsymbol{T}_{LIi} = \begin{bmatrix} c\theta_{109i}c\theta_{1110i}c\theta_{98i} - s\theta_{1110i}s\theta_{98i} & -c\theta_{109i}c\theta_{98i}s\theta_{1110i} - c\theta_{1110i}s\theta_{98i} & c\theta_{98i}s\theta_{109i} & 0\\ c\theta_{98i}s\theta_{1110i} + c\theta_{109i}c\theta_{1110i}s\theta_{98i} & c\theta_{1110i}c\theta_{98i} - c\theta_{109i}s\theta_{1110i}s\theta_{98i} & s\theta_{109i}s\theta_{98i} & 0\\ -c\theta_{1110i}\theta_{109i} & s\theta_{109i}s\theta_{1110i} & c\theta_{109i} & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.109}$$

$$\boldsymbol{T}_{LDi} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \tag{2.110}$$

Igualando los elementos (3,1) y (3,2) de las ecuaciones (2.109) y (2.110):

$$\begin{aligned} -c\theta_{1110i}\theta_{109i} &= d_{31} \\ s\theta_{109i}s\theta_{1110i} &= d_{32} \end{aligned} \tag{2.111}$$

Despejando cθ_{1110i} y sθ_{1110i}:

$$\begin{aligned} \mathrm{c}\theta_{1110i} &= -d_{31}csc\theta_{109i} \\ \mathrm{s}\theta_{1110i} &= d_{32}csc\theta_{109i} \end{aligned} \tag{2.112}$$

Formando la función tangente:

$$tan\theta_{1110i} = \frac{d_{32}}{-d_{31}} \tag{2.113}$$

Por lo tanto:

$$\theta_{1110i} = arctan\left(\frac{d_{32}}{-d_{31}}\right) \tag{2.114}$$

Dónde:

$$\begin{aligned} d_{31} = {} & s(\beta_{1413i} - \theta_{54})(c\theta_{21i}c\theta_{65i}s\beta_{54i} - s\theta_{21i}s\theta_{65i}) + c\theta_{43}c(\beta_{1413i} \\ & - \theta_{54})(-c\beta_{54i}c\theta_{65i}s\theta_{21} + c\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i} \\ & + c\theta_{21i}s\theta_{65i})) - c(\beta_{1413i} - \theta_{54})s\theta_{43}(c\beta_{54i}c\theta_{21}c\theta_{65i} \\ & + s\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i} + c\theta_{21i}s\theta_{65i})) \\ d_{32} = {} & c(\beta_{1413i} - \theta_{54})(-c\theta_{21i}c\theta_{65i}s\beta_{54i} + s\theta_{21i}s\theta_{65i}) + c\theta_{43}s(\beta_{1413i} \\ & - \theta_{54})(-c\beta_{54i}c\theta_{65i}s\theta_{21} + c\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i} \\ & + c\theta_{21i}s\theta_{65i})) - s\theta_{43}s(\beta_{1413i} - \theta_{54})(c\beta_{54i}c\theta_{21}c\theta_{65i} \\ & + s\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i} + c\theta_{21i}s\theta_{65i})) \end{aligned} \tag{2.115}$$

2.7.5 Solución del ángulo θ_{109i} de las cadenas i.

Despejando $\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i})$ de la ecuación (2.107):

$$\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i}) = \boldsymbol{T}_{48i}^{-1}\boldsymbol{T}_{24i}^{-1}\boldsymbol{T}_{02i}^{-1}\boldsymbol{T}_{03}\boldsymbol{T}_{311i}\boldsymbol{T}_{z6}^{-1}(\theta_{1110i}) \tag{2.116}$$

Renombrando el lado derecho de la ecuación (2.116) como:

$$\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i}) = \boldsymbol{T}_{LDi2} \tag{2.117}$$

Dónde:

$$\boldsymbol{T}_{z6}(\theta_{98i})\boldsymbol{T}_{z5}(\theta_{109i}) = \begin{bmatrix} c\theta_{109i}c\theta_{98i} & -s\theta_{98i} & c\theta_{98i}s\theta_{109i} & 0 \\ c\theta_{109i}s\theta_{98i} & c\theta_{98i} & s\theta_{109i}s\theta_{98i} & 0 \\ -s\theta_{109i} & 0 & c\theta_{109i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.118}$$

$$\boldsymbol{T}_{LDi2} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.119}$$

De las ecuaciones (2.118) y (2.119) se igualan los elementos (3,1) y (3,3):

$$\begin{aligned} -s\theta_{109i} &= e_{31} \\ c\theta_{109i} &= e_{33} \end{aligned} \tag{2.120}$$

Despejando $c\theta_{109i}$ y $s\theta_{109i}$ de las ecuaciones (2.120):

$$\begin{aligned} c\theta_{109i} &= e_{33} \\ s\theta_{109i} &= -e_{31} \end{aligned} \tag{2.121}$$

Formando la función tangente a partir de las ecuaciones (2.121):

$$tan\theta_{109i} = \frac{-e_{31}}{e_{33}} \tag{2.122}$$

Por lo tanto:

$$\theta_{109i} = arctan\left(\frac{-e_{31}}{e_{33}}\right) \tag{2.123}$$

Dónde:

$$
\begin{aligned}
e_{31} = -c\beta_{54i}c(\beta_{1413i} + \theta_{1110i} - \theta_{54})c\theta_{65i}s(\theta_{21} + \theta_{43}) + c\theta_{21i}(c(\beta_{1413i} \\
- \theta_{54})(c\theta_{65i}s\beta_{54i}s\theta_{1110i} + c\theta_{1110i}c(\theta_{21} + \theta_{43})s\theta_{65i}) \\
+ s(\beta_{1413i} - \theta_{54})(c\theta_{1110i}c\theta_{65i}s\beta_{54i} - c(\theta_{21} \\
+ \theta_{43})s\theta_{1110i}s\theta_{65i})) \\
+ s\theta_{21i}(-s\theta_{1110i}(c\theta_{21}c\theta_{43}c\theta_{65i}s\beta_{54i}s(\beta_{1413i} - \theta_{54}) \\
- c\theta_{65i}s\beta_{54i}s\theta_{21}s\theta_{43}s(\beta_{1413i} - \theta_{54}) + c(\beta_{1413i} - \theta_{54})s\theta_{65i}) \\
+ c\theta_{1110i}(c\theta_{21}c\theta_{43}c(\beta_{1413i} - \theta_{54})c\theta_{65i}s\beta_{54i} - c(\beta_{1413i} \\
- \theta_{54})c\theta_{65i}s\beta_{54i}s\theta_{21}s\theta_{43} - s(\beta_{1413i} - \theta_{54})s\theta_{65i})) \\
e_{33} = c\beta_{54i}c(\theta_{21} + \theta_{43})c\theta_{65i} + s(\theta_{21} + \theta_{43})(c\theta_{65i}s\beta_{54i}s\theta_{21i} + c\theta_{21i}s\theta_{65i})
\end{aligned}
\quad (2.124)
$$

2.7.6 Solución del ángulo θ_{98i} de las cadenas i.

Despejando $\boldsymbol{T}_{z6}(\theta_{98i})$ de la ecuación (2.107):

$$\boldsymbol{T}_{z6}(\theta_{98i}) = \boldsymbol{T}_{48i}^{-1}\boldsymbol{T}_{24i}^{-1}\boldsymbol{T}_{02i}^{-1}\boldsymbol{T}_{03}\boldsymbol{T}_{311i}\boldsymbol{T}_{z6}^{-1}(\theta_{1110i})\boldsymbol{T}_{z5}^{-1}(\theta_{109i}) \quad (2.125)$$

Renombrando el lado derecho de la ecuación como:

$$\boldsymbol{T}_{z6}(\theta_{98i}) = \boldsymbol{T}_{LDi3} \quad (2.126)$$

Dónde:

$$\boldsymbol{T}_{z6}(\theta_{98i}) = \begin{bmatrix} c\theta_{98i} & -s\theta_{98i} & 0 & 0 \\ s\theta_{98i} & c\theta_{98i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.127)$$

$$\boldsymbol{T}_{LDi3} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.128)$$

De las ecuaciones (2.127) y (2.128) se igualan los elementos (1,1) y (2,3)

$$\begin{aligned} s\theta_{98i} &= f_{21} \\ c\theta_{98i} &= f_{11} \end{aligned} \quad (2.129)$$

Formando la función tangente a partir de las ecuaciones (2.129):

$$tan\theta_{98i} = \frac{f_{21}}{f_{11}} \quad (2.130)$$

Por lo tanto:

$$\theta_{98i} = arctan\left(\frac{f_{21}}{f_{11}}\right) \tag{2.131}$$

Donde f_{11} y f_{21} se encuentran en el apéndice C.

2.8 Cálculo de velocidad de las cadenas i.

Para el cálculo de la velocidad se tiene como variables de entrada la velocidad del efector $(\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi})$ y las variables a calcular son las velocidades de los ángulos de las cadenas i es decir $\dot{\theta}_{21i}, \dot{\theta}_{65i}, \dot{\theta}_{76i}, \dot{\theta}_{1110i}, \dot{\theta}_{109i}, \dot{\theta}_{98i}$. Para esta sección se asume que la posición de los ángulos es conocida debido a que se calculó en la sección anterior.

2.8.1 Solución de la velocidad ángulo θ_{21i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.95) y se despeja $\dot{\theta}_{21i}$:

$$\dot{\theta}_{21i} = \frac{-\dot{C}_{1i} - \dot{A}_{1i}c\theta_{21i} - \dot{B}_{1i}s\theta_{21i}}{B_{1i}c\theta_{21i} - A_{1i}s\theta_{21i}} \tag{2.132}$$

Dónde:

$$\begin{aligned}
\dot{A}_{1i} = x_{43i}(&-2(\dot{\theta}_{21} + \dot{\theta}_{43})z_{1312i}c\theta_{21} + \theta_{43} - 2\dot{\theta}_{21}x_{32}s\theta_{21} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{1211i}s\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{65}s\theta_{21} + \theta_{43} - \theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})x_{1211i}s\beta_{1413i} \\
&+ \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})x_{65}s\theta_{21} + \theta_{43} + \theta_{54}) \\
\dot{B}_{1i} = -2&\dot{\theta}_{54}x_{43i}(x_{1211i}c\beta_{1413i} - \theta_{54} - x_{65}c\theta_{54}) \\
\dot{C}_{1i} = 2&\dot{\theta}_{21}x_{32}z_{10}c\theta_{21} + 2\dot{\theta}_{21}x_{32}z_{32i}c\theta_{21} + 2\dot{\theta}_{43}x_{32}z_{1312i}c\theta_{43} \\
&- 2\dot{\theta}_{54}signox_{1211i}y_{10i}c\beta_{1413i} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{1211i}z_{10}c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{1211i}z_{32i}c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{65}z_{10}c\theta_{21} + \theta_{43} - \theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{65}z_{32i}c\theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{1211i}z_{10}c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{1211i}z_{32i}c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} \\
&+ 2\dot{\theta}_{54}signox_{65}y_{10i}c\theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})x_{65}z_{10}c\theta_{21} \\
&+ \theta_{43} + \theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})x_{65}z_{32i}c\theta_{21} + \theta_{43} + \theta_{54} \\
&- 2(\dot{\theta}_{21} + \dot{\theta}_{43})z_{1312i}(z_{10} + z_{32i})s\theta_{21} + \theta_{43} + (\dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{1211i}x_{32}s\beta_{1413i} - \theta_{43} - \theta_{54} + (\dot{\theta}_{43} - \dot{\theta}_{54})x_{32}x_{65}s\theta_{43} \\
&- \theta_{54} - (\dot{\theta}_{43} - \dot{\theta}_{54})x_{1211i}x_{32}s\beta_{1413i} + \theta_{43} - \theta_{54} + (\dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{32}x_{65}s\theta_{43} + \theta_{54}
\end{aligned} \tag{2.133}$$

Sustituyendo $A_{1i}, B_{1i}, C_{1i}, \dot{A}_{1i}, \dot{B}_{1i}, \dot{C}_{1i}$ en (2.132), agrupando y simplificando:

$$\dot{\theta}_{21i} = \frac{1}{V_{44}}\left(\dot{\theta}_{21}V_{45} + \dot{\theta}_{43}V_{46} + \dot{\theta}_{54}V_{47}\right) \tag{2.134}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59) en (2.134) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{21i} = \frac{1}{V_{44}}\left(\dot{x}E_{31} + \dot{y}E_{32} + \dot{z}E_{33} + \dot{\psi}E_{34} + \dot{\theta}E_{35} + \dot{\phi}E_{36}\right) \tag{2.135}$$

Dónde:

$$\begin{aligned}
E_{31} &= \frac{V_2V_{45}}{V_1} + \frac{E_7V_{47}}{V_{15}} + \frac{E_1V_{46}}{V_8} & E_{34} &= \frac{E_{10}V_{47}}{V_{15}} + \frac{V_{45}V_5}{V_1} + \frac{E_4V_{46}}{V_8} \\
E_{32} &= \frac{V_3V_{45}}{V_1} + \frac{E_8V_{47}}{V_{15}} + \frac{E_2V_{46}}{V_8} & E_{35} &= \frac{E_{11}V_{47}}{V_{15}} + \frac{V_{45}V_6}{V_1} + \frac{E_5V_{46}}{V_8} \\
E_{33} &= \frac{V_4V_{45}}{V_1} + \frac{E_9V_{47}}{V_{15}} + \frac{E_3V_{46}}{V_8} & E_{36} &= \frac{E_{12}V_{47}}{V_{15}} + \frac{V_{45}V_7}{V_1} + \frac{E_6V_{46}}{V_8}
\end{aligned} \tag{2.136}$$

2.8.2 Solución de la velocidad ángulo θ_{65i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.103):

$$\dot{\theta}_{65i} = \frac{-\dot{Z}_i c\theta_{65i} - \dot{X}_i s\theta_{65i}}{X_i c\theta_{65i} - Z_i s\theta_{65i}} \tag{2.137}$$

Dónde:

$$\begin{aligned}
\dot{X}_i = \frac{1}{2}(&-2(\dot{\theta}_{21} + \dot{\theta}_{43})z_{1312i}c\theta_{21} + \theta_{43} - 2\dot{\theta}_{21}x_{32}s\theta_{21} + 2\dot{\theta}_{21i}x_{43i}s\theta_{21i} \\
&- (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})x_{1211i}s\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} \\
&+ \dot{\theta}_{43} - \dot{\theta}_{54})x_{65}s\theta_{21} + \theta_{43} - \theta_{54} + (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&- \dot{\theta}_{54})x_{1211i}s\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} \\
&+ \dot{\theta}_{54})x_{65}s\theta_{21} + \theta_{43} + \theta_{54}) \\
\dot{Y}_i = -c\beta_{54i}(&\dot{\theta}_{21i}x_{43i}c\theta_{21i} + \dot{\theta}_{54}x_{1211i}c\beta_{1413i} - \theta_{54} - \dot{\theta}_{54}x_{65}c\theta_{54}) \\
&+ \frac{1}{2}s\beta_{54i}(-2\dot{\theta}_{21}x_{32}c\theta_{21} + (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})x_{1211i}c\beta_{1413i} \\
&- \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})x_{65}c\theta_{21} + \theta_{43} - \theta_{54} \\
&+ (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})x_{1211i}c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} \\
&+ \dot{\theta}_{43} + \dot{\theta}_{54})x_{65}c\theta_{21} + \theta_{43} + \theta_{54} + 2(\dot{\theta}_{21} + \dot{\theta}_{43})z_{1312i}s\theta_{21} \\
&+ \theta_{43})
\end{aligned} \tag{2.138}$$

$$\dot{Z}_i = (\dot{\theta}_{21i} x_{43i} c\theta_{21i} + \dot{\theta}_{54} x_{1211i} c\beta_{1413i} - \theta_{54} - \dot{\theta}_{54} x_{65} c\theta_{54}) s\beta_{54\mathrm{i}}$$
$$+ \frac{1}{2} c\beta_{54\mathrm{i}} (-2\dot{\theta}_{21} x_{32} c\theta_{21} + (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}) x_{1211i} c\beta_{1413i}$$
$$- \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}) x_{65} c\theta_{21} + \theta_{43} - \theta_{54}$$
$$+ (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}) x_{1211i} c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21}$$
$$+ \dot{\theta}_{43} + \dot{\theta}_{54}) x_{65} c\theta_{21} + \theta_{43} + \theta_{54} + 2(\dot{\theta}_{21} + \dot{\theta}_{43}) z_{1312i} s\theta_{21}$$
$$+ \theta_{43})$$

Sustituyendo $X_i, Z_i, \dot{X}_i, \dot{Z}_i$ en (2.137), agrupando y simplificando:

$$\dot{\theta}_{65i} = \frac{1}{V_{48}} \left(\dot{\theta}_{21} V_{49} + \dot{\theta}_{43} V_{50} + \dot{\theta}_{54} V_{51} + \dot{\theta}_{21i} V_{52} \right) \tag{2.139}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.135) en (2.139) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{65i} = \frac{1}{V_{48}} \left(\dot{x} E_{37} + \dot{y} E_{38} + \dot{z} E_{39} + \dot{\psi} E_{40} + \dot{\theta} E_{41} + \dot{\phi} E_{42} \right) \tag{2.140}$$

Dónde:

$$\begin{aligned}
E_{37} &= \frac{V_2 V_{49}}{V_1} + \frac{E_7 V_{51}}{V_{15}} + \frac{E_{31} V_{52}}{V_{44}} + \frac{E_1 V_{50}}{V_8} \\
E_{38} &= \frac{V_3 V_{49}}{V_1} + \frac{E_8 V_{51}}{V_{15}} + \frac{E_{32} V_{52}}{V_{44}} + \frac{E_2 V_{50}}{V_8} \\
E_{39} &= \frac{V_4 V_{49}}{V_1} + \frac{E_9 V_{51}}{V_{15}} + \frac{E_{33} V_{52}}{V_{44}} + \frac{E_3 V_{50}}{V_8} \\
E_{40} &= \frac{V_{49} V_5}{V_1} + \frac{E_{10} V_{51}}{V_{15}} + \frac{E_{34} V_{52}}{V_{44}} + \frac{E_4 V_{50}}{V_8} \\
E_{41} &= \frac{E_{11} V_{51}}{V_{15}} + \frac{E_{35} V_{52}}{V_{44}} + \frac{V_{49} V_6}{V_1} + \frac{E_5 V_{50}}{V_8} \\
E_{42} &= \frac{E_{12} V_{51}}{V_{15}} + \frac{E_{36} V_{52}}{V_{44}} + \frac{V_{49} V_7}{V_1} + \frac{E_6 V_{50}}{V_8}
\end{aligned} \tag{2.141}$$

2.8.3 Solución de la velocidad ángulo θ_{76i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.105):

$$\dot{\theta}_{76i} = \frac{c\theta_{76i}{}^2 ((\dot{Y}_i X_i - \dot{X}_i Y_i) c\theta_{65i} - \dot{\theta}_{65i} X_i Y_i s\theta_{65i})}{X_i{}^2} \tag{2.142}$$

Donde X_i, Y_i se encuentran en las ecuaciones (2.100) y $\dot{X}_i, \dot{Y}_i$ en las ecuaciones (2.138). Sustituyendo $X_i, Y_i, \dot{X}_i, \dot{Y}_i$ en (2.142), agrupando y simplificando:

$$\dot{\theta}_{76i} = \frac{1}{V_{53}}\left(\dot{\theta}_{21}V_{54} + \dot{\theta}_{43}V_{55} + \dot{\theta}_{54}V_{56} + \dot{\theta}_{21i}V_{57} + \dot{\theta}_{65i}V_{58}\right) \tag{2.143}$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.135), (2.140) en (2.143) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\dot{\theta}_{76i} = \frac{1}{V_{53}}\left(\dot{x}E_{43} + \dot{y}E_{44} + \dot{z}E_{45} + \dot{\psi}E_{46} + \dot{\theta}E_{47} + \dot{\phi}E_{48}\right) \tag{2.144}$$

Dónde:

$$\begin{aligned}
E_{43} &= \frac{V_2V_{54}}{V_1} + \frac{E_7V_{56}}{V_{15}} + \frac{E_{31}V_{57}}{V_{44}} + \frac{E_{37}V_{58}}{V_{48}} + \frac{E_1V_{55}}{V_8}\\
E_{44} &= \frac{V_3V_{54}}{V_1} + \frac{E_8V_{56}}{V_{15}} + \frac{E_{32}V_{57}}{V_{44}} + \frac{E_{38}V_{58}}{V_{48}} + \frac{E_2V_{55}}{V_8}\\
E_{45} &= \frac{V_4V_{54}}{V_1} + \frac{E_9V_{56}}{V_{15}} + \frac{E_{33}V_{57}}{V_{44}} + \frac{E_{39}V_{58}}{V_{48}} + \frac{E_3V_{55}}{V_8}\\
E_{46} &= \frac{V_5V_{54}}{V_1} + \frac{E_{10}V_{56}}{V_{15}} + \frac{E_{34}V_{57}}{V_{44}} + \frac{E_{40}V_{58}}{V_{48}} + \frac{E_4V_{55}}{V_8}\\
E_{47} &= \frac{E_{11}V_{56}}{V_{15}} + \frac{E_{35}V_{57}}{V_{44}} + \frac{E_{41}V_{58}}{V_{48}} + \frac{V_{54}V_6}{V_1} + \frac{E_5V_{55}}{V_8}\\
E_{48} &= \frac{E_{12}V_{56}}{V_{15}} + \frac{E_{36}V_{57}}{V_{44}} + \frac{E_{42}V_{58}}{V_{48}} + \frac{V_{54}V_7}{V_1} + \frac{E_6V_{55}}{V_8}
\end{aligned} \tag{2.145}$$

2.8.4 Solución de la velocidad ángulo θ_{1110i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.114):

$$\dot{\theta}_{1110i} = \frac{(d_{32}\dot{d}_{31} - d_{31}\dot{d}_{32})c\theta_{1110i}{}^2}{d_{31}{}^2} \tag{2.146}$$

Donde d_{31} y d_{32} se encuentran en las ecuaciones (2.115) y además:

$$
\begin{aligned}
\dot{d}_{31} = {} & -s(\beta_{1413i}-\theta_{54})(c\theta_{65i}(\dot{\theta}_{65i}+\dot{\theta}_{21i}s\beta_{54i})s\theta_{21i}+c\theta_{21i}(\dot{\theta}_{21i} \\
& +\dot{\theta}_{65i}s\beta_{54i})s\theta_{65i})-\dot{\theta}_{54}c(\beta_{1413i}-\theta_{54})(c\theta_{21i}c\theta_{65i}s\beta_{54i} \\
& -s\theta_{21i}s\theta_{65i})+\dot{\theta}_{43}c(\beta_{1413i}-\theta_{54})s\theta_{43}(c\beta_{54i}c\theta_{65i}s\theta_{21} \\
& -c\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i}+c\theta_{21i}s\theta_{65i}))+\dot{\theta}_{54}c\theta_{43}s(\beta_{1413i} \\
& -\theta_{54})(-c\beta_{54i}c\theta_{65i}s\theta_{21}+c\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i}+c\theta_{21i}s\theta_{65i})) \\
& -\dot{\theta}_{43}c\theta_{43}c(\beta_{1413i}-\theta_{54})(c\beta_{54i}c\theta_{21}c\theta_{65i}+s\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i} \\
& +c\theta_{21i}s\theta_{65i}))-\dot{\theta}_{54}s\theta_{43}s(\beta_{1413i}-\theta_{54})(c\beta_{54i}c\theta_{21}c\theta_{65i} \\
& +s\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i}+c\theta_{21i}s\theta_{65i}))-c(\beta_{1413i} \\
& -\theta_{54})s\theta_{43}(c\theta_{21i}(c\theta_{65i}(\dot{\theta}_{65i}+\dot{\theta}_{21i}s\beta_{54i})s\theta_{21}+\dot{\theta}_{21}c\theta_{21}s\theta_{65i}) \\
& -c\beta_{54i}(\dot{\theta}_{21}c\theta_{65i}s\theta_{21}+\dot{\theta}_{65i}c\theta_{21}s\theta_{65i})+s\theta_{21i}(\dot{\theta}_{21}c\theta_{21}c\theta_{65i}s\beta_{54i} \\
& -(\dot{\theta}_{21i}+\dot{\theta}_{65i}s\beta_{54i})s\theta_{21}s\theta_{65i}))+c\theta_{43}c(\beta_{1413i} \\
& -\theta_{54})(-\dot{\theta}_{21}s\theta_{21}(c\theta_{65i}s\beta_{54i}s\theta_{21i}+c\theta_{21i}s\theta_{65i}) \\
& +c\beta_{54i}(-\dot{\theta}_{21}c\theta_{21}c\theta_{65i}+\dot{\theta}_{65i}s\theta_{21}s\theta_{65i})+c\theta_{21}(c\theta_{21i}c\theta_{65i}(\dot{\theta}_{65i} \\
& +\dot{\theta}_{21i}s\beta_{54i})-(\dot{\theta}_{21i}+\dot{\theta}_{65i}s\beta_{54i})s\theta_{21i}s\theta_{65i}))
\end{aligned}
\tag{2.147}
$$

$$
\begin{aligned}
\dot{d}_{32} = {} & c(\beta_{1413i}-\theta_{54})(c\theta_{65i}(\dot{\theta}_{65i}+(\dot{\theta}_{21i}-\dot{\theta}_{54}c(\theta_{21}+\theta_{43}))s\beta_{54i})s\theta_{21i} \\
& +c\theta_{21i}(\dot{\theta}_{21i}-\dot{\theta}_{54}c(\theta_{21}+\theta_{43})+\dot{\theta}_{65i}s\beta_{54i})s\theta_{65i})+s(\beta_{1413i} \\
& -\theta_{54})(-c\theta_{21i}(c\theta_{65i}(\dot{\theta}_{65i}s\theta_{21}s\theta_{43}+s\beta_{54i}(\dot{\theta}_{54}+\dot{\theta}_{21i}s\theta_{21}s\theta_{43})) \\
& +(\dot{\theta}_{21}+\dot{\theta}_{43})c\theta_{43}s\theta_{21}s\theta_{65i})+s\theta_{21i}(-(\dot{\theta}_{21} \\
& +\dot{\theta}_{43})c\theta_{43}c\theta_{65i}s\beta_{54i}s\theta_{21}+(\dot{\theta}_{54}+(\dot{\theta}_{21i} \\
& +\dot{\theta}_{65i}s\beta_{54i})s\theta_{21}s\theta_{43})s\theta_{65i})+c\theta_{21}(-s\theta_{21i}((\dot{\theta}_{21} \\
& +\dot{\theta}_{43})c\theta_{65i}s\beta_{54i}s\theta_{43}+c\theta_{43}(\dot{\theta}_{21i}+\dot{\theta}_{65i}s\beta_{54i})s\theta_{65i}) \\
& +c\theta_{21i}(c\theta_{43}c\theta_{65i}(\dot{\theta}_{65i}+\dot{\theta}_{21i}s\beta_{54i})-(\dot{\theta}_{21}+\dot{\theta}_{43})s\theta_{43}s\theta_{65i}))) \\
& +c\beta_{54i}(c\theta_{43}(\dot{\theta}_{54}c(\beta_{1413i}-\theta_{54})c\theta_{65i}s\theta_{21}-s(\beta_{1413i}-\theta_{54})((\dot{\theta}_{21} \\
& +\dot{\theta}_{43})c\theta_{21}c\theta_{65i}-\dot{\theta}_{65i}s\theta_{21}s\theta_{65i}))+s\theta_{43}((\dot{\theta}_{21} \\
& +\dot{\theta}_{43})c\theta_{65i}s\theta_{21}s(\beta_{1413i}-\theta_{54})+c\theta_{21}(\dot{\theta}_{54}c(\beta_{1413i}-\theta_{54})c\theta_{65i} \\
& +\dot{\theta}_{65i}s(\beta_{1413i}-\theta_{54})s\theta_{65i})))
\end{aligned}
$$

Sustituyendo las ecuaciones $d_{31}, d_{32}, \dot{d}_{31}, \dot{d}_{32}$ en (2.146), agrupando y simplificando:

$$
\dot{\theta}_{1110i} = \frac{1}{V_{59}}\left(\dot{\theta}_{21}V_{60}+\dot{\theta}_{43}V_{61}+\dot{\theta}_{54}V_{62}+\dot{\theta}_{21i}V_{63}+\dot{\theta}_{65i}V_{64}\right) \tag{2.148}
$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.135), (2.140), (2.144) en (2.148) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$
\dot{\theta}_{1110i} = \frac{1}{V_{59}}\left(\dot{x}E_{49}+\dot{y}E_{50}+\dot{z}E_{51}+\dot{\psi}E_{52}+\dot{\theta}E_{53}+\dot{\phi}E_{54}\right) \tag{2.149}
$$

Dónde:

$$E_{49}=\frac{V_2V_{60}}{V_1}+\frac{E_7V_{62}}{V_{15}}+\frac{E_{31}V_{63}}{V_{44}}+\frac{E_{37}V_{64}}{V_{48}}+\frac{E_1V_{61}}{V_8}$$
$$E_{50}=\frac{V_3V_{60}}{V_1}+\frac{E_8V_{62}}{V_{15}}+\frac{E_{32}V_{63}}{V_{44}}+\frac{E_{38}V_{64}}{V_{48}}+\frac{E_2V_{61}}{V_8}$$
$$E_{51}=\frac{V_4V_{60}}{V_1}+\frac{E_9V_{62}}{V_{15}}+\frac{E_{33}V_{63}}{V_{44}}+\frac{E_{39}V_{64}}{V_{48}}+\frac{E_3V_{61}}{V_8} \quad (2.150)$$
$$E_{52}=\frac{V_5V_{60}}{V_1}+\frac{E_{10}V_{62}}{V_{15}}+\frac{E_{34}V_{63}}{V_{44}}+\frac{E_{40}V_{64}}{V_{48}}+\frac{E_4V_{61}}{V_8}$$
$$E_{53}=\frac{V_6V_{60}}{V_1}+\frac{E_{11}V_{62}}{V_{15}}+\frac{E_{35}V_{63}}{V_{44}}+\frac{E_{41}V_{64}}{V_{48}}+\frac{E_5V_{61}}{V_8}$$
$$E_{54}=\frac{E_{12}V_{62}}{V_{15}}+\frac{E_{36}V_{63}}{V_{44}}+\frac{E_{42}V_{64}}{V_{48}}+\frac{V_{60}V_7}{V_1}+\frac{E_6V_{61}}{V_8}$$

2.8.5 Solución de la velocidad ángulo θ_{109i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.123):

$$\dot{\theta}_{109i}=\frac{(\dot{e}_{33}e_{31}-\dot{e}_{31}e_{33})c\theta_{109i}{}^2}{e_{33}{}^2} \quad (2.151)$$

Al derivar la ecuación (2.124) se obtiene $\dot{e}_{31}\ y\ \dot{e}_{33}$, que se encuentran en el apéndice C. Sustituyendo las ecuaciones (2.124) así como $\dot{e}_{31}\ y\ \dot{e}_{33}$ en la ecuación (2.151), simplificando y agrupando:

$$\dot{\theta}_{109i}=\frac{1}{V_{65}}\left(\dot{\theta}_{21}V_{66}+\dot{\theta}_{43}V_{66}+\dot{\theta}_{54}V_{67}+\dot{\theta}_{21i}V_{68}+\dot{\theta}_{65i}V_{69}+\dot{\theta}_{1110i}V_{70}\right) \quad (2.152)$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.135), (2.140), (2.144), (2.149) en (2.152) y al agrupar en $\ddot{x},\ddot{y},\ddot{z},\ddot{\psi},\ddot{\theta},\ddot{\phi}$ se tiene:

$$\dot{\theta}_{109i}=\frac{1}{V_{65}}\left(\dot{x}E_{55}+\dot{y}E_{56}+\dot{z}E_{57}+\dot{\psi}E_{58}+\dot{\theta}E_{59}+\dot{\phi}E_{60}\right) \quad (2.153)$$

Dónde:

$$
\begin{aligned}
E_{55} &= \frac{V_2V_{66}}{V_1} + \frac{E_7V_{67}}{V_{15}} + \frac{E_{31}V_{68}}{V_{44}} + \frac{E_{37}V_{69}}{V_{48}} + \frac{E_{49}V_{70}}{V_{59}} + \frac{E_1V_{66}}{V_8} \\
E_{56} &= \frac{V_3V_{66}}{V_1} + \frac{E_8V_{67}}{V_{15}} + \frac{E_{32}V_{68}}{V_{44}} + \frac{E_{38}V_{69}}{V_{48}} + \frac{E_{50}V_{70}}{V_{59}} + \frac{E_2V_{66}}{V_8} \\
E_{57} &= \frac{V_4V_{66}}{V_1} + \frac{E_9V_{67}}{V_{15}} + \frac{E_{33}V_{68}}{V_{44}} + \frac{E_{39}V_{69}}{V_{48}} + \frac{E_{51}V_{70}}{V_{59}} + \frac{E_3V_{66}}{V_8} \\
E_{58} &= \frac{V_5V_{66}}{V_1} + \frac{E_{10}V_{67}}{V_{15}} + \frac{E_{34}V_{68}}{V_{44}} + \frac{E_{40}V_{69}}{V_{48}} + \frac{E_{52}V_{70}}{V_{59}} + \frac{E_4V_{66}}{V_8} \\
E_{59} &= \frac{V_6V_{66}}{V_1} + \frac{E_{11}V_{67}}{V_{15}} + \frac{E_{35}V_{68}}{V_{44}} + \frac{E_{41}V_{69}}{V_{48}} + \frac{E_{53}V_{70}}{V_{59}} + \frac{E_5V_{66}}{V_8} \\
E_{60} &= \frac{E_{12}V_{67}}{V_{15}} + \frac{E_{36}V_{68}}{V_{44}} + \frac{E_{42}V_{69}}{V_{48}} + \frac{V_{66}V_7}{V_1} + \frac{E_{54}V_{70}}{V_{59}} + \frac{E_6V_{66}}{V_8}
\end{aligned} \tag{2.154}
$$

2.8.6 Solución de la velocidad ángulo θ98i de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.131):

$$
\dot{\theta}_{98i} = -\frac{(-\dot{f}_{21}f_{11} + \dot{f}_{11}f_{21})c\theta_{98i}{}^2}{f_{11}{}^2} \tag{2.155}
$$

Al derivar $f_{11}\ y\ f_{21}$ se obtiene $\dot{f}_{11}\ y\ \dot{f}_{21}$, que se encuentran en el apéndice C. Sustituyendo las ecuaciones $f_{11}\ y\ f_{21}$ así como $\dot{f}_{11}\ y\ \dot{f}_{21}$ en la ecuación (2.155), simplificando y agrupando:

$$
\begin{aligned}
\dot{\theta}_{98i} = \frac{1}{V_{71}}\Big(&\dot{\theta}_{21}V_{72} + \dot{\theta}_{43}V_{73} + \dot{\theta}_{54}V_{74} + \dot{\theta}_{21i}V_{75} + \dot{\theta}_{65i}V_{76} + \dot{\theta}_{76i}V_{77} \\
&+ \dot{\theta}_{1110i}V_{78} + \dot{\theta}_{109i}V_{79}\Big)
\end{aligned} \tag{2.156}
$$

Donde los coeficientes se encuentran en el apéndice D. Finalmente sustituyendo las ecuaciones (2.51), (2.55), (2.59), (2.135), (2.140), (2.144), (2.149), (2.153) en (2.156) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$
\dot{\theta}_{98i} = \frac{1}{V_{71}}\left(\dot{x}E_{61} + \dot{y}E_{62} + \dot{z}E_{63} + \dot{\psi}E_{64} + \dot{\theta}E_{65} + \dot{\phi}E_{66}\right) \tag{2.157}
$$

Dónde:

$$
\begin{aligned}
E_{61} &= \frac{V_2V_{72}}{V_1}+\frac{E_7V_{74}}{V_{15}}+\frac{E_{31}V_{75}}{V_{44}}+\frac{E_{37}V_{76}}{V_{48}}+\frac{E_{43}V_{77}}{V_{53}}+\frac{E_{49}V_{78}}{V_{59}}+\frac{E_{55}V_{79}}{V_{65}}+\frac{E_1V_{73}}{V_8}\\
E_{62} &= \frac{V_3V_{72}}{V_1}+\frac{E_8V_{74}}{V_{15}}+\frac{E_{32}V_{75}}{V_{44}}+\frac{E_{38}V_{76}}{V_{48}}+\frac{E_{44}V_{77}}{V_{53}}+\frac{E_{50}V_{78}}{V_{59}}+\frac{E_{56}V_{79}}{V_{65}}+\frac{E_2V_{73}}{V_8}\\
E_{63} &= \frac{V_4V_{72}}{V_1}+\frac{E_9V_{74}}{V_{15}}+\frac{E_{33}V_{75}}{V_{44}}+\frac{E_{39}V_{76}}{V_{48}}+\frac{E_{45}V_{77}}{V_{53}}+\frac{E_{51}V_{78}}{V_{59}}+\frac{E_{57}V_{79}}{V_{65}}+\frac{E_3V_{73}}{V_8}\\
E_{64} &= \frac{V_5V_{72}}{V_1}+\frac{E_{10}V_{74}}{V_{15}}+\frac{E_{34}V_{75}}{V_{44}}+\frac{E_{40}V_{76}}{V_{48}}+\frac{E_{46}V_{77}}{V_{53}}+\frac{E_{52}V_{78}}{V_{59}}+\frac{E_{58}V_{79}}{V_{65}}+\frac{E_4V_{73}}{V_8}\\
E_{65} &= \frac{V_6V_{72}}{V_1}+\frac{E_{11}V_{74}}{V_{15}}+\frac{E_{35}V_{75}}{V_{44}}+\frac{E_{41}V_{76}}{V_{48}}+\frac{E_{47}V_{77}}{V_{53}}+\frac{E_{53}V_{78}}{V_{59}}+\frac{E_{59}V_{79}}{V_{65}}+\frac{E_5V_{73}}{V_8}\\
E_{66} &= \frac{V_7V_{72}}{V_1}+\frac{E_{12}V_{74}}{V_{15}}+\frac{E_{36}V_{75}}{V_{44}}+\frac{E_{42}V_{76}}{V_{48}}+\frac{E_{48}V_{77}}{V_{53}}+\frac{E_{54}V_{78}}{V_{59}}+\frac{E_{60}V_{79}}{V_{65}}+\frac{E_6V_{73}}{V_8}
\end{aligned}
\tag{2.158}
$$

2.9 Calculo de la aceleración para las cadenas i.

Para el cálculo de la aceleración se tiene como variables de entrada la aceleración del efector $(\ddot{x},\ddot{y},\ddot{z},\ddot{\psi},\ddot{\theta},\ddot{\phi})$ y las variables a calcular son las aceleraciones de los ángulos de las cadenas i es decir $\ddot{\theta}_{21i},\ddot{\theta}_{65i},\ddot{\theta}_{76i},\ddot{\theta}_{1110i},\ddot{\theta}_{109i},\ddot{\theta}_{98i}$. Para esta sección se asume que la posición y la velocidad de los ángulos son conocidas debido a que se calcularon en secciones anteriores.

2.9.1 Solución de la aceleración ángulo θ_{21i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.132):

$$
\ddot{\theta}_{21i}=\frac{(-\ddot{C}_{1i}-\ddot{A}_{1i}c\theta_{21i}-2\dot{B}_{1i}\dot{\theta}_{21i}c\theta_{21i}+A_{1i}\dot{\theta}_{21i}^{\,2}c\theta_{21i}-\ddot{B}_{1i}s\theta_{21i}+2\dot{A}_{1i}\dot{\theta}_{21i}s\theta_{21i}+B_{1i}\dot{\theta}_{21i}^{\,2}s\theta_{21i})}{(B_{1i}c\theta_{21i}-A_{1i}s\theta_{21i})}
\tag{2.159}
$$

Dónde:

$$
\begin{aligned}
\ddot{A}_{1i} = x_{43i}(&-2x_{32}(\dot{\theta}_{21}^{\,2}c\theta_{21}+\ddot{\theta}_{21}s\theta_{21})-2z_{1312i}((\ddot{\theta}_{21}+\ddot{\theta}_{43})c\theta_{21}+\theta_{43}\\
&-(\dot{\theta}_{21}+\dot{\theta}_{43})^2s\theta_{21}+\theta_{43})\\
&-x_{1211i}(-(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54})^2c\beta_{1413i}-\theta_{21}-\theta_{43}-\theta_{54}+(\ddot{\theta}_{21}\\
&+\ddot{\theta}_{43}+\ddot{\theta}_{54})s\beta_{1413i}-\theta_{21}-\theta_{43}-\theta_{54})\\
&+x_{65}(-(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54})^2c\theta_{21}+\theta_{43}-\theta_{54}-(\ddot{\theta}_{21}+\ddot{\theta}_{43}\\
&-\ddot{\theta}_{54})s\theta_{21}+\theta_{43}-\theta_{54})-x_{1211i}(-(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54})^2c\beta_{1413i}\\
&+\theta_{21}+\theta_{43}-\theta_{54}-(\ddot{\theta}_{21}+\ddot{\theta}_{43}-\ddot{\theta}_{54})s\beta_{1413i}+\theta_{21}+\theta_{43}\\
&-\theta_{54})+x_{65}(-(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54})^2c\theta_{21}+\theta_{43}+\theta_{54}-(\ddot{\theta}_{21}\\
&+\ddot{\theta}_{43}+\ddot{\theta}_{54})s\theta_{21}+\theta_{43}+\theta_{54}))\\
\ddot{B}_{1i} = -2x_{43i}(&\ddot{\theta}_{54}x_{1211i}c\beta_{1413i}-\theta_{54}-\ddot{\theta}_{54}x_{65}c\theta_{54}+\dot{\theta}_{54}^{\,2}(x_{1211i}s\beta_{1413i}\\
&-\theta_{54}+x_{65}s\theta_{54}))
\end{aligned}
\tag{2.160}
$$

$$
\begin{aligned}
\ddot{C}_{1i} = {} & 2x_{32}z_{10}(\ddot{\theta}_{21}c\theta_{21} - \dot{\theta}_{21}{}^{2}s\theta_{21}) + 2x_{32}z_{32i}(\ddot{\theta}_{21}c\theta_{21} - \dot{\theta}_{21}{}^{2}s\theta_{21}) \\
& + 2x_{32}z_{1312i}(\ddot{\theta}_{43}c\theta_{43} - \dot{\theta}_{43}{}^{2}s\theta_{43}) + 2z_{1312i}(z_{10} \\
& + z_{32i})(-(\dot{\theta}_{21} + \dot{\theta}_{43})^{2}c\theta_{21} + \theta_{43} - (\ddot{\theta}_{21} + \ddot{\theta}_{43})s\theta_{21} + \theta_{43}) \\
& - 2\text{signo}x_{1211i}y_{10i}(\ddot{\theta}_{54}c\beta_{1413i} - \theta_{54} + \dot{\theta}_{54}{}^{2}s\beta_{1413i} - \theta_{54}) \\
& + x_{1211i}x_{32}(-(\dot{\theta}_{43} + \dot{\theta}_{54})^{2}c\beta_{1413i} - \theta_{43} - \theta_{54} + (\ddot{\theta}_{43} \\
& + \ddot{\theta}_{54})s\beta_{1413i} - \theta_{43} - \theta_{54}) + x_{1211i}z_{10}(-(\ddot{\theta}_{21} + \ddot{\theta}_{43} \\
& + \ddot{\theta}_{54})c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^{2}s\beta_{1413i} \\
& - \theta_{21} - \theta_{43} - \theta_{54}) + x_{1211i}z_{32i}(-(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54})c\beta_{1413i} \\
& - \theta_{21} - \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^{2}s\beta_{1413i} - \theta_{21} - \theta_{43} \\
& - \theta_{54}) - x_{32}x_{65}(-(\dot{\theta}_{43} - \dot{\theta}_{54})^{2}c\theta_{43} - \theta_{54} + (-\ddot{\theta}_{43} + \ddot{\theta}_{54})s\theta_{43} \\
& - \theta_{54}) + x_{1211i}x_{32}(-(\dot{\theta}_{43} - \dot{\theta}_{54})^{2}c\beta_{1413i} + \theta_{43} - \theta_{54} + (-\ddot{\theta}_{43} \\
& + \ddot{\theta}_{54})s\beta_{1413i} + \theta_{43} - \theta_{54}) + x_{65}z_{10}((\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})c\theta_{21} \\
& + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^{2}s\theta_{21} + \theta_{43} - \theta_{54}) \\
& + x_{65}z_{32i}((\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})c\theta_{21} + \theta_{43} - \theta_{54} \\
& - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^{2}s\theta_{21} + \theta_{43} - \theta_{54}) - x_{1211i}z_{10}((\ddot{\theta}_{21} + \ddot{\theta}_{43} \\
& - \ddot{\theta}_{54})c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^{2}s\beta_{1413i} \\
& + \theta_{21} + \theta_{43} - \theta_{54}) - x_{1211i}z_{32i}((\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})c\beta_{1413i} \\
& + \theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^{2}s\beta_{1413i} + \theta_{21} + \theta_{43} \\
& - \theta_{54}) + 2\text{signo}x_{65}y_{10i}(\ddot{\theta}_{54}c\theta_{54} - \dot{\theta}_{54}{}^{2}s\theta_{54}) \\
& - x_{32}x_{65}(-(\dot{\theta}_{43} + \dot{\theta}_{54})^{2}c\theta_{43} + \theta_{54} - (\ddot{\theta}_{43} + \ddot{\theta}_{54})s\theta_{43} + \theta_{54}) \\
& + x_{65}z_{10}((\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54})c\theta_{21} + \theta_{43} + \theta_{54} \\
& - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^{2}s\theta_{21} + \theta_{43} + \theta_{54}) + x_{65}z_{32i}((\ddot{\theta}_{21} + \ddot{\theta}_{43} \\
& + \ddot{\theta}_{54})c\theta_{21} + \theta_{43} + \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^{2}s\theta_{21} + \theta_{43} \\
& + \theta_{54})
\end{aligned}
$$

Sustituyendo $A_{1i}, B_{1i}, C_{1i}, \dot{A}_{1i}, \dot{B}_{1i}, \dot{C}_{1i}, \ddot{A}_{1i}, \ddot{B}_{1i}, \ddot{C}_{1i}$ en la ecuación (2.160) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{21i} = \frac{1}{V_{44}}\left(\ddot{x}G_{47} + \ddot{y}G_{48} + \ddot{z}G_{49} + \ddot{\psi}G_{50} + \ddot{\theta}G_{51} + \ddot{\phi}G_{52} + G_{53}\right) \qquad (2.161)$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.9.2 Solución de la aceleración ángulo θ_{65i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.137):

$$\ddot{\theta}_{65i} = \frac{-\ddot{Z}_i c\theta_{65i} - 2\dot{X}_i\dot{\theta}_{65i}c\theta_{65i} + \dot{\theta}_{65i}{}^{2}Z_i c\theta_{65i} - \ddot{X}_i s\theta_{65i} + 2\dot{Z}_i\dot{\theta}_{65i}s\theta_{65i} + \dot{\theta}_{65i}{}^{2}X_i s\theta_{65i}}{X_i c\theta_{65i} - Z_i s\theta_{65i}} \qquad (2.162)$$

Dónde:

$$\ddot{X}_i = \frac{1}{2}(-2x_{32}(\dot{\theta}_{21}^{\ 2}c\theta_{21} + \ddot{\theta}_{21}s\theta_{21}) + 2x_{43i}(\dot{\theta}_{21i}^{\ 2}c\theta_{21i} + \ddot{\theta}_{21i}s\theta_{21i})$$
$$- 2z_{1312i}((\ddot{\theta}_{21} + \ddot{\theta}_{43})c\theta_{21} + \theta_{43} - (\dot{\theta}_{21} + \dot{\theta}_{43})^2 s\theta_{21} + \theta_{43})$$
$$- x_{1211i}(-(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} + (\ddot{\theta}_{21}$$
$$+ \ddot{\theta}_{43} + \ddot{\theta}_{54})s\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54})$$
$$+ x_{65}(-(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 c\theta_{21} + \theta_{43} - \theta_{54} - (\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$- \ddot{\theta}_{54})s\theta_{21} + \theta_{43} - \theta_{54}) - x_{1211i}(-(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 c\beta_{1413i}$$
$$+ \theta_{21} + \theta_{43} - \theta_{54} - (\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})s\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54})$$
$$+ x_{65}(-(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 c\theta_{21} + \theta_{43} + \theta_{54} - (\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$+ \ddot{\theta}_{54})s\theta_{21} + \theta_{43} + \theta_{54}))$$
$$\ddot{Y}_i = -c\beta_{54i}(\ddot{\theta}_{21i}x_{43i}c\theta_{21i} + \ddot{\theta}_{54}x_{1211i}c\beta_{1413i} - \theta_{54} - \ddot{\theta}_{54}x_{65}c\theta_{54}$$
$$- \dot{\theta}_{21i}^{\ 2}x_{43i}s\theta_{21i} + \dot{\theta}_{54}^{\ 2}x_{1211i}s\beta_{1413i} - \theta_{54} + \dot{\theta}_{54}^{\ 2}x_{65}s\theta_{54})$$
$$+ \frac{1}{2}s\beta_{54i}(-2\ddot{\theta}_{21}x_{32}c\theta_{21} + 2\dot{\theta}_{21}^{\ 2}x_{32}s\theta_{21}$$
$$- 2z_{1312i}(-(\dot{\theta}_{21} + \dot{\theta}_{43})^2 c\theta_{21} + \theta_{43} - (\ddot{\theta}_{21} + \ddot{\theta}_{43})s\theta_{21} + \theta_{43})$$
$$- x_{1211i}(-(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54})c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54} \qquad (2.163)$$
$$- (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 s\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54}) - x_{65}((\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$- \ddot{\theta}_{54})c\theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 s\theta_{21} + \theta_{43} - \theta_{54})$$
$$+ x_{1211i}((\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}$$
$$- (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 s\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}) - x_{65}((\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$+ \ddot{\theta}_{54})c\theta_{21} + \theta_{43} + \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 s\theta_{21} + \theta_{43} + \theta_{54}))$$
$$\ddot{Z}_i = s\beta_{54i}(\ddot{\theta}_{21i}x_{43i}c\theta_{21i} + \ddot{\theta}_{54}x_{1211i}c\beta_{1413i} - \theta_{54} - \ddot{\theta}_{54}x_{65}c\theta_{54}$$
$$- \dot{\theta}_{21i}^{\ 2}x_{43i}s\theta_{21i} + \dot{\theta}_{54}^{\ 2}x_{1211i}s\beta_{1413i} - \theta_{54} + \dot{\theta}_{54}^{\ 2}x_{65}s\theta_{54})$$
$$+ \frac{1}{2}c\beta_{54i}(-2\ddot{\theta}_{21}x_{32}c\theta_{21} + 2\dot{\theta}_{21}^{\ 2}x_{32}s\theta_{21}$$
$$- 2z_{1312i}(-(\dot{\theta}_{21} + \dot{\theta}_{43})^2 c\theta_{21} + \theta_{43} - (\ddot{\theta}_{21} + \ddot{\theta}_{43})s\theta_{21} + \theta_{43})$$
$$- x_{1211i}(-(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54})c\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54}$$
$$- (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 s\beta_{1413i} - \theta_{21} - \theta_{43} - \theta_{54}) - x_{65}((\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$- \ddot{\theta}_{54})c\theta_{21} + \theta_{43} - \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 s\theta_{21} + \theta_{43} - \theta_{54})$$
$$+ x_{1211i}((\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54})c\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}$$
$$- (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54})^2 s\beta_{1413i} + \theta_{21} + \theta_{43} - \theta_{54}) - x_{65}((\ddot{\theta}_{21} + \ddot{\theta}_{43}$$
$$+ \ddot{\theta}_{54})c\theta_{21} + \theta_{43} + \theta_{54} - (\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54})^2 s\theta_{21} + \theta_{43} + \theta_{54}))$$

Sustituyendo $X_i, Z_i, \dot{X}_i, \dot{Z}_i, \ddot{X}_i, \ddot{Z}_i$ en la ecuación (2.162) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{65i} = \frac{1}{V_{48}}\left(\ddot{x}G_{54} + \ddot{y}G_{55} + \ddot{z}G_{56} + \ddot{\psi}G_{57} + \ddot{\theta}G_{58} + \ddot{\phi}G_{59} + G_{60}\right) \tag{2.164}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.9.3 Solución de la aceleración ángulo θ_{76i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.142):

$$\begin{aligned}\ddot{\theta}_{76i} = -\frac{1}{X^3}c\theta_{76i}{}^2((2\dot{X}\dot{Y}X - 2\dot{X}^2Y + X(-\ddot{Y}X + \ddot{X}Y + \dot{\theta}_{65i}{}^2 XY))c\theta_{65i} \\ + X((2\dot{Y}\dot{\theta}_{65i}X - 2\dot{X}\dot{\theta}_{65i}Y + \ddot{\theta}_{65i}XY)s\theta_{65i} \\ + 2\dot{\theta}_{76i}{}^2 X^2\mathrm{sec}\theta_{76i}{}^2\tan\theta_{76i}))\end{aligned} \tag{2.165}$$

Sustituyendo $X_i, Y_i, \dot{X}_i, \dot{Y}_i, \ddot{X}_i, \ddot{Y}_i$ en la ecuación (2.165) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{76i} = \frac{1}{G_{61}}\left(\ddot{x}G_{62} + \ddot{y}G_{63} + \ddot{z}G_{64} + \ddot{\psi}G_{65} + \ddot{\theta}G_{66} + \ddot{\phi}G_{67} + G_{68}\right) \tag{2.166}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.9.4 Solución de la aceleración ángulo θ_{1110i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.146):

$$\begin{aligned}\ddot{\theta}_{1110i} = -\frac{1}{d_{31}{}^3}c\theta_{1110i}{}^2(2d_{32}\dot{d}_{31}{}^2 - 2d_{31}\dot{d}_{31}\dot{d}_{32} - d_{31}d_{32}\ddot{d}_{31} + d_{31}{}^2\ddot{d}_{32} \\ + 2d_{31}{}^3\dot{\theta}_{1110i}{}^2\mathrm{sec}\theta_{1110i}{}^2\tan\theta_{1110i})\end{aligned} \tag{2.167}$$

Al derivar las ecuaciones (2.147) se obtienen $\ddot{d}_{31}$ y $\ddot{d}_{32}$, que se encuentran en el apéndice C. Sustituyendo $d_{31}, d_{32}, \dot{d}_{31}, \dot{d}_{32}, \ddot{d}_{31}, \ddot{d}_{32}$ en la ecuación (2.167) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{1110i} = \frac{1}{G_{69}}\left(\ddot{x}G_{70} + \ddot{y}G_{71} + \ddot{z}G_{72} + \ddot{\psi}G_{73} + \ddot{\theta}G_{74} + \ddot{\phi}G_{75} + G_{76}\right) \tag{2.168}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.9.5 Solución de la aceleración ángulo θ_{109i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.151):

$$\ddot{\theta}_{109i} = -\frac{1}{e_{33}{}^{3}} c\theta_{109i}{}^{2}(2\dot{e}_{33}{}^{2}e_{31} - 2\dot{e}_{31}\dot{e}_{33}e_{33} + e_{33}(-\ddot{e}_{33}e_{31} + \ddot{e}_{31}e_{33}) + 2\dot{\theta}_{109i}{}^{2}e_{33}{}^{3}\sec\theta_{109i}{}^{2}\tan\theta_{109i}) \tag{2.169}$$

Donde $\ddot{e}_{31}$ y $\ddot{e}_{33}$ se obtienen al derivar $\dot{e}_{31}$ y $\dot{e}_{33}$ y se encuentran en el apéndice C. Sustituyendo $e_{31}, e_{33}, \dot{e}_{31}, \dot{e}_{33}, \ddot{e}_{31}, \ddot{e}_{33}$ en la ecuación (2.169) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{109i} = \frac{1}{G_{77}}\left(\ddot{x}G_{78} + \ddot{y}G_{79} + \ddot{z}G_{80} + \ddot{\psi}G_{81} + \ddot{\theta}G_{82} + \ddot{\phi}G_{83} + G_{84}\right) \tag{2.170}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

2.9.6 Solución de la aceleración ángulo θ_{109i} de las cadenas i.

Se deriva con respecto al tiempo la ecuación (2.155):

$$\ddot{\theta}_{98i} = \frac{c\theta_{98i}{}^{2}(-2\dot{f}_{11}\dot{f}_{21}f_{11} + \ddot{f}_{21}f_{11}{}^{2} + 2\dot{f}_{11}{}^{2}f_{21} - \ddot{f}_{11}f_{11}f_{21} - 2\dot{\theta}_{98i}{}^{2}f_{11}{}^{3}\sec\theta_{98i}{}^{2}\tan\theta_{98i})}{f_{11}{}^{3}} \tag{2.171}$$

Donde $\ddot{f}_{11}$ y $\ddot{f}_{21}$ se obtienen al derivar $\dot{f}_{11}$ y $\dot{f}_{21}$ y se encuentran en el apéndice C. Sustituyendo $f_{11}, f_{21}, \dot{f}_{11}, \dot{f}_{21}, \ddot{f}_{11}, \ddot{f}_{21}$ en la ecuación (2.169) y al agrupar en $\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$ se tiene:

$$\ddot{\theta}_{98i} = \frac{1}{G_{85}}\left(\ddot{x}G_{86} + \ddot{y}G_{87} + \ddot{z}G_{88} + \ddot{\psi}G_{89} + \ddot{\theta}G_{90} + \ddot{\phi}G_{91} + G_{92}\right) \tag{2.172}$$

Donde los coeficientes de la ecuación se encuentran en el apéndice E.

Capítulo 3
Análisis Dinámico Formulación Euler-Lagrange

3.1 Introducción

En este capítulo, se considera la dinámica del robot, con el fin de poder determinar los torques aplicados por los actuadores en los eslabones de entrada para que el efector final alcance una trayectoria dada.

El método de Euler-Lagrange formula ecuaciones de movimiento usando un conjunto de coordenadas generalizadas (Spong et al., 1989). Esto permite eliminar todas o algunas de las fuerzas de restricción y permite manejar desplazamientos tanto desplazamientos lineales como angulares con un solo tipo de coordenadas. Con el entendimiento de la dinámica del manipulador, es posible diseñar un controlador con mejores características de ejecución que las realizadas con los típicos encontrados usando métodos heurísticos después de que el manipulador ha sido construido.
En este capítulo se empleará la siguiente notación:

La función Lagrangiana es definida como la diferencia entre la energía cinética y la energía potencial de un sistema como:

$$L = K - U \qquad 3.1$$

Donde K es la energía cinética definida como:

$$K = \frac{1}{2} m\boldsymbol{v}^T\boldsymbol{v} + \boldsymbol{\omega}^T\boldsymbol{I}\boldsymbol{\omega} \qquad 3.2$$

Y la energía potencial como:

$$U = -m\boldsymbol{g}^T\boldsymbol{r}_G \qquad 3.3$$

La energía cinética depende de la velocidad de los eslabones del manipulador, mientras que la energía potencial depende únicamente de la localización de los eslabones. La ecuación de Lagrange de movimiento es formulada en términos de la función Lagrangiana como:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j \qquad 3.4$$

El término Q_j es conocido como el vector de fuerzas generalizadas y se obtendrá a partir de expresiones, que involucren los torques y las coordenadas generalizadas.

3.2 Velocidad de Centros de Gravedad y Velocidad Angular para cadena 0

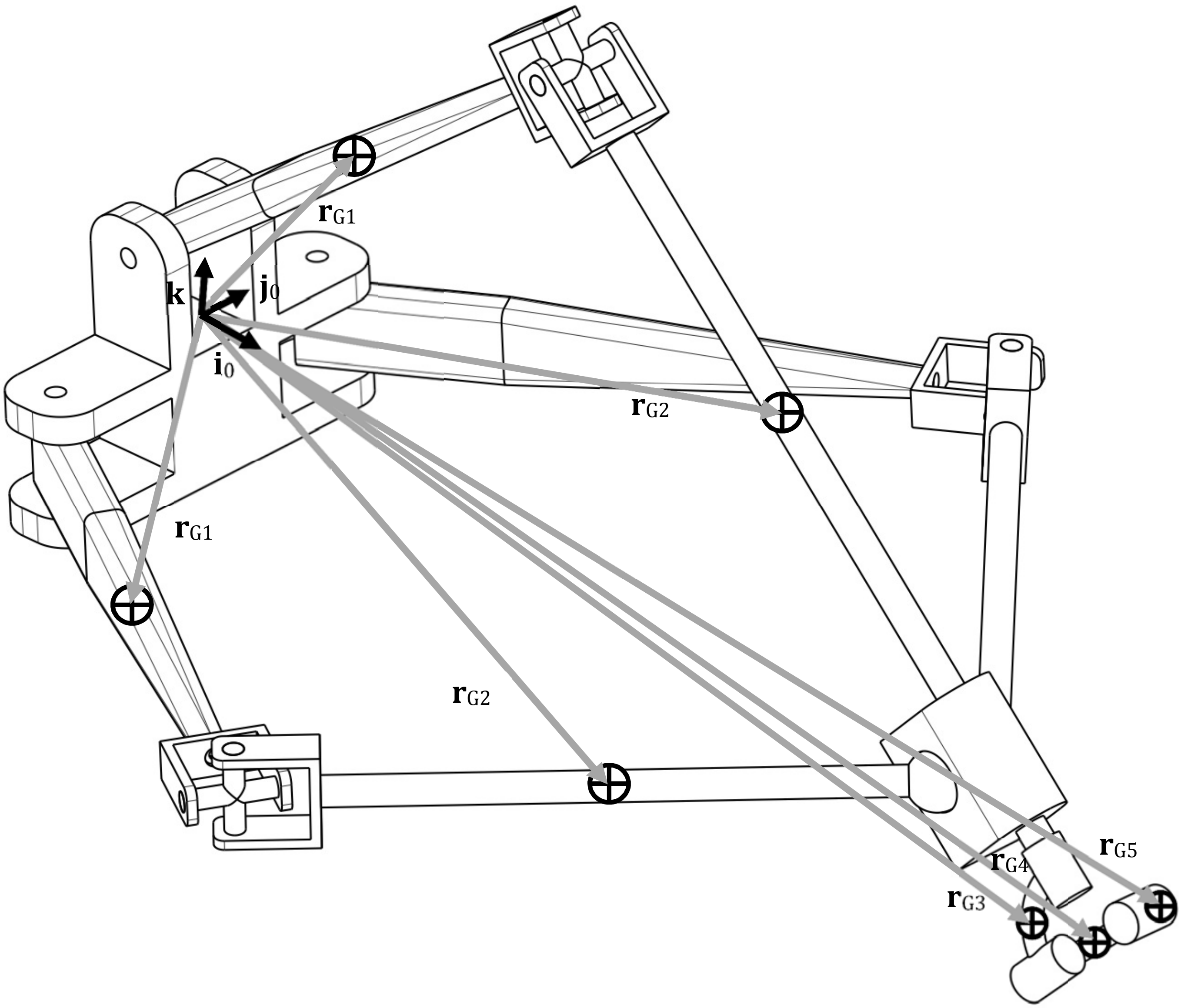


Figura 3-1 Centros de gravedad a partir de la base inercial.

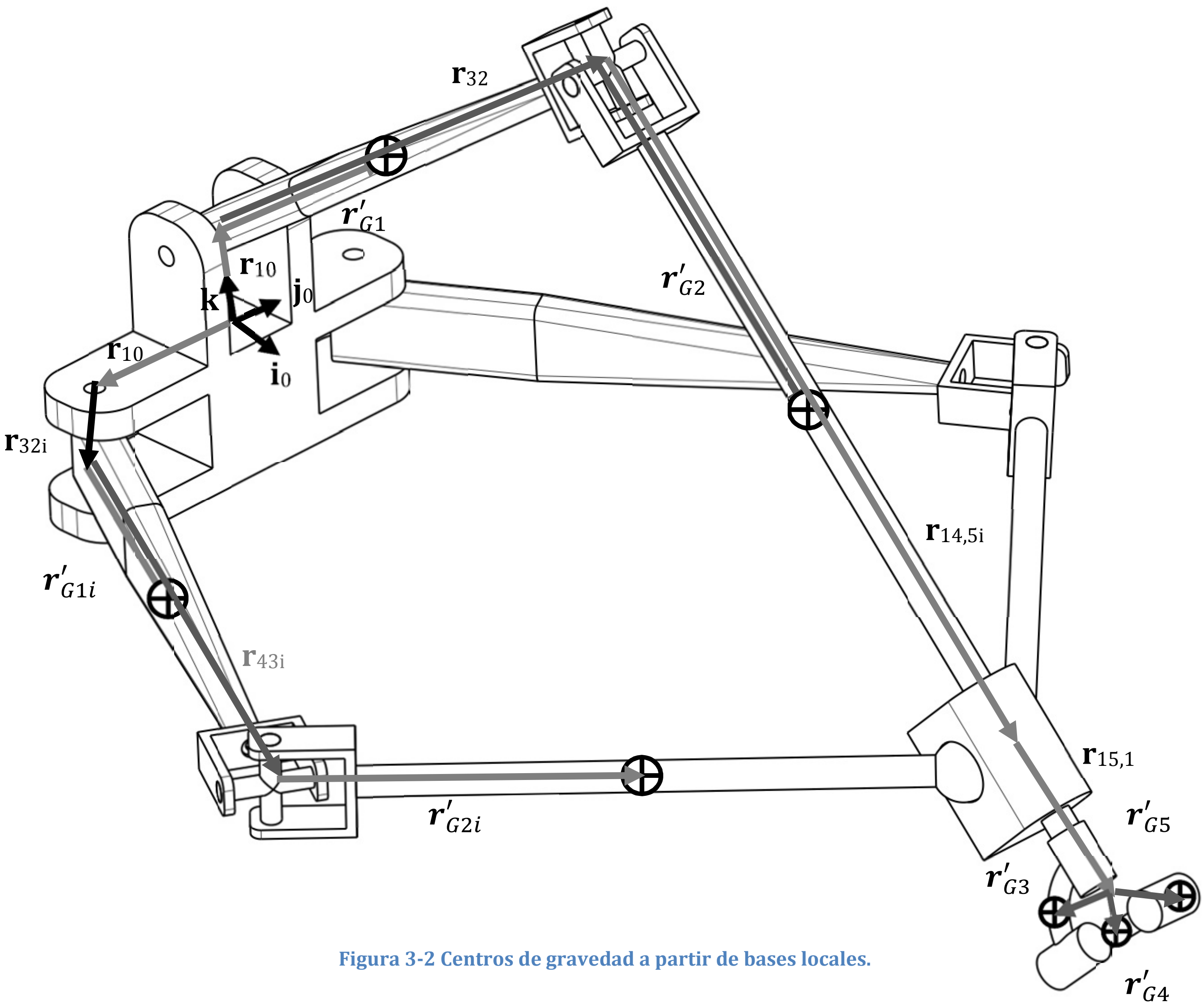


Figura 3-2 Centros de gravedad a partir de bases locales.

Con base en las figuras 3-1 y 3-2 se construye las ecuaciones vectoriales para la posición de los centros de gravedad de la cadena 0:

$$\boldsymbol{r}^0_{G1} = \boldsymbol{r}^0_{10} + \boldsymbol{r}^0_{G1'}$$
$$\boldsymbol{r}^0_{G2} = \boldsymbol{r}^0_{10} + \boldsymbol{r}^0_{32} + \boldsymbol{r}^0_{G2'}$$
$$\boldsymbol{r}^0_{G3} = \boldsymbol{r}^0_{10} + \boldsymbol{r}^0_{32} + \boldsymbol{r}^0_{14,5i} + \boldsymbol{r}^0_{15,14i} + \boldsymbol{r}^0_{G3'}$$
$$\boldsymbol{r}^0_{G4} = \boldsymbol{r}^0_{10} + \boldsymbol{r}^0_{32} + \boldsymbol{r}^0_{14,5i} + \boldsymbol{r}^0_{15,14i} + \boldsymbol{r}^0_{G4'}$$
$$\boldsymbol{r}^0_{G5} = \boldsymbol{r}^0_{10} + \boldsymbol{r}^0_{32} + \boldsymbol{r}^0_{14,5i} + \boldsymbol{r}^0_{15,14i} + \boldsymbol{r}^0_{G5'}$$

$$\boldsymbol{r}^0_{10} = z_{10}\boldsymbol{k}_0$$
$$\boldsymbol{r}^0_{32} = \boldsymbol{R}^0_2\boldsymbol{r}^2_{32}$$
$$\boldsymbol{r}^0_{14,5i} = \boldsymbol{R}^0_5\boldsymbol{r}^5_{14,5i}$$

$$\boldsymbol{r}_{15,14i}^{0} = \boldsymbol{R}_{14}^{0}\boldsymbol{r}_{15,14i}^{14}$$

$$\boldsymbol{r}_{G1'}^{0} = \boldsymbol{R}_{2}^{0}\boldsymbol{r}_{G1'}^{2}$$
$$\boldsymbol{r}_{G2'}^{0} = \boldsymbol{R}_{5}^{0}\boldsymbol{r}_{G2'}^{5}$$
$$\boldsymbol{r}_{G3'}^{0} = \boldsymbol{R}_{16}^{0}\boldsymbol{r}_{G3'}^{16}$$
$$\boldsymbol{r}_{G4'}^{0} = \boldsymbol{R}_{17}^{0}\boldsymbol{r}_{G4'}^{17}$$
$$\boldsymbol{r}_{G5'}^{0} = \boldsymbol{R}_{18}^{0}\boldsymbol{r}_{G5'}^{18}$$

$$\boldsymbol{R}_{2}^{0} = \boldsymbol{R}_{y}(\theta_{21})$$
$$\boldsymbol{R}_{5}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})\boldsymbol{R}_{z}(\theta_{54})$$
$$\boldsymbol{R}_{16}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})\boldsymbol{R}_{z}(\theta_{54})\boldsymbol{R}_{x}(\theta_{16,15})$$
$$\boldsymbol{R}_{17}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})\boldsymbol{R}_{z}(\theta_{54})\boldsymbol{R}_{x}(\theta_{16,15})\boldsymbol{R}_{z}(\theta_{17,16})$$
$$\boldsymbol{R}_{18}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})\boldsymbol{R}_{z}(\theta_{54})\boldsymbol{R}_{x}(\theta_{16,15})\boldsymbol{R}_{z}(\theta_{17,16})\boldsymbol{R}_{x}(\theta_{18,17})$$

$$\boldsymbol{r}_{32}^{2} = [x_{32}, 0,0]^{T}$$
$$\boldsymbol{r}_{14,5i}^{5} = [x_{14,5i}, 0,0]^{T}$$
$$\boldsymbol{r}_{15,14i}^{14} = [x_{15,14i}, 0,0]^{T}$$

$$\boldsymbol{r}_{G1'}^{2} = [x_{G1}, y_{G1}, z_{G1}]^{T}$$
$$\boldsymbol{r}_{G2'}^{5} = [x_{G2}, y_{G2}, z_{G2}]^{T}$$
$$\boldsymbol{r}_{G3'}^{16} = [x_{G3}, y_{G3}, z_{G3}]^{T}$$
$$\boldsymbol{r}_{G4'}^{17} = [x_{G4}, y_{G4}, z_{G4}]^{T}$$
$$\boldsymbol{r}_{G5'}^{18} = [x_{G5}, y_{G5}, z_{G5}]^{T}$$

Ahora se definen las ecuaciones para la velocidad de los centros de gravedad para la cadena 0:

$$\boldsymbol{v}_{G1}^{0} = \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{G1'}^{0}$$
$$\boldsymbol{v}_{G2}^{0} = \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{32}^{0} + \boldsymbol{v}_{G2'}^{0}$$
$$\boldsymbol{v}_{G3}^{0} = \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{32}^{0} + \boldsymbol{v}_{14,5i}^{0} + \boldsymbol{v}_{15,14i}^{0} + \boldsymbol{v}_{G3'}^{0}$$
$$\boldsymbol{v}_{G4}^{0} = \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{32}^{0} + \boldsymbol{v}_{14,5i}^{0} + \boldsymbol{v}_{15,14i}^{0} + \boldsymbol{v}_{G4'}^{0}$$
$$\boldsymbol{v}_{G5}^{0} = \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{32}^{0} + \boldsymbol{v}_{14,5i}^{0} + \boldsymbol{v}_{15,14i}^{0} + \boldsymbol{v}_{G5'}^{0}$$

$$\boldsymbol{v}_{10}^{0} = 0$$
$$\boldsymbol{v}_{32}^{0} = \boldsymbol{\omega}_{2}^{0} \times \boldsymbol{r}_{32}^{0}$$
$$\boldsymbol{v}_{14,5i}^{0} = \boldsymbol{\omega}_{5}^{0} \times \boldsymbol{r}_{14,5i}^{0}$$
$$\boldsymbol{v}_{15,14i}^{0} = \boldsymbol{\omega}_{5}^{0} \times \boldsymbol{r}_{15,14i}^{0}$$

$$\boldsymbol{v}_{G1'}^{0} = \boldsymbol{\omega}_{2}^{0} \times \boldsymbol{r}_{G1'}^{0}$$
$$\boldsymbol{v}_{G2'}^{0} = \boldsymbol{\omega}_{5}^{0} \times \boldsymbol{r}_{G2'}^{0}$$
$$\boldsymbol{v}_{G3'}^{0} = \boldsymbol{\omega}_{16}^{0} \times \boldsymbol{r}_{G3'}^{0}$$

$$\boldsymbol{v}_{G4'}^{0} = \boldsymbol{\omega}_{17}^{0} \times \boldsymbol{r}_{G4'}^{0}$$
$$\boldsymbol{v}_{G5'}^{0} = \boldsymbol{\omega}_{18}^{0} \times \boldsymbol{r}_{G5'}^{0}$$

$$\boldsymbol{\omega}_{2}^{0} = \boldsymbol{\omega}_{21}^{0}$$
$$\boldsymbol{\omega}_{5}^{0} = \boldsymbol{\omega}_{21}^{0} + \boldsymbol{\omega}_{43}^{0} + \boldsymbol{\omega}_{54}^{0}$$
$$\boldsymbol{\omega}_{16}^{0} = \boldsymbol{\omega}_{21}^{0} + \boldsymbol{\omega}_{43}^{0} + \boldsymbol{\omega}_{54}^{0} + \boldsymbol{\omega}_{16,15}^{0}$$
$$\boldsymbol{\omega}_{17}^{0} = \boldsymbol{\omega}_{21}^{0} + \boldsymbol{\omega}_{43}^{0} + \boldsymbol{\omega}_{54}^{0} + \boldsymbol{\omega}_{16,15}^{0} + \boldsymbol{\omega}_{17,16}^{0}$$
$$\boldsymbol{\omega}_{18}^{0} = \boldsymbol{\omega}_{21}^{0} + \boldsymbol{\omega}_{43}^{0} + \boldsymbol{\omega}_{54}^{0} + \boldsymbol{\omega}_{16,15}^{0} + \boldsymbol{\omega}_{17,16}^{0} + \boldsymbol{\omega}_{18,17}^{0}$$

$$\boldsymbol{\omega}_{21}^{0} = \dot{\theta}_{21}\, \boldsymbol{j}_{1}^{0}$$
$$\boldsymbol{\omega}_{43}^{0} = \dot{\theta}_{43}\, \boldsymbol{j}_{3}^{0}$$
$$\boldsymbol{\omega}_{54}^{0} = \dot{\theta}_{54}\, \boldsymbol{k}_{4}^{0}$$
$$\boldsymbol{\omega}_{1615}^{0} = \dot{\theta}_{1615}\, \boldsymbol{i}_{15}^{0}$$
$$\boldsymbol{\omega}_{1716}^{0} = \dot{\theta}_{1716}\, \boldsymbol{k}_{16}^{0}$$
$$\boldsymbol{\omega}_{1817}^{0} = \dot{\theta}_{1817}\, \boldsymbol{i}_{17}^{0}$$

$$\boldsymbol{j}_{1}^{0} = \boldsymbol{j}_{0}$$
$$\boldsymbol{j}_{3}^{0} = \boldsymbol{j}_{0}$$
$$\boldsymbol{k}_{4}^{0} = \boldsymbol{R}_{4}^{0}\boldsymbol{k}_{4}^{4}$$
$$\boldsymbol{i}_{15}^{0} = \boldsymbol{R}_{15}^{0}\boldsymbol{i}_{15}^{15}$$
$$\boldsymbol{k}_{16}^{0} = \boldsymbol{R}_{16}^{0}\boldsymbol{k}_{16}^{16}$$
$$\boldsymbol{i}_{17}^{0} = \boldsymbol{R}_{17}^{0}\boldsymbol{i}_{17}^{17}$$

$$\boldsymbol{R}_{4}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})$$
$$\boldsymbol{R}_{15}^{0} = \boldsymbol{R}_{y}(\theta_{21})\boldsymbol{R}_{y}(\theta_{43})\boldsymbol{R}_{z}(\theta_{54})$$

3.2.1 Velocidad de Centro de Gravedad del Cuerpo 1

$$\begin{aligned}\boldsymbol{v}_{G1}^{0} &= \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{G1'}^{0}\\ &= \boldsymbol{\omega}_{2}^{0} \times \boldsymbol{r}_{G1'}^{0}\\ &= \boldsymbol{\omega}_{21}^{0} \times \boldsymbol{r}_{G1'}^{0}\\ &= \left(\dot{\theta}_{21}\, \boldsymbol{j}_{1}^{0}\right) \times \boldsymbol{r}_{G1'}^{0}\\ &= \left(\boldsymbol{j}_{1}^{0} \times \boldsymbol{r}_{G1'}^{0}\right)\dot{\theta}_{21}\\ \boldsymbol{v}_{G1}^{0} &= \boldsymbol{k}_{1}\dot{\theta}_{21}\end{aligned} \qquad 3.5$$

Dónde:

$$\boldsymbol{k}_{1} = \boldsymbol{j}_{1}^{0} \times \boldsymbol{r}_{G1'}^{0} \qquad 3.6$$

De la ec. (2.51):

$$\dot{\theta}_{21} = \frac{1}{V_1}\left(V_2\dot{x} + V_3\dot{y} + V_4\dot{z} + V_5\dot{\psi} + V_6\dot{\theta} + V_7\dot{\phi}\right)$$

Renombrando:

$$\dot{\theta}_{21} = \boldsymbol{k}_2^T \dot{\boldsymbol{q}} \qquad 3.7$$

Dónde:

$$\boldsymbol{k}_2^T = \frac{1}{V_1}[V_2, V_3, V_4, V_5, V_6, V_7] \qquad 3.8$$
$$\dot{\boldsymbol{q}} = \left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]^T$$

Sustituyendo la ec. (3.7) en la ec. (3.5) con el fin de poner a esta última en función de las coordenadas generalizadas se tiene:

$$\boldsymbol{v}_{G1}^0 = \boldsymbol{k}_1(\boldsymbol{k}_2^T \dot{\boldsymbol{q}})$$
$$\boldsymbol{v}_{G1}^0 = (\boldsymbol{k}_1 \boldsymbol{k}_2^T)\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{v}_{G1}^0 = \boldsymbol{M}_1 \dot{\boldsymbol{q}} \qquad 3.9$$

Dónde:

$$\boldsymbol{M}_1 = \boldsymbol{k}_1 \boldsymbol{k}_2^T \qquad 3.10$$

3.2.2 Velocidad Angular del Cuerpo 1

$$\boldsymbol{\omega}_2^0 = \boldsymbol{\omega}_{21}^0$$
$$\boldsymbol{\omega}_{21}^0 = \boldsymbol{j}_1^0 \dot{\theta}_{21}$$

Sustituyendo (3.7) en la ec. anterior:

$$\boldsymbol{\omega}_{21}^0 = \boldsymbol{j}_1^0(\boldsymbol{k}_2^T \dot{\boldsymbol{q}})$$
$$\boldsymbol{\omega}_{21}^0 = (\boldsymbol{j}_1^0 \boldsymbol{k}_2^T)\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{\omega}_{21}^0 = \boldsymbol{M}_2 \dot{\boldsymbol{q}} \qquad 3.11$$

Dónde:

$$\boldsymbol{M}_2 = \boldsymbol{j}_1^0 \boldsymbol{k}_2^T \qquad 3.12$$

3.2.3 Velocidad de Centro de Gravedad del Cuerpo 2

$$\begin{aligned}
\boldsymbol{v}_{G2}^0 &= \boldsymbol{v}_{10}^0 + \boldsymbol{v}_{32}^0 + \boldsymbol{v}_{G2'}^0 \\
&= \boldsymbol{\omega}_2^0 \times \boldsymbol{r}_{32}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{G2'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times \boldsymbol{r}_{32}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{G2'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0) + (\boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{G2'}^0 \\
&= (\dot{\theta}_{21}\, \boldsymbol{j}_1^0) \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0) + (\dot{\theta}_{43}\, \boldsymbol{j}_3^0 + \dot{\theta}_{54}\, \boldsymbol{k}_4^0) \times \boldsymbol{r}_{G2'}^0 \\
&= (\, \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0\right))\dot{\theta}_{21} + (\, \boldsymbol{j}_3^0 \times \boldsymbol{r}_{G2'}^0)\dot{\theta}_{43} + (\, \boldsymbol{k}_4^0 \times \boldsymbol{r}_{G2'}^0)\dot{\theta}_{54} \\
\boldsymbol{v}_{G2}^0 &= \boldsymbol{k}_3 \dot{\theta}_{21} + \boldsymbol{k}_4 \dot{\theta}_{43} + \boldsymbol{k}_5 \dot{\theta}_{54}
\end{aligned} \qquad 3.13$$

Dónde:

$$\begin{aligned} \boldsymbol{k}_3 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0\right) \\ \boldsymbol{k}_4 &= \boldsymbol{j}_3^0 \times \boldsymbol{r}_{G2'}^0 \\ \boldsymbol{k}_5 &= \boldsymbol{k}_4^0 \times \boldsymbol{r}_{G2'}^0 \end{aligned} \quad 3.14$$

De las ecs. (2.55) y (2.59):

$$\dot{\theta}_{43} = \frac{1}{V_8}(E_1\dot{x} + E_2\dot{y} + E_3\dot{z} + E_4\dot{\psi} + E_5\dot{\theta} + E_6\dot{\phi})$$
$$\dot{\theta}_{54} = \frac{1}{V_{15}}(E_7\dot{x} + E_8\dot{y} + E_9\dot{z} + E_{10}\dot{\psi} + E_{11}\dot{\theta} + E_{12}\dot{\phi})$$

Renombrando:

$$\dot{\theta}_{43} = \boldsymbol{k}_6^T\dot{\boldsymbol{q}} \quad 3.15$$
$$\dot{\theta}_{54} = \boldsymbol{k}_7^T\dot{\boldsymbol{q}} \quad 3.16$$

Dónde:

$$\begin{aligned} \boldsymbol{k}_6^T &= \frac{1}{V_8}[E_1, E_2, E_3, E_4, E_5, E_6] \\ \boldsymbol{k}_7^T &= \frac{1}{V_{15}}[E_7, E_8, E_9, E_{10}, E_{11}, E_{12}] \end{aligned} \quad 3.17$$

Sustituyendo la ec. (3.7), (3.15) y (3.16) en la ec. (3.13):

$$\begin{aligned} \boldsymbol{v}_{G2}^0 &= \boldsymbol{k}_3\dot{\theta}_{21} + \boldsymbol{k}_4\dot{\theta}_{43} + \boldsymbol{k}_5\dot{\theta}_{54} \\ &= (\boldsymbol{k}_3\boldsymbol{k}_2^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_4\boldsymbol{k}_6^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_5\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} \\ \boldsymbol{v}_{G2}^0 &= (\boldsymbol{k}_3\boldsymbol{k}_2^T + \boldsymbol{k}_4\boldsymbol{k}_6^T + \boldsymbol{k}_5\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} \end{aligned}$$

Renombrando:

$$\boldsymbol{v}_{G2}^0 = \boldsymbol{M}_3\dot{\boldsymbol{q}} \quad 3.18$$

Dónde:

$$\boldsymbol{M}_3 = \boldsymbol{k}_3\boldsymbol{k}_2^T + \boldsymbol{k}_4\boldsymbol{k}_6^T + \boldsymbol{k}_5\boldsymbol{k}_7^T \quad 3.19$$

3.2.4 Velocidad Angular del Cuerpo 2

$$\begin{aligned} \boldsymbol{\omega}_5^0 &= \boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 \\ &= \boldsymbol{j}_1^0\dot{\theta}_{21} + \boldsymbol{j}_3^0\dot{\theta}_{43} + \boldsymbol{k}_4^0\dot{\theta}_{54} \\ &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T\dot{\boldsymbol{q}} + \boldsymbol{j}_3^0\boldsymbol{k}_6^T\dot{\boldsymbol{q}} + \boldsymbol{k}_4^0\boldsymbol{k}_7^T\dot{\boldsymbol{q}} \\ \boldsymbol{\omega}_5^0 &= (\boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} \end{aligned}$$

Renombrando:

$$\boldsymbol{\omega}_5^0 = \boldsymbol{M}_4\dot{\boldsymbol{q}} \quad 3.20$$

Dónde:

$$\boldsymbol{M}_4 = \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T \qquad 3.21$$

3.2.5 Velocidad de Centro de Gravedad del Cuerpo 3

$$\begin{aligned}
\boldsymbol{v}_{G3}^0 &= \boldsymbol{v}_{10}^0 + \boldsymbol{v}_{32}^0 + \boldsymbol{v}_{14,5i}^0 + \boldsymbol{v}_{15,14i}^0 + \boldsymbol{v}_{G3'}^0 \\
&= \boldsymbol{\omega}_2^0 \times \boldsymbol{r}_{32}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{14,5i}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{15,14i}^0 + \boldsymbol{\omega}_{16}^0 \times \boldsymbol{r}_{G3'}^0 \\
&= \boldsymbol{\omega}_2^0 \times \boldsymbol{r}_{32}^0 + \boldsymbol{\omega}_5^0 \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0) + \boldsymbol{\omega}_{16}^0 \times \boldsymbol{r}_{G3'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times \boldsymbol{r}_{32}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0) + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 \\
&\qquad + \boldsymbol{\omega}_{16,15}^0) \times \boldsymbol{r}_{G3'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0) + (\boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0) \\
&\qquad + \boldsymbol{\omega}_{16,15}^0 \times \boldsymbol{r}_{G3'}^0 \\
&= (\dot\theta_{21}\, \boldsymbol{j}_1^0) \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0) + (\dot\theta_{43}\, \boldsymbol{j}_3^0 + \dot\theta_{54}\, \boldsymbol{k}_4^0) \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 \\
&\qquad + \boldsymbol{r}_{G3'}^0) + (\dot\theta_{1615}\, \boldsymbol{i}_{15}^0) \times \boldsymbol{r}_{G3'}^0 \\
&= \left(\boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0\right)\right)\dot\theta_{21} + (\boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right))\dot\theta_{43} \\
&\qquad + (\boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right))\dot\theta_{54} + (\, \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G3'}^0)\dot\theta_{1615}
\end{aligned}$$

$$\boldsymbol{v}_{G3}^0 = \boldsymbol{k}_8\dot\theta_{21} + \boldsymbol{k}_9\dot\theta_{43} + \boldsymbol{k}_{10}\dot\theta_{54} + \boldsymbol{k}_{11}\dot\theta_{1615} \qquad 3.22$$

Dónde:

$$\begin{aligned}
\boldsymbol{k}_8 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0\right) \\
\boldsymbol{k}_9 &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right) \\
\boldsymbol{k}_{10} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right) \\
\boldsymbol{k}_{11} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G3'}^0
\end{aligned} \qquad 3.23$$

De la ec. (2.73):

$$\dot\theta_{1615} = \frac{1}{V_{36}}(E_{25}\dot x + E_{26}\dot y + E_{27}\dot z + E_{28}\dot\psi + E_{29}\dot\theta + E_{30}\dot\phi)$$

Renombrando:

$$\dot\theta_{1615} = \boldsymbol{k}_{12}^T\dot{\boldsymbol{q}} \qquad 3.24$$

Dónde:

$$\boldsymbol{k}_{12}^T = \frac{1}{V_{36}}[E_{25}, E_{26}, E_{27}, E_{28}, E_{29}, E_{30}] \qquad 3.25$$

Sustituyendo la ec. (3.7), (3.15), (3.16) y (3.24) en la ec. (3.22):

$$\begin{aligned}
\boldsymbol{v}_{G3}^0 &= \boldsymbol{k}_8\dot\theta_{21} + \boldsymbol{k}_9\dot\theta_{43} + \boldsymbol{k}_{10}\dot\theta_{54} + \boldsymbol{k}_{11}\dot\theta_{1615} \\
&= (\boldsymbol{k}_8\boldsymbol{k}_2^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_9\boldsymbol{k}_6^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{10}\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{11}\boldsymbol{k}_{12}^T)\dot{\boldsymbol{q}}
\end{aligned}$$

$$\boldsymbol{v}_{G3}^{0} = (\boldsymbol{k}_8\boldsymbol{k}_2^T + \boldsymbol{k}_9\boldsymbol{k}_6^T + \boldsymbol{k}_{10}\boldsymbol{k}_7^T + \boldsymbol{k}_{11}\boldsymbol{k}_{12}^T)\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{v}_{G3}^{0} = \boldsymbol{M}_5\dot{\boldsymbol{q}} \qquad 3.26$$

Dónde:

$$\boldsymbol{M}_5 = \boldsymbol{k}_8\boldsymbol{k}_2^T + \boldsymbol{k}_9\boldsymbol{k}_6^T + \boldsymbol{k}_{10}\boldsymbol{k}_7^T + \boldsymbol{k}_{11}\boldsymbol{k}_{12}^T \qquad 3.27$$

3.2.6 Velocidad Angular del Cuerpo 3

$$\begin{aligned}
\boldsymbol{\omega}_{16}^{0} &= \boldsymbol{\omega}_{21}^{0} + \boldsymbol{\omega}_{43}^{0} + \boldsymbol{\omega}_{54}^{0} + \boldsymbol{\omega}_{16,15}^{0} \\
&= \boldsymbol{j}_1^0\dot{\theta}_{21} + \boldsymbol{j}_3^0\dot{\theta}_{43} + \boldsymbol{k}_4^0\dot{\theta}_{54} + \boldsymbol{i}_{15}^0\dot{\theta}_{1615} \\
&= \boldsymbol{j}_1^0\boldsymbol{k}_2^T\dot{\boldsymbol{q}} + \boldsymbol{j}_3^0\boldsymbol{k}_6^T\dot{\boldsymbol{q}} + \boldsymbol{k}_4^0\boldsymbol{k}_7^T\dot{\boldsymbol{q}} + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T\dot{\boldsymbol{q}} \\
\boldsymbol{\omega}_{16}^{0} &= (\boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T)\dot{\boldsymbol{q}}
\end{aligned}$$

Renombrando:

$$\boldsymbol{\omega}_{16}^{0} = \boldsymbol{M}_6\dot{\boldsymbol{q}} \qquad 3.28$$

Dónde:

$$\boldsymbol{M}_6 = \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T \qquad 3.29$$

3.2.7 Velocidad de Centro de Gravedad del Cuerpo 4

$$\begin{aligned}
\boldsymbol{v}_{G4}^{0} &= \boldsymbol{v}_{10}^{0} + \boldsymbol{v}_{32}^{0} + \boldsymbol{v}_{14,5i}^{0} + \boldsymbol{v}_{15,14i}^{0} + \boldsymbol{v}_{G4'}^{0} \\
&= \boldsymbol{\omega}_2^0 \times \boldsymbol{r}_{32}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{14,5i}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{15,14i}^0 + \boldsymbol{\omega}_{17}^0 \times \boldsymbol{r}_{G4'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times \boldsymbol{r}_{32}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{14,5i}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{15,14i}^0 \\
&\qquad + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 + \boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0) \times \boldsymbol{r}_{G4'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) + (\boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 \\
&\qquad + \boldsymbol{r}_{G4'}^0) + (\boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0) \times \boldsymbol{r}_{G4'}^0 \\
&= (\dot{\theta}_{21}\,\boldsymbol{j}_1^0) \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) + (\dot{\theta}_{43}\,\boldsymbol{j}_3^0 + \dot{\theta}_{54}\,\boldsymbol{k}_4^0) \times (\boldsymbol{r}_{14,5i}^0 \\
&\qquad + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) + (\dot{\theta}_{1615}\,\boldsymbol{i}_{15}^0 + \dot{\theta}_{1716}\,\boldsymbol{k}_{16}^0) \times \boldsymbol{r}_{G4'}^0 \\
&= (\boldsymbol{j}_1^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0))\dot{\theta}_{21} + (\boldsymbol{j}_3^0 \\
&\qquad \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0))\dot{\theta}_{43} + (\boldsymbol{k}_4^0 \\
&\qquad \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0))\dot{\theta}_{54} + (\boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G4'}^0)\dot{\theta}_{1615} + (\boldsymbol{k}_{16}^0 \\
&\qquad \times \boldsymbol{r}_{G4'}^0)\dot{\theta}_{1716} \\
\boldsymbol{v}_{G4}^{0} &= \boldsymbol{k}_{13}\dot{\theta}_{21} + \boldsymbol{k}_{14}\dot{\theta}_{43} + \boldsymbol{k}_{15}\dot{\theta}_{54} + \boldsymbol{k}_{16}\dot{\theta}_{1615} + \boldsymbol{k}_{17}\dot{\theta}_{1716}
\end{aligned} \qquad 3.30$$

Dónde:

$$\begin{aligned}
\boldsymbol{k}_{13} &= \boldsymbol{j}_1^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) \\
\boldsymbol{k}_{14} &= \boldsymbol{j}_3^0 \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) \\
\boldsymbol{k}_{15} &= \boldsymbol{k}_4^0 \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0) \\
\boldsymbol{k}_{16} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G4'}^0 \\
\boldsymbol{k}_{17} &= \boldsymbol{k}_{16}^0 \times \boldsymbol{r}_{G4'}^0
\end{aligned} \qquad 3.31$$

De la ec. (2.69):

$$\dot{\theta}_{1716} = \frac{1}{V_{29}}(E_{19}\dot{x} + E_{20}\dot{y} + E_{21}\dot{z} + E_{22}\dot{\psi} + E_{23}\dot{\theta} + E_{24}\dot{\phi})$$

Renombrando:

$$\dot{\theta}_{1716} = \boldsymbol{k}_{18}^T\dot{\boldsymbol{q}} \qquad 3.32$$

Dónde:

$$\boldsymbol{k}_{18}^T = \frac{1}{V_{29}}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] \qquad 3.33$$

Sustituyendo las ecs. (3.7), (3.15), (3.16) ,(3.24) y (3.32) en la ec. (3.30):

$$\begin{aligned}\boldsymbol{v}_{G4}^0 &= \boldsymbol{k}_{13}\dot{\theta}_{21} + \boldsymbol{k}_{14}\dot{\theta}_{43} + \boldsymbol{k}_{15}\dot{\theta}_{54} + \boldsymbol{k}_{16}\dot{\theta}_{1615} + \boldsymbol{k}_{17}\dot{\theta}_{1716}\\ &= (\boldsymbol{k}_{13}\boldsymbol{k}_2^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{14}\boldsymbol{k}_6^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{15}\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{16}\boldsymbol{k}_{12}^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{17}\boldsymbol{k}_{18}^T)\dot{\boldsymbol{q}}\\ \boldsymbol{v}_{G4}^0 &= (\boldsymbol{k}_{13}\boldsymbol{k}_2^T + \boldsymbol{k}_{14}\boldsymbol{k}_6^T + \boldsymbol{k}_{15}\boldsymbol{k}_7^T + \boldsymbol{k}_{16}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{17}\boldsymbol{k}_{18}^T)\dot{\boldsymbol{q}}\end{aligned}$$

Renombrando:

$$\boldsymbol{v}_{G4}^0 = \boldsymbol{M}_7\dot{\boldsymbol{q}} \qquad 3.34$$

Dónde:

$$\boldsymbol{M}_7 = \boldsymbol{k}_{13}\boldsymbol{k}_2^T + \boldsymbol{k}_{14}\boldsymbol{k}_6^T + \boldsymbol{k}_{15}\boldsymbol{k}_7^T + \boldsymbol{k}_{16}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{17}\boldsymbol{k}_{18}^T \qquad 3.35$$

3.2.8 Velocidad Angular del Cuerpo 4

$$\begin{aligned}\boldsymbol{\omega}_{17}^0 &= \boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 + \boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0\\ &= \boldsymbol{j}_1^0\dot{\theta}_{21} + \boldsymbol{j}_3^0\dot{\theta}_{43} + \boldsymbol{k}_4^0\dot{\theta}_{54} + \boldsymbol{i}_{15}^0\dot{\theta}_{1615} + \boldsymbol{k}_{16}^0\dot{\theta}_{1716}\\ &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T\dot{\boldsymbol{q}} + \boldsymbol{j}_3^0\boldsymbol{k}_6^T\dot{\boldsymbol{q}} + \boldsymbol{k}_4^0\boldsymbol{k}_7^T\dot{\boldsymbol{q}} + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T\dot{\boldsymbol{q}} + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T\dot{\boldsymbol{q}}\\ &= (\boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T)\dot{\boldsymbol{q}}\end{aligned}$$

Renombrando:

$$\boldsymbol{\omega}_{17}^0 = \boldsymbol{M}_8\dot{\boldsymbol{q}} \qquad 3.36$$

Dónde:

$$\boldsymbol{M}_8 = \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T \qquad 3.37$$

3.2.9 Velocidad de Centro de Gravedad del Cuerpo 5

$$
\begin{aligned}
\boldsymbol{v}_{G5}^0 &= \boldsymbol{v}_{10}^0 + \boldsymbol{v}_{32}^0 + \boldsymbol{v}_{14,5i}^0 + \boldsymbol{v}_{15,14i}^0 + \boldsymbol{v}_{G5'}^0 \\
&= \boldsymbol{\omega}_2^0 \times \boldsymbol{r}_{32}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{14,5i}^0 + \boldsymbol{\omega}_5^0 \times \boldsymbol{r}_{15,14i}^0 + \boldsymbol{\omega}_{18}^0 \times \boldsymbol{r}_{G5'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times \boldsymbol{r}_{32}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{14,5i}^0 + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times \boldsymbol{r}_{15,14i}^0 \\
&\qquad + (\boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 + \boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0 + \boldsymbol{\omega}_{18,17}^0) \times \boldsymbol{r}_{G5'}^0 \\
&= \boldsymbol{\omega}_{21}^0 \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0) + (\boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0) \times (\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 \\
&\qquad + \boldsymbol{r}_{G5'}^0) + (\boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0 + \boldsymbol{\omega}_{18,17}^0) \times \boldsymbol{r}_{G5'}^0 \\
&= (\dot{\theta}_{21}\, \boldsymbol{j}_1^0) \times (\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0) + (\dot{\theta}_{43}\, \boldsymbol{j}_3^0 + \dot{\theta}_{54}\, \boldsymbol{k}_4^0) \times (\boldsymbol{r}_{14,5i}^0 \\
&\qquad + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0) + (\dot{\theta}_{1615}\, \boldsymbol{i}_{15}^0 + \dot{\theta}_{1716}\, \boldsymbol{k}_{16}^0 + \dot{\theta}_{1817}\, \boldsymbol{i}_{17}^0) \times \boldsymbol{r}_{G5'}^0 \\
&= (\,\boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right))\dot{\theta}_{21} + (\,\boldsymbol{j}_3^0 \\
&\qquad \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right))\dot{\theta}_{43} + (\boldsymbol{k}_4^0 \\
&\qquad \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right))\dot{\theta}_{54} + (\,\boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G5'}^0)\dot{\theta}_{1615} + (\boldsymbol{k}_{16}^0 \\
&\qquad \times \boldsymbol{r}_{G5'}^0)\dot{\theta}_{1716} + (\boldsymbol{i}_{17}^0 \times \boldsymbol{r}_{G5'}^0)\dot{\theta}_{1817} \\
\boldsymbol{v}_{G5}^0 &= \boldsymbol{k}_{19}\dot{\theta}_{21} + \boldsymbol{k}_{20}\dot{\theta}_{43} + \boldsymbol{k}_{21}\dot{\theta}_{54} + \boldsymbol{k}_{22}\dot{\theta}_{1615} + \boldsymbol{k}_{23}\dot{\theta}_{1716} + \boldsymbol{k}_{24}\dot{\theta}_{1817}
\end{aligned}
\tag{3.38}
$$

Dónde:

$$
\begin{aligned}
\boldsymbol{k}_{19} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{20} &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{21} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{22} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G5'}^0 \\
\boldsymbol{k}_{23} &= \mathbf{k}_{16}^0 \times \mathbf{r}_{G5'}^0 \\
\boldsymbol{k}_{24} &= \mathbf{i}_{17}^0 \times \mathbf{r}_{G5'}^0
\end{aligned}
\tag{3.39}
$$

De la ec. (2.64):

$$\dot{\theta}_{1817} = \frac{1}{V_{23}}\left(E_{13}\dot{x} + E_{14}\dot{y} + E_{15}\dot{z} + E_{16}\dot{\psi} + E_{17}\dot{\theta} + E_{18}\dot{\phi}\right)$$

Renombrando:

$$\dot{\theta}_{1817} = \boldsymbol{k}_{25}^T\dot{\boldsymbol{q}} \tag{3.40}$$

Dónde:

$$\boldsymbol{k}_{25}^T = \frac{1}{V_{23}}\left[E_{13}, E_{14}, E_{15}, E_{16}, E_{17}, E_{18}\right] \tag{3.41}$$

Sustituyendo las ecs. (3.7), (3.15), (3.16), (3.24), (3.32) y (3.40) en la ec. (3.38):

$$
\begin{aligned}
\boldsymbol{v}_{G5}^0 &= \boldsymbol{k}_{19}\dot{\theta}_{21} + \boldsymbol{k}_{20}\dot{\theta}_{43} + \boldsymbol{k}_{21}\dot{\theta}_{54} + \boldsymbol{k}_{22}\dot{\theta}_{1615} + \boldsymbol{k}_{23}\dot{\theta}_{1716} + \boldsymbol{k}_{24}\dot{\theta}_{1817} \\
&= (\boldsymbol{k}_{19}\boldsymbol{k}_2^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{20}\boldsymbol{k}_6^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{21}\boldsymbol{k}_7^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{22}\boldsymbol{k}_{12}^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{23}\boldsymbol{k}_{18}^T)\dot{\boldsymbol{q}} + (\boldsymbol{k}_{24}\boldsymbol{k}_{25}^T)\dot{\boldsymbol{q}}
\end{aligned}
$$

$$\boldsymbol{v}_{G5}^0 = (\boldsymbol{k}_{19}\boldsymbol{k}_2^T + \boldsymbol{k}_{20}\boldsymbol{k}_6^T + \boldsymbol{k}_{21}\boldsymbol{k}_7^T + \boldsymbol{k}_{22}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{23}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{24}\boldsymbol{k}_{25}^T)\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{v}_{G5}^0 = \boldsymbol{M}_9\dot{\boldsymbol{q}} \qquad 3.42$$

Dónde:

$$\boldsymbol{M}_9 = \boldsymbol{k}_{19}\boldsymbol{k}_2^T + \boldsymbol{k}_{20}\boldsymbol{k}_6^T + \boldsymbol{k}_{21}\boldsymbol{k}_7^T + \boldsymbol{k}_{22}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{23}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{24}\boldsymbol{k}_{25}^T \qquad 3.43$$

3.2.10 Velocidad Angular del Cuerpo 5

$$\begin{aligned}\boldsymbol{\omega}_{18}^0 &= \boldsymbol{\omega}_{21}^0 + \boldsymbol{\omega}_{43}^0 + \boldsymbol{\omega}_{54}^0 + \boldsymbol{\omega}_{16,15}^0 + \boldsymbol{\omega}_{17,16}^0 + \boldsymbol{\omega}_{18,17}^0 \\ &= \boldsymbol{j}_1^0\dot{\theta}_{21} + \boldsymbol{j}_3^0\dot{\theta}_{43} + \boldsymbol{k}_4^0\dot{\theta}_{54} + \boldsymbol{i}_{15}^0\dot{\theta}_{1615} + \boldsymbol{k}_{16}^0\dot{\theta}_{1716} + \boldsymbol{i}_{17}^0\dot{\theta}_{1817} \\ &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T\dot{\boldsymbol{q}} + \boldsymbol{j}_3^0\boldsymbol{k}_6^T\dot{\boldsymbol{q}} + \boldsymbol{k}_4^0\boldsymbol{k}_7^T\dot{\boldsymbol{q}} + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T\dot{\boldsymbol{q}} + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T\dot{\boldsymbol{q}} + \boldsymbol{i}_{17}^0\boldsymbol{k}_{25}^T\dot{\boldsymbol{q}} \\ \boldsymbol{\omega}_{18}^0 &= (\boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{i}_{17}^0\boldsymbol{k}_{25}^T)\dot{\boldsymbol{q}}\end{aligned}$$

Renombrando:

$$\boldsymbol{\omega}_{18}^0 = \boldsymbol{M}_{10}\dot{\boldsymbol{q}} \qquad 3.44$$

Dónde:

$$\boldsymbol{M}_{10} = \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{i}_{17}^0\boldsymbol{k}_{25}^T \qquad 3.45$$

Matrices de inercia medidas en la base inercial:

$$\begin{aligned}\boldsymbol{J}_1^0 &= \boldsymbol{R}_2^0\,\boldsymbol{J}_1^2\,{\boldsymbol{R}_2^0}^T \\ \boldsymbol{J}_2^0 &= \boldsymbol{R}_5^0\,\boldsymbol{J}_2^5\,{\boldsymbol{R}_5^0}^T \\ \boldsymbol{J}_3^0 &= \boldsymbol{R}_{16}^0\,\boldsymbol{J}_3^{16}\,{\boldsymbol{R}_{16}^0}^T \\ \boldsymbol{J}_4^0 &= \boldsymbol{R}_{17}^0\,\boldsymbol{J}_4^{17}\,{\boldsymbol{R}_{17}^0}^T \\ \boldsymbol{J}_5^0 &= \boldsymbol{R}_{18}^0\,\boldsymbol{J}_5^{18}\,{\boldsymbol{R}_{18}^0}^T\end{aligned}$$

3.3 Función Lagrangiana

Aplicando la ec. (3.1) se consigue de manera general la siguiente expresión:

$$L = \sum_{h=1}^{5}(K_h - U_h) + \sum_{i=1}^{2}\left(\sum_{k=1}^{2}(K_{ki} - U_{ki})\right) \qquad 3.46$$

h=número de cuerpos en la cadena 0.
i=número de la cadena 1 o 2.
k=número de cuerpos en la cadena h.

Expandiendo los términos del primer paréntesis:

$$L = \sum_{h=1}^{5}(K_h - U_h) + \sum_{i=1}^{2}\big((K_{1i} - U_{1i}) + (K_{2i} - U_{2i})\big) \quad 3.47$$

$$L = \sum_{h=1}^{5} L_h + \sum_{i=1}^{2}(L_{1i} + L_{2i}) \quad 3.48$$

Dónde:

$$L_1 = \frac{1}{2}\left(m_1 \boldsymbol{v}_{G1}^{0\,T} \boldsymbol{v}_{G1}^0 + \boldsymbol{\omega}_2^{0\,T} \boldsymbol{J}_1 \boldsymbol{\omega}_2^0\right) + m_1 \boldsymbol{g}^T \boldsymbol{r}_{G1}^0$$
$$L_2 = \frac{1}{2}\left(m_2 \boldsymbol{v}_{G2}^{0\,T} \boldsymbol{v}_{G2}^0 + \boldsymbol{\omega}_5^{0\,T} \boldsymbol{J}_2 \boldsymbol{\omega}_5^0\right) + m_2 \boldsymbol{g}^T \boldsymbol{r}_{G2}^0$$
$$L_3 = \frac{1}{2}\left(m_3 \boldsymbol{v}_{G3}^{0\,T} \boldsymbol{v}_{G3}^0 + \boldsymbol{\omega}_{16}^{0\,T} \boldsymbol{J}_3 \boldsymbol{\omega}_{16}^0\right) + m_3 \boldsymbol{g}^T \boldsymbol{r}_{G3}^0$$
$$L_4 = \frac{1}{2}\left(m_4 \boldsymbol{v}_{G4}^{0\,T} \boldsymbol{v}_{G4}^0 + \boldsymbol{\omega}_{17}^{0\,T} \boldsymbol{J}_4 \boldsymbol{\omega}_{17}^0\right) + m_4 \boldsymbol{g}^T \boldsymbol{r}_{G4}^0$$
$$L_5 = \frac{1}{2}\left(m_5 \boldsymbol{v}_{G5}^{0\,T} \boldsymbol{v}_{G5}^0 + \boldsymbol{\omega}_{18}^{0\,T} \boldsymbol{J}_5 \boldsymbol{\omega}_{18}^0\right) + m_5 \boldsymbol{g}^T \boldsymbol{r}_{G5}^0$$
$$L_{1i} = \frac{1}{2}\left(m_{1h} \boldsymbol{v}_{G1i}^{0\,T} \boldsymbol{v}_{G1i}^0 + \boldsymbol{\omega}_{2i}^{0\,T} \boldsymbol{J}_{1i} \boldsymbol{\omega}_{2i}^0\right) + m_{1i} \boldsymbol{g}^T \boldsymbol{r}_{G1i}^0$$
$$L_{2i} = \frac{1}{2}\left(m_{2h} \boldsymbol{v}_{G2i}^{0\,T} \boldsymbol{v}_{G2i}^0 + \boldsymbol{\omega}_{7i}^{0\,T} \boldsymbol{J}_{2i} \boldsymbol{\omega}_{7i}^0\right) + m_{2i} \boldsymbol{g}^T \boldsymbol{r}_{G2i}^0$$

Donde $\boldsymbol{g} = [0,0,-9.81]^T$

3.3.1 Desarrollo del Primer Término de la Ecuación de Lagrange para la cadena 0.

A partir de la ec. (3.4):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$$

Desarrollando el primer término de la ecuación anterior a partir de la ec. (3.47):

$$\frac{\partial L}{\partial \dot{q}_j} = \sum_{h=1}^{5} \frac{\partial L_h}{\partial \dot{q}_j} + \sum_{i=1}^{2}\left(\frac{\partial L_{1i}}{\partial \dot{q}_j} + \frac{\partial L_{2i}}{\partial \dot{q}_j}\right) \quad 3.49$$

Para j = 1,2,3,4,5,6,donde:

$$\dot{q}_1 = \dot{x} \qquad \dot{q}_2 = \dot{y} \qquad \dot{q}_3 = \dot{z}$$
$$\dot{q}_4 = \dot{\psi} \qquad \dot{q}_5 = \dot{\theta} \qquad \dot{q}_6 = \dot{\phi}$$

Desarrollando $\frac{\partial L_1}{\partial \dot{q}_j}$:

Se tiene:

$$L_1 = \frac{1}{2}\left(m_1 \boldsymbol{v}_{G1}^{0}{}^{T} \boldsymbol{v}_{G1}^{0} + \boldsymbol{\omega}_2^{0T} \boldsymbol{J}_1 \boldsymbol{\omega}_2^{0}\right) + m_1 \boldsymbol{g}^T \boldsymbol{r}_{G1}^{0}$$

Sustituyendo ec.(3.9) y ec.(3.11):

$$\begin{aligned}
L_1 &= \frac{1}{2}(m_1(\boldsymbol{M}_1\dot{\boldsymbol{q}})^T(\boldsymbol{M}_1\dot{\boldsymbol{q}}) + (\boldsymbol{M}_2\dot{\boldsymbol{q}})^T\boldsymbol{J}_1(\boldsymbol{M}_2\dot{\boldsymbol{q}})) + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0 \\
&= \frac{1}{2}\left(m_1\dot{\boldsymbol{q}}^T\boldsymbol{M}_1{}^T(\boldsymbol{M}_1\dot{\boldsymbol{q}}) + \dot{\boldsymbol{q}}^T\boldsymbol{M}_2{}^T\boldsymbol{J}_1(\boldsymbol{M}_2\dot{\boldsymbol{q}})\right) + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0 \\
&= \frac{1}{2}\left(m_1\dot{\boldsymbol{q}}^T(\boldsymbol{M}_1{}^T\boldsymbol{M}_1)\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T(\boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2)\dot{\boldsymbol{q}}\right) + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0 \\
&= \frac{1}{2}\dot{\boldsymbol{q}}^T(m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1 + \boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2)\dot{\boldsymbol{q}} + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0 \\
L_1 &= \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{N}_1\dot{\boldsymbol{q}} + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0 \qquad 3.50
\end{aligned}$$

Dónde:

$$\boldsymbol{N}_1 = m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1 + \boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2$$

Efectuando la derivada:

$$\begin{aligned}
\frac{\partial L_1}{\partial \dot{q}_j} &= \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_1\dot{\boldsymbol{q}}) + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0\right) \\
\frac{\partial L_1}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_1\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \qquad 3.51
\end{aligned}$$

Donde se cumple:

$$\begin{aligned}
\boldsymbol{N}_1{}^T &= \left(m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1 + \boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2\right)^T \\
&= \left(m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1\right)^T + \left(\boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2\right)^T \\
&= m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1 + \boldsymbol{M}_2{}^T\boldsymbol{J}_1{}^T\boldsymbol{M}_2 \\
&= m_1\boldsymbol{M}_1{}^T\boldsymbol{M}_1 + \boldsymbol{M}_2{}^T\boldsymbol{J}_1\boldsymbol{M}_2 \\
\boldsymbol{N}_1{}^T &= \boldsymbol{N}_1 \qquad 3.52
\end{aligned}$$

Tal que $\boldsymbol{J}_1 = \boldsymbol{J}_1{}^T$, ya que la matriz de inercia es simétrica. Además:

$$\begin{aligned}
\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} &= (\boldsymbol{N}_1\dot{\boldsymbol{q}})^T\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j} \\
&= \dot{\boldsymbol{q}}^T\boldsymbol{N}_1{}^T\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j} \\
\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} &= \dot{\boldsymbol{q}}^T\boldsymbol{N}_1\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}
\end{aligned}$$

Estas dos propiedades anteriores se ocupan para los demás cuerpos.

Simplificando:

$$\frac{\partial L_1}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_1\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right)$$
$$= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}}\right)$$
$$\frac{\partial L_1}{\partial \dot{q}_j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}} \qquad 3.53$$

Desarrollando $\frac{\partial L_2}{\partial \dot{q}_j}$:

Se tiene:

$$L_2 = \frac{1}{2}\left(m_2\boldsymbol{v}_{G2}^{0\;T}\boldsymbol{v}_{G2}^0 + \boldsymbol{\omega}_5^{0\;T}\boldsymbol{J}_2\boldsymbol{\omega}_5^0\right) + m_2\boldsymbol{g}^T\boldsymbol{r}_{G2}^0$$

Sustituyendo ec.(3.18) y ec.(3.20):

$$L_2 = \frac{1}{2}(m_2(\boldsymbol{M}_3\dot{\boldsymbol{q}})^T(\boldsymbol{M}_3\dot{\boldsymbol{q}}) + (\boldsymbol{M}_4\dot{\boldsymbol{q}})^T\boldsymbol{J}_1(\boldsymbol{M}_4\dot{\boldsymbol{q}})) + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^0$$
$$= \frac{1}{2}\dot{\boldsymbol{q}}^T(m_2\boldsymbol{M}_3{}^T\boldsymbol{M}_3 + \boldsymbol{M}_4{}^T\boldsymbol{J}_2\boldsymbol{M}_4)\dot{\boldsymbol{q}} + m_2\boldsymbol{g}^T\boldsymbol{r}_{G2}^0$$
$$L_2 = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{N}_2\dot{\boldsymbol{q}} + m_2\boldsymbol{g}^T\boldsymbol{r}_{G2}^0 \qquad 3.54$$

Dónde:

$$\boldsymbol{N}_2 = m_2\boldsymbol{M}_3{}^T\boldsymbol{M}_3 + \boldsymbol{M}_4{}^T\boldsymbol{J}_2\boldsymbol{M}_4$$

Efectuando la derivada:

$$\frac{\partial L_2}{\partial \dot{q}_j} = \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_2\dot{\boldsymbol{q}}) + m_2\boldsymbol{g}^T\boldsymbol{r}_{G2}^0\right)$$
$$\frac{\partial L_2}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_2\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \qquad 3.55$$

Simplificando:

$$\frac{\partial L_2}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_2\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right)$$
$$= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}}\right)$$
$$\frac{\partial L_2}{\partial \dot{q}_j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}} \qquad 3.56$$

Desarrollando $\frac{\partial L_3}{\partial \dot{q}_j}$:

Se tiene:

$$L_3 = \frac{1}{2}\left(m_3 \boldsymbol{v}_{G3}^{0}{}^T \boldsymbol{v}_{G3}^{0} + \boldsymbol{\omega}_{16}^{0}{}^T \boldsymbol{J}_3 \boldsymbol{\omega}_{16}^{0}\right) + m_3 \boldsymbol{g}^T \boldsymbol{r}_{G3}^{0}$$

Sustituyendo ec.(3.26) y ec.(3.28):

$$\begin{aligned} L_3 &= \frac{1}{2}(m_3(\boldsymbol{M}_5\dot{\boldsymbol{q}})^T(\boldsymbol{M}_5\dot{\boldsymbol{q}}) + (\boldsymbol{M}_6\dot{\boldsymbol{q}})^T\boldsymbol{J}_3(\boldsymbol{M}_6\dot{\boldsymbol{q}})) + m_3\boldsymbol{g}^T\boldsymbol{r}_{G3}^{0} \\ &= \frac{1}{2}\left(\dot{\boldsymbol{q}}^T(m_3\boldsymbol{M}_5{}^T\boldsymbol{M}_5 + \boldsymbol{M}_6{}^T\boldsymbol{J}_3\boldsymbol{M}_6)\dot{\boldsymbol{q}}\right) + m_3\boldsymbol{g}^T\boldsymbol{r}_{G3}^{0} \\ L_3 &= \frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_3\dot{\boldsymbol{q}}) + m_3\boldsymbol{g}^T\boldsymbol{r}_{G3}^{0} \end{aligned} \qquad 3.57$$

Dónde:

$$\boldsymbol{N}_3 = m_3\boldsymbol{M}_5{}^T\boldsymbol{M}_5 + \boldsymbol{M}_6{}^T\boldsymbol{J}_3\boldsymbol{M}_6$$

Efectuando la derivada:

$$\begin{aligned} \frac{\partial L_3}{\partial \dot{q}_j} &= \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_3\dot{\boldsymbol{q}}) + m_3\boldsymbol{g}^T\boldsymbol{r}_{G3}^{0}\right) \\ \frac{\partial L_3}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_3\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \end{aligned} \qquad 3.58$$

Simplificando:

$$\begin{aligned} \frac{\partial L_3}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_3\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \\ &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}}\right) \\ \frac{\partial L_3}{\partial \dot{q}_j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}} \end{aligned} \qquad 3.59$$

Desarrollando $\frac{\partial L_4}{\partial \dot{q}_j}$:

Se tiene:

$$L_4 = \frac{1}{2}\left(m_4 \boldsymbol{v}_{G4}^{0}{}^T \boldsymbol{v}_{G4}^{0} + \boldsymbol{\omega}_{17}^{0}{}^T \boldsymbol{J}_4 \boldsymbol{\omega}_{17}^{0}\right) + m_4 \boldsymbol{g}^T \boldsymbol{r}_{G4}^{0}$$

Sustituyendo ec.(3.34) y ec.(3.36):

$$\begin{aligned} L_4 &= \frac{1}{2}(m_4(\boldsymbol{M}_7\dot{\boldsymbol{q}})^T(\boldsymbol{M}_7\dot{\boldsymbol{q}}) + (\boldsymbol{M}_8\dot{\boldsymbol{q}})^T\boldsymbol{J}_4(\boldsymbol{M}_8\dot{\boldsymbol{q}})) + m_4\boldsymbol{g}^T\boldsymbol{r}_{G4}^{0} \\ &= \frac{1}{2}\left(\dot{\boldsymbol{q}}^T(m_4\boldsymbol{M}_7{}^T\boldsymbol{M}_7 + \boldsymbol{M}_8{}^T\boldsymbol{J}_4\boldsymbol{M}_8)\dot{\boldsymbol{q}}\right) + m_4\boldsymbol{g}^T\boldsymbol{r}_{G4}^{0} \\ L_4 &= \frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_4\dot{\boldsymbol{q}}) + m_4\boldsymbol{g}^T\boldsymbol{r}_{G4}^{0} \end{aligned} \qquad 3.60$$

Dónde:

$$\boldsymbol{N}_4 = m_4\boldsymbol{M}_7{}^T\boldsymbol{M}_7 + \boldsymbol{M}_8{}^T\boldsymbol{J}_4\boldsymbol{M}_8$$

Efectuando la derivada:

$$\frac{\partial L_4}{\partial \dot{q}_j} = \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_4\dot{\boldsymbol{q}}) + m_4\boldsymbol{g}^T\boldsymbol{r}_{G4}^0\right)$$
$$\frac{\partial L_4}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_4\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \qquad 3.61$$

Simplificando:

$$\frac{\partial L_4}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_4\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right)$$
$$= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}}\right)$$
$$\frac{\partial L_4}{\partial \dot{q}_j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}} \qquad 3.62$$

Desarrollando $\frac{\partial L_5}{\partial \dot{q}_j}$:

Se tiene:

$$L_5 = \frac{1}{2}\left(m_5\boldsymbol{v}_{G5}^0{}^T\boldsymbol{v}_{G5}^0 + \boldsymbol{\omega}_{18}^0{}^T\boldsymbol{J}_5\boldsymbol{\omega}_{18}^0\right) + m_5\boldsymbol{g}^T\boldsymbol{r}_{G5}^0$$

Sustituyendo ec.(3.42) y ec.(3.44):

$$L_5 = \frac{1}{2}(m_5(\boldsymbol{M}_9\dot{\boldsymbol{q}})^T(\boldsymbol{M}_9\dot{\boldsymbol{q}}) + (\boldsymbol{M}_{10}\dot{\boldsymbol{q}})^T\boldsymbol{J}_5(\boldsymbol{M}_{10}\dot{\boldsymbol{q}})) + m_5\boldsymbol{g}^T\boldsymbol{r}_{G5}^0$$
$$= \frac{1}{2}\left(\dot{\boldsymbol{q}}^T(m_5\boldsymbol{M}_9{}^T\boldsymbol{M}_9 + \boldsymbol{M}_{10}{}^T\boldsymbol{J}_5\boldsymbol{M}_{10})\dot{\boldsymbol{q}}\right) + m_5\boldsymbol{g}^T\boldsymbol{r}_{G5}^0$$
$$L_5 = \frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_5\dot{\boldsymbol{q}}) + m_5\boldsymbol{g}^T\boldsymbol{r}_{G5}^0 \qquad 3.63$$

Dónde:

$$\boldsymbol{N}_5 = m_5\boldsymbol{M}_9{}^T\boldsymbol{M}_9 + \boldsymbol{M}_{10}{}^T\boldsymbol{J}_5\boldsymbol{M}_{10}$$

Efectuando la derivada:

$$\frac{\partial L_5}{\partial \dot{q}_j} = \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T \boldsymbol{N}_5 \dot{\boldsymbol{q}}) + m_5 \boldsymbol{g}^T \boldsymbol{r}_{G5}^0\right)$$
$$\frac{\partial L_5}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j} \boldsymbol{N}_5 \dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \boldsymbol{N}_5 \frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \tag{3.64}$$

Simplificando:

$$\frac{\partial L_5}{\partial \dot{q}_j} = \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j} \boldsymbol{N}_5 \dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \boldsymbol{N}_5 \frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right)$$
$$= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j} \boldsymbol{N}_5 \dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j} \boldsymbol{N}_5 \dot{\boldsymbol{q}}\right)$$
$$\frac{\partial L_5}{\partial \dot{q}_j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j} \boldsymbol{N}_5 \dot{\boldsymbol{q}} \tag{3.65}$$

Al evaluar el término $\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}$, dependerá que valor tome *j*, de tal manera que se tienen los siguientes resultados para diferente valor del iterador *j*. De esta forma, para:

j=1

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_1} = \frac{\partial}{\partial \dot{q}_1}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right] = \frac{\partial}{\partial \dot{x}}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_1} = [1,0,0,0,0,0]$$

j=2

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_2} = \frac{\partial}{\partial \dot{q}_2}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right] = \frac{\partial}{\partial \dot{y}}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_2} = [0,1,0,0,0,0]$$

j=3

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_3} = \frac{\partial}{\partial \dot{q}_3}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right] = \frac{\partial}{\partial \dot{z}}\left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_3} = [0,0,1,0,0,0]$$

j=4

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_4} = \frac{\partial}{\partial \dot{q}_4}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}] = \frac{\partial}{\partial \dot{\psi}}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_4} = [0,0,0,1,0,0]$$

j=5

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_5} = \frac{\partial}{\partial \dot{q}_5}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}] = \frac{\partial}{\partial \dot{\theta}}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_5} = [0,0,0,0,1,0]$$

j=6

$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_6} = \frac{\partial}{\partial \dot{q}_6}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}] = \frac{\partial}{\partial \dot{\phi}}[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}]$$
$$\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_6} = [0,0,0,0,0,1]$$

Por lo tanto, se hace notar que al derivar el término $\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}$ respecto al tiempo se tiene:

$$\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right) = 0$$

Ahora, tomando la ecuación (3.48) y derivando respecto al tiempo se tiene:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = \frac{d}{dt}\left(\sum_{h=1}^{5} \frac{\partial L_h}{\partial \dot{q}_j} + \sum_{i=1}^{2}\left(\frac{\partial L_{1i}}{\partial \dot{q}_j} + \frac{\partial L_{2i}}{\partial \dot{q}_j}\right)\right) \qquad 3.66$$

El desarrollo de la derivada con respecto al tiempo de cada elemento se muestra a continuación:

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{q}_j}\right)$:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\dot{\boldsymbol{q}}\right) \\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_1\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_1}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\frac{d\dot{\boldsymbol{q}}}{dt}\right) \\
&= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_1\dot{\boldsymbol{q}}
\end{aligned}$$
$$\frac{d}{dt}\left(\frac{\partial L_1}{\partial \dot{q}_j}\right) = \mathbb{D}_{1j}\ddot{\boldsymbol{q}} + \mathbb{V}_{1j}\dot{\boldsymbol{q}} \qquad 3.67$$

Dónde:

$$\begin{aligned}\mathbb{D}_{1j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_1 \\ \mathbb{V}_{1j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_1\end{aligned} \qquad 3.68$$

Además:

$$\begin{aligned}
\boldsymbol{N}_1 &= m_1 {\boldsymbol{M}_1}^T \boldsymbol{M}_1 + {\boldsymbol{M}_2}^T \boldsymbol{J}_1 \boldsymbol{M}_2 \\
\boldsymbol{M}_1 &= \boldsymbol{k}_1 \boldsymbol{k}_2^T \\
\boldsymbol{M}_2 &= \boldsymbol{j}_1^0 \boldsymbol{k}_2^T \\
\boldsymbol{J}_1^0 &= \boldsymbol{R}_2^0\, \boldsymbol{J}_1^2\, {\boldsymbol{R}_2^0}^T \\
\boldsymbol{k}_1 &= \boldsymbol{j}_1^0 \times \boldsymbol{r}_{G1'}^0 \\
\boldsymbol{k}_2^T &= \frac{1}{V_1}[V_2, V_3, V_4, V_5, V_6, V_7] \\
\boldsymbol{r}_{G1'}^0 &= \boldsymbol{R}_2^0 \boldsymbol{r}_{G1'}^2 \\
\boldsymbol{R}_2^0 &= \boldsymbol{R}_y(\theta_{21}) \\
\boldsymbol{j}_1^0 &= \boldsymbol{j}_0 \\
\boldsymbol{r}_{G1'}^2 &= [x_{G1}, y_{G1}, z_{G1}]^T
\end{aligned}$$

Derivando:

$$\dot{\boldsymbol{N}}_1 = m_1\left({\dot{\boldsymbol{M}}_1}^T \boldsymbol{M}_1 + {\boldsymbol{M}_1}^T \dot{\boldsymbol{M}}_1\right) + \left({\dot{\boldsymbol{M}}_2}^T \boldsymbol{J}_1 \boldsymbol{M}_2 + {\boldsymbol{M}_2}^T \dot{\boldsymbol{J}}_1 \boldsymbol{M}_2 + {\boldsymbol{M}_2}^T \boldsymbol{J}_1 \dot{\boldsymbol{M}}_2\right)$$

A su vez:

$$\begin{aligned}
\dot{\boldsymbol{M}}_1 &= \dot{\boldsymbol{k}}_1 \boldsymbol{k}_2^T + \boldsymbol{k}_1 \dot{\boldsymbol{k}}_2^T \\
\dot{\boldsymbol{M}}_2 &= \boldsymbol{j}_1^0 \dot{\boldsymbol{k}}_2^T \\
\dot{\boldsymbol{J}}_1^0 &= \dot{\boldsymbol{R}}_2^0\, \boldsymbol{J}_1^2\, {\boldsymbol{R}_2^0}^T + \boldsymbol{R}_2^0\, \boldsymbol{J}_1^2\, {\dot{\boldsymbol{R}}_2^0}{}^T \\
\dot{\boldsymbol{k}}_1 &= \boldsymbol{j}_1^0 \times \dot{\boldsymbol{r}}_{G1'}^0 \\
\dot{\boldsymbol{k}}_2^T &= \frac{d}{dt}\left(V_1^{-1}[V_2, V_3, V_4, V_5, V_6, V_7]\right) \\
&= -V_1^{-2}\dot{V}_1[V_2, V_3, V_4, V_5, V_6, V_7] + V_1^{-1}\left[\dot{V}_2, \dot{V}_3, \dot{V}_4, \dot{V}_5, \dot{V}_6, \dot{V}_7\right] \\
\dot{\boldsymbol{k}}_2^T &= -\frac{1}{V_1^2}\dot{V}_1[V_2, V_3, V_4, V_5, V_6, V_7] + \frac{1}{V_1}\left[\dot{V}_2, \dot{V}_3, \dot{V}_4, \dot{V}_5, \dot{V}_6, \dot{V}_7\right] \\
\dot{\boldsymbol{r}}_{G1'}^0 &= \dot{\boldsymbol{R}}_2^0 \boldsymbol{r}_{G1'}^2 \\
\dot{\boldsymbol{R}}_2^0 &= \boldsymbol{B}_y(\theta_{21})\dot{\theta}_{21} \\
\boldsymbol{B}_y(\theta_{21}) &= \frac{d\boldsymbol{R}_y(\theta_{21})}{d\theta_{21}}
\end{aligned}$$

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{q}_j}\right)$:

$$\frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{q}_j}\right) = \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\dot{\boldsymbol{q}}\right)$$

$$= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_2\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_2}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\frac{d\dot{\boldsymbol{q}}}{dt}\right)$$

$$= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_2\dot{\boldsymbol{q}}$$

$$\frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{q}_j}\right) = \mathbb{D}_{2j}\ddot{\boldsymbol{q}} + \mathbb{V}_{2j}\dot{\boldsymbol{q}} \qquad 3.69$$

Dónde:

$$\mathbb{D}_{2j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_2$$
$$\mathbb{V}_{2j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_2 \qquad 3.70$$

Además:

$$\begin{aligned}
\boldsymbol{N}_2 &= m_2{\boldsymbol{M}_3}^T\boldsymbol{M}_3 + {\boldsymbol{M}_4}^T\boldsymbol{J}_2\boldsymbol{M}_4\\
\boldsymbol{M}_3 &= \boldsymbol{k}_3\boldsymbol{k}_2^T + \boldsymbol{k}_4\boldsymbol{k}_6^T + \boldsymbol{k}_5\boldsymbol{k}_7^T\\
\boldsymbol{M}_4 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T\\
\boldsymbol{J}_2^0 &= \boldsymbol{R}_5^0\,\boldsymbol{J}_2^5\,{\boldsymbol{R}_5^0}^T\\
\boldsymbol{k}_3 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0\right)\\
\boldsymbol{k}_4 &= \boldsymbol{j}_3^0 \times \boldsymbol{r}_{G2'}^0\\
\boldsymbol{k}_5 &= \boldsymbol{k}_4^0 \times \boldsymbol{r}_{G2'}^0\\
\boldsymbol{k}_6^T &= \frac{1}{V_8}[E_1, E_2, E_3, E_4, E_5, E_6]\\
\boldsymbol{k}_7^T &= \frac{1}{V_{15}}[E_7, E_8, E_9, E_{10}, E_{11}, E_{12}]\\
\boldsymbol{r}_{G2'}^0 &= \boldsymbol{R}_5^0\boldsymbol{r}_{G2'}^5\\
\boldsymbol{k}_4^0 &= \boldsymbol{R}_4^0\boldsymbol{k}_4^4\\
\boldsymbol{r}_{32}^0 &= \boldsymbol{R}_2^0\boldsymbol{r}_{32}^2\\
\boldsymbol{R}_4^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\\
\boldsymbol{R}_5^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\\
\boldsymbol{j}_3^0 &= \boldsymbol{j}_0
\end{aligned}$$

Derivando:

$$\dot{\boldsymbol{N}}_2 = m_2\left({\dot{\boldsymbol{M}}_3}^T\boldsymbol{M}_3 + {\boldsymbol{M}_3}^T\dot{\boldsymbol{M}}_3\right) + ({\dot{\boldsymbol{M}}_4}^T\boldsymbol{J}_2\boldsymbol{M}_4 + {\boldsymbol{M}_4}^T\dot{\boldsymbol{J}}_2\boldsymbol{M}_4 + {\boldsymbol{M}_4}^T\boldsymbol{J}_2\dot{\boldsymbol{M}}_4)$$

A su vez:

$$\begin{aligned}
\dot{\boldsymbol{M}}_3 &= \dot{\boldsymbol{k}}_3\boldsymbol{k}_2^T + \boldsymbol{k}_3\dot{\boldsymbol{k}}_2^T + \dot{\boldsymbol{k}}_4\boldsymbol{k}_6^T + \boldsymbol{k}_4\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_5\boldsymbol{k}_7^T + \boldsymbol{k}_5\dot{\boldsymbol{k}}_7^T\\
\dot{\boldsymbol{M}}_4 &= \boldsymbol{j}_1^0\dot{\boldsymbol{k}}_2^T + \boldsymbol{j}_3^0\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_4^0\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\dot{\boldsymbol{k}}_7^T
\end{aligned}$$

$$
\begin{aligned}
\dot{\boldsymbol{J}}_2^0 &= \dot{\boldsymbol{R}}_5^0 \boldsymbol{J}_2^5 \boldsymbol{R}_5^{0^T} + \boldsymbol{R}_5^0 \boldsymbol{J}_2^5 \dot{\boldsymbol{R}}_5^{0^T} \\
\dot{\boldsymbol{k}}_3 &= \boldsymbol{j}_1^0 \times \left(\dot{\boldsymbol{r}}_{32}^0 + \dot{\boldsymbol{r}}_{G2'}^0\right) \\
\dot{\boldsymbol{k}}_4 &= \boldsymbol{j}_3^0 \times \dot{\boldsymbol{r}}_{G2'}^0 \\
\dot{\boldsymbol{k}}_5 &= \dot{\boldsymbol{k}}_4^0 \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0 \times \dot{\boldsymbol{r}}_{G2'}^0 \\
\dot{\boldsymbol{k}}_6^T &= -\frac{1}{V_8^2}\dot{V}_8[E_1, E_2, E_3, E_4, E_5, E_6] + \frac{1}{V_8}\left[\dot{E}_1, \dot{E}_2, \dot{E}_3, \dot{E}_4, \dot{E}_5, \dot{E}_6\right] \\
\dot{\boldsymbol{k}}_7^T &= -\frac{1}{V_{15}^2}\dot{V}_{15}[E_7, E_8, E_9, E_{10}, E_{11}, E_{12}] + \frac{1}{V_{15}}\left[\dot{E}_7, \dot{E}_8, \dot{E}_9, \dot{E}_{10}, \dot{E}_{11}, \dot{E}_{12}\right] \\
\dot{\boldsymbol{r}}_{32}^0 &= \dot{\boldsymbol{R}}_2^0 \boldsymbol{r}_{32}^2 \\
\dot{\boldsymbol{r}}_{G2'}^0 &= \dot{\boldsymbol{R}}_5^0 \boldsymbol{r}_{G2'}^5 \\
\dot{\boldsymbol{k}}_4^0 &= \dot{\boldsymbol{R}}_4^0 \boldsymbol{k}_4^4 \\
\dot{\boldsymbol{R}}_5^0 &= \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\dot{\theta}_{21} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\dot{\theta}_{43} \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\dot{\theta}_{54} \\
\boldsymbol{B}_y(\theta_{43}) &= \frac{d\boldsymbol{R}_y(\theta_{43})}{d\theta_{43}} \\
\boldsymbol{B}_z(\theta_{54}) &= \frac{d\boldsymbol{R}_z(\theta_{54})}{d\theta_{54}} \\
\dot{\boldsymbol{R}}_4^0 &= \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\dot{\theta}_{21} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\dot{\theta}_{43}
\end{aligned}
$$

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_3}{\partial \dot{q}_j}\right)$:

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_3}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\dot{\boldsymbol{q}}\right) \\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_3\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_3}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\frac{d\dot{\boldsymbol{q}}}{dt}\right) \\
&= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_3\dot{\boldsymbol{q}} \\
\frac{d}{dt}\left(\frac{\partial L_3}{\partial \dot{q}_j}\right) &= \mathbb{D}_{3j}\ddot{\boldsymbol{q}} + \mathbb{V}_{3j}\dot{\boldsymbol{q}}
\end{aligned}
\qquad 3.71
$$

Dónde:

$$
\begin{aligned}
\mathbb{D}_{3j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_3 \\
\mathbb{V}_{3j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_3
\end{aligned}
\qquad 3.72
$$

Además:

$$
\begin{aligned}
\boldsymbol{N}_3 &= m_3\boldsymbol{M}_5{}^T\boldsymbol{M}_5 + \boldsymbol{M}_6{}^T\boldsymbol{J}_3\boldsymbol{M}_6 \\
\boldsymbol{M}_5 &= \boldsymbol{k}_8\boldsymbol{k}_2^T + \boldsymbol{k}_9\boldsymbol{k}_6^T + \boldsymbol{k}_{10}\boldsymbol{k}_7^T + \boldsymbol{k}_{11}\boldsymbol{k}_{12}^T
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{M}_6 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T \\
\boldsymbol{J}_3^0 &= \boldsymbol{R}_{16}^0\boldsymbol{J}_3^{16}\,{\boldsymbol{R}_{16}^0}^T \\
\boldsymbol{k}_8 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0\right) \\
\boldsymbol{k}_9 &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right) \\
\boldsymbol{k}_{10} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right) \\
\boldsymbol{k}_{11} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G3'}^0 \\
\boldsymbol{k}_{12}^T &= \frac{1}{V_{36}}[E_{25}, E_{26}, E_{27}, E_{28}, E_{29}, E_{30}] \\
\boldsymbol{i}_{15}^0 &= \boldsymbol{R}_{15}^0\boldsymbol{i}_{15}^{15} \\
\boldsymbol{r}_{14,5i}^0 &= \boldsymbol{R}_5^0\boldsymbol{r}_{14,5i}^5 \\
\boldsymbol{r}_{15,14i}^0 &= \boldsymbol{R}_{14}^0\boldsymbol{r}_{15,14i}^{14} \\
\boldsymbol{r}_{G3'}^0 &= \boldsymbol{R}_{16}^0\boldsymbol{r}_{G3'}^{16} \\
\boldsymbol{R}_{14}^0 &= \boldsymbol{R}_5^0 \\
\boldsymbol{R}_{15}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54}) \\
\boldsymbol{R}_{16}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x\left(\theta_{16,15}\right)
\end{aligned}
$$

Derivando:

$$
\dot{\boldsymbol{N}}_3 = m_3\left({\dot{\boldsymbol{M}}_5}^T\boldsymbol{M}_5 + {\boldsymbol{M}_5}^T\dot{\boldsymbol{M}}_5\right) + ({\dot{\boldsymbol{M}}_6}^T\boldsymbol{J}_3\boldsymbol{M}_6 + {\boldsymbol{M}_6}^T\dot{\boldsymbol{J}}_3\boldsymbol{M}_6 + {\boldsymbol{M}_6}^T\boldsymbol{J}_3\dot{\boldsymbol{M}}_6)
$$

A su vez:

$$
\begin{aligned}
\dot{\boldsymbol{M}}_5 &= \dot{\boldsymbol{k}}_8\boldsymbol{k}_2^T + \boldsymbol{k}_8\dot{\boldsymbol{k}}_2^T + \dot{\boldsymbol{k}}_9\boldsymbol{k}_6^T + \boldsymbol{k}_9\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_{10}\boldsymbol{k}_7^T + \boldsymbol{k}_{10}\dot{\boldsymbol{k}}_7^T + \dot{\boldsymbol{k}}_{11}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{11}\dot{\boldsymbol{k}}_{12}^T \\
\dot{\boldsymbol{M}}_6 &= \boldsymbol{j}_1^0\dot{\boldsymbol{k}}_2^T + \boldsymbol{j}_3^0\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_4^0\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\dot{\boldsymbol{k}}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0\dot{\boldsymbol{k}}_{12}^T \\
\dot{\boldsymbol{J}}_3^0 &= \dot{\boldsymbol{R}}_{16}^0\boldsymbol{J}_3^{16}\,{\boldsymbol{R}_{16}^0}^T + \boldsymbol{R}_{16}^0\boldsymbol{J}_3^{16}\,{\dot{\boldsymbol{R}}_{16}^0}^T \\
\dot{\boldsymbol{k}}_8 &= \boldsymbol{j}_1^0 \times \left(\dot{\boldsymbol{r}}_{32}^0 + \dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0\right) \\
\dot{\boldsymbol{k}}_9 &= \boldsymbol{j}_3^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G3'}^0\right) \\
\dot{\boldsymbol{k}}_{10} &= \dot{\boldsymbol{k}}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right) + \boldsymbol{k}_4^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G3'}^0\right) \\
\dot{\boldsymbol{k}}_{11} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G3'}^0 + \boldsymbol{i}_{15}^0 \times \dot{\boldsymbol{r}}_{G3'}^0 \\
\dot{\boldsymbol{k}}_{12}^T &= -\frac{1}{V_{36}^2}\dot{V}_{36}[E_{25}, E_{26}, E_{27}, E_{28}, E_{29}, E_{30}] + \frac{1}{V_{36}}\left[\dot{E}_{25}, \dot{E}_{26}, \dot{E}_{27}, \dot{E}_{28}, \dot{E}_{29}, \dot{E}_{30}\right] \\
\dot{\boldsymbol{r}}_{14,5i}^0 &= \dot{\boldsymbol{R}}_5^0\boldsymbol{r}_{14,5i}^5 \\
\dot{\boldsymbol{r}}_{15,14i}^0 &= \dot{\boldsymbol{R}}_{14}^0\boldsymbol{r}_{15,14i}^{14} \\
\dot{\boldsymbol{r}}_{G3'}^0 &= \dot{\boldsymbol{R}}_{16}^0\boldsymbol{r}_{G3'}^{16} \\
\boldsymbol{i}_{15}^0 &= \dot{\boldsymbol{R}}_{15}^0\boldsymbol{i}_{15}^{15} \\
\dot{\boldsymbol{R}}_{15}^0 &= \dot{\boldsymbol{R}}_5^0 \\
\dot{\boldsymbol{R}}_{16}^0 &= \boldsymbol{B}_y(\theta_{21})\dot{\theta}_{21}\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x\left(\theta_{16,15}\right) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\dot{\theta}_{43}\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x\left(\theta_{16,15}\right) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\dot{\theta}_{54}\boldsymbol{R}_x\left(\theta_{16,15}\right) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x\left(\theta_{16,15}\right)\dot{\theta}_{16,15}
\end{aligned}
$$

$$\boldsymbol{B}_x(\theta_{16,15}) = \frac{d\boldsymbol{R}_x(\theta_{16,15})}{d\theta_{16,15}}$$

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_4}{\partial \dot{q}_j}\right)$:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_4}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\dot{\boldsymbol{q}}\right) \\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_4\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_4}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\frac{d\dot{\boldsymbol{q}}}{dt}\right) \\
&= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_4\dot{\boldsymbol{q}} \\
\frac{d}{dt}\left(\frac{\partial L_4}{\partial \dot{q}_j}\right) &= \mathbb{D}_{4j}\ddot{\boldsymbol{q}} + \mathbb{V}_{4j}\dot{\boldsymbol{q}}
\end{aligned} \qquad 3.73$$

Dónde:

$$\begin{aligned}
\mathbb{D}_{4j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_4 \\
\mathbb{V}_{4j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_4
\end{aligned} \qquad 3.74$$

Además:

$$\begin{aligned}
\boldsymbol{N}_4 &= m_4\boldsymbol{M}_7{}^T\boldsymbol{M}_7 + \boldsymbol{M}_8{}^T\boldsymbol{J}_4\boldsymbol{M}_8 \\
\boldsymbol{M}_7 &= \boldsymbol{k}_{13}\boldsymbol{k}_2^T + \boldsymbol{k}_{14}\boldsymbol{k}_6^T + \boldsymbol{k}_{15}\boldsymbol{k}_7^T + \boldsymbol{k}_{16}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{17}\boldsymbol{k}_{18}^T \\
\boldsymbol{M}_8 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T \\
\boldsymbol{J}_4^0 &= \boldsymbol{R}_{17}^0\boldsymbol{J}_4^{17}\boldsymbol{R}_{17}^0{}^T \\
\boldsymbol{k}_{13} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{14} &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{15} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{16} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G4'}^0 \\
\boldsymbol{k}_{17} &= \boldsymbol{k}_{16}^0 \times \boldsymbol{r}_{G4'}^0 \\
\boldsymbol{k}_{18}^T &= \frac{1}{V_{29}}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] \\
\boldsymbol{r}_{G4'}^0 &= \boldsymbol{R}_{17}^0\boldsymbol{r}_{G4'}^{17} \\
\boldsymbol{k}_{16}^0 &= \boldsymbol{R}_{16}^0\boldsymbol{k}_{16}^{16} \\
\boldsymbol{R}_{17}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})
\end{aligned}$$

Derivando:

$$\dot{\boldsymbol{N}}_4 = m_4\left(\dot{\boldsymbol{M}}_7{}^T\boldsymbol{M}_7 + \boldsymbol{M}_7{}^T\dot{\boldsymbol{M}}_7\right) + (\dot{\boldsymbol{M}}_8{}^T\boldsymbol{J}_4\boldsymbol{M}_8 + \boldsymbol{M}_8{}^T\dot{\boldsymbol{J}}_4\boldsymbol{M}_8 + \boldsymbol{M}_8{}^T\boldsymbol{J}_4\dot{\boldsymbol{M}}_8)$$

A su vez:

$$\begin{aligned}
\dot{\boldsymbol{M}}_7 &= \dot{\boldsymbol{k}}_{13}\boldsymbol{k}_2^T + \boldsymbol{k}_{13}\dot{\boldsymbol{k}}_2^T + \dot{\boldsymbol{k}}_{14}\boldsymbol{k}_6^T + \boldsymbol{k}_{14}\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_{15}\boldsymbol{k}_7^T + \boldsymbol{k}_{15}\dot{\boldsymbol{k}}_7^T + \dot{\boldsymbol{k}}_{16}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}\dot{\boldsymbol{k}}_{12}^T \\
&\quad + \dot{\boldsymbol{k}}_{17}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{17}\dot{\boldsymbol{k}}_{18}^T \\
\dot{\boldsymbol{M}}_8 &= \boldsymbol{j}_1^0\dot{\boldsymbol{k}}_2^T + \boldsymbol{j}_3^0\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_4^0\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\dot{\boldsymbol{k}}_7^T + \dot{\boldsymbol{i}}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0\dot{\boldsymbol{k}}_{12}^T + \dot{\boldsymbol{k}}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{k}_{16}^0\dot{\boldsymbol{k}}_{18}^T \\
\dot{\boldsymbol{J}}_4^0 &= \dot{\boldsymbol{R}}_{17}^0\boldsymbol{J}_4^{17}\,\boldsymbol{R}_{17}^0{}^T + \boldsymbol{R}_{17}^0\boldsymbol{J}_4^{17}\,\dot{\boldsymbol{R}}_{17}^0{}^T \\
\dot{\boldsymbol{k}}_{13} &= \boldsymbol{j}_1^0 \times \left(\dot{\boldsymbol{r}}_{32}^0 + \dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G4'}^0\right) \\
\dot{\boldsymbol{k}}_{14} &= \boldsymbol{j}_3^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G4'}^0\right) \\
\dot{\boldsymbol{k}}_{15} &= \dot{\boldsymbol{k}}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) + \boldsymbol{k}_4^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G4'}^0\right) \\
\dot{\boldsymbol{k}}_{16} &= \dot{\boldsymbol{i}}_{15}^0 \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{i}_{15}^0 \times \dot{\boldsymbol{r}}_{G4'}^0 \\
\dot{\boldsymbol{k}}_{17} &= \dot{\boldsymbol{k}}_{16}^0 \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_{16}^0 \times \dot{\boldsymbol{r}}_{G4'}^0 \\
\dot{\boldsymbol{k}}_{16}^0 &= \dot{\boldsymbol{R}}_{16}^0\boldsymbol{k}_{16}^{16} \\
\dot{\boldsymbol{k}}_{18}^T &= -\frac{1}{V_{29}^2}\dot{V}_{29}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] + \frac{1}{V_{29}}\left[\dot{E}_{19}, \dot{E}_{20}, \dot{E}_{21}, \dot{E}_{22}, \dot{E}_{23}, \dot{E}_{24}\right] \\
\dot{\boldsymbol{r}}_{G4'}^0 &= \dot{\boldsymbol{R}}_{17}^0\boldsymbol{r}_{G4'}^{17} \\
\dot{\boldsymbol{R}}_{17}^0 &= \boldsymbol{B}_y(\theta_{21})\dot{\theta}_{21}\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\dot{\theta}_{43}\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\dot{\theta}_{54}\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\dot{\theta}_{16,15}\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\dot{\theta}_{17,16} \\
\boldsymbol{B}_z(\theta_{17,16}) &= \frac{d\boldsymbol{R}_z(\theta_{17,16})}{d\theta_{17,16}}
\end{aligned}$$

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_5}{\partial \dot{q}_j}\right)$:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_5}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_5\dot{\boldsymbol{q}}\right) \\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_5\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_5}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_5\frac{d\dot{\boldsymbol{q}}}{dt}\right) \\
&= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_5\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_5\dot{\boldsymbol{q}}
\end{aligned}$$

$$\frac{d}{dt}\left(\frac{\partial L_5}{\partial \dot{q}_j}\right) = \mathbb{D}_{5j}\ddot{\boldsymbol{q}} + \mathbb{V}_{5j}\dot{\boldsymbol{q}} \qquad 3.75$$

Dónde:

$$\mathbb{D}_{5j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_5$$
$$\mathbb{V}_{5j} = \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_5 \qquad 3.76$$

Además:

$$\begin{aligned}
\boldsymbol{N}_5 &= m_5\boldsymbol{M}_9{}^T\boldsymbol{M}_9 + \boldsymbol{M}_{10}{}^T\boldsymbol{J}_5\boldsymbol{M}_{10} \\
\boldsymbol{M}_9 &= \boldsymbol{k}_{19}\boldsymbol{k}_2^T + \boldsymbol{k}_{20}\boldsymbol{k}_6^T + \boldsymbol{k}_{21}\boldsymbol{k}_7^T + \boldsymbol{k}_{22}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{23}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{24}\boldsymbol{k}_{25}^T \\
\boldsymbol{M}_{10} &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{i}_{17}^0\boldsymbol{k}_{25}^T \\
\boldsymbol{J}_5^0 &= \boldsymbol{R}_{18}^0\,\boldsymbol{J}_5^{18}\,\boldsymbol{R}_{18}^0{}^T \\
\boldsymbol{k}_{19} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{20} &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{21} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) \\
\boldsymbol{k}_{22} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G5'}^0 \\
\boldsymbol{k}_{23} &= \boldsymbol{k}_{16}^0 \times \boldsymbol{r}_{G5'}^0 \\
\boldsymbol{k}_{24} &= \boldsymbol{i}_{17}^0 \times \boldsymbol{r}_{G5'}^0 \\
\boldsymbol{k}_{25}^T &= \frac{1}{V_{23}}[E_{13}, E_{14}, E_{15}, E_{16}, E_{17}, E_{18}] \\
\boldsymbol{i}_{17}^0 &= \boldsymbol{R}_{17}^0\boldsymbol{i}_{17}^{17} \\
\boldsymbol{r}_{G5'}^0 &= \boldsymbol{R}_{18}^0\boldsymbol{r}_{G5'}^{18} \\
\boldsymbol{R}_{18}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17}) \\
\boldsymbol{r}_{G5'}^{18} &= [x_{G5}, y_{G5}, z_{G5}]^T
\end{aligned}$$

Derivando:

$$\dot{\boldsymbol{N}}_5 = m_5\left(\dot{\boldsymbol{M}}_9{}^T\boldsymbol{M}_9 + \boldsymbol{M}_9{}^T\dot{\boldsymbol{M}}_9\right) + (\dot{\boldsymbol{M}}_{10}{}^T\boldsymbol{J}_5\boldsymbol{M}_{10} + \boldsymbol{M}_{10}{}^T\dot{\boldsymbol{J}}_5\boldsymbol{M}_{10} + \boldsymbol{M}_{10}{}^T\boldsymbol{J}_5\dot{\boldsymbol{M}}_{10})$$

A su vez:

$$\begin{aligned}
\dot{\boldsymbol{M}}_9 &= \dot{\boldsymbol{k}}_{19}\boldsymbol{k}_2^T + \boldsymbol{k}_{19}\dot{\boldsymbol{k}}_2^T + \dot{\boldsymbol{k}}_{20}\boldsymbol{k}_6^T + \boldsymbol{k}_{20}\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_{21}\boldsymbol{k}_7^T + \boldsymbol{k}_{21}\dot{\boldsymbol{k}}_7^T + \dot{\boldsymbol{k}}_{22}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{22}\dot{\boldsymbol{k}}_{12}^T + \dot{\boldsymbol{k}}_{23}\boldsymbol{k}_{18}^T \\
&\quad + \boldsymbol{k}_{23}\dot{\boldsymbol{k}}_{18}^T + \dot{\boldsymbol{k}}_{24}\boldsymbol{k}_{25}^T + \boldsymbol{k}_{24}\dot{\boldsymbol{k}}_{25}^T \\
\dot{\boldsymbol{M}}_{10} &= \boldsymbol{j}_1^0\dot{\boldsymbol{k}}_2^T + \boldsymbol{j}_3^0\dot{\boldsymbol{k}}_6^T + \dot{\boldsymbol{k}}_4^0\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\dot{\boldsymbol{k}}_7^T + \dot{\boldsymbol{i}}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0\dot{\boldsymbol{k}}_{12}^T + \dot{\boldsymbol{k}}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{k}_{16}^0\dot{\boldsymbol{k}}_{18}^T + \dot{\boldsymbol{i}}_{17}^0\boldsymbol{k}_{25}^T \\
&\quad + \boldsymbol{i}_{17}^0\dot{\boldsymbol{k}}_{25}^T \\
\dot{\boldsymbol{J}}_5^0 &= \dot{\boldsymbol{R}}_{18}^0\,\boldsymbol{J}_5^{18}\,\boldsymbol{R}_{18}^0{}^T + \boldsymbol{R}_{18}^0\,\boldsymbol{J}_5^{18}\,\dot{\boldsymbol{R}}_{18}^0{}^T \\
\dot{\boldsymbol{k}}_{19} &= \boldsymbol{j}_1^0 \times \left(\dot{\boldsymbol{r}}_{32}^0 + \dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G5'}^0\right) \\
\dot{\boldsymbol{k}}_{20} &= \boldsymbol{j}_3^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G5'}^0\right) \\
\dot{\boldsymbol{k}}_{21} &= \dot{\boldsymbol{k}}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) + \boldsymbol{k}_4^0 \times \left(\dot{\boldsymbol{r}}_{14,5i}^0 + \dot{\boldsymbol{r}}_{15,14i}^0 + \dot{\boldsymbol{r}}_{G5'}^0\right) \\
\dot{\boldsymbol{k}}_{22} &= \dot{\boldsymbol{i}}_{15}^0 \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{i}_{15}^0 \times \dot{\boldsymbol{r}}_{G5'}^0 \\
\dot{\boldsymbol{k}}_{23} &= \dot{\boldsymbol{k}}_{16}^0 \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{k}_{16}^0 \times \dot{\boldsymbol{r}}_{G5'}^0 \\
\dot{\boldsymbol{k}}_{24} &= \dot{\boldsymbol{i}}_{17}^0 \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{i}_{17}^0 \times \dot{\boldsymbol{r}}_{G5'}^0
\end{aligned}$$

$$\dot{\boldsymbol{k}}_{25}^{T} = -\frac{1}{V_{23}^{2}}\dot{V}_{23}[E_{13}, E_{14}, E_{15}, E_{16}, E_{17}, E_{18}] + \frac{1}{V_{23}}[\dot{E}_{13}, \dot{E}_{14}, \dot{E}_{15}, \dot{E}_{16}, \dot{E}_{17}, \dot{E}_{18}]$$
$$\dot{\boldsymbol{i}}_{17}^{0} = \dot{\boldsymbol{R}}_{17}^{0}\boldsymbol{i}_{17}^{17}$$
$$\dot{\boldsymbol{r}}_{G5'}^{0} = \dot{\boldsymbol{R}}_{18}^{0}\boldsymbol{r}_{G5'}^{18}$$
$$\begin{aligned}\dot{\boldsymbol{R}}_{18}^{0} = {} & \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\dot{\theta}_{21} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\dot{\theta}_{43} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\dot{\theta}_{54} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\dot{\theta}_{16,15} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\dot{\theta}_{17,16} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\dot{\theta}_{18,17}\end{aligned}$$

3.3.2. Desarrollo del Segundo Término de la Ecuación de Lagrange para la cadena 0

Tomando la ec. (3.47) y aplicando la derivada parcial con respecto a la variable de coordenadas cartesianas:

$$\frac{\partial L}{\partial q_j} = \sum_{h=1}^{5}\frac{\partial L_h}{\partial q_j} + \sum_{i=1}^{2}\left(\frac{\partial L_{1i}}{\partial q_j} + \frac{\partial L_{2i}}{\partial q_j}\right)$$

Desarrollando $\frac{\partial L_1}{\partial q_j}$

$$L_1 = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{N}_1\dot{\boldsymbol{q}} + m_1\boldsymbol{g}^T\boldsymbol{r}_{G1}^{0}$$

Dónde:

$$\begin{aligned}\boldsymbol{N}_1 &= m_1{\boldsymbol{M}_1}^T\boldsymbol{M}_1 + {\boldsymbol{M}_2}^T\boldsymbol{J}_1\boldsymbol{M}_2 \\ \boldsymbol{M}_1 &= \boldsymbol{k}_1\boldsymbol{k}_2^T \\ \boldsymbol{M}_2 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T \\ \boldsymbol{J}_1^0 &= \boldsymbol{R}_2^0\,\boldsymbol{J}_1^2\,{\boldsymbol{R}_2^0}^T \\ \boldsymbol{k}_1 &= \boldsymbol{j}_1^0 \times \boldsymbol{r}_{G1'}^0 \\ \boldsymbol{k}_2^T &= \frac{1}{V_1}[V_2, V_3, V_4, V_5, V_6, V_7] \\ \boldsymbol{r}_{G1'}^0 &= \boldsymbol{R}_2^0\boldsymbol{r}_{G1'}^2 \\ \boldsymbol{R}_2^0 &= \boldsymbol{R}_y(\theta_{21}) \\ \boldsymbol{j}_1^0 &= \boldsymbol{j}_0 \\ \boldsymbol{r}_{G1'}^2 &= [x_{G1}, y_{G1}, z_{G1}]^T\end{aligned}$$

Derivando con respecto a q_j :

$$\frac{\partial L_1}{\partial q_j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_1}{\partial q_j}\dot{\boldsymbol{q}} + m_1 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G1}^0}{\partial q_j}$$
$$\frac{\partial L_1}{\partial q_j} = {\mathbb{V}'_{1j}}^T \dot{\boldsymbol{q}} + \mathbb{C}_{1j} \qquad 3.77$$

Dónde:

$$\mathbb{V}'_{1j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_1}{\partial q_j}$$
$$\mathbb{C}_{1j} = m_1 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G1}^0}{\partial q_j} \qquad 3.78$$

A su vez:

$$\frac{\partial \boldsymbol{N}_1}{\partial q_j} = m_1\left(\frac{\partial {\boldsymbol{M}_1}^T}{\partial q_j}\boldsymbol{M}_1 + {\boldsymbol{M}_1}^T \frac{\partial \boldsymbol{M}_1}{\partial q_j}\right) + \frac{\partial {\boldsymbol{M}_2}^T}{\partial q_j}\boldsymbol{J}_1 \boldsymbol{M}_2 + {\boldsymbol{M}_2}^T \frac{\partial \boldsymbol{J}_1}{\partial q_j}\boldsymbol{M}_2 + {\boldsymbol{M}_2}^T \boldsymbol{J}_1 \frac{\partial \boldsymbol{M}_2}{\partial q_j}$$
$$\frac{\partial \boldsymbol{M}_1}{\partial q_j} = \frac{\partial \boldsymbol{k}_1}{\partial q_j}\boldsymbol{k}_2^T + \boldsymbol{k}_1 \frac{\partial \boldsymbol{k}_2^T}{\partial q_j}$$
$$\frac{\partial \boldsymbol{M}_2}{\partial q_j} = \boldsymbol{j}_1^0 \frac{\partial \boldsymbol{k}_2^T}{\partial q_j}$$
$$\frac{\partial \boldsymbol{J}_1^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_2^0}{\partial q_j}\boldsymbol{J}_1^2 {\boldsymbol{R}_2^0}^T + \boldsymbol{R}_2^0 \boldsymbol{J}_1^2 \frac{\partial {\boldsymbol{R}_2^0}^T}{\partial q_j}$$
$$\frac{\partial \boldsymbol{k}_1}{\partial q_j} = \boldsymbol{j}_1^0 \times \frac{\partial \boldsymbol{r}_{G1'}^0}{\partial q_j}$$
$$\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} = -\frac{1}{V_1^2}\frac{\partial V_1}{\partial q_j}[V_2, V_3, V_4, V_5, V_6, V_7] + \frac{1}{V_1}\left[\frac{\partial V_2}{\partial q_j}, \frac{\partial V_3}{\partial q_j}, \frac{\partial V_4}{\partial q_j}, \frac{\partial V_5}{\partial q_j}, \frac{\partial V_6}{\partial q_j}, \frac{\partial V_7}{\partial q_j}\right]$$
$$\frac{\partial \boldsymbol{r}_{10}^0}{\partial q_j} = 0$$
$$\frac{\partial \boldsymbol{r}_{G1'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_2^0}{\partial q_j}\boldsymbol{r}_{G1'}^2$$
$$\frac{\partial \boldsymbol{r}_{G1}^0}{\partial q_j} = \frac{\partial \boldsymbol{r}_{10}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G1'}^0}{\partial q_j}$$
$$\frac{\partial \boldsymbol{R}_2^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\frac{\partial \theta_{21}}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_1}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_1}{\partial q_j} = \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\frac{\partial \theta_{21}}{\partial q_j}\boldsymbol{r}_{G1'}^2 = \left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{G1'}^2\right)\frac{\partial \theta_{21}}{\partial q_j}$$
$$\frac{\partial \boldsymbol{k}_1}{\partial q_j} = \boldsymbol{J}_1 \frac{\partial \boldsymbol{\theta}}{\partial q_j} \qquad 3.79$$

Dónde:

$$\boldsymbol{J}_1 = \left[\left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{G1'}^2\right) \quad \boldsymbol{0} \quad \boldsymbol{0}\right]$$

Desarrollando $\frac{\partial \boldsymbol{k}_2^T}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} = \frac{\partial \boldsymbol{k}_2^T}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial \boldsymbol{k'}_2^T}{\partial q_j}$$
$$\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} = \boldsymbol{J}_2\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \frac{\partial \boldsymbol{k'}_2^T}{\partial q_j} \qquad 3.80$$

Dónde:

$$\boldsymbol{J}_2 = \left[\frac{\partial \boldsymbol{k}_2^T}{\partial \theta_{21}} \quad \boldsymbol{0} \quad \boldsymbol{0}\right]$$

De manera general cada V es función de las coordenadas generalizadas $\boldsymbol{q}$ y de los ángulos de la cadena, es decir:

$$V = V(\theta_{21}, \theta_{43}, \theta_{54}, \theta_{1615}, \theta_{1716}, \theta_{1817}, x, y, z, \psi, \theta, \phi)$$

Por lo tanto:

$$\frac{\partial V}{\partial q_j} = \frac{\partial V}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial V}{\partial \theta_{43}}\frac{\partial \theta_{43}}{\partial q_j} + \frac{\partial V}{\partial \theta_{54}}\frac{\partial \theta_{54}}{\partial q_j} + \frac{\partial V}{\partial \theta_{1615}}\frac{\partial \theta_{1615}}{\partial q_j} + \frac{\partial V}{\partial \theta_{1716}}\frac{\partial \theta_{1716}}{\partial q_j} + \frac{\partial V}{\partial \theta_{1817}}\frac{\partial \theta_{1817}}{\partial q_j} + \frac{\partial V}{\partial x}\frac{\partial x}{\partial q_j} + \frac{\partial V}{\partial y}\frac{\partial y}{\partial q_j} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial q_j} + \frac{\partial V}{\partial \psi}\frac{\partial \psi}{\partial q_j} + \frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial q_j} + \frac{\partial V}{\partial \phi}\frac{\partial \phi}{\partial q_j}$$

Los términos $\left(\frac{\partial \theta_{21}}{\partial q_j}, \frac{\partial \theta_{43}}{\partial q_j}, \frac{\partial \theta_{54}}{\partial q_j}, \frac{\partial \theta_{1615}}{\partial q_j}, \frac{\partial \theta_{1716}}{\partial q_j}, \frac{\partial \theta_{1817}}{\partial q_j}\right)$ se determinan en el apéndice A

Desarrollando $\frac{\partial L_2}{\partial q_j}$

$$L_2 = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{N}_2 \dot{\boldsymbol{q}} + m_2 \boldsymbol{g}^T \boldsymbol{r}_{G2}^0$$

Dónde:

$$\boldsymbol{N}_2 = m_2 {\boldsymbol{M}_3}^T \boldsymbol{M}_3 + {\boldsymbol{M}_4}^T \boldsymbol{J}_2 \boldsymbol{M}_4$$
$$\boldsymbol{M}_3 = \boldsymbol{k}_3\boldsymbol{k}_2^T + \boldsymbol{k}_4\boldsymbol{k}_6^T + \boldsymbol{k}_5\boldsymbol{k}_7^T$$

$$
\begin{aligned}
\boldsymbol{M}_4 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T \ + \ \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T \\
\boldsymbol{J}_2^0 &= \boldsymbol{R}_5^0\,\boldsymbol{J}_2^5\,\boldsymbol{R}_5^{0^T} \\
\boldsymbol{k}_3 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{G2'}^0\right) \\
\boldsymbol{k}_4 &= \boldsymbol{j}_3^0 \times \boldsymbol{r}_{G2'}^0 \\
\boldsymbol{k}_5 &= \boldsymbol{k}_4^0 \times \boldsymbol{r}_{G2'}^0 \\
\boldsymbol{k}_6^T &= \frac{1}{V_8}\,[E_1, E_2, E_3, E_4, E_5, E_6] \\
\boldsymbol{k}_7^T &= \frac{1}{V_{15}}\,[E_7, E_8, E_9, E_{10}, E_{11}, E_{12}] \\
\boldsymbol{k}_4^0 &= \boldsymbol{R}_4^0\boldsymbol{k}_4^4 \\
\boldsymbol{r}_{32}^0 &= \boldsymbol{R}_2^0\boldsymbol{r}_{32}^2 \\
\boldsymbol{r}_{G2'}^0 &= \boldsymbol{R}_5^0\boldsymbol{r}_{G2'}^5 \\
\boldsymbol{R}_4^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43}) \\
\boldsymbol{R}_5^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54}) \\
\boldsymbol{j}_3^0 &= \boldsymbol{j}_0 \\
\boldsymbol{r}_{32}^2 &= [x_{32}, 0{,}0]^T \\
\boldsymbol{r}_{G2'}^5 &= [x_{G2}, y_{G2}, z_{G2}]^T
\end{aligned}
$$

Derivando con respecto a q_j :

$$
\begin{aligned}
\frac{\partial L_2}{\partial q_j} &= \frac{1}{2}\dot{\boldsymbol{q}}^T\frac{\partial \boldsymbol{N}_2}{\partial q_j}\dot{\boldsymbol{q}} + m_2\boldsymbol{g}^T\frac{\partial \boldsymbol{r}_{G2}^0}{\partial q_j} \\
\frac{\partial L_2}{\partial q_j} &= \mathbb{V}'_{2j}\dot{\boldsymbol{q}} + \mathbb{C}_{2j}
\end{aligned} \qquad 3.81
$$

Dónde:

$$
\begin{aligned}
\mathbb{V}'_{2j} &= \frac{1}{2}\dot{\boldsymbol{q}}^T\frac{\partial \boldsymbol{N}_2}{\partial q_j} \\
\mathbb{C}_{2j} &= m_1\boldsymbol{g}^T\frac{\partial \boldsymbol{r}_{G2}^0}{\partial q_j}
\end{aligned} \qquad 3.82
$$

A su vez:

$$
\begin{aligned}
\frac{\partial \boldsymbol{N}_2}{\partial q_j} &= m_2\left(\frac{\partial \boldsymbol{M}_3}{\partial q_j}^T\boldsymbol{M}_3 + \boldsymbol{M}_3{}^T\frac{\partial \boldsymbol{M}_3}{\partial q_j}\right) + \frac{\partial \boldsymbol{M}_4}{\partial q_j}^T\boldsymbol{J}_2\boldsymbol{M}_4 + \boldsymbol{M}_4{}^T\frac{\partial \boldsymbol{J}_2}{\partial q_j}\boldsymbol{M}_4 + \boldsymbol{M}_4{}^T\boldsymbol{J}_2\frac{\partial \boldsymbol{M}_4}{\partial q_j} \\
\frac{\partial \boldsymbol{M}_3}{\partial q_j} &= \frac{\partial \boldsymbol{k}_3}{\partial q_j}\boldsymbol{k}_2^T + \boldsymbol{k}_3\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_4}{\partial q_j}\boldsymbol{k}_6^T + \boldsymbol{k}_4\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_5}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_5\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} \\
\frac{\partial \boldsymbol{M}_4}{\partial q_j} &= \boldsymbol{j}_1^0\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} \ + \ \boldsymbol{j}_3^0\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_4^0}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} \\
\frac{\partial \boldsymbol{J}_2^0}{\partial q_j} &= \frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{J}_2^5\,\boldsymbol{R}_5^{0^T} + \boldsymbol{R}_5^0\boldsymbol{J}_2^5\,\frac{\partial \boldsymbol{R}_5^{0^T}}{\partial q_j} \\
\frac{\partial \boldsymbol{k}_3}{\partial q_j} &= \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G2'}^0}{\partial q_j}\right)
\end{aligned}
$$

$$\frac{\partial \boldsymbol{k}_4}{\partial q_j} = \boldsymbol{j}_3^0 \times \frac{\partial \boldsymbol{r}_{G2'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_5}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0 \times \frac{\partial \boldsymbol{r}_{G2'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} = -\frac{1}{V_8^2}\frac{\partial V_8}{\partial q_j}[E_1, E_2, E_3, E_4, E_5, E_6] + \frac{1}{V_8}\left[\frac{\partial E_1}{\partial q_j}, \frac{\partial E_2}{\partial q_j}, \frac{\partial E_3}{\partial q_j}, \frac{\partial E_4}{\partial q_j}, \frac{\partial E_5}{\partial q_j}, \frac{\partial E_6}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} = -\frac{1}{V_{15}^2}\frac{\partial V_{15}}{\partial q_j}[E_7, E_8, E_9, E_{10}, E_{11}, E_{12}] + \frac{1}{V_8}\left[\frac{\partial E_7}{\partial q_j}, \frac{\partial E_8}{\partial q_j}, \frac{\partial E_9}{\partial q_j}, \frac{\partial E_{10}}{\partial q_j}, \frac{\partial E_{11}}{\partial q_j}, \frac{\partial E_{12}}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_2^0}{\partial q_j}\boldsymbol{r}_{32}^2$$

$$\frac{\partial \boldsymbol{r}_{G2'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{r}_{G2'}^5$$

$$\frac{\partial \boldsymbol{k}_4^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_4^0}{\partial q_j}\boldsymbol{k}_4^4$$

$$\begin{aligned}\frac{\partial \boldsymbol{R}_5^0}{\partial q_j} &= \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j}\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54}) + \boldsymbol{R}_y(\theta_{21})\frac{\partial \boldsymbol{R}_y(\theta_{43})}{\partial q_j}\boldsymbol{R}_z(\theta_{54}) \\ &\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\frac{\partial \boldsymbol{R}_z(\theta_{54})}{\partial q_j} \\ &= \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j} \\ &\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{R}_4^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j}\boldsymbol{R}_y(\theta_{43}) + \boldsymbol{R}_y(\theta_{21})\frac{\partial \boldsymbol{R}_y(\theta_{43})}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_4^0}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\frac{\partial \theta_{43}}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_3}{\partial q_j}$:

$$\begin{aligned}\frac{\partial \boldsymbol{k}_3}{\partial q_j} &= \left(\boldsymbol{j}_1^0 \times \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\right)\frac{\partial \theta_{21}}{\partial q_j} \\ &\quad + \left(\boldsymbol{j}_1^0 \times \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\right)\frac{\partial \theta_{43}}{\partial q_j} \\ &\quad + \left(\boldsymbol{j}_1^0 \times \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\right)\frac{\partial \theta_{54}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_3}{\partial q_j} = \boldsymbol{J}_3\frac{\partial \theta}{\partial q_j} \qquad 3.83$$

Dónde:

$$\boldsymbol{J}_3 = [\boldsymbol{J}_{31} \quad \boldsymbol{J}_{32} \quad \boldsymbol{J}_{33}]$$

A su vez:

$$\begin{aligned}
\boldsymbol{J}_{31} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\\
\boldsymbol{J}_{32} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\\
\boldsymbol{J}_{33} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)
\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_4}{\partial q_j}$:

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_4}{\partial q_j} = {} & \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{21}}{\partial q_j}\\
& + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{43}}{\partial q_j}\\
& + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{54}}{\partial q_j}\\
\frac{\partial \boldsymbol{k}_4}{\partial q_j} = {} & \boldsymbol{J}_4 \frac{\partial \boldsymbol{\theta}}{\partial q_j}
\end{aligned} \qquad 3.84$$

Dónde:

$$\boldsymbol{J}_4 = [\boldsymbol{J}_{41} \quad \boldsymbol{J}_{42} \quad \boldsymbol{J}_{43}]$$

A su vez:

$$\begin{aligned}
\boldsymbol{J}_{41} &= \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\\
\boldsymbol{J}_{42} &= \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\\
\boldsymbol{J}_{43} &= \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)
\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_5}{\partial q_j}$:

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_5}{\partial q_j} = {} & \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{21}}{\partial q_j}\\
& + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0\right.\\
& \left.\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{43}}{\partial q_j}\\
& + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5\right)\frac{\partial \theta_{54}}{\partial q_j}\\
\frac{\partial \boldsymbol{k}_5}{\partial q_j} = {} & \boldsymbol{J}_5 \frac{\partial \boldsymbol{\theta}}{\partial q_j}
\end{aligned} \qquad 3.85$$

Dónde:

$$\boldsymbol{J}_5 = [\boldsymbol{J}_{51} \quad \boldsymbol{J}_{52} \quad \boldsymbol{J}_{53}]$$

A su vez:

$$\boldsymbol{J}_{51} = \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5$$
$$\boldsymbol{J}_{52} = \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G2'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{G2'}^5$$
$$\boldsymbol{J}_{53} = \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{G2'}^5$$

Desarrollando $\frac{\partial \boldsymbol{k}_6^T}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} = \boldsymbol{J}_6 \frac{\partial \boldsymbol{\theta}}{\partial q_j} + \frac{\partial \boldsymbol{k'}_6^T}{\partial q_j} \qquad 3.86$$

Dónde:

$$\boldsymbol{J}_6 = \left[\frac{\partial \boldsymbol{k}_6^T}{\partial \theta_{21}} \quad \frac{\partial \boldsymbol{k}_6^T}{\partial \theta_{43}} \quad \frac{\partial \boldsymbol{k}_6^T}{\partial \theta_{54}}\right]$$

Desarrollando $\frac{\partial \boldsymbol{k}_7^T}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} = \boldsymbol{J}_7 \frac{\partial \boldsymbol{\theta}}{\partial q_j} + \frac{\partial \boldsymbol{k'}_7^T}{\partial q_j} \qquad 3.87$$

Dónde:

$$\boldsymbol{J}_7 = \left[\frac{\partial \boldsymbol{k}_7^T}{\partial \theta_{21}} \quad \frac{\partial \boldsymbol{k}_7^T}{\partial \theta_{43}} \quad \frac{\partial \boldsymbol{k}_7^T}{\partial \theta_{54}}\right]$$

Desarrollando $\frac{\partial L_3}{\partial q_j}$

$$L_3 = \frac{1}{2}(\dot{\boldsymbol{q}}^T \boldsymbol{N}_3 \dot{\boldsymbol{q}}) + m_3 \boldsymbol{g}^T \boldsymbol{r}_{G3}^0$$

Dónde:

$$\boldsymbol{N}_3 = m_3 {\boldsymbol{M}_5}^T \boldsymbol{M}_5 + {\boldsymbol{M}_6}^T \boldsymbol{J}_3 \boldsymbol{M}_6$$
$$\boldsymbol{M}_5 = \boldsymbol{k}_8 \boldsymbol{k}_2^T + \boldsymbol{k}_9 \boldsymbol{k}_6^T + \boldsymbol{k}_{10} \boldsymbol{k}_7^T + \boldsymbol{k}_{11} \boldsymbol{k}_{12}^T$$
$$\boldsymbol{M}_6 = \boldsymbol{j}_1^0 \boldsymbol{k}_2^T + \boldsymbol{j}_3^0 \boldsymbol{k}_6^T + \boldsymbol{k}_4^0 \boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0 \boldsymbol{k}_{12}^T$$

$$
\begin{aligned}
\boldsymbol{k}_8 &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0\right)\\
\boldsymbol{k}_9 &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right)\\
\boldsymbol{k}_{10} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0\right)\\
\boldsymbol{k}_{11} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G3'}^0\\
\boldsymbol{k}_{12}^T &= \frac{1}{V_{36}}\left[E_{25}, E_{26}, E_{27}, E_{28}, E_{29}, E_{30}\right]\\
\boldsymbol{i}_{15}^0 &= \boldsymbol{R}_{15}^0 \boldsymbol{i}_{15}^{15}\\
\boldsymbol{r}_{14,5i}^0 &= \boldsymbol{R}_5^0 \boldsymbol{r}_{14,5i}^5\\
\boldsymbol{r}_{15,14i}^0 &= \boldsymbol{R}_{14}^0 \boldsymbol{r}_{15,14i}^{14}\\
\boldsymbol{r}_{G3'}^0 &= \boldsymbol{R}_{16}^0 \boldsymbol{r}_{G3'}^{16}\\
\boldsymbol{R}_{14}^0 &= \boldsymbol{R}_5^0\\
\boldsymbol{R}_{15}^0 &= \boldsymbol{R}_5^0\\
\boldsymbol{R}_{16}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x\left(\theta_{16,15}\right)\\
\boldsymbol{r}_{14,5i}^5 &= \left[x_{14,5i}, 0{,}0\right]^T\\
\boldsymbol{r}_{15,14i}^{14} &= \left[x_{15,14i}, 0{,}0\right]^T\\
\boldsymbol{r}_{G3'}^{16} &= \left[x_{G3}, y_{G3}, z_{G3}\right]^T\\
\boldsymbol{J}_3^0 &= \boldsymbol{R}_{16}^0\, \boldsymbol{J}_3^{16}\, {\boldsymbol{R}_{16}^0}^T
\end{aligned}
$$

Derivando con respecto a q_j :

$$
\begin{aligned}
\frac{\partial L_3}{\partial q_j} &= \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_3}{\partial q_j}\dot{\boldsymbol{q}} + m_3 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G3}^0}{\partial q_j}\\
\frac{\partial L_3}{\partial q_j} &= \mathbb{V}'_{3j}\dot{\boldsymbol{q}} + \mathbb{C}_{3j}
\end{aligned}
\qquad 3.88
$$

Dónde:

$$
\begin{aligned}
\mathbb{V}'_{3j} &= \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_3}{\partial q_j}\\
\mathbb{C}_{3j} &= m_3 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G3}^0}{\partial q_j}
\end{aligned}
\qquad 3.89
$$

A su vez:

$$
\begin{aligned}
\frac{\partial \boldsymbol{N}_3}{\partial q_j} &= m_3\left(\frac{\partial {\boldsymbol{M}_5}^T}{\partial q_j}\boldsymbol{M}_5 + {\boldsymbol{M}_5}^T \frac{\partial \boldsymbol{M}_5}{\partial q_j}\right) + \frac{\partial {\boldsymbol{M}_6}^T}{\partial q_j}\boldsymbol{J}_3\boldsymbol{M}_6 + {\boldsymbol{M}_6}^T \frac{\partial \boldsymbol{J}_3}{\partial q_j}\boldsymbol{M}_6 + {\boldsymbol{M}_6}^T \boldsymbol{J}_3 \frac{\partial \boldsymbol{M}_6}{\partial q_j}\\
\frac{\partial \boldsymbol{M}_5}{\partial q_j} &= \frac{\partial \boldsymbol{k}_8}{\partial q_j}\boldsymbol{k}_2^T + \boldsymbol{k}_8 \frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_9}{\partial q_j}\boldsymbol{k}_6^T + \boldsymbol{k}_9 \frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{10}}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_{10}\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{11}}{\partial q_j}\boldsymbol{k}_{12}^T\\
&\quad + \boldsymbol{k}_{11}\frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j}
\end{aligned}
$$

$$\frac{\partial \boldsymbol{k}_8}{\partial q_j} = \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} \right)$$

$$\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_5^0}{\partial q_j} \boldsymbol{r}_{14,5i}^5$$

$$\frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j} \boldsymbol{r}_{15,14i}^{14}$$

$$\frac{\partial \boldsymbol{k}_9}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G3'}^0}{\partial q_j} \right)$$

$$\frac{\partial \boldsymbol{r}_{G3'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{16}^0}{\partial q_j} \boldsymbol{r}_{G3'}^{16}$$

$$\begin{aligned}
\frac{\partial \boldsymbol{R}_{16}^0}{\partial q_j} &= \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j} \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) + \boldsymbol{R}_y(\theta_{21}) \frac{\partial \boldsymbol{R}_y(\theta_{43})}{\partial q_j} \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \\
&\quad + \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \frac{\partial \boldsymbol{R}_z(\theta_{54})}{\partial q_j} \boldsymbol{R}_x(\theta_{16,15}) \\
&\quad + \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \frac{\partial \boldsymbol{R}_x(\theta_{16,15})}{\partial q_j} \\
&= \boldsymbol{B}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \frac{\partial \theta_{21}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21}) \boldsymbol{B}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \frac{\partial \theta_{43}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{B}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \frac{\partial \theta_{54}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{B}_x(\theta_{16,15}) \frac{\partial \theta_{16,15}}{\partial q_j}
\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{10}}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G3'}^0 \right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G3'}^0}{\partial q_j} \right)$$

$$\frac{\partial \boldsymbol{k}_{11}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \times \boldsymbol{r}_{G3'}^0 + \boldsymbol{i}_{15}^0 \times \frac{\partial \boldsymbol{r}_{G3'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{15}^0}{\partial q_j} \boldsymbol{i}_{15}^{15}$$

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j} &= -\frac{1}{V_{36}^2} \frac{\partial V_{36}}{\partial q_j} [E_{25}, E_{26}, E_{27}, E_{28}, E_{29}, E_{30}] \\
&\quad + \frac{1}{V_{36}} \left[\frac{\partial E_{25}}{\partial q_j}, \frac{\partial E_{26}}{\partial q_j}, \frac{\partial E_{27}}{\partial q_j}, \frac{\partial E_{28}}{\partial q_j}, \frac{\partial E_{29}}{\partial q_j}, \frac{\partial E_{30}}{\partial q_j} \right]
\end{aligned}$$

$$\frac{\partial \boldsymbol{M}_6}{\partial q_j} = \boldsymbol{j}_1^0 \frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \boldsymbol{j}_3^0 \frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \boldsymbol{k}_7^T + \boldsymbol{k}_4^0 \frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0 \frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{J}_3^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{16}^0}{\partial q_j} \boldsymbol{J}_3^{16} \, {\boldsymbol{R}_{16}^0}^T + \boldsymbol{R}_{16}^0 \boldsymbol{J}_3^{16} \, \frac{\partial {\boldsymbol{R}_{16}^0}^T}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_8}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_8}{\partial q_j} = \left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{21}}{\partial q_j} + \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{43}}{\partial q_j} + \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_8}{\partial q_j} = \boldsymbol{J}_8 \frac{\partial \boldsymbol{\theta}}{\partial q_j} \qquad 3.90$$

Dónde:

$$J_8 = [J_{81} \quad J_{82} \quad J_{83}]$$

A su vez:

$$J_{81} = \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$J_{82} = \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$J_{83} = \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$

Desarrollando $\frac{\partial \boldsymbol{k}_9}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_9}{\partial q_j} = \boldsymbol{J}_9 \frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_9' \frac{\partial \theta_{16,15}}{\partial q_j} \qquad 3.91$$

Dónde:

$$\boldsymbol{J}_9 = [\boldsymbol{J}_{91} \quad \boldsymbol{J}_{92} \quad \boldsymbol{J}_{93}]$$
$$\boldsymbol{J}_9' = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{r}_{G3'}^{16}\right)$$

A su vez:

$$\boldsymbol{J}_{91} = \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{r}_{G3'}^{16}$$
$$\boldsymbol{J}_{92} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{r}_{G3'}^{16}\right)$$

$$\boldsymbol{J}_{93} = \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{r}_{G3'}^{16}$$

Desarrollando $\frac{\partial L_4}{\partial q_j}$

$$L_4 = \frac{1}{2}(\dot{\boldsymbol{q}}^T \boldsymbol{N}_4 \dot{\boldsymbol{q}}) + m_4 \boldsymbol{g}^T \boldsymbol{r}_{G4}^0$$

Dónde:

$$\begin{aligned}
\boldsymbol{N}_4 &= m_4 \boldsymbol{M}_7{}^T \boldsymbol{M}_7 + \boldsymbol{M}_8{}^T \boldsymbol{J}_4 \boldsymbol{M}_8 \\
\boldsymbol{M}_7 &= \boldsymbol{k}_{13}\boldsymbol{k}_2^T + \boldsymbol{k}_{14}\boldsymbol{k}_6^T + \boldsymbol{k}_{15}\boldsymbol{k}_7^T + \boldsymbol{k}_{16}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{17}\boldsymbol{k}_{18}^T \\
\boldsymbol{M}_8 &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T \\
\boldsymbol{k}_{13} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{14} &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{15} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) \\
\boldsymbol{k}_{16} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G4'}^0 \\
\boldsymbol{k}_{17} &= \boldsymbol{k}_{16}^0 \times \boldsymbol{r}_{G4'}^0 \\
\boldsymbol{k}_{18}^T &= \frac{1}{V_{29}}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] \\
\boldsymbol{r}_{G4'}^0 &= \boldsymbol{R}_{17}^0 \boldsymbol{r}_{G4'}^{17} \\
\boldsymbol{k}_{16}^0 &= \boldsymbol{R}_{16}^0 \boldsymbol{k}_{16}^{16} \\
\boldsymbol{R}_{17}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
\boldsymbol{r}_{G4'}^{17} &= [x_{G4}, y_{G4}, z_{G4}]^T \\
\boldsymbol{J}_4^0 &= \boldsymbol{R}_{17}^0\, \boldsymbol{J}_4^{17}\, \boldsymbol{R}_{17}^0{}^T
\end{aligned}$$

Derivando con respecto a q_j :

$$\frac{\partial L_4}{\partial q_j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_4}{\partial q_j}\dot{\boldsymbol{q}} + m_4 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G4}^0}{\partial q_j}$$

$$\frac{\partial L_4}{\partial q_j} = \mathbb{V}'_{4j}\dot{\boldsymbol{q}} + \mathbb{C}_{4j} \qquad 3.92$$

Dónde:

$$\mathbb{V}'_{4j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_4}{\partial q_j}$$

$$\mathbb{C}_{4j} = m_4 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G4}^0}{\partial q_j} \qquad 3.93$$

A su vez:

$$\frac{\partial \boldsymbol{N}_4}{\partial q_j} = m_4\left(\frac{\partial \boldsymbol{M_7}^T}{\partial q_j}\boldsymbol{M}_7 + \boldsymbol{M}_7{}^T\frac{\partial \boldsymbol{M}_7}{\partial q_j}\right) + \frac{\partial \boldsymbol{M_8}^T}{\partial q_j}\boldsymbol{J}_4\boldsymbol{M}_8 + \boldsymbol{M}_8{}^T\frac{\partial \boldsymbol{J}_4}{\partial q_j}\boldsymbol{M}_8 + \boldsymbol{M}_8{}^T\boldsymbol{J}_4\frac{\partial \boldsymbol{M}_8}{\partial q_j}$$

$$\frac{\partial \boldsymbol{M}_7}{\partial q_j} = \frac{\partial \boldsymbol{k}_{13}}{\partial q_j}\boldsymbol{k}_2^T + \boldsymbol{k}_{13}\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{14}}{\partial q_j}\boldsymbol{k}_6^T + \boldsymbol{k}_{14}\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{15}}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_{15}\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{16}}{\partial q_j}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}\frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{17}}{\partial q_j}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{17}\frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{13}}{\partial q_j} = \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j}\boldsymbol{r}_{G4'}^{17}$$

$$\begin{aligned}
\frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j} &= \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j}\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\frac{\boldsymbol{R}_y(\theta_{43})}{\partial q_j}\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\frac{\partial \boldsymbol{R}_z(\theta_{54})}{\partial q_j}\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \boldsymbol{R}_x(\theta_{16,15})}{\partial q_j}\boldsymbol{R}_z(\theta_{17,16}) \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\frac{\partial \boldsymbol{R}_z(\theta_{17,16})}{\partial q_j} \\
&= \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j} \\
&\quad + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}
\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{k}_{16}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{i}_{15}^0 \times \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{16}^0}{\partial q_j} \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_{16}^0 \times \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{18}^{T}}{\partial q_j} = -\frac{1}{V_{29}^{2}}\frac{\partial V_{29}}{\partial q_j}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] + \frac{1}{V_{29}}\left[\frac{\partial E_{19}}{\partial q_j}, \frac{\partial E_{20}}{\partial q_j}, \frac{\partial E_{21}}{\partial q_j}, \frac{\partial E_{22}}{\partial q_j}, \frac{\partial E_{23}}{\partial q_j}, \frac{\partial E_{24}}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{k}_{16}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{16}^{0}}{\partial q_j}\boldsymbol{k}_{16}^{16}$$

$$\frac{\partial \boldsymbol{M}_{8}}{\partial q_j} = \boldsymbol{j}_1^0\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \boldsymbol{j}_3^0\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_4^0}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_4^0\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j}\boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0\frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{16}^0}{\partial q_j}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{16}^0\frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{J}_{4}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{17}^{0}}{\partial q_j}\boldsymbol{J}_4^{17}\,{\boldsymbol{R}_{17}^{0}}^{T} + \boldsymbol{R}_{17}^{0}\boldsymbol{J}_4^{17}\frac{\partial {\boldsymbol{R}_{17}^{0}}^{T}}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{13}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{13}}{\partial q_j} = \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{R}_2^0}{\partial q_j}\boldsymbol{r}_{32}^2 + \frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{r}_{14,5i}^5 + \frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j}\boldsymbol{r}_{15,14i}^{14} + \frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j}\boldsymbol{r}_{G4'}^{17}\right)$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{13}}{\partial q_j} = \boldsymbol{j}_1^0 \times \Bigg(&\boldsymbol{B}_y(\theta_{21})\frac{\partial \theta_{21}}{\partial q_j}\boldsymbol{r}_{32}^2 \\
&+ \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}\right. \\
&\left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\right)\boldsymbol{r}_{14,5i}^5 \\
&+ \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}\right. \\
&\left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\right)\boldsymbol{r}_{15,14i}^{14} \\
&+ \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j}\right. \\
&+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j} \\
&+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j} \\
&+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j} \\
&\left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}\right)\boldsymbol{r}_{G4'}^{17}\Bigg)\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \boldsymbol{k}_{13}}{\partial q_j} = \Bigg(& \Big(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{21}}{\partial q_j} \\
& + \Big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} \\
& + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{43}}{\partial q_j} \\
& + \Big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} \\
& + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{54}}{\partial q_j} \\
& + \Big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{16,15}}{\partial q_j} \\
& + \Big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{17,16}}{\partial q_j}\Bigg)
\end{aligned}
$$

$$
\frac{\partial \boldsymbol{k}_{13}}{\partial q_j} = \boldsymbol{J}_{13}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + J_{13}'\frac{\partial \theta_{16,15}}{\partial q_j} + J_{13}''\frac{\partial \theta_{17,16}}{\partial q_j} \qquad 3.94
$$

Dónde:

$$
\begin{aligned}
\boldsymbol{J}_{13} &= [J_{131} \quad J_{132} \quad J_{133}] \\
\boldsymbol{J}_{13}' &= \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right) \\
\boldsymbol{J}_{13}'' &= \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)
\end{aligned}
$$

A su vez:

$$
\begin{aligned}
\boldsymbol{J}_{131} = \Big(& \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big) \\
\boldsymbol{J}_{132} = \Big(& \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big) \\
\boldsymbol{J}_{133} = \Big(& \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_1^0 \\
& \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)
\end{aligned}
$$

Desarrollando $\frac{\partial \boldsymbol{k}_{14}}{\partial q_j}$:

$$
\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}\right)
$$

$$
\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{r}_{14,5i}^5 + \frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j}\boldsymbol{r}_{15,14i}^{14} + \frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j}\boldsymbol{r}_{G4'}^{17}\right)
$$

$$\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \boldsymbol{j}_3^0 \times \Bigg(\bigg(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\bigg)\boldsymbol{r}_{14,5i}^5$$
$$+ \bigg(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\bigg)\boldsymbol{r}_{15,14i}^{14}$$
$$+ \bigg(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j}$$
$$+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}\bigg)\boldsymbol{r}_{G4'}^{17}\Bigg)$$

$$\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \Bigg(\Big(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0$$
$$\times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{21}}{\partial q_j}$$
$$+ \Big(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$+ \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \Big(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$+ \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{54}}{\partial q_j}$$
$$+ \Big(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{16,15}}{\partial q_j}$$
$$+ \Big(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{17,16}}{\partial q_j}\Bigg)$$

$$\frac{\partial \boldsymbol{k}_{14}}{\partial q_j} = \boldsymbol{J}_{14}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}'_{14}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}''_{14}\frac{\partial \theta_{17,16}}{\partial q_j} \qquad 3.95$$

Dónde:

$$\boldsymbol{J}_{14} = [\boldsymbol{J}_{141} \quad \boldsymbol{J}_{142} \quad \boldsymbol{J}_{143}]$$
$$\boldsymbol{J}'_{14} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$
$$\boldsymbol{J}''_{14} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

A su vez:

$$\boldsymbol{J}_{141} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

$$\boldsymbol{J}_{142} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

$$\boldsymbol{J}_{143} = \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

Desarrollando $\frac{\partial \boldsymbol{k}_{15}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G4'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = \frac{\partial \boldsymbol{R}_4^0}{\partial q_j}\boldsymbol{k}_4^4 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{r}_{14,5i}^5 + \frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j}\boldsymbol{r}_{15,14i}^{14} + \frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j}\boldsymbol{r}_{G4'}^{17}\right)$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = {} & \left(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\right)\frac{\partial \theta_{21}}{\partial q_j} + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\right)\frac{\partial \theta_{43}}{\partial q_j}\right)\boldsymbol{k}_4^4 \\ & \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G4'}^0\right) + \boldsymbol{k}_4^0 \\ & \times \left(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}\right.\right. \\ & \left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\right)\boldsymbol{r}_{14,5i}^5 \\ & + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j}\right. \\ & \left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\right)\boldsymbol{r}_{15,14i}^{14} \\ & + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j}\right. \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j} \\ & \left.\left. + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}\right)\boldsymbol{r}_{G4'}^{17}\right)\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = \Big(\big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{14,5i}^0 + \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{15,14i}^0 + \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big)$$
$$\times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$+ \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{21}}{\partial q_j}$$
$$+ \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{14,5i}^0 + \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{15,14i}^0$$
$$+ \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \Big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{54}}{\partial q_j}$$
$$+ \Big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{16,15}}{\partial q_j}$$
$$+ \Big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)\frac{\partial \theta_{17,16}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{15}}{\partial q_j} = \boldsymbol{J}_{15}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{15}'\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{15}''\frac{\partial \theta_{17,16}}{\partial q_j} \qquad 3.96$$

Dónde:

$$\boldsymbol{J}_{15} = [J_{151} \quad J_{152} \quad J_{153}]$$
$$\boldsymbol{J}_{15}' = \Big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)$$
$$\boldsymbol{J}_{15}'' = \Big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)$$

$$\boldsymbol{J}_{151} = \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{14,5i}^0 + \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{15,14i}^0 + \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big)$$
$$\times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}$$
$$+ \boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}$$
$$\boldsymbol{J}_{152} = \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{14,5i}^0 + \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{15,14i}^0$$
$$+ \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4\big) \times \boldsymbol{r}_{G4'}^0 + \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\Big)$$
$$\boldsymbol{J}_{153} = \big(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} + \boldsymbol{k}_4^0$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$

$$\frac{\partial \boldsymbol{k}_{16}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{15}^{0}}{\partial q_j} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{i}_{15}^{0} \times \frac{\partial \boldsymbol{r}_{G4'}^{0}}{\partial q_j}$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{16}}{\partial q_j} = \Bigg(&\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}\Bigg)\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{i}_{15}^{0} \\ &\times \Bigg(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}\Bigg)\boldsymbol{r}_{G4'}^{17}\end{aligned}$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{16}}{\partial q_j} = \Big(&\big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\big) \\ &+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\Big)\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\big) \\ &+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\Big)\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\big) \\ &+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\Big)\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{16,15}}{\partial q_j} \\ &+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{17,16}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{16}}{\partial q_j} = \boldsymbol{J}_{16}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{16}'\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{16}''\frac{\partial \theta_{17,16}}{\partial q_j} \qquad 3.97$$

Dónde:

$$\begin{aligned}\boldsymbol{J}_{16} &= [J_{161} \quad J_{162} \quad J_{163}] \\ \boldsymbol{J}_{16}' &= \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big) \\ \boldsymbol{J}_{16}'' &= \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\end{aligned}$$

A su vez:

$$J_{161} = \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\right) + \left(\boldsymbol{i}_{15}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

$$J_{162} = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\right) + \left(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

$$J_{163} = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G4'}^{0}\right) + \left(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\right)$$

$$\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{16}^{0}}{\partial q_j} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0} \times \frac{\partial \boldsymbol{r}_{G4'}^{0}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{16}^{0}}{\partial q_j}\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0} \times \frac{\partial \boldsymbol{R}_{17}^{0}}{\partial q_j}\boldsymbol{r}_{G4'}^{17}$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \Bigg(&\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16}\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16}\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16}\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16}\frac{\partial \theta_{16,15}}{\partial q_j}\Bigg) \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0} \\ &\times \Bigg(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\frac{\partial \theta_{16,15}}{\partial q_j} \\ &+ \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\frac{\partial \theta_{17,16}}{\partial q_j}\Bigg)\boldsymbol{r}_{G4'}^{17}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{21}}{\partial q_j}$$
$$+ \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{43}}{\partial q_j}$$
$$+ \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{54}}{\partial q_j}$$
$$+ \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{16,15}}{\partial q_j}$$
$$+ \big(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)\frac{\partial \theta_{17,16}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{17}}{\partial q_j} = \boldsymbol{J}_{17}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}'_{17}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}''_{17}\frac{\partial \theta_{17,16}}{\partial q_j} \qquad 3.98$$

Dónde:

$$\boldsymbol{J}_{17} = [J_{171} \quad J_{172} \quad J_{173}]$$
$$\boldsymbol{J}'_{17} = \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$
$$\boldsymbol{J}''_{17} = \big(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$

A su vez:

$$\boldsymbol{J}_{171} = \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$

$$\boldsymbol{J}_{172} = \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$

$$\boldsymbol{J}_{173} = \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G4'}^{0} + \boldsymbol{k}_{16}^{0}$$
$$\times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{r}_{G4'}^{17}\big)$$

$$\frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j} = -\frac{1}{V_{29}^2}\frac{\partial V_{29}}{\partial q_j}[E_{19}, E_{20}, E_{21}, E_{22}, E_{23}, E_{24}] + \frac{1}{V_{29}}\left[\frac{\partial E_{19}}{\partial q_j}, \frac{\partial E_{20}}{\partial q_j}, \frac{\partial E_{21}}{\partial q_j}, \frac{\partial E_{22}}{\partial q_j}, \frac{\partial E_{23}}{\partial q_j}, \frac{\partial E_{24}}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j} = \boldsymbol{J}_{18}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \frac{\partial \boldsymbol{k'}_{18}^T}{\partial q_j} \qquad 3.99$$

Dónde:

$$\boldsymbol{J}_{18} = \begin{bmatrix}\frac{\partial \boldsymbol{k}_{18}^T}{\partial \theta_{21}} & \frac{\partial \boldsymbol{k}_{18}^T}{\partial \theta_{43}} & \frac{\partial \boldsymbol{k}_{18}^T}{\partial \theta_{54}}\end{bmatrix}$$

Desarrollando $\frac{\partial L_5}{\partial q_j}$

$$L_5 = \frac{1}{2}(\dot{\boldsymbol{q}}^T \boldsymbol{N}_5 \dot{\boldsymbol{q}}) + m_5 \boldsymbol{g}^T \boldsymbol{r}_{G5}^0$$

Dónde:

$$\begin{aligned}
\boldsymbol{N}_5 &= m_5 \boldsymbol{M}_9{}^T \boldsymbol{M}_9 + \boldsymbol{M}_{10}{}^T \boldsymbol{J}_5 \boldsymbol{M}_{10}\\
\boldsymbol{M}_9 &= \boldsymbol{k}_{19}\boldsymbol{k}_2^T + \boldsymbol{k}_{20}\boldsymbol{k}_6^T + \boldsymbol{k}_{21}\boldsymbol{k}_7^T + \boldsymbol{k}_{22}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{23}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{24}\boldsymbol{k}_{25}^T\\
\boldsymbol{M}_{10} &= \boldsymbol{j}_1^0\boldsymbol{k}_2^T + \boldsymbol{j}_3^0\boldsymbol{k}_6^T + \boldsymbol{k}_4^0\boldsymbol{k}_7^T + \boldsymbol{i}_{15}^0\boldsymbol{k}_{12}^T + \boldsymbol{k}_{16}^0\boldsymbol{k}_{18}^T + \boldsymbol{i}_{17}^0\boldsymbol{k}_{25}^T\\
\boldsymbol{k}_{19} &= \boldsymbol{j}_1^0 \times \left(\boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right)\\
\boldsymbol{k}_{20} &= \boldsymbol{j}_3^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right)\\
\boldsymbol{k}_{21} &= \boldsymbol{k}_4^0 \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right)\\
\boldsymbol{k}_{22} &= \boldsymbol{i}_{15}^0 \times \boldsymbol{r}_{G5'}^0\\
\boldsymbol{k}_{23} &= \boldsymbol{k}_{16}^0 \times \boldsymbol{r}_{G5'}^0\\
\boldsymbol{k}_{24} &= \boldsymbol{i}_{17}^0 \times \boldsymbol{r}_{G5'}^0\\
\boldsymbol{k}_{25}^T &= \frac{1}{V_{23}}[E_{13}, E_{14}, E_{15}, E_{16}, E_{17}, E_{18}]\\
\boldsymbol{i}_{17}^0 &= \boldsymbol{R}_{17}^0 \boldsymbol{i}_{17}^{17}\\
\boldsymbol{r}_{G5'}^0 &= \boldsymbol{R}_{18}^0 \boldsymbol{r}_{G5'}^{18}\\
\boldsymbol{R}_{18}^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\\
\boldsymbol{r}_{G5'}^{18} &= [x_{G5}, y_{G5}, z_{G5}]^T\\
\boldsymbol{J}_5^0 &= \boldsymbol{R}_{18}^0 \boldsymbol{J}_5^{18} \boldsymbol{R}_{18}^0{}^T
\end{aligned}$$

Derivando con respecto a q_j :

$$\frac{\partial L_5}{\partial q_j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_5}{\partial q_j}\dot{\boldsymbol{q}} + m_5 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G5}^0}{\partial q_j}$$
$$\frac{\partial L_5}{\partial q_j} = \mathbb{V}'_{5j}\dot{\boldsymbol{q}} + \mathbb{C}_{5j} \qquad 3.100$$

Dónde:

$$\mathbb{V}'_{5j} = \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_5}{\partial q_j}$$
$$\mathbb{C}_{5j} = m_5 \boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G5}^0}{\partial q_j} \qquad 3.101$$

Además:

$$\frac{\partial \boldsymbol{N}_5}{\partial q_j} = m_5\left(\frac{\partial {\boldsymbol{M}_9}^T}{\partial q_j}\boldsymbol{M}_9 + {\boldsymbol{M}_9}^T\frac{\partial \boldsymbol{M}_9}{\partial q_j}\right) + \frac{\partial {\boldsymbol{M}_{10}}^T}{\partial q_j}\boldsymbol{J}_5\boldsymbol{M}_{10} + {\boldsymbol{M}_{10}}^T\frac{\partial \boldsymbol{J}_5}{\partial q_j}\boldsymbol{M}_{10} + {\boldsymbol{M}_{10}}^T\boldsymbol{J}_5\frac{\partial \boldsymbol{M}_{10}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{M}_9}{\partial q_j} = \frac{\partial \boldsymbol{k}_{19}}{\partial q_j}\boldsymbol{k}_2^T + \boldsymbol{k}_{19}\frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{20}}{\partial q_j}\boldsymbol{k}_6^T + \boldsymbol{k}_{20}\frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{21}}{\partial q_j}\boldsymbol{k}_7^T + \boldsymbol{k}_{21}\frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{22}}{\partial q_j}\boldsymbol{k}_{12}^T + \boldsymbol{k}_{22}\frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{23}}{\partial q_j}\boldsymbol{k}_{18}^T + \boldsymbol{k}_{23}\frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{24}}{\partial q_j}\boldsymbol{k}_{25}^T + \boldsymbol{k}_{24}\frac{\partial \boldsymbol{k}_{25}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{19}}{\partial q_j} = \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{18}^0}{\partial q_j}\boldsymbol{r}_{G5'}^{18}$$

$$\begin{aligned}\frac{\partial \boldsymbol{R}_{18}^0}{\partial q_j} = {} & \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\frac{\partial \theta_{21}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\frac{\partial \theta_{43}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\frac{\partial \theta_{54}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\frac{\partial \theta_{16,15}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\frac{\partial \theta_{17,16}}{\partial q_j} \\ & + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\frac{\partial \theta_{18,17}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{20}}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{k}_{21}}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}\right)$$

$$\frac{\partial \boldsymbol{k}_{22}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{i}_{15}^0 \times \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{23}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{16}^0}{\partial q_j} \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{k}_{16}^0 \times \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{24}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{17}^0}{\partial q_j} \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{i}_{17}^0 \times \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{i}_{17}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{17}^0}{\partial q_j} \boldsymbol{i}_{17}^{17}$$

$$\frac{\partial \boldsymbol{k}_{25}^T}{\partial q_j} = -\frac{1}{V_{23}^2} \frac{\partial V_{23}}{\partial q_j} [E_{13}, E_{14}, E_{15}, E_{16}, E_{17}, E_{18}] + \frac{1}{V_{23}} \left[\frac{\partial E_{13}}{\partial q_j}, \frac{\partial E_{14}}{\partial q_j}, \frac{\partial E_{15}}{\partial q_j}, \frac{\partial E_{16}}{\partial q_j}, \frac{\partial E_{17}}{\partial q_j}, \frac{\partial E_{18}}{\partial q_j} \right]$$

$$\frac{\partial \boldsymbol{M}_{10}}{\partial q_j} = \boldsymbol{j}_1^0 \frac{\partial \boldsymbol{k}_2^T}{\partial q_j} + \boldsymbol{j}_3^0 \frac{\partial \boldsymbol{k}_6^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \boldsymbol{k}_7^T + \boldsymbol{k}_4^0 \frac{\partial \boldsymbol{k}_7^T}{\partial q_j} + \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \boldsymbol{k}_{12}^T + \boldsymbol{i}_{15}^0 \frac{\partial \boldsymbol{k}_{12}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{16}^0}{\partial q_j} \boldsymbol{k}_{18}^T + \boldsymbol{k}_{16}^0 \frac{\partial \boldsymbol{k}_{18}^T}{\partial q_j} + \frac{\partial \boldsymbol{i}_{17}^0}{\partial q_j} \boldsymbol{k}_{25}^T + \boldsymbol{i}_{17}^0 \frac{\partial \boldsymbol{k}_{25}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{J}_5^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{18}^0}{\partial q_j} \boldsymbol{J}_5^{18} {\boldsymbol{R}_{18}^0}^T + \boldsymbol{R}_{18}^0 \boldsymbol{J}_5^{18} \frac{\partial {\boldsymbol{R}_{18}^0}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{19}}{\partial q_j} = \boldsymbol{j}_1^0 \times \left(\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j} \right)$$

Desarrollando $\frac{\partial \boldsymbol{k}_{19}}{\partial q_j}$:

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_{19}}{\partial q_j} = \Big(& \big(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21}) \boldsymbol{r}_{32}^2\big) + \big(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{r}_{14,5i}^5\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{r}_{15,14i}^{14}\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \boldsymbol{R}_z(\theta_{17,16}) \boldsymbol{R}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big)\Big) \frac{\partial \theta_{21}}{\partial q_j} \\
& + \Big(\big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{B}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{r}_{14,5i}^5\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{B}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{r}_{15,14i}^{14}\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{B}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \boldsymbol{R}_z(\theta_{17,16}) \boldsymbol{R}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big)\Big) \frac{\partial \theta_{43}}{\partial q_j} \\
& + \Big(\big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{B}_z(\theta_{54}) \boldsymbol{r}_{14,5i}^5\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{B}_z(\theta_{54}) \boldsymbol{r}_{15,14i}^{14}\big) \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{B}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \boldsymbol{R}_z(\theta_{17,16}) \boldsymbol{R}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big)\Big) \frac{\partial \theta_{54}}{\partial q_j} \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{B}_x(\theta_{16,15}) \boldsymbol{R}_z(\theta_{17,16}) \boldsymbol{R}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big) \frac{\partial \theta_{16,15}}{\partial q_j} \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \boldsymbol{B}_z(\theta_{17,16}) \boldsymbol{R}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big) \frac{\partial \theta_{17,16}}{\partial q_j} \\
& + \big(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21}) \boldsymbol{R}_y(\theta_{43}) \boldsymbol{R}_z(\theta_{54}) \boldsymbol{R}_x(\theta_{16,15}) \boldsymbol{R}_z(\theta_{17,16}) \boldsymbol{B}_x(\theta_{18,17}) \boldsymbol{r}_{G5'}^{18}\big) \frac{\partial \theta_{18,17}}{\partial q_j}
\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{19}}{\partial q_j} = \boldsymbol{J}_{19}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{\mathbf{19}}^{'}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{\mathbf{19}}^{''}\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}_{\mathbf{19}}^{'''}\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.102$$

Dónde:

$$\boldsymbol{J}_{19} = [\boldsymbol{J}_{191} \quad \boldsymbol{J}_{192} \quad \boldsymbol{J}_{193}]$$
$$\boldsymbol{J}_{\mathbf{19}}^{'} = \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{\mathbf{19}}^{''} = \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{\mathbf{19}}^{'''} = \boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$

A su vez:

$$\begin{aligned}\boldsymbol{J}_{191} = &\left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2\right) + \left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ &+ \left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_1^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\ \boldsymbol{J}_{192} = &\left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\ \boldsymbol{J}_{193} = &\left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_1^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{20}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{20}}{\partial q_j} = \boldsymbol{j}_3^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}\right)$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{20}}{\partial q_j} = \Bigg(&\Big(\left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{16,15}}{\partial q_j} \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{17,16}}{\partial q_j} \\ &+ \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{18,17}}{\partial q_j}\Bigg)\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{20}}{\partial q_j} = \boldsymbol{J}_{20}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{\mathbf{20}}^{'}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{\mathbf{20}}^{''}\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}_{\mathbf{20}}^{'''}\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.103$$

Dónde:

$$\boldsymbol{J}_{20} = [\boldsymbol{J}_{201} \quad \boldsymbol{J}_{202} \quad \boldsymbol{J}_{203}]$$
$$\boldsymbol{J}_{\mathbf{20}}^{'} = \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{\mathbf{20}}^{''} = \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{\mathbf{20}}^{'''} = \boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$

A su vez:

$$\begin{aligned}\boldsymbol{J}_{201} = {} & \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{j}_3^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\end{aligned}$$
$$\begin{aligned}\boldsymbol{J}_{202} = {} & \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\end{aligned}$$
$$\begin{aligned}\boldsymbol{J}_{203} = {} & \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{j}_3^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{21}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{21}}{\partial q_j} = \frac{\partial \boldsymbol{k}_4^0}{\partial q_j} \times \left(\boldsymbol{r}_{14,5i}^0 + \boldsymbol{r}_{15,14i}^0 + \boldsymbol{r}_{G5'}^0\right) + \boldsymbol{k}_4^0 \times \left(\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}\right)$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{21}}{\partial q_j} = {} & \Big(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{14,5i}^0\right) + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{15,14i}^0\right) \\ & + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G5'}^0\right) + \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{21}}{\partial q_j} \\ & + \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{14,5i}^0\right) + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{15,14i}^0\right) \\ & + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G5'}^0\right) + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{43}}{\partial q_j} \\ & + \Big(\left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{54}}{\partial q_j} \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{16,15}}{\partial q_j} \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{17,16}}{\partial q_j} \\ & + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{18,17}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{21}}{\partial q_j} = \boldsymbol{J}_{21}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{21}^{'}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{21}^{''}\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}_{21}^{'''}\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.104$$

Dónde:

$$\begin{aligned}
\boldsymbol{J}_{21} &= [\boldsymbol{J}_{211} \quad \boldsymbol{J}_{212} \quad \boldsymbol{J}_{213}] \\
\boldsymbol{J}_{21}' &= \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18} \\
\boldsymbol{J}_{21}'' &= \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18} \\
\boldsymbol{J}_{21}''' &= \boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}
\end{aligned}$$

A su vez:

$$\begin{aligned}
\boldsymbol{J}_{211} = &\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{14,5i}^0\right) + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{15,14i}^0\right) \\
&+ \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G5'}^0\right) + \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\
&+ \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\
&+ \left(\boldsymbol{k}_4^0 \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}_{212} = &\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{14,5i}^0\right) + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{15,14i}^0\right) \\
&+ \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{k}_4^4 \times \boldsymbol{r}_{G5'}^0\right) + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) \\
&+ \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\
&+ \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}_{213} = &\left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right) + \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right) \\
&+ \left(\boldsymbol{k}_4^0 \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)
\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{22}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{22}}{\partial q_j} = \frac{\partial \boldsymbol{i}_{15}^0}{\partial q_j} \times \boldsymbol{r}_{G5'}^0 + \boldsymbol{i}_{15}^0 \times \frac{\partial \boldsymbol{r}_{G5'}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{22}}{\partial q_j} = \Big(\big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\big) + \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\Big)\frac{\partial \theta_{21}}{\partial q_j}$$

$$+ \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\big) + \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\Big)\frac{\partial \theta_{43}}{\partial q_j}$$

$$+ \Big(\big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\big) + \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\Big)\frac{\partial \theta_{54}}{\partial q_j}$$

$$+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\frac{\partial \theta_{16,15}}{\partial q_j}$$

$$+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\frac{\partial \theta_{17,16}}{\partial q_j}$$

$$+ \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)\frac{\partial \theta_{18,17}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{22}}{\partial q_j} = \boldsymbol{J}_{22}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{22}'\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{22}''\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}_{22}'''\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.105$$

Dónde:

$$\boldsymbol{J}_{22} = [\boldsymbol{J}_{221} \quad \boldsymbol{J}_{222} \quad \boldsymbol{J}_{223}]$$
$$\boldsymbol{J}_{22}' = \boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{22}'' = \boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$
$$\boldsymbol{J}_{22}''' = \boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}$$

A su vez:

$$\boldsymbol{J}_{221} = \big(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\big) + \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)$$
$$\boldsymbol{J}_{222} = \big(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\big) + \big(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\big)$$

$$\boldsymbol{J}_{223} = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{i}_{15}^{15} \times \boldsymbol{r}_{G5'}^{0}\right) + \left(\boldsymbol{i}_{15}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)$$

Desarrollando $\frac{\partial \boldsymbol{k}_{23}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{23}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{16}^{0}}{\partial q_j} \times \boldsymbol{r}_{G5'}^{0} + \boldsymbol{k}_{16}^{0} \times \frac{\partial \boldsymbol{r}_{G5'}^{0}}{\partial q_j}$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{23}}{\partial q_j} = &\Big(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G5'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{21}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G5'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{43}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G5'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{54}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G5'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\Big)\frac{\partial \theta_{16,15}}{\partial q_j} \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{17,16}}{\partial q_j} \\ &+ \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)\frac{\partial \theta_{18,17}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{23}}{\partial q_j} = \boldsymbol{J}_{23}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}_{23}'\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}_{23}''\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}_{23}'''\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.106$$

Dónde:

$$\boldsymbol{J}_{22} = [\boldsymbol{J}_{231} \quad \boldsymbol{J}_{232} \quad \boldsymbol{J}_{233}]$$

$$\boldsymbol{J}_{23}' = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{k}_{16}^{16} \times \boldsymbol{r}_{G5'}^{0}\right) + \left(\boldsymbol{k}_{16}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)$$

$$\boldsymbol{J}''_{23} = \boldsymbol{k}^0_{16} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}$$
$$\boldsymbol{J}'''_{23} = \boldsymbol{k}^0_{16} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}$$

A su vez:

$$\boldsymbol{J}_{231} = \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}^{16}_{16} \times \boldsymbol{r}^0_{G5'}\right) + \left(\boldsymbol{k}^0_{16} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)$$
$$\boldsymbol{J}_{232} = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}^{16}_{16} \times \boldsymbol{r}^0_{G5'}\right) + \left(\boldsymbol{k}^0_{16} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)$$
$$\boldsymbol{J}_{233} = \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{k}^{16}_{16} \times \boldsymbol{r}^0_{G5'}\right) + \left(\boldsymbol{k}^0_{16} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)$$

Desarrollando $\frac{\partial \boldsymbol{k}_{24}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{24}}{\partial q_j} = \frac{\partial \boldsymbol{i}^0_{17}}{\partial q_j} \times \boldsymbol{r}^0_{G5'} + \boldsymbol{i}^0_{17} \times \frac{\partial \boldsymbol{r}^0_{G5'}}{\partial q_j}$$

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_{24}}{\partial q_j} = &\Big(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}^{17}_{17} \times \boldsymbol{r}^0_{G5'}\right) \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\Big)\frac{\partial \theta_{21}}{\partial q_j} \\
&+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}^{17}_{17} \times \boldsymbol{r}^0_{G5'}\right) \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\Big)\frac{\partial \theta_{43}}{\partial q_j} \\
&+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}^{17}_{17} \times \boldsymbol{r}^0_{G5'}\right) \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\Big)\frac{\partial \theta_{54}}{\partial q_j} \\
&+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}^{17}_{17} \times \boldsymbol{r}^0_{G5'}\right) \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\Big)\frac{\partial \theta_{16,15}}{\partial q_j} \\
&+ \Big(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{i}^{17}_{17} \times \boldsymbol{r}^0_{G5'}\right) \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\Big)\frac{\partial \theta_{17,16}}{\partial q_j} \\
&+ \left(\boldsymbol{i}^0_{17} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}^{18}_{G5'}\right)\frac{\partial \theta_{18,17}}{\partial q_j}
\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{24}}{\partial q_j} = \boldsymbol{J}_{24}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}'_{24}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}''_{24}\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}'''_{24}\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.107$$

Dónde:

$$\begin{aligned}
\boldsymbol{J}_{24} &= [\boldsymbol{J}_{241} \quad \boldsymbol{J}_{242} \quad \boldsymbol{J}_{243}] \\
\boldsymbol{J}'_{24} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}_{17}^{17} \times \boldsymbol{r}_{G5'}^{0}\right) \\
&\qquad + \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{B}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}''_{24} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{i}_{17}^{17} \times \boldsymbol{r}_{G5'}^{0}\right) \\
&\qquad + \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{B}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}'''_{24} &= \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{B}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{J}_{241} &= \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}_{17}^{17} \times \boldsymbol{r}_{G5'}^{0}\right) \\
&\qquad + \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}_{242} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}_{17}^{17} \times \boldsymbol{r}_{G5'}^{0}\right) \\
&\qquad + \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right) \\
\boldsymbol{J}_{243} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{i}_{17}^{17} \times \boldsymbol{r}_{G5'}^{0}\right) \\
&\qquad + \left(\boldsymbol{i}_{17}^{0} \times \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_x(\theta_{16,15})\boldsymbol{R}_z(\theta_{17,16})\boldsymbol{R}_x(\theta_{18,17})\boldsymbol{r}_{G5'}^{18}\right)
\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{25}^T}{\partial q_j}$:

$$\begin{aligned}
\frac{\partial \boldsymbol{k}_{25}^T}{\partial q_j} = {} & \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{43}}\frac{\partial \theta_{43}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{54}}\frac{\partial \theta_{54}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{16,15}}\frac{\partial \theta_{16,15}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{17,16}}\frac{\partial \theta_{17,16}}{\partial q_j} \\
& + \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{18,17}}\frac{\partial \theta_{18,17}}{\partial q_j}
\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{25i}^T}{\partial q_j} = \boldsymbol{J}_{25}\frac{\partial \boldsymbol{\theta}}{\partial q_j} + \boldsymbol{J}'_{25}\frac{\partial \theta_{16,15}}{\partial q_j} + \boldsymbol{J}''_{25}\frac{\partial \theta_{17,16}}{\partial q_j} + \boldsymbol{J}'''_{25}\frac{\partial \theta_{18,17}}{\partial q_j} \qquad 3.108$$

Dónde:

$$\begin{aligned}
\boldsymbol{J}_{25} &= \left[\frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{21}} \quad \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{43}} \quad \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{54}}\right] \\
\boldsymbol{J}'_{25} &= \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{16,15}} \\
\boldsymbol{J}''_{25} &= \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{17,16}} \\
\boldsymbol{J}'''_{25} &= \frac{\partial \boldsymbol{k}_{25}^T}{\partial \theta_{18,17}}
\end{aligned}$$

3.4 Velocidad de Centros de Gravedad y Velocidad Angular para las cadenas i.

Al igual que con la cadena 0 se construyen las ecuaciones de lazo de los centros de gravedad on base en las figuras 3-1 y 3-2:

$$\boldsymbol{r}_{G1i}^{0} = \boldsymbol{r}_{10i}^{0} + \boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{G1i'}^{0}$$
$$\boldsymbol{r}_{G2i}^{0} = \boldsymbol{r}_{10i}^{0} + \boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}$$

$$\boldsymbol{r}_{10i}^{0} = signo\ y_{10}\boldsymbol{j}_{0}$$
$$\boldsymbol{r}_{32i}^{0} = \boldsymbol{R}_{2\mathrm{i}}^{0}\boldsymbol{r}_{32\mathrm{i}}^{2\mathrm{i}}$$
$$\boldsymbol{r}_{43i}^{0} = \boldsymbol{R}_{3\mathrm{i}}^{0}\boldsymbol{r}_{43i}^{3i}$$

$$\boldsymbol{r}_{G1i'}^{0} = \boldsymbol{R}_{3\mathrm{i}}^{0}\boldsymbol{r}_{G1i'}^{3i}$$
$$\boldsymbol{r}_{G2i'}^{0} = \boldsymbol{R}_{7\mathrm{i}}^{0}\boldsymbol{r}_{G2i'}^{7i}$$

$$\boldsymbol{R}_{2i}^{0} = \boldsymbol{R}_{z}(\theta_{21i})$$
$$\boldsymbol{R}_{3i}^{0} = \boldsymbol{R}_{2i}^{0}$$
$$\boldsymbol{R}_{7i}^{0} = \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})\boldsymbol{R}_{z}(\theta_{76i})$$

$$\boldsymbol{r}_{32i}^{2i} = [-z_{32i}, 0{,}0]^{T}$$
$$\boldsymbol{r}_{43i}^{3i} = [x_{43i}, 0{,}0]^{T}$$

$$\boldsymbol{r}_{G1i'}^{3i} = [x_{G1i}, y_{G1i}, z_{G1i}]^{T}$$
$$\boldsymbol{r}_{G2i'}^{7i} = [x_{G2i}, y_{G2i}, z_{G2i}]^{T}$$

$$\boldsymbol{v}_{G1i}^{0} = \boldsymbol{v}_{10i}^{0} + \boldsymbol{v}_{32i}^{0} + \boldsymbol{v}_{G1i'}^{0}$$
$$\boldsymbol{v}_{G2i}^{0} = \boldsymbol{v}_{10i}^{0} + \boldsymbol{v}_{32i}^{0} + \boldsymbol{v}_{43i}^{0} + \boldsymbol{v}_{G2i'}^{0}$$

$$\boldsymbol{v}_{10i}^{0} = 0$$
$$\boldsymbol{v}_{32i}^{0} = \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{32i}^{0}$$
$$\boldsymbol{v}_{43i}^{0} = \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{43i}^{0}$$

$$\boldsymbol{v}_{G1i'}^{0} = \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{G1i'}^{0}$$
$$\boldsymbol{v}_{G2i'}^{0} = \boldsymbol{\omega}_{7i}^{0} \times \boldsymbol{r}_{G2i'}^{0}$$

$$\boldsymbol{\omega}_{2i}^{0} = \boldsymbol{\omega}_{21i}^{0}$$
$$\boldsymbol{\omega}_{7i}^{0} = \boldsymbol{\omega}_{21i}^{0} + \boldsymbol{\omega}_{65i}^{0} + \boldsymbol{\omega}_{76i}^{0}$$

$$\boldsymbol{\omega}_{21i}^{0} = \dot{\theta}_{21i}\ \boldsymbol{k}_{1i}^{0}$$
$$\boldsymbol{\omega}_{65i}^{0} = \dot{\theta}_{65i}\ \boldsymbol{j}_{5i}^{0}$$
$$\boldsymbol{\omega}_{76i}^{0} = \dot{\theta}_{76i}\ \boldsymbol{k}_{6i}^{0}$$

$$\boldsymbol{k}_{1i}^{0} = \boldsymbol{k}_{0}$$
$$\boldsymbol{j}_{5i}^{0} = \boldsymbol{R}_{5i}^{0}\boldsymbol{j}_{5i}^{5i}$$
$$\boldsymbol{k}_{6i}^{0} = \boldsymbol{R}_{6i}^{0}\boldsymbol{k}_{6i}^{6i}$$

$$\boldsymbol{R}_{5\mathrm{i}}^{0} = \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})$$
$$\boldsymbol{R}_{6\mathrm{i}}^{0} = \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})$$

3.4.1 Velocidad de Centro de Gravedad del Cuerpo 1i

$$\begin{aligned}
\boldsymbol{v}_{G1i}^{0} &= \boldsymbol{v}_{10i}^{0} + \boldsymbol{v}_{32i}^{0} + \boldsymbol{v}_{G1i'}^{0} \\
&= \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{32i}^{0} + \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{G1i'}^{0} \\
&= \boldsymbol{\omega}_{21i}^{0} \times \boldsymbol{r}_{32i}^{0} + \boldsymbol{\omega}_{21i}^{0} \times \boldsymbol{r}_{G1i'}^{0} \\
&= \boldsymbol{\omega}_{21i}^{0} \times (\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{G1i'}^{0}) \\
&= \left(\dot{\theta}_{21i}\, \boldsymbol{k}_{1i}^{0}\right) \times (\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{G1i'}^{0}) \\
&= (\boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{G1i'}^{0}\right))\dot{\theta}_{21i}
\end{aligned}$$

$$\boldsymbol{v}_{G1i}^{0} = \boldsymbol{k}_{1i}\dot{\theta}_{21\mathrm{i}} \qquad 3.109$$

Dónde:

$$\boldsymbol{k}_{1i} = \boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{G1i'}^{0}\right) \qquad 3.110$$

De la ec. (2.40):

$$\dot{\theta}_{21\mathrm{i}} = \frac{1}{V_{44\mathrm{i}}}(E_{31\mathrm{i}}\dot{x} + E_{32\mathrm{i}}\dot{y} + E_{33\mathrm{i}}\dot{z} + E_{34\mathrm{i}}\dot{\psi} + E_{35\mathrm{i}}\dot{\theta} + E_{36\mathrm{i}}\dot{\phi})$$

Renombrando:

$$\dot{\theta}_{21\mathrm{i}} = \boldsymbol{k}_{2i}^{T}\dot{\boldsymbol{q}} \qquad 3.111$$

Dónde:

$$\begin{aligned}
\boldsymbol{k}_{2i}^{T} &= \frac{1}{V_{44i}}[E_{31i}, E_{32i}, E_{33i}, E_{34i}, E_{35i}, E_{36i}] \\
\dot{\boldsymbol{q}} &= \left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]^{T}
\end{aligned} \qquad 3.112$$

Sustituyendo la ec. (3.48) en la ec. (3.46) con el fin de poner a esta última en función de las coordenadas generalizadas se tiene:

$$\begin{aligned}
\boldsymbol{v}_{G1i}^{0} &= \boldsymbol{k}_{1i}\boldsymbol{k}_{2i}^{T}\dot{\boldsymbol{q}} \\
&= (\boldsymbol{k}_{1i}\boldsymbol{k}_{2i}^{T})\dot{\boldsymbol{q}}
\end{aligned}$$

Renombrando:

$$\boldsymbol{v}_{G1i}^{0} = \boldsymbol{M}_{1i}\dot{\boldsymbol{q}} \qquad 3.113$$

Dónde:

$$\boldsymbol{M}_{1i} = \boldsymbol{k}_{1i}\boldsymbol{k}_{2i}^{T} \qquad 3.114$$

3.4.2 Velocidad Angular del Cuerpo 1i

$$\boldsymbol{\omega}_{2i}^{0} = \boldsymbol{\omega}_{21i}^{0}$$
$$\boldsymbol{\omega}_{21i}^{0} = \boldsymbol{k}_{1i}^{0}\dot{\theta}_{21i}$$

Sustituyendo (3.48) en la ec. anterior:

$$\boldsymbol{\omega}_{21i}^{0} = \boldsymbol{k}_{1i}^{0}(\boldsymbol{k}_{2i}^{T}\dot{\boldsymbol{q}}) = (\boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T})\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{\omega}_{2i}^{0} = \boldsymbol{M}_{2i}\dot{\boldsymbol{q}} \qquad 3.115$$

Dónde:

$$\boldsymbol{M}_{2i} = \boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T} \qquad 3.116$$

3.4.3 Velocidad de Centro de Gravedad del Cuerpo 2i

$$\begin{aligned}
\boldsymbol{v}_{G2i}^{0} &= \boldsymbol{v}_{10i}^{0} + \boldsymbol{v}_{32i}^{0} + \boldsymbol{v}_{43i}^{0} + \boldsymbol{v}_{G2i'}^{0} \\
&= \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{32i}^{0} + \boldsymbol{\omega}_{2i}^{0} \times \boldsymbol{r}_{43i}^{0} + \boldsymbol{\omega}_{7i}^{0} \times \boldsymbol{r}_{G2i'}^{0} \\
&= \boldsymbol{\omega}_{21i}^{0} \times \boldsymbol{r}_{32i}^{0} + \boldsymbol{\omega}_{21i}^{0} \times \boldsymbol{r}_{43i}^{0} + (\boldsymbol{\omega}_{21i}^{0} + \boldsymbol{\omega}_{65i}^{0} + \boldsymbol{\omega}_{76i}^{0}) \times \boldsymbol{r}_{G2i'}^{0} \\
&= \left(\dot{\theta}_{21i}\, \boldsymbol{k}_{1i}^{0}\right) \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}\right) + \left(\dot{\theta}_{65i}\, \boldsymbol{j}_{5i}^{0} + \dot{\theta}_{76i}\, \boldsymbol{k}_{6i}^{0}\right) \times \boldsymbol{r}_{G2i'}^{0} \\
&= (\, \boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}\right))\dot{\theta}_{21i} + \left(\boldsymbol{j}_{5i}^{0} \times \boldsymbol{r}_{G2i'}^{0}\right)\dot{\theta}_{65i} + (\, \boldsymbol{k}_{6i}^{0} \\
&\qquad \times \boldsymbol{r}_{G2i'}^{0}\,)\dot{\theta}_{76i} \qquad 3.117 \\
\boldsymbol{v}_{G2i}^{0} &= \boldsymbol{k}_{3i}\dot{\theta}_{21\mathrm{i}} + \boldsymbol{k}_{4i}\dot{\theta}_{65i} + \boldsymbol{k}_{5i}\dot{\theta}_{76i}
\end{aligned}$$

Dónde:

$$\begin{aligned}
\boldsymbol{k}_{3i} &= \boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}\right) \\
\boldsymbol{k}_{4i} &= \boldsymbol{j}_{5i}^{0} \times \boldsymbol{r}_{G2i'}^{0} \\
\boldsymbol{k}_{5i} &= \boldsymbol{k}_{6i}^{0} \times \boldsymbol{r}_{G2i'}^{0}
\end{aligned} \qquad 3.118$$

De la ec. (2.40):

$$\dot{\theta}_{65\mathrm{i}} = \frac{1}{V_{48\mathrm{i}}}(E_{37\mathrm{i}}\dot{x} + E_{38\mathrm{i}}\dot{y} + E_{39\mathrm{i}}\dot{z} + E_{40\mathrm{i}}\dot{\psi} + E_{41\mathrm{i}}\dot{\theta} + E_{42\mathrm{i}}\dot{\phi})$$
$$\dot{\theta}_{76\mathrm{i}} = \frac{1}{V_{53\mathrm{i}}}(E_{43\mathrm{i}}\dot{x} + E_{44\mathrm{i}}\dot{y} + E_{45\mathrm{i}}\dot{z} + E_{46\mathrm{i}}\dot{\psi} + E_{47\mathrm{i}}\dot{\theta} + E_{48\mathrm{i}}\dot{\phi})$$

Renombrando:

$$\begin{aligned}
\dot{\theta}_{65\mathrm{i}} &= \boldsymbol{k}_{6i}^{T}\dot{\boldsymbol{q}} \\
\dot{\theta}_{76\mathrm{i}} &= \boldsymbol{k}_{7i}^{T}\dot{\boldsymbol{q}}
\end{aligned} \qquad 3.119$$

Dónde:

$$\boldsymbol{k}_{6i}^{T} = \frac{1}{V_{48i}}[E_{37i}, E_{38i}, E_{39i}, E_{40i}, E_{41i}, E_{42i}]$$
$$\boldsymbol{k}_{7i}^{T} = \frac{1}{V_{53i}}[E_{43i}, E_{44i}, E_{45i}, E_{46i}, E_{47i}, E_{48i}] \qquad 3.120$$
$$\dot{\boldsymbol{q}} = \left[\dot{x}, \dot{y}, \dot{z}, \dot{\psi}, \dot{\theta}, \dot{\phi}\right]^{T}$$

Sustituyendo la ec.(3.48) y (3.56) en la ec. (3.54) con el fin de poner a esta última en función de las coordenadas generalizadas se tiene:

$$\boldsymbol{v}_{G2i}^{0} = \boldsymbol{k}_{3i}\boldsymbol{k}_{2i}^{T}\dot{\boldsymbol{q}} + \boldsymbol{k}_{4i}\boldsymbol{k}_{6i}^{T}\dot{\boldsymbol{q}} + \boldsymbol{k}_{5i}\boldsymbol{k}_{7i}^{T}\dot{\boldsymbol{q}}$$
$$\boldsymbol{v}_{G2i}^{0} = (\boldsymbol{k}_{3i}\boldsymbol{k}_{2i}^{T} + \boldsymbol{k}_{4i}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{5i}\boldsymbol{k}_{7i}^{T})\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{v}_{G2i}^{0} = \boldsymbol{M}_{3i}\dot{\boldsymbol{q}} \qquad 3.121$$

Dónde:

$$\boldsymbol{M}_{3i} = \boldsymbol{k}_{3i}\boldsymbol{k}_{2i}^{T} + \boldsymbol{k}_{4i}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{5i}\boldsymbol{k}_{7i}^{T} \qquad 3.122$$

3.4.4 Velocidad Angular del Cuerpo 2i

$$\boldsymbol{\omega}_{7i}^{0} = \boldsymbol{\omega}_{21i}^{0} + \boldsymbol{\omega}_{65i}^{0} + \boldsymbol{\omega}_{76i}^{0}$$
$$\boldsymbol{\omega}_{7i}^{0} = \boldsymbol{k}_{1i}^{0}\dot{\theta}_{21i} + \boldsymbol{j}_{5i}^{0}\dot{\theta}_{65i} + \boldsymbol{k}_{6i}^{0}\dot{\theta}_{76i}$$

Sustituyendo (3.48) y (3.56) en la ec. anterior:

$$\boldsymbol{\omega}_{7i}^{0} = \boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T}\dot{\boldsymbol{q}} + \boldsymbol{j}_{5i}^{0}\boldsymbol{k}_{6i}^{T}\dot{\boldsymbol{q}} + \boldsymbol{k}_{6i}^{0}\boldsymbol{k}_{7i}^{T}\dot{\boldsymbol{q}}$$
$$\boldsymbol{\omega}_{7i}^{0} = (\boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T} + \boldsymbol{j}_{5i}^{0}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{6i}^{0}\boldsymbol{k}_{7i}^{T})\dot{\boldsymbol{q}}$$

Renombrando:

$$\boldsymbol{\omega}_{7i}^{0} = \boldsymbol{M}_{4i}\dot{\boldsymbol{q}} \qquad 3.123$$

Dónde:

$$\boldsymbol{M}_{4i} = \boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T} + \boldsymbol{j}_{5i}^{0}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{6i}^{0}\boldsymbol{k}_{7i}^{T} \qquad 3.124$$

Matrices de inercia medidas en la base inercial:

$$\boldsymbol{J}_{1i}^{0} = \boldsymbol{R}_{2i}^{0}\,\boldsymbol{J}_{1i}^{2i}\,{\boldsymbol{R}_{2i}^{0}}^{T}$$
$$\boldsymbol{J}_{2i}^{0} = \boldsymbol{R}_{7i}^{0}\,\boldsymbol{J}_{2i}^{7i}\,{\boldsymbol{R}_{7i}^{0}}^{T}$$

3.5 Función Lagrangiana para cadenas i.

3.5.1 Desarrollo del Primer Término de la Ecuación de Lagrange para las cadenas i.

Desarrollando $\frac{\partial L_{1i}}{\partial \dot{q}_j}$:

Se tiene:

$$L_{1i} = \frac{1}{2}\left(m_{1i}\boldsymbol{v}_{G1i}^{0\ T}\boldsymbol{v}_{G1i}^{0} + \boldsymbol{\omega}_{2i}^{0\ T}\boldsymbol{J}_{1i}\boldsymbol{\omega}_{2i}^{0}\right) + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0$$

Sustituyendo ec.(3.113) y ec.(3.115):

$$\begin{aligned}
L_{1i} &= \frac{1}{2}(m_{1i}(\boldsymbol{M}_{1i}\dot{\boldsymbol{q}})^T(\boldsymbol{M}_{1i}\dot{\boldsymbol{q}}) + (\boldsymbol{M}_{2i}\dot{\boldsymbol{q}})^T\boldsymbol{J}_{1i}(\boldsymbol{M}_{2i}\dot{\boldsymbol{q}})) + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0 \\
&= \frac{1}{2}\left(m_{1i}\dot{\boldsymbol{q}}^T\boldsymbol{M}_{1i}^{\ T}(\boldsymbol{M}_{1i}\dot{\boldsymbol{q}}) + \dot{\boldsymbol{q}}^T\boldsymbol{M}_{2i}^{\ T}\boldsymbol{J}_{1i}(\boldsymbol{M}_{2i}\dot{\boldsymbol{q}})\right) + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0 \\
&= \frac{1}{2}\left(m_{1i}\dot{\boldsymbol{q}}^T(\boldsymbol{M}_{1i}^{\ T}\boldsymbol{M}_{1i})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T(\boldsymbol{M}_{2i}^{\ T}\boldsymbol{J}_{1i}\boldsymbol{M}_{2i})\dot{\boldsymbol{q}}\right) + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0 \\
&= \frac{1}{2}\dot{\boldsymbol{q}}^T(m_{1i}\boldsymbol{M}_{1i}^{\ T}\boldsymbol{M}_{1i} + \boldsymbol{M}_{2i}^{\ T}\boldsymbol{J}_{1i}\boldsymbol{M}_{2i})\dot{\boldsymbol{q}} + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0 \\
L_{1i} &= \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0 \qquad 3.125
\end{aligned}$$

Dónde:

$$\boldsymbol{N}_{1i} = m_{1i}\boldsymbol{M}_{1i}^{\ T}\boldsymbol{M}_{1i} + \boldsymbol{M}_{2i}^{\ T}\boldsymbol{J}_{1i}\boldsymbol{M}_{2i}$$

Efectuando la derivada:

$$\begin{aligned}
\frac{\partial L_{1i}}{\partial \dot{q}_j} &= \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{N}_{1i}\dot{\boldsymbol{q}}) + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0\right) \\
\frac{\partial L_{1i}}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_{1i}\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \qquad 3.126
\end{aligned}$$

Simplificando:

$$\begin{aligned}
\frac{\partial L_{1i}}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{N}_{1i}\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \\
&= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}}\right) \\
\frac{\partial L_{1i}}{\partial \dot{q}_j} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} \qquad 3.127
\end{aligned}$$

Desarrollando $\frac{\partial L_{2i}}{\partial \dot{q}_j}$:

Se tiene:

$$L_{2i} = \frac{1}{2}\left(m_{2i}\boldsymbol{v}_{G2i}^{0}{}^{T}\boldsymbol{v}_{G2i}^{0} + \boldsymbol{\omega}_{7i}^{0}{}^{T}\boldsymbol{J}_{2i}\boldsymbol{\omega}_{7i}^{0}\right) + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0}$$

Sustituyendo ec.(3.121) y ec.(3.123):

$$\begin{aligned} L_{2i} &= \frac{1}{2}(m_{2i}(\boldsymbol{M}_{3i}\dot{\boldsymbol{q}})^{T}(\boldsymbol{M}_{3i}\dot{\boldsymbol{q}}) + (\boldsymbol{M}_{4i}\dot{\boldsymbol{q}})^{T}\boldsymbol{J}_{1i}(\boldsymbol{M}_{4i}\dot{\boldsymbol{q}})) + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0} \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^{T}(m_{2i}\boldsymbol{M}_{3i}{}^{T}\boldsymbol{M}_{3i} + \boldsymbol{M}_{4i}{}^{T}\boldsymbol{J}_{2i}\boldsymbol{M}_{4i})\dot{\boldsymbol{q}} + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0} \\ L_{2i} &= \frac{1}{2}\dot{\boldsymbol{q}}^{T}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0} \end{aligned} \tag{3.128}$$

Dónde:

$$\boldsymbol{N}_{2i} = m_{2i}\boldsymbol{M}_{3i}{}^{T}\boldsymbol{M}_{3i} + \boldsymbol{M}_{4i}{}^{T}\boldsymbol{J}_{2i}\boldsymbol{M}_{4i}$$

Efectuando la derivada:

$$\begin{aligned} \frac{\partial L_{2i}}{\partial \dot{q}_j} &= \frac{\partial}{\partial \dot{q}_j}\left(\frac{1}{2}(\dot{\boldsymbol{q}}^{T}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}}) + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0}\right) \\ \frac{\partial L_{2i}}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{T}\boldsymbol{N}_{2i}\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \end{aligned} \tag{3.129}$$

Simplificando:

$$\begin{aligned} \frac{\partial L_{2i}}{\partial \dot{q}_j} &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{T}\boldsymbol{N}_{2i}\frac{\partial \dot{\boldsymbol{q}}}{\partial \dot{q}_j}\right) \\ &= \frac{1}{2}\left(\frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}}\right) \\ \frac{\partial L_{2i}}{\partial \dot{q}_j} &= \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} \end{aligned} \tag{3.130}$$

El desarrollo de la derivada con respecto al tiempo de cada elemento se muestra a continuación:

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_{1i}}{\partial \dot{q}_j}\right)$:

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_{1i}}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\dot{\boldsymbol{q}}\right)\\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\right)\boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_{1i}}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\frac{d\dot{\boldsymbol{q}}}{dt}\right)\\
&= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_{1i}\dot{\boldsymbol{q}}\\
\frac{d}{dt}\left(\frac{\partial L_{1i}}{\partial \dot{q}_j}\right) &= \mathbb{D}_{1ij}\ddot{\boldsymbol{q}} + \mathbb{V}_{1ij}\dot{\boldsymbol{q}}
\end{aligned}
\qquad 3.131
$$

Dónde:

$$
\begin{aligned}
\mathbb{D}_{1ij} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\boldsymbol{N}_{1i}\\
\mathbb{V}_{1ij} &= \frac{\partial \dot{\boldsymbol{q}}^T}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_{1i}
\end{aligned}
\qquad 3.132
$$

Además:

$$
\begin{aligned}
\boldsymbol{N}_{1i} &= m_{1i}\boldsymbol{M}_{1i}{}^T\boldsymbol{M}_{1i} + \boldsymbol{M}_{2i}{}^T\boldsymbol{J}_{1i}\boldsymbol{M}_{2i}\\
\boldsymbol{M}_{1i} &= \boldsymbol{k}_{1i}\boldsymbol{k}_{2i}^T\\
\boldsymbol{M}_{2i} &= \boldsymbol{k}_{1i}^0\boldsymbol{k}_{2i}^T\\
\boldsymbol{k}_{1i} &= \boldsymbol{k}_{1i}^0 \times \left(\boldsymbol{r}_{32i}^0 + \boldsymbol{r}_{G1i'}^0\right)\\
\boldsymbol{k}_{2i}^T &= \frac{1}{V_{44}}\left[E_{31}, E_{32}, E_{33}, E_{34}, E_{35}, E_{36}\right]\\
\boldsymbol{k}_{1i}^0 &= \boldsymbol{k}_0\\
\boldsymbol{r}_{32i}^0 &= \boldsymbol{R}_{2i}^0\boldsymbol{r}_{32i}^{2i}\\
\boldsymbol{r}_{G1i'}^0 &= \boldsymbol{R}_{3i}^0\boldsymbol{r}_{G1i'}^{3i}\\
\boldsymbol{R}_{2i}^0 &= \boldsymbol{R}_z(\theta_{21i})\\
\boldsymbol{R}_{3i}^0 &= \boldsymbol{R}_{2i}^0\\
\boldsymbol{r}_{32i}^{2i} &= [-z_{32i}, 0,0]^T\\
\boldsymbol{r}_{43i}^{3i} &= [x_{43i}, 0,0]^T\\
\boldsymbol{r}_{G1i'}^{3i} &= [x_{G1i}, y_{G1i}, z_{G1i}]^T\\
\boldsymbol{J}_{1i}^0 &= \boldsymbol{R}_{2i}^0\,\boldsymbol{J}_{1i}^{2i}\,\boldsymbol{R}_{2i}^0{}^T
\end{aligned}
$$

Derivando:

$$
\dot{\boldsymbol{N}}_{1i} = m_{1i}\left(\dot{\boldsymbol{M}}_{1i}{}^T\boldsymbol{M}_{1i} + \boldsymbol{M}_{1i}{}^T\dot{\boldsymbol{M}}_{1i}\right) + \left(\dot{\boldsymbol{M}}_{2i}{}^T\boldsymbol{J}_{1i}\boldsymbol{M}_{2i} + \boldsymbol{M}_{2i}{}^T\dot{\boldsymbol{J}}_{1i}\boldsymbol{M}_{2i} + \boldsymbol{M}_{2i}{}^T\boldsymbol{J}_{1i}\dot{\boldsymbol{M}}_{2i}\right)
$$

A su vez:

$$
\begin{aligned}
\dot{\boldsymbol{M}}_{1i} &= \dot{\boldsymbol{k}}_{1i}\boldsymbol{k}_{2i}^T + \boldsymbol{k}_{1i}\dot{\boldsymbol{k}}_{2i}^T\\
\dot{\boldsymbol{k}}_{1i} &= \boldsymbol{k}_{1i}^0 \times \left(\dot{\boldsymbol{r}}_{32i}^0 + \dot{\boldsymbol{r}}_{G1i'}^0\right)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\boldsymbol{r}}_{32i}^{0} &= \dot{\boldsymbol{R}}_{2i}^{0}\boldsymbol{r}_{32i}^{2i} \\
\dot{\boldsymbol{R}}_{2i}^{0} &= \boldsymbol{B}_{z}(\theta_{21i})\dot{\theta}_{21i} \\
\boldsymbol{B}_{z}(\theta_{21i}) &= \frac{d\boldsymbol{R}_{z}(\theta_{21i})}{d\theta_{21i}} \\
\dot{\boldsymbol{r}}_{G1i'}^{0} &= \dot{\boldsymbol{R}}_{3i}^{0}\boldsymbol{r}_{G1i'}^{3i} \\
\dot{\boldsymbol{R}}_{3i}^{0} &= \dot{\boldsymbol{R}}_{2i}^{0} \\
\dot{\boldsymbol{k}}_{2i}^{T} &= -\frac{1}{V_{44}^{2}}\dot{V}_{44}[E_{31}, E_{32}, E_{33}, E_{34}, E_{35}, E_{36}] + \frac{1}{V_{44}}\left[\dot{E}_{31}, \dot{E}_{32}, \dot{E}_{33}, \dot{E}_{34}, \dot{E}_{35}, \dot{E}_{36}\right] \\
\dot{\boldsymbol{M}}_{2i} &= \boldsymbol{k}_{1i}^{0}\dot{\boldsymbol{k}}_{2i}^{T} \\
\dot{\boldsymbol{j}}_{1i}^{0} &= \dot{\boldsymbol{R}}_{2i}^{0}\,\boldsymbol{J}_{1i}^{2i}\,\boldsymbol{R}_{2i}^{0\;T} + \boldsymbol{R}_{2i}^{0}\,\boldsymbol{J}_{1i}^{2i}\,\dot{\boldsymbol{R}}_{2i}^{0\;T}
\end{aligned}
$$

Desarrollando $\frac{d}{dt}\left(\frac{\partial L_{2i}}{\partial \dot{q}_j}\right)$:

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\partial L_{2i}}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}}\right) \\
&= \left(\frac{d}{dt}\left(\frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\right)\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\frac{d\boldsymbol{N}_{2i}}{dt}\dot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\frac{d\dot{\boldsymbol{q}}}{dt}\right) \\
&= \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i}\ddot{\boldsymbol{q}} + \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_{2i}\dot{\boldsymbol{q}} \\
\frac{d}{dt}\left(\frac{\partial L_{2i}}{\partial \dot{q}_j}\right) &= \mathbb{D}_{2ij}\ddot{\boldsymbol{q}} + \mathbb{V}_{2ij}\dot{\boldsymbol{q}}
\end{aligned}
\qquad 3.133
$$

Dónde:

$$
\begin{aligned}
\mathbb{D}_{2ij} &= \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\boldsymbol{N}_{2i} \\
\mathbb{V}_{2ij} &= \frac{\partial \dot{\boldsymbol{q}}^{T}}{\partial \dot{q}_j}\dot{\boldsymbol{N}}_{2i}
\end{aligned}
\qquad 3.134
$$

Además:

$$
\begin{aligned}
\boldsymbol{N}_{2i} &= m_{2i}\boldsymbol{M}_{3i}{}^{T}\boldsymbol{M}_{3i} + \boldsymbol{M}_{4i}{}^{T}\boldsymbol{J}_{2i}\boldsymbol{M}_{4i} \\
\boldsymbol{M}_{3i} &= \boldsymbol{k}_{3i}\boldsymbol{k}_{2i}^{T} + \boldsymbol{k}_{4i}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{5i}\boldsymbol{k}_{7i}^{T} \\
\boldsymbol{M}_{4i} &= \boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T} + \boldsymbol{j}_{5i}^{0}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{6i}^{0}\boldsymbol{k}_{7i}^{T} \\
\boldsymbol{k}_{3i} &= \boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}\right) \\
\boldsymbol{k}_{4i} &= \boldsymbol{j}_{5i}^{0} \times \boldsymbol{r}_{G2i'}^{0} \\
\boldsymbol{k}_{5i} &= \boldsymbol{k}_{6i}^{0} \times \boldsymbol{r}_{G2i'}^{0} \\
\boldsymbol{k}_{6i}^{T} &= \frac{1}{V_{48}}[E_{37}, = E_{38}, E_{39}, E_{40}, E_{41}, E_{42}] \\
\boldsymbol{k}_{7i}^{T} &= \frac{1}{V_{53}}[E_{43}, E_{44}, E_{45}, E_{46}, E_{47}, E_{48}] \\
\boldsymbol{j}_{5i}^{0} &= \boldsymbol{R}_{5i}^{0}\boldsymbol{j}_{5i}^{5i}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{k}_{6i}^{0} &= \boldsymbol{R}_{6i}^{0}\boldsymbol{k}_{6i}^{6i} \\
\boldsymbol{R}_{5\mathrm{i}}^{0} &= \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i}) \\
\boldsymbol{R}_{6\mathrm{i}}^{0} &= \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i}) \\
\boldsymbol{r}_{43i}^{0} &= \boldsymbol{R}_{3\mathrm{i}}^{0}\boldsymbol{r}_{43i}^{3i} \\
\boldsymbol{R}_{3i}^{0} &= \boldsymbol{R}_{2i}^{0} \\
\boldsymbol{r}_{G2i'}^{0} &= \boldsymbol{R}_{7\mathrm{i}}^{0}\boldsymbol{r}_{G2i'}^{7i} \\
\boldsymbol{R}_{7\mathrm{i}}^{0} &= \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})\boldsymbol{R}_{z}(\theta_{76i}) \\
\boldsymbol{r}_{43i}^{3i} &= [x_{43i}, 0{,}0]^{T} \\
\boldsymbol{r}_{G2i'}^{7i} &= [x_{G2i}, y_{G2i}, z_{G2i}]^{T} \\
\boldsymbol{J}_{2i}^{0} &= \boldsymbol{R}_{7i}^{0}\,\boldsymbol{J}_{2i}^{7i}\,{\boldsymbol{R}_{7i}^{0}}^{T}
\end{aligned}
$$

Derivando:

$$
\dot{\boldsymbol{N}}_{2i} = m_{2i}\left({\dot{\boldsymbol{M}}_{3i}}^{T}\boldsymbol{M}_{3i} + {\boldsymbol{M}_{3i}}^{T}\dot{\boldsymbol{M}}_{3i}\right) + \left({\dot{\boldsymbol{M}}_{4i}}^{T}\boldsymbol{J}_{2i}\boldsymbol{M}_{4i} + {\boldsymbol{M}_{4i}}^{T}\dot{\boldsymbol{J}}_{2i}\boldsymbol{M}_{4i} + {\boldsymbol{M}_{4i}}^{T}\boldsymbol{J}_{2i}\dot{\boldsymbol{M}}_{4i}\right)
$$

A su vez:

$$
\begin{aligned}
\dot{\boldsymbol{M}}_{3i} &= \dot{\boldsymbol{k}}_{3i}\boldsymbol{k}_{2i}^{T} + \boldsymbol{k}_{3i}\dot{\boldsymbol{k}}_{2i}^{T} + \dot{\boldsymbol{k}}_{4i}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{4i}\dot{\boldsymbol{k}}_{6i}^{T} + \dot{\boldsymbol{k}}_{5i}\boldsymbol{k}_{7i}^{T} + \boldsymbol{k}_{5i}\dot{\boldsymbol{k}}_{7i}^{T} \\
\dot{\boldsymbol{M}}_{4i} &= \boldsymbol{k}_{1i}^{0}\dot{\boldsymbol{k}}_{2i}^{T} + \ \dot{\boldsymbol{j}}_{5i}^{0}\boldsymbol{k}_{6i}^{T} + \ \boldsymbol{j}_{5i}^{0}\dot{\boldsymbol{k}}_{6i}^{T} + \ \dot{\boldsymbol{k}}_{6i}^{0}\boldsymbol{k}_{7i}^{T} + \ \boldsymbol{k}_{6i}^{0}\dot{\boldsymbol{k}}_{7i}^{T} \\
\dot{\boldsymbol{k}}_{3i} &= \boldsymbol{k}_{1i}^{0} \times \left(\dot{\boldsymbol{r}}_{32i}^{0} + \dot{\boldsymbol{r}}_{43i}^{0} + \dot{\boldsymbol{r}}_{G2i'}^{0}\right) \\
\dot{\boldsymbol{r}}_{43i}^{0} &= \dot{\boldsymbol{R}}_{3\mathrm{i}}^{0}\boldsymbol{r}_{43i}^{3i} \\
\dot{\boldsymbol{r}}_{G2i'}^{0} &= \dot{\boldsymbol{R}}_{7\mathrm{i}}^{0}\boldsymbol{r}_{G2i'}^{7i} \\
\dot{\boldsymbol{R}}_{7\mathrm{i}}^{0} &= \boldsymbol{B}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})\boldsymbol{R}_{z}(\theta_{76i})\dot{\theta}_{21i} \\
&\quad + \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{B}_{y}(\theta_{65i})\boldsymbol{R}_{z}(\theta_{76i})\dot{\theta}_{65i} \\
&\quad + \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})\boldsymbol{B}_{z}(\theta_{76i})\dot{\theta}_{76i}
\end{aligned}
$$

$$
\boldsymbol{B}_{y}(\theta_{65i}) = \frac{d\boldsymbol{R}_{y}(\theta_{65i})}{d\theta_{65i}}
$$

$$
\boldsymbol{B}_{z}(\theta_{76i}) = \frac{d\boldsymbol{R}_{z}(\theta_{76i})}{d\theta_{76i}}
$$

$$
\begin{aligned}
\dot{\boldsymbol{k}}_{4i} &= \dot{\boldsymbol{j}}_{5i}^{0} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{j}_{5i}^{0} \times \dot{\boldsymbol{r}}_{G2i'}^{0} \\
\dot{\boldsymbol{j}}_{5i}^{0} &= \dot{\boldsymbol{R}}_{5i}^{0}\boldsymbol{j}_{5i}^{5i} \\
\dot{\boldsymbol{R}}_{5\mathrm{i}}^{0} &= \boldsymbol{B}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\dot{\theta}_{21i} \\
\dot{\boldsymbol{k}}_{5i} &= \dot{\boldsymbol{k}}_{6i}^{0} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{k}_{6i}^{0} \times \dot{\boldsymbol{r}}_{G2i'}^{0} \\
\dot{\boldsymbol{k}}_{6i}^{0} &= \dot{\boldsymbol{R}}_{6i}^{0}\boldsymbol{k}_{6i}^{6i} \\
\dot{\boldsymbol{R}}_{6i}^{0} &= \boldsymbol{B}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})\dot{\theta}_{21i} + \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{B}_{y}(\theta_{65i})\dot{\theta}_{65i} \\
\dot{\boldsymbol{k}}_{6i}^{T} &= -\frac{1}{V_{48}^{2}}\dot{V}_{48}[E_{37}, E_{38}, E_{39}, E_{40}, E_{41}, E_{42}] + \frac{1}{V_{48}}\left[\dot{E}_{37}, \dot{E}_{38}, \dot{E}_{39}, \dot{E}_{40}, \dot{E}_{41}, \dot{E}_{42}\right] \\
\boldsymbol{k}_{7i}^{T} &= \ -\frac{1}{V_{53}^{2}}\dot{V}_{53}[E_{43}, E_{44}, E_{45}, E_{46}, E_{47}, E_{48}] + \frac{1}{V_{53}}\left[\dot{E}_{43}, \dot{E}_{44}, \dot{E}_{45}, \dot{E}_{46}, \dot{E}_{47}, \dot{E}_{48}\right] \\
\dot{\boldsymbol{J}}_{2i}^{0} &= \dot{\boldsymbol{R}}_{7i}^{0}\,\boldsymbol{J}_{2i}^{7i}\,{\boldsymbol{R}_{7i}^{0}}^{T} + \boldsymbol{R}_{7i}^{0}\,\boldsymbol{J}_{2i}^{7i}\,{\dot{\boldsymbol{R}}_{7i}^{0}}^{T}
\end{aligned}
$$

3.5.2. Desarrollo del Segundo Término de la Ecuación de Lagrange para las cadenas i.

Desarrollando $\frac{\partial L_{1i}}{\partial q_j}$

$$L_{1i} = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{N}_{1i}\dot{\boldsymbol{q}} + m_{1i}\boldsymbol{g}^T\boldsymbol{r}_{G1i}^0$$

Dónde:

$$\begin{aligned}
\boldsymbol{N}_{1i} &= m_{1i}\boldsymbol{M}_{1i}{}^T\boldsymbol{M}_{1i} + \boldsymbol{M}_{2i}{}^T\boldsymbol{J}_{1i}\boldsymbol{M}_{2i}\\
\boldsymbol{M}_{1i} &= \boldsymbol{k}_{1i}\boldsymbol{k}_{2i}^T\\
\boldsymbol{M}_{2i} &= \boldsymbol{k}_{1i}^0\boldsymbol{k}_{2i}^T\\
\boldsymbol{k}_{1i} &= \boldsymbol{k}_{1i}^0 \times \left(\boldsymbol{r}_{32i}^0 + \boldsymbol{r}_{G1i'}^0\right)\\
\boldsymbol{k}_{2i}^T &= \frac{1}{V_{44}}[E_{31}, E_{32}, E_{33}, E_{34}, E_{35}, E_{36}]\\
\boldsymbol{k}_{1i}^0 &= \boldsymbol{k}_0\\
\boldsymbol{r}_{32i}^0 &= \boldsymbol{R}_{2i}^0\boldsymbol{r}_{32i}^{2i}\\
\boldsymbol{r}_{G1i'}^0 &= \boldsymbol{R}_{3i}^0\boldsymbol{r}_{G1i'}^{3i}\\
\boldsymbol{R}_{2i}^0 &= \boldsymbol{R}_z(\theta_{21i})\\
\boldsymbol{R}_{3i}^0 &= \boldsymbol{R}_{2i}^0\\
\boldsymbol{r}_{32i}^{2i} &= [-z_{32i}, 0,0]^T\\
\boldsymbol{r}_{43i}^{3i} &= [x_{43i}, 0,0]^T\\
\boldsymbol{r}_{G1i'}^{3i} &= [x_{G1i}, y_{G1i}, z_{G1i}]^T\\
\boldsymbol{J}_{1i}^0 &= \boldsymbol{R}_{2i}^0\,\boldsymbol{J}_{1i}^{2i}\,\boldsymbol{R}_{2i}^0{}^T
\end{aligned}$$

Derivando con respecto a q_j :

$$\frac{\partial L_{1i}}{\partial q_j} = \frac{1}{2}\dot{\boldsymbol{q}}^T\frac{\partial \boldsymbol{N}_{1i}}{\partial q_j}\dot{\boldsymbol{q}} + m_{1i}\boldsymbol{g}^T\frac{\partial \boldsymbol{r}_{G1i}^0}{\partial q_j}$$

$$\frac{\partial L_{1i}}{\partial q_j} = \mathbb{V}'_{1ij}\dot{\boldsymbol{q}} + \mathbb{C}_{1ij} \qquad 3.135$$

Dónde:

$$\mathbb{V}'_{1ij} = \frac{1}{2}\dot{\boldsymbol{q}}^T\frac{\partial \boldsymbol{N}_{1i}}{\partial q_j}$$
$$\mathbb{C}_{1ij} = m_{1i}\boldsymbol{g}^T\frac{\partial \boldsymbol{r}_{G1i}^0}{\partial q_j} \qquad 3.136$$

A su vez:

$$\frac{\partial \boldsymbol{N}_{1i}}{\partial q_j} = m_{1i}\left(\frac{\partial \boldsymbol{M}_{1i}{}^T}{\partial q_j}\boldsymbol{M}_{1i} + \boldsymbol{M}_{1i}{}^T\frac{\partial \boldsymbol{M}_{1i}}{\partial q_j}\right) + \frac{\partial \boldsymbol{M}_{2i}{}^T}{\partial q_j}\boldsymbol{J}_{1i}\boldsymbol{M}_{2i} + \boldsymbol{M}_{2i}{}^T\frac{\partial \boldsymbol{J}_{1i}}{\partial q_j}\boldsymbol{M}_{2i} + \boldsymbol{M}_{2i}{}^T\boldsymbol{J}_{1i}\frac{\partial \boldsymbol{M}_{2i}}{\partial q_j}$$

Además:

$$\frac{\partial \boldsymbol{M}_{1i}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} \boldsymbol{k}_{2i}^T + \boldsymbol{k}_{1i} \frac{\partial \boldsymbol{k}_{2i}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{M}_{2i}}{\partial q_j} = \boldsymbol{k}_{1i}^0 \frac{\partial \boldsymbol{k}_{2i}^T}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \boldsymbol{k}_{1i}^0 \times \left(\frac{\partial \boldsymbol{r}_{32i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G1i'}^0}{\partial q_j} \right)$$

$$\frac{\partial \boldsymbol{k}_{2i}^T}{\partial q_j} = -\frac{1}{V_{44}^2} \frac{\partial V_{44}}{\partial q_j} [E_{31}, E_{32}, E_{33}, E_{34}, E_{35}, E_{36}] + \frac{1}{V_{44}} \left[\frac{\partial E_{31}}{\partial q_j}, \frac{\partial E_{32}}{\partial q_j}, \frac{\partial E_{33}}{\partial q_j}, \frac{\partial E_{34}}{\partial q_j}, \frac{\partial E_{35}}{\partial q_j}, \frac{\partial E_{36}}{\partial q_j} \right]$$

$$\frac{\partial \boldsymbol{r}_{32i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j} \boldsymbol{r}_{32i}^{2i}$$

$$\frac{\partial \boldsymbol{r}_{G1i'}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{3i}^0}{\partial q_j} \boldsymbol{r}_{G1i'}^{3i}$$

$$\frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j} = \boldsymbol{B}_z(\theta_{21i}) \ \frac{\partial \theta_{21i}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{3i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{J}_{1i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j} \ \boldsymbol{J}_{1i}^{2i} \, {\boldsymbol{R}_{2i}^0}^T + \boldsymbol{R}_{2i}^0 \boldsymbol{J}_{1i}^{2i} \ \frac{\partial {\boldsymbol{R}_{2i}^0}^T}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \boldsymbol{k}_{1i}^0 \times \left(\frac{\partial \boldsymbol{r}_{32i}^0}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G1i'}^0}{\partial q_j} \right)$$

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \boldsymbol{k}_{1i}^0 \times \left(\frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j} \boldsymbol{r}_{32i}^{2i} + \frac{\partial \boldsymbol{R}_{3i}^0}{\partial q_j} \boldsymbol{r}_{G1i'}^{3i} \right)$$

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \boldsymbol{k}_{1i}^0 \times \left(\boldsymbol{B}_z(\theta_{21i}) \ \frac{\partial \theta_{21i}}{\partial q_j} \boldsymbol{r}_{32i}^{2i} + \boldsymbol{B}_z(\theta_{21i}) \ \frac{\partial \theta_{21i}}{\partial q_j} \boldsymbol{r}_{G1i'}^{3i} \right)$$

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \left(\boldsymbol{k}_{1i}^0 \times \boldsymbol{B}_z(\theta_{21i}) \boldsymbol{r}_{32i}^{2i} + \boldsymbol{k}_{1i}^0 \times \boldsymbol{B}_z(\theta_{21i}) \ \boldsymbol{r}_{G1i'}^{3i} \right) \frac{\partial \theta_{21i}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{1i}}{\partial q_j} = \boldsymbol{J}_{1i} \frac{\partial \boldsymbol{\beta}}{\partial q_j} \qquad 3.137$$

Dónde:

$$\boldsymbol{J}_{1i} = \left[\boldsymbol{k}_{1i}^0 \times \boldsymbol{B}_z(\theta_{21i}) \boldsymbol{r}_{32i}^{2i} + \boldsymbol{k}_{1i}^0 \times \boldsymbol{B}_z(\theta_{21i}) \ \boldsymbol{r}_{G1i'}^{3i} \quad \boldsymbol{0} \quad \boldsymbol{0} \right]$$

Desarrollando $\frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{21i}}\frac{\partial \theta_{21i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{65i}}\frac{\partial \theta_{65i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{76i}}\frac{\partial \theta_{76i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{43}}\frac{\partial \theta_{43}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{54}}\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial q_j} = \boldsymbol{J}_{2i}\frac{\partial \boldsymbol{\beta}}{\partial q_j} + \boldsymbol{J}'_{2i}\frac{\partial \boldsymbol{\theta}}{\partial q_j} \qquad 3.138$$

Dónde:

$$\boldsymbol{J}_{2i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{21i}} & \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{65i}} & \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{76i}} \end{bmatrix}$$
$$\boldsymbol{J}'_{2i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{21}} & \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{43}} & \frac{\partial \boldsymbol{k}_{2i}^{T}}{\partial \theta_{56}} \end{bmatrix}$$

Desarrollando $\frac{\partial L_{2i}}{\partial q_j}$

$$L_{2i} = \frac{1}{2}\dot{\boldsymbol{q}}^{T}\boldsymbol{N}_{2i}\dot{\boldsymbol{q}} + m_{2i}\boldsymbol{g}^{T}\boldsymbol{r}_{G2i}^{0}$$

Dónde:

$$\boldsymbol{N}_{2i} = m_{2i}\boldsymbol{M}_{3i}{}^{T}\boldsymbol{M}_{3i} + \boldsymbol{M}_{4i}{}^{T}\boldsymbol{J}_{2i}\boldsymbol{M}_{4i}$$
$$\boldsymbol{M}_{3i} = \boldsymbol{k}_{3i}\boldsymbol{k}_{2i}^{T} + \boldsymbol{k}_{4i}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{5i}\boldsymbol{k}_{7i}^{T}$$
$$\boldsymbol{M}_{4i} = \boldsymbol{k}_{1i}^{0}\boldsymbol{k}_{2i}^{T} + \boldsymbol{j}_{5i}^{0}\boldsymbol{k}_{6i}^{T} + \boldsymbol{k}_{6i}^{0}\boldsymbol{k}_{7i}^{T}$$
$$\boldsymbol{k}_{3i} = \boldsymbol{k}_{1i}^{0} \times \left(\boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{G2i'}^{0}\right)$$
$$\boldsymbol{k}_{4i} = \boldsymbol{j}_{5i}^{0} \times \boldsymbol{r}_{G2i'}^{0}$$
$$\boldsymbol{k}_{5i} = \boldsymbol{k}_{6i}^{0} \times \boldsymbol{r}_{G2i'}^{0}$$
$$\boldsymbol{k}_{6i}^{T} = \frac{1}{V_{48}}[E_{37}, E_{38}, E_{39}, E_{40}, E_{41}, E_{42}]$$
$$\boldsymbol{k}_{7i}^{T} = \frac{1}{V_{53}}[E_{43}, E_{44}, E_{45}, E_{46}, E_{47}, E_{48}]$$
$$\boldsymbol{j}_{5i}^{0} = \boldsymbol{R}_{5i}^{0}\boldsymbol{j}_{5i}^{5i}$$
$$\boldsymbol{k}_{6i}^{0} = \boldsymbol{R}_{6i}^{0}\boldsymbol{k}_{6i}^{6i}$$
$$\boldsymbol{R}_{5\mathrm{i}}^{0} = \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})$$
$$\boldsymbol{R}_{6\mathrm{i}}^{0} = \boldsymbol{R}_{z}(\theta_{21i})\boldsymbol{R}_{x}(\beta_{54i})\boldsymbol{R}_{y}(\theta_{65i})$$
$$\boldsymbol{r}_{43i}^{0} = \boldsymbol{R}_{3\mathrm{i}}^{0}\boldsymbol{r}_{43i}^{3i}$$
$$\boldsymbol{R}_{3i}^{0} = \boldsymbol{R}_{2i}^{0}$$
$$\boldsymbol{r}_{G2i'}^{0} = \boldsymbol{R}_{7\mathrm{i}}^{0}\boldsymbol{r}_{G2i'}^{7i}$$

$$
\begin{aligned}
\boldsymbol{R}_{7\mathrm{i}}^{0} &= \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i}) \\
\boldsymbol{r}_{43i}^{3i} &= [x_{43i}, 0,0]^T \\
\boldsymbol{r}_{G2i'}^{7i} &= [x_{G2i}, y_{G2i}, z_{G2i}]^T \\
\boldsymbol{J}_{2i}^{0} &= \boldsymbol{R}_{7i}^{0}\,\boldsymbol{J}_{2i}^{7i}\,\boldsymbol{R}_{7i}^{0\;T}
\end{aligned}
$$

Derivando con respecto a q_j :

$$
\begin{aligned}
\frac{\partial L_2}{\partial q_j} &= \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_{2i}}{\partial q_j}\dot{\boldsymbol{q}} + m_{2i}\boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G2i}^{0}}{\partial q_j} \\
\frac{\partial L_{2i}}{\partial q_j} &= \mathbb{V}'_{2ij}\dot{\boldsymbol{q}} + \mathbb{C}_{2ij}
\end{aligned}
\qquad 3.139
$$

Dónde:

$$
\begin{aligned}
\mathbb{V}'_{2ij} &= \frac{1}{2}\dot{\boldsymbol{q}}^T \frac{\partial \boldsymbol{N}_{2i}}{\partial q_j} \\
\mathbb{C}_{2ij} &= m_{2i}\boldsymbol{g}^T \frac{\partial \boldsymbol{r}_{G2i}^{0}}{\partial q_j}
\end{aligned}
\qquad 3.140
$$

Además:

$$
\begin{aligned}
\frac{\partial \boldsymbol{N}_{2i}}{\partial q_j} = m_{2i}\left(\frac{\partial \boldsymbol{M}_{3i}}{\partial q_j}^T \boldsymbol{M}_{3i} + \boldsymbol{M}_{3i}{}^T \frac{\partial \boldsymbol{M}_{3i}}{\partial q_j}\right) + \frac{\partial \boldsymbol{M}_{4i}}{\partial q_j}^T \boldsymbol{J}_{2i}\boldsymbol{M}_{4i} + \boldsymbol{M}_{4i}{}^T \frac{\partial \boldsymbol{J}_{2i}}{\partial q_j}\boldsymbol{M}_{4i} \\
+ \boldsymbol{M}_{4i}{}^T \boldsymbol{J}_{2i}\frac{\partial \boldsymbol{M}_{4i}}{\partial q_j}
\end{aligned}
$$

A su vez:

$$
\begin{aligned}
\frac{\partial \boldsymbol{M}_{3i}}{\partial q_j} &= \frac{\partial \boldsymbol{k}_{3i}}{\partial q_j}\boldsymbol{k}_{2i}^T + \boldsymbol{k}_{3i}\frac{\partial \boldsymbol{k}_{2i}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{4i}}{\partial q_j}\boldsymbol{k}_{6i}^T + \boldsymbol{k}_{4i}\frac{\partial \boldsymbol{k}_{6i}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{5i}}{\partial q_j}\boldsymbol{k}_{7i}^T + \boldsymbol{k}_{5i}\frac{\partial \boldsymbol{k}_{7i}^T}{\partial q_j} \\
\frac{\partial \boldsymbol{M}_{4i}}{\partial q_j} &= \boldsymbol{k}_{1i}^{0}\frac{\partial \boldsymbol{k}_{2i}^T}{\partial q_j} + \frac{\partial \boldsymbol{j}_{5i}^{0}}{\partial q_j}\boldsymbol{k}_{6i}^T + \boldsymbol{j}_{5i}^{0}\frac{\partial \boldsymbol{k}_{6i}^T}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^{0}}{\partial q_j}\boldsymbol{k}_{7i}^T + \boldsymbol{k}_{6i}^{0}\frac{\partial \boldsymbol{k}_{7i}^T}{\partial q_j} \\
\frac{\partial \boldsymbol{k}_{3i}}{\partial q_j} &= \boldsymbol{k}_{1i}^{0} \times \left(\frac{\partial \boldsymbol{r}_{32i}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{43i}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j}\right) \\
\frac{\partial \boldsymbol{k}_{4i}}{\partial q_j} &= \frac{\partial \boldsymbol{j}_{5i}^{0}}{\partial q_j} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{j}_{5i}^{0} \times \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j} \\
\frac{\partial \boldsymbol{k}_{5i}}{\partial q_j} &= \frac{\partial \boldsymbol{k}_{6i}^{0}}{\partial q_j} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{k}_{6i}^{0} \times \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j} \\
\frac{\partial \boldsymbol{k}_{6i}^{0}}{\partial q_j} &= \frac{\partial \boldsymbol{R}_{6i}^{0}}{\partial q_j}\boldsymbol{k}_{6i}^{6i}
\end{aligned}
$$

$$\frac{\partial \boldsymbol{j}_{5i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{5i}^{0}}{\partial q_j} \boldsymbol{j}_{5i}^{5i}$$

$$\frac{\partial \boldsymbol{k}_{6i}^{T}}{\partial q_j} = -\frac{1}{V_{48}^{2}} \frac{\partial V_{48}}{\partial q_j} [E_{37}, E_{38}, E_{39}, E_{40}, E_{41}, E_{42}] + \frac{1}{V_{48}} \left[\frac{\partial E_{37}}{\partial q_j}, \frac{\partial E_{38}}{\partial q_j}, \frac{\partial E_{39}}{\partial q_j}, \frac{\partial E_{40}}{\partial q_j}, \frac{\partial E_{41}}{\partial q_j}, \frac{\partial E_{42}}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{k}_{7i}^{T}}{\partial q_j} = -\frac{1}{V_{53}^{2}} \frac{\partial V_{53}}{\partial q_j} [E_{43}, E_{44}, E_{45}, E_{46}, E_{47}, E_{48}] + \frac{1}{V_{53}} \left[\frac{\partial E_{43}}{\partial q_j}, \frac{\partial E_{44}}{\partial q_j}, \frac{\partial E_{45}}{\partial q_j}, \frac{\partial E_{46}}{\partial q_j}, \frac{\partial E_{47}}{\partial q_j}, \frac{\partial E_{48}}{\partial q_j}\right]$$

$$\frac{\partial \boldsymbol{r}_{43i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{3\mathrm{i}}^{0}}{\partial q_j} \boldsymbol{r}_{43i}^{3i}$$

$$\frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{7\mathrm{i}}^{0}}{\partial q_j} \boldsymbol{r}_{G2i'}^{7i}$$

$$\frac{\partial \boldsymbol{R}_{5i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{z}(\theta_{21i})}{\partial q_j} \boldsymbol{R}_{x}(\beta_{54i}) + \boldsymbol{R}_{z}(\theta_{21i}) \frac{\partial \boldsymbol{R}_{x}(\beta_{54i})}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{5i}^{0}}{\partial q_j} = \boldsymbol{B}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \frac{\partial \theta_{21i}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{6i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{z}(\theta_{21i})}{\partial q_j} \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) + \boldsymbol{R}_{z}(\theta_{21i}) \frac{\partial \boldsymbol{R}_{x}(\beta_{54i})}{\partial q_j} \boldsymbol{R}_{y}(\theta_{65i}) + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \frac{\partial \boldsymbol{R}_{y}(\theta_{65i})}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{6i}^{0}}{\partial q_j} = \boldsymbol{B}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) \frac{\partial \theta_{21i}}{\partial q_j} + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{B}_{y}(\theta_{65i}) \frac{\partial \theta_{65i}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{7\mathrm{i}}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{z}(\theta_{21i})}{\partial q_j} \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) \boldsymbol{R}_{z}(\theta_{76i}) + \boldsymbol{R}_{z}(\theta_{21i}) \frac{\partial \boldsymbol{R}_{x}(\beta_{54i})}{\partial q_j} \boldsymbol{R}_{y}(\theta_{65i}) \boldsymbol{R}_{z}(\theta_{76i}) + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \frac{\partial \boldsymbol{R}_{y}(\theta_{65i})}{\partial q_j} \boldsymbol{R}_{z}(\theta_{76i}) + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) \frac{\partial \boldsymbol{R}_{z}(\theta_{76i})}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{7\mathrm{i}}^{0}}{\partial q_j} = \boldsymbol{B}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) \boldsymbol{R}_{z}(\theta_{76i}) \frac{\partial \theta_{21i}}{\partial q_j} + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{B}_{y}(\theta_{65i}) \boldsymbol{R}_{z}(\theta_{76i}) \frac{\partial \theta_{65i}}{\partial q_j} + \boldsymbol{R}_{z}(\theta_{21i}) \boldsymbol{R}_{x}(\beta_{54i}) \boldsymbol{R}_{y}(\theta_{65i}) \boldsymbol{B}_{z}(\theta_{76i}) \frac{\partial \theta_{76i}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{J}_{2i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{7i}^{0}}{\partial q_j} \boldsymbol{J}_{2i}^{7i} \boldsymbol{R}_{7i}^{0\;T} + \boldsymbol{R}_{7i}^{0} \boldsymbol{J}_{2i}^{7i} \frac{\partial \boldsymbol{R}_{7i}^{0\;T}}{\partial q_j}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{3i}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{3i}}{\partial q_j} = \boldsymbol{k}_{1i}^{0} \times \left(\frac{\partial \boldsymbol{r}_{32i}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{43i}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j} \right)$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{3i}}{\partial q_j} = \Bigg(& \big(\boldsymbol{k}_{1i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\, \boldsymbol{r}_{32i}^{2i} + \boldsymbol{k}_{1i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\, \boldsymbol{r}_{43i}^{3i} + \boldsymbol{k}_{1i}^{0} \\ & \times \boldsymbol{B}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big) \frac{\partial \theta_{21i}}{\partial q_j} \\ & + \big(\boldsymbol{k}_{1i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{B}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big) \frac{\partial \theta_{65i}}{\partial q_j} \\ & + \big(\boldsymbol{k}_{1i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{B}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big) \frac{\partial \theta_{76i}}{\partial q_j} \Bigg)\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{3i}}{\partial q_j} = \boldsymbol{J}_{3i} \frac{\partial \boldsymbol{\beta}}{\partial q_j} \qquad 3.141$$

Dónde:

$$\boldsymbol{J}_{3i} = [\boldsymbol{J}_{31i} \quad \boldsymbol{J}_{32i} \quad \boldsymbol{J}_{33i}]$$

A su vez:

$$\begin{aligned}\boldsymbol{J}_{31i} &= \boldsymbol{k}_{1i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\, \boldsymbol{r}_{32i}^{2i} + \boldsymbol{k}_{1i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\, \boldsymbol{r}_{43i}^{3i} + \boldsymbol{k}_{1i}^{0} \\ &\qquad \times \boldsymbol{B}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i} \\ \boldsymbol{J}_{32i} &= \boldsymbol{k}_{1i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{B}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i} \\ \boldsymbol{J}_{33i} &= \boldsymbol{k}_{1i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{B}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{4i}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{4i}}{\partial q_j} = \frac{\partial \boldsymbol{j}_{5i}^{0}}{\partial q_j} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{j}_{5i}^{0} \times \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j}$$

$$\begin{aligned}\frac{\partial \boldsymbol{k}_{4i}}{\partial q_j} = \Big(& \big(\boldsymbol{B}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{j}_{5i}^{5i} \times \boldsymbol{r}_{G2i'}^{0}\big) \\ & + \big(\boldsymbol{j}_{5i}^{0} \times \boldsymbol{B}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big)\Big) \frac{\partial \theta_{21i}}{\partial q_j} \\ & + \big(\boldsymbol{j}_{5i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{B}_y(\theta_{65i}) \boldsymbol{R}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big) \frac{\partial \theta_{65i}}{\partial q_j} \\ & + \big(\boldsymbol{j}_{5i}^{0} \times \boldsymbol{R}_z(\theta_{21i}) \boldsymbol{R}_x(\beta_{54i}) \boldsymbol{R}_y(\theta_{65i}) \boldsymbol{B}_z(\theta_{76i}) \boldsymbol{r}_{G2i'}^{7i}\big) \frac{\partial \theta_{76i}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{4i}}{\partial q_j} = \boldsymbol{J}_{4i} \frac{\partial \boldsymbol{\beta}}{\partial q_j} \qquad 3.142$$

Dónde:

$$\boldsymbol{J}_{4i} = [\boldsymbol{J}_{41i} \quad \boldsymbol{J}_{42i} \quad \boldsymbol{J}_{43i}]$$

A su vez:

$$\boldsymbol{J}_{41i} = \left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{j}_{5i}^{5i} \times \boldsymbol{r}_{G2i'}^{0}\right) + \left(\boldsymbol{j}_{5i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right)$$
$$\boldsymbol{J}_{42i} = \boldsymbol{j}_{5i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}$$
$$\boldsymbol{J}_{43i} = \boldsymbol{j}_{5i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{5i}}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{5i}}{\partial q_j} = \frac{\partial \boldsymbol{k}_{6i}^{0}}{\partial q_j} \times \boldsymbol{r}_{G2i'}^{0} + \boldsymbol{k}_{6i}^{0} \times \frac{\partial \boldsymbol{r}_{G2i'}^{0}}{\partial q_j}$$
$$\begin{aligned}\frac{\partial \boldsymbol{k}_{5i}}{\partial q_j} = &\Big(\left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{k}_{6i}^{6i} \times \boldsymbol{r}_{G2i'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{6i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right)\Big)\frac{\partial \theta_{21i}}{\partial q_j} \\ &+ \Big(\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{k}_{6i}^{6i} \times \boldsymbol{r}_{G2i'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{6i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right)\Big)\frac{\partial \theta_{65i}}{\partial q_j} \\ &+ \left(\boldsymbol{k}_{6i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right)\frac{\partial \theta_{76i}}{\partial q_j}\end{aligned}$$

$$\frac{\partial \boldsymbol{k}_{5i}}{\partial q_j} = \boldsymbol{J}_{5i} \frac{\partial \boldsymbol{\beta}}{\partial q_j} \qquad 3.143$$

Dónde:

$$\boldsymbol{J}_{5i} = [\boldsymbol{J}_{51i} \quad \boldsymbol{J}_{52i} \quad \boldsymbol{J}_{53i}]$$

A su vez:

$$\begin{aligned}\boldsymbol{J}_{51i} = &\left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{k}_{6i}^{6i} \times \boldsymbol{r}_{G2i'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{6i}^{0} \times \boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right) \\ \boldsymbol{J}_{52i} = &\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{k}_{6i}^{6i} \times \boldsymbol{r}_{G2i'}^{0}\right) \\ &+ \left(\boldsymbol{k}_{6i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\right) \\ \boldsymbol{J}_{53i} = &\ \boldsymbol{k}_{6i}^{0} \times \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{G2i'}^{7i}\end{aligned}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{6i}^T}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{6i}^T}{\partial q_j} = \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{21i}}\frac{\partial \theta_{21i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{65i}}\frac{\partial \theta_{65i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{76i}}\frac{\partial \theta_{76i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{43}}\frac{\partial \theta_{43}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{54}}\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{6i}^T}{\partial q_j} = \boldsymbol{J}_{6i}\frac{\partial \boldsymbol{\beta}}{\partial q_j} + \boldsymbol{J}'_{6i}\frac{\partial \boldsymbol{\theta}}{\partial q_j} \qquad 3.144$$

Dónde:

$$\boldsymbol{J}_{6i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{21i}} & \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{65i}} & \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{76i}} \end{bmatrix}$$
$$\boldsymbol{J}'_{6i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{21}} & \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{43}} & \frac{\partial \boldsymbol{k}_{6i}^T}{\partial \theta_{56}} \end{bmatrix}$$

Desarrollando $\frac{\partial \boldsymbol{k}_{7i}^T}{\partial q_j}$:

$$\frac{\partial \boldsymbol{k}_{7i}^T}{\partial q_j} = \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{21i}}\frac{\partial \theta_{21i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{65i}}\frac{\partial \theta_{65i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{76i}}\frac{\partial \theta_{76i}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{21}}\frac{\partial \theta_{21}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{43}}\frac{\partial \theta_{43}}{\partial q_j} + \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{54}}\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{k}_{7i}^T}{\partial q_j} = \boldsymbol{J}_{7i}\frac{\partial \boldsymbol{\beta}}{\partial q_j} + \boldsymbol{J}'_{7i}\frac{\partial \boldsymbol{\theta}}{\partial q_j} \qquad 3.145$$

Dónde:

$$\boldsymbol{J}_{7i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{21i}} & \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{65i}} & \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{76i}} \end{bmatrix}$$
$$\boldsymbol{J}'_{7i} = \begin{bmatrix} \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{21}} & \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{43}} & \frac{\partial \boldsymbol{k}_{7i}^T}{\partial \theta_{56}} \end{bmatrix}$$

3.6 Fuerzas Generalizadas.

La formulación de Lagrange considera el uso de fuerzas generalizadas es decir fuerzas y torques aplicadas externamente por los actuadores, de modo que es necesario desarrollar expresiones que sean compatibles con el Lagrangiano, esto se obtiene a partir de la expresión de trabajo virtual. Considerando los torques ejercidos por los actuadores en la base del robot:

$$\delta W = \sum_{i=1}^{3} \boldsymbol{F}_i^T \delta \boldsymbol{R}_i + \boldsymbol{M}_i^T \delta \boldsymbol{Q}_i$$
$$= \boldsymbol{\tau}_1^T \delta \boldsymbol{Q}_1 + \boldsymbol{\tau}_2^T \delta \boldsymbol{Q}_2 + \boldsymbol{\tau}_3^T \delta \boldsymbol{Q}_3 \quad 3.146$$

Obteniendo los términos $\boldsymbol{\tau}_i$:

$$\boldsymbol{\tau}_1 = \tau_1 \boldsymbol{j}_0$$
$$\boldsymbol{\tau}_2 = \tau_2 \boldsymbol{k}_0$$
$$\boldsymbol{\tau}_3 = \tau_3 \boldsymbol{k}_0$$

Las velocidades angulares se relacionan con los desplazamientos virtuales, esto es:

$$\delta \boldsymbol{Q} = \frac{\partial \boldsymbol{\omega}}{\partial \dot{\theta}} \delta \theta$$

$$\delta \boldsymbol{Q}_1 = \frac{\partial \boldsymbol{\omega}_2^0}{\partial \dot{\theta}_{21}} \delta \theta_{21}$$
$$= \frac{\partial \boldsymbol{\omega}_{21}^0}{\partial \dot{\theta}_{21}} \delta \theta_{21} = \frac{\partial \dot{\theta}_{21}\, \boldsymbol{j}_1^0}{\partial \dot{\theta}_{21}} \delta \theta_{21} \quad 3.147$$
$$\delta \boldsymbol{Q}_1 = \boldsymbol{j}_0 \delta \theta_{21}$$

$$\delta \boldsymbol{Q}_2 = \frac{\partial \boldsymbol{\omega}_{21}^0}{\partial \dot{\theta}_{211}} \delta \theta_{211}$$
$$= \frac{\partial \boldsymbol{\omega}_{211}^0}{\partial \dot{\theta}_{211}} \delta \theta_{211} = \frac{\partial \dot{\theta}_{211}\, \boldsymbol{k}_{11}^0}{\partial \dot{\theta}_{211}} \delta \theta_{211} \quad 3.148$$
$$\delta \boldsymbol{Q}_2 = \boldsymbol{k}_0 \delta \theta_{211}$$

$$\delta \boldsymbol{Q}_3 = \frac{\partial \boldsymbol{\omega}_{22}^0}{\partial \dot{\theta}_{212}} \delta \theta_{212}$$
$$= \frac{\partial \boldsymbol{\omega}_{212}^0}{\partial \dot{\theta}_{212}} \delta \theta_{212} = \frac{\partial \dot{\theta}_{212}\, \boldsymbol{k}_{12}^0}{\partial \dot{\theta}_{212}} \delta \theta_{212} \quad 3.149$$
$$\delta \boldsymbol{Q}_3 = \boldsymbol{k}_0 \delta \theta_{212}$$

$$\delta W = \boldsymbol{\tau}_1^T \delta \boldsymbol{Q}_{21} + \boldsymbol{\tau}_2^T \delta \boldsymbol{Q}_{211} + \boldsymbol{\tau}_3^T \delta \boldsymbol{Q}_{212}$$
$$= (\tau_1 \boldsymbol{j}_0)^T (\boldsymbol{j}_0 \delta \theta_{21}) + (\tau_2 \boldsymbol{k}_0)^T (\boldsymbol{k}_0 \delta \theta_{211}) + (\tau_3 \boldsymbol{k}_0)^T (\boldsymbol{k}_0 \delta \theta_{212})$$
$$= \tau_1 \delta \theta_{21} + \tau_2 \delta \theta_{211} + \tau_3 \delta \theta_{212}$$
$$\delta W = [\tau_1\ \tau_2\ \tau_3] \begin{bmatrix} \delta \theta_{21} \\ \delta \theta_{211} \\ \delta \theta_{212} \end{bmatrix} \quad 3.150$$

Usando:

$$\delta\theta_{21} = \boldsymbol{k}_2^T\delta\boldsymbol{q}$$
$$\delta\theta_{21i} = \boldsymbol{k}_{2i}^T\delta\boldsymbol{q}$$

Sustituyendo:

$$\begin{aligned}\delta W &= [\tau_1\ \tau_2\ \tau_3]\begin{bmatrix}\boldsymbol{k}_2^T\\ \boldsymbol{k}_{21}^T\\ \boldsymbol{k}_{22}^T\end{bmatrix}\delta\boldsymbol{q}\\ &= \boldsymbol{\tau}^T\boldsymbol{J}_\tau\delta\boldsymbol{q}\\ \delta W &= \boldsymbol{Q}^T\delta\boldsymbol{q}\end{aligned} \quad 3.151$$

Dónde:

$$\begin{aligned}\boldsymbol{\tau}^T &= [\tau_1 \quad \tau_2 \quad \tau_3]\\ \boldsymbol{J}_\tau &= [\boldsymbol{k}_2^T\ \boldsymbol{k}_{21}^T\ \boldsymbol{k}_{22}^T]^T\\ \boldsymbol{k}_2^T &= \frac{1}{V_1}[V_2, V_3, V_4, V_5, V_6, V_7]\\ \boldsymbol{k}_{2i}^T &= \frac{1}{V_{44i}}[V_{31i}, V_{32i}, V_{33i}, V_{34i}, V_{35i}, V_{36i}]\\ \delta\boldsymbol{q} &= [\delta x, \delta y, \delta z, \delta\psi, \delta\theta, \delta\phi]^T\end{aligned}$$

$$\begin{aligned}\boldsymbol{Q}^T &= \boldsymbol{\tau}^T\boldsymbol{J}_\tau\\ \boldsymbol{Q} &= \boldsymbol{J}_\tau{}^T\boldsymbol{\tau}\end{aligned} \quad 3.152$$

3.7 Sustitución de los términos de la ecuación de Lagrange.

Por Ultimo se tiene la ecuación (3.4):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$$

Dónde:

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) &= \frac{d}{dt}\left(\sum_{h=1}^{5} L_h + \sum_{i=1}^{2}(L_{1i} + L_{2i})\right)\\ &= \mathbb{D}_{1j}\ddot{\boldsymbol{q}} + \mathbb{V}_{1j}\dot{\boldsymbol{q}} + \mathbb{D}_{2j}\ddot{\boldsymbol{q}} + \mathbb{V}_{2j}\dot{\boldsymbol{q}} + \mathbb{D}_{3j}\ddot{\boldsymbol{q}} + \mathbb{V}_{3j}\dot{\boldsymbol{q}} + \mathbb{D}_{4j}\ddot{\boldsymbol{q}} + \mathbb{V}_{4j}\dot{\boldsymbol{q}} + \mathbb{D}_{5j}\ddot{\boldsymbol{q}}\\ &\quad + \mathbb{V}_{5j}\dot{\boldsymbol{q}} + \sum_{i=1}^{2}\left(\mathbb{D}_{1ij}\ddot{\boldsymbol{q}} + \mathbb{V}_{1ij}\dot{\boldsymbol{q}} + \mathbb{D}_{2ij}\ddot{\boldsymbol{q}} + \mathbb{V}_{2ij}\dot{\boldsymbol{q}}\right)\\ &= \left[\mathbb{D}_{1j} + \mathbb{D}_{2j} + \mathbb{D}_{3j} + \mathbb{D}_{4j} + \mathbb{D}_{5j} + \sum_{i=1}^{2}\left(\mathbb{D}_{1ij} + \mathbb{D}_{2ij}\right)\right]\ddot{\boldsymbol{q}}\\ &\quad + \left[\mathbb{V}_{1j} + \mathbb{V}_{2j} + \mathbb{V}_{3j} + \mathbb{V}_{4j} + \mathbb{V}_{5j} + \sum_{i=1}^{2}\left(\mathbb{V}_{1ij} + \mathbb{V}_{2ij}\right)\right]\dot{\boldsymbol{q}}\end{aligned}$$

$$\frac{\partial L}{\partial q_j} = \sum_{h=1}^{5} \frac{\partial L_h}{\partial q_j} + \sum_{i=1}^{2} \left(\frac{\partial L_{1i}}{\partial q_j} + \frac{\partial L_{2i}}{\partial q_j} \right)$$

$$\frac{\partial L}{\partial q_j} = \mathbb{V}'_{1j}\dot{\boldsymbol{q}} + \mathbb{C}_{1j} + \mathbb{V}'_{2j}\dot{\boldsymbol{q}} + \mathbb{C}_{2j} + \mathbb{V}'_{3j}\dot{\boldsymbol{q}} + \mathbb{C}_{3j} + \mathbb{V}'_{4j}\dot{\boldsymbol{q}} + \mathbb{C}_{4j} + \mathbb{V}'_{5j}\dot{\boldsymbol{q}} + \mathbb{C}_{5j} + \sum_{i=1}^{2} \left(\mathbb{V}'_{1ij}\dot{\boldsymbol{q}} + \mathbb{C}_{1ij} + \mathbb{V}'_{2ij}\dot{\boldsymbol{q}} + \mathbb{C}_{2ij} \right)$$

$$= \left[\mathbb{V}'_{1j} + \mathbb{V}'_{2j} + \mathbb{V}'_{3j} + \mathbb{V}'_{4j} + \mathbb{V}'_{5j} + \sum_{i=1}^{2} \left(\mathbb{V}'_{1ij} + \mathbb{V}'_{2ij} \right) \right] \dot{\boldsymbol{q}} + \left[\mathbb{C}_{1j} + \mathbb{C}_{2j} + \mathbb{C}_{3j} + \mathbb{C}_{4j} + \mathbb{C}_{5j} + \sum_{i=1}^{2} \left(\mathbb{C}_{1ij} + \mathbb{C}_{2ij} \right) \right]$$

Sustituyendo en la ecuación (3.4):

$$\left[\mathbb{D}_{1j} + \mathbb{D}_{2j} + \mathbb{D}_{3j} + \mathbb{D}_{4j} + \mathbb{D}_{5j} + \sum_{i=1}^{2} \left(\mathbb{D}_{1ij} + \mathbb{D}_{2ij} \right) \right] \ddot{\boldsymbol{q}} + \left[\mathbb{V}_{1j} + \mathbb{V}_{2j} + \mathbb{V}_{3j} + \mathbb{V}_{4j} + \mathbb{V}_{5j} + \sum_{i=1}^{2} \left(\mathbb{V}_{1ij} + \mathbb{V}_{2ij} \right) \right] \dot{\boldsymbol{q}} - \left[\mathbb{V}'_{1j} + \mathbb{V}'_{2j} + \mathbb{V}'_{3j} + \mathbb{V}'_{4j} + \mathbb{V}'_{5j} + \sum_{i=1}^{2} \left(\mathbb{V}'_{1ij} + \mathbb{V}'_{2ij} \right) \right] \dot{\boldsymbol{q}} - \left[\mathbb{C}_{1j} + \mathbb{C}_{2j} + \mathbb{C}_{3j} + \mathbb{C}_{4j} + \mathbb{C}_{5j} + \sum_{i=1}^{2} \left(\mathbb{C}_{1ij} + \mathbb{C}_{2ij} \right) \right] = \boldsymbol{Q}_j$$

$$\left[\mathbb{D}_{1j} + \mathbb{D}_{2j} + \mathbb{D}_{3j} + \mathbb{D}_{4j} + \mathbb{D}_{5j} + \sum_{i=1}^{2} \left(\mathbb{D}_{1ij} + \mathbb{D}_{2ij} \right) \right] \ddot{\boldsymbol{q}} + \left[\mathbb{V}_{1j} + \mathbb{V}_{2j} + \mathbb{V}_{3j} + \mathbb{V}_{4j} + \mathbb{V}_{5j} + \sum_{i=1}^{2} \left(\mathbb{V}_{1ij} + \mathbb{V}_{2ij} \right) - \mathbb{V}'_{1j} - \mathbb{V}'_{1j} - \mathbb{V}'_{3j} - \mathbb{V}'_{4j} - \mathbb{V}'_{5j} - \sum_{i=1}^{2} \left(\mathbb{V}'_{1ij} + \mathbb{V}'_{2ij} \right) \right] \dot{\boldsymbol{q}} - \left[\mathbb{C}_{1j} + \mathbb{C}_{2j} + \mathbb{C}_{3j} + \mathbb{C}_{4j} + \mathbb{C}_{5j} + \sum_{i=1}^{2} \left(\mathbb{C}_{1ij} + \mathbb{C}_{2ij} \right) \right] = \boldsymbol{Q}_j$$

Finalmente:

$$D_j \ddot{\boldsymbol{q}} + V_j \dot{\boldsymbol{q}} + C_j = \boldsymbol{Q}_j \qquad 3.153$$

Dónde:

$$D_j = \mathbb{D}_{1j} + \mathbb{D}_{2j} + \mathbb{D}_{3j} + \mathbb{D}_{4j} + \mathbb{D}_{5j} + \sum_{i=1}^{2}\left(\mathbb{D}_{1ij} + \mathbb{D}_{2ij}\right)$$

$$V_j{}^T = \mathbb{V}_{1j} + \mathbb{V}_{2j} + \mathbb{V}_{3j} + \mathbb{V}_{4j} + \mathbb{V}_{5j} + \sum_{i=1}^{2}\left(\mathbb{V}_{1ij} + \mathbb{V}_{2ij}\right) - \mathbb{V}'_{1j} - \mathbb{V}'_{1j} - \mathbb{V}'_{3j} - \mathbb{V}'_{4j} - \mathbb{V}'_{5j}$$
$$- \sum_{i=1}^{2}\left(\mathbb{V}'_{1ij} + \mathbb{V}'_{2ij}\right)$$

$$C_j = -\mathbb{C}_{1j} - \mathbb{C}_{2j} - \mathbb{C}_{3j} - \mathbb{C}_{4j} - \mathbb{C}_{5j} - \sum_{i=1}^{2}\left(-\mathbb{C}_{1ij} - \mathbb{C}_{2ij}\right)$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} \ddot{\boldsymbol{q}} + \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \dot{\boldsymbol{q}} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix}$$

$$D_j'^T \ddot{\boldsymbol{q}} + V_j'^T \dot{\boldsymbol{q}} + C_j' = \boldsymbol{Q}$$
$$D_j'^T \ddot{\boldsymbol{q}} + V_j'^T \dot{\boldsymbol{q}} + C_j' = \boldsymbol{J}_\tau{}^T \boldsymbol{\tau}$$
$$\boldsymbol{J}_\tau{}^{-T} D_j'^T \ddot{\boldsymbol{q}} + \boldsymbol{J}_\tau{}^{-T} V_j'^T \dot{\boldsymbol{q}} + \boldsymbol{J}_\tau{}^{-T} C_j' = \boldsymbol{\tau}$$

Finalmente:

$$D_j{}^T \ddot{\boldsymbol{q}} + V_j{}^T \dot{\boldsymbol{q}} + C_j = \boldsymbol{\tau} \qquad 3.154$$

Dónde:

$$D_j{}^T = \boldsymbol{J}_\tau{}^{-T} D_j'^T$$
$$V_j = \boldsymbol{J}_\tau{}^{-T} V_j'^T$$
$$C_j = \boldsymbol{J}_\tau{}^{-T} C_j'$$

Capítulo 4
Resultados

A continuación, se muestran los resultados del análisis cinemático y dinámico del robot paralelo para la trayectoria descrita en el apéndice B.

4.1 Gráficas de Posición

Los parámetros geométricos relacionados con la estructura del robot son los siguientes:

Cadena 0:

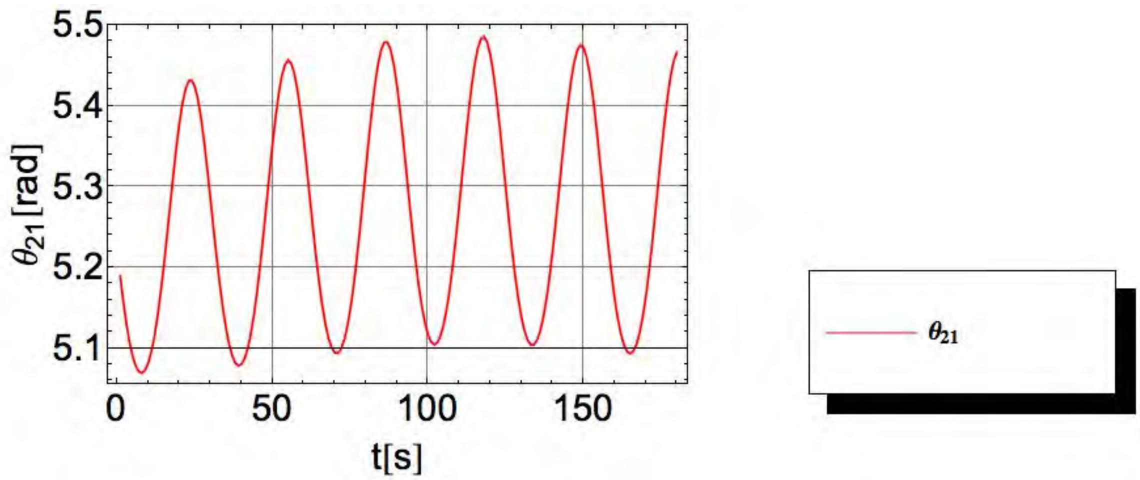
$z_{10} = .05\,m$
$x_{32} = .33\,m$
$x_{65} = .37\,m$

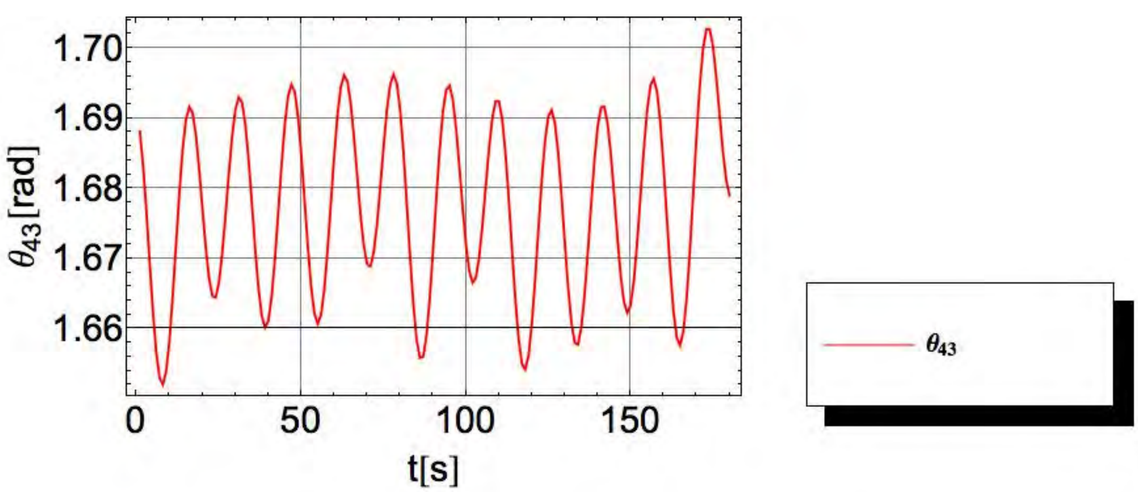
Cadena 1:

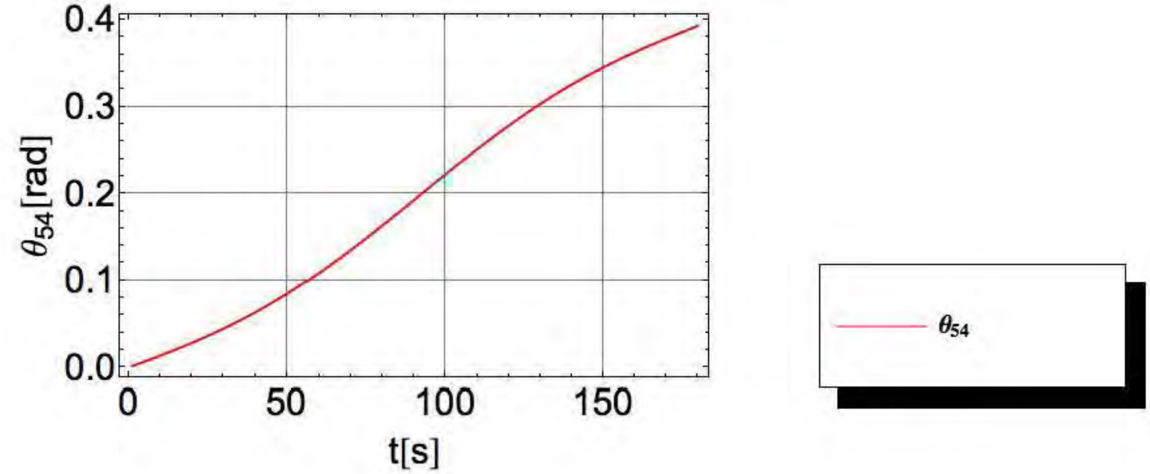
$y_{101} = .085\,m$
$z_{321} = .035\,m$
$x_{431} = .33\,m$
$\beta_{541} = 0°$
$x_{871} = .325\,m$
$x_{12111} = .015\,m$
$z_{13121} = .02\,m$
$\beta_{14131} = 270°$
$x_{15141} = .1\,m$
$x_{19181} = .06\,m$

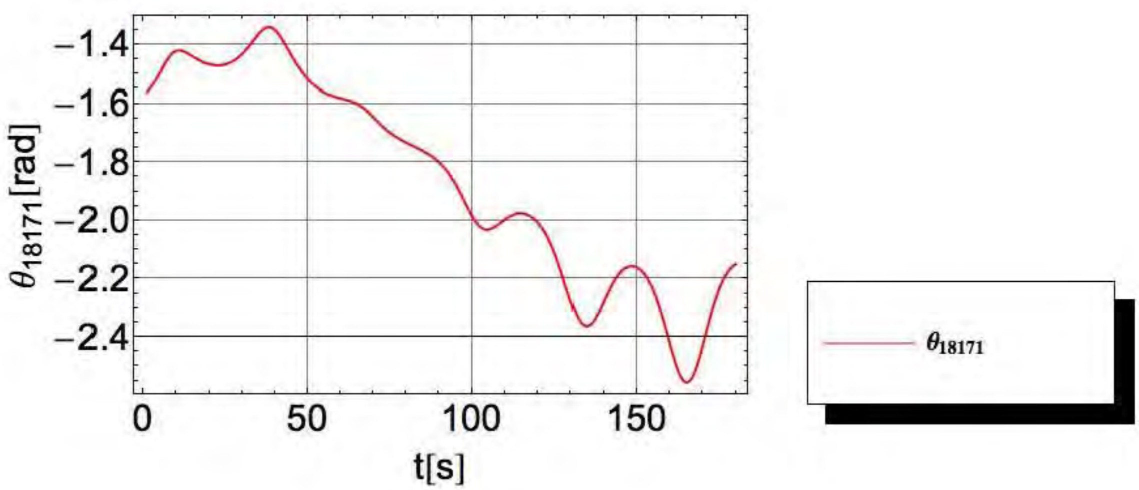
Cadena 2:

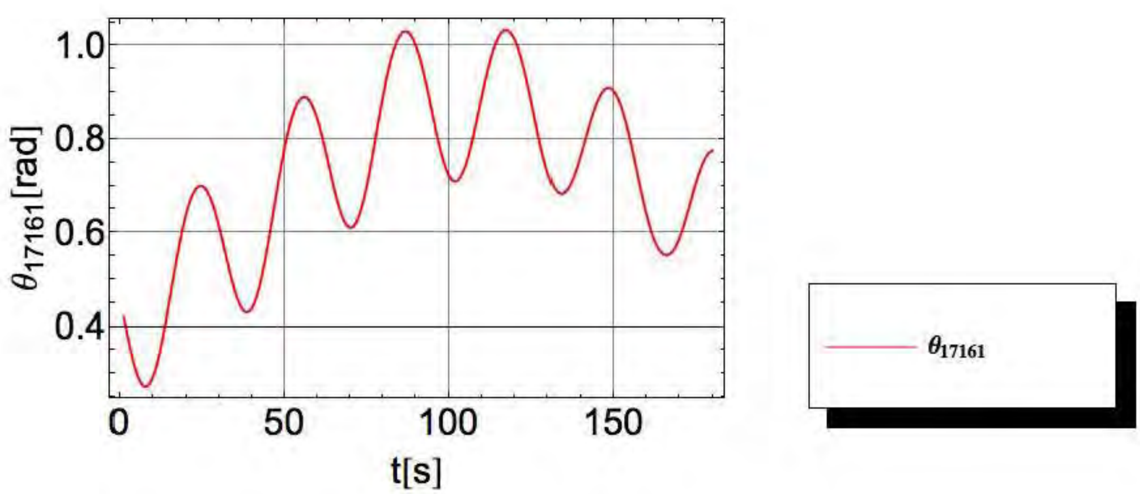
$y_{102} = .085\,m$
$z_{322} = .035\,m$
$x_{432} = .33\,m$
$\beta_{542} = 0°$
$x_{872} = .325\,m$
$x_{12112} = .015\,m$
$z_{13122} = .02\,m$
$\beta_{14132} = 90°$
$x_{15142} = .1\,m$
$x_{19182} = .06\,m$

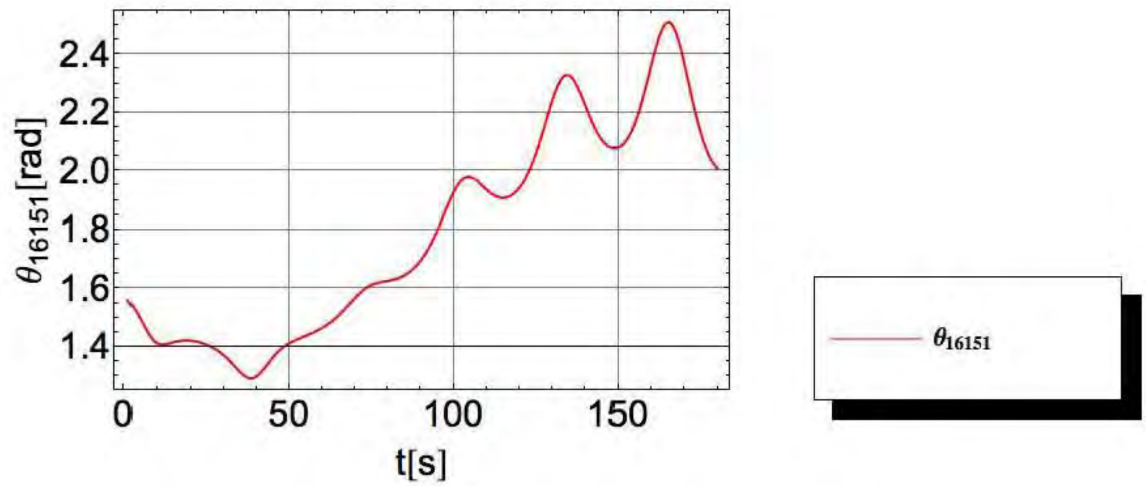


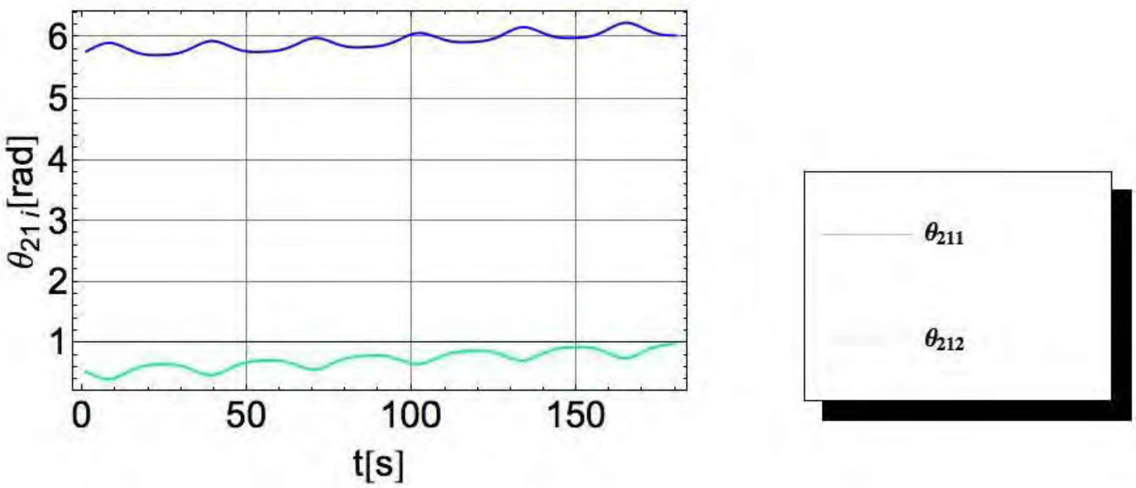


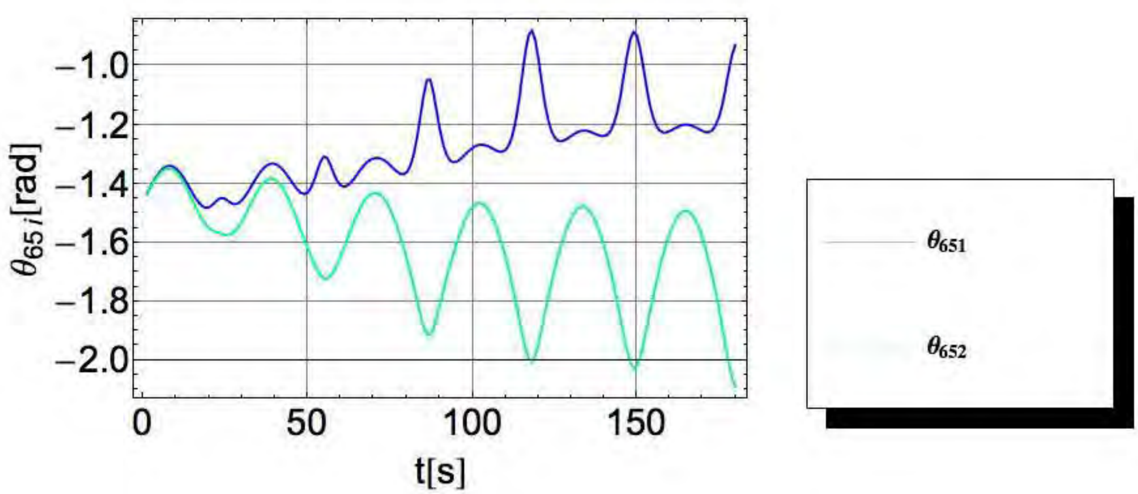

0.4
0.3
0.2
0.1
0.0
θ54[rad]
0
50
100
150
t[s]
θ54

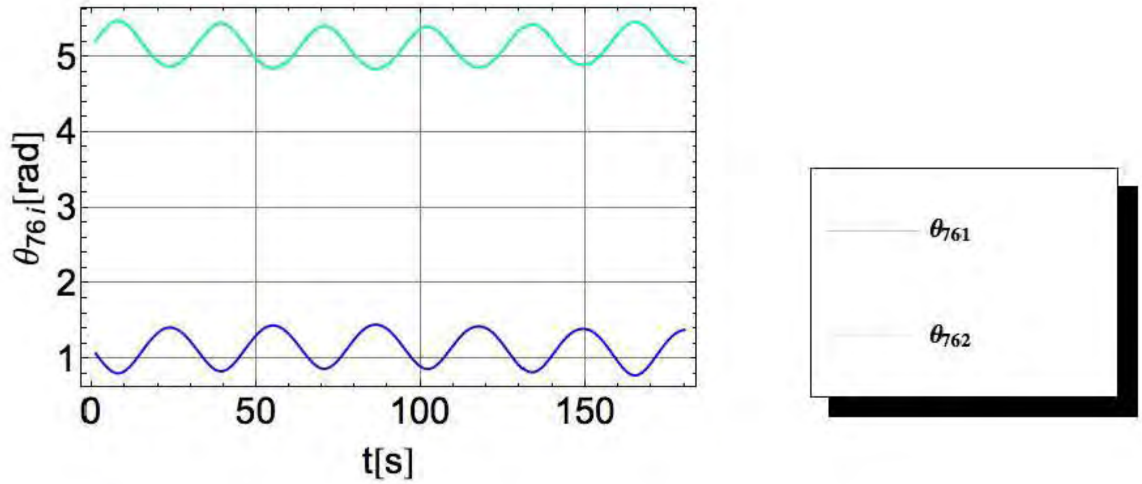

−1.4
−1.6
−1.8
−2.0
−2.2
−2.4
θ18171[rad]
0
50
100
150
t[s]
θ18171

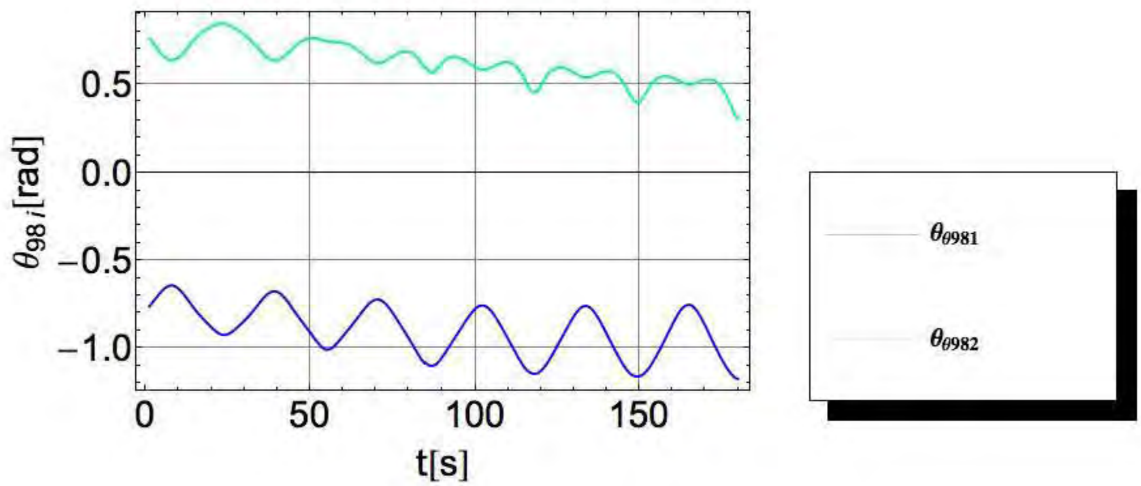

1.0
0.8
0.6
0.4
θ17161[rad]
0
50
100
150
t[s]
θ17161

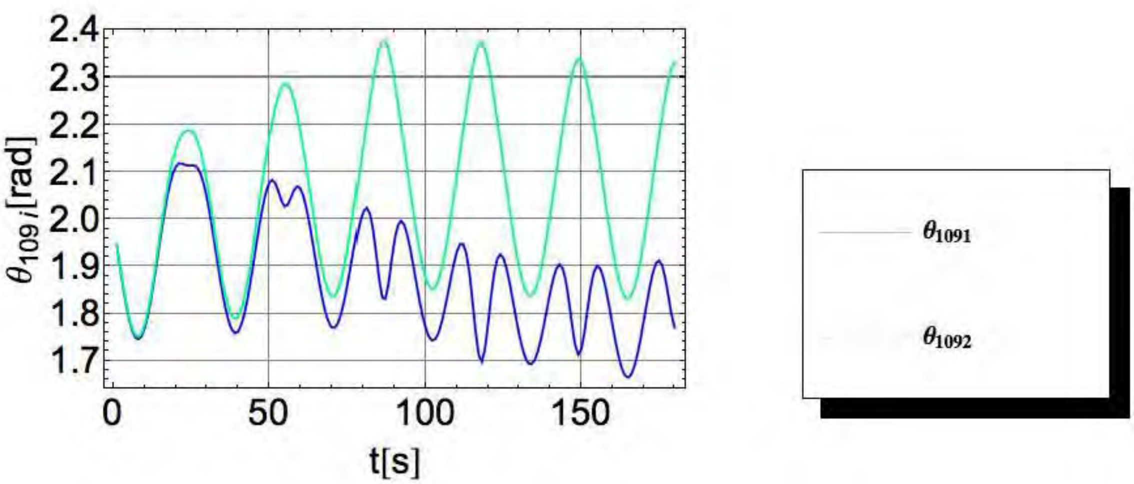

2.4
2.2
2.0
1.8
1.6
1.4
θ_{16151}[rad]
0
50
100
150
t[s]
θ_{16151}

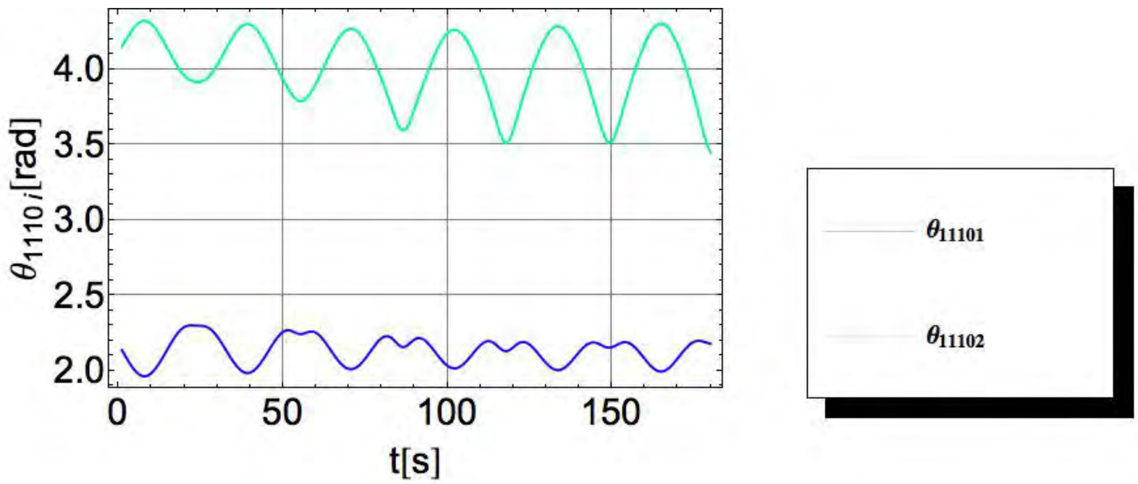

6
5
4
3
2
1
θ_{21i}[rad]
0
50
100
150
t[s]
θ_{211}
θ_{212}


−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
θ_{65i}[rad]
0
50
100
150
t[s]
θ_{651}
θ_{652}

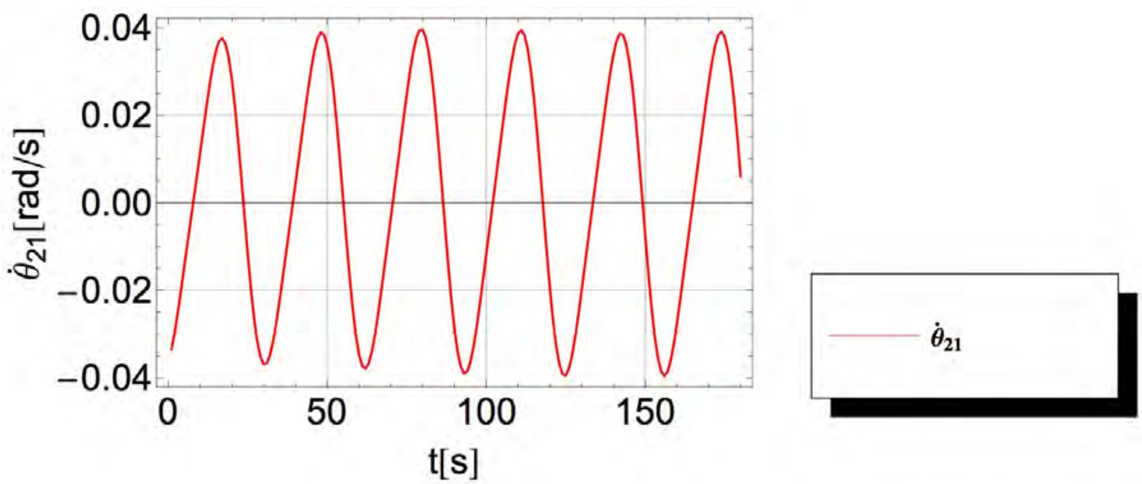

θ_{76i}[rad]
5
4
3
2
1
0
50
100
150
t[s]
θ_{761}
θ_{762}

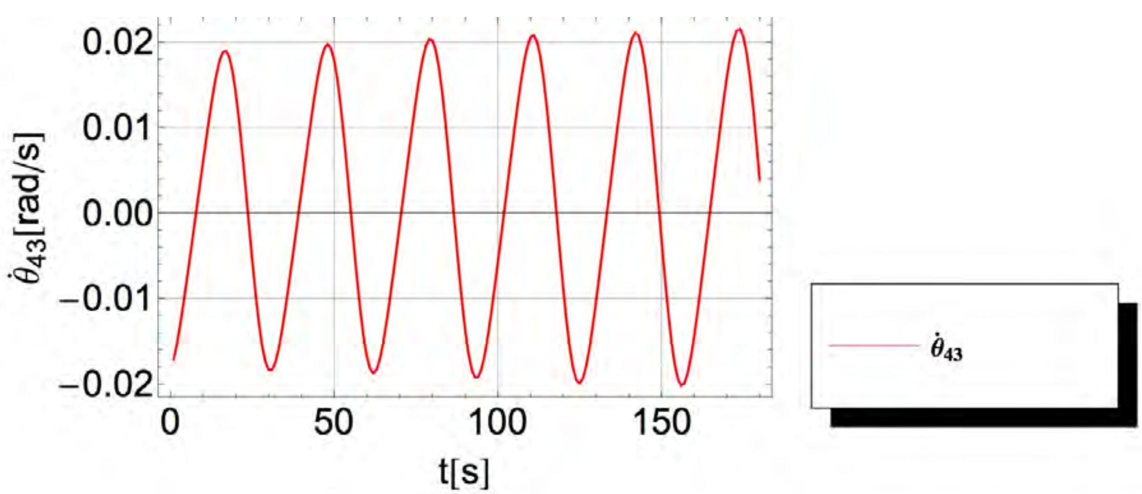

θ_{98i}[rad]
0.5
0.0
−0.5
−1.0
0
50
100
150
t[s]
$\theta_{\theta 981}$
$\theta_{\theta 982}$

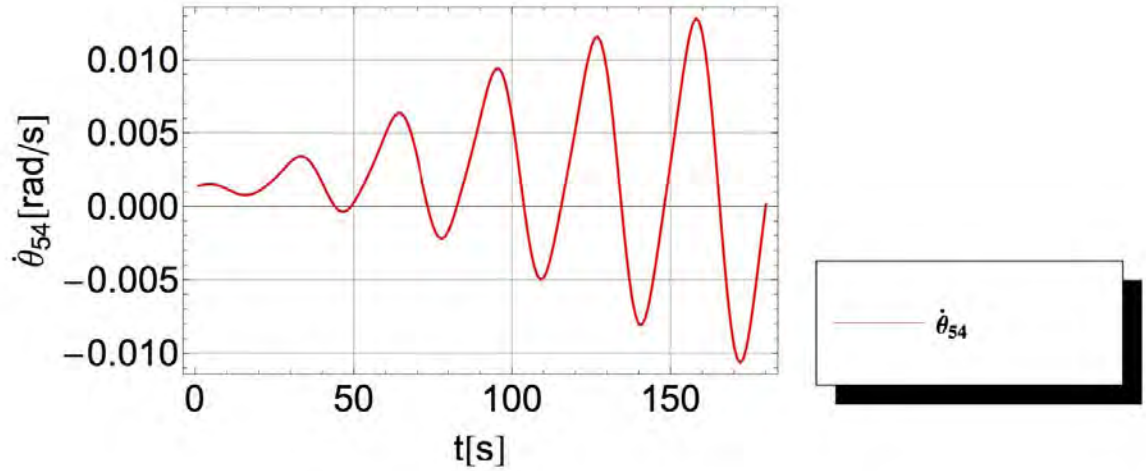

θ_{109i}[rad]
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
0
50
100
150
t[s]
θ_{1091}
θ_{1092}

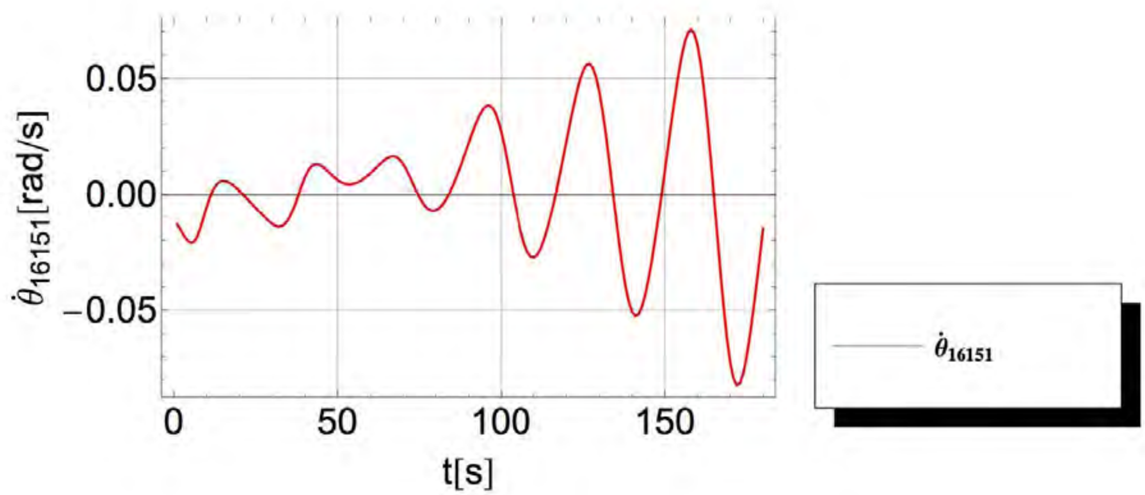


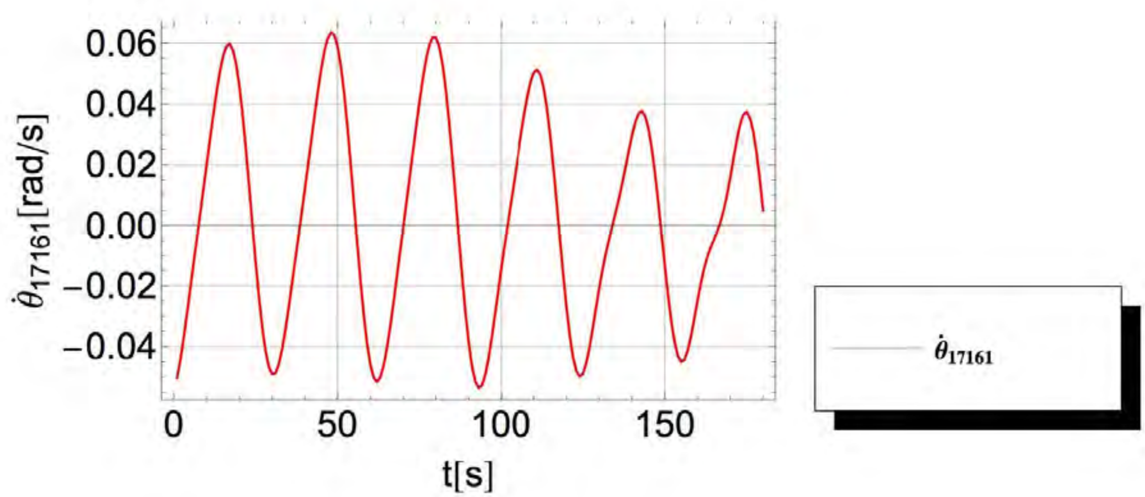
4.2 Gráficas de Velocidad

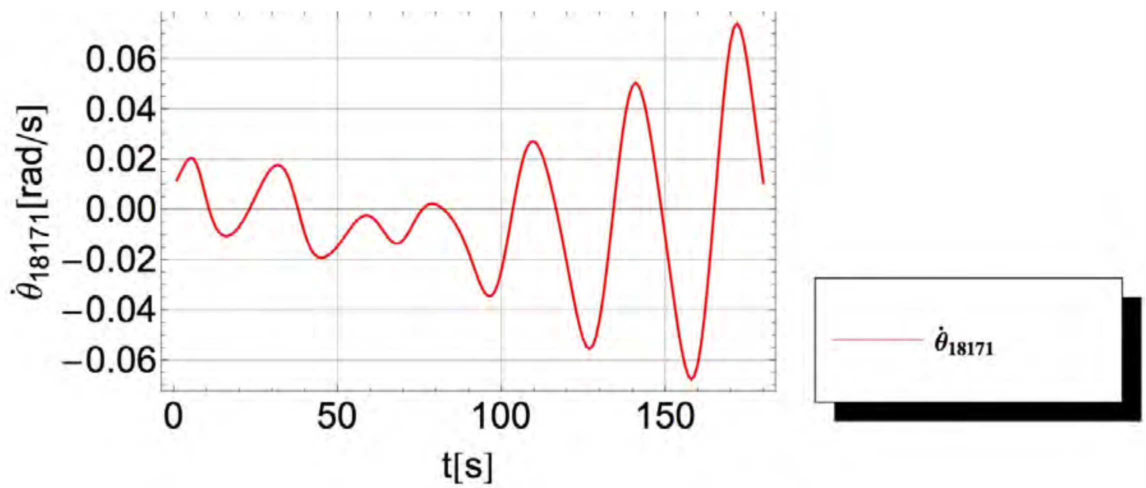





0.010
0.005
0.000
−0.005
−0.010
$\dot{\theta}_{54}$[rad/s]
0
50
100
150
t[s]
$\dot{\theta}_{54}$

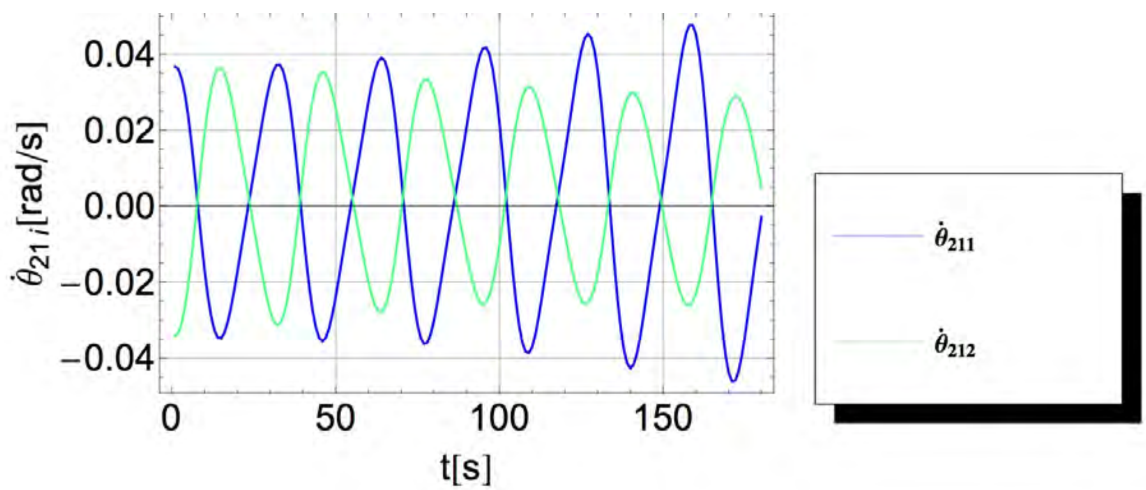

0.05
0.00
−0.05
$\dot{\theta}_{16151}$[rad/s]
0
50
100
150
t[s]
$\dot{\theta}_{16151}$


0.06
0.04
0.02
0.00
−0.02
−0.04
$\dot{\theta}_{17161}$[rad/s]
0
50
100
150
t[s]
$\dot{\theta}_{17161}$

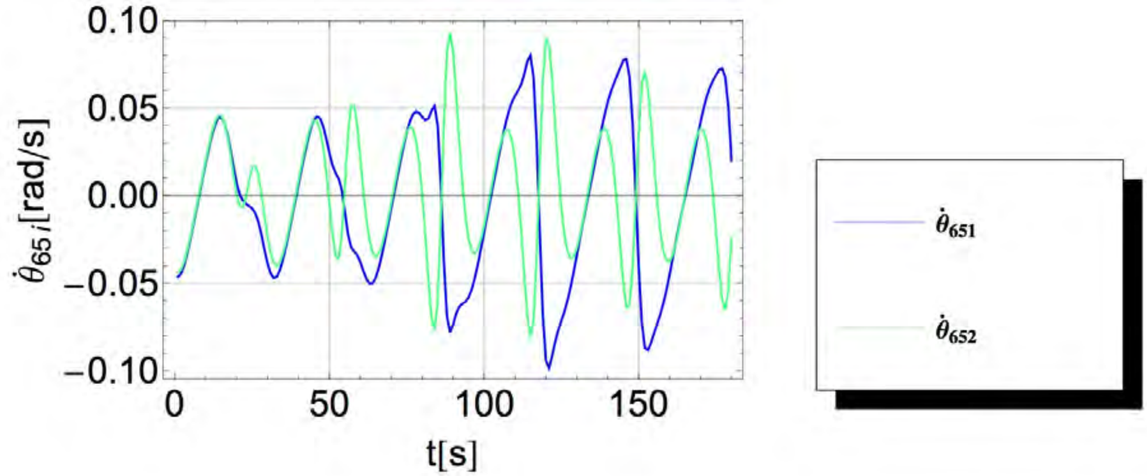


0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
0
50
100
150
t[s]
$\dot{\theta}_{18171}$[rad/s]
$\dot{\theta}_{18171}$

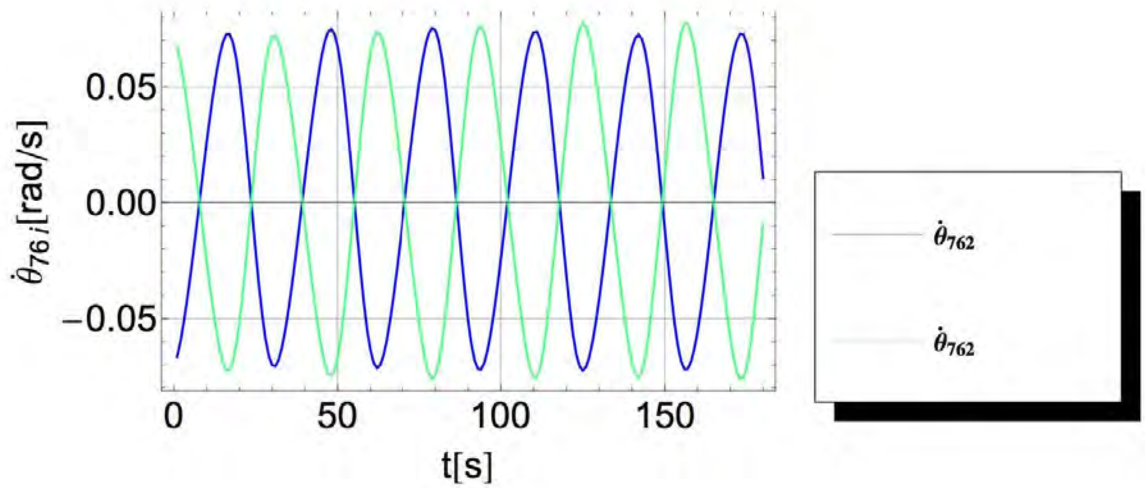
Cadena i



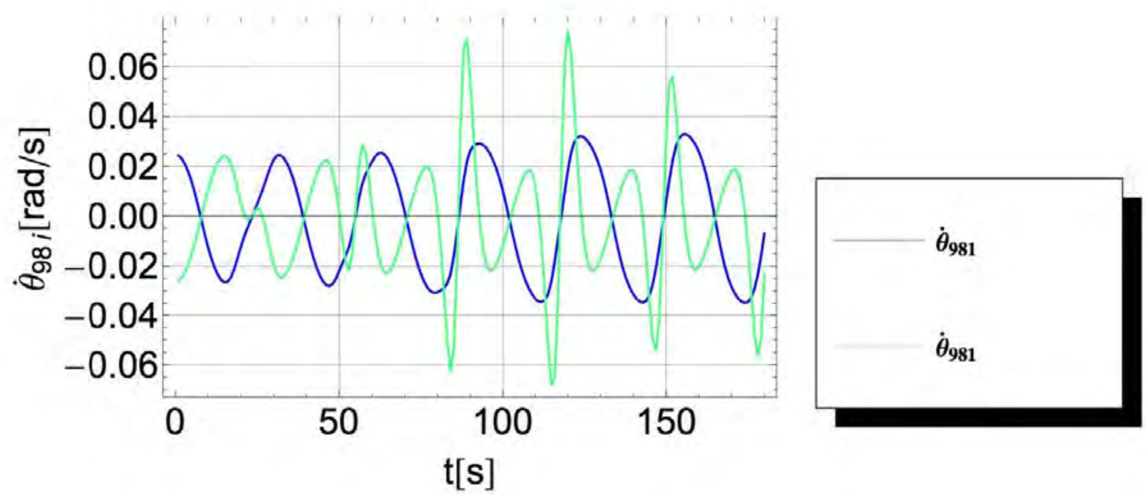
0.04
0.02
0.00
−0.02
−0.04
0
50
100
150
t[s]
$\dot{\theta}_{21\,i}$[rad/s]
$\dot{\theta}_{211}$
$\dot{\theta}_{212}$



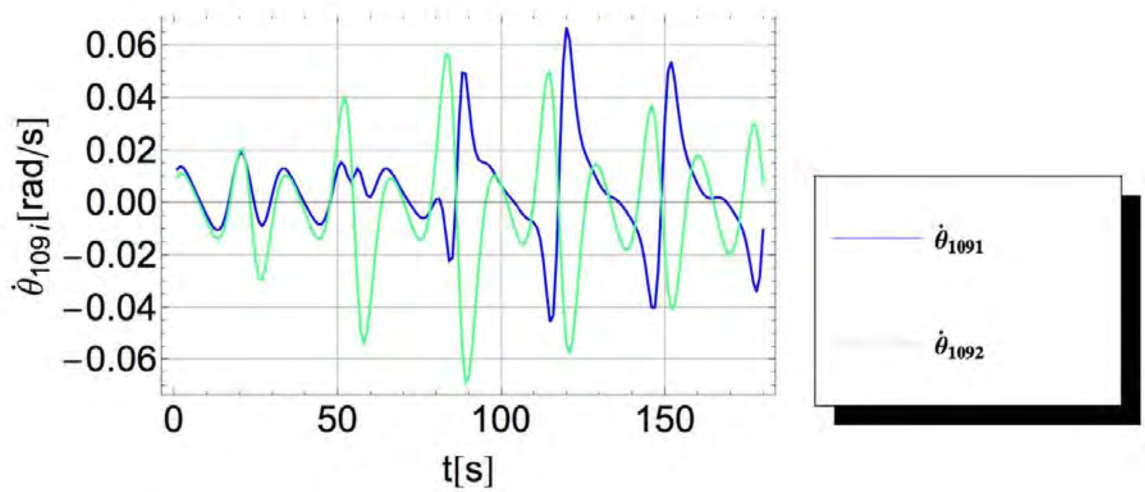
0.10
0.05
0.00
−0.05
−0.10
0
50
100
150
t[s]
$\dot{\theta}_{65\,i}$[rad/s]
$\dot{\theta}_{651}$
$\dot{\theta}_{652}$



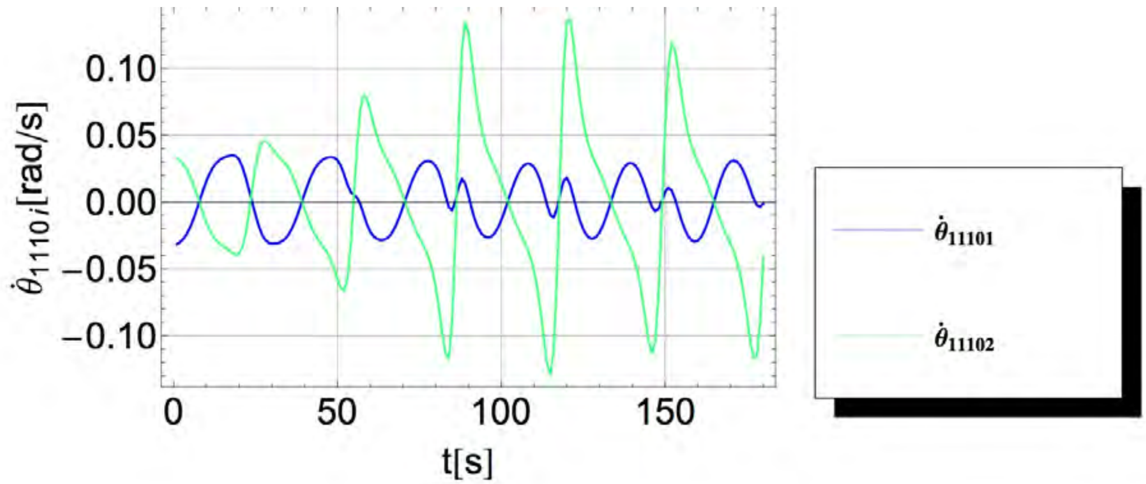
0.05
0.00
−0.05
0
50
100
150
t[s]
$\dot{\theta}_{76\,i}$[rad/s]
$\dot{\theta}_{762}$
$\dot{\theta}_{762}$



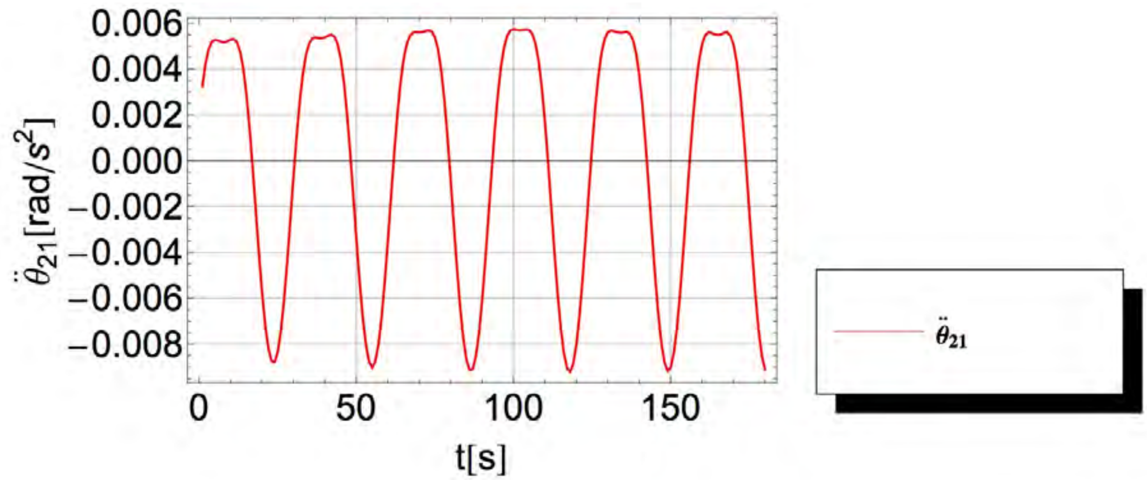
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
0
50
100
150
t[s]
$\dot{\theta}_{98\,i}$[rad/s]
$\dot{\theta}_{981}$
$\dot{\theta}_{981}$

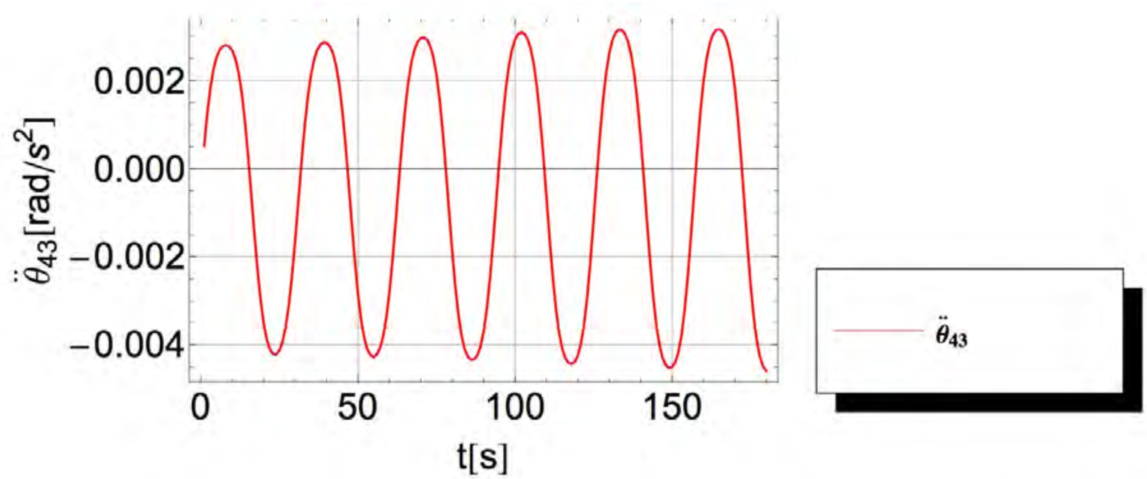


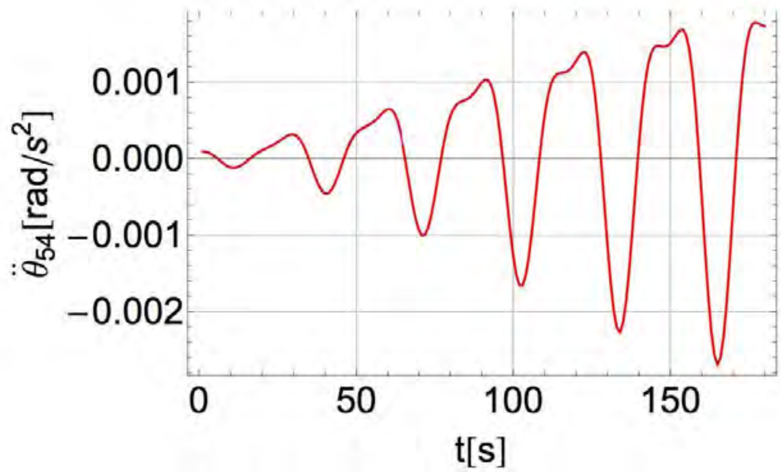
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
0
50
100
150
t[s]
$\dot{\theta}_{109\,i}$[rad/s]
$\dot{\theta}_{1091}$
$\dot{\theta}_{1092}$



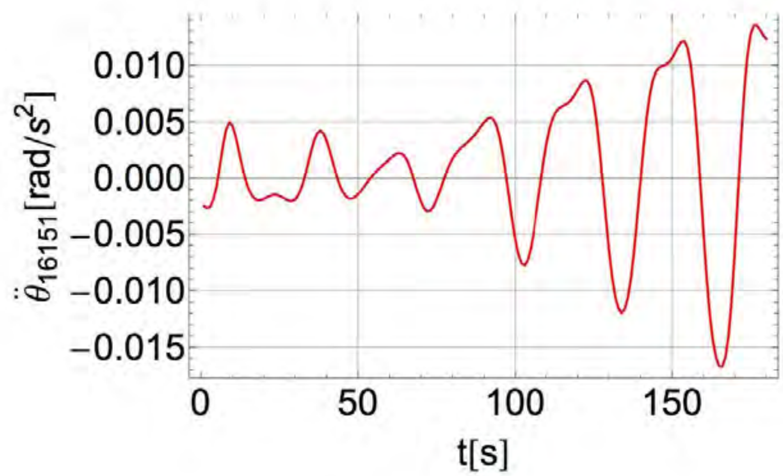
4.3 Gráficas de Aceleración



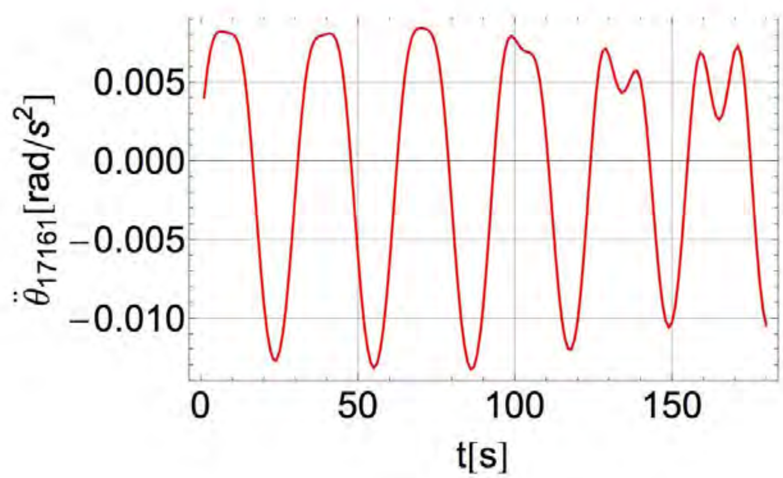




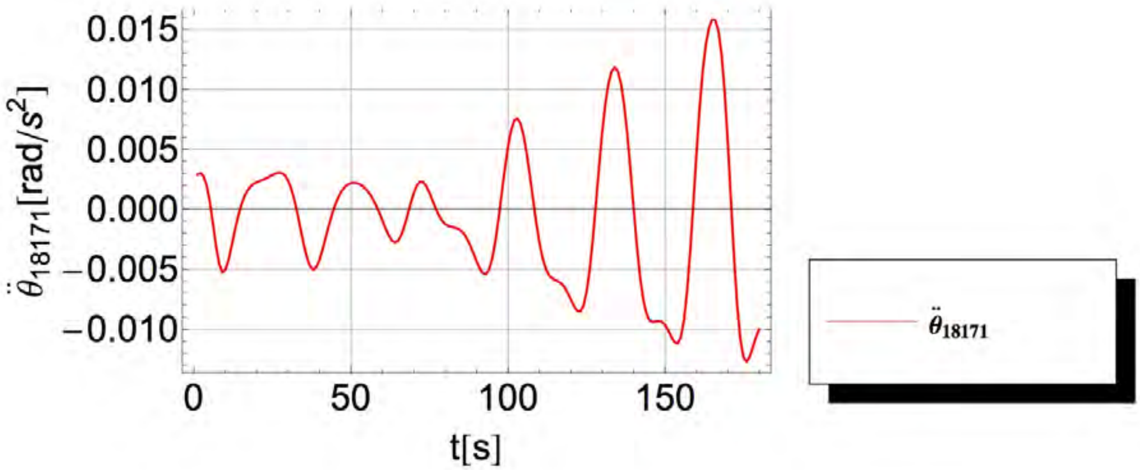
$\ddot{\theta}_{54}$



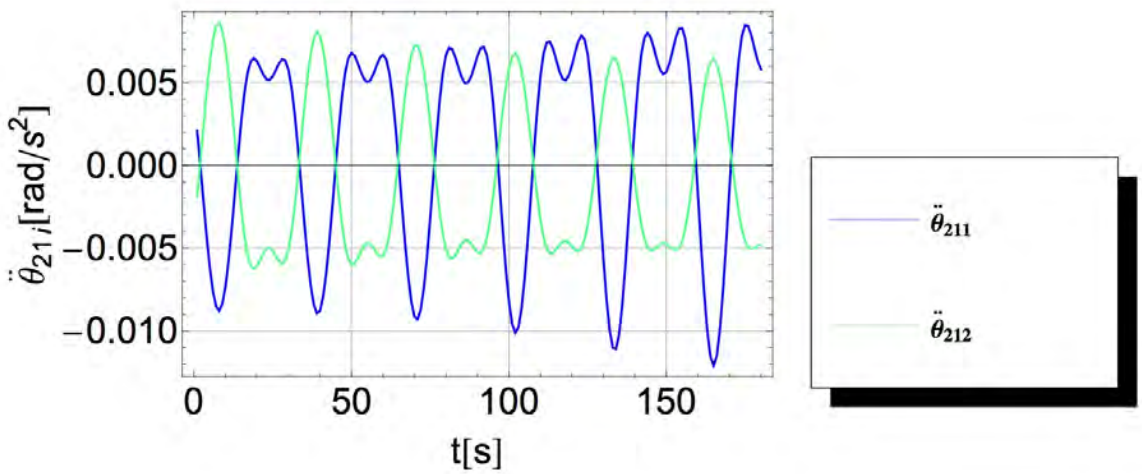
$\ddot{\theta}_{16151}$

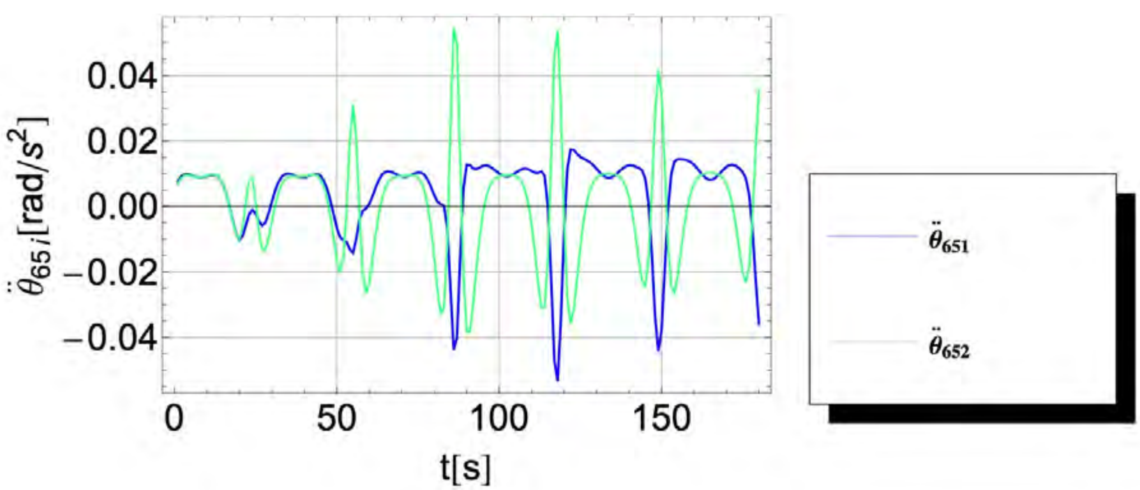


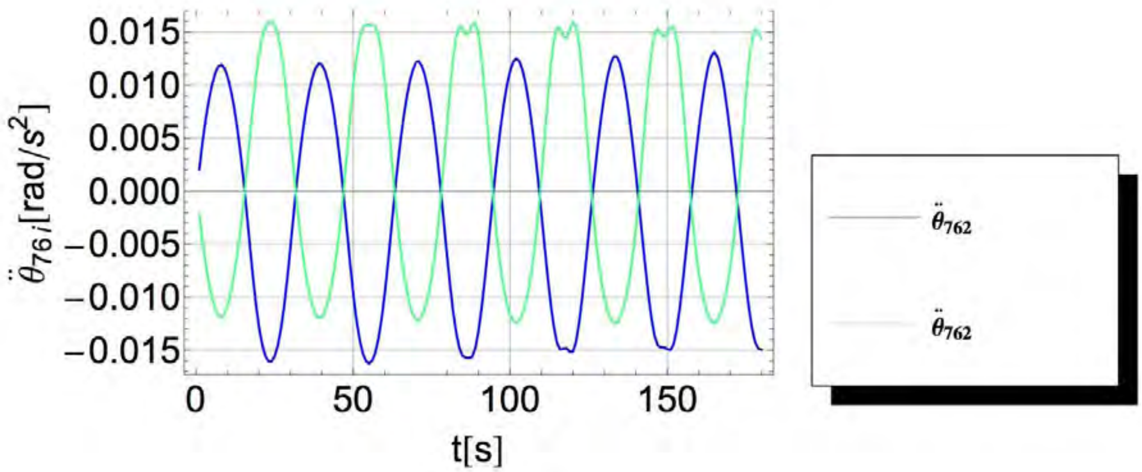
$\ddot{\theta}_{17161}$


0.015
0.010
0.005
0.000
−0.005
−0.010
$\ddot{\theta}_{18171}$[rad/s^2]
0
50
100
150
t[s]
$\ddot{\theta}_{18171}$

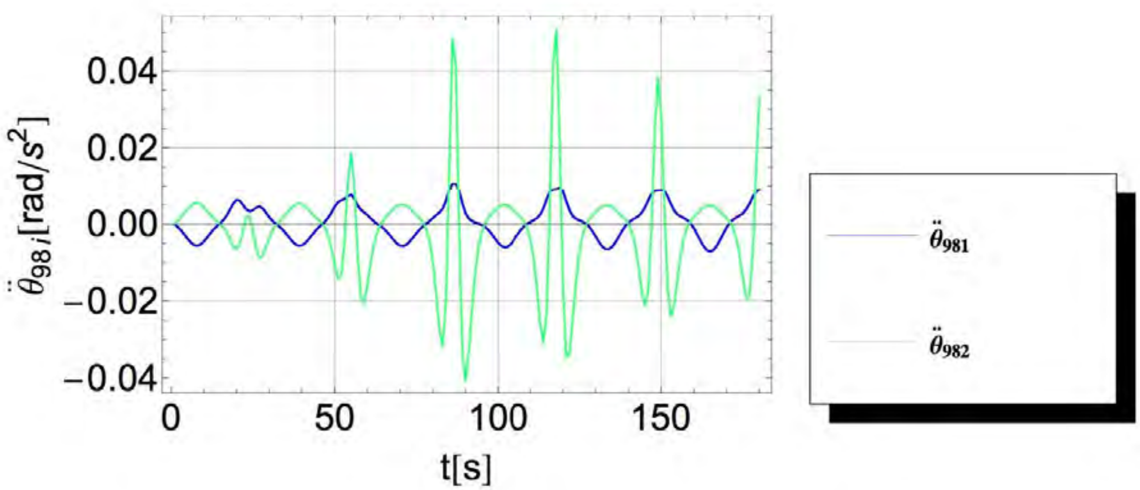
Cadena i


0.005
0.000
−0.005
−0.010
$\ddot{\theta}_{21\,i}$[rad/s^2]
0
50
100
150
t[s]
$\ddot{\theta}_{211}$
$\ddot{\theta}_{212}$

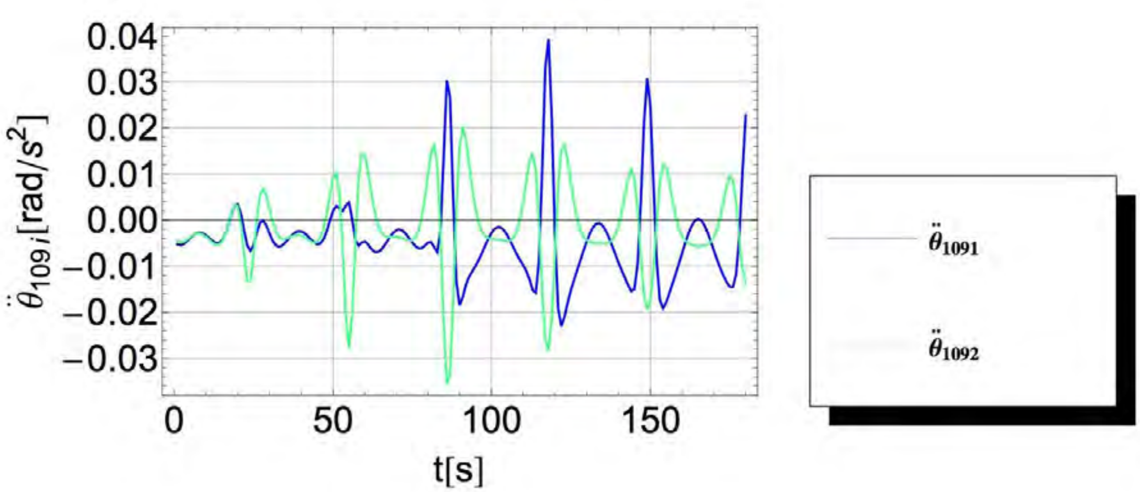

0.04
0.02
0.00
−0.02
−0.04
$\ddot{\theta}_{65\,i}$[rad/s^2]
0
50
100
150
t[s]
$\ddot{\theta}_{651}$
$\ddot{\theta}_{652}$



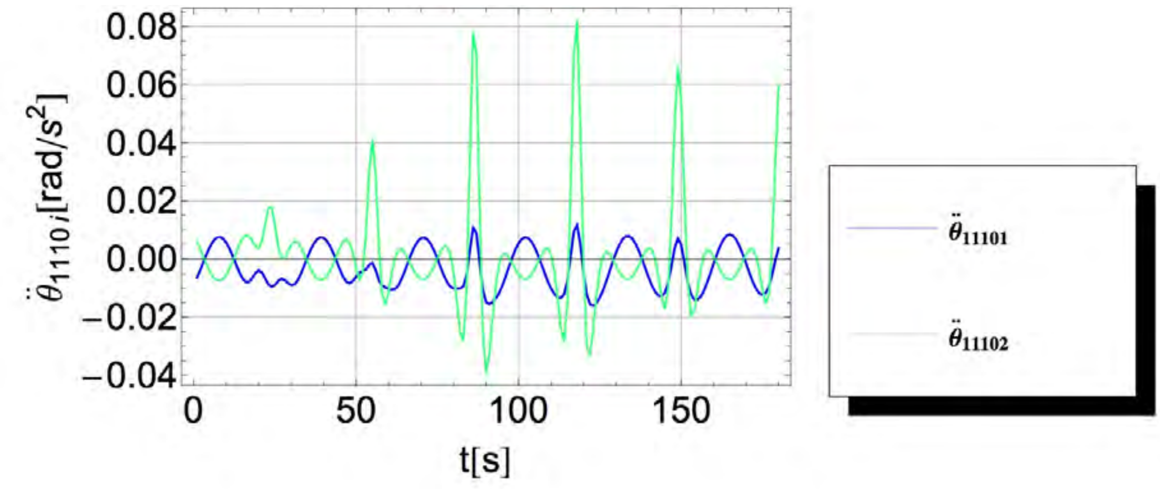
$\ddot{\theta}_{76i}$[rad/s^2]
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015
0
50
100
150
t[s]
$\ddot{\theta}_{762}$
$\ddot{\theta}_{762}$



$\ddot{\theta}_{98i}$[rad/s^2]
0.04
0.02
0.00
−0.02
−0.04
0
50
100
150
t[s]
$\ddot{\theta}_{981}$
$\ddot{\theta}_{982}$



$\ddot{\theta}_{109i}$[rad/s^2]
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
0
50
100
150
t[s]
$\ddot{\theta}_{1091}$
$\ddot{\theta}_{1092}$



4.4 Solución del método Euler-Lagrange

Para la solución del método de Euler-Lagrange se evaluó la ecuación (3.154) con cada uno de los términos que la componen. Para la obtención de los datos de masa e inercia se un software de CAD y la simulación de un material. Dichos datos se presentan a continuación:

Cadena 0:

$$\boldsymbol{J}_1^2 = \begin{bmatrix} 20.988 & 0 & 0 \\ 0 & 3027.487 & 0 \\ 0 & 0 & 3016.058 \end{bmatrix} kg * mm^2$$

$$\boldsymbol{J}_2^5 = \begin{bmatrix} 199.423 & -5.176 & 1381.812 \\ -5.176 & 45670 & 0.314 \\ 1381.812 & 0.314 & 45670 \end{bmatrix} kg * mm^2$$

$$\boldsymbol{J}_3^{16} = \begin{bmatrix} 1.681 & 0 & 0 \\ 0 & 95.822 & 0 \\ 0 & 0 & 95.822 \end{bmatrix} kg * mm^2$$

$$\boldsymbol{J}_4^{17} = \begin{bmatrix} 1.681 & 0 & 0 \\ 0 & 95.822 & 0 \\ 0 & 0 & 95.822 \end{bmatrix} kg * mm^2$$

$$\boldsymbol{J}_5^{18} = \begin{bmatrix} 1.681 & 0 & 0 \\ 0 & 95.822 & 0 \\ 0 & 0 & 95.822 \end{bmatrix} kg * mm^2$$

$$m_1 = 0.166\ kg$$
$$m_2 = 0.188\ kg$$
$$m_3 = 0.04\ kg$$
$$m_4 = 0.035\ kg$$
$$m_5 = 0.035\ kg$$

Cadenas i:

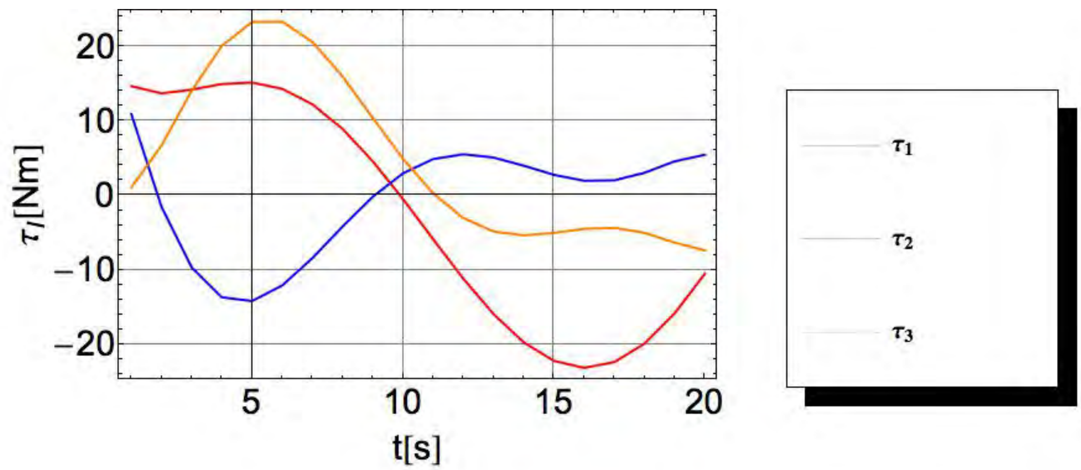
$$\boldsymbol{J}_1^{2i} = \begin{bmatrix} 20.988 & 0 & 0 \\ 0 & 3027.487 & 0 \\ 0 & 0 & 3016.058 \end{bmatrix} kg * mm^2$$

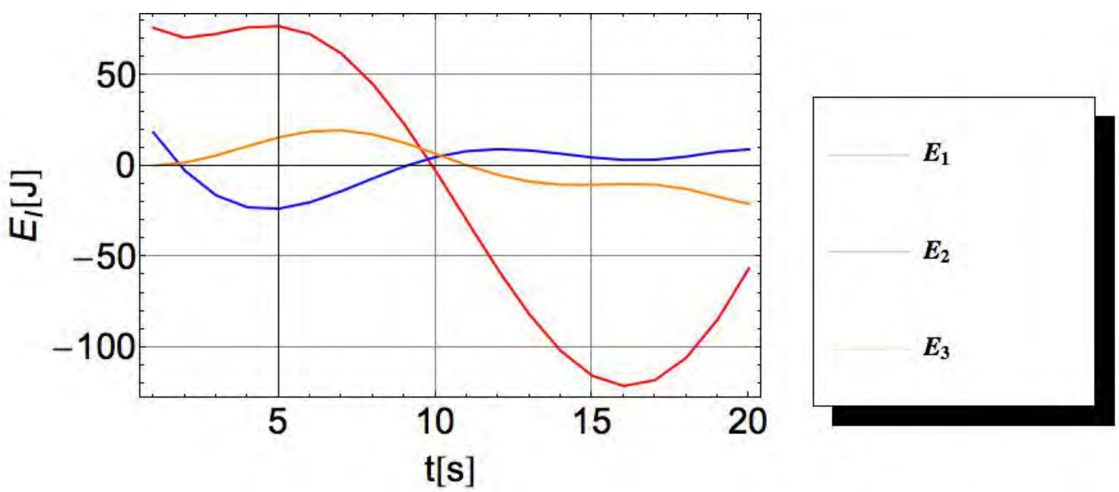
$$\boldsymbol{J}_2^{7i} = \begin{bmatrix} 2.62 & 0 & 0 \\ 0 & 2322.307 & 0 \\ 0 & 0 & 2322.307 \end{bmatrix} kg * mm^2$$

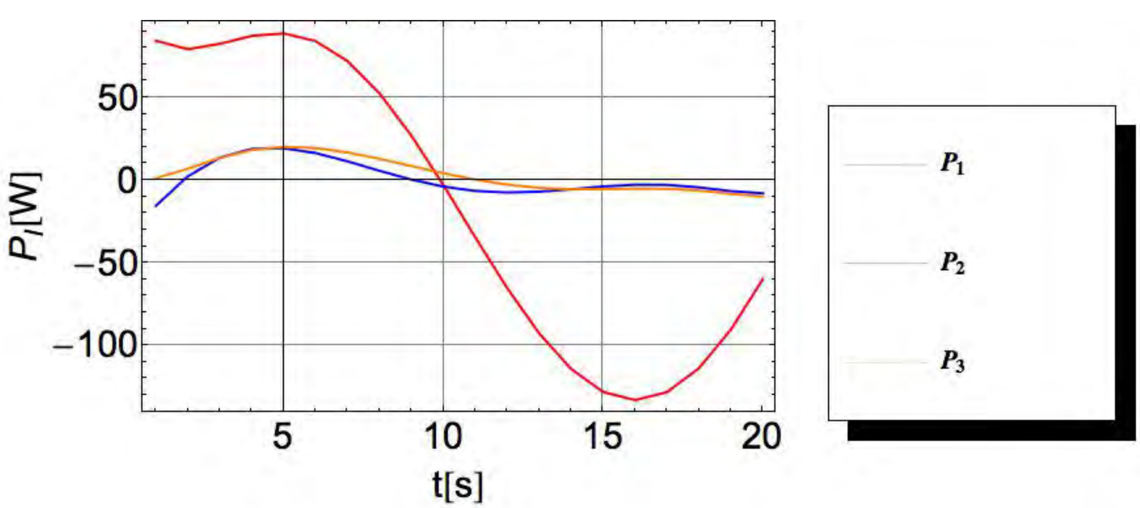
$$m_{1i} = 0.166\ kg$$
$$m_{2i} = 0.062\ kg$$

Para i=1,2

Con base en los datos antes mencionados y en las ecuaciones obtenidas se obtuvieron las gráficas correspondientes al comportamiento dinámico de los actuadores presentes en la base del robot ya que estos son los que soportan la carga para mover el mismo. Las gráficas obtenidas corresponden al torque, trabajo y potencia para la trayectoria descrita en el apéndice A.







Capítulo 5
Conclusiones

Con el presente trabajo fue posible aplicar métodos para el análisis cinemático y dinámico de una configuración geométrica espacial. Dichos métodos permitieron la obtención de variables importantes para evaluar el funcionamiento del robot y de esta manera evaluar si es viable su construcción para realizar el propósito para el que fue diseñado.

En el caso de la cinemática se empleó el método de transformaciones homogéneas, el cual es muy versátil para analizar configuraciones geométricas compuestas por diferentes tipos de juntas. El método permite la descomposición de dichas juntas en traslaciones y rotaciones en el espacio lo cual es útil para el análisis de cada uno de los ángulos por separado. Sin embargo una de las desventajas del método es que las ecuaciones obtenidas fueron muy complejas lo cual derivó en dificultad para su manejo y en la gran capacidad de procesamiento necesaria para la realización de los cálculos.

Dado que el propósito original del robot fue el de pintar partes en una línea industrial, la trayectoria se eligió para simular el movimiento que tendría al realizar esta tarea. De esta manera fue posible observar el comportamiento detallado de posición, velocidad y aceleración de cada ángulo al trazar la trayectoria descrita. Con base en las gráficas cinemáticas obtenidas se observa que los resultados son coherentes con la trayectoria senoidal propuesta ya que se observa un comportamiento cíclico.

Para la dinámica el método empleado fue el de Euler-Lagrange el cual es un método que no contempla las fuerzas y momentos de reacción en cada uno de los eslabones de las cadenas cinemáticas sino solo las fuerzas requeridas para que el sistema realice un movimiento específico. Al observar las gráficas dinámicas se pueden apreciar los límites superiores e inferiores lo cual es útil para la selección de los actuadores. Para la aplicación del método fue necesario conocer los parámetros físicos involucrados en la dinámica como son las masas los momentos de inercia y las distancias a los centros de gravedad los cuales se obtuvieron con el uso de un software CAD.

Por lo anterior se puede concluir que el método de Euler-Lagrange se puede usar cuando se desea conocer las especificaciones de los actuadores y cuando no es crítico el conocimiento de las fuerzas involucradas en los elementos de unión como son los pernos y rodamientos. Debido a esto y a la selección de la trayectoria se puede decir que los resultados dinámicos obtenidos se pueden usar para la selección real de los actuadores. Así mismo el modelo dinámico obtenido se puede usar en un trabajo posterior para la formulación de un esquema de control.

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Apéndice A.

Desarrollo de la ecuación de lazo.

Con el fin de obtener el término $\frac{\partial \boldsymbol{\theta}}{\partial q_j}$ se procederá a generar una ecuación de lazo que nos permitirá relacionar el vector de derivadas parciales respecto a q_j de los ángulos $\theta_{21}, \theta_{43}, \theta_{54}$ con la derivada parcial respecto a q_j del vector de coordenadas generalizadas, esto es:

$$\boldsymbol{J}_{\theta i} \frac{\partial \boldsymbol{\theta}}{\partial q_j} = \boldsymbol{J}_{qi} \frac{\partial \boldsymbol{q}}{\partial q_j}$$

A partir de la figura 2-6 se tiene la ecuación de lazo:

$$\boldsymbol{r}_{10}^{0} + \boldsymbol{r}_{32}^{0} + \boldsymbol{r}_{14,5i}^{0} + \boldsymbol{r}_{15,14i}^{0} = \boldsymbol{r}_{p}^{0} - \boldsymbol{r}_{1918}^{0} \tag{A.1}$$

Dónde:

$$\begin{aligned}
\boldsymbol{r}_{10}^{0} &= z_{10}\boldsymbol{k}_0 \\
\boldsymbol{r}_{32}^{0} &= \boldsymbol{R}_2^0 \boldsymbol{r}_{32}^{2} \\
\boldsymbol{r}_{14,5i}^{0} &= \boldsymbol{R}_5^0 \boldsymbol{r}_{14,5i}^{5} \\
\boldsymbol{r}_{15,14i}^{0} &= \boldsymbol{R}_{14}^0 \boldsymbol{r}_{15,14i}^{14} \\
\boldsymbol{r}_{p}^{0} &= [x, y, z]^T \\
\boldsymbol{r}_{1918}^{0} &= \boldsymbol{R}_p^0 \boldsymbol{r}_{1918}^{p}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{R}_2^0 &= \boldsymbol{R}_y(\theta_{21}) \\
\boldsymbol{R}_5^0 &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54}) \\
\boldsymbol{R}_{14}^0 &= \boldsymbol{R}_5^0 \\
\boldsymbol{R}_p^0 &= \boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{r}_{32}^{2} &= [x_{32}, 0{,}0]^T \\
\boldsymbol{r}_{14,5i}^{5} &= \left[x_{14,5i}, 0{,}0\right]^T \\
\boldsymbol{r}_{15,14i}^{14} &= \left[x_{15,14i}, 0{,}0\right]^T \\
\boldsymbol{r}_{1918}^{p} &= [-x_{1918}, 0{,}0]^T
\end{aligned}$$

Derivando la ecuación (A.1) con respecto a q_j:

$$\frac{\partial}{\partial q_j}\left(\boldsymbol{r}_{10}^{0} + \boldsymbol{r}_{32}^{0} + \boldsymbol{r}_{14,5i}^{0} + \boldsymbol{r}_{15,14i}^{0}\right) = \frac{\partial}{\partial q_j}\left(\boldsymbol{r}_{p}^{0} - \boldsymbol{r}_{1918}^{0}\right)$$

$$\frac{\partial \boldsymbol{r}_{10}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{32}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{14,5i}^{0}}{\partial q_j} + \frac{\partial \boldsymbol{r}_{15,14i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{r}_{p}^{0}}{\partial q_j} - \frac{\partial \boldsymbol{r}_{1918}^{0}}{\partial q_j} \tag{A.2}$$

Obteniendo cada uno de los componentes de la ec. anterior, primero escribimos las parciales de las matrices previamente obtenidas:

$$\frac{\partial \boldsymbol{R}_2^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\frac{\partial \theta_{21}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_5^0}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_5^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_p^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_x(\theta)}{\partial q_j}\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi) + \boldsymbol{R}_x(\theta)\frac{\partial \boldsymbol{R}_y(\psi)}{\partial q_j}\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi) + \boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\frac{\partial \boldsymbol{R}_z(\phi)}{\partial q_j}$$
$$= \boldsymbol{B}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)\frac{\partial \theta}{\partial q_j} + \boldsymbol{R}_x(\theta)\boldsymbol{B}_y(\psi)\boldsymbol{R}_z(\phi)\frac{\partial \psi}{\partial q_j} + \boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{B}_z(\phi)\frac{\partial \phi}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{10}^0}{\partial q_j} = 0$$

$$\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} = \boldsymbol{R}_2^0\boldsymbol{r}_{32}^2 = \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2\right)\frac{\partial \theta_{21}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} = \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right)\frac{\partial \theta_{21}}{\partial q_j} + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right)\frac{\partial \theta_{43}}{\partial q_j} + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5\right)\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{15,14i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{14}^0}{\partial q_j}\boldsymbol{r}_{15,14i}^{14}$$
$$= \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{21}}{\partial q_j} + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{43}}{\partial q_j} + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_p^0}{\partial q_j} = \frac{\partial \boldsymbol{r}_p^0}{\partial x}\frac{\partial x}{\partial q_j} + \frac{\partial \boldsymbol{r}_p^0}{\partial y}\frac{\partial y}{\partial q_j} + \frac{\partial \boldsymbol{r}_p^0}{\partial z}\frac{\partial z}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{1918}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_p^0}{\partial q_j}\boldsymbol{r}_{1918}^p$$

$$= \left(\boldsymbol{B}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\theta}{\partial q_j} + \left(\boldsymbol{R}_x(\theta)\boldsymbol{B}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\psi}{\partial q_j} + \left(\boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{B}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\phi}{\partial q_j}$$

Sustituyendo los términos anteriores en la ec. (A.1):

$$\begin{aligned}
&\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^{2}\right)\frac{\partial\theta_{21}}{\partial q_j} + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{21}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{43}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{54}}{\partial q_j} \\
&\quad + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{21}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{43}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{54}}{\partial q_j} \\
&\quad = \frac{\partial\boldsymbol{r}_p^0}{\partial x}\frac{\partial x}{\partial q_j} + \frac{\partial\boldsymbol{r}_p^0}{\partial y}\frac{\partial y}{\partial q_j} + \frac{\partial\boldsymbol{r}_p^0}{\partial z}\frac{\partial z}{\partial q_j} - \left(\boldsymbol{B}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\theta}{\partial q_j} \\
&\quad - \left(\boldsymbol{R}_x(\theta)\boldsymbol{B}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\psi}{\partial q_j} - \left(\boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{B}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\phi}{\partial q_j}
\end{aligned}$$

Simplificando y agrupando los términos semejantes:

$$\begin{aligned}
&\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^{2} + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5} + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{21}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{43}}{\partial q_j} \\
&\quad + \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14}\right)\frac{\partial\theta_{54}}{\partial q_j} \\
&\quad = \frac{\partial\boldsymbol{r}_p^0}{\partial q_j} - \left(\boldsymbol{B}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\theta}{\partial q_j} - \left(\boldsymbol{R}_x(\theta)\boldsymbol{B}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\psi}{\partial q_j} \\
&\quad - \left(\boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{B}_z(\phi)\boldsymbol{r}_{1918}^{p}\right)\frac{\partial\phi}{\partial q_j}
\end{aligned}$$

Renombrando y acomodando matricialmente:

$$\boldsymbol{r}_{Ai}\frac{\partial\theta_{21}}{\partial q_j} + \boldsymbol{r}_{Bi}\frac{\partial\theta_{43}}{\partial q_j} + \boldsymbol{r}_{Ci}\frac{\partial\theta_{54}}{\partial q_j} = \boldsymbol{I}_{3x3}\frac{\partial\boldsymbol{r}_p^0}{\partial q_j} + \boldsymbol{r}_{Di}\frac{\partial\theta}{\partial q_j} + \boldsymbol{r}_{Ei}\frac{\partial\psi}{\partial q_j} + \boldsymbol{r}_{Fi}\frac{\partial\phi}{\partial q_j}$$

$$
\begin{bmatrix} \boldsymbol{r}_{Ai} & \boldsymbol{r}_{Bi} & \boldsymbol{r}_{Ci} \end{bmatrix} \begin{bmatrix} \frac{\partial \theta_{21}}{\partial q_j} \\ \frac{\partial \theta_{43}}{\partial q_j} \\ \frac{\partial \theta_{54}}{\partial q_j} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{3x3} & \boldsymbol{r}_{Di} & \boldsymbol{r}_{Ei} & \boldsymbol{r}_{Fi} \end{bmatrix} \begin{bmatrix} \frac{\partial \boldsymbol{r}_p^0}{\partial q_j} \\ \frac{\partial \theta}{\partial q_j} \\ \frac{\partial \psi}{\partial q_j} \\ \frac{\partial \phi}{\partial q_j} \end{bmatrix}
$$

Finalmente se tiene:

$$
\boldsymbol{J}_{\theta i} \frac{\partial \boldsymbol{\theta}}{\partial q_j} = \boldsymbol{J}_{qi} \frac{\partial \boldsymbol{q}}{\partial q_j} \tag{A.3}
$$

Donde:

$$
\begin{aligned}
\boldsymbol{r}_{Ai} &= \boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2 + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} \\
\boldsymbol{r}_{Bi} &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} \\
\boldsymbol{r}_{Ci} &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^5 + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{15,14i}^{14} \\
\boldsymbol{r}_{Di} &= -\left(\boldsymbol{B}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^p\right) \\
\boldsymbol{r}_{Ei} &= -\left(\boldsymbol{R}_x(\theta)\boldsymbol{B}_y(\psi)\boldsymbol{R}_z(\phi)\boldsymbol{r}_{1918}^p\right) \\
\boldsymbol{r}_{Fi} &= -\left(\boldsymbol{R}_x(\theta)\boldsymbol{R}_y(\psi)\boldsymbol{B}_z(\phi)\boldsymbol{r}_{1918}^p\right)
\end{aligned}
$$

Ahora se genera una ecuación de lazo que nos permitirá relacionar el vector de derivadas parciales respecto a q_j de los ángulos $\theta_{21i}, \theta_{65i}, \theta_{76i}$con la derivada parcial respecto a q_j.

Con base en la figura 2-12 se tiene la ecuación de lazo siguiente:

$$
\boldsymbol{r}_{10i}^0 + \boldsymbol{r}_{32i}^0 + \boldsymbol{r}_{43i}^0 + \boldsymbol{r}_{87i}^0 = \boldsymbol{r}_{10}^0 + \boldsymbol{r}_{32}^0 + \boldsymbol{r}_{14,5i}^0 - \boldsymbol{r}_{13,12i}^0 - \boldsymbol{r}_{12,11i}^0 \tag{A.4}
$$

Dónde:

$$
\begin{aligned}
\boldsymbol{r}_{10i}^0 &= signo\ y_{10}\boldsymbol{j}_0 \\
\boldsymbol{r}_{32i}^0 &= \boldsymbol{R}_{2\mathrm{i}}^0\boldsymbol{r}_{32\mathrm{i}}^{2\mathrm{i}} \\
\boldsymbol{r}_{43i}^0 &= \boldsymbol{R}_{3\mathrm{i}}^0\boldsymbol{r}_{43i}^{3i} \\
\boldsymbol{r}_{87i}^0 &= \boldsymbol{R}_{7\mathrm{i}}^0\boldsymbol{r}_{87i}^{7i} \\
\boldsymbol{r}_{10}^0 &= z_{10}\boldsymbol{k}_0 \\
\boldsymbol{r}_{32}^0 &= \boldsymbol{R}_2^0\boldsymbol{r}_{32}^2 \\
\boldsymbol{r}_{14,5i}^0 &= \boldsymbol{R}_5^0\boldsymbol{r}_{14,5i}^5 \\
\boldsymbol{r}_{13,12i}^0 &= \boldsymbol{R}_{12\mathrm{i}}^0\boldsymbol{r}_{13,12i}^{12i} \\
\boldsymbol{r}_{12,11i}^0 &= \boldsymbol{R}_{11\mathrm{i}}^0\boldsymbol{r}_{12,11i}^{11i}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{R}_{2i}^{0} &= \boldsymbol{R}_z(\theta_{21i}) \\
\boldsymbol{R}_{3i}^{0} &= \boldsymbol{R}_{2i}^{0} \\
\boldsymbol{R}_{7i}^{0} &= \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i}) \\
\boldsymbol{R}_{2}^{0} &= \boldsymbol{R}_y(\theta_{21}) \\
\boldsymbol{R}_{5}^{0} &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54}) \\
\boldsymbol{R}_{11\mathrm{i}}^{0} &= \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i}) \\
\boldsymbol{R}_{12\mathrm{i}}^{0} &= \boldsymbol{R}_{11\mathrm{i}}^{0}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{r}_{32i}^{2i} &= [-z_{32i}, 0{,}0]^T \\
\boldsymbol{r}_{43i}^{3i} &= [x_{43i}, 0{,}0]^T \\
\boldsymbol{r}_{87i}^{7i} &= [x_{87i}, 0{,}0]^T \\
\boldsymbol{r}_{32}^{2} &= [x_{32}, 0{,}0]^T \\
\boldsymbol{r}_{14,5i}^{5} &= [x_{14,5i}, 0{,}0]^T \\
\boldsymbol{r}_{12,11i}^{0} &= [-x_{12,11i}, 0{,}0]^T \\
\boldsymbol{r}_{13,12i}^{0} &= [-z_{13,12i}, 0{,}0]^T
\end{aligned}
$$

Derivando la ecuación (A.4) con respecto a q_j

$$
\frac{\partial}{\partial q_j}(\boldsymbol{r}_{10i}^{0} + \boldsymbol{r}_{32i}^{0} + \boldsymbol{r}_{43i}^{0} + \boldsymbol{r}_{87i}^{0}) = \frac{\partial}{\partial q_j}(\boldsymbol{r}_{10}^{0} + \boldsymbol{r}_{32}^{0} + \boldsymbol{r}_{14,5i}^{0} - \boldsymbol{r}_{13,12i}^{0} - \boldsymbol{r}_{12,11i}^{0})
$$

$$
\begin{aligned}
&\frac{\partial}{\partial q_j}\boldsymbol{r}_{10i}^{0} + \frac{\partial}{\partial q_j}\boldsymbol{r}_{32i}^{0} + \frac{\partial}{\partial q_j}\boldsymbol{r}_{43i}^{0} + \frac{\partial}{\partial q_j}\boldsymbol{r}_{87i}^{0} \\
&\qquad = \frac{\partial}{\partial q_j}\boldsymbol{r}_{10}^{0} + \frac{\partial}{\partial q_j}\boldsymbol{r}_{32}^{0} + \frac{\partial}{\partial q_j}\boldsymbol{r}_{14,5i}^{0} - \frac{\partial}{\partial q_j}\boldsymbol{r}_{13,12i}^{0} - \frac{\partial}{\partial q_j}\boldsymbol{r}_{12,11i}^{0}
\end{aligned} \tag{A.5}
$$

Obteniendo cada uno de los componentes de la ec. anterior, primero escribimos las parciales de las matrices previamente obtenidas:

$$
\frac{\partial \boldsymbol{R}_{2i}^{0}}{\partial q_j} = \boldsymbol{B}_z(\theta_{21i})\ \frac{\partial \theta_{21i}}{\partial q_j}
$$

$$
\frac{\partial \boldsymbol{R}_{3i}^{0}}{\partial q_j} = \frac{\partial \boldsymbol{R}_{2i}^{0}}{\partial q_j}
$$

$$
\begin{aligned}
\frac{\partial \boldsymbol{R}_{7\mathrm{i}}^{0}}{\partial q_j} = {} & \frac{\partial \boldsymbol{R}_z(\theta_{21i})}{\partial q_j}\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i}) + \boldsymbol{R}_z(\theta_{21i})\frac{\partial \boldsymbol{R}_x(\beta_{54i})}{\partial q_j}\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i}) \\
& + \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\frac{\partial \boldsymbol{R}_y(\theta_{65i})}{\partial q_j}\boldsymbol{R}_z(\theta_{76i}) \\
& + \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\frac{\partial \boldsymbol{R}_z(\theta_{76i})}{\partial q_j}
\end{aligned}
$$

$$\frac{\partial \boldsymbol{R}_2^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_y(\theta_{21})}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\frac{\partial \theta_{21}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_5^0}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\frac{\partial \theta_{43}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{11\mathrm{i}}^0}{\partial q_j} = \boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\frac{\partial \theta_{21}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\frac{\partial \theta_{43}}{\partial q_j} + \boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\frac{\partial \theta_{54}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{R}_{12\mathrm{i}}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{11\mathrm{i}}^0}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{10i}^0}{\partial q_j} = 0$$

$$\frac{\partial \boldsymbol{r}_{32i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{2i}^0}{\partial q_j}\boldsymbol{r}_{32i}^{2i}$$

$$\frac{\partial \boldsymbol{r}_{43i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{3\mathrm{i}}^0}{\partial q_j}\boldsymbol{r}_{43i}^{3i}$$

$$\frac{\partial \boldsymbol{r}_{87i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{7\mathrm{i}}^0}{\partial q_j}\boldsymbol{r}_{87i}^{7i}$$

$$\frac{\partial \boldsymbol{r}_{10}^0}{\partial q_j} = 0$$

$$\frac{\partial \boldsymbol{r}_{32}^0}{\partial q_j} = \boldsymbol{R}_2^0\boldsymbol{r}_{32}^2 = \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^2\right)\frac{\partial \theta_{21}}{\partial q_j}$$

$$\frac{\partial \boldsymbol{r}_{14,5i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_5^0}{\partial q_j}\boldsymbol{r}_{14,5i}^5$$

$$\frac{\partial \boldsymbol{r}_{12,11i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{11\mathrm{i}}^0}{\partial q_j}\boldsymbol{r}_{12,11i}^5$$

$$\frac{\partial \boldsymbol{r}_{13,12i}^0}{\partial q_j} = \frac{\partial \boldsymbol{R}_{12\mathrm{i}}^0}{\partial q_j}\boldsymbol{r}_{13,12i}^5$$

Sustituyendo los términos anteriores en la ec. (A.4):

$$
\begin{aligned}
&\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{32i}^{2i}\right)\frac{\partial\theta_{21i}}{\partial q_j}+\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{43i}^{3i}\right)\frac{\partial\theta_{21i}}{\partial q_j}\\
&\quad+\left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\frac{\partial\theta_{21i}}{\partial q_j}\\
&\quad+\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\frac{\partial\theta_{65i}}{\partial q_j}\\
&\quad+\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\frac{\partial\theta_{76i}}{\partial q_j}\\
&\quad=\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^{2}\right)\frac{\partial\theta_{21}}{\partial q_j}+\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{21}}{\partial q_j}\\
&\quad+\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{43}}{\partial q_j}\\
&\quad+\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\frac{\partial\theta_{54}}{\partial q_j}\\
&\quad-\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)\frac{\partial\theta_{21}}{\partial q_j}\\
&\quad-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)\frac{\partial\theta_{43}}{\partial q_j}\\
&\quad-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)\frac{\partial\theta_{54}}{\partial q_j}\\
&\quad-\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\frac{\partial\theta_{21}}{\partial q_j}\\
&\quad-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\frac{\partial\theta_{43}}{\partial q_j}\\
&\quad-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\frac{\partial\theta_{54}}{\partial q_j}
\end{aligned}
$$

Simplificando y agrupando los términos semejantes:

$$\left(\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{32i}^{2i}\right)+\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{43i}^{3i}\right)+\left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\right)\frac{\partial\theta_{21i}}{\partial q_j}$$
$$+\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\frac{\partial\theta_{65i}}{\partial q_j}$$
$$+\left(\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)\frac{\partial\theta_{76i}}{\partial q_j}$$
$$=\left(\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^{2}\right)+\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\right.$$
$$-\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)$$
$$\left.-\left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\right)\frac{\partial\theta_{21}}{\partial q_j}$$
$$+\left(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\right.$$
$$-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)$$
$$\left.-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\right)\frac{\partial\theta_{43}}{\partial q_j}$$
$$+\left(\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right)\right.$$
$$-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right)$$
$$\left.-\left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)\right)\frac{\partial\theta_{54}}{\partial q_j}$$

Renombrando y acomodando matricialmente:

$$\boldsymbol{r}_{Gi}\frac{\partial\theta_{21i}}{\partial q_j}+\boldsymbol{r}_{Hi}\frac{\partial\theta_{65i}}{\partial q_j}+\boldsymbol{r}_{Ii}\frac{\partial\theta_{76i}}{\partial q_j}=\boldsymbol{r}_{Ji}\frac{\partial\theta_{21}}{\partial q_j}+\boldsymbol{r}_{Ki}\frac{\partial\theta_{43}}{\partial q_j}+\boldsymbol{r}_{Li}\frac{\partial\theta_{54}}{\partial q_j}$$
$$\begin{bmatrix}\boldsymbol{r}_{Gi} & \boldsymbol{r}_{Hi} & \boldsymbol{r}_{Ii}\end{bmatrix}\begin{bmatrix}\frac{\partial\theta_{21i}}{\partial q_j}\\ \frac{\partial\theta_{65i}}{\partial q_j}\\ \frac{\partial\theta_{76i}}{\partial q_j}\end{bmatrix}=\begin{bmatrix}\boldsymbol{r}_{Ji} & \boldsymbol{r}_{Ki} & \boldsymbol{r}_{Li}\end{bmatrix}\begin{bmatrix}\frac{\partial\theta_{21}}{\partial q_j}\\ \frac{\partial\theta_{43}}{\partial q_j}\\ \frac{\partial\theta_{54}}{\partial q_j}\end{bmatrix}$$

Finalmente se tiene:

$$\boldsymbol{J}_{\beta i}\frac{\partial\boldsymbol{\beta}}{\partial q_j}=\boldsymbol{J}_{ki}\frac{\partial\boldsymbol{\theta}}{\partial q_j}\qquad\text{(A.6)}$$

Donde:

$$\boldsymbol{r}_{Gi}=\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{32i}^{2i}\right)+\left(\boldsymbol{B}_z(\theta_{21i})\,\boldsymbol{r}_{43i}^{3i}\right)+\left(\boldsymbol{B}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}\right)$$
$$\boldsymbol{r}_{Hi}=\boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{B}_y(\theta_{65i})\boldsymbol{R}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i}$$

$$
\begin{aligned}
\boldsymbol{r}_{Ii} &= \boldsymbol{R}_z(\theta_{21i})\boldsymbol{R}_x(\beta_{54i})\boldsymbol{R}_y(\theta_{65i})\boldsymbol{B}_z(\theta_{76i})\boldsymbol{r}_{87i}^{7i} \\
\boldsymbol{r}_{Ji} &= \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{r}_{32}^{2}\right) + \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right) \\
&\qquad - \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right) \\
&\qquad - \left(\boldsymbol{B}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right) \\
\boldsymbol{r}_{Ki} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right) - \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right) \\
&\qquad - \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{B}_y(\theta_{43})\boldsymbol{R}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right) \\
\boldsymbol{r}_{Li} &= \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{r}_{14,5i}^{5}\right) - \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{13,12i}^{5}\right) \\
&\qquad - \left(\boldsymbol{R}_y(\theta_{21})\boldsymbol{R}_y(\theta_{43})\boldsymbol{B}_z(\theta_{54})\boldsymbol{R}_z(\beta_{14,13i})\boldsymbol{r}_{12,11i}^{5}\right)
\end{aligned}
$$

Despejando y sustituyendo:

$$
\begin{aligned}
\frac{\partial \boldsymbol{\theta}}{\partial q_j} &= \boldsymbol{J}_{\theta i}^{-1}\boldsymbol{J}_{qi}\frac{\partial \boldsymbol{q}}{\partial q_j} \\
\boldsymbol{J}_{\beta i}\frac{\partial \boldsymbol{\beta}}{\partial q_j} &= \boldsymbol{J}_{ki}\boldsymbol{J}_{\theta i}^{-1}\boldsymbol{J}_{qi}\frac{\partial \boldsymbol{q}}{\partial q_j}
\end{aligned}
\tag{A.7}
$$

Apéndice B
Trayectoria

La trayectoria que recorre el efector final del robot se describe a continuación. Está compuesta por una función senoidal y una parábola por lo que tiene las siguientes ecuaciones paramétricas:

$$x = -0.001t^2$$
$$y = t$$
$$z = 100\ sen(0.3t)$$

Para t de $(-60\pi, 60\pi)$

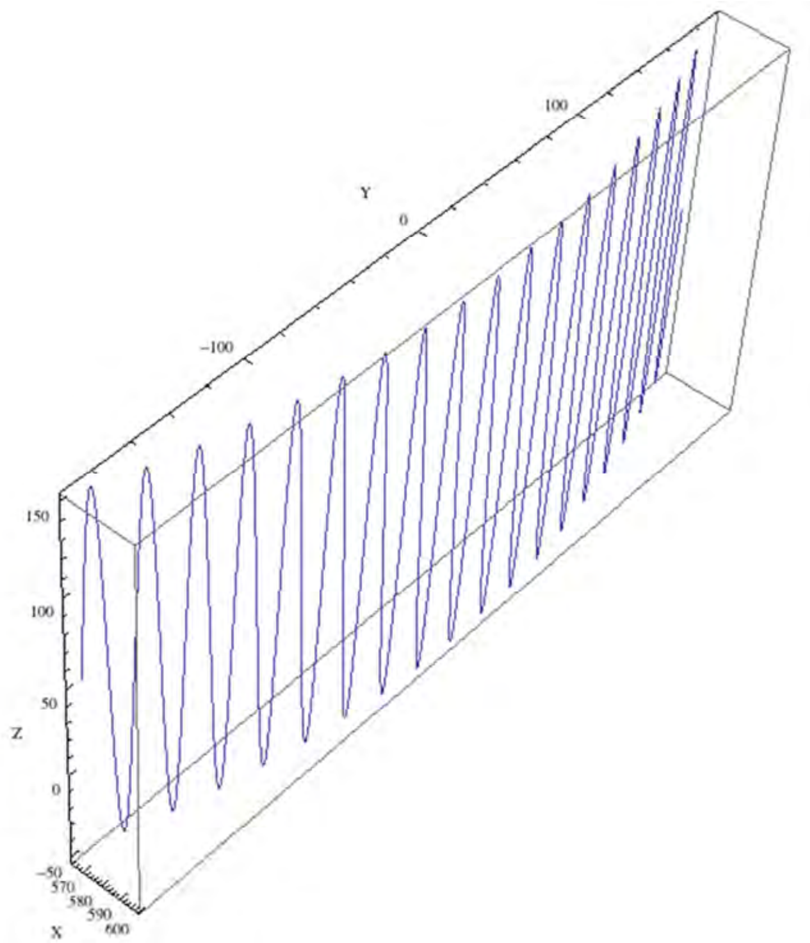

Figura B1 Trayectoria trazada por el efector final.

Apéndice C

Coeficientes de la ecuación (2.71)

$$
\begin{aligned}
\dot{c}_{22} = {} & \dot\theta_{1716}\, c\,\theta_{1716}\, (-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - \\
& \dot\theta_{1716}\, s\,\theta_{1716}\, (s\,\theta_{1817}\, (c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\, (-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + \\
& c\,\theta_{1817}\, (c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\, (c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))) + \\
& s\,\theta_{1716}\, (-\dot\theta_{54}\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + (\dot\theta_{21}+\dot\theta_{43})\, c\,\phi\, c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54} + \dot\phi\, c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + \\
& \dot\psi\, c(\theta_{21}+\theta_{43})\, c\,\phi\, s\,\theta_{54}\, s\,\psi + \dot\theta_{54}\, c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + (\dot\theta_{21}+\dot\theta_{43})\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54} \\
& (s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) - \dot\theta_{54}\, s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi) + c\,\theta_{54}\, (c\,\theta\, c\,\phi\, (\dot\phi + \dot\theta\, s\,\psi) + s\,\theta\, (\dot\psi\, c\,\phi\, c\,\psi - s\,\phi\, (\dot\theta + \dot\phi\, s\,\psi))) + \\
& s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (c\,\phi\, s\,\theta\, (\dot\phi + \dot\theta\, s\,\psi) + c\,\theta\, (-\dot\psi\, c\,\phi\, c\,\psi + s\,\phi\, (\dot\theta + \dot\phi\, s\,\psi)))) + \\
& c\,\theta_{1716}\, (\dot\theta_{1817}\, c\,\theta_{1817}\, (c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\, (-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) - \\
& \dot\theta_{1817}\, s\,\theta_{1817}\, (c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\, (c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi)) + \\
& s\,\theta_{1817}\, (\dot\theta\, c\,\theta\, c\,\theta_{54}\, c\,\psi - \dot\theta_{54}\, c\,\psi\, s\,\theta\, s\,\theta_{54} - \dot\psi\, c\,\theta_{54}\, s\,\theta\, s\,\psi + \dot\theta_{54}\, c\,\theta_{54}\, (-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi) + \\
& s\,\theta_{54}\, ((\dot\psi - (\dot\theta_{21}+\dot\theta_{43})\, c\,\theta)\, c(\theta_{21}+\theta_{43})\, c\,\psi + s(\theta_{21}+\theta_{43})\, (\dot\theta\, c\,\psi\, s\,\theta - (\dot\theta_{21}+\dot\theta_{43} - \dot\psi\, c\,\theta)\, s\,\psi))) + c\,\theta_{1817} \\
& (\dot\phi\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + \dot\theta_{54}\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\psi\, s\,\phi - (\dot\theta_{21}+\dot\theta_{43})\, c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, s\,\phi - \dot\psi\, c(\theta_{21}+\theta_{43}) \\
& s\,\theta_{54}\, s\,\phi\, s\,\psi + \dot\theta_{54}\, c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + (\dot\theta_{21}+\dot\theta_{43})\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) - \\
& \dot\theta_{54}\, s\,\theta_{54}\, (c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\, (c\,\phi\, s\,\theta\, (\dot\theta + \dot\phi\, s\,\psi) + s\,\phi\, (\dot\psi\, c\,\psi\, s\,\theta + c\,\theta\, (\dot\phi + \dot\theta\, s\,\psi))) + \\
& s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\, (-s\,\theta\, s\,\phi\, (\dot\phi + \dot\theta\, s\,\psi) + c\,\theta\, (\dot\psi\, c\,\psi\, s\,\phi + c\,\phi\, (\dot\theta + \dot\phi\, s\,\psi))))))
\end{aligned}
$$

$$
\begin{aligned}
\dot{c}_{22} = {} & \dot\theta_{1716}\, c\,\theta_{1716}\, (c\,\phi\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, (s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi)) - \dot\theta_{1716}\, s\,\theta_{1716} \\
& (-s\,\theta_{1817}\, (c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\psi + s(\theta_{21}+\theta_{43})\, s\,\psi) + c\,\theta_{1817}\, (-c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\phi + c(\theta_{21}+\theta_{43})\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi))) + \\
& s\,\theta_{1716}\, (-s(\theta_{21}+\theta_{43})\, (\dot\phi\, c\,\psi\, s\,\phi + (\dot\theta_{21}+\dot\theta_{43})\, s\,\theta\, s\,\phi + (\dot\psi - (\dot\theta_{21}+\dot\theta_{43})\, c\,\theta)\, c\,\phi\, s\,\psi) + \\
& c(\theta_{21}+\theta_{43})\, (c\,\theta\, s\,\phi\, (\dot\theta + \dot\phi\, s\,\psi) + c\,\phi\, ((\dot\theta_{21}+\dot\theta_{43} - \dot\psi\, c\,\theta)\, c\,\psi + s\,\theta\, (\dot\phi + \dot\theta\, s\,\psi)))) + \\
& c\,\theta_{1716}\, (-c(\theta_{21}+\theta_{43})\, (\dot\theta_{1817}\, c\,\phi\, s\,\theta\, s\,\theta_{1817} + \dot\phi\, c\,\theta_{1817}\, s\,\theta\, s\,\phi + c\,\psi\, (-\dot\theta\, s\,\theta\, s\,\theta_{1817} + (\dot\theta_{21}+\dot\theta_{43})\, c\,\theta_{1817}\, s\,\phi) + \\
& \dot\theta_{21}\, s\,\theta_{1817}\, s\,\psi + \dot\theta_{43}\, s\,\theta_{1817}\, s\,\psi + \dot\theta\, c\,\theta_{1817}\, s\,\theta\, s\,\phi\, s\,\psi) - \\
& s(\theta_{21}+\theta_{43})\, (c\,\psi\, s\,\theta_{1817}\, (\dot\psi - \dot\theta_{1817}\, s\,\phi) + c\,\theta_{1817}\, (c\,\phi\, (\dot\phi\, c\,\psi + (\dot\theta_{21}+\dot\theta_{43})\, s\,\theta) + (\dot\theta_{1817} - \dot\psi\, s\,\phi)\, s\,\psi)) + \\
& c\,\theta\, (s\,\theta_{1817}\, ((\dot\theta_{21}+\dot\theta_{43})\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, (\dot\psi - \dot\theta_{1817}\, s\,\phi)\, s\,\psi) + \\
& c\,\theta_{1817}\, (-(\dot\theta_{21}+\dot\theta_{43})\, s(\theta_{21}+\theta_{43})\, s\,\phi\, s\,\psi + c(\theta_{21}+\theta_{43})\, (c\,\psi\, (-\dot\theta_{1817} + \dot\psi\, s\,\phi) + c\,\phi\, (\dot\theta + \dot\phi\, s\,\psi))))))
\end{aligned}
$$

Coeficientes de la ecuación (2.83)

$$
\begin{aligned}
\ddot{b}_{12} = {} & (\ddot\theta_{54}\, c\,\theta_{54} - \dot\theta_{54}^{2}\, s\,\theta_{54})\, (c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) + \\
& (\dot\theta_{54}^{2}\, c\,\theta_{54} + \ddot\theta_{54}\, s\,\theta_{54})\, (c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\, (c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)) - \\
& 2\, \dot\theta_{54}\, c\,\theta_{54}\, (c\,\phi\, s\,\theta\, (\dot\theta + \dot\phi\, s\,\psi) + s\,\phi\, (\dot\psi\, c\,\psi\, s\,\theta + c\,\theta\, (\dot\phi + \dot\theta\, s\,\psi))) + \\
& s\,\theta_{54}\, (-c\,\theta\, (s\,\phi\, (\ddot\phi + 2\, \dot\theta\, \dot\psi\, c\,\psi + \ddot\theta\, s\,\psi) + c\,\phi\, (\dot\theta^{2} + \dot\phi^{2} + 2\, \dot\theta\, \dot\phi\, s\,\psi)) + \\
& s\,\theta\, (-c\,\phi\, (\ddot\theta + 2\, \dot\phi\, \dot\psi\, c\,\psi + \ddot\phi\, s\,\psi) + s\,\phi\, (2\, \dot\theta\, \dot\phi - \ddot\psi\, c\,\psi + (\dot\theta^{2} + \dot\phi^{2} + \dot\psi^{2})\, s\,\psi))) - c\,\theta_{54} \\
& (c\,\phi\, s\,\theta\, ((\ddot\theta_{21}+\ddot\theta_{43})\, c(\theta_{21}+\theta_{43}) - (\dot\theta_{21}+\dot\theta_{43})^{2}\, s(\theta_{21}+\theta_{43})) + 2\, (\dot\theta_{21}+\dot\theta_{43})\, c(\theta_{21}+\theta_{43})\, (\dot\theta\, c\,\theta\, c\,\phi - \dot\phi\, s\,\theta\, s\,\phi) + s(\theta_{21}+\theta_{43}) \\
& (-s\,\theta\, ((\dot\theta^{2} + \dot\phi^{2})\, c\,\phi + \ddot\phi\, s\,\phi) + c\,\theta\, (\ddot\theta\, c\,\phi - 2\, \dot\theta\, \dot\phi\, s\,\phi)) + (\ddot\phi\, c\,\phi - \dot\phi^{2}\, s\,\phi)\, (c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi) +
\end{aligned}
$$

$$2\,\dot\phi\,c\,\phi\left(-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\psi\,c\,\theta\right)c\,\psi\,s(\theta_{21}+\theta_{43})+\left(\left(-\dot\psi+\left(\dot\theta_{21}+\dot\theta_{43}\right)c\,\theta\right)c(\theta_{21}+\theta_{43})-\dot\theta\,s\,\theta\,s(\theta_{21}+\theta_{43})\right)s\,\psi\right)+$$
$$s\,\phi\left(-s(\theta_{21}+\theta_{43})\left(c\,\psi\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\psi\,c\,\theta+2\,\dot\theta\,\dot\psi\,s\,\theta\right)+\right.\right.$$
$$\left.\left(-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\psi+\left(\dot\theta^2+\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\psi^2\right)c\,\theta+\ddot\theta\,s\,\theta\right)s\,\psi\right)-c(\theta_{21}+\theta_{43})$$
$$\left.\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\psi^2-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\psi\,c\,\theta\right)c\,\psi+\left(\ddot\psi-\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c\,\theta+2\,\dot\theta\left(\dot\theta_{21}+\dot\theta_{43}\right)s\,\theta\right)s\,\psi\right)\right)+$$
$$2\,\dot\theta_{54}\,s\,\theta_{54}\left(c(\theta_{21}+\theta_{43})\left(c\,\phi\left(\dot\phi\,c\,\psi+\left(\dot\theta_{21}+\dot\theta_{43}\right)s\,\theta\right)+\left(-\dot\psi+\left(\dot\theta_{21}+\dot\theta_{43}\right)c\,\theta\right)s\,\phi\,s\,\psi\right)+\right.$$
$$\left.s(\theta_{21}+\theta_{43})\left(-s\,\phi\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)c\,\psi+s\,\theta\left(\dot\phi+\dot\theta\,s\,\psi\right)\right)+c\,\theta\left(\dot\psi\,c\,\psi\,s\,\phi+c\,\phi\left(\dot\theta+\dot\phi\,s\,\psi\right)\right)\right)\right)\Big)$$

$$\ddot b_{13}=2\,\dot\psi\,c\,\psi\left(-\left(\dot\theta_{21}+\dot\theta_{43}\right)c\,\theta_{54}\,s(\theta_{21}+\theta_{43})-\dot\theta_{54}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\right)+$$
$$c\,\psi\,s(\theta_{21}+\theta_{43})\left(c\,\theta\left(\left(\dot\theta^2+\dot\theta_{54}{}^2\right)c\,\theta_{54}+\ddot\theta_{54}\,s\,\theta_{54}\right)+s\,\theta\left(\ddot\theta\,c\,\theta_{54}-2\,\dot\theta\,\dot\theta_{54}\,s\,\theta_{54}\right)\right)+$$
$$c\,\psi\left(-c\,\theta\left(2\,\dot\theta\,\dot\theta_{54}\,c\,\theta_{54}+\ddot\theta\,s\,\theta_{54}\right)+s\,\theta\left(-\ddot\theta_{54}\,c\,\theta_{54}+\left(\dot\theta^2+\dot\theta_{54}{}^2\right)s\,\theta_{54}\right)\right)+2\,\dot\psi\left(\dot\theta_{54}\,c\,\theta_{54}\,s\,\theta+\dot\theta\,c\,\theta\,s\,\theta_{54}\right)s\,\psi+$$
$$\left(-c(\theta_{21}+\theta_{43})\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\theta_{54}{}^2\right)c\,\theta_{54}+\ddot\theta_{54}\,s\,\theta_{54}\right)-s(\theta_{21}+\theta_{43})\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c\,\theta_{54}-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{54}\,s\,\theta_{54}\right)\right)$$
$$s\,\psi+s\,\theta\,s\,\theta_{54}\left(\dot\psi^2\,c\,\psi+\ddot\psi\,s\,\psi\right)+c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\left(\ddot\psi\,c\,\psi-\dot\psi^2\,s\,\psi\right)+$$
$$2\left(\dot\theta\,c\,\theta_{54}\,s\,\theta+\dot\theta_{54}\,c\,\theta\,s\,\theta_{54}\right)\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)c(\theta_{21}+\theta_{43})\,c\,\psi-\dot\psi\,s(\theta_{21}+\theta_{43})\,s\,\psi\right)-$$
$$c\,\theta\,c\,\theta_{54}\left(-s(\theta_{21}+\theta_{43})\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\psi^2\right)c\,\psi+\ddot\psi\,s\,\psi\right)+c(\theta_{21}+\theta_{43})\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c\,\psi-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\psi\,s\,\psi\right)\right)$$

Coeficientes de la ecuación (2.85)

$$\ddot b_{11}=c\,\phi\,c\,\psi\left(-c(\theta_{21}+\theta_{43})\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\theta_{54}{}^2\right)c\,\theta_{54}+\ddot\theta_{54}\,s\,\theta_{54}\right)-\right.$$
$$\left.s(\theta_{21}+\theta_{43})\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c\,\theta_{54}-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{54}\,s\,\theta_{54}\right)\right)+$$
$$2\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)c\,\theta_{54}\,s(\theta_{21}+\theta_{43})+\dot\theta_{54}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\right)\left(\dot\phi\,c\,\psi\,s\,\phi+\dot\psi\,c\,\phi\,s\,\psi\right)+$$
$$\left(-s(\theta_{21}+\theta_{43})\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\theta_{54}{}^2\right)c\,\theta_{54}+\ddot\theta_{54}\,s\,\theta_{54}\right)+c(\theta_{21}+\theta_{43})\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c\,\theta_{54}-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{54}\,s\,\theta_{54}\right)\right)$$
$$\left(-s\,\theta\,s\,\phi+c\,\theta\,c\,\phi\,s\,\psi\right)+\left(\ddot\theta_{54}\,c\,\theta_{54}-\dot\theta_{54}{}^2\,s\,\theta_{54}\right)\left(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi\right)-$$
$$c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\left(c\,\phi\left(\left(\dot\phi^2+\dot\psi^2\right)c\,\psi+\ddot\psi\,s\,\psi\right)+s\,\phi\left(\ddot\phi\,c\,\psi-2\,\dot\phi\,\dot\psi\,s\,\psi\right)\right)+$$
$$2\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)c(\theta_{21}+\theta_{43})\,c\,\theta_{54}-\dot\theta_{54}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\right)\left(-c\,\phi\,s\,\theta\left(\dot\phi+\dot\theta\,s\,\psi\right)+c\,\theta\left(\dot\psi\,c\,\phi\,c\,\psi-s\,\phi\left(\dot\theta+\dot\phi\,s\,\psi\right)\right)\right)+$$
$$2\,\dot\theta_{54}\,c\,\theta_{54}\left(c\,\theta\,c\,\phi\left(\dot\phi+\dot\theta\,s\,\psi\right)+s\,\theta\left(\dot\psi\,c\,\phi\,c\,\psi-s\,\phi\left(\dot\theta+\dot\phi\,s\,\psi\right)\right)\right)-$$
$$c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\left(s\,\theta\left(c\,\phi\left(\ddot\phi+2\,\dot\theta\,\dot\psi\,c\,\psi+\ddot\theta\,s\,\psi\right)-s\,\phi\left(\dot\theta^2+\dot\phi^2+2\,\dot\theta\,\dot\phi\,s\,\psi\right)\right)+\right.$$
$$\left.c\,\theta\left(s\,\phi\left(\ddot\theta+2\,\dot\phi\,\dot\psi\,c\,\psi+\ddot\phi\,s\,\psi\right)+c\,\phi\left(2\,\dot\theta\,\dot\phi-\ddot\psi\,c\,\psi+\left(\dot\theta^2+\dot\phi^2+\dot\psi^2\right)s\,\psi\right)\right)\right)+$$
$$s\,\theta_{54}\left(c\,\theta\left(c\,\phi\left(\ddot\phi+2\,\dot\theta\,\dot\psi\,c\,\psi+\ddot\theta\,s\,\psi\right)-s\,\phi\left(\dot\theta^2+\dot\phi^2+2\,\dot\theta\,\dot\phi\,s\,\psi\right)\right)-\right.$$
$$\left.s\,\theta\left(s\,\phi\left(\ddot\theta+2\,\dot\phi\,\dot\psi\,c\,\psi+\ddot\phi\,s\,\psi\right)+c\,\phi\left(2\,\dot\theta\,\dot\phi-\ddot\psi\,c\,\psi+\left(\dot\theta^2+\dot\phi^2+\dot\psi^2\right)s\,\psi\right)\right)\right)$$

Coeficientes de la ecuación (2.87)

$$\ddot c_{22}=\left(\ddot\theta_{1716}\,c\,\theta_{1716}-\dot\theta_{1716}{}^2\,s\,\theta_{1716}\right)$$
$$\left(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\left(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi\right)+c\,\theta_{54}\left(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi\right)\right)+$$

$$\left(-\dot{\theta}_{1716}{}^{2}\,\mathrm{c}\,\theta_{1716}-\ddot{\theta}_{1716}\,\mathrm{s}\,\theta_{1716}\right)(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$

$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)))+$$

$$2\,\dot{\theta}_{1716}\,\mathrm{c}\,\theta_{1716}\,\left(-\dot{\theta}_{54}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}+\dot{\phi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\right.$$

$$\dot{\psi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}$$

$$(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)-\dot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\left(\mathrm{c}\,\theta\,\mathrm{c}\,\phi\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\theta\left(\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi-\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$\left.\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(-\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi+\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)\right)+\mathrm{s}\,\theta_{1716}$$

$$\left(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\phi}^{2}\right)\mathrm{c}\,\phi+\ddot{\phi}\,\mathrm{s}\,\phi\right)+\mathrm{s}(\theta_{21}+\theta_{43})\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\phi-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\phi}\,\mathrm{s}\,\phi\right)\right)+\right.$$

$$\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\theta_{54}\right)+\right.$$

$$\left.\mathrm{s}(\theta_{21}+\theta_{43})\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{s}\,\theta_{54}\right)\right)(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)-$$

$$\left(\dot{\theta}_{54}{}^{2}\,\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)+2\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})+\dot{\phi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\right)$$

$$\left(\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi-\dot{\psi}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi\right)-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\left(-\mathrm{s}\,\theta_{54}\left(\left(\dot{\theta}_{54}{}^{2}+\dot{\psi}^{2}\right)\mathrm{c}\,\psi+\ddot{\psi}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta_{54}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\psi-2\,\dot{\theta}_{54}\,\dot{\psi}\,\mathrm{s}\,\psi\right)\right)-$$

$$2\,\dot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\left(\mathrm{c}\,\theta\,\mathrm{c}\,\phi\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\theta\left(\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi-\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$2\left(\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\right)\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(-\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi+\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\left(\mathrm{s}\,\theta\left(\mathrm{c}\,\phi\left(\ddot{\phi}+2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\theta}\,\mathrm{s}\,\psi\right)-\mathrm{s}\,\phi\left(\dot{\theta}^{2}+\dot{\phi}^{2}+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)\right)+\right.$$

$$\left.\mathrm{c}\,\theta\left(\mathrm{s}\,\phi\left(\ddot{\theta}+2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\phi\left(2\,\dot{\theta}\,\dot{\phi}-\ddot{\psi}\,\mathrm{c}\,\psi+\left(\dot{\theta}^{2}+\dot{\phi}^{2}+\dot{\psi}^{2}\right)\mathrm{s}\,\psi\right)\right)\right)+$$

$$\mathrm{c}\,\theta_{54}\left(\mathrm{c}\,\theta\left(\mathrm{c}\,\phi\left(\ddot{\phi}+2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\theta}\,\mathrm{s}\,\psi\right)-\mathrm{s}\,\phi\left(\dot{\theta}^{2}+\dot{\phi}^{2}+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)\right)-\right.$$

$$\left.\left.\mathrm{s}\,\theta\left(\mathrm{s}\,\phi\left(\ddot{\theta}+2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\phi\left(2\,\dot{\theta}\,\dot{\phi}-\ddot{\psi}\,\mathrm{c}\,\psi+\left(\dot{\theta}^{2}+\dot{\phi}^{2}+\dot{\psi}^{2}\right)\mathrm{s}\,\psi\right)\right)\right)\right)-$$

$$2\,\dot{\theta}_{1716}\,\mathrm{s}\,\theta_{1716}\,(\dot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))-$$

$$\dot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))+$$

$$\mathrm{s}\,\theta_{1817}\,(\dot{\theta}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi-\dot{\theta}_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)+$$

$$\mathrm{s}\,\theta_{54}\,\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\left(\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right))+\mathrm{c}\,\theta_{1817}$$

$$\left(\dot{\phi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\dot{\theta}_{54}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi-\dot{\psi}\,\mathrm{c}(\theta_{21}+\theta_{43})\right.$$

$$\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-$$

$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\phi\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$\left.\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\left(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)\right))+$$

$$\mathrm{c}\,\theta_{1716}\left(\left(\ddot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}-\dot{\theta}_{1817}{}^{2}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\right.$$

$$\left(-\dot{\theta}_{1817}{}^{2}\,\mathrm{c}\,\theta_{1817}-\ddot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\right)$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))+$$

$$2\,\dot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}\,(\dot{\theta}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi-\dot{\theta}_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)+$$

$$\mathrm{s}\,\theta_{54}\,\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\left(\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right))+$$

$$\mathrm{s}\,\theta_{1817}\left(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\left(\ddot{\theta}\,\mathrm{c}\,\theta-\dot{\theta}^{2}\,\mathrm{s}\,\theta\right)+\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\theta_{54}\right)(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-\right.$$

$$2\,\dot{\theta}\,\mathrm{c}\,\theta\left(\dot{\theta}_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi\right)-\mathrm{s}\,\theta\left(\mathrm{c}\,\theta_{54}\left(\left(\dot{\theta}_{54}{}^{2}+\dot{\psi}^{2}\right)\mathrm{c}\,\psi+\ddot{\psi}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\theta_{54}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\psi-2\,\dot{\theta}_{54}\,\dot{\psi}\,\mathrm{s}\,\psi\right)\right)+$$

$$2\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\left(\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right)+$$

$$\mathrm{s}\,\theta_{54}\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(\mathrm{c}\,\psi\left(\ddot{\psi}-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\theta+2\,\dot{\theta}\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta\right)-\right.\right.$$

$$\left(\dot\theta_{21}{}^{2} + 2\,\dot\theta_{21}\,\dot\theta_{43} + \dot\theta_{43}{}^{2} + \dot\psi^{2} - 2\,(\dot\theta_{21} + \dot\theta_{43})\,\dot\psi\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\big) + \mathrm{s}(\theta_{21} + \theta_{43})\,\big(\mathrm{c}\,\psi\,\big(-2\,(\dot\theta_{21} + \dot\theta_{43})\,\dot\psi +$$

$$\left(\dot\theta^{2} + \dot\theta_{21}{}^{2} + 2\,\dot\theta_{21}\,\dot\theta_{43} + \dot\theta_{43}{}^{2} + \dot\psi^{2}\right)\mathrm{c}\,\theta + \ddot\theta\,\mathrm{s}\,\theta\big) - \left(\ddot\theta_{21} + \ddot\theta_{43} - \ddot\psi\,\mathrm{c}\,\theta + 2\,\dot\theta\,\dot\psi\,\mathrm{s}\,\theta\right)\mathrm{s}\,\psi\big)\big)\big) -$$

$$2\,\dot\theta_{1817}\,\mathrm{s}\,\theta_{1817}\,\big(\dot\phi\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54} + \dot\theta_{54}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi - (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi -$$

$$\dot\psi\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi + \dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) +$$

$$(\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) - \dot\theta_{54}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) -$$

$$\mathrm{c}\,\theta_{54}\,\big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\phi\,(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi))\big) +$$

$$\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,\big(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta\,(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi))\big)\big) +$$

$$\mathrm{c}\,\theta_{1817}\,\Big(\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\psi\,\Big(\mathrm{c}\,\theta_{54}\,\big(2\,\dot\theta_{54}\,\dot\phi\,\mathrm{c}\,\phi + \ddot\theta_{54}\,\mathrm{s}\,\phi\big) + \mathrm{s}\,\theta_{54}\,\big(\ddot\phi\,\mathrm{c}\,\phi - \big(\dot\theta_{54}{}^{2} + \dot\phi^{2}\big)\,\mathrm{s}\,\phi\big)\Big) + 2\,(\dot\phi\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54} + \dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\phi)$$

$$\big(-(\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21} + \theta_{43}) - \dot\psi\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\psi\big) + \Big(\mathrm{c}(\theta_{21} + \theta_{43})\,\big(2\,(\dot\theta_{21} + \dot\theta_{43})\,\dot\theta_{54}\,\mathrm{c}\,\theta_{54} + (\ddot\theta_{21} + \ddot\theta_{43})\,\mathrm{s}\,\theta_{54}\big) +$$

$$\mathrm{s}(\theta_{21} + \theta_{43})\,\big(\ddot\theta_{54}\,\mathrm{c}\,\theta_{54} - \big(\dot\theta_{21}{}^{2} + 2\,\dot\theta_{21}\,\dot\theta_{43} + \dot\theta_{43}{}^{2} + \dot\theta_{54}{}^{2}\big)\,\mathrm{s}\,\theta_{54}\big)\Big)\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) -$$

$$\big(\dot\theta_{54}{}^{2}\,\mathrm{c}\,\theta_{54} + \ddot\theta_{54}\,\mathrm{s}\,\theta_{54}\big)\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\big(-\mathrm{c}(\theta_{21} + \theta_{43})$$

$$\big(\big(\dot\theta_{21}{}^{2} + 2\,\dot\theta_{21}\,\dot\theta_{43} + \dot\theta_{43}{}^{2} + \dot\psi^{2}\big)\,\mathrm{c}\,\psi + \ddot\psi\,\mathrm{s}\,\psi\big) - \mathrm{s}(\theta_{21} + \theta_{43})\,\big((\ddot\theta_{21} + \ddot\theta_{43})\,\mathrm{c}\,\psi - 2\,(\dot\theta_{21} + \dot\theta_{43})\,\dot\psi\,\mathrm{s}\,\psi\big)\big) +$$

$$2\,\dot\theta_{54}\,\mathrm{s}\,\theta_{54}\,\big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\phi\,(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi))\big) +$$

$$2\,\big(\dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21} + \theta_{43}) + (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\big)\,\big(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta\,(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi))\big) +$$

$$\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,\Big(-\mathrm{s}\,\theta\,\Big(\mathrm{s}\,\phi\,\big(\ddot\phi + 2\,\dot\theta\,\dot\psi\,\mathrm{c}\,\psi + \ddot\theta\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\phi\,\big(\dot\theta^{2} + \dot\phi^{2} + 2\,\dot\theta\,\dot\phi\,\mathrm{s}\,\psi\big)\Big) +$$

$$\mathrm{c}\,\theta\,\Big(\mathrm{c}\,\phi\,\big(\ddot\theta + 2\,\dot\phi\,\dot\psi\,\mathrm{c}\,\psi + \ddot\phi\,\mathrm{s}\,\psi\big) - \mathrm{s}\,\phi\,\big(2\,\dot\theta\,\dot\phi - \ddot\psi\,\mathrm{c}\,\psi + \big(\dot\theta^{2} + \dot\phi^{2} + \dot\psi^{2}\big)\,\mathrm{s}\,\psi\big)\Big)\Big) +$$

$$\mathrm{c}\,\theta_{54}\,\Big(-\mathrm{c}\,\theta\,\Big(\mathrm{s}\,\phi\,\big(\ddot\phi + 2\,\dot\theta\,\dot\psi\,\mathrm{c}\,\psi + \ddot\theta\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\phi\,\big(\dot\theta^{2} + \dot\phi^{2} + 2\,\dot\theta\,\dot\phi\,\mathrm{s}\,\psi\big)\Big) +$$

$$\mathrm{s}\,\theta\,\Big(-\mathrm{c}\,\phi\,\big(\ddot\theta + 2\,\dot\phi\,\dot\psi\,\mathrm{c}\,\psi + \ddot\phi\,\mathrm{s}\,\psi\big) + \mathrm{s}\,\phi\,\big(2\,\dot\theta\,\dot\phi - \ddot\psi\,\mathrm{c}\,\psi + \big(\dot\theta^{2} + \dot\phi^{2} + \dot\psi^{2}\big)\,\mathrm{s}\,\psi\big)\Big)\Big)\Big)\Big)$$

$$\ddot{\mathrm{c}}_{32} = \big(\ddot\theta_{1716}\,\mathrm{c}\,\theta_{1716} - \dot\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{1716}\big)\,(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{c}(\theta_{21} + \theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)) + \big(\dot\theta_{1716}{}^{2}\,\mathrm{c}\,\theta_{1716} + \ddot\theta_{1716}\,\mathrm{s}\,\theta_{1716}\big)$$

$$(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\psi + \mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\psi) - \mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\phi + \mathrm{c}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))) +$$

$$2\,\dot\theta_{1716}\,\mathrm{c}\,\theta_{1716}\,\big(-\mathrm{s}(\theta_{21} + \theta_{43})\,\big(\dot\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi + (\dot\psi - (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}\,\theta)\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi\big) +$$

$$\mathrm{c}(\theta_{21} + \theta_{43})\,\big(\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\phi\,((\dot\theta_{21} + \dot\theta_{43} - \dot\psi\,\mathrm{c}\,\theta)\,\mathrm{c}\,\psi + \mathrm{s}\,\theta\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi))\big)\big) +$$

$$\mathrm{s}\,\theta_{1716}\,\Big(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\big((\ddot\theta_{21} + \ddot\theta_{43})\,\mathrm{c}(\theta_{21} + \theta_{43}) - (\dot\theta_{21} + \dot\theta_{43})^{2}\,\mathrm{s}(\theta_{21} + \theta_{43})\big) - 2\,(\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}(\theta_{21} + \theta_{43})\,(\dot\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \dot\psi\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) +$$

$$\big(-(\dot\theta_{21} + \dot\theta_{43})^{2}\,\mathrm{c}(\theta_{21} + \theta_{43}) - (\ddot\theta_{21} + \ddot\theta_{43})\,\mathrm{s}(\theta_{21} + \theta_{43})\big)\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) -$$

$$\mathrm{s}(\theta_{21} + \theta_{43})\,\big(\mathrm{c}\,\phi\,\big(\big(\dot\phi^{2} + \dot\psi^{2}\big)\,\mathrm{c}\,\psi + \ddot\psi\,\mathrm{s}\,\psi\big) + \mathrm{s}\,\phi\,\big(\ddot\phi\,\mathrm{c}\,\psi - 2\,\dot\phi\,\dot\psi\,\mathrm{s}\,\psi\big)\big) -$$

$$2\,(\dot\theta_{21} + \dot\theta_{43})\,\mathrm{s}(\theta_{21} + \theta_{43})\,\big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,(\dot\phi + \dot\theta\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta\,(-\dot\psi\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{s}\,\phi\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi))\big) +$$

$$\mathrm{c}(\theta_{21} + \theta_{43})\,\Big(\mathrm{s}\,\theta\,\Big(\mathrm{c}\,\phi\,\big(\ddot\phi + 2\,\dot\theta\,\dot\psi\,\mathrm{c}\,\psi + \ddot\theta\,\mathrm{s}\,\psi\big) - \mathrm{s}\,\phi\,\big(\dot\theta^{2} + \dot\phi^{2} + 2\,\dot\theta\,\dot\phi\,\mathrm{s}\,\psi\big)\Big) +$$

$$\mathrm{c}\,\theta\,\Big(\mathrm{s}\,\phi\,\big(\ddot\theta + 2\,\dot\phi\,\dot\psi\,\mathrm{c}\,\psi + \ddot\phi\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\phi\,\big(2\,\dot\theta\,\dot\phi - \ddot\psi\,\mathrm{c}\,\psi + \big(\dot\theta^{2} + \dot\phi^{2} + \dot\psi^{2}\big)\,\mathrm{s}\,\psi\big)\Big)\Big)\Big) -$$

$$2\,\dot\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,\big(-\mathrm{c}(\theta_{21} + \theta_{43})\,\big(\dot\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817} + \dot\phi\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\psi\,(-\dot\theta\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817} + (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi) +$$

$$\dot\theta_{21}\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi + \dot\theta_{43}\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi + \dot\theta\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi\big) -$$

$$\mathrm{s}(\theta_{21} + \theta_{43})\,\big(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,(\dot\psi - \dot\theta_{1817}\,\mathrm{s}\,\phi) + \mathrm{c}\,\theta_{1817}\,\big(\mathrm{c}\,\phi\,(\dot\phi\,\mathrm{c}\,\psi + (\dot\theta_{21} + \dot\theta_{43})\,\mathrm{s}\,\theta) + (\dot\theta_{1817} - \dot\psi\,\mathrm{s}\,\phi)\,\mathrm{s}\,\psi\big)\big) +$$

$$\mathrm{c}\,\theta\,\big(\mathrm{s}\,\theta_{1817}\,((\dot\theta_{21} + \dot\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{c}(\theta_{21} + \theta_{43})\,(\dot\psi - \dot\theta_{1817}\,\mathrm{s}\,\phi)\,\mathrm{s}\,\psi) +$$

$$\mathrm{c}\,\theta_{1817}\,\big(-(\dot\theta_{21} + \dot\theta_{43})\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi + \mathrm{c}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\psi\,(-\dot\theta_{1817} + \dot\psi\,\mathrm{s}\,\phi) + \mathrm{c}\,\phi\,(\dot\theta + \dot\phi\,\mathrm{s}\,\psi))\big)\big)\big) +$$

$$\mathrm{c}\,\theta_{1716}\left(\left(-\ddot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}+\dot{\theta}_{1817}{}^{2}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-\right.$$

$$2\,\dot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\left(-\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right)-$$

$$\mathrm{s}\,\theta_{1817}\left(\mathrm{s}(\theta_{21}+\theta_{43})\left(\mathrm{c}\,\psi\left(\ddot{\psi}-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\theta+2\,\dot{\theta}\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta\right)-\right.\right.$$

$$\left.\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\psi}^{2}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)-\mathrm{c}(\theta_{21}+\theta_{43})$$

$$\left.\left(\mathrm{c}\,\psi\left(-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}+\left(\dot{\theta}^{2}+\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\psi}^{2}\right)\mathrm{c}\,\theta+\ddot{\theta}\,\mathrm{s}\,\theta\right)-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}-\ddot{\psi}\,\mathrm{c}\,\theta+2\,\dot{\theta}\,\dot{\psi}\,\mathrm{s}\,\theta\right)\mathrm{s}\,\psi\right)\right)+$$

$$\left(\dot{\theta}_{1817}{}^{2}\,\mathrm{c}\,\theta_{1817}+\ddot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi-\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))-$$

$$2\,\dot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\left(-\mathrm{c}\,\phi\left(\dot{\phi}\,\mathrm{c}\,\psi+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta\right)\mathrm{s}(\theta_{21}+\theta_{43})-\right.$$

$$\mathrm{s}\,\phi\left(-\dot{\psi}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi+\mathrm{c}(\theta_{21}+\theta_{43})\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\psi+\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$\left.\mathrm{c}\,\theta\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}(\theta_{21}+\theta_{43})\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)\right)+\mathrm{c}\,\theta_{1817}$$

$$\left(-\mathrm{c}\,\psi\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\phi}\,\mathrm{c}\,\phi+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\phi\right)+\mathrm{s}(\theta_{21}+\theta_{43})\left(\ddot{\phi}\,\mathrm{c}\,\phi-\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\phi}^{2}\right)\mathrm{s}\,\phi\right)\right)+\right.$$

$$2\,\dot{\psi}\left(\dot{\phi}\,\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\right)\mathrm{s}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\left(\dot{\psi}^{2}\,\mathrm{c}\,\psi+\ddot{\psi}\,\mathrm{s}\,\psi\right)+$$

$$\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^{2}\mathrm{c}(\theta_{21}+\theta_{43})-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\right)(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-$$

$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\left(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)+$$

$$\mathrm{c}(\theta_{21}+\theta_{43})\left(-\mathrm{s}\,\theta\left(\mathrm{s}\,\phi\left(\ddot{\phi}+2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\phi\left(\dot{\theta}^{2}+\dot{\phi}^{2}+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)\right)+\right.$$

$$\left.\left.\left.\mathrm{c}\,\theta\left(\mathrm{c}\,\phi\left(\ddot{\theta}+2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\psi+\ddot{\phi}\,\mathrm{s}\,\psi\right)-\mathrm{s}\,\phi\left(2\,\dot{\theta}\,\dot{\phi}-\ddot{\psi}\,\mathrm{c}\,\psi+\left(\dot{\theta}^{2}+\dot{\phi}^{2}+\dot{\psi}^{2}\right)\mathrm{s}\,\psi\right)\right)\right)\right)\right)$$

Coeficientes de la ecuación (2.131)

$$\mathrm{f}_{11}=\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})-$$

$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(-\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-$$

$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))))-$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\beta_{54\,\mathrm{i}}$$

$$(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))))$$

$$\mathrm{f}_{21}=\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(-\mathrm{s}\,\theta_{43}$$

$$(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))+$$

$$\mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})$$

$$(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))$$

$$\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))-$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$
$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$
$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))))$$

Coeficientes de la ecuación (2.151)

$$\dot{\mathrm{e}}_{31} = -\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} -\right.$$
$$\mathrm{s}(\theta_{21} + \theta_{43})\left(\left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\Big) +$$
$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\Big(-\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot\theta_{65\,\mathrm{i}} + \left(\dot\theta_{21\,\mathrm{i}} + \left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}(\theta_{21} + \theta_{43})\right)\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54}) +\right.$$
$$\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\Big(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) +$$
$$\left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big) +$$
$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\Big(-\left(\dot\theta_{21\,\mathrm{i}} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})$$
$$\left(\left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) +$$
$$\mathrm{c}(\theta_{21} + \theta_{43})\left(\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right) + \left(-\dot\theta_{1110\,\mathrm{i}} + \dot\theta_{54}\right)\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\Big)\Big) +$$
$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\Big(-\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\left(\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot\theta_{65\,\mathrm{i}} + \left(\dot\theta_{21\,\mathrm{i}} + \left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}(\theta_{21} + \theta_{43})\right)\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}} +\right.$$
$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(\dot\theta_{21\,\mathrm{i}} + \left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}(\theta_{21} + \theta_{43}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big) + \mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})$$
$$\Big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(-\left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right) + \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) +$$
$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\Big(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \left(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43}) - \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}} +$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\Big)\Big)\Big)$$

$$\dot{\mathrm{e}}_{33} = -\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) -$$
$$\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +$$
$$\mathrm{s}(\theta_{21} + \theta_{43})\left(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right) - \left(\dot\theta_{21\,\mathrm{i}} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)$$

Coeficientes de la ecuación (2.155)

$$\dot{\mathrm{f}}_{11} = \dot\theta_{109\,\mathrm{i}}\,\mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) -$$
$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} - \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$
$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} - \mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) +$$
$$\mathrm{s}\,\theta_{109\,\mathrm{i}}\Big(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} - \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) -$$
$$\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} - \dot\theta_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{76\,\mathrm{i}} -$$
$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\Big(\mathrm{c}\,\theta_{21}\left(\mathrm{s}\,\theta_{43}\left(\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} - \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right) +\right.$$
$$\mathrm{c}\,\theta_{43}\left(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} + \left(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}} - \dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\Big) +$$
$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{43}\left(\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} - \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right) -\right.$$
$$\mathrm{s}\,\theta_{43}\left(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} + \left(\left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}} - \dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\Big)\Big) -$$
$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\Big(\mathrm{s}\,\theta_{21}\Big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\left(\dot\theta_{76\,\mathrm{i}} - \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) + \mathrm{c}\,\theta_{43}\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} +$$
$$\left(\dot\theta_{21} + \dot\theta_{43} - \dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\Big)\Big) + \mathrm{c}\,\theta_{21}\Big(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(-\dot\theta_{76\,\mathrm{i}} + \left(\dot\theta_{21} + \dot\theta_{43}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right) +$$
$$\mathrm{s}\,\theta_{43}\left(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \left(\dot\theta_{21} + \dot\theta_{43} - \dot\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\Big)\Big)\Big) -$$
$$\dot\theta_{109\,\mathrm{i}}\,\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(-\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) +$$
$$\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} - \mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) +$$
$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) +$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))))-$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}$$

$$(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))))+$$

$$c\,\theta_{109\,i}\,(-\dot{\theta}_{1110\,i}\,s\,\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))))-$$

$$\dot{\theta}_{1110\,i}\,c\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}$$

$$(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))+$$

$$c\,\theta_{1110\,i}\,\big(\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$\dot{\theta}_{43}\,c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+\dot{\theta}_{54}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+$$

$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))-$$

$$s(\beta_{1413\,i}-\theta_{54})\,\big(-s\,\theta_{21\,i}\,\big(c\,\theta_{76\,i}\,\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{65\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)+c\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-$$

$$\dot{\theta}_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+c\,\theta_{21\,i}\,\big(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)\big)-$$

$$\dot{\theta}_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+$$

$$\dot{\theta}_{54}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,\big(\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})-$$

$$\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))-$$

$$c\,\theta_{21}\,\big(c\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+c\,\theta_{65\,i}\,\big(c\,\theta_{76\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}+$$

$$\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+s\,\theta_{65\,i}\,\big(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)\big)-$$

$$s\,\theta_{21}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,\big(-\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)\big)\big)+$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,\big(-c\,\theta_{21}\,c\,\theta_{76\,i}\,\big(\dot{\theta}_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\big)\big)+$$

$$s\,\theta_{21}\,\big(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{65\,i}+s\,\beta_{54\,i}\,\big(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\big)\,s\,\theta_{76\,i}+$$

$$c\,\theta_{65\,i}\,\big(c\,\theta_{76\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)\big)+$$

$$c\,\beta_{54\,i}\,\big(s\,\theta_{21}\,\big(c\,\theta_{76\,i}\,\big(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}-\dot{\theta}_{21}\,s\,\theta_{65\,i}\big)+\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+$$

$$c\,\theta_{21}\,\big(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\big(\dot{\theta}_{21}\,s\,\theta_{21\,i}-\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\big)\,s\,\theta_{76\,i}\big)\big)\big)\big)-$$

$$s\,\theta_{1110\,i}\,\big(\dot{\theta}_{54}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$c(\beta_{1413\,i}-\theta_{54})\,\big(-s\,\theta_{21\,i}\,\big(c\,\theta_{76\,i}\,\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{65\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)+c\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-$$

$$\dot{\theta}_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+c\,\theta_{21\,i}\,\big(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)\big)-$$

$$\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+$$

$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))+$$

$$s(\beta_{1413\,i}-\theta_{54})\,\big(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) + \dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43} +$$
$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})) +$$
$$\mathrm{c}\,\theta_{21}\,(-\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}\,(\dot{\theta}_{43}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i} - \dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})) -$$
$$\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{43} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i} +$$
$$\mathrm{c}\,\theta_{65\,i}\,(\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$
$$\mathrm{c}\,\beta_{54\,i}\,(-\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} - \dot{\theta}_{43}\,\mathrm{s}\,\theta_{65\,i}) + \dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$
$$\mathrm{s}\,\theta_{43}\,(\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} + (\dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) +$$
$$\mathrm{s}\,\theta_{21}\,(-\dot{\theta}_{43}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} +$$
$$\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} - \dot{\theta}_{43}\,\mathrm{s}\,\theta_{65\,i}) + \dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$
$$\mathrm{c}\,\theta_{43}\,(\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} + (\dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) +$$
$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,(-\dot{\theta}_{76\,i} + \dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) + \mathrm{s}\,\theta_{43}\,(\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} +$$
$$\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + (\dot{\theta}_{43} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))))))$$

$$\dot{\mathrm{f}}_{21} = \dot{\theta}_{109\,i}\,\mathrm{c}\,\theta_{109\,i}\,(-\mathrm{s}\,\theta_{43}$$
$$(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) + (\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) +$$
$$\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) + \mathrm{s}\,\theta_{109\,i}\,(-\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}$$
$$(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) + (\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) +$$
$$\mathrm{c}\,\theta_{43}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} -$$
$$(\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$
$$\mathrm{c}\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}))) - \mathrm{s}\,\theta_{43}$$
$$(-\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{21} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) -$$
$$\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$
$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$
$$\mathrm{c}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) -$$
$$\dot{\theta}_{109\,i}\,\mathrm{s}\,\theta_{109\,i}\,(\mathrm{c}\,\theta_{1110\,i}\,(\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$
$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) -$$
$$\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$
$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) +$$
$$\mathrm{c}\,\theta_{109\,i}\,(-\dot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$
$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))) -$$

$$\dot{\theta}_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} - (\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}) +$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))) +$$

$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\big(\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} - (\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) +$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +$$

$$\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} + \big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}} + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big) - \big(\dot{\theta}_{65\,\mathrm{i}} + \dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big) +$$

$$\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}) +$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}) +$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) -$$

$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) -$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) - \mathrm{s}\,\theta_{21}$$

$$\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} + \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) - \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} -$$

$$\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}} + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}} + \dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big) -$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) +$$

$$\mathrm{c}\,\theta_{21}\,\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)\big) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\big(-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} + \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}} +$$

$$\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21} - \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big) - \mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}} + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} +$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} - \big(\dot{\theta}_{65\,\mathrm{i}} + \dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big) +$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\mathrm{s}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big) + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big) +$$

$$\mathrm{c}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} + \big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)\big)\big) -$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\big(\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} - (\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) +$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}) +$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) +$$

$$\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}) +$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\big(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big) +$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}} + \dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big) +$$

$$\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \big(\dot{\theta}_{65\,\mathrm{i}} + \dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big) +$$

$$\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) -$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + (\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \theta_{54})\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}) - \mathrm{s}\,\theta_{21}$$

$$\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}} + \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - \mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) - \dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}} -$$

$$\left(c\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+c\,\theta_{21\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)-$$
$$\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$
$$c\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\right)-$$
$$c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\left(-s\,\theta_{21}\left(\dot{\theta}_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)s\,\theta_{76\,i}+\right.$$
$$c\,\theta_{21}\left(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\left(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)-c\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}\,s\,\theta_{76\,i}+\right.$$
$$c\,\theta_{21\,i}\left(\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)+$$
$$c\,\beta_{54\,i}\left(s\,\theta_{21}\left(c\,\theta_{76\,i}\left(-\dot{\theta}_{21}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.c\,\theta_{21}\left(\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+\left(-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{21}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)\right)\right)$$

Coeficientes de la ecuación (2.167)

$$\ddot{d}_{31}=2\,\dot{\theta}_{54}\,c\,(\beta_{1413\,i}-\theta_{54})\left(c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+c\,\theta_{21\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)-$$
$$\left(\ddot{\theta}_{54}\,c\,(\beta_{1413\,i}-\theta_{54})+\dot{\theta}_{54}^{\,2}\,s\,(\beta_{1413\,i}-\theta_{54})\right)(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-s\,\theta_{21\,i}\,s\,\theta_{65\,i})+$$
$$\left(-c\,\theta_{43}\left(\left(\dot{\theta}_{43}^{\,2}+\dot{\theta}_{54}^{\,2}\right)c\,(\beta_{1413\,i}-\theta_{54})-\ddot{\theta}_{54}\,s\,(\beta_{1413\,i}-\theta_{54})\right)-s\,\theta_{43}\left(\ddot{\theta}_{43}\,c\,(\beta_{1413\,i}-\theta_{54})+2\,\dot{\theta}_{43}\,\dot{\theta}_{54}\,s\,(\beta_{1413\,i}-\theta_{54})\right)\right)$$
$$(-c\,\beta_{54\,i}\,c\,\theta_{65\,i}\,s\,\theta_{21}+c\,\theta_{21}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i}))+$$
$$s\,\theta_{43}\left(\dot{\theta}_{54}^{\,2}\,c\,(\beta_{1413\,i}-\theta_{54})-\ddot{\theta}_{54}\,s\,(\beta_{1413\,i}-\theta_{54})\right)(c\,\beta_{54\,i}\,c\,\theta_{21}\,c\,\theta_{65\,i}+s\,\theta_{21}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i}))-$$
$$s\,(\beta_{1413\,i}-\theta_{54})\left(c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)s\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)+\right.$$
$$\left.s\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)\right)+$$
$$2\left(-\dot{\theta}_{43}\,c\,(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}+\dot{\theta}_{54}\,c\,\theta_{43}\,s\,(\beta_{1413\,i}-\theta_{54})\right)$$
$$\left(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i})+c\,\beta_{54\,i}\left(-\dot{\theta}_{21}\,c\,\theta_{21}\,c\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}\right)+\right.$$
$$\left.c\,\theta_{21}\left(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)+$$
$$c\,\theta_{43}\,c\,(\beta_{1413\,i}-\theta_{54})\left(c\,\beta_{54\,i}\left(s\,\theta_{21}\left(\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)c\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,s\,\theta_{65\,i}\right)+c\,\theta_{21}\left(-\ddot{\theta}_{21}\,c\,\theta_{65\,i}+2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,i}\,s\,\theta_{65\,i}\right)\right)-\right.$$
$$s\,\theta_{21}\left(c\,\theta_{21\,i}\left(2\,\dot{\theta}_{21}\,c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)+\ddot{\theta}_{21}\,s\,\theta_{65\,i}\right)+\right.$$
$$\left.s\,\theta_{21\,i}\left(\ddot{\theta}_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-2\,\dot{\theta}_{21}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)\right)+$$
$$c\,\theta_{21}\left(-s\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)s\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)+\right.$$
$$\left.\left.c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)\right)\right)-$$
$$2\,\dot{\theta}_{54}\,s\,(\beta_{1413\,i}-\theta_{54})\left(\dot{\theta}_{43}\,c\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{21}\,c\,\theta_{65\,i}+s\,\theta_{21}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i}))+\right.$$
$$s\,\theta_{43}\left(c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21}+\dot{\theta}_{21}\,c\,\theta_{21}\,s\,\theta_{65\,i}\right)-c\,\beta_{54\,i}\left(\dot{\theta}_{21}\,c\,\theta_{65\,i}\,s\,\theta_{21}+\dot{\theta}_{65\,i}\,c\,\theta_{21}\,s\,\theta_{65\,i}\right)+\right.$$
$$\left.\left.s\,\theta_{21\,i}\left(\dot{\theta}_{21}\,c\,\theta_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21}\,s\,\theta_{65\,i}\right)\right)\right)-$$
$$c\,(\beta_{1413\,i}-\theta_{54})\left(\left(\ddot{\theta}_{43}\,c\,\theta_{43}-\dot{\theta}_{43}^{\,2}\,s\,\theta_{43}\right)(c\,\beta_{54\,i}\,c\,\theta_{21}\,c\,\theta_{65\,i}+s\,\theta_{21}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i}))+\right.$$
$$2\,\dot{\theta}_{43}\,c\,\theta_{43}\left(c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21}+\dot{\theta}_{21}\,c\,\theta_{21}\,s\,\theta_{65\,i}\right)-c\,\beta_{54\,i}\left(\dot{\theta}_{21}\,c\,\theta_{65\,i}\,s\,\theta_{21}+\dot{\theta}_{65\,i}\,c\,\theta_{21}\,s\,\theta_{65\,i}\right)+\right.$$
$$\left.s\,\theta_{21\,i}\left(\dot{\theta}_{21}\,c\,\theta_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21}\,s\,\theta_{65\,i}\right)\right)+$$
$$s\,\theta_{43}\left(-c\,\beta_{54\,i}\left(c\,\theta_{21}\left(\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)c\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,s\,\theta_{65\,i}\right)+s\,\theta_{21}\left(\ddot{\theta}_{21}\,c\,\theta_{65\,i}-2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,i}\,s\,\theta_{65\,i}\right)\right)+\right.$$
$$c\,\theta_{21}\left(c\,\theta_{21\,i}\left(2\,\dot{\theta}_{21}\,c\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)+\ddot{\theta}_{21}\,s\,\theta_{65\,i}\right)+\right.$$
$$\left.s\,\theta_{21\,i}\left(\ddot{\theta}_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-2\,\dot{\theta}_{21}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)\Big)\Big)$$

$$\ddot{\mathrm{d}}_{31}=2\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+$$

$$\left(\dot{\theta}_{54}^{\,2}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})-\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+$$

$$\left(\mathrm{s}\,\theta_{43}\left(-2\,\dot{\theta}_{43}\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+\ddot{\theta}_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)+\mathrm{c}\,\theta_{43}\left(\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+\left(\dot{\theta}_{43}^{\,2}+\dot{\theta}_{54}^{\,2}\right)\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)\right)$$

$$\left(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}-\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\left(-\ddot{\theta}_{43}\,\mathrm{c}\,\theta_{43}+\dot{\theta}_{43}^{\,2}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$2\left(\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)$$

$$\left(\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}-\dot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{21}\left(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{c}\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{c}\,\theta_{21}\left(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}+2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)-\right.$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-2\,\dot{\theta}_{21}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{c}\,\theta_{21}\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)\right)-$$

$$2\,\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\left(-\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)-\right.$$

$$\left.\left.\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21\,i}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\right)\right)-$$

$$\mathrm{s}\,\theta_{43}\left(-\left(\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+\dot{\theta}_{54}^{\,2}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)\left(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)-\right.$$

$$2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)-\right.$$

$$\left.\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21\,i}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(-\mathrm{c}\,\beta_{54\,i}\left(\mathrm{c}\,\theta_{21}\left(\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{c}\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}-2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-2\,\dot{\theta}_{21}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\left.\left.\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot{\theta}_{21}^{\,2}+\dot{\theta}_{21\,i}^{\,2}+\dot{\theta}_{65\,i}^{\,2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)\right)\right)$$

Coeficientes de la ecuación (2.169)

$$\ddot{\mathrm{e}}_{31}=-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}(\theta_{21}+\theta_{43})\left(-\left(\dot{\theta}_{1110\,i}-\dot{\theta}_{54}\right)^{2}\mathrm{c}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})+\left(-\ddot{\theta}_{1110\,i}+\ddot{\theta}_{54}\right)\mathrm{s}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\right)+$$

$$2\left(\dot{\theta}_{1110\,i}-\dot{\theta}_{54}\right)\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,i}-\dot{\theta}_{65\,i}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,i}\right)-$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}+\theta_{1110\,\mathrm{i}}-\theta_{54})\,\Big(-\mathrm{s}(\theta_{21}+\theta_{43})\,\Big(\Big(\dot\theta_{21}{}^{2}+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^{2}+\dot\theta_{65\,\mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{c}(\theta_{21}+\theta_{43})\,\Big(\Big(\ddot\theta_{21}+\ddot\theta_{43}\Big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-2\,\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)+$$

$$\Big(-\dot\theta_{21\,\mathrm{i}}{}^{2}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\ddot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\Big)\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))+\Big(\ddot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\dot\theta_{21\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\Big)$$

$$(-\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$2\,\dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-$$

$$\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-$$

$$\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(-\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\Big(-\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-$$

$$\mathrm{c}(\theta_{21}+\theta_{43})\,\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\Big)\Big)+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\Big(\Big(-\dot\theta_{54}{}^{2}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\ddot\theta_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big)\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-$$

$$\Big(\ddot\theta_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\dot\theta_{54}{}^{2}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big)\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$2\,\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-$$

$$\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\Big(\ddot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}-\dot\theta_{1110\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\Big)+2\,\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}$$

$$\Big(-\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}-\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)-2\,\dot\theta_{1110\,\mathrm{i}}\,\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+$$

$$\Big(-\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\Big(\Big(\dot\theta_{1110\,\mathrm{i}}{}^{2}+\Big(\dot\theta_{21}+\dot\theta_{43}\Big)^{2}\Big)\,\mathrm{c}(\theta_{21}+\theta_{43})+\Big(\ddot\theta_{21}+\ddot\theta_{43}\Big)\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)+$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(-\ddot\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})+2\,\dot\theta_{1110\,\mathrm{i}}\,\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-$$

$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(\dot\theta_{65\,\mathrm{i}}{}^{2}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\Big(\ddot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\dot\theta_{65\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)-$$

$$2\,\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(-\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\Big(-\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)$$

$$\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}(\theta_{21}+\theta_{43})\,\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(\Big(\dot\theta_{21}+\dot\theta_{43}\Big)^{2}\,\mathrm{c}(\theta_{21}+\theta_{43})+\Big(\ddot\theta_{21}+\ddot\theta_{43}\Big)\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+$$

$$2\,\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{s}(\theta_{21}+\theta_{43})\,\Big(\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{1110\,\mathrm{i}}$$

$$\Big(\Big(\dot\theta_{1110\,\mathrm{i}}{}^{2}+\dot\theta_{65\,\mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(\ddot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-2\,\dot\theta_{1110\,\mathrm{i}}\,\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)+\mathrm{c}(\theta_{21}+\theta_{43})$$

$$\Big(-\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\Big(2\,\dot\theta_{1110\,\mathrm{i}}\,\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(-\ddot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\Big(\dot\theta_{1110\,\mathrm{i}}{}^{2}+\dot\theta_{65\,\mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)\Big)\Big)\Big)+$$

$$2\,\dot\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\Big(-\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(\dot\theta_{65\,\mathrm{i}}+\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\Big)\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\Big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)+$$

$$\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\Big)+$$

$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}+\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(-\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(\dot\theta_{65\,\mathrm{i}}+\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\Big)\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\Big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)+$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\Big(\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}+\Big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}-\dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\Big(\Big(\dot\theta_{21}+\dot\theta_{43}\Big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)\Big)\Big)+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(\Big(-\ddot\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}+\dot\theta_{1110\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\Big)\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})-$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\left(\dot{\theta}_{1110\,\mathrm{i}}{}^{2}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}+\ddot{\theta}_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\right)$$

$$(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\Big(-\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\Big(\ddot{\theta}_{65\,\mathrm{i}}+2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+$$

$$\ddot{\theta}_{54}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{43}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\right)\Big)-$$

$$\left(\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\Big(-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}-\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{65\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}$$

$$\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{43}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\theta_{43}\right)\Big)+$$

$$\Big(\ddot{\theta}_{54}+2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\right)\Big)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)-$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\Big(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\Big(\ddot{\theta}_{65\,\mathrm{i}}+2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\ddot{\theta}_{54}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{43}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\right)\Big)-$$

$$\left(\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\Big(-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,\mathrm{i}}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}-\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{65\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}$$

$$\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{43}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\theta_{43}\right)\Big)+$$

$$\Big(\ddot{\theta}_{54}+2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\right)\Big)\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)+$$

$$2\,\dot{\theta}_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\Big(\Big(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}$$

$$\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})-\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(-\dot{\theta}_{54}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\right)\Big)\Big)-$$

$$2\,\dot{\theta}_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\Big(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}-\dot{\theta}_{54}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\dot{\theta}_{54}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)-$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}-\left(\dot{\theta}_{54}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\Big)\Big)\Big)$$

$$\ddot{\mathrm{e}}_{33}=\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^{2}\mathrm{c}(\theta_{21}+\theta_{43})-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\right)+$$

$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\left(\dot{\theta}_{65\,\mathrm{i}}{}^{2}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)+$$

$$\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^{2}\mathrm{s}(\theta_{21}+\theta_{43})\right)(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\left(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)-\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)-$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\Big(\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\right)\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\left(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\Big)$$

Coeficientes de la ecuación (2.171)

$$\ddot{f}_{11} = (\ddot{\theta}_{109\,i}\, c\,\theta_{109\,i} - \dot{\theta}_{109\,i}^{\,2}\, s\,\theta_{109\,i})\,(c\,\theta_{21\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i}\, s(\theta_{21} + \theta_{43}) -$$

$$s\,\beta_{54\,i}\,(c\,\theta_{43}\, c\,\theta_{76\,i}\, s\,\theta_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + c\,\theta_{21}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{43}\, s\,\theta_{65\,i} - c\,\theta_{21}\, c\,\theta_{43}\, s\,\theta_{76\,i} + s\,\theta_{21}\, s\,\theta_{43}\, s\,\theta_{76\,i}) -$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\, c\,\theta_{43}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i} - c\,\theta_{76\,i}\, s\,\theta_{21}\, s\,\theta_{43}\, s\,\theta_{65\,i} + c\,\theta_{43}\, s\,\theta_{21}\, s\,\theta_{21\,i}\, s\,\theta_{76\,i} + c\,\theta_{21}\, s\,\theta_{21\,i}\, s\,\theta_{43}\, s\,\theta_{76\,i})) +$$

$$2\,\dot{\theta}_{109\,i}\, c\,\theta_{109\,i}\,(((\dot{\theta}_{21} + \dot{\theta}_{43})\, c\,\theta_{21\,i}\, c(\theta_{21} + \theta_{43})\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} - \dot{\theta}_{21\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}\, s(\theta_{21} + \theta_{43}) -$$

$$\dot{\theta}_{65\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}\, s(\theta_{21} + \theta_{43})\, s\,\theta_{65\,i} - \dot{\theta}_{76\,i}\, c\,\theta_{21\,i}\, c\,\theta_{65\,i}\, s(\theta_{21} + \theta_{43})\, s\,\theta_{76\,i} -$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,(s\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot{\theta}_{76\,i}\, s\,\theta_{21\,i} - (\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{65\,i}) + \dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} + ((\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{21\,i} - \dot{\theta}_{76\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})) +$$

$$s\,\theta_{21}\,(c\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot{\theta}_{76\,i}\, s\,\theta_{21\,i} - (\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{65\,i}) + \dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, s\,\theta_{76\,i}) -$$

$$s\,\theta_{43}\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} + ((\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{21\,i} - \dot{\theta}_{76\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}))) -$$

$$s\,\beta_{54\,i}\,(s\,\theta_{21}\,(c\,\theta_{76\,i}\, s\,\theta_{43}\,(\dot{\theta}_{76\,i} - (\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) + c\,\theta_{43}\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + \dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i} +$$

$$(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})) + c\,\theta_{21}\,(c\,\theta_{43}\, c\,\theta_{76\,i}\,(-\dot{\theta}_{76\,i} + (\dot{\theta}_{21} + \dot{\theta}_{43})\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) +$$

$$s\,\theta_{43}\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + \dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i} + (\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})))) +$$

$$s\,\theta_{109\,i}\,(-c\,\theta_{76\,i}\, s(\theta_{21} + \theta_{43})\,(c\,\theta_{21\,i}\,((\dot{\theta}_{21\,i}^{\,2} + \dot{\theta}_{65\,i}^{\,2})\, c\,\theta_{65\,i} + \ddot{\theta}_{65\,i}\, s\,\theta_{65\,i}) + s\,\theta_{21\,i}\,(\ddot{\theta}_{21\,i}\, c\,\theta_{65\,i} - 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\, s\,\theta_{65\,i})) +$$

$$2\,(-\dot{\theta}_{21\,i}\, c\,\theta_{65\,i}\, s\,\theta_{21\,i} - \dot{\theta}_{65\,i}\, c\,\theta_{21\,i}\, s\,\theta_{65\,i})\,((\dot{\theta}_{21} + \dot{\theta}_{43})\, c(\theta_{21} + \theta_{43})\, c\,\theta_{76\,i} - \dot{\theta}_{76\,i}\, s(\theta_{21} + \theta_{43})\, s\,\theta_{76\,i}) +$$

$$c\,\theta_{21\,i}\, c\,\theta_{65\,i}\,(-s(\theta_{21} + \theta_{43})\,((\dot{\theta}_{21}^{\,2} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}^{\,2} + \dot{\theta}_{76\,i}^{\,2})\, c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\, s\,\theta_{76\,i}) +$$

$$c(\theta_{21} + \theta_{43})\,((\ddot{\theta}_{21} + \ddot{\theta}_{43})\, c\,\theta_{76\,i} - 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{76\,i}\, s\,\theta_{76\,i})) -$$

$$c\,\beta_{54\,i}\,(-c\,\theta_{76\,i}\,(c\,\theta_{21}\,((\dot{\theta}_{21}^{\,2} + \dot{\theta}_{43}^{\,2})\, c\,\theta_{43} + \ddot{\theta}_{43}\, s\,\theta_{43}) + s\,\theta_{21}\,(\ddot{\theta}_{21}\, c\,\theta_{43} - 2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\, s\,\theta_{43}))\, s\,\theta_{65\,i} +$$

$$c\,\theta_{76\,i}\, s\,\theta_{21}\,(-c\,\theta_{43}\,(2\,\dot{\theta}_{43}\,\dot{\theta}_{65\,i}\, c\,\theta_{65\,i} + \ddot{\theta}_{43}\, s\,\theta_{65\,i}) + s\,\theta_{43}\,(-\ddot{\theta}_{65\,i}\, c\,\theta_{65\,i} + (\dot{\theta}_{43}^{\,2} + \dot{\theta}_{65\,i}^{\,2})\, s\,\theta_{65\,i})) -$$

$$(s\,\theta_{21}\,(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\,i}\, c\,\theta_{21\,i} + \ddot{\theta}_{21}\, s\,\theta_{21\,i}) + c\,\theta_{21}\,(-\ddot{\theta}_{21\,i}\, c\,\theta_{21\,i} + (\dot{\theta}_{21}^{\,2} + \dot{\theta}_{21\,i}^{\,2})\, s\,\theta_{21\,i}))\, s\,\theta_{43}\, s\,\theta_{76\,i} +$$

$$s\,\theta_{21\,i}\,(-s\,\theta_{21}\,((\dot{\theta}_{21}^{\,2} + \dot{\theta}_{43}^{\,2})\, c\,\theta_{43} + \ddot{\theta}_{43}\, s\,\theta_{43}) + c\,\theta_{21}\,(\ddot{\theta}_{21}\, c\,\theta_{43} - 2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\, s\,\theta_{43}))\, s\,\theta_{76\,i} +$$

$$2\,(\dot{\theta}_{21}\, c\,\theta_{21}\, c\,\theta_{43} - \dot{\theta}_{43}\, s\,\theta_{21}\, s\,\theta_{43})\,(\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + \dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, s\,\theta_{76\,i}) +$$

$$2\,(\dot{\theta}_{21\,i}\, c\,\theta_{21}\, c\,\theta_{21\,i} - \dot{\theta}_{21}\, s\,\theta_{21}\, s\,\theta_{21\,i})\,(\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{43} + \dot{\theta}_{43}\, c\,\theta_{43}\, s\,\theta_{76\,i}) +$$

$$2\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{43} + \dot{\theta}_{43}\, c\,\theta_{43}\, s\,\theta_{65\,i})\,(-\dot{\theta}_{21}\, c\,\theta_{21}\, c\,\theta_{76\,i} + \dot{\theta}_{76\,i}\, s\,\theta_{21}\, s\,\theta_{76\,i}) -$$

$$2\,(\dot{\theta}_{21}\, c\,\theta_{43}\, s\,\theta_{21} + \dot{\theta}_{43}\, c\,\theta_{21}\, s\,\theta_{43})\,(\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} - \dot{\theta}_{76\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i}) +$$

$$s\,\theta_{43}\, s\,\theta_{65\,i}\,(s\,\theta_{21}\,((\dot{\theta}_{21}^{\,2} + \dot{\theta}_{76\,i}^{\,2})\, c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\, s\,\theta_{76\,i}) + c\,\theta_{21}\,(-\ddot{\theta}_{21}\, c\,\theta_{76\,i} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{76\,i}\, s\,\theta_{76\,i})) +$$

$$c\,\theta_{21}\, c\,\theta_{43}\,(-s\,\theta_{65\,i}\,((\dot{\theta}_{65\,i}^{\,2} + \dot{\theta}_{76\,i}^{\,2})\, c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\, s\,\theta_{76\,i}) + c\,\theta_{65\,i}\,(\ddot{\theta}_{65\,i}\, c\,\theta_{76\,i} - 2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\, s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\, s\,\theta_{21}\,(c\,\theta_{21\,i}\,(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\, c\,\theta_{76\,i} + \ddot{\theta}_{21\,i}\, s\,\theta_{76\,i}) + s\,\theta_{21\,i}\,(\ddot{\theta}_{76\,i}\, c\,\theta_{76\,i} - (\dot{\theta}_{21\,i}^{\,2} + \dot{\theta}_{76\,i}^{\,2})\, s\,\theta_{76\,i})) +$$

$$c\,\theta_{21}\, s\,\theta_{21\,i}\,(c\,\theta_{43}\,(2\,\dot{\theta}_{43}\,\dot{\theta}_{76\,i}\, c\,\theta_{76\,i} + \ddot{\theta}_{43}\, s\,\theta_{76\,i}) + s\,\theta_{43}\,(\ddot{\theta}_{76\,i}\, c\,\theta_{76\,i} - (\dot{\theta}_{43}^{\,2} + \dot{\theta}_{76\,i}^{\,2})\, s\,\theta_{76\,i}))) -$$

$$s\,\beta_{54\,i}\,(c\,\theta_{21}\,(-s\,\theta_{43}\,(c\,\theta_{76\,i}\,(-2\,\dot{\theta}_{21}\,\dot{\theta}_{76\,i} - 2\,\dot{\theta}_{43}\,\dot{\theta}_{76\,i} - \ddot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{21\,i} + \dot{\theta}_{21}^{\,2}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + \dot{\theta}_{21\,i}^{\,2}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} +$$

$$2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + \dot{\theta}_{43}^{\,2}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + \dot{\theta}_{65\,i}^{\,2}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + \dot{\theta}_{76\,i}^{\,2}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} -$$

$$c\,\theta_{21\,i}\,(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\, c\,\theta_{65\,i} + \ddot{\theta}_{21\,i}\, s\,\theta_{65\,i})) - (\ddot{\theta}_{21} + \ddot{\theta}_{43} - 2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\, c\,\theta_{65\,i}\, s\,\theta_{21\,i} - 2\,\dot{\theta}_{21\,i}$$

$$\dot{\theta}_{76\,i}\, c\,\theta_{21\,i}\, s\,\theta_{65\,i} - \ddot{\theta}_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}) + c\,\theta_{43}\,(c\,\theta_{76\,i}\,(-\ddot{\theta}_{76\,i} + 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{65\,i}$$

$$c\,\theta_{65\,i}\, s\,\theta_{21\,i} + 2\,\dot{\theta}_{21\,i}\,(\dot{\theta}_{21} + \dot{\theta}_{43})\, c\,\theta_{21\,i}\, s\,\theta_{65\,i} + \ddot{\theta}_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i} + \ddot{\theta}_{43}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) +$$

$$(\dot{\theta}_{21}^{\,2} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}^{\,2} + \dot{\theta}_{76\,i}^{\,2} - 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})) -$$

$$s\theta_{21}\,(c\theta_{43}\,(c\theta_{76\,i}\,(-2\,\dot\theta_{21}\,\dot\theta_{76\,i}-2\,\dot\theta_{43}\,\dot\theta_{76\,i}-\ddot\theta_{65\,i}\,c\theta_{65\,i}\,s\theta_{21\,i}+\dot\theta_{21}{}^{2}\,s\theta_{21\,i}\,s\theta_{65\,i}+$$
$$\dot\theta_{21\,i}{}^{2}\,s\theta_{21\,i}\,s\theta_{65\,i}+2\,\dot\theta_{21}\,\dot\theta_{43}\,s\theta_{21\,i}\,s\theta_{65\,i}+\dot\theta_{43}{}^{2}\,s\theta_{21\,i}\,s\theta_{65\,i}+$$
$$\dot\theta_{65\,i}{}^{2}\,s\theta_{21\,i}\,s\theta_{65\,i}+\dot\theta_{76\,i}{}^{2}\,s\theta_{21\,i}\,s\theta_{65\,i}-c\theta_{21\,i}\,(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,c\theta_{65\,i}+\ddot\theta_{21\,i}\,s\theta_{65\,i}))-$$
$$(\ddot\theta_{21}+\ddot\theta_{43}-2\,\dot\theta_{65\,i}\,\dot\theta_{76\,i}\,c\theta_{65\,i}\,s\theta_{21\,i}-2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,c\theta_{21\,i}\,s\theta_{65\,i}-\ddot\theta_{76\,i}\,s\theta_{21\,i}\,s\theta_{65\,i})\,s\theta_{76\,i})+$$
$$s\theta_{43}\,(c\theta_{76\,i}\,(-\ddot\theta_{76\,i}+2\,(\dot\theta_{21}+\dot\theta_{43})\,\dot\theta_{65\,i}\,c\theta_{65\,i}\,s\theta_{21\,i}+2\,\dot\theta_{21\,i}\,(\dot\theta_{21}+\dot\theta_{43})\,c\theta_{21\,i}\,s\theta_{65\,i}+$$
$$\ddot\theta_{21}\,s\theta_{21\,i}\,s\theta_{65\,i}+\ddot\theta_{43}\,s\theta_{21\,i}\,s\theta_{65\,i})+(\dot\theta_{21}{}^{2}+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^{2}+\dot\theta_{76\,i}{}^{2}-$$
$$2\,(\dot\theta_{21}+\dot\theta_{43})\,\dot\theta_{76\,i}\,s\theta_{21\,i}\,s\theta_{65\,i})\,s\theta_{76\,i})))))+(-\dot\theta_{109\,i}{}^{2}\,c\theta_{109\,i}-\ddot\theta_{109\,i}\,s\theta_{109\,i})$$
$$(c\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+c\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))+$$
$$c(\beta_{1413\,i}-\theta_{54})\,s\theta_{43}\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21}+s\beta_{54\,i}\,(c\theta_{76\,i}\,s\theta_{21}\,s\theta_{21\,i}\,s\theta_{65\,i}-c\theta_{21}\,s\theta_{76\,i})+$$
$$c\beta_{54\,i}\,(c\theta_{21}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21}\,s\theta_{21\,i}\,s\theta_{76\,i}))+$$
$$c\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\theta_{21}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i})+$$
$$c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}-s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))))-$$
$$s\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+c\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))+$$
$$s(\beta_{1413\,i}-\theta_{54})\,(c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{43}\,c\theta_{65\,i}\,c\theta_{76\,i}-c\theta_{43}\,s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i})+$$
$$s\theta_{43}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i}))+s\theta_{21}\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{43}+s\beta_{54\,i}$$
$$(c\theta_{76\,i}\,s\theta_{21\,i}\,s\theta_{43}\,s\theta_{65\,i}-c\theta_{43}\,s\theta_{76\,i})+c\beta_{54\,i}\,(c\theta_{43}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21\,i}\,s\theta_{43}\,s\theta_{76\,i})))))-$$
$$2\,\dot\theta_{109\,i}\,s\theta_{109\,i}\,(-\dot\theta_{1110\,i}\,s\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+c\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))+$$
$$c(\beta_{1413\,i}-\theta_{54})\,s\theta_{43}\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21}+s\beta_{54\,i}\,(c\theta_{76\,i}\,s\theta_{21}\,s\theta_{21\,i}\,s\theta_{65\,i}-c\theta_{21}\,s\theta_{76\,i})+$$
$$c\beta_{54\,i}\,(c\theta_{21}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21}\,s\theta_{21\,i}\,s\theta_{76\,i}))+$$
$$c\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\theta_{21}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i})+$$
$$c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}-s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))))-$$
$$\dot\theta_{1110\,i}\,c\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+c\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))+$$
$$s(\beta_{1413\,i}-\theta_{54})\,(c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{43}\,c\theta_{65\,i}\,c\theta_{76\,i}-c\theta_{43}\,s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i})+$$
$$s\theta_{43}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i}))+s\theta_{21}\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{43}+s\beta_{54\,i}$$
$$(c\theta_{76\,i}\,s\theta_{21\,i}\,s\theta_{43}\,s\theta_{65\,i}-c\theta_{43}\,s\theta_{76\,i})+c\beta_{54\,i}\,(c\theta_{43}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21\,i}\,s\theta_{43}\,s\theta_{76\,i}))))+$$
$$c\theta_{1110\,i}\,(\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+c\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))+$$
$$\dot\theta_{43}\,c\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21}+s\beta_{54\,i}\,(c\theta_{76\,i}\,s\theta_{21}\,s\theta_{21\,i}\,s\theta_{65\,i}-c\theta_{21}\,s\theta_{76\,i})+$$
$$c\beta_{54\,i}\,(c\theta_{21}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21}\,s\theta_{21\,i}\,s\theta_{76\,i}))+\dot\theta_{54}\,s\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(-c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}\,s\theta_{21}+$$
$$s\beta_{54\,i}\,(c\theta_{76\,i}\,s\theta_{21}\,s\theta_{21\,i}\,s\theta_{65\,i}-c\theta_{21}\,s\theta_{76\,i})+c\beta_{54\,i}\,(c\theta_{21}\,c\theta_{76\,i}\,s\theta_{65\,i}+s\theta_{21}\,s\theta_{21\,i}\,s\theta_{76\,i}))-$$
$$s(\beta_{1413\,i}-\theta_{54})\,(-s\theta_{21\,i}\,(c\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\beta_{54\,i})\,s\theta_{65\,i}+\dot\theta_{76\,i}\,c\theta_{65\,i}\,s\theta_{76\,i})+c\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\theta_{21\,i}\,c\theta_{76\,i}-$$
$$\dot\theta_{21\,i}\,s\theta_{21\,i}\,s\theta_{76\,i})+c\theta_{21\,i}\,(c\theta_{65\,i}\,c\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\beta_{54\,i})-\dot\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}\,s\theta_{76\,i}))-$$
$$\dot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\theta_{43}\,(s\theta_{21}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i})+$$
$$c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}-s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i})))+$$
$$\dot\theta_{54}\,c\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(s\theta_{21}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i})+$$
$$c\theta_{21}\,(c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}-s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i})))+$$
$$c\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(\dot\theta_{21}\,c\theta_{21}\,(c\beta_{54\,i}\,c\theta_{76\,i}\,s\theta_{65\,i}-s\beta_{54\,i}\,s\theta_{76\,i})-$$
$$\dot\theta_{21}\,s\theta_{21}\,(c\theta_{21\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}-s\theta_{21\,i}\,(c\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{65\,i}+c\beta_{54\,i}\,s\theta_{76\,i}))-$$
$$c\theta_{21}\,(c\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\theta_{76\,i}\,s\theta_{21\,i}+\dot\theta_{21\,i}\,c\theta_{21\,i}\,s\theta_{76\,i})+c\theta_{65\,i}\,(c\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\beta_{54\,i})\,s\theta_{21\,i}+$$
$$\dot\theta_{76\,i}\,c\theta_{21\,i}\,s\theta_{76\,i})+s\theta_{65\,i}\,(c\theta_{21\,i}\,c\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\beta_{54\,i})-\dot\theta_{76\,i}\,s\beta_{54\,i}\,s\theta_{21\,i}\,s\theta_{76\,i}))-$$
$$s\theta_{21}\,(\dot\theta_{76\,i}\,c\theta_{76\,i}\,s\beta_{54\,i}+c\beta_{54\,i}\,(-\dot\theta_{65\,i}\,c\theta_{65\,i}\,c\theta_{76\,i}+\dot\theta_{76\,i}\,s\theta_{65\,i}\,s\theta_{76\,i})))+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\left(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{21}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}+\right.$$

$$\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{21}\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)\right)-$$

$$\mathrm{s}\,\theta_{1110\,i}\left(\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\right.$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}-\right.\right.$$

$$\left.\left.\dot\theta_{21\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)-$$

$$\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}+$$

$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})+\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})))+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+\right.$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}+$$

$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})+\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{c}\,\theta_{21}\left(-\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}\left(\dot\theta_{43}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)-\right.$$

$$\mathrm{c}\,\theta_{43}\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{43}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}+\right.$$

$$\left.\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{c}\,\beta_{54\,i}\left(-\mathrm{c}\,\theta_{43}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{43}\,\mathrm{s}\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\mathrm{s}\,\theta_{43}\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(\dot\theta_{43}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(-\dot\theta_{43}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}+\right.$$

$$\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}+\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{43}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{43}\,\mathrm{s}\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{43}\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(\dot\theta_{43}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{s}\,\beta_{54\,i}\left(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\left(-\dot\theta_{76\,i}+\dot\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{43}\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\left(\dot\theta_{43}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)\right)\right)\right)+$$

$$\mathrm{c}\,\theta_{109\,i}\left(\left(-\dot\theta_{1110\,i}{}^{2}\,\mathrm{c}\,\theta_{1110\,i}-\ddot\theta_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\right)(-\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\right.$$

$$\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-$$

$$\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))))+\left(-\ddot\theta_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}+\dot\theta_{1110\,i}{}^{2}\,\mathrm{s}\,\theta_{1110\,i}\right)$$

$$(\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\beta_{54\,i}$$

$$(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})+\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}))))-$$

$$2\,\dot\theta_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\left(\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\right.$$

$$\dot\theta_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\dot\theta_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}))-$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$
\begin{aligned}
&\quad c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\\
&\quad c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\Big)-\\
&\dot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+\\
&\quad c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+\\
&\dot\theta_{54}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+\\
&\quad c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+\\
&c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\left(\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})-\right.\\
&\quad \dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))-\\
&\quad c\,\theta_{21}\left(c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\right.\right.\\
&\qquad\qquad \left.\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+s\,\theta_{65\,i}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\Big)-\\
&\quad s\,\theta_{21}\left(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\left(-\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot\theta_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\Big)+\\
&c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\left(-c\,\theta_{21}\,c\,\theta_{76\,i}\left(\dot\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)+\right.\\
&\quad s\,\theta_{21}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{21}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}+\right.\\
&\qquad c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\Big)+\\
&\quad c\,\beta_{54\,i}\left(s\,\theta_{21}\left(c\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-\dot\theta_{21}\,s\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad c\,\theta_{21}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot\theta_{21}\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\Big)\Big)\Big)+c\,\theta_{1110\,i}\\
&\Big(\left(\ddot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}{}^{2}\,s(\beta_{1413\,i}-\theta_{54})\right)(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad\Big(-s\,\theta_{43}\left(\left(\dot\theta_{43}{}^{2}+\dot\theta_{54}{}^{2}\right)c(\beta_{1413\,i}-\theta_{54})-\ddot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\right)+\\
&\quad c\,\theta_{43}\left(\ddot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})+2\,\dot\theta_{43}\,\dot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\right)\Big)(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+\\
&\quad s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+\\
&2\,\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})\left(-s\,\theta_{21\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\quad c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\\
&\quad c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\Big)+\\
&\Big(-c\,\theta_{43}\left(\left(\dot\theta_{43}{}^{2}+\dot\theta_{54}{}^{2}\right)c(\beta_{1413\,i}-\theta_{54})-\ddot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\right)-\\
&\quad s\,\theta_{43}\left(\ddot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})+2\,\dot\theta_{43}\,\dot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\right)\Big)(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+\\
&\quad c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))-s(\beta_{1413\,i}-\theta_{54})\\
&\quad\Big(c\,\beta_{54\,i}\left(-s\,\theta_{21\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,c\,\theta_{76\,i}+\ddot\theta_{21\,i}\,s\,\theta_{76\,i}\right)+c\,\theta_{21\,i}\left(\ddot\theta_{76\,i}\,c\,\theta_{76\,i}-\left(\dot\theta_{21\,i}{}^{2}+\dot\theta_{76\,i}{}^{2}\right)s\,\theta_{76\,i}\right)\right)-\\
&\quad s\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}+\dot\theta_{76\,i}{}^{2}+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,s\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\Big)+\\
&\quad c\,\theta_{21\,i}\left(-s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}+\dot\theta_{76\,i}{}^{2}\right)s\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,s\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\Big)\Big)+\\
&2\left(-\dot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}+\dot\theta_{54}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\right)\left(\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})-\right.\\
&\quad \dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))-\\
&\quad c\,\theta_{21}\left(c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\right.\right.\\
&\qquad\qquad \left.\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+s\,\theta_{65\,i}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\Big)-\\
&\quad s\,\theta_{21}\left(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\left(-\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot\theta_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\Big)+\\
&2\left(\dot\theta_{43}\,c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\right)
\end{aligned}
$$

$$\left(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot\theta_{21}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}+\right.$$

$$\left.\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{21}\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)+\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}$$

$$\left(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2\right)\mathrm{c}\,\theta_{65\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21\,i}\left(\ddot\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$2\left(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}\right)-$$

$$\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\left(-\mathrm{s}\,\theta_{21}\left(\left(\dot\theta_{21}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{c}\,\theta_{76\,i}+\ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\theta_{21}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{76\,i}-2\,\dot\theta_{21}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{s}\,\beta_{54\,i}\left(2\,\dot\theta_{21}\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}+\ddot\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\ddot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}-\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\left(\dot\theta_{21}{}^2\,\mathrm{c}\,\theta_{21}+\ddot\theta_{21}\,\mathrm{s}\,\theta_{21}\right)\mathrm{s}\,\theta_{76\,i}+\mathrm{c}\,\theta_{21}\left(-\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}+\dot\theta_{76\,i}{}^2\,\mathrm{s}\,\theta_{76\,i}\right)+$$

$$2\left(\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i}+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}\right)+$$

$$\left.\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\left(-\mathrm{s}\,\theta_{21}\left(\left(\dot\theta_{21}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{c}\,\theta_{76\,i}+\ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\theta_{21}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{76\,i}-2\,\dot\theta_{21}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\right)+$$

$$\mathrm{c}\,\beta_{54\,i}\left(2\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{21}+\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\right)+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\left(\ddot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}-\dot\theta_{65\,i}{}^2\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left(\mathrm{c}\,\theta_{21}\left(2\,\dot\theta_{21}\,\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}+\ddot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\right)+\mathrm{s}\,\theta_{21}\left(\ddot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}-\left(\dot\theta_{21}{}^2+\dot\theta_{21\,i}{}^2\right)\mathrm{s}\,\theta_{21\,i}\right)\right)\mathrm{s}\,\theta_{76\,i}+$$

$$\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\left(\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}-\dot\theta_{76\,i}{}^2\,\mathrm{s}\,\theta_{76\,i}\right)-2\,\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}\right)-$$

$$\left.\left.\mathrm{s}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{21}\left(\left(\dot\theta_{21}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{c}\,\theta_{76\,i}+\ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{s}\,\theta_{21}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{76\,i}-2\,\dot\theta_{21}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\right)\right)+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{21}-\dot\theta_{21}{}^2\,\mathrm{s}\,\theta_{21}\right)(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})-\right.$$

$$\left(\dot\theta_{21}{}^2\,\mathrm{c}\,\theta_{21}+\ddot\theta_{21}\,\mathrm{s}\,\theta_{21}\right)(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$2\,\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\right.\right.$$

$$\left.\left.\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{s}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)-$$

$$2\,\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\left(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\left(-\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}+\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{s}\,\beta_{54\,i}\left(-\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}+\dot\theta_{76\,i}{}^2\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\mathrm{c}\,\beta_{54\,i}\left(-\mathrm{s}\,\theta_{65\,i}\left(\left(\dot\theta_{65\,i}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{c}\,\theta_{76\,i}+\ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{c}\,\theta_{65\,i}\left(\ddot\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-2\,\dot\theta_{65\,i}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\right)+$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\beta_{54\,i}\left(-\mathrm{c}\,\theta_{21\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}+\ddot\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\mathrm{s}\,\theta_{21\,i}\left(-\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(\dot\theta_{21\,i}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{s}\,\theta_{76\,i}\right)\right)-\right.$$

$$\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2+\dot\theta_{76\,i}{}^2+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{21\,i}\left(\mathrm{s}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2+\dot\theta_{76\,i}{}^2\right)\mathrm{s}\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}\right)-\right.$$

$$\left.\left.\left.\mathrm{c}\,\theta_{65\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)\right)-$$

$$2\,\dot\theta_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\left(\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+\right.$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}+\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\mathrm{c}\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}-\dot\theta_{21\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+$$

$$\left.\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)-$$

$$\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$
$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+$$
$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))+$$
$$s(\beta_{1413\,i}-\theta_{54})\left(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+\right.$$
$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\dot{\theta}_{21}\,c\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+$$
$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))+$$
$$c\,\theta_{21}\left(-c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot{\theta}_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}-\dot{\theta}_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)-\right.$$
$$c\,\theta_{43}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{43}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}+\right.$$
$$\left.c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\right)+$$
$$c\,\beta_{54\,i}\left(-c\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}-\dot{\theta}_{43}\,s\,\theta_{65\,i}\right)+\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.\left.s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot{\theta}_{43}\,s\,\theta_{21\,i}-\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)+$$
$$s\,\theta_{21}\left(-\dot{\theta}_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}+\dot{\theta}_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}+\right.$$
$$\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\left(s\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}-\dot{\theta}_{43}\,s\,\theta_{65\,i}\right)+\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.c\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot{\theta}_{43}\,s\,\theta_{21\,i}-\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)+$$
$$s\,\beta_{54\,i}\left(c\,\theta_{43}\,c\,\theta_{76\,i}\left(-\dot{\theta}_{76\,i}+\dot{\theta}_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)+s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\right.\right.$$
$$\left.\left.\left.\left.\left.\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\left(\dot{\theta}_{43}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)\right)\right)-s\,\theta_{1110\,i}$$
$$\left(\left(-\dot{\theta}_{54}{}^{2}\,c(\beta_{1413\,i}-\theta_{54})+\ddot{\theta}_{54}\,s(\beta_{1413\,i}-\theta_{54})\right)(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\right.$$
$$2\,\dot{\theta}_{54}\,s(\beta_{1413\,i}-\theta_{54})\left(-s\,\theta_{21\,i}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$c\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot{\theta}_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+$$
$$\left.c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)+c(\beta_{1413\,i}-\theta_{54})$$
$$\left(c\,\beta_{54\,i}\left(-s\,\theta_{21\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}+\ddot{\theta}_{21\,i}\,s\,\theta_{76\,i}\right)+c\,\theta_{21\,i}\left(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}-\left(\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}\right)s\,\theta_{76\,i}\right)\right)-\right.$$
$$s\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)+\ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)-2\,\dot{\theta}_{76\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\right)+$$
$$c\,\theta_{21\,i}\left(-s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+\left(\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}\right)s\,\beta_{54\,i}\right)+\ddot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.\left.c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)-2\,\dot{\theta}_{76\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\right)\right)+$$
$$\left(-\ddot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})-\dot{\theta}_{54}{}^{2}\,s(\beta_{1413\,i}-\theta_{54})\right)(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-$$
$$c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$
$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+$$
$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))-$$
$$2\,\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\left(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+\right.$$
$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\dot{\theta}_{21}\,c\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+$$
$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))+$$
$$c\,\theta_{21}\left(-c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot{\theta}_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}-\dot{\theta}_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)-\right.$$
$$c\,\theta_{43}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{43}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}+\right.$$
$$\left.c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\right)+$$
$$c\,\beta_{54\,i}\left(-c\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}-\dot{\theta}_{43}\,s\,\theta_{65\,i}\right)+\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.$$
$$\left.\left.s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot{\theta}_{43}\,s\,\theta_{21\,i}-\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)+$$
$$s\,\theta_{21}\left(-\dot{\theta}_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}+\dot{\theta}_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}+\right.$$

$$\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\,(s\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i} - \dot\theta_{43}\,s\,\theta_{65\,i}) + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + (\dot\theta_{43}\,s\,\theta_{21\,i} - \dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) +$$

$$s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,(-\dot\theta_{76\,i} + \dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) + s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + (\dot\theta_{43} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))) + s(\beta_{1413\,i} - \theta_{54})$$

$$((-\dot\theta_{21}^{\,2}\,c\,\theta_{21} - \ddot\theta_{21}\,s\,\theta_{21})\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i})) + (\ddot\theta_{21}\,c\,\theta_{21} - \dot\theta_{21}^{\,2}\,s\,\theta_{21})\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} +$$

$$s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) + c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) -$$

$$2\,\dot\theta_{21}\,s\,\theta_{21}\,(-c\,\theta_{76\,i}\,s\,\theta_{43}\,(\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i} + s\,\beta_{54\,i}\,(\dot\theta_{76\,i} - \dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})) -$$

$$c\,\theta_{43}\,(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i} + s\,\beta_{54\,i}\,(\dot\theta_{43} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i} +$$

$$c\,\theta_{65\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i} + \dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\beta_{54\,i}\,(-c\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i} - \dot\theta_{43}\,s\,\theta_{65\,i}) + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + (\dot\theta_{43}\,s\,\theta_{21\,i} - \dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) + 2\,\dot\theta_{21}\,c\,\theta_{21}$$

$$(-\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \dot\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43} + \dot\theta_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} +$$

$$\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\,(s\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i} - \dot\theta_{43}\,s\,\theta_{65\,i}) + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + (\dot\theta_{43}\,s\,\theta_{21\,i} - \dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) +$$

$$s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,(-\dot\theta_{76\,i} + \dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) + s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + (\dot\theta_{43} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) + s\,\theta_{21}$$

$$(c\,\theta_{76\,i}\,s\,\theta_{43}\,(c\,\theta_{21\,i}\,((\dot\theta_{21\,i}^{\,2} + \dot\theta_{65\,i}^{\,2})\,c\,\theta_{65\,i} + \ddot\theta_{65\,i}\,s\,\theta_{65\,i}) + s\,\theta_{21\,i}\,(\ddot\theta_{21\,i}\,c\,\theta_{65\,i} - 2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,s\,\theta_{65\,i})) +$$

$$2\,(\dot\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{21\,i} + \dot\theta_{65\,i}\,c\,\theta_{21\,i}\,s\,\theta_{65\,i})\,(\dot\theta_{43}\,c\,\theta_{43}\,c\,\theta_{76\,i} - \dot\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,(-s\,\theta_{43}\,((\dot\theta_{43}^{\,2} + \dot\theta_{76\,i}^{\,2})\,c\,\theta_{76\,i} + \ddot\theta_{76\,i}\,s\,\theta_{76\,i}) + c\,\theta_{43}\,(\ddot\theta_{43}\,c\,\theta_{76\,i} - 2\,\dot\theta_{43}\,\dot\theta_{76\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\beta_{54\,i}\,(2\,\dot\theta_{43}\,\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,(c\,\theta_{43}\,(2\,\dot\theta_{43}\,\dot\theta_{65\,i}\,c\,\theta_{65\,i} + \ddot\theta_{43}\,s\,\theta_{65\,i}) +$$

$$s\,\theta_{43}\,(\ddot\theta_{65\,i}\,c\,\theta_{65\,i} - (\dot\theta_{43}^{\,2} + \dot\theta_{65\,i}^{\,2})\,s\,\theta_{65\,i})) + (\dot\theta_{43}^{\,2}\,c\,\theta_{43} + \ddot\theta_{43}\,s\,\theta_{43})\,s\,\theta_{76\,i} +$$

$$c\,\theta_{43}\,(-\ddot\theta_{76\,i}\,c\,\theta_{76\,i} + \dot\theta_{76\,i}^{\,2}\,s\,\theta_{76\,i}) + 2\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43} + \dot\theta_{43}\,c\,\theta_{43}\,s\,\theta_{65\,i})$$

$$(\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}) + s\,\theta_{43}\,s\,\theta_{65\,i}\,(-s\,\theta_{21\,i}\,((\dot\theta_{21\,i}^{\,2} + \dot\theta_{76\,i}^{\,2})\,c\,\theta_{76\,i} +$$

$$\ddot\theta_{76\,i}\,s\,\theta_{76\,i}) + c\,\theta_{21\,i}\,(\ddot\theta_{21\,i}\,c\,\theta_{76\,i} - 2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,s\,\theta_{76\,i}))) + c\,\beta_{54\,i}$$

$$(2\,\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{43}\,c\,\theta_{43}\,s\,\theta_{21\,i} + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{43}) + c\,\theta_{43}\,c\,\theta_{76\,i}\,(\ddot\theta_{65\,i}\,c\,\theta_{65\,i} - \dot\theta_{65\,i}^{\,2}\,s\,\theta_{65\,i}) +$$

$$(c\,\theta_{21\,i}\,(2\,\dot\theta_{21\,i}\,\dot\theta_{43}\,c\,\theta_{43} + \ddot\theta_{21\,i}\,s\,\theta_{43}) + s\,\theta_{21\,i}\,(\ddot\theta_{43}\,c\,\theta_{43} - (\dot\theta_{21\,i}^{\,2} + \dot\theta_{43}^{\,2})\,s\,\theta_{43}))\,s\,\theta_{76\,i} +$$

$$s\,\theta_{21\,i}\,s\,\theta_{43}\,(\ddot\theta_{76\,i}\,c\,\theta_{76\,i} - \dot\theta_{76\,i}^{\,2}\,s\,\theta_{76\,i}) - 2\,\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,(\dot\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{43} + \dot\theta_{76\,i}\,c\,\theta_{43}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{65\,i}\,(c\,\theta_{43}\,((\dot\theta_{43}^{\,2} + \dot\theta_{76\,i}^{\,2})\,c\,\theta_{76\,i} + \ddot\theta_{76\,i}\,s\,\theta_{76\,i}) + s\,\theta_{43}\,(\ddot\theta_{43}\,c\,\theta_{76\,i} - 2\,\dot\theta_{43}\,\dot\theta_{76\,i}\,s\,\theta_{76\,i}))))) +$$

$$c\,\theta_{21}\,(-c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,((\dot\theta_{21\,i}^{\,2} + \dot\theta_{43}^{\,2})\,c\,\theta_{43} + \ddot\theta_{43}\,s\,\theta_{43}) + s\,\theta_{21\,i}\,(\ddot\theta_{21\,i}\,c\,\theta_{43} - 2\,\dot\theta_{21\,i}\,\dot\theta_{43}\,s\,\theta_{43})) +$$

$$s\,\theta_{21\,i}\,(\dot\theta_{43}^{\,2}\,c\,\theta_{43} + \ddot\theta_{43}\,s\,\theta_{43})\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$2\,(\dot\theta_{21\,i}\,c\,\theta_{43}\,s\,\theta_{21\,i} + \dot\theta_{43}\,c\,\theta_{21\,i}\,s\,\theta_{43})\,(\dot\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \dot\theta_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}) +$$

$$(\ddot\theta_{43}\,c\,\theta_{43} - \dot\theta_{43}^{\,2}\,s\,\theta_{43})\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i}) - c\,\theta_{21\,i}\,c\,\theta_{43}$$

$$(c\,\theta_{65\,i}\,((\dot\theta_{65\,i}^{\,2} + \dot\theta_{76\,i}^{\,2})\,c\,\theta_{76\,i} + \ddot\theta_{76\,i}\,s\,\theta_{76\,i}) + s\,\theta_{65\,i}\,(\ddot\theta_{65\,i}\,c\,\theta_{76\,i} - 2\,\dot\theta_{65\,i}\,\dot\theta_{76\,i}\,s\,\theta_{76\,i})) -$$

$$2\,\dot{\theta}_{43}\,c\,\theta_{43}\left(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\left(-\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)+$$

$$s\,\theta_{43}\left(s\,\beta_{54\,i}\left(-\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{76\,i}{}^{2}\,s\,\theta_{76\,i}\right)+c\,\beta_{54\,i}\left(-s\,\theta_{65\,i}\left(\left(\dot{\theta}_{65\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}\right)c\,\theta_{76\,i}+\ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)+\right.\right.$$

$$\left.\left.c\,\theta_{65\,i}\left(\ddot{\theta}_{65\,i}\,c\,\theta_{76\,i}-2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)\right)\right)\Big)+$$

$$2\,\dot{\theta}_{43}\,s\,\theta_{43}\left(\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{21\,i}\right.$$

$$\left.\left(\dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,c\,\theta_{76\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\right)\Big)-c\,\theta_{43}$$

$$\left(c\,\beta_{54\,i}\left(c\,\theta_{21\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}+\ddot{\theta}_{21\,i}\,s\,\theta_{76\,i}\right)+s\,\theta_{21\,i}\left(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}-\left(\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}\right)s\,\theta_{76\,i}\right)\right)-\right.$$

$$s\,\beta_{54\,i}\left(-c\,\theta_{21\,i}\left(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+s\,\theta_{65\,i}\left(\ddot{\theta}_{21\,i}\,c\,\theta_{76\,i}-2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)\right)+\right.$$

$$s\,\theta_{21\,i}\left(s\,\theta_{65\,i}\left(\left(\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2}+\dot{\theta}_{76\,i}{}^{2}\right)c\,\theta_{76\,i}+\ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\left.c\,\theta_{65\,i}\left(-\ddot{\theta}_{65\,i}\,c\,\theta_{76\,i}+2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i}\right)\right)\right)\right)\Big)\Big)\Big)\Big)$$

$$\ddot{f}_{21}=\left(\ddot{\theta}_{109\,i}\,c\,\theta_{109\,i}-\dot{\theta}_{109\,i}{}^{2}\,s\,\theta_{109\,i}\right)(-s\,\theta_{43}$$

$$(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})+$$

$$c\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))+2\,\dot{\theta}_{109\,i}\,c\,\theta_{109\,i}\left(-\dot{\theta}_{43}\,c\,\theta_{43}\right.$$

$$(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$

$$\dot{\theta}_{43}\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))+$$

$$c\,\theta_{43}\left(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-\right.$$

$$s\,\theta_{21}\left(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}-\right.$$

$$\left.\left(c\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+c\,\theta_{21\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)-$$

$$\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$

$$\left.c\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\right)-s\,\theta_{43}$$

$$\left(-s\,\theta_{21}\left(\dot{\theta}_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)s\,\theta_{76\,i}+c\,\theta_{21}\left(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\left(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)-\right.\right.$$

$$\left.c\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}\,s\,\theta_{76\,i}+c\,\theta_{21\,i}\left(\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)+$$

$$c\,\beta_{54\,i}\left(s\,\theta_{21}\left(c\,\theta_{76\,i}\left(-\dot{\theta}_{21}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\left.c\,\theta_{21}\left(\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+\left(-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{21}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)\right)+\left(-\dot{\theta}_{109\,i}{}^{2}\,c\,\theta_{109\,i}-\ddot{\theta}_{109\,i}\,s\,\theta_{109\,i}\right)$$

$$(c\,\theta_{1110\,i}\,(s(\beta_{1413\,i}-\theta_{54})\,(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})-$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))-$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})-$$

$$c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$

$$s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))+$$

$$s\,\theta_{109\,i}\left(\left(-\ddot{\theta}_{43}\,c\,\theta_{43}+\dot{\theta}_{43}{}^{2}\,s\,\theta_{43}\right)(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+\right.$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{21}-\dot{\theta}_{21}^{\;2}\,\mathrm{s}\,\theta_{21})+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(-\mathrm{s}\,\theta_{21}\,(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-(\dot{\theta}_{21}^{\;2}+\dot{\theta}_{21\,\mathrm{i}}^{\;2})\,\mathrm{s}\,\theta_{21\,\mathrm{i}}))))+$$

$$2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$2\,\dot{\theta}_{76\,\mathrm{i}}\,(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,(2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+\mathrm{s}\,\theta_{21}\,(\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-(\dot{\theta}_{21}^{\;2}+\dot{\theta}_{65\,\mathrm{i}}^{\;2})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))+$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})+\ddot{\theta}_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+2\,\dot{\theta}_{21}\,(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21}^{\;2}+\dot{\theta}_{21\,\mathrm{i}}^{\;2}+\dot{\theta}_{65\,\mathrm{i}}^{\;2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})+(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})-(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+(\dot{\theta}_{21}^{\;2}+\dot{\theta}_{21\,\mathrm{i}}^{\;2}+\dot{\theta}_{65\,\mathrm{i}}^{\;2})\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})))$$

$$\mathrm{s}\,\theta_{76\,\mathrm{i}}-(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})\,(\dot{\theta}_{76\,\mathrm{i}}^{\;2}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\dot{\theta}_{76\,\mathrm{i}}^{\;2}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))-$$

$$(\dot{\theta}_{43}^{\;2}\,\mathrm{c}\,\theta_{43}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{43})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))-$$

$$2\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}-$$

$$(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))-$$

$$2\,\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,(-\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{76\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{21}-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))+$$

$$\mathrm{c}\,\theta_{43}\,(-(\dot{\theta}_{21}^{\;2}\,\mathrm{c}\,\theta_{21}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}$$

$$(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}-$$

$$(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{21}+\dot{\theta}_{21}^{\;2}\,\mathrm{s}\,\theta_{21})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$2\,\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{c}\,\theta_{21}\,(-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\dot{\theta}_{76\,\mathrm{i}}^{\;2}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(2\,\dot{\theta}_{65\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{s}\,\theta_{65\,\mathrm{i}}\,(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\dot{\theta}_{65\,\mathrm{i}}^{\;2}+\dot{\theta}_{76\,\mathrm{i}}^{\;2})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,((\dot{\theta}_{21\,\mathrm{i}}^{\;2}+\dot{\theta}_{76\,\mathrm{i}}^{\;2})\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(-\mathrm{s}\,\theta_{65\,\mathrm{i}}\,(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})+(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\dot{\theta}_{21\,\mathrm{i}}^{\;2}+\dot{\theta}_{65\,\mathrm{i}}^{\;2}+\dot{\theta}_{76\,\mathrm{i}}^{\;2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\big(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\Big)+$$

$$\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\Big(-\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\Big(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\Big(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\Big)\Big)\Big)\Big)\Big)-$$

$$2\,\dot{\theta}_{109\,\mathrm{i}}\,\mathrm{s}\,\theta_{109\,\mathrm{i}}\,\Big(-\dot{\theta}_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))-$$

$$\dot{\theta}_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})))+$$

$$\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\big(\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))-$$

$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{s}\,\theta_{21}$$

$$\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}-$$

$$\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)\big)-$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\mathrm{s}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)\big)\big)-$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\big(\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)+$$

$$\dot{\theta}_{54}\, c(\beta_{1413\,i}-\theta_{54})\, s\,\theta_{43}\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}))-$$
$$\dot{\theta}_{43}\, c\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}))-$$
$$s\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, \big(-\dot{\theta}_{21}\, s\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-s\,\theta_{21}$$
$$\big(\dot{\theta}_{21\,i}\, c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\, c\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{76\,i}-$$
$$\big(c\,\theta_{65\,i}\, \big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, \big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{65\,i}\big)\, s\,\theta_{76\,i}\big)-$$
$$\dot{\theta}_{21}\, c\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})+$$
$$c\,\theta_{21}\, \big(-\dot{\theta}_{76\,i}\, s\,\beta_{54\,i}\, s\,\theta_{76\,i}+c\,\beta_{54\,i}\, \big(\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}\big)\big)\big)-$$
$$c\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, \big(-s\,\theta_{21}\, \big(\dot{\theta}_{21}\, c\,\theta_{21\,i}\, c\,\theta_{65\,i}+s\,\beta_{54\,i}\, \big(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}\big)\big)\, s\,\theta_{76\,i}+$$
$$c\,\theta_{21}\, \big(c\,\theta_{76\,i}\, s\,\beta_{54\,i}\, \big(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}\big)-c\,\theta_{65\,i}\, \big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{21\,i}\, s\,\theta_{76\,i}+$$
$$c\,\theta_{21\,i}\, \big(\dot{\theta}_{76\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i}-\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{65\,i}\, s\,\theta_{76\,i}\big)\big)+$$
$$c\,\beta_{54\,i}\, \big(s\,\theta_{21}\, \big(c\,\theta_{76\,i}\, \big(-\dot{\theta}_{21}\, s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\, s\,\theta_{65\,i}\big)+\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}\big)+$$
$$c\,\theta_{21}\, \big(\dot{\theta}_{21\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}+\big(-\dot{\theta}_{76\,i}\, s\,\theta_{21\,i}+\dot{\theta}_{21}\, s\,\theta_{65\,i}\big)\, s\,\theta_{76\,i}\big)\big)\big)\big)+$$
$$c\,\theta_{109\,i}\, \big(\big(-\dot{\theta}_{1110\,i}^{\;2}\, c\,\theta_{1110\,i}-\ddot{\theta}_{1110\,i}\, s\,\theta_{1110\,i}\big)\, (s(\beta_{1413\,i}-\theta_{54})\, (-c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}+(c\,\theta_{65\,i}\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, s\,\beta_{54\,i}\, s\,\theta_{65\,i})$$
$$s\,\theta_{76\,i})-c\,\theta_{43}\, c(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21}+c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i}+c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i})-$$
$$c(\beta_{1413\,i}-\theta_{54})\, s\,\theta_{43}\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+$$
$$(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})))+\big(-\ddot{\theta}_{1110\,i}\, c\,\theta_{1110\,i}+\dot{\theta}_{1110\,i}^{\;2}\, s\,\theta_{1110\,i}\big)$$
$$(c(\beta_{1413\,i}-\theta_{54})\, (c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}-(c\,\theta_{65\,i}\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, s\,\beta_{54\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})-$$
$$c\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21}+c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i}+c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i})-$$
$$s\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})))-$$
$$2\,\dot{\theta}_{1110\,i}\, s\,\theta_{1110\,i}\, \big(\dot{\theta}_{54}\, c(\beta_{1413\,i}-\theta_{54})\, (c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}-(c\,\theta_{65\,i}\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, s\,\beta_{54\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})+$$
$$s(\beta_{1413\,i}-\theta_{54})\, \big(\dot{\theta}_{21\,i}\, c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, (c\,\theta_{65\,i}\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, s\,\beta_{54\,i}\, s\,\theta_{65\,i})+$$
$$\dot{\theta}_{76\,i}\, c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, s\,\theta_{76\,i}+\big(c\,\theta_{21\,i}\, c\,\theta_{65\,i}\, \big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\, s\,\beta_{54\,i}\big)-\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}\big)\, s\,\theta_{76\,i}\big)+$$
$$\dot{\theta}_{43}\, c(\beta_{1413\,i}-\theta_{54})\, s\,\theta_{43}\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21}+c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i}+c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i})-$$
$$\dot{\theta}_{54}\, c\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21}+c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i}+c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i})-$$
$$\dot{\theta}_{43}\, c\,\theta_{43}\, c(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}))-$$
$$\dot{\theta}_{54}\, s\,\theta_{43}\, s(\beta_{1413\,i}-\theta_{54})\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}))-$$
$$c(\beta_{1413\,i}-\theta_{54})\, s\,\theta_{43}\, \big(-\dot{\theta}_{21}\, s\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}+c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})-s\,\theta_{21}$$
$$\big(\dot{\theta}_{21\,i}\, c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\, c\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{76\,i}-$$
$$\big(c\,\theta_{65\,i}\, \big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{21\,i}+c\,\theta_{21\,i}\, \big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\, s\,\beta_{54\,i}\big)\, s\,\theta_{65\,i}\big)\, s\,\theta_{76\,i}\big)-$$
$$\dot{\theta}_{21}\, c\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i}+(c\,\theta_{21\,i}\, c\,\theta_{65\,i}-s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})+$$
$$c\,\theta_{21}\, \big(-\dot{\theta}_{76\,i}\, s\,\beta_{54\,i}\, s\,\theta_{76\,i}+c\,\beta_{54\,i}\, \big(\dot{\theta}_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}\big)\big)\big)-$$
$$c\,\theta_{43}\, c(\beta_{1413\,i}-\theta_{54})\, \big(-s\,\theta_{21}\, \big(\dot{\theta}_{21}\, c\,\theta_{21\,i}\, c\,\theta_{65\,i}+s\,\beta_{54\,i}\, \big(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}\big)\big)\, s\,\theta_{76\,i}+$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,(\dot\theta_{21} - \dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) -$$

$$2\,\dot\theta_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\,(\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\dot\theta_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\dot\theta_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{65\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\dot\theta_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) - \mathrm{s}\,\theta_{21}$$

$$(\dot\theta_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot\theta_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} -$$

$$(\mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(-\dot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}))) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{s}\,\theta_{21}\,(\dot\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot\theta_{76\,i} - \dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,(\dot\theta_{21} - \dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) + \mathrm{c}\,\theta_{1110\,i}$$

$$((\ddot\theta_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54}) + \dot\theta_{54}^{\,2}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54}))\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$2\,\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\dot\theta_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}) +$$

$$\dot\theta_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\dot\theta_{43}^{\,2}\,\mathrm{c}\,\theta_{43} + \ddot\theta_{43}\,\mathrm{s}\,\theta_{43})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{43}\,(\dot\theta_{54}^{\,2}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54}) - \ddot\theta_{54}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54}))\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21\,i}\,((\dot\theta_{21\,i}^{\,2} + \dot\theta_{76\,i}^{\,2})\,\mathrm{c}\,\theta_{76\,i} + \ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{s}\,\theta_{21\,i}\,(\ddot\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - 2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}\,\theta_{21\,i}\,(-\mathrm{s}\,\theta_{65\,i}\,(2\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}) + (\ddot\theta_{65\,i} + \ddot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{65\,i}\,(\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{21\,i}^{\,2} + \dot\theta_{65\,i}^{\,2} + \dot\theta_{76\,i}^{\,2} + 2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{65\,i}\,(2\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) + (\ddot\theta_{21\,i} + \ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{65\,i}\,(\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} - (2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i} + (\dot\theta_{21\,i}^{\,2} + \dot\theta_{65\,i}^{\,2} + \dot\theta_{76\,i}^{\,2})\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}))) -$$

$$2\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$\mathrm{s}\,\theta_{43}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+$$

$$\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}-$$

$$\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)\big)\big)+$$

$$2\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\big(\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\mathrm{s}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}})\big)\big)\big)\big)-$$

$$\mathrm{c}\,\theta_{43}\,\big(\big(-\dot{\theta}_{54}{}^{2}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\big)\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{21}-\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\theta_{21}\big)+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(-\mathrm{s}\,\theta_{21}\,\big(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{21}\,\big(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\big)\big)\big)+2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}$$

$$\big(\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\big)-$$

$$2\,\dot{\theta}_{76\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\big)\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\big(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{21}\,\big(2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\mathrm{s}\,\theta_{21}\,\big(\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)+\mathrm{s}\,\theta_{21}$$

$$\big(\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\ddot{\theta}_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+2\,\dot{\theta}_{21}$$

$$\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)-\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+$$

$$2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\big(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}$$

$$\big(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\big)\big)$$

$$\mathrm{s}\,\theta_{76\,\mathrm{i}}-(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})\,\big(\dot{\theta}_{76\,\mathrm{i}}{}^{2}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\big(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\dot{\theta}_{76\,\mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)+$$

$$2\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)\big)+$$

$$\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\mathrm{s}\,\theta_{21}\,\big(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\big(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{76\,\mathrm{i}})\big)\big)\big)\big)-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\big(\ddot{\theta}_{43}\,\mathrm{c}\,\theta_{43}-\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\theta_{43}\big)\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+$$

$$2\,\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\big(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{s}\,\theta_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+$$

$$\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} -$$

$$(c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}))) +$$

$$s\,\theta_{43}\,(-(\dot{\theta}_{21}{}^{2}\,c\,\theta_{21} + \ddot{\theta}_{21}\,s\,\theta_{21})\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - 2\,\dot{\theta}_{21}\,c\,\theta_{21}$$

$$(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}$$

$$s\,\theta_{76\,i} - (c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) +$$

$$(-\ddot{\theta}_{21}\,c\,\theta_{21} + \dot{\theta}_{21}{}^{2}\,s\,\theta_{21})\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$2\,\dot{\theta}_{21}\,s\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{21}\,(-s\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}{}^{2}\,c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}) + c\,\beta_{54\,i}\,(c\,\theta_{65\,i}\,(2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i} + \ddot{\theta}_{65\,i}\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{65\,i}\,(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i} - (\dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,s\,\theta_{76\,i}))) - s\,\theta_{21}\,(c\,\beta_{54\,i}\,(-s\,\theta_{21\,i}$$

$$((\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}) + c\,\theta_{21\,i}\,(\ddot{\theta}_{21\,i}\,c\,\theta_{76\,i} - 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{21\,i}\,(-s\,\theta_{65\,i}\,(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i}) + (\ddot{\theta}_{65\,i} + \ddot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{65\,i}\,(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i} - (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2} + 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21\,i}\,(-c\,\theta_{65\,i}\,(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i}) + (\ddot{\theta}_{21\,i} + \ddot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) + s\,\theta_{65\,i}$$

$$(-\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i} + (2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i} + (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,s\,\beta_{54\,i})\,s\,\theta_{76\,i})))))) -$$

$$s\,\theta_{1110\,i}\,((-\dot{\theta}_{54}{}^{2}\,c(\beta_{1413\,i} - \theta_{54}) + \ddot{\theta}_{54}\,s(\beta_{1413\,i} - \theta_{54}))\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} -$$

$$(c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) +$$

$$(\dot{\theta}_{43}{}^{2}\,c\,\theta_{43} + \ddot{\theta}_{43}\,s\,\theta_{43})\,s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$2\,\dot{\theta}_{54}\,s(\beta_{1413\,i} - \theta_{54})\,(c\,\beta_{54\,i}\,(\dot{\theta}_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{65\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{65\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i} - (\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$(-\ddot{\theta}_{43}\,c\,\theta_{43} + \dot{\theta}_{43}{}^{2}\,s\,\theta_{43})\,s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) + c(\beta_{1413\,i} - \theta_{54})$$

$$(-c\,\beta_{54\,i}\,(c\,\theta_{21\,i}\,((\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}) + s\,\theta_{21\,i}\,(\ddot{\theta}_{21\,i}\,c\,\theta_{76\,i} - 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21\,i}\,(s\,\theta_{65\,i}\,(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i}) + (\ddot{\theta}_{65\,i} + \ddot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{65\,i}\,(-\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i} + (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2} + 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{21\,i}\,(-c\,\theta_{65\,i}\,(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i}) + (\ddot{\theta}_{21\,i} + \ddot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{65\,i}\,(-\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i} + (2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i} + (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}))) -$$

$$2\,\dot{\theta}_{43}\,c\,\theta_{43}\,(-\dot{\theta}_{54}\,c(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) +$$

$$s(\beta_{1413\,i} - \theta_{54})\,(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} +$$

$$\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} -$$

$$(c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$
\begin{aligned}
&\dot\theta_{21}\, c\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}) + \\
&c\,\theta_{21}\, (-\dot\theta_{76\,i}\, s\,\beta_{54\,i}\, s\,\theta_{76\,i} + c\,\beta_{54\,i}\, (\dot\theta_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i} + \dot\theta_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i})))) - \\
&c\,\theta_{43}\, (-(\ddot\theta_{54}\, c(\beta_{1413\,i} - \theta_{54}) + \dot\theta_{54}{}^{2}\, s(\beta_{1413\,i} - \theta_{54}))\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21} + c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i}) + \\
&(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i} + c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i}) + \\
&s(\beta_{1413\,i} - \theta_{54})\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, (\ddot\theta_{21}\, c\,\theta_{21} - \dot\theta_{21}{}^{2}\, s\,\theta_{21}) + c\,\beta_{54\,i}\, (-s\,\theta_{21}\, (2\,\dot\theta_{21}\, \dot\theta_{21\,i}\, c\,\theta_{21\,i} + \ddot\theta_{21}\, s\,\theta_{21\,i}) + \\
&c\,\theta_{21}\, (\ddot\theta_{21\,i}\, c\,\theta_{21\,i} - (\dot\theta_{21}{}^{2} + \dot\theta_{21\,i}{}^{2})\, s\,\theta_{21\,i}))) + 2\,\dot\theta_{76\,i}\, c\,\theta_{76\,i} \\
&(\dot\theta_{65\,i}\, c\,\beta_{54\,i}\, c\,\theta_{65\,i}\, s\,\theta_{21} + \dot\theta_{21}\, c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{65\,i} - c\,\theta_{21}\, (c\,\theta_{65\,i}\, (\dot\theta_{21\,i} + \dot\theta_{65\,i}\, s\,\beta_{54\,i})\, s\,\theta_{21\,i} + \\
&c\,\theta_{21\,i}\, (\dot\theta_{65\,i} + \dot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i}) - \dot\theta_{21}\, s\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})) - \\
&2\,\dot\theta_{76\,i}\, (\dot\theta_{21}\, c\,\theta_{21}\, s\,\beta_{54\,i} + c\,\beta_{54\,i}\, (\dot\theta_{21\,i}\, c\,\theta_{21}\, c\,\theta_{21\,i} - \dot\theta_{21}\, s\,\theta_{21}\, s\,\theta_{21\,i}))\, s\,\theta_{76\,i} + \\
&(c\,\beta_{54\,i}\, (c\,\theta_{21}\, (2\,\dot\theta_{21}\, \dot\theta_{65\,i}\, c\,\theta_{65\,i} + \ddot\theta_{21}\, s\,\theta_{65\,i}) + s\,\theta_{21}\, (\ddot\theta_{65\,i}\, c\,\theta_{65\,i} - (\dot\theta_{21}{}^{2} + \dot\theta_{65\,i}{}^{2})\, s\,\theta_{65\,i})) + s\,\theta_{21} \\
&(s\,\theta_{21\,i}\, (2\,\dot\theta_{21}\, c\,\theta_{65\,i}\, (\dot\theta_{21\,i} + \dot\theta_{65\,i}\, s\,\beta_{54\,i}) + \ddot\theta_{21}\, s\,\beta_{54\,i}\, s\,\theta_{65\,i}) + c\,\theta_{21\,i}\, (-\ddot\theta_{21}\, c\,\theta_{65\,i} + 2\,\dot\theta_{21} \\
&(\dot\theta_{65\,i} + \dot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i})) - c\,\theta_{21}\, (c\,\theta_{21\,i}\, (c\,\theta_{65\,i}\, (\dot\theta_{21}{}^{2} + \dot\theta_{21\,i}{}^{2} + \dot\theta_{65\,i}{}^{2} + \\
&2\,\dot\theta_{21\,i}\, \dot\theta_{65\,i}\, s\,\beta_{54\,i}) + (\ddot\theta_{65\,i} + \ddot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i}) + s\,\theta_{21\,i}\, (c\,\theta_{65\,i} \\
&(\ddot\theta_{21\,i} + \ddot\theta_{65\,i}\, s\,\beta_{54\,i}) - (2\,\dot\theta_{21\,i}\, \dot\theta_{65\,i} + (\dot\theta_{21}{}^{2} + \dot\theta_{21\,i}{}^{2} + \dot\theta_{65\,i}{}^{2})\, s\,\beta_{54\,i})\, s\,\theta_{65\,i}))) \\
&s\,\theta_{76\,i} - (s\,\beta_{54\,i}\, s\,\theta_{21} + c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i})\, (\dot\theta_{76\,i}{}^{2}\, c\,\theta_{76\,i} + \ddot\theta_{76\,i}\, s\,\theta_{76\,i}) + \\
&(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i} + c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, (\ddot\theta_{76\,i}\, c\,\theta_{76\,i} - \dot\theta_{76\,i}{}^{2}\, s\,\theta_{76\,i})) - \\
&2\,\dot\theta_{54}\, c(\beta_{1413\,i} - \theta_{54})\, (-s\,\theta_{21}\, (\dot\theta_{21}\, c\,\theta_{21\,i}\, c\,\theta_{65\,i} + s\,\beta_{54\,i}\, (\dot\theta_{76\,i} - \dot\theta_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i} + \\
&c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}\, (\dot\theta_{21} - \dot\theta_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) - c\,\theta_{65\,i}\, (\dot\theta_{21\,i} + \dot\theta_{65\,i}\, s\,\beta_{54\,i})\, s\,\theta_{21\,i}\, s\,\theta_{76\,i} + \\
&c\,\theta_{21\,i}\, (\dot\theta_{76\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})) + \\
&c\,\beta_{54\,i}\, (s\,\theta_{21}\, (c\,\theta_{76\,i}\, (-\dot\theta_{21}\, s\,\theta_{21\,i} + \dot\theta_{76\,i}\, s\,\theta_{65\,i}) + \dot\theta_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}) + \\
&c\,\theta_{21}\, (\dot\theta_{21\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i} + (-\dot\theta_{76\,i}\, s\,\theta_{21\,i} + \dot\theta_{21}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})))) + \\
&2\,\dot\theta_{43}\, s\,\theta_{43}\, (-\dot\theta_{54}\, c(\beta_{1413\,i} - \theta_{54})\, (c\,\theta_{76\,i}\, (s\,\beta_{54\,i}\, s\,\theta_{21} + c\,\beta_{54\,i}\, c\,\theta_{21}\, s\,\theta_{21\,i}) + \\
&(c\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{65\,i} + c\,\theta_{21}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i}) + \\
&s(\beta_{1413\,i} - \theta_{54})\, (-s\,\theta_{21}\, (\dot\theta_{21}\, c\,\theta_{21\,i}\, c\,\theta_{65\,i} + s\,\beta_{54\,i}\, (\dot\theta_{76\,i} - \dot\theta_{21}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}))\, s\,\theta_{76\,i} + \\
&c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i}\, (\dot\theta_{21} - \dot\theta_{76\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) - c\,\theta_{65\,i}\, (\dot\theta_{21\,i} + \dot\theta_{65\,i}\, s\,\beta_{54\,i})\, s\,\theta_{21\,i}\, s\,\theta_{76\,i} + \\
&c\,\theta_{21\,i}\, (\dot\theta_{76\,i}\, c\,\theta_{65\,i}\, c\,\theta_{76\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i}\, s\,\theta_{76\,i})) + \\
&c\,\beta_{54\,i}\, (s\,\theta_{21}\, (c\,\theta_{76\,i}\, (-\dot\theta_{21}\, s\,\theta_{21\,i} + \dot\theta_{76\,i}\, s\,\theta_{65\,i}) + \dot\theta_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}) + \\
&c\,\theta_{21}\, (\dot\theta_{21\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i} + (-\dot\theta_{76\,i}\, s\,\theta_{21\,i} + \dot\theta_{21}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})))) - \\
&s\,\theta_{43}\, (-(\ddot\theta_{54}\, c(\beta_{1413\,i} - \theta_{54}) + \dot\theta_{54}{}^{2}\, s(\beta_{1413\,i} - \theta_{54}))\, (c\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i} + c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i}) - \\
&s\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i})) - \\
&2\,\dot\theta_{54}\, c(\beta_{1413\,i} - \theta_{54})\, (-\dot\theta_{21}\, s\,\theta_{21}\, (c\,\theta_{76\,i}\, s\,\beta_{54\,i} + c\,\beta_{54\,i}\, s\,\theta_{65\,i}\, s\,\theta_{76\,i}) - s\,\theta_{21} \\
&(\dot\theta_{21\,i}\, c\,\beta_{54\,i}\, c\,\theta_{21\,i}\, c\,\theta_{76\,i} + \dot\theta_{76\,i}\, c\,\theta_{76\,i}\, (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i}) - \dot\theta_{76\,i}\, c\,\beta_{54\,i}\, s\,\theta_{21\,i} \\
&s\,\theta_{76\,i} - (c\,\theta_{65\,i}\, (\dot\theta_{21\,i} + \dot\theta_{65\,i}\, s\,\beta_{54\,i})\, s\,\theta_{21\,i} + c\,\theta_{21\,i}\, (\dot\theta_{65\,i} + \dot\theta_{21\,i}\, s\,\beta_{54\,i})\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}) - \\
&\dot\theta_{21}\, c\,\theta_{21}\, (c\,\beta_{54\,i}\, c\,\theta_{76\,i}\, s\,\theta_{21\,i} + (c\,\theta_{21\,i}\, c\,\theta_{65\,i} - s\,\beta_{54\,i}\, s\,\theta_{21\,i}\, s\,\theta_{65\,i})\, s\,\theta_{76\,i}) + \\
&c\,\theta_{21}\, (-\dot\theta_{76\,i}\, s\,\beta_{54\,i}\, s\,\theta_{76\,i} + c\,\beta_{54\,i}\, (\dot\theta_{76\,i}\, c\,\theta_{76\,i}\, s\,\theta_{65\,i} + \dot\theta_{65\,i}\, c\,\theta_{65\,i}\, s\,\theta_{76\,i}))) +
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{s}(\beta_{1413\ \mathrm{i}} - \theta_{54}) \Big(-\big(\dot{\theta}_{21}{}^{2}\, \mathrm{c}\,\theta_{21} + \ddot{\theta}_{21}\, \mathrm{s}\,\theta_{21}\big) (\mathrm{c}\,\theta_{76\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}} + \mathrm{c}\,\beta_{54\ \mathrm{i}}\, \mathrm{s}\,\theta_{65\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}) - 2\, \dot{\theta}_{21}\, \mathrm{c}\,\theta_{21} \\
&\quad \big(\dot{\theta}_{21\ \mathrm{i}}\, \mathrm{c}\,\beta_{54\ \mathrm{i}}\, \mathrm{c}\,\theta_{21\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} + \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} (\mathrm{c}\,\theta_{21\ \mathrm{i}}\, \mathrm{c}\,\theta_{65\ \mathrm{i}} - \mathrm{s}\,\beta_{54\ \mathrm{i}}\, \mathrm{s}\,\theta_{21\ \mathrm{i}}\, \mathrm{s}\,\theta_{65\ \mathrm{i}}) - \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\beta_{54\ \mathrm{i}}\, \mathrm{s}\,\theta_{21\ \mathrm{i}} \\
&\qquad \mathrm{s}\,\theta_{76\ \mathrm{i}} - \big(\mathrm{c}\,\theta_{65\ \mathrm{i}} \big(\dot{\theta}_{21\ \mathrm{i}} + \dot{\theta}_{65\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{21\ \mathrm{i}} + \mathrm{c}\,\theta_{21\ \mathrm{i}} \big(\dot{\theta}_{65\ \mathrm{i}} + \dot{\theta}_{21\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{65\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \\
&\quad \big(-\ddot{\theta}_{21}\, \mathrm{c}\,\theta_{21} + \dot{\theta}_{21}{}^{2}\, \mathrm{s}\,\theta_{21}\big) (\mathrm{c}\,\beta_{54\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}}\, \mathrm{s}\,\theta_{21\ \mathrm{i}} + (\mathrm{c}\,\theta_{21\ \mathrm{i}}\, \mathrm{c}\,\theta_{65\ \mathrm{i}} - \mathrm{s}\,\beta_{54\ \mathrm{i}}\, \mathrm{s}\,\theta_{21\ \mathrm{i}}\, \mathrm{s}\,\theta_{65\ \mathrm{i}})\, \mathrm{s}\,\theta_{76\ \mathrm{i}}) - \\
&\quad 2\, \dot{\theta}_{21}\, \mathrm{s}\,\theta_{21} \big(-\dot{\theta}_{76\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}} + \mathrm{c}\,\beta_{54\ \mathrm{i}} \big(\dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}}\, \mathrm{s}\,\theta_{65\ \mathrm{i}} + \dot{\theta}_{65\ \mathrm{i}}\, \mathrm{c}\,\theta_{65\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big)\big) + \\
&\quad \mathrm{c}\,\theta_{21} \Big(-\mathrm{s}\,\beta_{54\ \mathrm{i}} \big(\dot{\theta}_{76\ \mathrm{i}}{}^{2}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} + \ddot{\theta}_{76\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \mathrm{c}\,\beta_{54\ \mathrm{i}} \Big(\mathrm{c}\,\theta_{65\ \mathrm{i}} \big(2\, \dot{\theta}_{65\ \mathrm{i}}\, \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} + \ddot{\theta}_{65\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \\
&\qquad \mathrm{s}\,\theta_{65\ \mathrm{i}} \big(\ddot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} - \big(\dot{\theta}_{65\ \mathrm{i}}{}^{2} + \dot{\theta}_{76\ \mathrm{i}}{}^{2}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big)\Big)\Big) - \mathrm{s}\,\theta_{21} \Big(\mathrm{c}\,\beta_{54\ \mathrm{i}} \Big(-\mathrm{s}\,\theta_{21\ \mathrm{i}} \\
&\qquad \big(\big(\dot{\theta}_{21\ \mathrm{i}}{}^{2} + \dot{\theta}_{76\ \mathrm{i}}{}^{2}\big)\, \mathrm{c}\,\theta_{76\ \mathrm{i}} + \ddot{\theta}_{76\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \mathrm{c}\,\theta_{21\ \mathrm{i}} \big(\ddot{\theta}_{21\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} - 2\, \dot{\theta}_{21\ \mathrm{i}}\, \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big)\Big) + \\
&\quad \mathrm{c}\,\theta_{21\ \mathrm{i}} \Big(-\mathrm{s}\,\theta_{65\ \mathrm{i}} \big(2\, \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} \big(\dot{\theta}_{65\ \mathrm{i}} + \dot{\theta}_{21\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big) + \big(\ddot{\theta}_{65\ \mathrm{i}} + \ddot{\theta}_{21\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \\
&\qquad \mathrm{c}\,\theta_{65\ \mathrm{i}} \big(\ddot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} - \big(\dot{\theta}_{21\ \mathrm{i}}{}^{2} + \dot{\theta}_{65\ \mathrm{i}}{}^{2} + \dot{\theta}_{76\ \mathrm{i}}{}^{2} + 2\, \dot{\theta}_{21\ \mathrm{i}}\, \dot{\theta}_{65\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big)\Big) + \\
&\quad \mathrm{s}\,\theta_{21\ \mathrm{i}} \Big(-\mathrm{c}\,\theta_{65\ \mathrm{i}} \big(2\, \dot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}} \big(\dot{\theta}_{21\ \mathrm{i}} + \dot{\theta}_{65\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big) + \big(\ddot{\theta}_{21\ \mathrm{i}} + \ddot{\theta}_{65\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big) + \mathrm{s}\,\theta_{65\ \mathrm{i}} \\
&\qquad \big(-\ddot{\theta}_{76\ \mathrm{i}}\, \mathrm{c}\,\theta_{76\ \mathrm{i}}\, \mathrm{s}\,\beta_{54\ \mathrm{i}} + \big(2\, \dot{\theta}_{21\ \mathrm{i}}\, \dot{\theta}_{65\ \mathrm{i}} + \big(\dot{\theta}_{21\ \mathrm{i}}{}^{2} + \dot{\theta}_{65\ \mathrm{i}}{}^{2} + \dot{\theta}_{76\ \mathrm{i}}{}^{2}\big)\, \mathrm{s}\,\beta_{54\ \mathrm{i}}\big)\, \mathrm{s}\,\theta_{76\ \mathrm{i}}\big)\Big)\Big)\Big)\Big)
\end{aligned}
$$

Apéndice D

Coeficientes de la ecuación (2.51)

$V_1 = -2\,x_{32}\,(x - x_{1918}\,c\,\phi\,c\,\psi)\,s\,\theta_{21} - 2\,x_{32}\,c\,\theta_{21}\,(z - z_{10} - x_{1918}\,s\,\theta\,s\,\phi + x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi)$

$V_2 = 2\,(x - x_{32}\,c\,\theta_{21} - x_{1918}\,c\,\phi\,c\,\psi)$

$V_3 = 2\,(y - x_{1918}\,c\,\theta\,s\,\phi - x_{1918}\,c\,\phi\,s\,\theta\,s\,\psi)$

$V_4 = 2\,(z - z_{10} + x_{32}\,s\,\theta_{21} - x_{1918}\,s\,\theta\,s\,\phi + x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi)$

$V_5 = 2\,x_{1918}\,c\,\phi\,(-y\,c\,\psi\,s\,\theta + c\,\theta\,c\,\psi\,(z - z_{10} + x_{32}\,s\,\theta_{21}) + (x - x_{32}\,c\,\theta_{21})\,s\,\psi)$

$V_6 = -2\,x_{1918}\,(c\,\theta\,((z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\phi + y\,c\,\phi\,s\,\psi) + s\,\theta\,(-y\,s\,\phi + c\,\phi\,(z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\psi))$

$V_7 = 2\,x_{1918}\,(-c\,\phi\,s\,\theta\,(z - z_{10} + x_{32}\,s\,\theta_{21}) + s\,\phi\,((x - x_{32}\,c\,\theta_{21})\,c\,\psi + y\,s\,\theta\,s\,\psi) - c\,\theta\,(y\,c\,\phi + (z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\phi\,s\,\psi))$

Coeficientes de la ecuación (2.54)

$V_8 = c\,\theta_{43}\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\,c\,\phi\,c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\,s\,\theta\,s\,\phi - x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi)) -$
$\quad s\,\theta_{43}\,((x - x_{1918}\,c\,\phi\,c\,\psi)\,s\,\theta_{21} + c\,\theta_{21}\,(z - z_{10} - x_{1918}\,s\,\theta\,s\,\phi + x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))$

$V_9 = -s\,\theta_{43}$

$V_{10} = -c\,\theta_{43}$

$V_{11} = -x_{1918}\,c\,\phi\,(c\,\theta\,c\,\theta_{43}\,c\,\psi + s\,\theta_{43}\,s\,\psi)$

$V_{12} = x_{1918}\,c\,\theta_{43}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)$

$V_{13} = x_{1918}\,(-c\,\psi\,s\,\theta_{43}\,s\,\phi + c\,\theta_{43}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))$

$V_{14} = -x_{32}\,c(\theta_{21} - \theta_{43})$

Coeficientes de la ecuación (2.58)

$V_{15} = (-x_{32} + c\,\theta_{21}\,(x - x_{1918}\,c\,\phi\,c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\,s\,\theta\,s\,\phi - x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))^2$

$V_{16} = -c\,\theta_{43}\,c\,\theta_{54}{}^2\,(y - x_{1918}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi))$

$V_{17} = c\,\theta_{43}\,c\,\theta_{54}{}^2\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\,c\,\phi\,c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\,s\,\theta\,s\,\phi - x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))$

$V_{18} = x_{1918}\,c\,\theta_{43}\,c\,\theta_{54}{}^2\,c\,\phi\,(s\,\psi\,(-y + x_{1918}\,c\,\theta\,s\,\phi + x_{1918}\,c\,\phi\,s\,\theta\,s\,\psi) -$
$\quad c\,\psi\,s\,\theta\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\,c\,\phi\,c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\,s\,\theta\,s\,\phi - x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi)))$

$$V_{19} = x_{1918}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\, (s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi)\, (-x_{32} + c\,\theta_{21}\, (x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\, (-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))$$

$$\begin{aligned} V_{20} = x_{1918}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\, (&c\,\psi\, s\,\phi\, (-y + x_{1918}\, c\,\theta\, s\,\phi + x_{1918}\, c\,\phi\, s\,\theta\, s\,\psi) - \\ &c\,\theta\, c\,\phi\, (-x_{32} + c\,\theta_{21}\, (x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\, (-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) + \\ &s\,\theta\, s\,\phi\, s\,\psi\, (-x_{32} + c\,\theta_{21}\, (x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\, (-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))) \end{aligned}$$

$$V_{21} = -x_{32}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\, s\,\theta_{21}\, (y - x_{1918}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\begin{aligned} V_{22} = \\ &c\,\theta_{54}{}^{2}\, s\,\theta_{43}\, (-x_{32} + c\,\theta_{21}\, (x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\, (-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))\, (y - x_{1918}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) \end{aligned}$$

Coeficientes de la ecuación (2.63)

$$V_{23} = (s\,\theta_{54}\, (c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\, (c\,\phi\, s\,\theta\, s(\theta_{21} + \theta_{43}) + s\,\phi\, (c(\theta_{21} + \theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21} + \theta_{43})\, s\,\psi)))^{2}$$

$$\begin{aligned} V_{24} = \\ &c\,\theta_{1817}{}^{2} \Big(c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi\, (c\,\theta_{54}\, s\,\theta\, s(\theta_{21} + \theta_{43}) - c\,\theta\, s\,\theta_{54}) + c(\theta_{21} + \theta_{43})^{2}\, c\,\theta_{54}{}^{2}\, s\,\phi + (c\,\theta\, c\,\theta_{54}\, s(\theta_{21} + \theta_{43}) + s\,\theta\, s\,\theta_{54}) \\ &\quad (s\,\theta\, (s\,\theta_{54}\, s\,\phi + c\,\theta_{54}\, c\,\phi\, s(\theta_{21} + \theta_{43})\, s\,\psi) + c\,\theta\, (c\,\theta_{43}\, c\,\theta_{54}\, s\,\theta_{21}\, s\,\phi + c\,\theta_{21}\, c\,\theta_{54}\, s\,\theta_{43}\, s\,\phi - c\,\phi\, s\,\theta_{54}\, s\,\psi))\Big) \end{aligned}$$

$$\begin{aligned} V_{25} = c\,\theta_{1817}{}^{2} \Big(&c\,\theta^{2}\, c\,\phi\, c\,\psi \left(c\,\theta_{54}{}^{2}\, s(\theta_{21} + \theta_{43})^{2} + s\,\theta_{54}{}^{2}\right) - c\,\theta\, c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, (s\,\theta_{54}\, s\,\phi + c\,\theta_{54}\, c\,\phi\, s(\theta_{21} + \theta_{43})\, s\,\psi) + \\ &s\,\theta \Big(c\,\phi\, c\,\psi\, s\,\theta\, s\,\theta_{54}{}^{2} + c\,\theta_{54}{}^{2}\, s(\theta_{21} + \theta_{43})\, (c\,\phi\, c\,\psi\, s\,\theta\, s(\theta_{21} + \theta_{43}) + c(\theta_{21} + \theta_{43})\, s\,\phi) - c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, c\,\phi\, s\,\theta_{54}\, s\,\psi\Big)\Big) \end{aligned}$$

$$\begin{aligned} V_{26} = &c\,\theta_{1817}{}^{2}\, (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21} + \theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, s\,\psi) \\ &(c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21} + \theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) \end{aligned}$$

$$\begin{aligned} V_{27} = -c\,\theta_{1817}{}^{2}\, c\,\theta_{54}\, (&(c\,\theta\, c\,\theta_{54} + c\,\theta_{43}\, s\,\theta\, s\,\theta_{21}\, s\,\theta_{54})\, s\,\phi + \\ &c\,\phi\, (c\,\psi\, s\,\theta_{21}\, s\,\theta_{43}\, s\,\theta_{54} + c\,\theta_{54}\, s\,\theta\, s\,\psi - c\,\theta\, c\,\theta_{43}\, s\,\theta_{21}\, s\,\theta_{54}\, s\,\psi) - c\,\theta_{21}\, s\,\theta_{54}\, (c\,\theta_{43}\, c\,\phi\, c\,\psi + s\,\theta_{43}\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi))) \end{aligned}$$

$$V_{28} = -c\,\theta_{1817}{}^{2}\, (c\,\theta_{43}\, (c\,\theta_{21}\, s\,\theta\, s\,\phi + c\,\phi\, (c\,\psi\, s\,\theta_{21} - c\,\theta\, c\,\theta_{21}\, s\,\psi)) + s\,\theta_{43}\, (c\,\theta_{21}\, c\,\phi\, c\,\psi + s\,\theta_{21}\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi)))$$

Coeficientes de la ecuación (2.68)

$$V_{29} = (c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21} + \theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^{2}$$

$$\begin{aligned} V_{30} = &c\,\theta_{1716}{}^{2}\, \csc(\theta_{1817}) \\ &\Big(c(\theta_{21} + \theta_{43})^{2}\, c\,\theta_{54}{}^{2}\, c\,\phi + c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, c\,\psi\, (-c\,\theta_{54}\, s\,\theta\, s(\theta_{21} + \theta_{43}) + c\,\theta\, s\,\theta_{54})\, s\,\phi + (c\,\theta\, c\,\theta_{54}\, s(\theta_{21} + \theta_{43}) + s\,\theta\, s\,\theta_{54}) \\ &\quad (s\,\theta\, (c\,\phi\, s\,\theta_{54} - c\,\theta_{54}\, s(\theta_{21} + \theta_{43})\, s\,\phi\, s\,\psi) + c\,\theta\, (c\,\theta_{43}\, c\,\theta_{54}\, c\,\phi\, s\,\theta_{21} + c\,\theta_{21}\, c\,\theta_{54}\, c\,\phi\, s\,\theta_{43} + s\,\theta_{54}\, s\,\phi\, s\,\psi))\Big) \end{aligned}$$

$$\begin{aligned} V_{31} = &-c\,\theta_{1716}{}^{2}\, \csc(\theta_{1817}) \\ &\left(-\frac{1}{8}\, (-6 + 2\, c(2\, (\theta_{21} + \theta_{43})) + c(2\, (\theta_{21} + \theta_{43} - \theta_{54})) + 2\, c(2\, \theta_{54}) + c(2\, (\theta_{21} + \theta_{43} + \theta_{54})))\, c\,\psi\, s\,\phi - c(\theta_{21} + \theta_{43})\, c\,\theta_{54} \right. \\ &\quad \left. (c\,\theta_{43}\, c\,\theta_{54}\, s\,\theta_{21}\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{21}\, c\,\theta_{54}\, s\,\theta_{43}\, (c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + s\,\theta_{54}\, (-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi))\right) \end{aligned}$$

$$V_{32} = -c\,\theta_{1716}{}^{2}\, \csc(\theta_{1817})\, (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21} + \theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21} + \theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\phi+\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))$$

$$\mathrm{V}_{33}=-\mathrm{c}\,\theta_{1716}{}^2\,\mathrm{c}\,\theta_{54}\,\mathrm{csc}(\theta_{1817})\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54}-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi)-\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\theta\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))$$

$$\mathrm{V}_{34}=\mathrm{c}\,\theta_{1716}{}^2\,\mathrm{csc}(\theta_{1817})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\phi\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi-\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)))$$

$$\mathrm{V}_{35}=\mathrm{c}\,\theta_{1716}{}^2\,\cot(\theta_{1817})\,\mathrm{csc}\,(\theta_{1817})\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}\,(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)\,(\mathrm{c}\,(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi+\mathrm{c}\,\theta_{54}\,\mathrm{s}\,(\theta_{21}+\theta_{43})\,(-\mathrm{s}\theta\,\mathrm{s}\phi+\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))$$

Coeficientes de la ecuación (2.72)

$$\mathrm{V}_{36}=(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^2$$

$$\mathrm{V}_{37}=\mathrm{c}\,\theta_{1615}{}^2\Big(\mathrm{c}\,\theta^2\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi+\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\phi)\,(-\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}+\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta\Big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}{}^2\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}-\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}(2\,\phi)+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{c}(2\,\phi)\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{c}(2\,\phi)\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}{}^2\Big(-\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\theta_{1817}{}^2\,\mathrm{s}\,\theta_{54}+\mathrm{c}\,\theta_{1817}{}^2\,\mathrm{s}\,\phi\,(\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)\Big)\Big)+\mathrm{s}\,\theta\Big(-\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}{}^2\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi)+\mathrm{s}\,\phi\,(-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\Big(\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi\,(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1817}\Big(\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,\Big(\mathrm{c}\,\phi^2\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}-\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi^2+\mathrm{c}\,\theta_{43}\,\mathrm{s}(2\,\phi)\Big)+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,(-\mathrm{c}\,\theta_{43}\,\mathrm{c}(2\,\phi)\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,\phi))-\mathrm{c}(2\,\phi)\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi\Big)\Big)-\mathrm{c}\,\theta_{1716}{}^2\,\Big(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}-\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,\Big(-\mathrm{c}\,\theta_{1817}{}^2\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\psi^2\,\mathrm{s}(\theta_{21}+\theta_{43})\,\Big(\mathrm{s}\,\theta_{1817}{}^2+\mathrm{c}\,\theta_{1817}{}^2\,\mathrm{s}\,\phi^2\Big)+\mathrm{s}\,\psi\,\Big(-\mathrm{c}\,\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817}+\mathrm{s}\,\theta_{1817}{}^2\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{1817}{}^2\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi^2\,\mathrm{s}\,\psi\Big)\Big)\Big)\Big)\Big)$$

$$\mathrm{V}_{38}=\mathrm{c}\,\theta_{1615}{}^2\Big(\mathrm{c}\,\theta\,(-\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}+\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi))\,(\mathrm{s}\,\theta_{1716}\,(\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}+\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}-\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)))+\mathrm{c}\,\theta^2\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\Big(2\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi)+\mathrm{s}\,\theta_{1716}{}^2\,\Big(\mathrm{s}\,\phi^2+\mathrm{c}\,\phi^2\,\mathrm{s}\,\psi^2\Big)+\mathrm{c}\,\theta_{1716}{}^2\,\Big(\mathrm{c}\,\psi^2\,\mathrm{s}\,\theta_{1817}{}^2-2\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{1817}{}^2\,\Big(\mathrm{c}\,\phi^2+\mathrm{s}\,\phi^2\,\mathrm{s}\,\psi^2\Big)\Big)\Big)+\mathrm{s}\,\theta\Big(\mathrm{s}\,\theta_{1716}{}^2\,\Big(-\mathrm{c}\,\phi^2\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{54}\,\Big(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi^2+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi^2\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi^2\Big)\Big)+\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,\Big(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\Big(\mathrm{c}\,\theta_{54}\,\Big(\mathrm{c}\,\phi^2\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi^2+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}(2\,\phi)\Big)+\mathrm{s}\,\theta_{54}\,\mathrm{s}(2\,\phi)\,\mathrm{s}\,\psi\Big)-\mathrm{s}\,\theta_{1817}\,\Big(\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\phi\,\Big(\mathrm{c}\,\psi^2\,\mathrm{s}\,\theta_{54}-2\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi-\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi^2+\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,\psi)\Big)\Big)\Big)-\mathrm{c}\,\theta_{1716}{}^2\,\Big(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}{}^2\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1817}{}^2\,\Big(\mathrm{c}(\theta_{21}+\theta_{43})^2\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi^2\,\mathrm{s}\,\psi+\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\Big(\mathrm{c}\,\phi^2+\mathrm{s}\,\phi^2\,\mathrm{s}\,\psi^2\Big)\Big)+\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta_{1817}\,\Big(-\mathrm{c}\,\psi^2\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\psi+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi^2-\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi\,\mathrm{s}(2\,\psi)+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}(2\,\psi))\Big)\Big)\Big)\Big)$$

$$\mathrm{V}_{39}=$$
$$\mathrm{c}\,\theta_{1615}{}^2\,\Big(\mathrm{c}\,\theta^2\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\Big(\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}+\mathrm{s}\,\theta_{1716}{}^2\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{1716}{}^2\,\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\psi)\Big)+$$

$$\mathrm{c}\,\theta\Big(-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}\,(\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1716}{}^{2}\,\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\theta_{1817}\,(-\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}+\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))\Big)-$$
$$\mathrm{s}\,\theta\Big(\mathrm{s}\,\theta_{1716}{}^{2}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}$$
$$(\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\phi)+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,(-\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi)-\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1716}{}^{2}$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}$$
$$\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\psi+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi-\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\psi))))\Big)\Big)$$

$$\mathrm{V}_{40}=-\mathrm{c}\,\theta_{1615}{}^{2}\Bigg(\mathrm{s}\,\theta_{1716}{}^{2}\,\big(\mathrm{s}\,\theta^{2}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi^{2}+\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta^{2}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\phi^{2}\,\mathrm{c}\,\psi\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)\big)+$$
$$\mathrm{c}\,\theta^{2}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}-\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi-\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1716}\,(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}+\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)))+$$
$$\mathrm{c}\,\theta_{1716}{}^{2}\,\big(\mathrm{s}\,\theta_{1817}{}^{2}\,\mathrm{s}\,\psi\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi)+$$
$$\mathrm{c}\,\theta_{1817}{}^{2}\,\big(\mathrm{c}\,\phi^{2}\,\mathrm{s}\,\theta^{2}\,\mathrm{s}\,\theta_{54}-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta^{2}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}\,\psi\,\mathrm{s}\,\phi^{2}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)\big)+$$
$$\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}(2\,\psi)\,\mathrm{s}\,\phi)+\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}(2\,\psi))\big)+$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\Bigg(-\mathrm{c}(\theta_{21}+\theta_{43})^{2}\,\mathrm{s}\,\theta_{54}\left(\frac{1}{2}\,\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(2\,\theta)+\mathrm{c}(2\,\psi))\,\mathrm{s}(2\,\phi)+\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}(2\,\psi)\right)+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta$$
$$\big(-\mathrm{c}\,\psi^{2}\,\mathrm{s}\,\theta_{1817}+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi^{2}+\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi\,\mathrm{s}(2\,\psi)\big)+\mathrm{s}(\theta_{21}+\theta_{43})\Bigg(\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta^{2}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{c}(2\,\phi)\,\mathrm{s}\,\psi)-$$
$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\left(\frac{1}{2}\,\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(2\,\theta)+\mathrm{c}(2\,\psi))\,\mathrm{s}(2\,\phi)+\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}(2\,\psi)\right)\Bigg)\Bigg)+$$
$$\mathrm{c}\,\theta\Big(-\mathrm{s}\,\theta_{1716}\,\big(\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\theta_{21}\,\big(-\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\phi-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi^{2}+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\phi^{2}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi^{2}\big)+$$
$$\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta_{1716}\,\mathrm{s}(2\,\phi)\,\mathrm{s}\,\psi+2\,\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{1817}\,\mathrm{c}(2\,\phi)\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1716}\,\big(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{43}\,\big(-\mathrm{s}\,\phi^{2}+\mathrm{c}\,\phi^{2}\,\mathrm{s}\,\psi^{2}\big)\big)\big)+$$
$$\mathrm{c}\,\theta_{1716}{}^{2}\,\big(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{43}+2\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54})+$$
$$\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi+2\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi)\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{1817}{}^{2}$$
$$\big(\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}(2\,\phi)\,\mathrm{s}\,\psi+\mathrm{c}\,\theta_{54}\,\big(\mathrm{c}\,\phi^{2}\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi^{2}\,\mathrm{s}\,\psi^{2}\big)\big)\big)+$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1716}\,\big(\mathrm{c}\,\theta_{1817}\,\big(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}(2\,\phi)\,\mathrm{c}\,\psi+\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}(2\,\phi)\,\big(1+\mathrm{s}\,\psi^{2}\big)\big)+$$
$$\mathrm{s}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi-\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}(2\,\psi))\big)\Big)\Bigg)$$

$$\mathrm{V}_{41}=-\mathrm{c}\,\theta_{1615}{}^{2}\,(\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}$$
$$(-\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)-\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}$$
$$(-\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}+\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}\,(-\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54})+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\phi+\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$\mathrm{V}_{42}=-\frac{1}{2}\,\mathrm{c}\,\theta_{1615}{}^{2}$$
$$\big(-\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\phi+$$
$$\mathrm{c}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(2\,\theta_{1716})\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi+\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\psi-$$
$$\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\psi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi-\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1716}{}^{2}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi-$$
$$2\,\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,(\mathrm{c}\,\theta\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{43}+\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54})+$$

$c\,\psi\,(s\,\theta\,s\,\theta_{1817}\,s\,\theta_{54} + c\,\theta_{1817}\,c\,\theta_{54}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\phi) + c\,\theta_{54}\,s\,\theta_{1817}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\psi) +$
$c\,\theta_{1716}{}^2\,(-c\,\theta_{54}\,s\,\theta_{21}\,(c\,\phi\,c\,\psi\,s\,\theta_{43} + c\,\theta_{43}\,s\,\theta\,s\,\phi - c\,\theta\,c\,\theta_{43}\,c\,\phi\,s\,\psi) + s\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi) +$
$2\,c\,\theta_{21}\,c\,\theta_{54}\,(c\,\theta_{43}\,c\,\phi\,c\,\psi + s\,\theta_{43}\,(-s\,\theta\,s\,\phi + c\,\theta\,c\,\phi\,s\,\psi))) + c\,\theta_{1817}\,s(2\,\theta_{1716})$
$(s\,\theta\,s\,\theta_{54}\,s\,\phi\,s\,\psi + c\,\theta_{21}\,c\,\theta_{54}\,s\,\theta_{43}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{43}\,c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s\,\theta_{21} + s\,\phi\,(c\,\theta_{21}\,c\,\psi + c\,\theta\,s\,\theta_{21}\,s\,\psi))))\Big)$

$$V_{43} = c\,\theta_{1615}{}^2\,(c\,\theta_{21}\,c\,\theta_{54}\,c\,\phi\,s\,\theta\,s\,\theta_{1817}\,s\,\theta_{43} + c\,\theta_{1817}\,c\,\psi\,s\,\theta\,s\,\theta_{54} - c\,\theta_{54}\,c\,\psi\,s\,\theta_{1817}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\phi + c\,\theta_{1817}\,c\,\theta_{54}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\psi +$$
$$s\,\theta\,s\,\theta_{1817}\,s\,\theta_{54}\,s\,\phi\,s\,\psi + c\,\theta_{43}\,c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s\,\theta_{1817}\,s\,\theta_{21} + c\,\theta_{21}\,(c\,\psi\,s\,\theta_{1817}\,s\,\phi - c\,\theta_{1817}\,s\,\psi)) +$$
$$c\,\theta\,(c\,\theta_{1817}\,c\,\theta_{54}\,c\,\psi\,s\,(\theta_{21} + \theta_{43}) + s\,\theta_{1817}\,(-c\phi\,s\,\theta_{54} + c\,\theta_{54}\,s\,(\theta_{21} + \theta_{43})\,s\,\phi\,s\,\psi)))$$

Coeficientes de la ecuación (2.134)

$$V_{44} =$$
$$-\,x_{43\ i}\,s\,\theta_{21\ i}\,(2\,x_{32}\,c\,\theta_{21} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) + x_{65}\,c(\theta_{21} + \theta_{43} - \theta_{54}) - x_{1211\ i}\,c(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) +$$
$$x_{65}\,c(\theta_{21} + \theta_{43} + \theta_{54}) - 2\,z_{1312\ i}\,s(\theta_{21} + \theta_{43})) + 2\,x_{43\ i}\,c\,\theta_{21\ i}\,(-\text{signo}\ y_{10\ i} + x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})$$

$$V_{45} = -2\,x_{32}\,(z_{10} + z_{32\ i})\,c\,\theta_{21} + (z_{10} + z_{32\ i})\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) - x_{65}\,c(\theta_{21} + \theta_{43} - \theta_{54}) +$$
$$x_{1211\ i}\,c(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) - x_{65}\,c(\theta_{21} + \theta_{43} + \theta_{54}) + 2\,z_{1312\ i}\,s(\theta_{21} + \theta_{43})) +$$
$$2\,x_{43\ i}\,c\,\theta_{21\ i}\,(z_{1312\ i}\,c(\theta_{21} + \theta_{43}) + x_{32}\,s\,\theta_{21} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s(\theta_{21} + \theta_{43}))$$

$$V_{46} = -2\,x_{32}\,z_{1312\ i}\,c\,\theta_{43} + x_{1211\ i}\,z_{10}\,c\,(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) +$$
$$x_{1211\ i}\,z_{32\ i}\,c\,(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) - x_{65}\,z_{10}\,c\,(\theta_{21} + \theta_{43} - \theta_{54}) - x_{65}\,z_{32\ i}\,c\,(\theta_{21} + \theta_{43} - \theta_{54}) +$$
$$x_{1211\ i}\,z_{10}\,c\,(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) + x_{1211\ i}\,z_{32\ i}\,c\,(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) - x_{65}\,z_{10}\,c\,(\theta_{21} + \theta_{43} + \theta_{54}) -$$
$$x_{65}\,z_{32\ i}\,c\,(\theta_{21} + \theta_{43} + \theta_{54}) + 2\,z_{10}\,z_{1312\ i}\,s\,(\theta_{21} + \theta_{43}) + 2\,z_{1312\ i}\,z_{32\ i}\,s\,(\theta_{21} + \theta_{43}) +$$
$$2\,x_{43\ i}\,c\,\theta_{21\ i}\,(z_{1312\ i}\,c\,(\theta_{21} + \theta_{43}) + (-x_{1211\ i}\,c\,(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,(\theta_{21} + \theta_{43})) -$$
$$x_{1211\ i}\,x_{32}\,s\,(\beta_{1413\ i} - \theta_{43} - \theta_{54}) - x_{32}\,x_{65}\,s\,(\theta_{43} - \theta_{54}) + x_{1211\ i}\,x_{32}\,s\,(\beta_{1413\ i} + \theta_{43} - \theta_{54}) - x_{32}\,x_{65}\,s\,(\theta_{43} + \theta_{54})$$

$$V_{47} = x_{1211\ i}\,(z_{10} + z_{32\ i})\,c\,(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) +$$
$$x_{65}\,z_{10}\,c\,(\theta_{21} + \theta_{43} - \theta_{54}) + x_{65}\,z_{32\ i}\,c\,(\theta_{21} + \theta_{43} - \theta_{54}) - x_{1211\ i}\,z_{10}\,c\,(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) -$$
$$x_{1211\ i}\,z_{32\ i}\,c\,(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) - 2\,\text{signo}\ x_{65}\,y_{10\ i}\,c\,\theta_{54} - x_{65}\,z_{10}\,c\,(\theta_{21} + \theta_{43} + \theta_{54}) -$$
$$x_{65}\,z_{32\ i}\,c\,(\theta_{21} + \theta_{43} + \theta_{54}) - 2\,x_{43\ i}\,x_{65}\,c\,\theta_{54}\,s\,\theta_{21\ i} + 2\,x_{1211\ i}\,c\,(\beta_{1413\ i} - \theta_{54})\,(\text{signo}\ y_{10\ i} + x_{43\ i}\,s\,\theta_{21\ i}) -$$
$$x_{1211\ i}\,x_{32}\,s\,(\beta_{1413\ i} - \theta_{43} - \theta_{54}) + x_{1211\ i}\,x_{43\ i}\,c\,\theta_{21\ i}\,s\,(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54}) +$$
$$x_{32}\,x_{65}\,s\,(\theta_{43} - \theta_{54}) - x_{1211\ i}\,x_{32}\,s\,(\beta_{1413\ i} + \theta_{43} - \theta_{54}) - x_{43\ i}\,x_{65}\,c\,\theta_{21\ i}\,s\,(\theta_{21} + \theta_{43} - \theta_{54}) +$$
$$x_{1211\ i}\,x_{43\ i}\,c\,\theta_{21\ i}\,s\,(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54}) - x_{32}\,x_{65}\,s\,(\theta_{43} + \theta_{54}) + x_{43\ i}\,x_{65}\,c\,\theta_{21\ i}\,s\,(\theta_{21} + \theta_{43} + \theta_{54})$$

Coeficientes de la ecuación (2.139)

$$V_{48} = c\,\theta_{65\ i}$$
$$(-\,x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) - c\,\theta_{21\ i}\,s\,\theta_{21}$$
$$(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) + x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) -$$
$$(z_{32\ i}\,c\,\beta_{54\ i} + \text{signo}\ y_{10\ i}\,c\,\theta_{21\ i}\,s\,\beta_{54\ i} + c\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\theta_{21}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i} +$$
$$(c\,\beta_{54\ i}\,s\,\theta_{21} - c\,\theta_{21}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) -$$
$$(c\,\beta_{54\ i}\,c\,\theta_{21} + s\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\beta_{54\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54}))\,s\,\theta_{65\ i}$$

$$V_{49}=\frac{1}{2}\,(c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,(2\,x_{32}\,c\,\theta_{21}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})+\ x_{65}\,c(\theta_{21}+\theta_{43}-\theta_{54})-$$

$$x_{1211\ i}\,c(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})+\ x_{65}\,c(\theta_{21}+\theta_{43}+\theta_{54})-2\ z_{1312\ i}\,s(\theta_{21}+\theta_{43}))+$$

$$2\,(\ z_{1312\ i}\,c(\theta_{21}+\theta_{43})+x_{32}\,s\,\theta_{21}+(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})\,s(\theta_{21}+\theta_{43}))\,s\,\theta_{65\ i})$$

$$V_{50}=\frac{1}{2}\,(-c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,(\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})-\ x_{65}\,c(\theta_{21}+\theta_{43}-\theta_{54})+$$

$$x_{1211\ i}\,c(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})-\ x_{65}\,c(\theta_{21}+\theta_{43}+\theta_{54})+2\ z_{1312\ i}\,s(\theta_{21}+\theta_{43}))+$$

$$2\,(\ z_{1312\ i}\,c(\theta_{21}+\theta_{43})+(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})\,s(\theta_{21}+\theta_{43}))\,s\,\theta_{65\ i})$$

$$V_{51}=\frac{1}{2}\,(-2\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}+2\ x_{65}\,c\,\theta_{54}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}-$$

$$2\,(\ x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{54})\,(c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,s(\theta_{21}+\theta_{43})-c(\theta_{21}+\theta_{43})\,s\,\theta_{65\ i}))$$

$$V_{52}=-\,x_{43\ i}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}+s\,\theta_{21\ i}\,s\,\theta_{65\ i})$$

Coeficientes de la ecuación (2.143)

$$V_{53}=(\ x_{43\ i}+\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(-x_{32}+c\,\theta_{43}\,(\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})-\ x_{65}\,c\,\theta_{54})+\ z_{1312\ i}\,s\,\theta_{43})+c\,\theta_{21\ i}\,s\,\theta_{21}$$

$$(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})-\ x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})-\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})^{2}$$

$$V_{54}=\frac{1}{2}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}{}^{2}\,(-2\,x_{32}\,c\,\theta_{21}\,s\,\beta_{54\ i}$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})\,s\,\beta_{54\ i}$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-\ x_{65}\,c(\theta_{21}+\theta_{43}-\theta_{54})\,s\,\beta_{54\ i}$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+\ x_{1211\ i}\,c(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})\,s\,\beta_{54\ i}$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-\ x_{65}\,c(\theta_{21}+\theta_{43}+\theta_{54})\,s\,\beta_{54\ i}$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+2\ z_{1312\ i}\,s\,\beta_{54\ i}\,s(\theta_{21}+\theta_{43})$$

$$(-\,x_{43\ i}-\text{signo}\ \ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})-\ z_{1312\ i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(\ z_{1312\ i}\,c\,\theta_{43}-\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+\ x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-$$

$$2\ z_{1312\ i}\,c(\theta_{21}+\theta_{43})\,(\text{signo}\ \ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-\ z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}-$$

$$(s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(\ x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})-\ x_{65}\,c\,\theta_{54})+\ z_{1312\ i}\,s\,\theta_{43})+$$

$$(c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(\ z_{1312\ i}\,c\,\theta_{43}+(-\,x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,c\,\theta_{54})\,s\,\theta_{43})-$$

$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(\ x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+\ x_{65}\,s\,\theta_{54}))-$$

$$2\,x_{32}\,s\,\theta_{21}\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})) - x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54})$$
$$(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})) -$$
$$x_{65}\,s(\theta_{21} + \theta_{43} - \theta_{54})\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})) + x_{1211\ i}\,s(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54})$$
$$(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})) -$$
$$x_{65}\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54}))\,s(\theta_{21} + \theta_{43} + \theta_{54}))$$

$$V_{55} = \frac{1}{2}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}{}^{2}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54})\,s\,\beta_{54\ i}$$
$$(-x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) +$$
$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) - x_{65}\,c(\theta_{21} + \theta_{43} - \theta_{54})\,s\,\beta_{54\ i}$$
$$(-x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) +$$
$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) + x_{1211\ i}\,c(\beta_{1413\ i} + \theta_{21} + \theta_{43} - \theta_{54})\,s\,\beta_{54\ i}$$
$$(-x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) +$$
$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) - x_{65}\,c(\theta_{21} + \theta_{43} + \theta_{54})\,s\,\beta_{54\ i}$$
$$(-x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) +$$
$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) + 2\,z_{1312\ i}\,s\,\beta_{54\ i}\,s(\theta_{21} + \theta_{43})$$
$$(-x_{43\ i} - \text{signo}\ y_{10\ i}\,s\,\theta_{21\ i} + c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54}) - z_{1312\ i}\,s\,\theta_{43}) -$$
$$c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43} - x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54})\,s\,\theta_{43} + x_{65}\,c\,\theta_{54}\,s\,\theta_{43}) +$$
$$x_{1211\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54}) -$$
$$2\,z_{1312\ i}\,c(\theta_{21} + \theta_{43})\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$
$$c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{54}) + x_{65}\,s\,\theta_{54})) - x_{1211\ i}\,s(\beta_{1413\ i} - \theta_{21} - \theta_{43} - \theta_{54})$$
$$(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i} - z_{32\ i}\,s\,\beta_{54\ i} - s\,\beta_{54\ i}\,(z_{10} - x_{32}\,s\,\theta_{21}) + x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i} -$$
$$(s\,\beta_{54\ i}\,s\,\theta_{21} + c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) + z_{1312\ i}\,s\,\theta_{43}) +$$
$$(c\,\theta_{21}\,s\,\beta_{54\ i} - c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43} + (-x_{1211\ i}\,c(\beta_{1413\ i} - \theta_{54}) + x_{65}\,c\,\theta_{54})\,s\,\theta_{43}) -$$

$$
\begin{aligned}
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))- \\
& x_{65}\,s(\theta_{21}+\theta_{43}-\theta_{54})\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})- \\
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))+x_{1211\ i}\,s(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54}) \\
& (\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})- \\
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))- \\
& x_{65}\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})- \\
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))\,s(\theta_{21}+\theta_{43}+\theta_{54}))
\end{aligned}
$$

$$
\begin{aligned}
V_{56} = & \frac{1}{2}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}{}^{2}\,(-2\ x_{1211\ i}\,c\,\beta_{54\ i}\,c\,(\beta_{1413\ i}-\theta_{54}) \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+2\ x_{65}\,c\,\beta_{54\ i}\,c\,\theta_{54} \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})\,s\,\beta_{54\ i} \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})+x_{65}\,c\,(\theta_{21}+\theta_{43}-\theta_{54})\,s\,\beta_{54\ i} \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-x_{1211\ i}\,c\,(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})\,s\,\beta_{54\ i} \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-x_{65}\,c\,(\theta_{21}+\theta_{43}+\theta_{54})\,s\,\beta_{54\ i} \\
& (-x_{43\ i}-\text{signo}\ y_{10\ i}\,s\,\theta_{21\ i}+c\,\theta_{21}\,c\,\theta_{21\ i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\ i}\,s\,\theta_{43})- \\
& c\,\theta_{21\ i}\,s\,\theta_{21}\,(z_{1312\ i}\,c\,\theta_{43}-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+ \\
& x_{1211\ i}\,s\,\theta_{21\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{21\ i}\,s\,\theta_{54})-x_{1211\ i}\,s\,(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54}) \\
& (\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})- \\
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))+ \\
& x_{65}\,s\,(\theta_{21}+\theta_{43}-\theta_{54})\,(\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})- \\
& c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,(x_{1211\ i}\,s\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,s\,\theta_{54}))-x_{1211\ i}\,s\,(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54}) \\
& (\text{signo}\ y_{10\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}-z_{32\ i}\,s\,\beta_{54\ i}-s\,\beta_{54\ i}\,(z_{10}-x_{32}\,s\,\theta_{21})+x_{32}\,c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i}- \\
& (s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})\,(c\,\theta_{43}\,(x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\ i}\,s\,\theta_{43})+ \\
& (c\,\theta_{21}\,s\,\beta_{54\ i}-c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i})\,(z_{1312\ i}\,c\,\theta_{43}+(-x_{1211\ i}\,c\,(\beta_{1413\ i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})-
\end{aligned}
$$

$$\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}\,(\text{x}_{1211\ \text{i}}\,\text{s}\,(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{s}\,\theta_{54}))-$$
$$\text{x}_{65}\,(\text{signo}\ \text{y}_{10\ \text{i}}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}-\text{z}_{32\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}-\text{s}\,\beta_{54\ \text{i}}\,(\text{z}_{10}-\text{x}_{32}\,\text{s}\,\theta_{21})+\text{x}_{32}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}}-$$
$$(\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}+\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{c}\,\theta_{43}\,(\text{x}_{1211\ \text{i}}\,\text{c}\,(\beta_{1413\ \text{i}}-\theta_{54})-\text{x}_{65}\,\text{c}\,\theta_{54})+\text{z}_{1312\ \text{i}}\,\text{s}\,\theta_{43})+$$
$$(\text{c}\,\theta_{21}\,\text{s}\,\beta_{54\ \text{i}}-\text{c}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{z}_{1312\ \text{i}}\,\text{c}\,\theta_{43}+(-\text{x}_{1211\ \text{i}}\,\text{c}\,(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{c}\,\theta_{54})\,\text{s}\,\theta_{43})-$$
$$\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}\,(\text{x}_{1211\ \text{i}}\,\text{s}\,(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{s}\,\theta_{54}))\,\text{s}\,(\theta_{21}+\theta_{43}+\theta_{54}))$$

$$\text{V}_{57}=\text{x}_{43\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}\,\text{c}\,\theta_{76\ \text{i}}{}^{2}\,(-\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}$$
$$(-\text{x}_{43\ \text{i}}-\text{signo}\ \text{y}_{10\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21}\,\text{c}\,\theta_{21\ \text{i}}\,(\text{x}_{32}+\text{c}\,\theta_{43}\,(-\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{c}\,\theta_{54})-\text{z}_{1312\ \text{i}}\,\text{s}\,\theta_{43})-$$
$$\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{21}\,(\text{z}_{1312\ \text{i}}\,\text{c}\,\theta_{43}-\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})\,\text{s}\,\theta_{43}+\text{x}_{65}\,\text{c}\,\theta_{54}\,\text{s}\,\theta_{43})+$$
$$\text{x}_{1211\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{54})+$$
$$\text{s}\,\theta_{21\ \text{i}}\,(\text{signo}\ \text{y}_{10\ \text{i}}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}-\text{z}_{32\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}-\text{s}\,\beta_{54\ \text{i}}\,(\text{z}_{10}-\text{x}_{32}\,\text{s}\,\theta_{21})+\text{x}_{32}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}}-$$
$$(\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}+\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{c}\,\theta_{43}\,(\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})-\text{x}_{65}\,\text{c}\,\theta_{54})+\text{z}_{1312\ \text{i}}\,\text{s}\,\theta_{43})+$$
$$(\text{c}\,\theta_{21}\,\text{s}\,\beta_{54\ \text{i}}-\text{c}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{z}_{1312\ \text{i}}\,\text{c}\,\theta_{43}+(-\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{c}\,\theta_{54})\,\text{s}\,\theta_{43})-$$
$$\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}\,(\text{x}_{1211\ \text{i}}\,\text{s}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{s}\,\theta_{54})))$$

$$\text{V}_{58}=\text{c}\,\theta_{76\ \text{i}}{}^{2}$$
$$(\text{x}_{43\ \text{i}}+\text{signo}\ \text{y}_{10\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21}\,\text{c}\,\theta_{21\ \text{i}}\,(-\text{x}_{32}+\text{c}\,\theta_{43}\,(\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})-\text{x}_{65}\,\text{c}\,\theta_{54})+\text{z}_{1312\ \text{i}}\,\text{s}\,\theta_{43})+\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{21}$$
$$(\text{z}_{1312\ \text{i}}\,\text{c}\,\theta_{43}-\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})\,\text{s}\,\theta_{43}+\text{x}_{65}\,\text{c}\,\theta_{54}\,\text{s}\,\theta_{43})-\text{x}_{1211\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}(\beta_{1413\ \text{i}}-\theta_{54})-\text{x}_{65}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{54})$$
$$(-\text{signo}\ \text{y}_{10\ \text{i}}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}+\text{z}_{32\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}+\text{s}\,\beta_{54\ \text{i}}\,(\text{z}_{10}-\text{x}_{32}\,\text{s}\,\theta_{21})-\text{x}_{32}\,\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}}+$$
$$(\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}+\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{c}\,\theta_{43}\,(\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})-\text{x}_{65}\,\text{c}\,\theta_{54})+\text{z}_{1312\ \text{i}}\,\text{s}\,\theta_{43})-$$
$$(\text{c}\,\theta_{21}\,\text{s}\,\beta_{54\ \text{i}}-\text{c}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{21\ \text{i}})\,(\text{z}_{1312\ \text{i}}\,\text{c}\,\theta_{43}+(-\text{x}_{1211\ \text{i}}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{c}\,\theta_{54})\,\text{s}\,\theta_{43})+$$
$$\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21\ \text{i}}\,(\text{x}_{1211\ \text{i}}\,\text{s}(\beta_{1413\ \text{i}}-\theta_{54})+\text{x}_{65}\,\text{s}\,\theta_{54}))\,\text{s}\,\theta_{65\ \text{i}}$$

Coeficientes de la ecuación (2.148)

$$\text{V}_{59}=(\text{s}(\beta_{1413\ \text{i}}-\theta_{54})\,(\text{c}\,\theta_{21\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}-\text{s}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}})+$$
$$\text{c}\,\theta_{43}\,\text{c}(\beta_{1413\ \text{i}}-\theta_{54})\,(-\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\theta_{21}+\text{c}\,\theta_{21}\,(\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}}))-$$
$$\text{c}(\beta_{1413\ \text{i}}-\theta_{54})\,\text{s}\,\theta_{43}\,(\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{21}\,\text{c}\,\theta_{65\ \text{i}}+\text{s}\,\theta_{21}\,(\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}})))^{2}$$

$$\text{V}_{60}=$$
$$\text{c}\,\theta_{1110\ \text{i}}{}^{2}\,(\text{c}\,\theta_{21\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}-\text{s}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}})\,(\text{c}\,\beta_{54\ \text{i}}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\ \text{i}}+\text{s}(\theta_{21}+\theta_{43})\,(\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}}))$$

$$\text{V}_{61}=$$
$$\text{c}\,\theta_{1110\ \text{i}}{}^{2}\,(\text{c}\,\theta_{21\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}-\text{s}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}})\,(\text{c}\,\beta_{54\ \text{i}}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\ \text{i}}+\text{s}(\theta_{21}+\theta_{43})\,(\text{c}\,\theta_{65\ \text{i}}\,\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}+\text{c}\,\theta_{21\ \text{i}}\,\text{s}\,\theta_{65\ \text{i}}))$$

$$\text{V}_{62}=\text{c}\,\theta_{1110\ \text{i}}{}^{2}$$
$$\Bigg(-\frac{1}{8}\,\text{c}\,\theta_{21\ \text{i}}{}^{2}\,(-4+2\,\text{c}(2\ \beta_{54\ \text{i}})-2\,\text{c}(2\,(\theta_{21}+\theta_{43}))+\text{c}(2\,(\beta_{54\ \text{i}}-\theta_{65\ \text{i}}))+\text{c}(2\,(\theta_{21}+\theta_{43}-\theta_{65\ \text{i}}))+\text{c}(2\,(\beta_{54\ \text{i}}+\theta_{65\ \text{i}}))+$$
$$\text{c}(2\,(\theta_{21}+\theta_{43}+\theta_{65\ \text{i}})))+\text{c}\,\beta_{54\ \text{i}}{}^{2}\,\text{c}\,\theta_{65\ \text{i}}{}^{2}\,\text{s}(\theta_{21}+\theta_{43})^{2}-\text{c}\,\beta_{54\ \text{i}}\,\text{c}\,\theta_{65\ \text{i}}{}^{2}\,\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}(2\,(\theta_{21}+\theta_{43}))+$$
$$\text{s}\,\theta_{21\ \text{i}}{}^{2}\left(\text{c}\,\theta_{21}{}^{2}\,\text{c}\,\theta_{43}{}^{2}\,\text{c}\,\theta_{65\ \text{i}}{}^{2}\,\text{s}\,\beta_{54\ \text{i}}{}^{2}-2\,\text{c}\,\theta_{21}\,\text{c}\,\theta_{43}\,\text{c}\,\theta_{65\ \text{i}}{}^{2}\,\text{s}\,\beta_{54\ \text{i}}{}^{2}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{43}+\text{c}\,\theta_{65\ \text{i}}{}^{2}\,\text{s}\,\beta_{54\ \text{i}}{}^{2}\,\text{s}\,\theta_{21}{}^{2}\,\text{s}\,\theta_{43}{}^{2}+\text{s}\,\theta_{65\ \text{i}}{}^{2}\right)-$$
$$\text{c}\,\theta_{21\ \text{i}}\,\text{s}(\theta_{21}+\theta_{43})\,(\text{c}\,\beta_{54\ \text{i}}\,\text{c}(\theta_{21}+\theta_{43})+\text{s}\,\beta_{54\ \text{i}}\,\text{s}\,\theta_{21\ \text{i}}\,\text{s}(\theta_{21}+\theta_{43}))\,\text{s}(2\,\theta_{65\ \text{i}})\Bigg)$$

$$\text{V}_{63}=\frac{1}{8}\,\text{c}\,\theta_{1110\ \text{i}}{}^{2}\left(\text{s}\,\theta_{21}\left(-(-6+2\,\text{c}(2\ \beta_{54\ \text{i}})+\text{c}(2\,(\beta_{54\ \text{i}}-\theta_{65\ \text{i}}))+2\,\text{c}(2\,\theta_{65\ \text{i}})+\text{c}(2\,(\beta_{54\ \text{i}}+\theta_{65\ \text{i}}))\right)\,\text{s}\,\theta_{43}+\right.$$

$$2\,c\,\theta_{43}\left(\left(1+c\,\theta_{65\ i}{}^{2}\right)s(2\ \beta_{54\ i})\,s\,\theta_{21\ i}-2\,c\,\beta_{54\ i}\,s\,\theta_{65\ i}\,(-2\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\right)\Big)+$$

$$c\,\theta_{21}\left(c\,\theta_{43}\,(-6+2\,c(2\ \beta_{54\ i})+c(2\,(\beta_{54\ i}-\theta_{65\ i}))+2\,c(2\,\theta_{65\ i})+c(2\,(\beta_{54\ i}+\theta_{65\ i})))+\right.$$

$$\left.2\,s\,\theta_{43}\left(\left(1+c\,\theta_{65\ i}{}^{2}\right)s(2\ \beta_{54\ i})\,s\,\theta_{21\ i}-2\,c\,\beta_{54\ i}\,s\,\theta_{65\ i}\,(-2\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\right)\right)\Big)\Big)$$

$$V_{64}=c\,\theta_{1110\ i}{}^{2}\,(s\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{43}\,s\,\theta_{21\ i}+s\,\beta_{54\ i}\,s\,\theta_{43})+c\,\theta_{21}\,(-c\,\theta_{43}\,s\,\beta_{54\ i}+c\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}))$$

Coeficientes de la ecuación (2.152)

$$V_{65}=(c\,\beta_{54\ i}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i}+s(\theta_{21}+\theta_{43})\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\theta_{65\ i}))^{2}$$

$$V_{66}=c\,\theta_{109\ i}{}^{2}\Big(c\,\beta_{54\ i}{}^{2}\,c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})\,c\,\theta_{65\ i}{}^{2}-$$

$$c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,s(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}-s\,\theta_{21\ i}\,s\,\theta_{65\ i})+$$

$$(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\theta_{65\ i})\Big(c\,\theta_{21\ i}\Big(c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})+$$

$$c(\theta_{21}+\theta_{43})^{2}\,c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})\,s\,\theta_{65\ i}+c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})\,s(\theta_{21}+\theta_{43})^{2}\,s\,\theta_{65\ i}\Big)+$$

$$\frac{1}{4}\,s\,\theta_{21\ i}\,(-c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}-\theta_{65\ i})-c(\beta_{1413\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}-\theta_{65\ i})+$$

$$c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}+\theta_{65\ i})+c(\beta_{1413\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}+\theta_{65\ i})-$$

$$s(\beta_{1413\ i}-\ \beta_{54\ i}+\theta_{1110\ i}-\theta_{54}-\theta_{65\ i})+s(\beta_{1413\ i}+\ \beta_{54\ i}+\theta_{1110\ i}-\theta_{54}-\theta_{65\ i})-$$

$$s(\beta_{1413\ i}-\ \beta_{54\ i}+\theta_{1110\ i}-\theta_{54}+\theta_{65\ i})+s(\beta_{1413\ i}+\ \beta_{54\ i}+\theta_{1110\ i}-\theta_{54}+\theta_{65\ i}))\Big)\Big)$$

$$V_{67}=c\,\theta_{109\ i}{}^{2}\,(-c\,\beta_{54\ i}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i}-s(\theta_{21}+\theta_{43})\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\theta_{65\ i}))$$

$$(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{1110\ i}\,s(\beta_{1413\ i}-\theta_{54})-c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,s(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})-$$

$$s\,\theta_{1110\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i}+c(\theta_{21}+\theta_{43})\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{1110\ i}\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\theta_{65\ i})+$$

$$c\,\theta_{1110\ i}\,(s\,\theta_{21\ i}\,(c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s(\beta_{1413\ i}-\theta_{54})+c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i})+$$

$$c\,\theta_{21\ i}\,(-c(\beta_{1413\ i}-\theta_{54})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}+c(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i})))$$

$$V_{68}=$$

$$c\,\theta_{109\ i}{}^{2}\Big(-\frac{1}{8}\,(-6+2\,c(2\ \beta_{54\ i})+c(2\,(\beta_{54\ i}-\theta_{65\ i}))+2\,c(2\,\theta_{65\ i})+c(2\,(\beta_{54\ i}+\theta_{65\ i})))\,s(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})+$$

$$c\,\beta_{54\ i}\,c\,\theta_{65\ i}\Big(c(\theta_{21}+\theta_{43})\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{1110\ i}\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\theta_{65\ i})+$$

$$c(\theta_{21}+\theta_{43})^{2}\,s\,\theta_{1110\ i}\,s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}-s\,\theta_{21\ i}\,s\,\theta_{65\ i})+$$

$$c(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54})\,s(\theta_{21}+\theta_{43})^{2}\,(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}+s\,\theta_{21\ i}\,s\,\theta_{65\ i})+$$

$$c\,\theta_{1110\ i}\,c(\theta_{21}+\theta_{43})\,(s\,\theta_{21\ i}\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s(\beta_{1413\ i}-\theta_{54})+c(\theta_{21}+\theta_{43})\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i})+$$

$$c\,\theta_{21\ i}\,(-c(\theta_{21}+\theta_{43})\,c(\beta_{1413\ i}-\theta_{54})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}+s(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i}))\Big)\Big)$$

$$V_{69}=$$

$$c\,\theta_{109\ i}{}^{2}\,(s\,\beta_{1413\ i}\,(c(\theta_{1110\ i}-\theta_{54})\,s\,\beta_{54\ i}\,s(\theta_{21}+\theta_{43})+c\,\beta_{54\ i}\,(c(\theta_{21}+\theta_{43})\,c(\theta_{1110\ i}-\theta_{54})\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s(\theta_{1110\ i}-\theta_{54})))-$$

$$c\,\beta_{1413\ i}\,(-s\,\beta_{54\ i}\,s(\theta_{21}+\theta_{43})\,s(\theta_{1110\ i}-\theta_{54})+c\,\beta_{54\ i}\,(s\,\theta_{1110\ i}\,(-c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,s\,\theta_{21\ i}+$$

$$c\,\theta_{54}\,s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{43}+c\,\theta_{21\ i}\,s\,\theta_{54})+c\,\theta_{1110\ i}\,(c\,\theta_{21\ i}\,c\,\theta_{54}+c(\theta_{21}+\theta_{43})\,s\,\theta_{21\ i}\,s\,\theta_{54}))))$$

$$V_{70} = -c\,\theta_{109\ i}{}^2\,(c\,\beta_{54\ i}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i} + s(\theta_{21}+\theta_{43})\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i} + c\,\theta_{21\ i}\,s\,\theta_{65\ i}))$$
$$(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{1110\ i}\,s(\beta_{1413\ i}-\theta_{54}) + c\,\beta_{54\ i}\,c\,\theta_{65\ i}\,s(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}+\theta_{1110\ i}-\theta_{54}) +$$
$$s\,\theta_{1110\ i}\,s\,\theta_{21\ i}\,s(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i} - c(\theta_{21}+\theta_{43})\,c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{1110\ i}\,(c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i} + c\,\theta_{21\ i}\,s\,\theta_{65\ i}) -$$
$$c\,\theta_{1110\ i}\,(s\,\theta_{21\ i}\,(c(\theta_{21}+\theta_{43})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i}\,s(\beta_{1413\ i}-\theta_{54}) + c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i}) +$$
$$c\,\theta_{21\ i}\,(-c(\beta_{1413\ i}-\theta_{54})\,c\,\theta_{65\ i}\,s\,\beta_{54\ i} + c(\theta_{21}+\theta_{43})\,s(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{65\ i})))$$

Coeficientes de la ecuación (2.156)

$$V_{71} = (s\,\theta_{109\ i}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s(\theta_{21}+\theta_{43}) -$$
$$s\,\beta_{54\ i}\,(c\,\theta_{43}\,c\,\theta_{76\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i} + c\,\theta_{21}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{65\ i} - c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\ i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\ i}) -$$
$$c\,\beta_{54\ i}\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i} - c\,\theta_{76\ i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\ i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i} + c\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{76\ i})) +$$
$$c\,\theta_{109\ i}\,(c\,\theta_{1110\ i}\,(-s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i} + c\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i} + c\,\beta_{54\ i}\,s\,\theta_{76\ i})) +$$
$$c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21} + s\,\beta_{54\ i}\,(c\,\theta_{76\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i} - c\,\theta_{21}\,s\,\theta_{76\ i}) +$$
$$c\,\beta_{54\ i}\,(c\,\theta_{21}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i} + s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i})) +$$
$$c\,\theta_{43}\,c(\beta_{1413\ i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i} - s\,\beta_{54\ i}\,s\,\theta_{76\ i}) +$$
$$c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i} - s\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i} + c\,\beta_{54\ i}\,s\,\theta_{76\ i})))) -$$
$$s\,\theta_{1110\ i}\,(c(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i} + c\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i} + c\,\beta_{54\ i}\,s\,\theta_{76\ i})) +$$
$$s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{43}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i} - c\,\theta_{43}\,s\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i} + c\,\beta_{54\ i}\,s\,\theta_{76\ i}) +$$
$$s\,\theta_{43}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i} - s\,\beta_{54\ i}\,s\,\theta_{76\ i})) +$$
$$s\,\theta_{21}\,(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{43} + s\,\beta_{54\ i}\,(c\,\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{65\ i} - c\,\theta_{43}\,s\,\theta_{76\ i}) +$$
$$c\,\beta_{54\ i}\,(c\,\theta_{43}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i} + s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{76\ i}))))))^2$$

$$V_{72} = \frac{1}{128}\,c\,\theta_{98\ i}{}^2\,(8\,c(2\,\theta_{109\ i}-\theta_{21\ i}-\theta_{65\ i}) + 48\,c(\theta_{21\ i}-\theta_{65\ i}) -$$
$$8\,c(2\,\theta_{109\ i}+\theta_{21\ i}-\theta_{65\ i}) - 8\,c(2\,\beta_{1413\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$4\,c(2\,\beta_{1413\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) - 4\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$8\,c(2\,\beta_{1413\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) + 4\,c(2\,\beta_{1413\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$4\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i}) - 4\,c(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$4\,c(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$4\,c(2\,\beta_{1413\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$4\,c(2\,\beta_{1413\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$4\,c(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$4\,c(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$
$$2\,c(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i}) +$$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$

$4\,\mathrm{c}(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,\mathrm{c}(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) - 8\,\mathrm{c}(2\,\theta_{109\,i} - \theta_{21\,i} + \theta_{65\,i}) -$

$48\,\mathrm{c}(\theta_{21\,i} + \theta_{65\,i}) + 8\,\mathrm{c}(2\,\theta_{109\,i} + \theta_{21\,i} + \theta_{65\,i}) + 8\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$

$4\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) + 4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$

$8\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$4\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$

$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}-\theta_{21\ i}+\theta_{43}-2\,\theta_{54}+\theta_{65\ i})-$
$4\,c(2\,\beta_{1413\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{21\ i}+\theta_{43}-2\,\theta_{54}+\theta_{65\ i})-$
$2\,c(2\,\beta_{1413\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{21\ i}+\theta_{43}-2\,\theta_{54}+\theta_{65\ i})-$
$2\,c(2\,\beta_{1413\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21}+\theta_{21\ i}+\theta_{43}-2\,\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}-\theta_{21\ i}+\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}-\theta_{21\ i}+\theta_{43}-\theta_{54}+\theta_{65\ i})-$
$4\,c(\beta_{1413\ i}-2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{21\ i}+\theta_{43}-\theta_{54}+\theta_{65\ i})+$
$4\,c(\beta_{1413\ i}+2\,\theta_{109\ i}+\theta_{1110\ i}+\theta_{21}+\theta_{21\ i}+\theta_{43}-\theta_{54}+\theta_{65\ i})-24\,s(\beta_{54\ i}-\theta_{21\ i}-\theta_{65\ i})+$
$4\,s(\beta_{54\ i}-2\,\theta_{109\ i}-\theta_{21\ i}-\theta_{65\ i})+4\,s(\beta_{54\ i}+2\,\theta_{109\ i}-\theta_{21\ i}-\theta_{65\ i})-$
$24\,s(\beta_{54\ i}+\theta_{21\ i}-\theta_{65\ i})+4\,s(\beta_{54\ i}-2\,\theta_{109\ i}+\theta_{21\ i}-\theta_{65\ i})+4\,s(\beta_{54\ i}+2\,\theta_{109\ i}+\theta_{21\ i}-\theta_{65\ i})+$
$4\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-4\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})+$
$4\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-4\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}+\theta_{21\ i}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$s(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$s(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}-\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$s(2\,\beta_{1413\ i}-\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$s(2\,\beta_{1413\ i}+\beta_{54\ i}-2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$
$s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$s(2\,\beta_{1413\ i}+\beta_{54\ i}+2\,\theta_{109\ i}+2\,\theta_{1110\ i}-\theta_{21}+\theta_{21\ i}-\theta_{43}-2\,\theta_{54}-\theta_{65\ i})-$
$2\,s(2\,\beta_{1413\ i}-\beta_{54\ i}+2\,\theta_{1110\ i}+\theta_{21}-\theta_{21\ i}+\theta_{43}-2\,\theta_{54}-\theta_{65\ i})+$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} - \theta_{65\,\mathrm{i}}) -$

$24\,\mathrm{s}(\beta_{54\,\mathrm{i}} - \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) + 4\,\mathrm{s}(\beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} - \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) + 4\,\mathrm{s}(\beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} - \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) -$

$24\,\mathrm{s}(\beta_{54\,\mathrm{i}} + \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) + 4\,\mathrm{s}(\beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) + 4\,\mathrm{s}(\beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{21\,\mathrm{i}} + \theta_{65\,\mathrm{i}}) +$

$4\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) - 4\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$4\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) - 4\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} + \theta_{21\,\mathrm{i}} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) -$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$\mathrm{s}(2\,\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + 2\,\theta_{1110\,\mathrm{i}} - \theta_{21} - \theta_{21\,\mathrm{i}} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,\mathrm{i}}) +$

$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}))$

$V_{73} = \frac{1}{128}\,c\,\theta_{98\,i}{}^2\,(8\,c(2\,\theta_{109\,i} - \theta_{21\,i} - \theta_{65\,i}) + 48\,c(\theta_{21\,i} - \theta_{65\,i}) -$
$8\,c(2\,\theta_{109\,i} + \theta_{21\,i} - \theta_{65\,i}) - 8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) + 4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$

$2\,c(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$2\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$4\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$4\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) - 8\,c(2\,\theta_{109\ i} - \theta_{21\ i} + \theta_{65\ i}) -$
$48\,c(\theta_{21\ i} + \theta_{65\ i}) + 8\,c(2\,\theta_{109\ i} + \theta_{21\ i} + \theta_{65\ i}) + 8\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} - \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) +$
$4\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) + 4\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) -$
$8\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} + \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) - 4\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21\ i} - 2\,\theta_{54} + \theta_{65\ i}) - 4\,c(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,c(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,c(2\,\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,c(2\,\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,c(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,c(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$

$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$4\,c(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$4\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) - 24\,s(\beta_{54\,i} - \theta_{21\,i} - \theta_{65\,i}) +$
$4\,s(\beta_{54\,i} - 2\,\theta_{109\,i} - \theta_{21\,i} - \theta_{65\,i}) + 4\,s(\beta_{54\,i} + 2\,\theta_{109\,i} - \theta_{21\,i} - \theta_{65\,i}) -$
$24\,s(\beta_{54\,i} + \theta_{21\,i} - \theta_{65\,i}) + 4\,s(\beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{21\,i} - \theta_{65\,i}) + 4\,s(\beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{21\,i} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$

$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$24\,s(\beta_{54\,i} - \theta_{21\,i} + \theta_{65\,i}) + 4\,s(\beta_{54\,i} - 2\,\theta_{109\,i} - \theta_{21\,i} + \theta_{65\,i}) + 4\,s(\beta_{54\,i} + 2\,\theta_{109\,i} - \theta_{21\,i} + \theta_{65\,i}) -$
$24\,s(\beta_{54\,i} + \theta_{21\,i} + \theta_{65\,i}) + 4\,s(\beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{21\,i} + \theta_{65\,i}) + 4\,s(\beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{21\,i} + \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$

$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$
$2\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$
$2\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) -$

$$2\,\mathrm{s}(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}) +$$
$$2\,\mathrm{s}(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} + \theta_{65\,i}))$$

$$\mathrm{V}_{74} = \frac{1}{2}\,\mathrm{c}\,\theta_{98\,i}{}^{2}\,\big(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{21\,i} +$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{109\,i}{}^{2}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43} -$$
$$\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{109\,i}{}^{2}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43} +$$
$$\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\big(2\,\mathrm{c}\,\theta_{109\,i}{}^{2}\,\mathrm{c}(\theta_{21} + \theta_{43}) + \mathrm{c}(\beta_{1413\,i} + \theta_{1110\,i} - \theta_{54})\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}(\theta_{21} + \theta_{43})\big) +$$
$$\mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54} +$$
$$\mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54} +$$
$$\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} -$$
$$\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{109\,i}{}^{2}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} +$$
$$\mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{109\,i}{}^{2}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} +$$
$$\mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i} +$$
$$\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}(2\,\theta_{109\,i})\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i} +$$
$$\mathrm{c}\,\theta_{109\,i}{}^{2}\,\mathrm{s}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - 2\,\mathrm{c}\,\theta_{109\,i}\,\mathrm{s}\,\theta_{109\,i}\,(\mathrm{c}\,\theta_{1110\,i}$$
$$(\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\beta_{1413\,i}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i}) +$$
$$\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{1413\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{1413\,i}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i})) +$$
$$\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{s}\,\beta_{1413\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})) +$$
$$\mathrm{c}\,\beta_{1413\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i})))))\big)$$

$$\mathrm{V}_{75} = -\frac{1}{256}\,\mathrm{c}\,\theta_{98\,i}{}^{2}$$
$$(80\,\mathrm{c}(\beta_{54\,i} - \theta_{65\,i}) + 8\,\mathrm{c}(\beta_{54\,i} - 2\,\theta_{109\,i} - \theta_{65\,i}) + 8\,\mathrm{c}(\beta_{54\,i} + 2\,\theta_{109\,i} - \theta_{65\,i}) - 8\,\mathrm{c}(\beta_{54\,i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{65\,i}) +$$
$$12\,\mathrm{c}(\beta_{54\,i} - 2\,\theta_{109\,i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{65\,i}) + 12\,\mathrm{c}(\beta_{54\,i} + 2\,\theta_{109\,i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{65\,i}) -$$
$$12\,\mathrm{c}(2\,\theta_{109\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - \theta_{65\,i}) - 12\,\mathrm{c}(2\,\theta_{109\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - \theta_{65\,i}) -$$
$$8\,\mathrm{c}(\beta_{54\,i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{65\,i}) + 12\,\mathrm{c}(\beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{65\,i}) +$$
$$12\,\mathrm{c}(\beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{65\,i}) - 8\,\mathrm{c}(2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{65\,i}) +$$
$$12\,\mathrm{c}(2\,\theta_{109\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{65\,i}) - 8\,\mathrm{c}(2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{65\,i}) +$$
$$12\,\mathrm{c}(2\,\theta_{109\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{65\,i}) - 8\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$8\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) - 4\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$4\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$$
$$2\,\mathrm{c}(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$$

$8\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$8\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$4\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$8\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$4\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$8\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} - \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} - \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} - \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$4\ c(2\ \beta_{1413\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} - 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) +$
$2\ c(2\ \beta_{1413\ i} + 2\ \theta_{109\ i} + 2\ \theta_{1110\ i} + 2\ \theta_{21} + \theta_{21\ i} + 2\ \theta_{43} - 2\ \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} - \beta_{54\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + \beta_{54\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - \beta_{54\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} + \beta_{54\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - \theta_{21\ i} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} - \theta_{21\ i} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} + \theta_{21\ i} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - 2\ \theta_{21} + \theta_{21\ i} - 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - \beta_{54\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} + \beta_{54\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} - \beta_{54\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$
$8\ c(\beta_{1413\ i} + \beta_{54\ i} + 2\ \theta_{109\ i} + \theta_{1110\ i} + 2\ \theta_{21} + 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) +$
$8\ c(\beta_{1413\ i} - 2\ \theta_{109\ i} + \theta_{1110\ i} + 2\ \theta_{21} - \theta_{21\ i} + 2\ \theta_{43} - \theta_{54} - \theta_{65\ i}) -$

$8\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$8\,c(\beta_{1413\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$8\,c(\beta_{1413\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) + 80\,c(\beta_{54\,i} + \theta_{65\,i}) + 8\,c(\beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{65\,i}) +$
$8\,c(\beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{65\,i}) - 8\,c(\beta_{54\,i} - 2\,\theta_{21} - 2\,\theta_{43} + \theta_{65\,i}) + 12\,c(\beta_{54\,i} - 2\,\theta_{109\,i} - 2\,\theta_{21} - 2\,\theta_{43} + \theta_{65\,i}) +$
$12\,c(\beta_{54\,i} + 2\,\theta_{109\,i} - 2\,\theta_{21} - 2\,\theta_{43} + \theta_{65\,i}) + 12\,c(2\,\theta_{109\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) +$
$12\,c(2\,\theta_{109\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) - 8\,c(\beta_{54\,i} + 2\,\theta_{21} + 2\,\theta_{43} + \theta_{65\,i}) +$
$12\,c(\beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{21} + 2\,\theta_{43} + \theta_{65\,i}) + 12\,c(\beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{21} + 2\,\theta_{43} + \theta_{65\,i}) +$
$8\,c(2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) - 12\,c(2\,\theta_{109\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) +$
$8\,c(2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) - 12\,c(2\,\theta_{109\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) -$
$8\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 8\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) - 4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$8\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,c(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,c(2\,\beta_{1413\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,c(2\,\beta_{1413\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$$4\,c(2\,\beta_{1413\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$$
$$2\,c(2\,\beta_{1413\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$$
$$8\,c(\beta_{1413\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$8\,c(\beta_{1413\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$$
$$4\,s(\beta_{54\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) - 6\,s(\beta_{54\ i} - 2\,\theta_{109\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) -$$
$$6\,s(\beta_{54\ i} + 2\,\theta_{109\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) - 4\,s(\beta_{54\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) +$$
$$6\,s(\beta_{54\ i} - 2\,\theta_{109\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) + 6\,s(\beta_{54\ i} + 2\,\theta_{109\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{65\ i}) -$$
$$4\,s(\beta_{54\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) + 6\,s(\beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) +$$
$$6\,s(\beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) + 4\,s(\beta_{54\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) -$$
$$6\,s(\beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) - 6\,s(\beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - \theta_{65\ i}) +$$
$$2\,s(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$2\,s(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$$
$$s(2\,\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$s(2\,\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$$
$$s(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$s(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$2\,s(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$$
$$2\,s(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$s(2\,\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$$
$$s(2\,\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$
$$s(2\,\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) +$$
$$s(2\,\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\ i}) -$$

$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - 2\,\theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$

$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} - \theta_{21} + \theta_{21\,i} - \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} - \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + \theta_{21} + \theta_{21\,i} + \theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) -$
$4\,s(\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + \theta_{1110\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} - \theta_{54} - \theta_{65\,i}) +$
$4\,s(\beta_{54\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) - 6\,s(\beta_{54\,i} - 2\,\theta_{109\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) -$
$6\,s(\beta_{54\,i} + 2\,\theta_{109\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) - 4\,s(\beta_{54\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) +$
$6\,s(\beta_{54\,i} - 2\,\theta_{109\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) + 6\,s(\beta_{54\,i} + 2\,\theta_{109\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} + \theta_{65\,i}) -$
$4\,s(\beta_{54\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) + 6\,s(\beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) +$
$6\,s(\beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{21} - \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) + 4\,s(\beta_{54\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) -$
$6\,s(\beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) - 6\,s(\beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{21} + \theta_{21\,i} + 2\,\theta_{43} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} - \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - 2\,\theta_{21} + \theta_{21\,i} - 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$4\,s(2\,\beta_{1413\,i} - \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$4\,s(2\,\beta_{1413\,i} + \beta_{54\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$
$2\,s(2\,\beta_{1413\,i} - \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) +$
$2\,s(2\,\beta_{1413\,i} + \beta_{54\,i} - 2\,\theta_{109\,i} + 2\,\theta_{1110\,i} - \theta_{21} - \theta_{21\,i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\,i}) -$

$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$4\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$4\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} - \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$4\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + \theta_{21} + \theta_{21\ i} + \theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$s(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$s(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} - \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$2\,s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$2\,s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$s(2\ \beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$s(2\ \beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) +$
$s(2\ \beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$s(2\ \beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + 2\,\theta_{1110\ i} + 2\,\theta_{21} + \theta_{21\ i} + 2\,\theta_{43} - 2\,\theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} - \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - 2\,\theta_{21} + \theta_{21\ i} - 2\,\theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) -$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} + \beta_{54\ i} + 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} - \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) +$
$4\,s(\beta_{1413\ i} - \beta_{54\ i} - 2\,\theta_{109\ i} + \theta_{1110\ i} - \theta_{21} + \theta_{21\ i} - \theta_{43} - \theta_{54} + \theta_{65\ i}) -$

$$
\begin{aligned}
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{21} + \theta_{21\,\mathrm{i}} - \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} - \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + \theta_{21} + \theta_{21\,\mathrm{i}} + \theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} - \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} - \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} - \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} - \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} + \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} - 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} + \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) -\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} - \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} + \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}) +\\
&4\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \beta_{54\,\mathrm{i}} + 2\,\theta_{109\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} + 2\,\theta_{21} + \theta_{21\,\mathrm{i}} + 2\,\theta_{43} - \theta_{54} + \theta_{65\,\mathrm{i}}))
\end{aligned}
$$

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\begin{aligned}
\mathrm{V}_{76} = -\mathrm{c}\,\theta_{98\,\mathrm{i}}{}^{2} \Bigg(&-\frac{1}{2}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{s}(\theta_{109\,\mathrm{i}} - \theta_{21} - \theta_{43}) - 2\,\mathrm{c}\,\theta_{109\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54})\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{s}(\theta_{109\,\mathrm{i}} + \theta_{21} + \theta_{43})) + \mathrm{c}\,\beta_{54\,\mathrm{i}}\\
&(\mathrm{s}\,\theta_{109\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54})\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}} + \theta_{1110\,\mathrm{i}} - \theta_{54})))\Bigg)\\
&(\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} + \mathrm{s}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})) +\\
&\mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(-\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{54} +\\
&\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}))\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} - \mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})) +\\
&\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{1413\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))) + \mathrm{s}\,\theta_{1110\,\mathrm{i}}\\
&(\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})) -\\
&\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54} + \mathrm{s}\,\theta_{21\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\\
&\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) - \mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})))))
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{V}_{77} = -\mathrm{c}\,\theta_{98\,\mathrm{i}}{}^{2} \Bigg(&-2\,\mathrm{c}\,\theta_{109\,\mathrm{i}}\,\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} + \mathrm{s}(\theta_{21} + \theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\\
&(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(-\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{54} +\\
&\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}))\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} - \mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})) +\\
&\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}\,\beta_{1413\,\mathrm{i}}\,\mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} + \mathrm{c}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))) + \mathrm{s}\,\theta_{1110\,\mathrm{i}}\\
&(\mathrm{s}\,\beta_{1413\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) +\\
&\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + \mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43}) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})) -\\
&\mathrm{c}\,\beta_{1413\,\mathrm{i}}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21} + \theta_{43})\,\mathrm{s}\,\theta_{54} + \mathrm{s}\,\theta_{21\,\mathrm{i}}\,(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{54} + \mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}
\end{aligned}
$$

$$\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{54}+\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i})-\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\theta_{65\,i}))))+$$

$$\mathrm{s}\,\theta_{109\,i}{}^2\Big(\mathrm{s}\,\theta_{21}{}^2\,\big(\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{c}\,\theta_{43}{}^2\,\mathrm{c}\,\theta_{65\,i}{}^2+\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}{}^2\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}(2\,\theta_{43})+2\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}$$

$$(-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i}+$$

$$\mathrm{s}\,\beta_{54\,i}{}^2\,\big(\mathrm{s}\,\theta_{43}{}^2+\mathrm{c}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)+\mathrm{c}\,\beta_{54\,i}{}^2\,\big(\mathrm{c}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2+\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)\big)+$$

$$\mathrm{c}\,\theta_{21}{}^2\,\big(-2\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+\mathrm{c}\,\theta_{43}{}^2\,\big(\mathrm{s}\,\beta_{54\,i}{}^2+\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)+$$

$$\mathrm{s}\,\theta_{43}{}^2\,\big(\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{c}\,\theta_{65\,i}{}^2-2\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21\,i}{}^2\,\big(\mathrm{c}\,\beta_{54\,i}{}^2+\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)\big)\big)+2\,\mathrm{c}\,\theta_{21}$$

$$\mathrm{s}\,\theta_{21}\Big(-\frac{1}{4}\,\mathrm{c}\,\beta_{54\,i}{}^2\,(\mathrm{c}(2\,\theta_{21\,i})-\mathrm{c}(2\,\theta_{65\,i}))\,\mathrm{s}(2\,\theta_{43})-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}(2\,\theta_{43})\,\mathrm{c}\,\theta_{65\,i}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{43}\,\big(\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{c}\,\theta_{65\,i}{}^2-2\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}{}^2\,\big(-1+\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)\big)\Big)\Big)+$$

$$\frac{1}{16}\,\mathrm{c}\,\theta_{109\,i}{}^2\,\big(4\,\mathrm{c}\,\beta_{54\,i}{}^2\,\big(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,(\mathrm{c}(2\,\beta_{1413\,i}-\theta_{1110\,i}-2\,\theta_{54})+3\,\mathrm{c}(2\,\beta_{1413\,i}+\theta_{1110\,i}-2\,\theta_{54}))\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}(2\,\theta_{21\,i})-$$

$$2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}+2\,\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2+$$

$$4\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2-2\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2+$$

$$2\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))-2\,\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$2\,\mathrm{c}\,\theta_{21\,i}{}^2\,\big(2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{1110\,i}{}^2+\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))\big)+$$

$$4\,\mathrm{c}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{65\,i}{}^2+2\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{65\,i}{}^2-$$

$$2\,\mathrm{c}\,\theta_{43}{}^2\,\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))\,\mathrm{s}\,\theta_{65\,i}{}^2-$$

$$\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))\,\mathrm{s}\,\theta_{65\,i}{}^2+$$

$$2\,\mathrm{c}\,\theta_{21}{}^2\,\big(2\,\mathrm{s}\,\theta_{1110\,i}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2-\mathrm{s}(2\,\theta_{1110\,i})\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))\big)\big(\mathrm{c}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2+\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2\big)\big)+$$

$$\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{1110\,i}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(2\,\mathrm{s}(\theta_{1110\,i}-\theta_{21}-\theta_{21\,i}-\theta_{43})+2\,\mathrm{s}(\theta_{1110\,i}-\theta_{21}+\theta_{21\,i}-\theta_{43})+$$

$$2\,\mathrm{s}(\theta_{1110\,i}+\theta_{21}-\theta_{21\,i}+\theta_{43})+2\,\mathrm{s}(\theta_{1110\,i}+\theta_{21}+\theta_{21\,i}+\theta_{43})+2\,\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}-\theta_{21\,i}-2\,\theta_{54})+$$

$$6\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}-\theta_{21\,i}-2\,\theta_{54})-2\,\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}+\theta_{21\,i}-2\,\theta_{54})-$$

$$6\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}+\theta_{21\,i}-2\,\theta_{54})-\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}-\theta_{21}-\theta_{21\,i}-\theta_{43}-2\,\theta_{54})-$$

$$3\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}-\theta_{21}-\theta_{21\,i}-\theta_{43}-2\,\theta_{54})-\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}-\theta_{21}+\theta_{21\,i}-\theta_{43}-2\,\theta_{54})-$$

$$3\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}-\theta_{21}+\theta_{21\,i}-\theta_{43}-2\,\theta_{54})-\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}+\theta_{21}-\theta_{21\,i}+\theta_{43}-2\,\theta_{54})-$$

$$3\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}+\theta_{21}-\theta_{21\,i}+\theta_{43}-2\,\theta_{54})-\mathrm{s}(2\,\beta_{1413\,i}-\theta_{1110\,i}+\theta_{21}+\theta_{21\,i}+\theta_{43}-2\,\theta_{54})-$$

$$3\,\mathrm{s}(2\,\beta_{1413\,i}+\theta_{1110\,i}+\theta_{21}+\theta_{21\,i}+\theta_{43}-2\,\theta_{54}))\,\mathrm{s}(2\,\theta_{65\,i})+$$

$$4\,\mathrm{c}\,\theta_{1110\,i}{}^2\,\big(4\,\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{c}\,\theta_{65\,i}{}^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{43}{}^2-2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\beta_{54\,i})\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}{}^2+$$

$$4\,\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{43}{}^2-2\,\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{c}\,\theta_{65\,i}{}^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{43})+$$

$$2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{43})-2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}(2\,\theta_{43})-$$

$$2\,\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}(2\,\theta_{43})+4\,\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2+$$

$$4\,\mathrm{c}\,\theta_{65\,i}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2-2\,\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$2\,\mathrm{c}\,\theta_{65\,i}{}^2\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))+2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\beta_{54\,i})\,\mathrm{s}(2\,\theta_{21})$$

$$\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2+4\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}\,\theta_{65\,i}{}^2+$$

$$2\,\mathrm{c}\,\beta_{54\,i}{}^2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i}{}^2+2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21}{}^2$$

$$\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i}{}^2-2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{21\,i}{}^2\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i}{}^2+$$

$$4\,\mathrm{c}\,\theta_{21\,i}{}^2\,\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{65\,i}{}^2-2\,\mathrm{s}\,\beta_{54\,i}{}^2\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(2\,(\beta_{1413\,i}-\theta_{54}))\,\mathrm{s}\,\theta_{65\,i}{}^2-$$

$$2\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}(2\,\theta_{65\,i})-2\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}{}^2$$

$$\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}\,\theta_{43}{}^2\,\mathrm{s}(2\,\theta_{65\,i})-2\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\theta_{21}{}^2\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(2\,\theta_{65\,i})+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{21})\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}(2\,\theta_{43})\,\mathrm{s}(2\,\theta_{65\,i})+2\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(2\,\theta_{21\,i})\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})^2\,\mathrm{s}(2\,\theta_{65\,i})+$$

$$2\,c\,\theta_{21\,i}{}^2\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})-2\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}{}^2\,s\,\theta_{43}\,s(2\,(\beta_{1413\,i}-\theta_{54}))$$

$$s(2\,\theta_{65\,i})+2\,c\,\theta_{43}\,s\,\theta_{21}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\left(c\,\theta_{21\,i}\,c\,\theta_{65\,i}{}^2\,s(2\,\beta_{54\,i})-c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s(2\,\theta_{65\,i})\right)\Big)+$$

$$2\,c\,\theta_{21}{}^2\,c(\beta_{1413\,i}-\theta_{54})^2\,\Big(2\,c\,\theta_{21\,i}{}^2\,c\,\theta_{43}{}^2\,c\,\theta_{65\,i}{}^2+2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{43}{}^2+s(2\,\beta_{54\,i})\,s\,\theta_{21\,i}\,s(2\,\theta_{43})+$$

$$2\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{21\,i}{}^2\,s\,\theta_{65\,i}{}^2-s(2\,\beta_{54\,i})\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s\,\theta_{65\,i}{}^2+2\,c\,\beta_{54\,i}{}^2\,\Big(c\,\theta_{43}{}^2\,s\,\theta_{21\,i}{}^2+$$

$$s\,\theta_{43}{}^2\,s\,\theta_{65\,i}{}^2\Big)-c\,\theta_{43}{}^2\,s\,\beta_{54\,i}\,s(2\,\theta_{21\,i})\,s(2\,\theta_{65\,i})+c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(2\,\theta_{65\,i})\Big)+$$

$$2\,c\,\theta_{21}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,\Big(c\,\beta_{54\,i}{}^2\,c\,\theta_{43}\,s(2\,\theta_{21\,i})+c\,\theta_{21\,i}\,c\,\theta_{65\,i}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{43}-c\,\beta_{54\,i}\,s\,\theta_{21\,i}$$

$$s\,\theta_{43}\,s(2\,\theta_{65\,i})-c\,\theta_{43}\,\left(c\,\theta_{65\,i}{}^2\,s(2\,\theta_{21\,i})-s\,\beta_{54\,i}{}^2\,s(2\,\theta_{21\,i})\,s\,\theta_{65\,i}{}^2+c(2\,\theta_{21\,i})\,s\,\beta_{54\,i}\,s(2\,\theta_{65\,i})\right)\Big)+$$

$$2\,c\,\theta_{43}{}^2\,c(\beta_{1413\,i}-\theta_{54})^2\,\Big(2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{21}{}^2+c\,\theta_{65\,i}{}^2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{21})\,s\,\theta_{21\,i}+$$

$$c\,\beta_{54\,i}\,\left(2\,c\,\beta_{54\,i}\,s\,\theta_{21}{}^2\,s\,\theta_{65\,i}{}^2+c\,\theta_{21\,i}\,s(2\,\theta_{21})\,s(2\,\theta_{65\,i})\right)\Big)\Big)\Big)+$$

$$2\,\Big(2\,c\,\theta_{21\,i}\,(c(2\,\beta_{1413\,i}-\theta_{1110\,i}-2\,\theta_{54})+3\,c(2\,\beta_{1413\,i}+\theta_{1110\,i}-2\,\theta_{54}))\,c\,\theta_{65\,i}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}\,s(\theta_{21}+\theta_{43})+$$

$$8\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2+4\,c\,\theta_{43}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})$$

$$s\,\theta_{21\,i}\,s(\beta_{1413\,i}-\theta_{54})^2+4\,c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}{}^2\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21\,i})\,s(\beta_{1413\,i}-\theta_{54})^2-$$

$$4\,c\,\theta_{65\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}\,s(2\,\theta_{21\,i})\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})^2+8\,c\,\theta_{21}{}^2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2$$

$$s\,\theta_{43}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2-4\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s\,\theta_{43}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2+$$

$$4\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})\,s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2+4\,c\,\theta_{21}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21\,i}$$

$$s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2-4\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2-$$

$$4\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))-2\,c\,\theta_{43}{}^2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})$$

$$s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s(2\,(\beta_{1413\,i}-\theta_{54}))+4\,c\,\theta_{65\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21\,i}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$4\,c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))-4\,c\,\theta_{65\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}$$

$$s(2\,\theta_{21\,i})\,s\,\theta_{43}\,s(2\,(\beta_{1413\,i}-\theta_{54}))-4\,c\,\theta_{21}{}^2\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{43}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s\,\theta_{43}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))-2\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})$$

$$s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))-2\,c\,\theta_{21}{}^2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))+$$

$$2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))-4\,c\,\theta_{43}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})$$

$$s\,\theta_{21\,i}\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+8\,c\,\theta_{21}{}^2\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21\,i}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2-$$

$$4\,c\,\theta_{21}\,c\,\theta_{43}\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21\,i})\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+4\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}\,s(2\,\theta_{21\,i})$$

$$s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+4\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s\,\theta_{43}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+$$

$$8\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}{}^2\,s\,\theta_{43}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2-4\,c\,\theta_{21}{}^2\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21\,i}$$

$$s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+4\,s(2\,\beta_{54\,i})\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2-$$

$$4\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})\,s\,\theta_{21\,i}{}^2\,s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2\,s\,\theta_{65\,i}{}^2+$$

$$2\,c\,\theta_{43}{}^2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$4\,c\,\theta_{21}{}^2\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21\,i}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$4\,c\,\theta_{21}\,c\,\theta_{43}\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2+$$

$$4\,s\,\beta_{54\,i}{}^2\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}\,s(2\,\theta_{21\,i})\,s\,\theta_{43}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})\,s\,\theta_{21\,i}\,s\,\theta_{43}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$4\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}{}^2\,s\,\theta_{43}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2+$$

$$2\,c\,\theta_{21}{}^2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$2\,s(2\,\beta_{54\,i})\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}{}^2\,s\,\theta_{21\,i}\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2+$$

$$2\,s\,\beta_{54\,i}{}^2\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})\,s\,\theta_{21\,i}{}^2\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^2-$$

$$4\,c\,\theta_{21}\,c\,\theta_{43}\,s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s\,\theta_{21\,i}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2\,s(2\,\theta_{65\,i})-$$

$$4\,c\,\theta_{21}{}^2\,c\,\theta_{43}{}^2\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21\,i})\,s(\beta_{1413\,i}-\theta_{54})^2\,s(2\,\theta_{65\,i})+$$

$$4\,s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}\,s\,\theta_{21\,i}{}^2\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})^2\,s(2\,\theta_{65\,i})-$$

$$4\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21}{}^2\,s(2\,\theta_{21\,i})\,s\,\theta_{43}{}^2\,s(\beta_{1413\,i}-\theta_{54})^2\,s(2\,\theta_{65\,i})+$$

$$2\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^2\,s(2\,\theta_{21})\,s(2\,\theta_{21\,i})\,s(2\,\theta_{43})\,s(\beta_{1413\,i}-\theta_{54})^2\,s(2\,\theta_{65\,i})-$$

$$4\,c\,\theta_{21}\,c\,\theta_{43}\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^2\,s\,\theta_{21\,i}{}^2\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})+$$

$$2\,s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})+$$
$$2\,c\,\theta_{21}{}^{2}\,c\,\theta_{43}{}^{2}\,s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})+$$
$$4\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^{2}\,s\,\theta_{21}\,s\,\theta_{21\,i}{}^{2}\,s\,\theta_{43}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})+$$
$$2\,s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}{}^{2}\,s(2\,\theta_{21\,i})\,s\,\theta_{43}{}^{2}\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})-$$
$$s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21})\,s(2\,\theta_{21\,i})\,s(2\,\theta_{43})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s(2\,\theta_{65\,i})+$$
$$2\,c\,\theta_{21\,i}{}^{2}\Big((c(\theta_{21}-\theta_{43})+3\,c(\theta_{21}+\theta_{43}))\,(c(\beta_{1413\,i}-\theta_{1110\,i}-\theta_{54})+3\,c(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54}))$$
$$c\,\theta_{65\,i}{}^{2}\,s\,\theta_{1110\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})+$$
$$2\,c\,\theta_{21}{}^{2}\,c\,\theta_{43}{}^{2}\,c\,\theta_{65\,i}{}^{2}\left(2\,s\,\theta_{1110\,i}{}^{2}\,s(\beta_{1413\,i}-\theta_{54})^{2}-s(2\,\theta_{1110\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\right)-$$
$$c\,\theta_{21}\,c\,\theta_{43}\,(c(2\,\beta_{1413\,i}-\theta_{1110\,i}-2\,\theta_{54})+3\,c(2\,\beta_{1413\,i}+\theta_{1110\,i}-2\,\theta_{54}))\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}\,s(2\,\theta_{65\,i})+$$
$$2\,s\,\beta_{54\,i}\Big(s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s(2\,(\beta_{1413\,i}-\theta_{54}))\,s\,\theta_{65\,i}{}^{2}-$$
$$2\,s\,\theta_{1110\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,s(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\,s(2\,\theta_{65\,i})\Big)\Big)+$$
$$4\,c(\beta_{1413\,i}-\theta_{54})^{2}\Big(c\,\theta_{65\,i}{}^{2}\left(2\,s\,\theta_{1110\,i}{}^{2}\,s\,\theta_{21\,i}{}^{2}-c(\theta_{21}+\theta_{43})\,s(2\,\theta_{1110\,i})\,s(2\,\theta_{21\,i})\right)+s\,\beta_{54\,i}$$
$$\Big(-s\,\beta_{54\,i}\,s(2\,\theta_{1110\,i})\,s\,\theta_{21}\,s(2\,\theta_{21\,i})\,s\,\theta_{43}\,s\,\theta_{65\,i}{}^{2}+s\,\theta_{1110\,i}{}^{2}\,s(2\,\theta_{21\,i})\,s(2\,\theta_{65\,i})-s(2\,\theta_{1110\,i})\,s\,\theta_{21}$$
$$s\,\theta_{21\,i}{}^{2}\,s\,\theta_{43}\,s(2\,\theta_{65\,i})+c\,\theta_{21}\,c\,\theta_{43}\,s(2\,\theta_{1110\,i})\left(s\,\beta_{54\,i}\,s(2\,\theta_{21\,i})\,s\,\theta_{65\,i}{}^{2}+s\,\theta_{21\,i}{}^{2}\,s(2\,\theta_{65\,i})\right)+$$
$$c\,\theta_{21\,i}{}^{2}\left(2\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}{}^{2}\,s\,\theta_{65\,i}{}^{2}+s(2\,\theta_{1110\,i})\,s\,\theta_{21}\,s\,\theta_{43}\,s(2\,\theta_{65\,i})\right)\Big)\Big)\Big)\Big)\Big)\Big)$$

$$V_{78}=-\frac{1}{2}\,c\,\theta_{98\,i}{}^{2}\Big(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{1413\,i}\,s\,\beta_{54\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{21\,i}+$$
$$c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}-c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{109\,i}{}^{2}\,s\,\theta_{21}\,s\,\theta_{21\,i}+c\,\theta_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}-$$
$$c\,\theta_{21}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{109\,i}{}^{2}\,s\,\theta_{21\,i}\,s\,\theta_{43}+c\,\beta_{1413\,i}\,c\,\theta_{1110\,i}\,c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}+$$
$$c\,\beta_{54\,i}\,c\,\theta_{65\,i}\left(2\,c\,\theta_{109\,i}{}^{2}\,c(\theta_{21}+\theta_{43})+c(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\,s(2\,\theta_{109\,i})\,s(\theta_{21}+\theta_{43})\right)+$$
$$c\,\beta_{1413\,i}\,c\,\theta_{1110\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{54}+c\,\theta_{1110\,i}\,c\,\theta_{65\,i}\,s\,\beta_{1413\,i}\,s\,\beta_{54\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{54}+$$
$$c\,\beta_{1413\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{54}+$$
$$c\,\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{54}\,s\,\beta_{1413\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{65\,i}+c\,\theta_{21\,i}\,c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{65\,i}-$$
$$c\,\theta_{21\,i}\,c\,\theta_{43}\,s\,\theta_{109\,i}{}^{2}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{1110\,i}\,c\,\theta_{54}\,s\,\beta_{1413\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}+$$
$$c\,\beta_{1413\,i}\,c\,\theta_{54}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}+c\,\theta_{21}\,c\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{21}\,c\,\theta_{21\,i}\,s\,\theta_{109\,i}{}^{2}\,s\,\theta_{43}\,s\,\theta_{65\,i}+$$
$$c\,\beta_{1413\,i}\,c\,\theta_{1110\,i}\,c\,\theta_{21\,i}\,c\,\theta_{54}\,s(2\,\theta_{109\,i})\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i}+s\,\beta_{1413\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{21\,i}\,s\,\theta_{54}\,s\,\theta_{65\,i}+$$
$$c\,\theta_{1110\,i}\,c\,\theta_{21\,i}\,s\,\beta_{1413\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{54}\,s\,\theta_{65\,i}+c\,\beta_{1413\,i}\,c\,\theta_{21\,i}\,s(2\,\theta_{109\,i})\,s\,\theta_{1110\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{54}\,s\,\theta_{65\,i}+$$
$$c\,\theta_{109\,i}{}^{2}\,s(\theta_{21}+\theta_{43})\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i})-2\,c\,\theta_{109\,i}\,s\,\theta_{109\,i}\,(c\,\theta_{1110\,i}$$
$$(s\,\theta_{21\,i}\,(c\,\beta_{1413\,i}\,c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}+c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{1413\,i}\,s\,\beta_{54\,i}\,s\,\theta_{54}+c\,\beta_{1413\,i}\,s\,\theta_{54}\,s\,\theta_{65\,i})+$$
$$c\,\theta_{21\,i}\,(c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{1413\,i}\,s\,\beta_{54\,i}+c\,\beta_{1413\,i}\,c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,s\,\theta_{65\,i}+c\,\theta_{21}\,c\,\theta_{43}\,s\,\beta_{1413\,i}\,s\,\theta_{54}\,s\,\theta_{65\,i}))+$$
$$s\,\theta_{1110\,i}\,(s\,\beta_{1413\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{54}+c\,\theta_{54}\,s\,\theta_{21}\,s\,\theta_{43}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i}))+$$
$$c\,\beta_{1413\,i}\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{54}+c\,\theta_{21\,i}\,(c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}+c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{54}\,s\,\theta_{65\,i}))))\Big)$$

$$V_{79}=$$
$$c\,\theta_{98\,i}{}^{2}\,(s\,\beta_{1413\,i}\,(c\,\beta_{54\,i}\,c(\theta_{1110\,i}-\theta_{54})\,c\,\theta_{65\,i}\,s(\theta_{21}+\theta_{43})-c\,\theta_{21\,i}\,(c\,\theta_{54}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{1110\,i}+c\,\theta_{1110\,i}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{65\,i})+$$
$$s\,\theta_{54}\,(-c\,\theta_{1110\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}+c(\theta_{21}+\theta_{43})\,s\,\theta_{1110\,i}\,s\,\theta_{65\,i}))+$$
$$s\,\theta_{21\,i}\,(s\,\theta_{1110\,i}\,(-c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{54}+c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{54}+c\,\theta_{54}\,s\,\theta_{65\,i})-$$
$$c\,\theta_{1110\,i}\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{43}+s\,\theta_{54}\,s\,\theta_{65\,i})))+$$
$$c\,\beta_{1413\,i}\,(s\,\theta_{1110\,i}\,(-c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\theta_{65\,i})+s\,\theta_{54}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}-s\,\theta_{21\,i}\,s\,\theta_{65\,i})+$$
$$c\,\theta_{54}\,(c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}+c\,\beta_{54\,i}\,c\,\theta_{65\,i}\,s(\theta_{21}+\theta_{43})+c\,\theta_{21\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i}))-$$
$$c\,\theta_{1110\,i}\,(c\,\beta_{54\,i}\,c\,\theta_{65\,i}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}+s\,\theta_{21\,i}\,(-c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{54}+$$
$$c\,\theta_{65\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{54}+c\,\theta_{54}\,s\,\theta_{65\,i})-c\,\theta_{21\,i}\,(c\,\theta_{54}\,c\,\theta_{65\,i}\,s\,\beta_{54\,i}+c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\theta_{65\,i}))))$$

Apéndice E

Coeficientes de la ecuación (2.77)

$$G_1 = 2\,(x - x_{32}\,c\,\theta_{21} - x_{1918}\,c\,\phi\,c\,\psi)$$

$$G_2 = 2\,(y - x_{1918}\,c\,\theta\,s\,\phi - x_{1918}\,c\,\phi\,s\,\theta\,s\,\psi)$$

$$G_3 = 2\,(z - z_{10} + x_{32}\,s\,\theta_{21} - x_{1918}\,s\,\theta\,s\,\phi + x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi)$$

$$G_4 = 2\,x_{1918}\,c\,\phi\,(-y\,c\,\psi\,s\,\theta + c\,\theta\,c\,\psi\,(z - z_{10} + x_{32}\,s\,\theta_{21}) + (x - x_{32}\,c\,\theta_{21})\,s\,\psi)$$

$$G_5 = -2\,x_{1918}\,(c\,\theta\,((z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\phi + y\,c\,\phi\,s\,\psi) + s\,\theta\,(-y\,s\,\phi + c\,\phi\,(z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\psi))$$

$$G_6 = 2\,x_{1918}\,(-c\,\phi\,s\,\theta\,(z - z_{10} + x_{32}\,s\,\theta_{21}) + s\,\phi\,((x - x_{32}\,c\,\theta_{21})\,c\,\psi + y\,s\,\theta\,s\,\psi) - c\,\theta\,(y\,c\,\phi + (z - z_{10} + x_{32}\,s\,\theta_{21})\,s\,\phi\,s\,\psi))$$

$$\begin{aligned}
G_7 = 2\Big(&\dot{x}^2 + \dot{y}^2 + \dot{z}^2 - 2\,\dot{y}\,\dot{\phi}\,x_{1918}\,c\,\theta\,c\,\phi + 2\,\dot{\theta}\,\dot{\phi}\,x_{1918}\,z_{10}\,c\,\theta\,c\,\phi + \dot{\psi}^2\,x\,x_{1918}\,c\,\phi\,c\,\psi - \left(\dot{\phi}^2 + \dot{\psi}^2\right)x_{1918}\,x_{32}\,c\,\theta_{21}\,c\,\phi\,c\,\psi +\\
&\dot{\theta}_{21}{}^2\,x_{32}\,c\,\theta_{21}\,(x - x_{1918}\,c\,\phi\,c\,\psi) - 2\,\dot{z}\,\dot{\phi}\,x_{1918}\,c\,\phi\,s\,\theta - 2\,\dot{\psi}\,x_{1918}\,c\,\phi\,c\,\psi\left(\left(-\dot{z} + \dot{\theta}\,y\right)c\,\theta + \left(\dot{y} + \dot{\theta}\,(z - z_{10})\right)s\,\theta\right) -\\
&2\,\dot{\theta}\,\dot{\phi}\,x_{1918}\,x_{32}\,c\,\theta\,c\,\phi\,s\,\theta_{21} - 2\,\dot{\theta}\,\dot{\psi}\,x_{1918}\,x_{32}\,c\,\phi\,c\,\psi\,s\,\theta\,s\,\theta_{21} + \dot{\theta}^2\,x_{1918}\,y\,c\,\theta\,s\,\phi + \dot{\phi}^2\,x_{1918}\,y\,c\,\theta\,s\,\phi +\\
&\dot{\theta}^2\,x_{1918}\,z\,s\,\theta\,s\,\phi + \dot{\phi}^2\,x_{1918}\,z\,s\,\theta\,s\,\phi - \left(\dot{\theta}^2 + \dot{\phi}^2\right)x_{1918}\,z_{10}\,s\,\theta\,s\,\phi - 2\,\dot{\phi}\,\dot{\psi}\,x_{1918}\,x_{32}\,c\,\theta\,c\,\psi\,s\,\theta_{21}\,s\,\phi +\\
&\dot{\theta}^2\,x_{1918}\,x_{32}\,s\,\theta\,s\,\theta_{21}\,s\,\phi + \dot{\phi}^2\,x_{1918}\,x_{32}\,s\,\theta\,s\,\theta_{21}\,s\,\phi + 2\,\dot{\theta}\,x_{1918}\,s\,\theta\left(\dot{\phi}\,y\,c\,\phi + \dot{y}\,s\,\phi\right) - 2\,\dot{\theta}\,x_{1918}\,c\,\theta\left(\dot{\phi}\,z\,c\,\phi + \dot{z}\,s\,\phi\right) +\\
&2\,\dot{x}\,\dot{\psi}\,x_{1918}\,c\,\phi\,s\,\psi - 2\,\dot{y}\,\dot{\theta}\,x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi - \dot{\theta}^2\,x_{1918}\,z\,c\,\theta\,c\,\phi\,s\,\psi + \dot{\theta}^2\,x_{1918}\,z_{10}\,c\,\theta\,c\,\phi\,s\,\psi - 2\,\dot{z}\,\dot{\theta}\,x_{1918}\,c\,\phi\,s\,\theta\,s\,\psi +\\
&\dot{\theta}^2\,x_{1918}\,y\,c\,\phi\,s\,\theta\,s\,\psi + \dot{\psi}^2\,x_{1918}\,c\,\phi\,((-z + z_{10})\,c\,\theta + y\,s\,\theta)\,s\,\psi - \left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2\right)x_{1918}\,x_{32}\,c\,\theta\,c\,\phi\,s\,\theta_{21}\,s\,\psi +\\
&2\,\dot{\phi}\,\dot{\psi}\,x_{1918}\,x_{32}\,c\,\theta_{21}\,s\,\phi\,s\,\psi + 2\,\dot{\theta}\,\dot{\phi}\,x_{1918}\,x_{32}\,s\,\theta\,s\,\theta_{21}\,s\,\phi\,s\,\psi + 2\,\dot{\theta}_{21}\,x_{32}\,s\,\theta_{21}\left(\dot{x} + \dot{\phi}\,x_{1918}\,c\,\psi\,s\,\phi + \dot{\psi}\,x_{1918}\,c\,\phi\,s\,\psi\right) -\\
&\dot{\theta}_{21}{}^2\,x_{32}\,s\,\theta_{21}\,(z - z_{10} - x_{1918}\,s\,\theta\,s\,\phi + x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi) + \dot{\phi}^2\,x_{1918}\,c\,\phi\,(x\,c\,\psi + ((-z + z_{10})\,c\,\theta + y\,s\,\theta)\,s\,\psi) +\\
&2\,\dot{\phi}\,x_{1918}\,s\,\phi\left(\dot{x}\,c\,\psi + \dot{\psi}\,c\,\psi\,((-z + z_{10})\,c\,\theta + y\,s\,\theta) - \dot{\psi}\,x\,s\,\psi + \left(\left(-\dot{z} + \dot{\theta}\,y\right)c\,\theta + \left(\dot{y} + \dot{\theta}\,(z - z_{10})\right)s\,\theta\right)s\,\psi\right) +\\
&2\,\dot{\theta}_{21}\,x_{32}\,c\,\theta_{21}\left(\dot{z} - x_{1918}\,c\,\phi\,s\,\theta\left(\dot{\phi} + \dot{\theta}\,s\,\psi\right) + x_{1918}\,c\,\theta\left(\dot{\psi}\,c\,\phi\,c\,\psi - s\,\phi\left(\dot{\theta} + \dot{\phi}\,s\,\psi\right)\right)\right)\Big)
\end{aligned}$$

Coeficientes de la ecuación (2.80)

$$G_8 = -s\,\theta_{43}$$

$$G_9 = -c\,\theta_{43}$$

$$G_{10} = 0$$

$$G_{11} = -x_{1918}\,c\,\phi\,(c\,\theta\,c\,\theta_{43}\,c\,\psi + s\,\theta_{43}\,s\,\psi)$$

$$G_{12} = x_{1918}\,c\,\theta_{43}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)$$

$$G_{13} = x_{1918}\,(-c\,\psi\,s\,\theta_{43}\,s\,\phi + c\,\theta_{43}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))$$

$$
\begin{aligned}
G_{14} = {} & -\ddot{\theta}_{21}\, x_{32}\, c\,\theta_{21}\, c\,\theta_{43} + 2\,\dot{\theta}\,\dot{\phi}\, x_{1918}\, c\,\theta\, c\,\theta_{43}\, c\,\phi + 2\,\dot{\theta}\,\dot{\psi}\, x_{1918}\, c\,\theta_{43}\, c\,\phi\, c\,\psi\, s\,\theta + \dot{\theta}_{21}{}^{2}\, x_{32}\, c\,\theta_{43}\, s\,\theta_{21} - \\
& \dot{\theta}_{21}{}^{2}\, x_{32}\, c\,\theta_{21}\, s\,\theta_{43} - \left(\dot{\phi}^{2} + \dot{\psi}^{2}\right) x_{1918}\, c\,\phi\, c\,\psi\, s\,\theta_{43} - \ddot{\theta}_{21}\, x_{32}\, s\,\theta_{21}\, s\,\theta_{43} + 2\,\dot{\phi}\,\dot{\psi}\, x_{1918}\, c\,\theta\, c\,\theta_{43}\, c\,\psi\, s\,\phi - \\
& \dot{\theta}^{2}\, x_{1918}\, c\,\theta_{43}\, s\,\theta\, s\,\phi - \dot{\phi}^{2}\, x_{1918}\, c\,\theta_{43}\, s\,\theta\, s\,\phi + \left(\dot{\theta}^{2} + \dot{\phi}^{2} + \dot{\psi}^{2}\right) x_{1918}\, c\,\theta\, c\,\theta_{43}\, c\,\phi\, s\,\psi - 2\,\dot{\theta}\,\dot{\phi}\, x_{1918}\, c\,\theta_{43}\, s\,\theta\, s\,\phi\, s\,\psi + \\
& 2\,\dot{\phi}\,\dot{\psi}\, x_{1918}\, s\,\theta_{43}\, s\,\phi\, s\,\psi - 2\,\dot{\theta}_{43}\, c\,\theta_{43}\left(\dot{x} + \dot{\theta}_{21}\, x_{32}\, s\,\theta_{21} + \dot{\phi}\, x_{1918}\, c\,\psi\, s\,\phi + \dot{\psi}\, x_{1918}\, c\,\phi\, s\,\psi\right) + \\
& \dot{\theta}_{43}{}^{2}\, s\,\theta_{43}\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) + \\
& \dot{\theta}_{43}{}^{2}\, c\,\theta_{43}\,((x - x_{1918}\, c\,\phi\, c\,\psi)\, s\,\theta_{21} + c\,\theta_{21}\,(z - z_{10} - x_{1918}\, s\,\theta\, s\,\phi + x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) + \\
& 2\,\dot{\theta}_{43}\, s\,\theta_{43}\left(\dot{z} + \dot{\theta}_{21}\, x_{32}\, c\,\theta_{21} - \dot{\phi}\, x_{1918}\, c\,\phi\, s\,\theta - \dot{\theta}\, x_{1918}\, c\,\phi\, s\,\theta\, s\,\psi + x_{1918}\, c\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)
\end{aligned}
$$

Coeficientes de la ecuación (2.82)

$$
G_{15} = (-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{3}
$$

$$
\begin{aligned}
G_{16} = {} & -c\,\theta_{43}\, c\,\theta_{54}{}^{2} \\
& (-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))\,(y - x_{1918}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))
\end{aligned}
$$

$$
G_{17} = c\,\theta_{43}\, c\,\theta_{54}{}^{2}\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2}
$$

$$
G_{18} = 0
$$

$$
\begin{aligned}
G_{19} = {} & x_{1918}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\, c\,\phi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) \\
& (s\,\psi\,(-y + x_{1918}\, c\,\theta\, s\,\phi + x_{1918}\, c\,\phi\, s\,\theta\, s\,\psi) - \\
& \quad c\,\psi\, s\,\theta\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)))
\end{aligned}
$$

$$
G_{20} = x_{1918}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi)\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2}
$$

$$
\begin{aligned}
G_{21} = {} & x_{1918}\, c\,\theta_{43}\, c\,\theta_{54}{}^{2}\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) \\
& (c\,\psi\, s\,\phi\,(-y + x_{1918}\, c\,\theta\, s\,\phi + x_{1918}\, c\,\phi\, s\,\theta\, s\,\psi) - \\
& \quad c\,\theta\, c\,\phi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) + \\
& \quad s\,\theta\, s\,\phi\, s\,\psi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)))
\end{aligned}
$$

$$
\begin{aligned}
G_{22} = {} & \\
& c\,\theta_{54}{}^{2}\Big(-2\,\dot{\theta}\,\dot{\psi}\, x_{1918}\, c\,\theta\, c\,\theta_{43}\, c\,\phi\, c\,\psi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2} + 2\,\dot{\theta}\,\dot{\phi} \\
& \quad x_{1918}\, c\,\theta_{43}\, c\,\phi\, s\,\theta\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2} + 2\,\dot{\phi}\,\dot{\psi}\, x_{1918} \\
& \quad c\,\theta_{43}\, c\,\psi\, s\,\theta\, s\,\phi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2} + \left(\dot{\theta}^{2} + \dot{\phi}^{2} + \dot{\psi}^{2}\right) \\
& \quad x_{1918}\, c\,\theta_{43}\, c\,\phi\, s\,\theta\, s\,\psi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2} + x_{1918}\, c\,\theta \\
& \quad c\,\theta_{43}\, s\,\phi\left(\dot{\theta}^{2} + \dot{\phi}^{2} + 2\,\dot{\theta}\,\dot{\phi}\, s\,\psi\right)(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))^{2} + \\
& \quad 2\, c\,\theta_{43}\left(\dot{x} + \dot{\theta}_{21}\, x_{32}\, s\,\theta_{21} + \dot{\phi}\, x_{1918}\, c\,\psi\, s\,\phi + \dot{\psi}\, x_{1918}\, c\,\phi\, s\,\psi\right)^{2}(y - x_{1918}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - \dot{\theta}_{21}{}^{2}\, x_{32}\, c\,\theta_{21}\, c\,\theta_{43} \\
& \quad (-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))\,(y - x_{1918}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - \\
& \quad \left(\dot{\phi}^{2} + \dot{\psi}^{2}\right) x_{1918}\, c\,\theta_{43}\, c\,\phi\, c\,\psi\,(-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi)) \\
& \quad (y - x_{1918}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - \ddot{\theta}_{21}\, x_{32}\, c\,\theta_{43}\, s\,\theta_{21} \\
& \quad (-x_{32} + c\,\theta_{21}\,(x - x_{1918}\, c\,\phi\, c\,\psi) + s\,\theta_{21}\,(-z + z_{10} + x_{1918}\, s\,\theta\, s\,\phi - x_{1918}\, c\,\theta\, c\,\phi\, s\,\psi))\,(y - x_{1918}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +
\end{aligned}
$$

$$2\,\dot{\phi}\,\dot{\psi}\,x_{1918}\,c\,\theta_{43}\,s\,\phi\,s\,\psi\,(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))$$
$$(y-x_{1918}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+2\,\dot{\theta}_{43}\,s\,\theta_{43}\,\left(\dot{x}+\dot{\theta}_{21}\,x_{32}\,s\,\theta_{21}+\dot{\phi}\,x_{1918}\,c\,\psi\,s\,\phi+\dot{\psi}\,x_{1918}\,c\,\phi\,s\,\psi\right)$$
$$(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))\,(y-x_{1918}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))-$$
$$\dot{\theta}_{43}{}^{2}\,c\,\theta_{43}\,(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))^{2}$$
$$(y-x_{1918}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))-$$
$$\ddot{\theta}_{43}\,s\,\theta_{43}\,(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))^{2}$$
$$(y-x_{1918}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))-2\,c\,\theta_{43}\,\left(\dot{x}+\dot{\theta}_{21}\,x_{32}\,s\,\theta_{21}+\dot{\phi}\,x_{1918}\,c\,\psi\,s\,\phi+\dot{\psi}\,x_{1918}\,c\,\phi\,s\,\psi\right)$$
$$(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))$$
$$\left(\dot{y}-x_{1918}\left(c\,\theta\,c\,\phi\left(\dot{\phi}+\dot{\theta}\,s\,\psi\right)+s\,\theta\left(\dot{\psi}\,c\,\phi\,c\,\psi-s\,\phi\left(\dot{\theta}+\dot{\phi}\,s\,\psi\right)\right)\right)\right)-$$
$$2\,\dot{\theta}_{43}\,s\,\theta_{43}\,(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))^{2}$$
$$\left(\dot{y}-x_{1918}\left(c\,\theta\,c\,\phi\left(\dot{\phi}+\dot{\theta}\,s\,\psi\right)+s\,\theta\left(\dot{\psi}\,c\,\phi\,c\,\psi-s\,\phi\left(\dot{\theta}+\dot{\phi}\,s\,\psi\right)\right)\right)\right)-$$
$$2\,\dot{\theta}_{54}{}^{2}\,\sec(\theta_{54})^{2}\,(-x_{32}+c\,\theta_{21}\,(x-x_{1918}\,c\,\phi\,c\,\psi)+s\,\theta_{21}\,(-z+z_{10}+x_{1918}\,s\,\theta\,s\,\phi-x_{1918}\,c\,\theta\,c\,\phi\,s\,\psi))^{3}\,\tan(\theta_{54})\Big)$$

Coeficientes de la ecuación (2.82)

$$G_{23}=(s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))^{3}$$

$$G_{24}=0$$

$$G_{25}=0$$

$$G_{26}=0$$

$$G_{27}=-c\,\theta_{1817}{}^{2}\,(s\,\theta_{54}\,(-c\,\theta\,c\,\phi+s\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))$$
$$\Big(c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\phi\,c\,\psi\,(c\,\theta_{54}\,s\,\theta\,s(\theta_{21}+\theta_{43})-c\,\theta\,s\,\theta_{54})+c(\theta_{21}+\theta_{43})^{2}\,c\,\theta_{54}{}^{2}\,s\,\phi+(c\,\theta\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})+s\,\theta\,s\,\theta_{54})$$
$$(s\,\theta\,(s\,\theta_{54}\,s\,\phi+c\,\theta_{54}\,c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\psi)+c\,\theta\,(c\,\theta_{43}\,c\,\theta_{54}\,s\,\theta_{21}\,s\,\phi+c\,\theta_{21}\,c\,\theta_{54}\,s\,\theta_{43}\,s\,\phi-c\,\phi\,s\,\theta_{54}\,s\,\psi))\Big)$$

$$G_{28}=-c\,\theta_{1817}{}^{2}\,\left(c\,\theta^{2}\,c\,\phi\,c\,\psi\left(c\,\theta_{54}{}^{2}\,s(\theta_{21}+\theta_{43})^{2}+s\,\theta_{54}{}^{2}\right)-c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,(s\,\theta_{54}\,s\,\phi+c\,\theta_{54}\,c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\psi)+\right.$$
$$\left.s\,\theta\left(c\,\phi\,c\,\psi\,s\,\theta\,s\,\theta_{54}{}^{2}+c\,\theta_{54}{}^{2}\,s(\theta_{21}+\theta_{43})\,(c\,\phi\,c\,\psi\,s\,\theta\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\phi)-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\phi\,s\,\theta_{54}\,s\,\psi\right)\right)$$
$$(s\,\theta_{54}\,(-c\,\theta\,c\,\phi+s\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))$$

$$G_{29}=-c\,\theta_{1817}{}^{2}\,(c\,\theta\,c\,\theta_{54}\,c\,\psi\,s(\theta_{21}+\theta_{43})+c\,\psi\,s\,\theta\,s\,\theta_{54}-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\psi)$$
$$(c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\phi\,c\,\psi+c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,(-s\,\theta\,s\,\phi+c\,\theta\,c\,\phi\,s\,\psi)+s\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))$$
$$(s\,\theta_{54}\,(-c\,\theta\,c\,\phi+s\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))$$

$$G_{30}=-c\,\theta_{1817}{}^{2}\left(\left(\dot{\theta}^{2}+\dot{\phi}^{2}\right)c\,\theta_{54}\,c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})\,(c\,\theta\,c\,\theta_{54}\,c\,\psi\,s(\theta_{21}+\theta_{43})+c\,\psi\,s\,\theta\,s\,\theta_{54}-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\psi)\right.$$
$$(s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))-c\,\theta_{54}\,c\,\phi\,s\,\theta$$
$$\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)c(\theta_{21}+\theta_{43})-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^{2}s(\theta_{21}+\theta_{43})\right)(c\,\theta\,c\,\theta_{54}\,c\,\psi\,s(\theta_{21}+\theta_{43})+c\,\psi\,s\,\theta\,s\,\theta_{54}-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\psi)$$
$$(s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))+$$
$$\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\psi}^{2}-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\,c\,\theta\right)c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\psi\,s\,\phi$$
$$(c\,\theta\,c\,\theta_{54}\,c\,\psi\,s(\theta_{21}+\theta_{43})+c\,\psi\,s\,\theta\,s\,\theta_{54}-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\psi)$$
$$(s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-c\,\theta_{54}\,(c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+s\,\phi\,(c(\theta_{21}+\theta_{43})\,c\,\psi+c\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\psi)))+$$

$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\ddot{\theta}_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))-$$

$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))-$$

$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\left(\dot{\theta}\,\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\dot{\phi}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\right)(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$2\,\dot{\theta}\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\left(\dot{\theta}^{2}+\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\psi}^{2}\right)\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi$$

$$(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\left(\dot{\theta}^{2}+\dot{\phi}^{2}+\dot{\psi}^{2}\right)\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))-$$

$$\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\left(\dot{\theta}^{2}+\dot{\phi}^{2}+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\dot{\phi}^{2}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))-$$

$$2\,\dot{\phi}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\left(\left(-\dot{\psi}+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}(\theta_{21}+\theta_{43})-\dot{\theta}\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\right)\mathrm{s}\,\psi\right)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\theta_{54}\right)(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\left(\dot{\theta}_{54}{}^{2}\,\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$

$$(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))-2\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}$$

$$(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\phi\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))+$$

$$\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\left(\left(\dot{\theta}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))^{2}+$$

$$\mathrm{c}\,\psi\,\mathrm{s}\,\theta\left(-\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}+\left(\dot{\theta}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{s}\,\theta_{54}\right)$$

$$(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)-\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))^{2}+$$

$$\left(-\mathrm{c}(\theta_{21}+\theta_{43})\left(\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)-\right.$$

$$s(\theta_{21}+\theta_{43})\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\theta_{54} - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\, s\,\theta_{54}\right)\Big) s\,\psi$$

$$(s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 -$$

$$c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\psi - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\, s\,\psi\right)$$

$$(s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 +$$

$$2\left(\dot{\theta}\, c\,\theta_{54}\, s\,\theta + \dot{\theta}_{54}\, c\,\theta\, s\,\theta_{54}\right)\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c(\theta_{21}+\theta_{43})\, c\,\psi - \dot{\psi}\, s(\theta_{21}+\theta_{43})\, s\,\psi\right)$$

$$(s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 +$$

$$2\,\dot{\phi}\,\dot{\psi}\, c\,\phi\, c\,\psi\, s\,\theta\, s\,\theta_{54}\,(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi))) +$$

$$\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\phi\, s\,\psi\,(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi))) +$$

$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\, c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, s\,\phi\, s\,\psi\,(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi))) - 2\,\dot{\theta}\,\dot{\theta}_{54}\, c\,\theta$$

$$c\,\theta_{54}\, c\,\psi\,(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 +$$

$$\left(\dot{\theta}_{21}{}^2 + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}{}^2 + \dot{\psi}^2\right) c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43})$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 +$$

$$\dot{\psi}^2\, c\,\psi\, s\,\theta\, s\,\theta_{54}\,(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 -$$

$$2\,\dot{\theta}\,\dot{\theta}_{54}\, c\,\psi\, s\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54}$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 -$$

$$2\,\dot{\psi}\, c\,\psi\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta_{54}\, s(\theta_{21}+\theta_{43}) + \dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\right)$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 - \dot{\psi}^2$$

$$c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 + 2$$

$$\dot{\psi}\left(\dot{\theta}_{54}\, c\,\theta_{54}\, s\,\theta + \dot{\theta}\, c\,\theta\, s\,\theta_{54}\right) s\,\psi$$

$$(s\,\theta_{54}\,(-c\,\theta\, c\,\phi + s\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))^2 +$$

$$2\,\dot{\theta}_{54}\, s\,\theta_{54}\,(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))$$

$$\Big(c(\theta_{21}+\theta_{43})\left(c\,\phi\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + \left(-\dot{\psi} + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) s\,\phi\, s\,\psi\right) +$$

$$s(\theta_{21}+\theta_{43})\left(-s\,\phi\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\psi + s\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right) + c\,\theta\left(\dot{\psi}\, c\,\psi\, s\,\phi + c\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\Big) -$$

$$2\,\Big(s\,\theta_{54}\left(-c\,\theta\, c\,\psi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) + \dot{\psi}\, s\,\theta\, s\,\psi\right) - c\,\theta_{54}\,\Big(c\,\psi\, s\,\theta\left(\dot{\theta}_{54} - \dot{\theta}\, s(\theta_{21}+\theta_{43})\right) +$$

$$\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) s(\theta_{21}+\theta_{43})\, s\,\psi\Big) + c(\theta_{21}+\theta_{43})\left(\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c\,\theta_{54}\, c\,\psi - \dot{\theta}_{54}\, s\,\theta_{54}\, s\,\psi\right)\Big)$$

$$(s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)))$$

$$\Big(\dot{\theta}_{54}\, c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) + \dot{\theta}_{54}\, s\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)) -$$

$$s\,\theta_{54}\left(c\,\phi\, s\,\theta\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right) + s\,\phi\left(\dot{\psi}\, c\,\psi\, s\,\theta + c\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right)\right) -$$

$$c\,\theta_{54}\,\Big(c(\theta_{21}+\theta_{43})\left(c\,\phi\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + \left(-\dot{\psi} + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) s\,\phi\, s\,\psi\right) +$$

$$s(\theta_{21}+\theta_{43})\left(-s\,\phi\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\psi + s\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right) + c\,\theta\left(\dot{\psi}\, c\,\psi\, s\,\phi + c\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\Big)\Big) +$$

$$2\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$\Big(\dot{\theta}_{54}\, c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) + \dot{\theta}_{54}\, s\,\theta_{54}\,(c\,\phi\, s\,\theta\, s(\theta_{21}+\theta_{43}) + s\,\phi\,(c(\theta_{21}+\theta_{43})\, c\,\psi + c\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\psi)) -$$

$$s\,\theta_{54}\left(c\,\phi\, s\,\theta\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right) + s\,\phi\left(\dot{\psi}\, c\,\psi\, s\,\theta + c\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right)\right) -$$

$$c\,\theta_{54}\,\Big(c(\theta_{21}+\theta_{43})\left(c\,\phi\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + \left(-\dot{\psi} + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) s\,\phi\, s\,\psi\right) + s(\theta_{21}+\theta_{43})$$

$$\left(-s\,\phi\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\psi + s\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right) + c\,\theta\left(\dot{\psi}\, c\,\psi\, s\,\phi + c\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\Big)\Big)^2 + 2\,\dot{\theta}_{1817}{}^2\, \sec(\theta_{1817})^2$$

$$\left(\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) - \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi + \mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))\right)^3 \tan(\theta_{1817})\Big)$$

Coeficientes de la ecuación (2.86)

$$G_{31} = \left(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)\right)^3$$

$$G_{32} = 0$$

$$G_{33} = 0$$

$$G_{34} = 0$$

$$G_{35} = \mathrm{c}\,\theta_{1716}{}^2\,\csc(\theta_{1817})\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))$$
$$\Big(\mathrm{c}(\theta_{21}+\theta_{43})^2\,\mathrm{c}\,\theta_{54}{}^2\,\mathrm{c}\,\phi + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,(-\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\theta\,\mathrm{s}\,\theta_{54})\,\mathrm{s}\,\phi + (\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54})$$
$$(\mathrm{s}\,\theta\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54} - \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{43} + \mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))\Big)$$

$$G_{36} = \mathrm{c}\,\theta_{1716}{}^2\,\csc(\theta_{1817})\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))$$
$$\Big(\frac{1}{8}\,(-6 + 2\,\mathrm{c}(2\,(\theta_{21}+\theta_{43})) + \mathrm{c}(2\,(\theta_{21}+\theta_{43}-\theta_{54})) + 2\,\mathrm{c}(2\,\theta_{54}) + \mathrm{c}(2\,(\theta_{21}+\theta_{43}+\theta_{54})))\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}$$
$$(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\phi + \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))\Big)$$

$$G_{37} = -\mathrm{c}\,\theta_{1716}{}^2\,\csc(\theta_{1817})\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))$$
$$(\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\phi + \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{s}\,\phi\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi + \mathrm{c}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)))$$

$$G_{38} = \mathrm{c}\,\theta_{1716}{}^2\,\Big(-2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\csc(\theta_{1817})\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$
$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\csc(\theta_{1817})\,\mathrm{s}\,\theta\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) +$$
$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\csc(\theta_{1817})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$
$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\phi\,\csc(\theta_{1817})\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) +$$
$$\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\csc(\theta_{1817})\,\Big(-\mathrm{c}(\theta_{21}+\theta_{43})\,\Big(\Big(\dot{\theta}_{21}{}^2 + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}{}^2 + \dot{\theta}_{54}{}^2\Big)\,\mathrm{c}\,\theta_{54} + \ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\Big) - \mathrm{s}(\theta_{21}+\theta_{43})$$
$$\Big(\Big(\ddot{\theta}_{21} + \ddot{\theta}_{43}\Big)\,\mathrm{c}\,\theta_{54} - 2\,\Big(\dot{\theta}_{21} + \dot{\theta}_{43}\Big)\,\dot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\Big)\Big)\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$
$$2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\csc(\theta_{1817})\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$
$$2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\psi\,\csc(\theta_{1817})\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$
$$\Big(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2\Big)\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\csc(\theta_{1817})\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi\,(\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi)$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$

$$
\begin{aligned}
&\left(\dot{\theta}^2+\dot{\phi}^2+\dot{\psi}^2\right) c\,\phi\, \csc(\theta_{1817})\, s\,\theta\, s\,\theta_{54}\, s\,\psi\, (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&2\,\dot{\phi}\,\dot{\psi}\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, \csc(\theta_{1817})\, s\,\phi\, s\,\psi\, (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&c\,\theta_{54}\, \csc(\theta_{1817})\, s\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\phi \left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\, s\,\psi\right) (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - \\
&c\,\theta\, \csc(\theta_{1817})\, s\,\theta_{54}\, s\,\phi \left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\, s\,\psi\right) (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&\left(\dot{\phi}^2+\dot{\psi}^2\right) c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi\, \csc(\theta_{1817})\, (-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&2\, \csc(\theta_{1817}) \left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta_{54}\, s(\theta_{21}+\theta_{43}) + \dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\right) \\
&\quad (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi) \left(\dot{\phi}\, c\,\psi\, s\,\phi + \dot{\psi}\, c\,\phi\, s\,\psi\right) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&\csc(\theta_{1817}) \left(-s(\theta_{21}+\theta_{43}) \left(\left(\dot{\theta}_{21}^{\,2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}^{\,2}+\dot{\theta}_{54}^{\,2}\right) c\,\theta_{54} + \ddot{\theta}_{54}\, s\,\theta_{54}\right) + \right. \\
&\qquad \left. c(\theta_{21}+\theta_{43}) \left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\theta_{54} - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) \dot{\theta}_{54}\, s\,\theta_{54}\right)\right) \\
&\quad (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + \\
&\csc(\theta_{1817}) \left(\ddot{\theta}_{54}\, c\,\theta_{54} - \dot{\theta}_{54}^{\,2}\, s\,\theta_{54}\right) (c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) - 2\,\dot{\theta}\,\dot{\theta}_{54}\, c\,\theta\, c\,\theta_{54} \\
&\quad c\,\psi\, \csc(\theta_{1817})\, (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&\left(\dot{\theta}_{21}^{\,2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}^{\,2}+\dot{\psi}^2\right) c\,\theta\, c\,\theta_{54}\, c\,\psi\, \csc(\theta_{1817})\, s(\theta_{21}+\theta_{43}) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \dot{\psi}^2\, c\,\psi\, \csc(\theta_{1817}) \\
&\quad s\,\theta\, s\,\theta_{54}\, (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 - \\
&2\,\dot{\theta}\,\dot{\theta}_{54}\, c\,\psi\, \csc(\theta_{1817})\, s\,\theta\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54} \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&c\,\theta\, c\,\psi\, \csc(\theta_{1817})\, s(\theta_{21}+\theta_{43}) \left(\left(\dot{\theta}^2+\dot{\theta}_{54}^{\,2}\right) c\,\theta_{54} + \ddot{\theta}_{54}\, s\,\theta_{54}\right) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&c\,\psi\, \csc(\theta_{1817})\, s\,\theta \left(-\ddot{\theta}_{54}\, c\,\theta_{54} + \left(\dot{\theta}^2+\dot{\theta}_{54}^{\,2}\right) s\,\theta_{54}\right) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&2\,\dot{\psi}\, c\,\psi\, \csc(\theta_{1817}) \left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta_{54}\, s(\theta_{21}+\theta_{43}) - \dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\right) \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 - \dot{\psi}^2\, c(\theta_{21}+\theta_{43}) \\
&\quad c\,\theta_{54}\, \csc(\theta_{1817})\, s\,\psi\, (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&2\,\dot{\psi}\, \csc(\theta_{1817}) \left(\dot{\theta}_{54}\, c\,\theta_{54}\, s\,\theta + \dot{\theta}\, c\,\theta\, s\,\theta_{54}\right) s\,\psi \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + \\
&\csc(\theta_{1817}) \left(-c(\theta_{21}+\theta_{43}) \left(\left(\dot{\theta}_{21}^{\,2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}^{\,2}+\dot{\theta}_{54}^{\,2}\right) c\,\theta_{54} + \ddot{\theta}_{54}\, s\,\theta_{54}\right) - \right. \\
&\qquad \left. s(\theta_{21}+\theta_{43}) \left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\theta_{54} - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) \dot{\theta}_{54}\, s\,\theta_{54}\right)\right) s\,\psi \\
&\quad (c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\, (-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\, (c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 - \\
&c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, \csc(\theta_{1817}) \left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right) c\,\psi - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) \dot{\psi}\, s\,\psi\right)
\end{aligned}
$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 +$$

$$\dot{\theta}_{1817}{}^2 \cot(\theta_{1817})^2 \csc(\theta_{1817})\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 +$$

$$\dot{\theta}_{1817}{}^2 \csc(\theta_{1817})^3\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 +$$

$$2\csc(\theta_{1817})\left(\dot{\theta}\, c\,\theta_{54}\, s\,\theta + \dot{\theta}_{54}\, c\,\theta\, s\,\theta_{54}\right)\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c(\theta_{21}+\theta_{43})\, c\,\psi - \dot{\psi}\, s(\theta_{21}+\theta_{43})\, s\,\psi\right)$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^2 + 2\csc(\theta_{1817})$$

$$\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c(\theta_{21}+\theta_{43})\, c\,\theta_{54} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\right)(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\left(-c\,\phi\, s\,\theta\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right) + c\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right) +$$

$$2\,\dot{\theta}_{54}\, c\,\theta_{54} \csc(\theta_{1817})\,(c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c\,\psi\, s\,\theta\, s\,\theta_{54} - c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\left(c\,\theta\, c\,\phi\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right) + s\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right) -$$

$$2\csc(\theta_{1817})\,(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\left(s\,\theta_{54}\left(-c\,\theta\, c\,\psi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) + \dot{\psi}\, s\,\theta\, s\,\psi\right) - c\,\theta_{54}\left(c\,\psi\, s\,\theta\left(\dot{\theta}_{54} - \dot{\theta}\, s(\theta_{21}+\theta_{43})\right) +\right.\right.$$

$$\left.\left.\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) s(\theta_{21}+\theta_{43})\, s\,\psi\right) + c(\theta_{21}+\theta_{43})\left(\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c\,\theta_{54}\, c\,\psi - \dot{\theta}_{54}\, s\,\theta_{54}\, s\,\psi\right)\right)$$

$$\left(s\,\theta_{54}\left(-\dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi + c\,\theta\, c\,\phi\left(\dot{\phi} + \left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) s\,\psi\right) +\right.\right.$$

$$\left.s\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43}) + \dot{\phi}\, s\,\psi\right)\right)\right) - c\,\theta_{54}\left(c\,\phi\left(\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) c\,\psi\, s(\theta_{21}+\theta_{43}) +\right.\right.$$

$$\left.\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c(\theta_{21}+\theta_{43})\, s\,\psi + s\,\theta\left(-\dot{\theta}_{54}\, s\,\psi + s(\theta_{21}+\theta_{43})\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right)\right) +$$

$$\left.\left.\left.s\,\phi\left(c(\theta_{21}+\theta_{43})\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + c\,\theta\left(-\dot{\theta}_{54} + s(\theta_{21}+\theta_{43})\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\right)\right)\right) +$$

$$2\csc(\theta_{1817})\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$\left(s\,\theta_{54}\left(-\dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi + c\,\theta\, c\,\phi\left(\dot{\phi} + \left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) s\,\psi\right) +\right.\right.$$

$$\left.s\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43}) + \dot{\phi}\, s\,\psi\right)\right)\right) - c\,\theta_{54}\left(c\,\phi\left(\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) c\,\psi\, s(\theta_{21}+\theta_{43}) +\right.\right.$$

$$\left.\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c(\theta_{21}+\theta_{43})\, s\,\psi + s\,\theta\left(-\dot{\theta}_{54}\, s\,\psi + s(\theta_{21}+\theta_{43})\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right)\right) +$$

$$\left.\left.\left.s\,\phi\left(c(\theta_{21}+\theta_{43})\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + c\,\theta\left(-\dot{\theta}_{54} + s(\theta_{21}+\theta_{43})\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\right)\right)\right)^2 -$$

$$\cot(\theta_{1817}) \csc(\theta_{1817})\,(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\left(\ddot{\theta}_{1817}\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)\right.$$

$$(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$2\,\dot{\theta}_{1817}\,(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))$$

$$\left(s\,\theta_{54}\left(-c\,\theta\, c\,\psi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) + \dot{\psi}\, s\,\theta\, s\,\psi\right) - c\,\theta_{54}\left(c\,\psi\, s\,\theta\left(\dot{\theta}_{54} - \dot{\theta}\, s(\theta_{21}+\theta_{43})\right) +\right.\right.$$

$$\left.\left.\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) s(\theta_{21}+\theta_{43})\, s\,\psi\right) + c(\theta_{21}+\theta_{43})\left(\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c\,\theta_{54}\, c\,\psi - \dot{\theta}_{54}\, s\,\theta_{54}\, s\,\psi\right)\right) -$$

$$2\,\dot{\theta}_{1817}\,(-c\,\theta\, c\,\theta_{54}\, c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\psi\, s\,\theta\, s\,\theta_{54} + c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, s\,\psi)$$

$$\left(s\,\theta_{54}\left(-\dot{\theta}_{54}\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi + c\,\theta\, c\,\phi\left(\dot{\phi} + \left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43})\right) s\,\psi\right) +\right.\right.$$

$$\left.s\,\theta\left(\dot{\psi}\, c\,\phi\, c\,\psi - s\,\phi\left(\dot{\theta} - \dot{\theta}_{54}\, s(\theta_{21}+\theta_{43}) + \dot{\phi}\, s\,\psi\right)\right)\right) - c\,\theta_{54}\left(c\,\phi\left(\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\psi}\, c\,\theta\right) c\,\psi\, s(\theta_{21}+\theta_{43}) +\right.\right.$$

$$\left.\left(\dot{\psi} - \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) c\,\theta\right) c(\theta_{21}+\theta_{43})\, s\,\psi + s\,\theta\left(-\dot{\theta}_{54}\, s\,\psi + s(\theta_{21}+\theta_{43})\left(\dot{\phi} + \dot{\theta}\, s\,\psi\right)\right)\right) +$$

$$\left.\left.\left.\left.s\,\phi\left(c(\theta_{21}+\theta_{43})\left(\dot{\phi}\, c\,\psi + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right) s\,\theta\right) + c\,\theta\left(-\dot{\theta}_{54} + s(\theta_{21}+\theta_{43})\left(\dot{\theta} + \dot{\phi}\, s\,\psi\right)\right)\right)\right)\right)\right)\right) -$$

$$2\,\dot{\theta}_{1716}{}^2 \sec(\theta_{1716})^2\,(c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(-s\,\theta\, s\,\phi + c\,\theta\, c\,\phi\, s\,\psi) + s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi))^3$$

$$\left.\tan(\theta_{1716})\right)$$

Coeficientes de la ecuación (2.88)

$$
\begin{aligned}
G_{39} = {} & (s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) + \\
& c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + \\
& c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^3
\end{aligned}
$$

$$G_{40} = 0$$

$$G_{41} = 0$$

$$G_{42} = 0$$

$$
\begin{aligned}
G_{43} = {} & c\,\theta_{1615}{}^2\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s\,\theta\,(s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi + c\,\theta_{54}\,c\,\phi\,s\,\psi)) + \\
& c\,\theta_{1716}\,(s\,\theta_{54}\,(c\,\theta_{1817}\,c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43}) + c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\phi + c(\theta_{21}+\theta_{43})\,s\,\theta_{1817}\,s\,\psi) + \\
& c\,\theta_{54}\,s\,\theta\,(c\,\psi\,s\,\theta_{1817} - c\,\theta_{1817}\,s\,\phi\,s\,\psi)) + c\,\theta\,(s\,\theta_{1716}\,(c\,\theta_{54}\,s\,\phi - c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\psi) + \\
& c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\theta_{54}\,c\,\phi - c\,\psi\,s\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54} + c\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi))) \\
& \big(c\,\theta^2\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,(c\,\theta_{1716}\,c\,\theta_{1817}\,c\,\phi + s\,\theta_{1716}\,s\,\phi)\,(-c\,\phi\,c\,\psi\,s\,\theta_{1716} + c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\psi\,s\,\phi + s\,\theta_{1817}\,s\,\psi)) + \\
& c\,\theta\,\big(c\,\phi\,s\,\theta_{1716}{}^2\,(c\,\phi\,s\,\theta_{54} - c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi) - c\,\theta_{1716}\,s\,\theta_{1716}\,(c\,\theta_{1817}\,s\,\theta_{54}\,s(2\,\phi) + c\,\theta_{43}\,c\,\theta_{54}\,s\,\theta_{21} \\
& (c\,\psi\,s\,\theta_{1817}\,s\,\phi + c\,\theta_{1817}\,c(2\,\phi)\,s\,\psi) + c\,\theta_{21}\,c\,\theta_{54}\,s\,\theta_{43}\,(c\,\psi\,s\,\theta_{1817}\,s\,\phi + c\,\theta_{1817}\,c(2\,\phi)\,s\,\psi)) + c\,\theta_{1716}{}^2 \\
& \big(-c\,\theta_{1817}\,c\,\theta_{54}\,c\,\phi\,c\,\psi\,s\,\theta_{1817}\,s(\theta_{21}+\theta_{43}) + s\,\theta_{1817}{}^2\,s\,\theta_{54} + c\,\theta_{1817}{}^2\,s\,\phi\,(s\,\theta_{54}\,s\,\phi + c\,\theta_{54}\,c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\psi)\big)\big) + \\
& s\,\theta\,\big(-c\,\phi\,s\,\theta_{1716}{}^2\,(c\,\theta_{21}\,c\,\theta_{54}\,c\,\phi\,s\,\theta_{43} + c\,\theta_{43}\,c\,\theta_{54}\,(c\,\phi\,s\,\theta_{21} + c\,\theta_{21}\,c\,\psi\,s\,\theta\,s\,\phi) + s\,\phi \\
& (-c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{21}\,s\,\theta_{43} + s\,\theta_{54}\,s\,\psi)) + c\,\theta_{1716}\,s\,\theta_{1716}\,\big(s\,\theta_{1817}\,s\,\phi\,(-c\,\psi\,s\,\theta_{54} + c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\theta\,s\,\psi) + \\
& c\,\theta_{1817}\,\big(c\,\theta_{54}\,s\,\theta_{21}\,\big(c\,\phi^2\,c\,\psi\,s\,\theta\,s\,\theta_{43} - c\,\psi\,s\,\theta\,s\,\theta_{43}\,s\,\phi^2 + c\,\theta_{43}\,s(2\,\phi)\big) + c\,\theta_{21}\,c\,\theta_{54}\,(-c\,\theta_{43}\,c(2\,\phi)\,c\,\psi\,s\,\theta + \\
& s\,\theta_{43}\,s(2\,\phi)) - c(2\,\phi)\,s\,\theta_{54}\,s\,\psi\big)\big) - c\,\theta_{1716}{}^2\,\big(c\,\theta_{1817}\,c\,\phi\,s\,\theta_{54}\,(c\,\psi\,s\,\theta_{1817} - c\,\theta_{1817}\,s\,\phi\,s\,\psi) + \\
& c\,\theta_{54}\,\big(-c\,\theta_{1817}{}^2\,c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta\,s\,\phi + c\,\psi^2\,s(\theta_{21}+\theta_{43})\,\big(s\,\theta_{1817}{}^2 + c\,\theta_{1817}{}^2\,s\,\phi^2\big) + \\
& s\,\psi\,\big(-c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,s\,\theta\,s\,\theta_{1817} + s\,\theta_{1817}{}^2\,s(\theta_{21}+\theta_{43})\,s\,\psi + c\,\theta_{1817}{}^2\,s(\theta_{21}+\theta_{43})\,s\,\phi^2\,s\,\psi\big)\big)\big)\big)\big)
\end{aligned}
$$

$$
\begin{aligned}
G_{44} = {} & c\,\theta_{1615}{}^2\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s\,\theta\,(s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi + c\,\theta_{54}\,c\,\phi\,s\,\psi)) + \\
& c\,\theta_{1716}\,(s\,\theta_{54}\,(c\,\theta_{1817}\,c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43}) + c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\phi + c(\theta_{21}+\theta_{43})\,s\,\theta_{1817}\,s\,\psi) + \\
& c\,\theta_{54}\,s\,\theta\,(c\,\psi\,s\,\theta_{1817} - c\,\theta_{1817}\,s\,\phi\,s\,\psi)) + c\,\theta\,(s\,\theta_{1716}\,(c\,\theta_{54}\,s\,\phi - c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\psi) + \\
& c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\theta_{54}\,c\,\phi - c\,\psi\,s\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54} + c\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi))) \\
& \big(c\,\theta\,(-c\,\phi\,c\,\psi\,s\,\theta_{1716} + c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\psi\,s\,\phi + s\,\theta_{1817}\,s\,\psi))\,(s\,\theta_{1716}\,(s\,\theta_{54}\,s\,\phi + c\,\theta_{54}\,c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\psi) + \\
& c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\phi\,s\,\theta_{54} + c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,(c\,\psi\,s\,\theta_{1817} - c\,\theta_{1817}\,s\,\phi\,s\,\psi))) + \\
& c\,\theta^2\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,\big(2\,c\,\theta_{1716}\,c\,\phi\,c\,\psi\,s\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\psi\,s\,\phi + s\,\theta_{1817}\,s\,\psi) + s\,\theta_{1716}{}^2\,\big(s\,\phi^2 + c\,\phi^2\,s\,\psi^2\big) + \\
& c\,\theta_{1716}{}^2\,\big(c\,\psi^2\,s\,\theta_{1817}{}^2 - 2\,c\,\theta_{1817}\,c\,\psi\,s\,\theta_{1817}\,s\,\phi\,s\,\psi + c\,\theta_{1817}{}^2\,\big(c\,\phi^2 + s\,\phi^2\,s\,\psi^2\big)\big)\big) + \\
& s\,\theta\,\big(s\,\theta_{1716}{}^2\,\big(-c\,\phi^2\,c\,\psi\,s\,\theta_{54}\,s\,\psi + c\,\theta_{54}\,\big(c\,\phi\,c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi^2 + c(\theta_{21}+\theta_{43})\,c\,\phi^2\,s\,\theta\,s\,\psi^2\big)\big) + \\
& c\,\theta_{1716}\,s\,\theta_{1716}\,\big(c\,\theta_{1817}\,c\,\psi\,\big(c\,\theta_{54}\,\big(c\,\phi^2\,s(\theta_{21}+\theta_{43}) - s(\theta_{21}+\theta_{43})\,s\,\phi^2 + c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta\,s(2\,\phi)\big) + \\
& s\,\theta_{54}\,s(2\,\phi)\,s\,\psi\big) - s\,\theta_{1817}\,\big(c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi + \\
& c\,\phi\,\big(c\,\psi^2\,s\,\theta_{54} - 2\,c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\psi - s\,\theta_{54}\,s\,\psi^2 + c\,\theta_{54}\,s\,\theta\,s\,\theta_{21}\,s\,\theta_{43}\,s(2\,\psi)\big)\big)\big) - \\
& c\,\theta_{1716}{}^2\,\big(-c\,\psi\,s\,\theta_{1817}{}^2\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{54}\,c\,\psi\,s\,\theta - c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{21}\,s\,\theta_{43} + s\,\theta_{54}\,s\,\psi) + \\
& c\,\theta_{1817}{}^2\,\big(c(\theta_{21}+\theta_{43})^2\,c\,\psi\,s\,\theta_{54}\,s\,\phi^2\,s\,\psi + c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi\,(c\,\theta_{54}\,c\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi) - \\
& c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\theta\,\big(c\,\phi^2 + s\,\phi^2\,s\,\psi^2\big)\big) + c\,\theta_{1817}\,s\,\theta_{1817}\,\big(-c\,\psi^2\,s\,\theta_{54}\,s\,\phi + c\,\theta_{21}\,c\,\theta_{54}\,c\,\phi\,s\,\theta_{43}\,s\,\psi + \\
& s\,\theta_{54}\,s\,\phi\,s\,\psi^2 - c\,\theta_{54}\,s\,\theta\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\phi\,s(2\,\psi) + c\,\theta_{43}\,c\,\theta_{54}\,(c\,\phi\,s\,\theta_{21}\,s\,\psi + c\,\theta_{21}\,s\,\theta\,s\,\phi\,s(2\,\psi))\big)\big)\big)\big)
\end{aligned}
$$

$$
G_{45} = c\,\theta_{1615}{}^2\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s\,\theta\,(s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi + c\,\theta_{54}\,c\,\phi\,s\,\psi)) +
$$

$$c\,\theta_{1716}\,(s\,\theta_{54}\,(c\,\theta_{1817}\,c\,\phi\,s\,\theta\,s(\theta_{21}+\theta_{43})+c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\phi+c(\theta_{21}+\theta_{43})\,s\,\theta_{1817}\,s\,\psi)+$$
$$c\,\theta_{54}\,s\,\theta\,(c\,\psi\,s\,\theta_{1817}-c\,\theta_{1817}\,s\,\phi\,s\,\psi))+c\,\theta\,(s\,\theta_{1716}\,(c\,\theta_{54}\,s\,\phi-c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\psi)+$$
$$c\,\theta_{1716}\,(c\,\theta_{1817}\,c\,\theta_{54}\,c\,\phi-c\,\psi\,s\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}+c\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi)))$$
$$\left(c\,\theta^2\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\left(c\,\theta_{1716}\,c\,\phi\,c\,\psi\,s\,\theta_{1716}\,s\,\theta_{1817}+s\,\theta_{1716}{}^2\,s\,\psi+c\,\theta_{1716}{}^2\,c\,\theta_{1817}\,(-c\,\psi\,s\,\theta_{1817}\,s\,\phi+c\,\theta_{1817}\,s\,\psi)\right)+\right.$$
$$c\,\theta\left(-c\,\theta_{54}\,c\,\psi\,s\,\theta_{1716}{}^2\,s(\theta_{21}+\theta_{43})+c\,\theta_{1716}\,s\,\theta_{1716}\,s\,\theta_{1817}\,(s\,\theta_{54}\,s\,\phi+c\,\theta_{54}\,c\,\phi\,s(\theta_{21}+\theta_{43})\,s\,\psi)-\right.$$
$$\left.c\,\theta_{1716}{}^2\,c\,\theta_{1817}\,(c\,\theta_{1817}\,c\,\theta_{54}\,c\,\psi\,s(\theta_{21}+\theta_{43})+s\,\theta_{1817}\,(-c\,\phi\,s\,\theta_{54}+c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi))\right)-$$
$$s\,\theta\left(s\,\theta_{1716}{}^2\,(c\,\psi\,s\,\theta_{54}-c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,s\,\theta\,s\,\psi)+c\,\theta_{1716}\,s\,\theta_{1716}\,s\,\theta_{1817}\right.$$
$$(c\,\theta_{54}\,s\,\theta_{21}\,(c\,\phi\,c\,\psi\,s\,\theta\,s\,\theta_{43}+c\,\theta_{43}\,s\,\phi)+c\,\theta_{21}\,c\,\theta_{54}\,(-c\,\theta_{43}\,c\,\phi\,c\,\psi\,s\,\theta+s\,\theta_{43}\,s\,\phi)-c\,\phi\,s\,\theta_{54}\,s\,\psi)+c\,\theta_{1716}{}^2$$
$$c\,\theta_{1817}\,(c\,\theta_{21}\,c\,\theta_{54}\,c\,\phi\,s\,\theta_{1817}\,s\,\theta_{43}+c\,\theta_{1817}\,c\,\psi\,s\,\theta_{54}-c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{1817}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\phi+c\,\theta_{1817}\,c\,\theta_{54}\,s\,\theta\,s\,\theta_{21}$$
$$\left.\left.s\,\theta_{43}\,s\,\psi+s\,\theta_{1817}\,s\,\theta_{54}\,s\,\phi\,s\,\psi+c\,\theta_{43}\,c\,\theta_{54}\,(c\,\phi\,s\,\theta_{1817}\,s\,\theta_{21}+c\,\theta_{21}\,s\,\theta\,(c\,\psi\,s\,\theta_{1817}\,s\,\phi-c\,\theta_{1817}\,s\,\psi))))\right)\right)$$

$$G_{46}=c\,\theta_{1615}{}^2$$
$$\left(-2\,\dot\phi\,\dot\psi\,c\,\theta_{1716}\,c\,\theta_{1817}\,c\,\theta_{54}\,c\,\phi\,c\,\psi\,s\,\theta\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+c\,\theta_{1716}\right.$$
$$(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$2\,\dot\theta\,\dot\psi\,c\,\theta\,c\,\theta_{54}\,c\,\phi\,c\,\psi\,s\,\theta_{1716}\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))-$$
$$2\,\dot\theta\,\dot\phi\,c\,\theta_{54}\,c\,\phi\,s\,\theta\,s\,\theta_{1716}\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))-$$
$$\dot\theta^2\,c\,\theta_{1716}\,c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{1817}\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))-$$
$$\left(\dot\theta_{54}{}^2+\dot\psi^2\right)c\,\theta_{1716}\,c\,\theta_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{1817}\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\phi^2\right)c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{1716}\,s\,\theta_{54}$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$

$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\left(\dot{\theta}_{54}{}^{2}+\dot{\psi}^{2}\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\theta_{54}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+$$
$$\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{1716}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+2\,\dot{\phi}\,\dot{\psi}$$
$$\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$

$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\left(\dot{\theta}^2+\dot{\theta}_{21}{}^2+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^2+\dot{\psi}^2\right)\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$2\,\dot{\theta}\,\dot{\psi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$2\,\dot{\phi}\,\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\phi\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\left(\dot{\theta}_{54}{}^2+\dot{\phi}^2\right)\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\left(\dot{\theta}_{21}{}^2+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^2+\dot{\psi}^2\right)\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$2\,\dot{\theta}\,\dot{\phi}\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) - 2\,\dot{\theta}\,\dot{\psi}$$

$$c\,\theta_{1716}\,c\,\theta_{1817}\,c\,\psi\,s\,\theta\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) +$$

$$2\,\dot{\phi}\,\dot{\psi}\,c\,\theta\,c\,\psi\,s\,\theta_{1716}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) +$$

$$c\,\theta_{1716}\,c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\psi\,\left(2\,\dot{\theta}_{54}\,\dot{\phi}\,c\,\phi + \ddot{\theta}_{54}\,s\,\phi\right)$$

$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) +$$

$$c\,\psi\,s\,\theta_{1716}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43}\right)c\,\phi - 2\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)\dot{\phi}\,s\,\phi\right)$$

$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) -$$

$$\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2\right)c\,\theta_{54}\,c\,\phi\,s\,\theta\,s\,\theta_{1716}\,s\,\psi\,(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) -$$

$$\left(\dot{\theta}_{21}^{\,2} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}^{\,2} + \dot{\psi}^2 - 2\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)\dot{\psi}\,c\,\theta\right)c\,\theta_{1716}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{1817}\,s\,\theta_{54}\,s\,\psi$$

$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi + c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43}) - c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi)))) +$$

$$\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2\right)c\,\theta\,c\,\phi\,s\,\theta_{1716}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\psi$$

$$(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$\ddot{\theta}_{21}\,c\theta_{1716}\,s\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\theta_{54}\,s\psi\,(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$\ddot{\theta}_{43}\,c\theta_{1716}\,s\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\theta_{54}\,s\psi\,(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$2\,\dot{\theta}\,\dot{\psi}\,c\theta_{1716}\,s\theta\,s\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\theta_{54}\,s\psi\,(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))+$$
$$\left(\dot{\theta}^2+\dot{\phi}^2+\dot{\psi}^2\right)c\theta_{1716}\,c\theta_{1817}\,c\theta_{54}\,s\theta\,s\phi\,s\psi\,(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$\left(\dot{\theta}^2+\dot{\phi}^2+\dot{\psi}^2\right)c\theta\,c\theta_{1716}\,c\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\theta_{54}\,s\phi\,s\psi$$
$$(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$c\theta\,c\theta_{1716}\,c\theta_{1817}\,c\theta_{54}\,c\phi\left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\,s\psi\right)(-s\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\theta\,s\phi+$$
$$c\phi\,(c\psi\,s(\theta_{21}+\theta_{43})-c\theta\,c(\theta_{21}+\theta_{43})\,s\psi))+c\theta_{1716}\,(s\theta_{1817}\,(c\theta\,c(\theta_{21}+\theta_{43})\,c\psi+s(\theta_{21}+\theta_{43})\,s\psi)-$$
$$c\theta_{1817}\,(-c\psi\,s(\theta_{21}+\theta_{43})\,s\phi+c(\theta_{21}+\theta_{43})\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi))))$$
$$(s\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\phi\,c\psi\,s\theta_{54}+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(s\theta\,s\phi-c\theta\,c\phi\,s\psi)+c\theta_{54}\,(c\theta\,s\phi+c\phi\,s\theta\,s\psi))+$$
$$c\theta_{1716}\,(s\theta_{1817}\,(c\theta_{54}\,c\psi\,s\theta+s\theta_{54}\,(-c\theta\,c\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\psi))+$$
$$c\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\psi\,s\theta_{54}\,s\phi+s(\theta_{21}+\theta_{43})\,s\theta_{54}\,(c\phi\,s\theta+c\theta\,s\phi\,s\psi)+c\theta_{54}\,(c\theta\,c\phi-s\theta\,s\phi\,s\psi))))-$$
$$c\theta_{1716}\,c\theta_{1817}\,c\phi\,s\theta\,s(\theta_{21}+\theta_{43})\,s\theta_{54}\left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\,s\psi\right)$$

$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}\,\phi\left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1716}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\left(\dot{\theta}^2+\dot{\phi}^2+2\,\dot{\theta}\,\dot{\phi}\,\mathrm{s}\,\psi\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{c}\,\psi-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\,\mathrm{s}\,\psi\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{1716}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\psi-2\,\dot{\theta}_{54}\,\dot{\psi}\,\mathrm{s}\,\psi\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\psi-2\,\dot{\theta}_{54}\,\dot{\psi}\,\mathrm{s}\,\psi\right)(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\dot{\theta}_{54}{}^2\,\mathrm{s}\,\theta_{54}\right)(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\left(\dot{\phi}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\phi\right)\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\dot{\psi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-2\,\dot{\theta}\,\mathrm{c}\,\theta$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}\left(\dot{\theta}_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\dot{\psi}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\psi\right)(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\mathrm{s}\,\theta_{1716}\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\theta_{54}\right)+\mathrm{s}(\theta_{21}+\theta_{43})\right.$$
$$\left.\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{s}\,\theta_{54}\right)\right)$$
$$(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{s}\,\theta_{1716}\left(\dot{\theta}_{54}{}^{2}\,\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\mathrm{s}\,\theta_{1716}\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\phi\,\mathrm{s}(\theta_{21}+\theta_{43})+\dot{\phi}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\right)\left(\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi-\dot{\psi}\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\left(\mathrm{c}(\theta_{21}+\theta_{43})\left(2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\theta}_{54}\,\mathrm{c}\,\theta_{54}+\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}\,\theta_{54}\right)+\right.$$
$$\left.\mathrm{s}(\theta_{21}+\theta_{43})\left(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{54}-\left(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{s}\,\theta_{54}\right)\right)$$
$$(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))-$$
$$\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\left(\dot{\theta}_{54}{}^{2}\,\mathrm{c}\,\theta_{54}+\ddot{\theta}_{54}\,\mathrm{s}\,\theta_{54}\right)(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta_{1817}\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\left(\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\mathrm{c}\,\theta_{1716}\left(\ddot{\theta}_{1817}\,\mathrm{c}\,\theta_{1817}-\dot{\theta}_{1817}{}^{2}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\left(\ddot{\theta}_{1716}\,\mathrm{c}\,\theta_{1716}-\dot{\theta}_{1716}{}^{2}\,\mathrm{s}\,\theta_{1716}\right)(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+$$
$$\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$\mathrm{c}\,\theta_{1716}\left(-\dot{\theta}_{1817}{}^{2}\,\mathrm{c}\,\theta_{1817}-\ddot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+$$
$$\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))+$$
$$2\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta_{54}\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{s}\,\phi\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)$$
$$(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$2\,c\,\theta_{1716}\,c\,\theta_{1817}\left(\dot{\theta}_{54}\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\right)$$
$$\left(-s\,\theta\,s\,\phi\left(\dot{\phi}+\dot{\theta}\,s\,\psi\right)+c\,\theta\left(\dot{\psi}\,c\,\psi\,s\,\phi+c\,\phi\left(\dot{\theta}+\dot{\phi}\,s\,\psi\right)\right)\right)$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))-$$
$$2\,\dot{\theta}_{54}\,s\,\theta_{1716}\,s\,\theta_{54}\left(c\,\theta\,c\,\phi\left(\dot{\phi}+\dot{\theta}\,s\,\psi\right)+s\,\theta\left(\dot{\psi}\,c\,\phi\,c\,\psi-s\,\phi\left(\dot{\theta}+\dot{\phi}\,s\,\psi\right)\right)\right)$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$2\,s\,\theta_{1716}\left(\dot{\theta}_{54}\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\right)\left(c\,\phi\,s\,\theta\left(\dot{\phi}+\dot{\theta}\,s\,\psi\right)+c\,\theta\left(-\dot{\psi}\,c\,\phi\,c\,\psi+s\,\phi\left(\dot{\theta}+\dot{\phi}\,s\,\psi\right)\right)\right)$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$2\,\dot{\theta}_{1817}\,c\,\theta_{1716}\,c\,\theta_{1817}\left(\dot{\theta}\,c\,\theta\,c\,\theta_{54}\,c\,\psi-\dot{\theta}_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{54}-\dot{\psi}\,c\,\theta_{54}\,s\,\theta\,s\,\psi+\right.$$
$$\dot{\theta}_{54}\,c\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi)+$$
$$\left.s\,\theta_{54}\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c\,\theta\right)c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\left(\dot{\theta}\,c\,\psi\,s\,\theta-\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,c\,\theta\right)s\,\psi\right)\right)\right)$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))+$$
$$\left(-\dot{\theta}_{1716}{}^{2}\,c\,\theta_{1716}-\ddot{\theta}_{1716}\,s\,\theta_{1716}\right)(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)))$$
$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\phi+c\,\phi\,(c\,\psi\,s(\theta_{21}+\theta_{43})-c\,\theta\,c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)-$$
$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$
$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))-$$
$$2\,\dot{\theta}_{1817}\,c\,\theta_{1716}\,s\,\theta_{1817}\left(\dot{\phi}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+\dot{\theta}_{54}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\psi\,s\,\phi-\right.$$
$$\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi-\dot{\psi}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi+$$

$$\dot\theta_{54}\, c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + (\dot\theta_{21}+\dot\theta_{43})\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) -$$

$$\dot\theta_{54}\, s\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi) - c\,\theta_{54}\,(c\,\phi\, s\,\theta\,(\dot\theta+\dot\phi\, s\,\psi) + s\,\phi\,(\dot\psi\, c\,\psi\, s\,\theta + c\,\theta\,(\dot\phi+\dot\theta\, s\,\psi))) +$$

$$s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(-s\,\theta\, s\,\phi\,(\dot\phi+\dot\theta\, s\,\psi) + c\,\theta\,(\dot\psi\, c\,\psi\, s\,\phi + c\,\phi\,(\dot\theta+\dot\phi\, s\,\psi))))$$

$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\, s\,\theta\, s\,\phi + c\,\phi\,(c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\theta\, c(\theta_{21}+\theta_{43})\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\psi + s(\theta_{21}+\theta_{43})\, s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi)))) +$$

$$2\,\dot\theta_{1716}\, c\,\theta_{1716}\,(-\dot\theta_{54}\, c(\theta_{21}+\theta_{43})\, c\,\theta_{54}\, c\,\phi\, c\,\psi + (\dot\theta_{21}+\dot\theta_{43})\, c\,\phi\, c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\theta_{54} + \dot\phi\, c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi +$$

$$\dot\psi\, c(\theta_{21}+\theta_{43})\, c\,\phi\, s\,\theta_{54}\, s\,\psi + \dot\theta_{54}\, c\,\theta_{54}\, s(\theta_{21}+\theta_{43})\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) +$$

$$(\dot\theta_{21}+\dot\theta_{43})\, c(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) - \dot\theta_{54}\, s\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi) +$$

$$c\,\theta_{54}\,(c\,\theta\, c\,\phi\,(\dot\phi+\dot\theta\, s\,\psi) + s\,\theta\,(\dot\psi\, c\,\phi\, c\,\psi - s\,\phi\,(\dot\theta+\dot\phi\, s\,\psi))) +$$

$$s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta\,(\dot\phi+\dot\theta\, s\,\psi) + c\,\theta\,(-\dot\psi\, c\,\phi\, c\,\psi + s\,\phi\,(\dot\theta+\dot\phi\, s\,\psi))))$$

$$(-s\,\theta_{1716}\,(c(\theta_{21}+\theta_{43})\, s\,\theta\, s\,\phi + c\,\phi\,(c\,\psi\, s(\theta_{21}+\theta_{43}) - c\,\theta\, c(\theta_{21}+\theta_{43})\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\psi + s(\theta_{21}+\theta_{43})\, s\,\psi) -$$

$$c\,\theta_{1817}\,(-c\,\psi\, s(\theta_{21}+\theta_{43})\, s\,\phi + c(\theta_{21}+\theta_{43})\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) +$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi)))) +$$

$$2\,\dot\phi\,\dot\psi\, c\,\theta\, c\,\theta_{1716}\, c\,\theta_{1817}\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 +$$

$$2\,\dot\theta\,\dot\phi\, c\,\theta\, c(\theta_{21}+\theta_{43})\, c\,\phi\, s\,\theta_{1716}\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) +$$

$$c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + c\,\theta_{1716}$$

$$(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi +$$

$$s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 + 2\,\dot\theta\,\dot\psi\, c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi$$

$$s\,\theta\, s\,\theta_{1716}\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 -$$

$$2\,(\dot\theta_{21}+\dot\theta_{43})\,\dot\psi\, c\,\theta_{1716}\, c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{1817}\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} +$$

$$s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 +$$

$$(\dot\theta^2 + \dot\theta_{21}^{\,2} + 2\,\dot\theta_{21}\,\dot\theta_{43} + \dot\theta_{43}^{\,2} + \dot\psi^2)\, c\,\theta\, c\,\theta_{1716}\, c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{1817}$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) + c\,\theta_{1716}$$

$$(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi +$$

$$s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 - (\dot\phi^2 + \dot\psi^2)\, c\,\phi\, c\,\psi\, s\,\theta_{1716}$$

$$s(\theta_{21}+\theta_{43})\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(s\,\theta\, s\,\phi - c\,\theta\, c\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, s\,\phi + c\,\phi\, s\,\theta\, s\,\psi)) +$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\, c\,\psi\, s\,\theta + s\,\theta_{54}\,(-c\,\theta\, c\,\psi\, s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\, s\,\psi)) + c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\, c\,\psi\, s\,\theta_{54}\, s\,\phi + s(\theta_{21}+\theta_{43})\, s\,\theta_{54}\,(c\,\phi\, s\,\theta + c\,\theta\, s\,\phi\, s\,\psi) + c\,\theta_{54}\,(c\,\theta\, c\,\phi - s\,\theta\, s\,\phi\, s\,\psi))))^2 +$$

$$(\ddot\theta_{21}+\ddot\theta_{43})\, c\,\theta\, c\,\theta_{1716}\, c\,\psi\, s\,\theta_{1817}\, s(\theta_{21}+\theta_{43})\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\, c\,\phi\, c\,\psi\, s\,\theta_{54} +$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-$$

$$2\,\dot\theta\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\theta_{1716}\,\mathrm{c}\psi\,\mathrm{s}\theta\,\mathrm{s}\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2+$$

$$\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{1716}\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})-\left(\dot\theta_{21}+\dot\theta_{43}\right)^2\mathrm{s}(\theta_{21}+\theta_{43})\right)$$

$$(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-$$

$$2\,\dot\theta\,\dot\phi\,\mathrm{c}\theta\,\mathrm{c}\theta_{1716}\,\mathrm{c}\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\phi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}$$

$$(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-$$

$$2\,\dot\theta\,\dot\psi\,\mathrm{c}\theta_{1716}\,\mathrm{c}\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta\,\mathrm{s}\phi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}$$

$$(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+\mathrm{c}\theta_{1716}$$

$$(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2+2\,\dot\phi\,\dot\psi\,\mathrm{c}\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi$$

$$\mathrm{s}\theta_{1716}\,\mathrm{s}\phi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2+$$

$$\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\phi^2\right)\mathrm{c}\theta_{1716}\,\mathrm{c}\theta_{1817}\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\phi$$

$$(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2+$$

$$\dot\psi^2\,\mathrm{c}\theta_{1716}\,\mathrm{c}\theta_{1817}\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\phi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}$$

$$(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-$$

$$\mathrm{c}\theta_{1716}\,\mathrm{c}\theta_{1817}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\left(2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\phi\,\mathrm{c}\phi+\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\mathrm{s}\phi\right)$$

$$(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2+$$

$$\left(\dot\theta^2+\dot\phi^2+\dot\psi^2\right)\mathrm{c}\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{s}\theta_{1716}\,\mathrm{s}\psi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$\mathrm{c}\theta_{1716}\,(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-$$

$$\ddot\theta_{21}\,\mathrm{c}\theta_{1716}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{1817}\,\mathrm{s}\psi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+$$

$$\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+\mathrm{c}\theta_{1716}$$

$$(\mathrm{s}\theta_{1817}\,(\mathrm{c}\theta_{54}\,\mathrm{c}\psi\,\mathrm{s}\theta+\mathrm{s}\theta_{54}\,(-\mathrm{c}\theta\,\mathrm{c}\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\psi))+\mathrm{c}\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}\,\mathrm{s}\phi+$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{c}\phi\,\mathrm{s}\theta+\mathrm{c}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{c}\phi-\mathrm{s}\theta\,\mathrm{s}\phi\,\mathrm{s}\psi))))^2-\ddot\theta_{43}\,\mathrm{c}\theta_{1716}\,\mathrm{c}(\theta_{21}+\theta_{43})$$

$$\mathrm{s}\theta_{1817}\,\mathrm{s}\psi\,(\mathrm{s}\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\phi\,\mathrm{c}\psi\,\mathrm{s}\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\theta_{54}\,(\mathrm{s}\theta\,\mathrm{s}\phi-\mathrm{c}\theta\,\mathrm{c}\phi\,\mathrm{s}\psi)+\mathrm{c}\theta_{54}\,(\mathrm{c}\theta\,\mathrm{s}\phi+\mathrm{c}\phi\,\mathrm{s}\theta\,\mathrm{s}\psi))+$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 -$$
$$2\,\dot{\theta}\,\dot{\psi}\,c\,\theta_{1716}\,c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\theta_{1817}\,s\,\psi\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}$$
$$(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$
$$\left(\dot{\theta}_{21}{}^2 + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}{}^2 + \dot{\psi}^2 - 2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\dot{\psi}\,c\,\theta\right)c\,\theta_{1716}\,s\,\theta_{1817}\,s(\theta_{21}+\theta_{43})\,s\,\psi$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 -$$
$$\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\psi}^2\right)c\,\theta\,c\,\theta_{1716}\,c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$
$$2\,\dot{\phi}\,\dot{\psi}\,s\,\theta_{1716}\,s(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) +$$
$$c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$
$$2\,\dot{\psi}\,c\,\theta_{1716}\,c\,\theta_{1817}\left(\dot{\phi}\,c\,\phi\,s(\theta_{21}+\theta_{43}) + \left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c(\theta_{21}+\theta_{43})\,s\,\phi\right)s\,\psi$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 -$$
$$c\,\theta_{1716}\,c\,\theta_{1817}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,s\,\theta\left(\dot{\theta}^2 + \dot{\phi}^2 + 2\,\dot{\theta}\,\dot{\phi}\,s\,\psi\right)$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 -$$
$$c(\theta_{21}+\theta_{43})\,s\,\theta\,s\,\theta_{1716}\,s\,\phi\left(\dot{\theta}^2 + \dot{\phi}^2 + 2\,\dot{\theta}\,\dot{\phi}\,s\,\psi\right)(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} +$$
$$s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 -$$
$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)c(\theta_{21}+\theta_{43})\,s\,\theta_{1716}\left(\dot{\phi}\,c\,\psi\,s\,\phi + \dot{\psi}\,c\,\phi\,s\,\psi\right)$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$
$$s\,\theta_{1716}\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^2 c(\theta_{21}+\theta_{43}) - \left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)s(\theta_{21}+\theta_{43})\right)(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi)$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$
$$c\,\theta_{1716}\left(-\ddot{\theta}_{1817}\,c\,\theta_{1817} + \dot{\theta}_{1817}{}^2\,s\,\theta_{1817}\right)(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi + s(\theta_{21}+\theta_{43})\,s\,\psi)$$
$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54} + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi - c\,\theta\,c\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,s\,\phi + c\,\phi\,s\,\theta\,s\,\psi)) +$$
$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta + s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43}) + c(\theta_{21}+\theta_{43})\,s\,\psi)) + c\,\theta_{1817}$$
$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi + s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta + c\,\theta\,s\,\phi\,s\,\psi) + c\,\theta_{54}\,(c\,\theta\,c\,\phi - s\,\theta\,s\,\phi\,s\,\psi))))^2 +$$

$$\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)^{2}\mathrm{c}(\theta_{21}+\theta_{43})-\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\right)(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}-$$
$$2\,\dot{\theta}_{1817}\,\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\left(\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\left(-\dot{\theta}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{s}\,\psi\right)\right)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}+$$
$$\left(\ddot{\theta}_{1716}\,\mathrm{c}\,\theta_{1716}-\dot{\theta}_{1716}{}^{2}\,\mathrm{s}\,\theta_{1716}\right)(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}+$$
$$\mathrm{c}\,\theta_{1716}\left(\dot{\theta}_{1817}{}^{2}\,\mathrm{c}\,\theta_{1817}+\ddot{\theta}_{1817}\,\mathrm{s}\,\theta_{1817}\right)(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi-\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}-$$
$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta_{1716}\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}(\theta_{21}+\theta_{43})\left(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}-$$
$$2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta_{1716}\,\mathrm{s}(\theta_{21}+\theta_{43})\left(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\theta\left(-\dot{\psi}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi+\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}+$$
$$\left(\dot{\theta}_{1716}{}^{2}\,\mathrm{c}\,\theta_{1716}+\ddot{\theta}_{1716}\,\mathrm{s}\,\theta_{1716}\right)(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)))$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}+$$
$$2\,\dot{\theta}_{1716}\,\mathrm{c}\,\theta_{1716}\left(-\mathrm{s}(\theta_{21}+\theta_{43})\left(\dot{\phi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\left(\dot{\psi}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\theta\right)\mathrm{c}\,\phi\,\mathrm{s}\,\psi\right)+\right.$$
$$\left.\mathrm{c}(\theta_{21}+\theta_{43})\left(\mathrm{c}\,\theta\,\mathrm{s}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)+\mathrm{c}\,\phi\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,\mathrm{c}\,\theta\right)\mathrm{c}\,\psi+\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)\right)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}-$$
$$2\,\dot{\theta}_{1817}\,\mathrm{c}\,\theta_{1716}\,\mathrm{s}\,\theta_{1817}\left(-\mathrm{c}\,\phi\left(\dot{\phi}\,\mathrm{c}\,\psi+\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}\,\theta\right)\mathrm{s}(\theta_{21}+\theta_{43})-\right.$$
$$\mathrm{s}\,\phi\left(-\dot{\psi}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi+\mathrm{c}(\theta_{21}+\theta_{43})\left(\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{c}\,\psi+\mathrm{s}\,\theta\left(\dot{\phi}+\dot{\theta}\,\mathrm{s}\,\psi\right)\right)\right)+$$
$$\left.\mathrm{c}\,\theta\left(-\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi+\mathrm{c}(\theta_{21}+\theta_{43})\left(\dot{\psi}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\left(\dot{\theta}+\dot{\phi}\,\mathrm{s}\,\psi\right)\right)\right)\right)$$
$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi-\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta+\mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})+\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+\mathrm{c}\,\theta_{1817}$$
$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)+\mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{2}-$$
$$2\,\dot{\theta}_{1716}\,\mathrm{s}\,\theta_{1716}\,(-\mathrm{s}\,\theta_{1716}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi+\mathrm{c}\,\phi\,(\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi))+$$
$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi+\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)-$$
$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi+\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta+\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+$$

$$c\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))$$

$$(\dot{\theta}_{1817}\,c\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))-$$

$$\dot{\theta}_{1817}\,s\,\theta_{1817}\,(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))+$$

$$s\,\theta_{1817}\,(\dot{\theta}\,c\,\theta\,c\,\theta_{54}\,c\,\psi-\dot{\theta}_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{54}-\dot{\psi}\,c\,\theta_{54}\,s\,\theta\,s\,\psi+\dot{\theta}_{54}\,c\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi)+$$

$$s\,\theta_{54}\,((\dot{\psi}-(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\theta)\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,(\dot{\theta}\,c\,\psi\,s\,\theta-(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,c\,\theta)\,s\,\psi)))+$$

$$c\,\theta_{1817}\,(\dot{\phi}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+\dot{\theta}_{54}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\psi\,s\,\phi-(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi-$$

$$\dot{\psi}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi+\dot{\theta}_{54}\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+$$

$$(\dot{\theta}_{21}+\dot{\theta}_{43})\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)-\dot{\theta}_{54}\,s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-$$

$$c\,\theta_{54}\,(c\,\phi\,s\,\theta\,(\dot{\theta}+\dot{\phi}\,s\,\psi)+s\,\phi\,(\dot{\psi}\,c\,\psi\,s\,\theta+c\,\theta\,(\dot{\phi}+\dot{\theta}\,s\,\psi)))+$$

$$s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(-s\,\theta\,s\,\phi\,(\dot{\phi}+\dot{\theta}\,s\,\psi)+c\,\theta\,(\dot{\psi}\,c\,\psi\,s\,\phi+c\,\phi\,(\dot{\theta}+\dot{\phi}\,s\,\psi))))))-$$

$$2\,\dot{\theta}_{1716}\,s\,\theta_{1716}\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$

$$c\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))))^{2}$$

$$(-c(\theta_{21}+\theta_{43})\,(\dot{\theta}_{1817}\,c\,\phi\,s\,\theta\,s\,\theta_{1817}+\dot{\phi}\,c\,\theta_{1817}\,s\,\theta\,s\,\phi+c\,\psi\,(-\dot{\theta}\,s\,\theta\,s\,\theta_{1817}+(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\theta_{1817}\,s\,\phi)+$$

$$\dot{\theta}_{21}\,s\,\theta_{1817}\,s\,\psi+\dot{\theta}_{43}\,s\,\theta_{1817}\,s\,\psi+\dot{\theta}\,c\,\theta_{1817}\,s\,\theta\,s\,\phi\,s\,\psi)-$$

$$s(\theta_{21}+\theta_{43})\,(c\,\psi\,s\,\theta_{1817}\,(\dot{\psi}-\dot{\theta}_{1817}\,s\,\phi)+c\,\theta_{1817}\,(c\,\phi\,(\dot{\phi}\,c\,\psi+(\dot{\theta}_{21}+\dot{\theta}_{43})\,s\,\theta)+(\dot{\theta}_{1817}-\dot{\psi}\,s\,\phi)\,s\,\psi))+$$

$$c\,\theta\,(s\,\theta_{1817}\,((\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,(\dot{\psi}-\dot{\theta}_{1817}\,s\,\phi)\,s\,\psi)+$$

$$c\,\theta_{1817}\,(-(\dot{\theta}_{21}+\dot{\theta}_{43})\,s(\theta_{21}+\theta_{43})\,s\,\phi\,s\,\psi+c(\theta_{21}+\theta_{43})\,(c\,\psi\,(-\dot{\theta}_{1817}+\dot{\psi}\,s\,\phi)+c\,\phi\,(\dot{\theta}+\dot{\phi}\,s\,\psi)))))+$$

$$2\,(s\,\theta_{1716}\,(c\,\phi\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi))+$$

$$c\,\theta_{1716}\,(-s\,\theta_{1817}\,(c\,\theta\,c(\theta_{21}+\theta_{43})\,c\,\psi+s(\theta_{21}+\theta_{43})\,s\,\psi)+$$

$$c\,\theta_{1817}\,(-c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\phi+c(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi))))$$

$$(\dot{\theta}_{1716}\,c\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))-$$

$$\dot{\theta}_{1716}\,s\,\theta_{1716}\,(s\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))+c\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)))+$$

$$s\,\theta_{1716}\,(-\dot{\theta}_{54}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\phi\,c\,\psi+(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\phi\,c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}+$$

$$\dot{\phi}\,c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+\dot{\psi}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,s\,\theta_{54}\,s\,\psi+$$

$$\dot{\theta}_{54}\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+(\dot{\theta}_{21}+\dot{\theta}_{43})\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)-$$

$$\dot{\theta}_{54}\,s\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi\,(\dot{\phi}+\dot{\theta}\,s\,\psi)+s\,\theta\,(\dot{\psi}\,c\,\phi\,c\,\psi-s\,\phi\,(\dot{\theta}+\dot{\phi}\,s\,\psi)))+$$

$$s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta\,(\dot{\phi}+\dot{\theta}\,s\,\psi)+c\,\theta\,(-\dot{\psi}\,c\,\phi\,c\,\psi+s\,\phi\,(\dot{\theta}+\dot{\phi}\,s\,\psi))))+$$

$$c\,\theta_{1716}\,(\dot{\theta}_{1817}\,c\,\theta_{1817}\,(c\,\theta_{54}\,c\,\psi\,s\,\theta+s\,\theta_{54}\,(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi))-\dot{\theta}_{1817}\,s\,\theta_{1817}$$

$$(c(\theta_{21}+\theta_{43})\,c\,\psi\,s\,\theta_{54}\,s\,\phi+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi))+$$

$$s\,\theta_{1817}\,(\dot{\theta}\,c\,\theta\,c\,\theta_{54}\,c\,\psi-\dot{\theta}_{54}\,c\,\psi\,s\,\theta\,s\,\theta_{54}-\dot{\psi}\,c\,\theta_{54}\,s\,\theta\,s\,\psi+\dot{\theta}_{54}\,c\,\theta_{54}$$

$$(-c\,\theta\,c\,\psi\,s(\theta_{21}+\theta_{43})+c(\theta_{21}+\theta_{43})\,s\,\psi)+s\,\theta_{54}\,((\dot{\psi}-(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\theta)\,c(\theta_{21}+\theta_{43})\,c\,\psi+$$

$$s(\theta_{21}+\theta_{43})\,(\dot{\theta}\,c\,\psi\,s\,\theta-(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\psi}\,c\,\theta)\,s\,\psi)))+c\,\theta_{1817}\,(\dot{\phi}\,c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+$$

$$\dot{\theta}_{54}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{54}\,c\,\psi\,s\,\phi-(\dot{\theta}_{21}+\dot{\theta}_{43})\,c\,\psi\,s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi-\dot{\psi}\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,s\,\phi\,s\,\psi+$$

$$\dot{\theta}_{54}\,c\,\theta_{54}\,s(\theta_{21}+\theta_{43})\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)+(\dot{\theta}_{21}+\dot{\theta}_{43})\,c(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(c\,\phi\,s\,\theta+c\,\theta\,s\,\phi\,s\,\psi)-$$

$$\dot{\theta}_{54}\,s\,\theta_{54}\,(c\,\theta\,c\,\phi-s\,\theta\,s\,\phi\,s\,\psi)-c\,\theta_{54}\,(c\,\phi\,s\,\theta\,(\dot{\theta}+\dot{\phi}\,s\,\psi)+s\,\phi\,(\dot{\psi}\,c\,\psi\,s\,\theta+c\,\theta\,(\dot{\phi}+\dot{\theta}\,s\,\psi)))+$$

$$s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(-s\,\theta\,s\,\phi\,(\dot{\phi}+\dot{\theta}\,s\,\psi)+c\,\theta\,(\dot{\psi}\,c\,\psi\,s\,\phi+c\,\phi\,(\dot{\theta}+\dot{\phi}\,s\,\psi)))))))^{2}-$$

$$2\,(s\,\theta_{1716}\,(-c(\theta_{21}+\theta_{43})\,c\,\phi\,c\,\psi\,s\,\theta_{54}+s(\theta_{21}+\theta_{43})\,s\,\theta_{54}\,(s\,\theta\,s\,\phi-c\,\theta\,c\,\phi\,s\,\psi)+c\,\theta_{54}\,(c\,\theta\,s\,\phi+c\,\phi\,s\,\theta\,s\,\psi))+$$

$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)) +$$

$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))$$

$$\Big(\dot\theta_{1716}\,\mathrm{c}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54} + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) -$$

$$\dot\theta_{1716}\,\mathrm{s}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)) +$$

$$\mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))) +$$

$$\mathrm{s}\,\theta_{1716}\,\big(-\dot\theta_{54}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54} + \dot\phi\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi +$$

$$\dot\psi\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\psi + \dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) +$$

$$\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) - \dot\theta_{54}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi) +$$

$$\mathrm{c}\,\theta_{54}\,\big(\mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\big(\dot\phi + \dot\theta\,\mathrm{s}\,\psi\big) + \mathrm{s}\,\theta\,\big(\dot\psi\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi - \mathrm{s}\,\phi\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big)\big)\big) +$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\big(\dot\phi + \dot\theta\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\theta\,\big(-\dot\psi\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi + \mathrm{s}\,\phi\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big)\big)\big)\big) +$$

$$\mathrm{c}\,\theta_{1716}\,\big(\dot\theta_{1817}\,\mathrm{c}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)) - \dot\theta_{1817}\,\mathrm{s}\,\theta_{1817}$$

$$(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi)) + \mathrm{s}\,\theta_{1817}$$

$$\big(\dot\theta\,\mathrm{c}\,\theta\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi - \dot\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{54} - \dot\psi\,\mathrm{c}\,\theta_{54}\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi + \dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi) +$$

$$\mathrm{s}\,\theta_{54}\,\big(\big(\dot\psi - \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi + \mathrm{s}(\theta_{21}+\theta_{43})\,\big(\dot\theta\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta - \big(\dot\theta_{21}+\dot\theta_{43} - \dot\psi\,\mathrm{c}\,\theta\big)\,\mathrm{s}\,\psi\big)\big)\big) +$$

$$\mathrm{c}\,\theta_{1817}\,\big(\dot\phi\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54} + \dot\theta_{54}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi - \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi -$$

$$\dot\psi\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi + \dot\theta_{54}\,\mathrm{c}\,\theta_{54}\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) +$$

$$\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) - \dot\theta_{54}\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) -$$

$$\mathrm{c}\,\theta_{54}\,\big(\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big) + \mathrm{s}\,\phi\,\big(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\big(\dot\phi + \dot\theta\,\mathrm{s}\,\psi\big)\big)\big) +$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,\big(-\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\big(\dot\phi + \dot\theta\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\theta\,\big(\dot\psi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big)\big)\big)\big)\big)\Big)$$

$$\Big(\dot\theta_{1716}\,\mathrm{c}\,\theta_{1716}\,(\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi)) - \dot\theta_{1716}\,\mathrm{s}\,\theta_{1716}$$

$$(-\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi) +$$

$$\mathrm{c}\,\theta_{1817}\,(-\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\phi + \mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))) +$$

$$\mathrm{s}\,\theta_{1716}\,\big(-\mathrm{s}(\theta_{21}+\theta_{43})\,\big(\dot\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\phi + \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \big(\dot\psi - \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta\big)\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi\big) +$$

$$\mathrm{c}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\phi\,\big(\big(\dot\theta_{21}+\dot\theta_{43} - \dot\psi\,\mathrm{c}\,\theta\big)\,\mathrm{c}\,\psi + \mathrm{s}\,\theta\,\big(\dot\phi + \dot\theta\,\mathrm{s}\,\psi\big)\big)\big)\big) +$$

$$\mathrm{c}\,\theta_{1716}\,\big(-\mathrm{c}(\theta_{21}+\theta_{43})\,\big(\dot\theta_{1817}\,\mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817} + \dot\phi\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\psi\,\big(-\dot\theta\,\mathrm{s}\,\theta\,\mathrm{s}\,\theta_{1817} + \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\phi\big) +$$

$$\dot\theta_{21}\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi + \dot\theta_{43}\,\mathrm{s}\,\theta_{1817}\,\mathrm{s}\,\psi + \dot\theta\,\mathrm{c}\,\theta_{1817}\,\mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi\big) -$$

$$\mathrm{s}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{1817}\,\big(\dot\psi - \dot\theta_{1817}\,\mathrm{s}\,\phi\big) + \mathrm{c}\,\theta_{1817}\,\big(\mathrm{c}\,\phi\,\big(\dot\phi\,\mathrm{c}\,\psi + \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{s}\,\theta\big) + \big(\dot\theta_{1817} - \dot\psi\,\mathrm{s}\,\phi\big)\,\mathrm{s}\,\psi\big)\big) +$$

$$\mathrm{c}\,\theta\,\big(\mathrm{s}\,\theta_{1817}\,\big(\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\big(\dot\psi - \dot\theta_{1817}\,\mathrm{s}\,\phi\big)\,\mathrm{s}\,\psi\big) + \mathrm{c}\,\theta_{1817}\,\big(-\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{s}(\theta_{21}+\theta_{43})$$

$$\mathrm{s}\,\phi\,\mathrm{s}\,\psi + \mathrm{c}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}\,\psi\,\big(-\dot\theta_{1817} + \dot\psi\,\mathrm{s}\,\phi\big) + \mathrm{c}\,\phi\,\big(\dot\theta + \dot\phi\,\mathrm{s}\,\psi\big)\big)\big)\big)\big)\big) - 2\,\dot\theta_{1615}{}^{2}\,\sec(\theta_{1615})^{2}$$

$$(\mathrm{s}\,\theta_{1716}\,(-\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\phi\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54} + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{s}\,\theta\,\mathrm{s}\,\phi - \mathrm{c}\,\theta\,\mathrm{c}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{s}\,\phi + \mathrm{c}\,\phi\,\mathrm{s}\,\theta\,\mathrm{s}\,\psi)) +$$

$$\mathrm{c}\,\theta_{1716}\,(\mathrm{s}\,\theta_{1817}\,(\mathrm{c}\,\theta_{54}\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta + \mathrm{s}\,\theta_{54}\,(-\mathrm{c}\,\theta\,\mathrm{c}\,\psi\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\psi)) + \mathrm{c}\,\theta_{1817}\,(\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\psi\,\mathrm{s}\,\theta_{54}$$

$$\mathrm{s}\,\phi + \mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{54}\,(\mathrm{c}\,\phi\,\mathrm{s}\,\theta + \mathrm{c}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi) + \mathrm{c}\,\theta_{54}\,(\mathrm{c}\,\theta\,\mathrm{c}\,\phi - \mathrm{s}\,\theta\,\mathrm{s}\,\phi\,\mathrm{s}\,\psi))))^{3}\,\tan(\theta_{1615})\Big)$$

Coeficientes de la ecuación (2.161)

$G_{47} = 0$

$G_{48} = 0$

$G_{49} = 0$

$$G_{50} = 0$$

$$G_{51} = 0$$

$$G_{52} = 0$$

$$G_{53} = 4\,\dot{\theta}_{21\,i}\,\dot{\theta}_{54}\,x_{43\,i}\,c\,\theta_{21\,i}\,(x_{1211\,i}\,c(\beta_{1413\,i} - \theta_{54}) - x_{65}\,c\,\theta_{54}) -$$

$$2\,x_{32}\,z_{10}\left(\ddot{\theta}_{21}\,c\,\theta_{21} - \dot{\theta}_{21}{}^{2}\,s\,\theta_{21}\right) - 2\,x_{32}\,z_{32\,i}\left(\ddot{\theta}_{21}\,c\,\theta_{21} - \dot{\theta}_{21}{}^{2}\,s\,\theta_{21}\right) - 2\,x_{32}\,z_{1312\,i}\left(\ddot{\theta}_{43}\,c\,\theta_{43} - \dot{\theta}_{43}{}^{2}\,s\,\theta_{43}\right) -$$

$$2\,z_{1312\,i}\,(z_{10} + z_{32\,i})\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)^{2} c(\theta_{21} + \theta_{43}) - \left(\ddot{\theta}_{21} + \ddot{\theta}_{43}\right) s(\theta_{21} + \theta_{43})\right) +$$

$$\dot{\theta}_{21\,i}{}^{2}\,x_{43\,i}\,c\,\theta_{21\,i}\,(2\,x_{32}\,c\,\theta_{21} - x_{1211\,i}\,c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) + x_{65}\,c(\theta_{21} + \theta_{43} - \theta_{54}) -$$

$$x_{1211\,i}\,c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) + x_{65}\,c(\theta_{21} + \theta_{43} + \theta_{54}) - 2\,z_{1312\,i}\,s(\theta_{21} + \theta_{43})) +$$

$$2\,\mathrm{signo}\,x_{1211\,i}\,y_{10\,i}\left(\ddot{\theta}_{54}\,c(\beta_{1413\,i} - \theta_{54}) + \dot{\theta}_{54}{}^{2}\,s(\beta_{1413\,i} - \theta_{54})\right) -$$

$$x_{1211\,i}\,x_{32}\left(-\left(\dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} c(\beta_{1413\,i} - \theta_{43} - \theta_{54}) + \left(\ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\beta_{1413\,i} - \theta_{43} - \theta_{54})\right) -$$

$$x_{1211\,i}\,z_{10}\left(-\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54})\right) -$$

$$x_{1211\,i}\,z_{32\,i}\left(-\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54})\right) +$$

$$x_{32}\,x_{65}\left(-\left(\dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} c(\theta_{43} - \theta_{54}) + \left(-\ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\theta_{43} - \theta_{54})\right) -$$

$$x_{1211\,i}\,x_{32}\left(-\left(\dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} c(\beta_{1413\,i} + \theta_{43} - \theta_{54}) + \left(-\ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\beta_{1413\,i} + \theta_{43} - \theta_{54})\right) -$$

$$x_{65}\,z_{10}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} s(\theta_{21} + \theta_{43} - \theta_{54})\right) -$$

$$x_{65}\,z_{32\,i}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} s(\theta_{21} + \theta_{43} - \theta_{54})\right) +$$

$$x_{1211\,i}\,z_{10}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54})\right) +$$

$$x_{1211\,i}\,z_{32\,i}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54})\right) -$$

$$2\,\mathrm{signo}\,x_{65}\,y_{10\,i}\left(\ddot{\theta}_{54}\,c\,\theta_{54} - \dot{\theta}_{54}{}^{2}\,s\,\theta_{54}\right) + 2\,\dot{\theta}_{21\,i}{}^{2}\,x_{43\,i}\,s\,\theta_{21\,i}\,(-\mathrm{signo}\,y_{10\,i} + x_{1211\,i}\,s(\beta_{1413\,i} - \theta_{54}) + x_{65}\,s\,\theta_{54}) +$$

$$2\,x_{43\,i}\,s\,\theta_{21\,i}\left(\ddot{\theta}_{54}\,x_{1211\,i}\,c(\beta_{1413\,i} - \theta_{54}) - \ddot{\theta}_{54}\,x_{65}\,c\,\theta_{54} + \dot{\theta}_{54}{}^{2}\,(x_{1211\,i}\,s(\beta_{1413\,i} - \theta_{54}) + x_{65}\,s\,\theta_{54})\right) +$$

$$x_{32}\,x_{65}\left(-\left(\dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} c(\theta_{43} + \theta_{54}) - \left(\ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\theta_{43} + \theta_{54})\right) -$$

$$x_{65}\,z_{10}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} + \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} s(\theta_{21} + \theta_{43} + \theta_{54})\right) -$$

$$x_{65}\,z_{32\,i}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} + \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} s(\theta_{21} + \theta_{43} + \theta_{54})\right) +$$

$$2\,\dot{\theta}_{21\,i}\,x_{43\,i}\,s\,\theta_{21\,i}\left(-2\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right) z_{1312\,i}\,c(\theta_{21} + \theta_{43}) - 2\,\dot{\theta}_{21}\,x_{32}\,s\,\theta_{21} -\right.$$

$$\left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{1211\,i}\,s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{65}\,s(\theta_{21} + \theta_{43} - \theta_{54}) +$$

$$\left.\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{1211\,i}\,s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{65}\,s(\theta_{21} + \theta_{43} + \theta_{54})\right) -$$

$$x_{43\,i}\,c\,\theta_{21\,i}\left(-2\,x_{32}\left(\dot{\theta}_{21}{}^{2}\,c\,\theta_{21} + \ddot{\theta}_{21}\,s\,\theta_{21}\right) - 2\,z_{1312\,i}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43}\right) c(\theta_{21} + \theta_{43}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)^{2} s(\theta_{21} + \theta_{43})\right) -\right.$$

$$x_{1211\,i}\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) + \left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54})\right) +$$

$$x_{65}\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} c(\theta_{21} + \theta_{43} - \theta_{54}) - \left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) s(\theta_{21} + \theta_{43} - \theta_{54})\right) -$$

$$x_{1211\,i}\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^{2} c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54})\right) +$$

$$\left.x_{65}\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^{2} c(\theta_{21} + \theta_{43} + \theta_{54}) - \left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) s(\theta_{21} + \theta_{43} + \theta_{54})\right)\right)$$

Coeficientes de la ecuación (2.164)

$G_{54} = 0$

$G_{55} = 0$

$G_{56} = 0$

$G_{57} = 0$

$G_{58} = 0$

$G_{59} = 0$

$$G_{60} = \dot{\theta}_{65\,i}^{\,2}\, c\,\theta_{65\,i}\,(z_{32\,i}\, c\,\beta_{54\,i} + \text{signo}\, y_{10\,i}\, c\,\theta_{21\,i}\, s\,\beta_{54\,i} + c\,\beta_{54\,i}\,(z_{10} - x_{32}\, s\,\theta_{21}) + x_{32}\, c\,\theta_{21}\, s\,\beta_{54\,i}\, s\,\theta_{21\,i} +$$
$$(c\,\beta_{54\,i}\, s\,\theta_{21} - c\,\theta_{21}\, s\,\beta_{54\,i}\, s\,\theta_{21\,i})\,(c\,\theta_{43}\,(x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54}) - x_{65}\, c\,\theta_{54}) + z_{1312\,i}\, s\,\theta_{43}) -$$
$$(c\,\beta_{54\,i}\, c\,\theta_{21} + s\,\beta_{54\,i}\, s\,\theta_{21}\, s\,\theta_{21\,i})\,(z_{1312\,i}\, c\,\theta_{43} + (-x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54}) + x_{65}\, c\,\theta_{54})\, s\,\theta_{43}) -$$
$$c\,\theta_{21\,i}\, s\,\beta_{54\,i}\,(x_{1211\,i}\, s(\beta_{1413\,i} - \theta_{54}) + x_{65}\, s\,\theta_{54})) -$$
$$\dot{\theta}_{65\,i}\, c\,\theta_{65\,i}\left(-2\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right) z_{1312\,i}\, c(\theta_{21} + \theta_{43}) - 2\,\dot{\theta}_{21}\, x_{32}\, s\,\theta_{21} + 2\,\dot{\theta}_{21\,i}\, x_{43\,i}\, s\,\theta_{21\,i} -\right.$$
$$\left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{1211\,i}\, s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{65}\, s(\theta_{21} + \theta_{43} - \theta_{54}) +$$
$$\left.\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{1211\,i}\, s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{65}\, s(\theta_{21} + \theta_{43} + \theta_{54})\right) +$$
$$c\,\theta_{65\,i}\left(-s\,\beta_{54\,i}\left(\ddot{\theta}_{21\,i}\, x_{43\,i}\, c\,\theta_{21\,i} + \ddot{\theta}_{54}\, x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54}) - \ddot{\theta}_{54}\, x_{65}\, c\,\theta_{54} -\right.\right.$$
$$\left.\dot{\theta}_{21\,i}^{\,2}\, x_{43\,i}\, s\,\theta_{21\,i} + \dot{\theta}_{54}^{\,2}\, x_{1211\,i}\, s(\beta_{1413\,i} - \theta_{54}) + \dot{\theta}_{54}^{\,2}\, x_{65}\, s\,\theta_{54}\right) -$$
$$\frac{1}{2}\, c\,\beta_{54\,i}\left(-2\,\ddot{\theta}_{21}\, x_{32}\, c\,\theta_{21} + 2\,\dot{\theta}_{21}^{\,2}\, x_{32}\, s\,\theta_{21} - 2\, z_{1312\,i}\left(-\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)^2 c(\theta_{21} + \theta_{43}) - \left(\ddot{\theta}_{21} + \ddot{\theta}_{43}\right) s(\theta_{21} + \theta_{43})\right) -\right.$$
$$x_{1211\,i}\left(-\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^2 s(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54})\right) -$$
$$x_{65}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^2 s(\theta_{21} + \theta_{43} - \theta_{54})\right) +$$
$$x_{1211\,i}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} - \ddot{\theta}_{54}\right) c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right)^2 s(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54})\right) -$$
$$\left.\left.x_{65}\left(\left(\ddot{\theta}_{21} + \ddot{\theta}_{43} + \ddot{\theta}_{54}\right) c(\theta_{21} + \theta_{43} + \theta_{54}) - \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right)^2 s(\theta_{21} + \theta_{43} + \theta_{54})\right)\right)\right) +$$
$$2\,\dot{\theta}_{65\,i}\left(\left(\dot{\theta}_{21\,i}\, x_{43\,i}\, c\,\theta_{21\,i} + \dot{\theta}_{54}\, x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54}) - \dot{\theta}_{54}\, x_{65}\, c\,\theta_{54}\right) s\,\beta_{54\,i} +\right.$$
$$\frac{1}{2}\, c\,\beta_{54\,i}\left(-2\,\dot{\theta}_{21}\, x_{32}\, c\,\theta_{21} + \left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{21} - \theta_{43} - \theta_{54}) -\right.$$
$$\left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{65}\, c(\theta_{21} + \theta_{43} - \theta_{54}) + \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{54}\right) x_{1211\,i}\, c(\beta_{1413\,i} + \theta_{21} + \theta_{43} - \theta_{54}) -$$
$$\left.\left.\left(\dot{\theta}_{21} + \dot{\theta}_{43} + \dot{\theta}_{54}\right) x_{65}\, c(\theta_{21} + \theta_{43} + \theta_{54}) + 2\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right) z_{1312\,i}\, s(\theta_{21} + \theta_{43})\right)\right) s\,\theta_{65\,i} +$$
$$\dot{\theta}_{65\,i}^{\,2}\,(-x_{43\,i} - \text{signo}\, y_{10\,i}\, s\,\theta_{21\,i} + c\,\theta_{21}\, c\,\theta_{21\,i}\,(x_{32} + c\,\theta_{43}\,(-x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54}) + x_{65}\, c\,\theta_{54}) - z_{1312\,i}\, s\,\theta_{43}) -$$
$$c\,\theta_{21\,i}\, s\,\theta_{21}\,(z_{1312\,i}\, c\,\theta_{43} - x_{1211\,i}\, c(\beta_{1413\,i} - \theta_{54})\, s\,\theta_{43} + x_{65}\, c\,\theta_{54}\, s\,\theta_{43}) +$$

$$\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\mathrm{x}_{65}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{54})\ \mathrm{s}\,\theta_{65\ \mathrm{i}}-$$

$$\frac{1}{2}\left(-2\,\mathrm{x}_{32}\left(\dot\theta_{21}{}^2\ \mathrm{c}\,\theta_{21}+\ddot\theta_{21}\ \mathrm{s}\,\theta_{21}\right)+2\,\mathrm{x}_{43\ \mathrm{i}}\left(\dot\theta_{21\ \mathrm{i}}{}^2\ \mathrm{c}\,\theta_{21\ \mathrm{i}}+\ddot\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\right)-\right.$$

$$2\,\mathrm{z}_{1312\ \mathrm{i}}\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\mathrm{c}(\theta_{21}+\theta_{43})-\left(\dot\theta_{21}+\dot\theta_{43}\right)^2\mathrm{s}(\theta_{21}+\theta_{43})\right)-$$

$$\mathrm{x}_{1211\ \mathrm{i}}\left(-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)^2\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{21}-\theta_{43}-\theta_{54})+\left(\ddot\theta_{21}+\ddot\theta_{43}+\ddot\theta_{54}\right)\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{21}-\theta_{43}-\theta_{54})\right)+$$

$$\mathrm{x}_{65}\left(-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^2\mathrm{c}(\theta_{21}+\theta_{43}-\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)\mathrm{s}(\theta_{21}+\theta_{43}-\theta_{54})\right)-$$

$$\mathrm{x}_{1211\ \mathrm{i}}\left(-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^2\mathrm{c}(\beta_{1413\ \mathrm{i}}+\theta_{21}+\theta_{43}-\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)\mathrm{s}(\beta_{1413\ \mathrm{i}}+\theta_{21}+\theta_{43}-\theta_{54})\right)+$$

$$\left.\mathrm{x}_{65}\left(-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)^2\mathrm{c}(\theta_{21}+\theta_{43}+\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}+\ddot\theta_{54}\right)\mathrm{s}(\theta_{21}+\theta_{43}+\theta_{54})\right)\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}$$

Coeficientes de la ecuación (2.166)

$$\mathrm{G}_{61}=(-\mathrm{x}_{43\ \mathrm{i}}-\mathrm{signo}\ \mathrm{y}_{10\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}+\mathrm{c}\,\theta_{21}\ \mathrm{c}\,\theta_{21\ \mathrm{i}}\ (\mathrm{x}_{32}+\mathrm{c}\,\theta_{43}\ (-\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54})-\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{s}\,\theta_{43})-\mathrm{c}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{21}$$
$$(\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{c}\,\theta_{43}-\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\ \mathrm{s}\,\theta_{43}+\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54}\ \mathrm{s}\,\theta_{43})+\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\mathrm{x}_{65}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{54})^3$$

$$\mathrm{G}_{62}=0$$

$$\mathrm{G}_{63}=0$$

$$\mathrm{G}_{64}=0$$

$$\mathrm{G}_{65}=0$$

$$\mathrm{G}_{66}=0$$

$$\mathrm{G}_{67}=0$$

$$\mathrm{G}_{68}=$$

$$-\mathrm{c}\,\theta_{76\ \mathrm{i}}{}^2\left(\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\left(\left(-\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\dot\theta_{21\ \mathrm{i}}\ \mathrm{x}_{43\ \mathrm{i}}\ \mathrm{c}\,\theta_{21\ \mathrm{i}}+\dot\theta_{54}\ \mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})-\dot\theta_{54}\ \mathrm{x}_{65}\ \mathrm{c}\,\theta_{54}\right)+\frac{1}{2}\ \mathrm{s}\,\beta_{54\ \mathrm{i}}\left(-2\,\dot\theta_{21}\ \mathrm{x}_{32}\ \mathrm{c}\,\theta_{21}+\left(\dot\theta_{21}+\right.\right.\right.\right.\right.\right.$$

$$\left.\dot\theta_{43}+\dot\theta_{54}\right)\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{21}-\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)\mathrm{x}_{65}\ \mathrm{c}(\theta_{21}+\theta_{43}-\theta_{54})+$$

$$\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}+\theta_{21}+\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)\mathrm{x}_{65}$$

$$\left.\left.\mathrm{c}(\theta_{21}+\theta_{43}+\theta_{54})+2\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{s}(\theta_{21}+\theta_{43})\right)\right)$$

$$(-\mathrm{x}_{43\ \mathrm{i}}-\mathrm{signo}\ \mathrm{y}_{10\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}+\mathrm{c}\,\theta_{21}\ \mathrm{c}\,\theta_{21\ \mathrm{i}}\ (\mathrm{x}_{32}+\mathrm{c}\,\theta_{43}\ (-\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54})-\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{s}\,\theta_{43})-$$

$$\mathrm{c}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{21}\ (\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{c}\,\theta_{43}-\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\ \mathrm{s}\,\theta_{43}+\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54}\ \mathrm{s}\,\theta_{43})+\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})+$$

$$\mathrm{x}_{65}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}\ \mathrm{s}\,\theta_{54})\left(-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{c}(\theta_{21}+\theta_{43})-2\,\dot\theta_{21}\ \mathrm{x}_{32}\ \mathrm{s}\,\theta_{21}+2\,\dot\theta_{21\ \mathrm{i}}\ \mathrm{x}_{43\ \mathrm{i}}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}-\right.$$

$$\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{21}-\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)\mathrm{x}_{65}\ \mathrm{s}(\theta_{21}+\theta_{43}-\theta_{54})+$$

$$\left.\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{s}(\beta_{1413\ \mathrm{i}}+\theta_{21}+\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)\mathrm{x}_{65}\ \mathrm{s}(\theta_{21}+\theta_{43}+\theta_{54})\right)-$$

$$\frac{1}{2}\ (-\mathrm{signo}\ \mathrm{y}_{10\ \mathrm{i}}\ \mathrm{c}\,\beta_{54\ \mathrm{i}}\ \mathrm{c}\,\theta_{21\ \mathrm{i}}+\mathrm{z}_{32\ \mathrm{i}}\ \mathrm{s}\,\beta_{54\ \mathrm{i}}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\ (\mathrm{z}_{10}-\mathrm{x}_{32}\ \mathrm{s}\,\theta_{21})-\mathrm{x}_{32}\ \mathrm{c}\,\beta_{54\ \mathrm{i}}\ \mathrm{c}\,\theta_{21}\ \mathrm{s}\,\theta_{21\ \mathrm{i}}+$$

$$(\mathrm{s}\,\beta_{54\ \mathrm{i}}\ \mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\ \mathrm{c}\,\theta_{21}\ \mathrm{s}\,\theta_{21\ \mathrm{i}})\ (\mathrm{c}\,\theta_{43}\ (\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})-\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54})+\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{s}\,\theta_{43})-$$

$$(\mathrm{c}\,\theta_{21}\ \mathrm{s}\,\beta_{54\ \mathrm{i}}-\mathrm{c}\,\beta_{54\ \mathrm{i}}\ \mathrm{s}\,\theta_{21}\ \mathrm{s}\,\theta_{21\ \mathrm{i}})\ (\mathrm{z}_{1312\ \mathrm{i}}\ \mathrm{c}\,\theta_{43}+(-\mathrm{x}_{1211\ \mathrm{i}}\ \mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\mathrm{x}_{65}\ \mathrm{c}\,\theta_{54})\ \mathrm{s}\,\theta_{43})+$$

$$c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,(x_{1211\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{54}))$$

$$\left(-2\left(\dot\theta_{21}+\dot\theta_{43}\right)z_{1312\,i}\,c(\theta_{21}+\theta_{43})-2\,\dot\theta_{21}\,x_{32}\,s\,\theta_{21}+2\,\dot\theta_{21\,i}\,x_{43\,i}\,s\,\theta_{21\,i}-\right.$$

$$\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)x_{1211\,i}\,s(\beta_{1413\,i}-\theta_{21}-\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)x_{65}\,s(\theta_{21}+\theta_{43}-\theta_{54})+$$

$$\left.\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)x_{1211\,i}\,s(\beta_{1413\,i}+\theta_{21}+\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)x_{65}\,s(\theta_{21}+\theta_{43}+\theta_{54})\right)^{2}+$$

$$(-x_{43\,i}-\text{signo}\,y_{10\,i}\,s\,\theta_{21\,i}+c\,\theta_{21}\,c\,\theta_{21\,i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\,i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\,i}\,s\,\theta_{21}\,(z_{1312\,i}\,c\,\theta_{43}-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\,i}\,s\,\theta_{21\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{21\,i}\,s\,\theta_{54})$$

$$\left(\dot\theta_{65\,i}^{\;2}\,(-x_{43\,i}-\text{signo}\,y_{10\,i}\,s\,\theta_{21\,i}+c\,\theta_{21}\,c\,\theta_{21\,i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})+x_{65}\,c\,\theta_{54})-\right.$$

$$z_{1312\,i}\,s\,\theta_{43})-c\,\theta_{21\,i}\,s\,\theta_{21}\,(z_{1312\,i}\,c\,\theta_{43}-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}+$$

$$x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+x_{1211\,i}\,s\,\theta_{21\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{21\,i}\,s\,\theta_{54})$$

$$(-\text{signo}\,y_{10\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}+z_{32\,i}\,s\,\beta_{54\,i}+s\,\beta_{54\,i}\,(z_{10}-x_{32}\,s\,\theta_{21})-x_{32}\,c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}+$$

$$(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,(c\,\theta_{43}\,(x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\,i}\,s\,\theta_{43})-$$

$$(c\,\theta_{21}\,s\,\beta_{54\,i}-c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i})\,(z_{1312\,i}\,c\,\theta_{43}+(-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})+$$

$$c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,(x_{1211\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{54}))+$$

$$\frac{1}{2}\,(-\text{signo}\,y_{10\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}+z_{32\,i}\,s\,\beta_{54\,i}+s\,\beta_{54\,i}\,(z_{10}-x_{32}\,s\,\theta_{21})-x_{32}\,c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}+$$

$$(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,(c\,\theta_{43}\,(x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})-x_{65}\,c\,\theta_{54})+z_{1312\,i}\,s\,\theta_{43})-$$

$$(c\,\theta_{21}\,s\,\beta_{54\,i}-c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i})\,(z_{1312\,i}\,c\,\theta_{43}+(-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})+x_{65}\,c\,\theta_{54})\,s\,\theta_{43})+$$

$$c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,(x_{1211\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{54}))$$

$$\left(-2\,x_{32}\left(\dot\theta_{21}^{\;2}\,c\,\theta_{21}+\ddot\theta_{21}\,s\,\theta_{21}\right)+2\,x_{43\,i}\left(\dot\theta_{21\,i}^{\;2}\,c\,\theta_{21\,i}+\ddot\theta_{21\,i}\,s\,\theta_{21\,i}\right)-\right.$$

$$2\,z_{1312\,i}\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)c(\theta_{21}+\theta_{43})-\left(\dot\theta_{21}+\dot\theta_{43}\right)^{2}s(\theta_{21}+\theta_{43})\right)-x_{1211\,i}$$

$$\left(-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)^{2}c(\beta_{1413\,i}-\theta_{21}-\theta_{43}-\theta_{54})+\left(\ddot\theta_{21}+\ddot\theta_{43}+\ddot\theta_{54}\right)s(\beta_{1413\,i}-\theta_{21}-\theta_{43}-\theta_{54})\right)+$$

$$x_{65}\left(-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^{2}c(\theta_{21}+\theta_{43}-\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)s(\theta_{21}+\theta_{43}-\theta_{54})\right)-x_{1211\,i}$$

$$\left(-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^{2}c(\beta_{1413\,i}+\theta_{21}+\theta_{43}-\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)s(\beta_{1413\,i}+\theta_{21}+\theta_{43}-\theta_{54})\right)+$$

$$\left.x_{65}\left(-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)^{2}c(\theta_{21}+\theta_{43}+\theta_{54})-\left(\ddot\theta_{21}+\ddot\theta_{43}+\ddot\theta_{54}\right)s(\theta_{21}+\theta_{43}+\theta_{54})\right)\right)-$$

$$(-x_{43\,i}-\text{signo}\,y_{10\,i}\,s\,\theta_{21\,i}+c\,\theta_{21}\,c\,\theta_{21\,i}\,(x_{32}+c\,\theta_{43}\,(-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})+x_{65}\,c\,\theta_{54})-z_{1312\,i}\,s\,\theta_{43})-$$

$$c\,\theta_{21\,i}\,s\,\theta_{21}\,(z_{1312\,i}\,c\,\theta_{43}-x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}+x_{65}\,c\,\theta_{54}\,s\,\theta_{43})+$$

$$x_{1211\,i}\,s\,\theta_{21\,i}\,s(\beta_{1413\,i}-\theta_{54})+x_{65}\,s\,\theta_{21\,i}\,s\,\theta_{54})$$

$$\left(-c\,\beta_{54\,i}\left(\ddot\theta_{21\,i}\,x_{43\,i}\,c\,\theta_{21\,i}+\ddot\theta_{54}\,x_{1211\,i}\,c(\beta_{1413\,i}-\theta_{54})-\ddot\theta_{54}\,x_{65}\,c\,\theta_{54}-\dot\theta_{21\,i}^{\;2}\,x_{43\,i}\,s\,\theta_{21\,i}+\right.\right.$$

$$\left.\dot\theta_{54}^{\;2}\,x_{1211\,i}\,s(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}^{\;2}\,x_{65}\,s\,\theta_{54}\right)+\frac{1}{2}\,s\,\beta_{54\,i}\left(-2\,\ddot\theta_{21}\,x_{32}\,c\,\theta_{21}+2\,\dot\theta_{21}^{\;2}\,x_{32}\,s\,\theta_{21}-\right.$$

$$2\,z_{1312\,i}\left(-\left(\dot\theta_{21}+\dot\theta_{43}\right)^{2}c(\theta_{21}+\theta_{43})-\left(\ddot\theta_{21}+\ddot\theta_{43}\right)s(\theta_{21}+\theta_{43})\right)-x_{1211\,i}\left(\left(-\ddot\theta_{21}-\ddot\theta_{43}-\ddot\theta_{54}\right)\right.$$

$$\left.c(\beta_{1413\,i}-\theta_{21}-\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}+\dot\theta_{54}\right)^{2}s(\beta_{1413\,i}-\theta_{21}-\theta_{43}-\theta_{54})\right)-$$

$$x_{65}\left(\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)c(\theta_{21}+\theta_{43}-\theta_{54})-\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^{2}s(\theta_{21}+\theta_{43}-\theta_{54})\right)+$$

$$x_{1211\,i}\left(\left(\ddot\theta_{21}+\ddot\theta_{43}-\ddot\theta_{54}\right)c(\beta_{1413\,i}+\theta_{21}+\theta_{43}-\theta_{54})-\right.$$

$$\left.\left(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{54}\right)^{2}s(\beta_{1413\,i}+\theta_{21}+\theta_{43}-\theta_{54})\right)-$$

$$x_{65}\left(\left(\ddot{\theta}_{21}+\ddot{\theta}_{43}+\ddot{\theta}_{54}\right)c(\theta_{21}+\theta_{43}+\theta_{54})-\left(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54}\right)^2 s(\theta_{21}+\theta_{43}+\theta_{54})\right)\Bigg)\Bigg)\Bigg)+$$

$$(-x_{43\ i}-\text{signo}\ y_{10\ i}\ s\,\theta_{21\ i}+c\,\theta_{21}\ c\,\theta_{21\ i}\ (x_{32}+c\,\theta_{43}\ (-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})-z_{1312\ i}\ s\,\theta_{43})-$$
$$c\,\theta_{21\ i}\ s\,\theta_{21}$$
$$(z_{1312\ i}\ c\,\theta_{43}-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})\ s\,\theta_{43}+x_{65}\ c\,\theta_{54}\ s\,\theta_{43})+$$
$$x_{1211\ i}\ s\,\theta_{21\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{21\ i}\ s\,\theta_{54})$$
$$\Bigg(\Bigg(2\,\dot{\theta}_{65\ i}\Bigg(-c\,\beta_{54\ i}\left(\dot{\theta}_{21\ i}\ x_{43\ i}\ c\,\theta_{21\ i}+\dot{\theta}_{54}\ x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})-\dot{\theta}_{54}\ x_{65}\ c\,\theta_{54}\right)+$$
$$\frac{1}{2}\ s\,\beta_{54\ i}\Big(-2\,\dot{\theta}_{21}\ x_{32}\ c\,\theta_{21}+\left(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54}\right)x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})-$$
$$\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54}\right)x_{65}\ c(\theta_{21}+\theta_{43}-\theta_{54})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54}\right)x_{1211\ i}\ c(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})-$$
$$\left(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54}\right)x_{65}\ c(\theta_{21}+\theta_{43}+\theta_{54})+2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)z_{1312\ i}\ s(\theta_{21}+\theta_{43})\Big)\Bigg)$$
$$(-x_{43\ i}-\text{signo}\ y_{10\ i}\ s\,\theta_{21\ i}+c\,\theta_{21}\ c\,\theta_{21\ i}\ (x_{32}+c\,\theta_{43}\ (-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})-$$
$$z_{1312\ i}\ s\,\theta_{43})-c\,\theta_{21\ i}\ s\,\theta_{21}\ (z_{1312\ i}\ c\,\theta_{43}-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})\ s\,\theta_{43}+x_{65}\ c\,\theta_{54}\ s\,\theta_{43})+$$
$$x_{1211\ i}\ s\,\theta_{21\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{21\ i}\ s\,\theta_{54})+\ddot{\theta}_{65\ i}\ (-x_{43\ i}-\text{signo}\ y_{10\ i}\ s\,\theta_{21\ i}+$$
$$c\,\theta_{21}\ c\,\theta_{21\ i}\ (x_{32}+c\,\theta_{43}\ (-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})-z_{1312\ i}\ s\,\theta_{43})-$$
$$c\,\theta_{21\ i}\ s\,\theta_{21}\ (z_{1312\ i}\ c\,\theta_{43}-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})\ s\,\theta_{43}+x_{65}\ c\,\theta_{54}\ s\,\theta_{43})+$$
$$x_{1211\ i}\ s\,\theta_{21\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{21\ i}\ s\,\theta_{54})$$
$$(-\text{signo}\ y_{10\ i}\ c\,\beta_{54\ i}\ c\,\theta_{21\ i}+z_{32\ i}\ s\,\beta_{54\ i}+s\,\beta_{54\ i}\ (z_{10}-x_{32}\ s\,\theta_{21})-x_{32}\ c\,\beta_{54\ i}\ c\,\theta_{21}\ s\,\theta_{21\ i}+$$
$$(s\,\beta_{54\ i}\ s\,\theta_{21}+c\,\beta_{54\ i}\ c\,\theta_{21}\ s\,\theta_{21\ i})\ (c\,\theta_{43}\ (x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})-x_{65}\ c\,\theta_{54})+z_{1312\ i}\ s\,\theta_{43})-$$
$$(c\,\theta_{21}\ s\,\beta_{54\ i}-c\,\beta_{54\ i}\ s\,\theta_{21}\ s\,\theta_{21\ i})\ (z_{1312\ i}\ c\,\theta_{43}+(-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})\ s\,\theta_{43})+$$
$$c\,\beta_{54\ i}\ c\,\theta_{21\ i}\ (x_{1211\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{54}))-$$
$$\dot{\theta}_{65\ i}\ (-\text{signo}\ y_{10\ i}\ c\,\beta_{54\ i}\ c\,\theta_{21\ i}+z_{32\ i}\ s\,\beta_{54\ i}+s\,\beta_{54\ i}\ (z_{10}-x_{32}\ s\,\theta_{21})-x_{32}\ c\,\beta_{54\ i}\ c\,\theta_{21}\ s\,\theta_{21\ i}+$$
$$(s\,\beta_{54\ i}\ s\,\theta_{21}+c\,\beta_{54\ i}\ c\,\theta_{21}\ s\,\theta_{21\ i})\ (c\,\theta_{43}\ (x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})-x_{65}\ c\,\theta_{54})+z_{1312\ i}\ s\,\theta_{43})-$$
$$(c\,\theta_{21}\ s\,\beta_{54\ i}-c\,\beta_{54\ i}\ s\,\theta_{21}\ s\,\theta_{21\ i})\ (z_{1312\ i}\ c\,\theta_{43}+(-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})\ s\,\theta_{43})+$$
$$c\,\beta_{54\ i}\ c\,\theta_{21\ i}\ (x_{1211\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{54}))\Big(-2\left(\dot{\theta}_{21}+\dot{\theta}_{43}\right)z_{1312\ i}\ c(\theta_{21}+\theta_{43})-$$
$$2\,\dot{\theta}_{21}\ x_{32}\ s\,\theta_{21}+2\,\dot{\theta}_{21\ i}\ x_{43\ i}\ s\,\theta_{21\ i}-\left(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54}\right)x_{1211\ i}\ s(\beta_{1413\ i}-\theta_{21}-\theta_{43}-\theta_{54})-$$
$$\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54}\right)x_{65}\ s(\theta_{21}+\theta_{43}-\theta_{54})+\left(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{54}\right)x_{1211\ i}\ s(\beta_{1413\ i}+\theta_{21}+\theta_{43}-\theta_{54})-$$
$$\left(\dot{\theta}_{21}+\dot{\theta}_{43}+\dot{\theta}_{54}\right)x_{65}\ s(\theta_{21}+\theta_{43}+\theta_{54})\Big)\Bigg)\ s\,\theta_{65\ i}+2\,\dot{\theta}_{76\ i}^{\ 2}\ \sec(\theta_{76\ i})^2$$
$$(-x_{43\ i}-\text{signo}\ y_{10\ i}\ s\,\theta_{21\ i}+c\,\theta_{21}\ c\,\theta_{21\ i}\ (x_{32}+c\,\theta_{43}\ (-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})+x_{65}\ c\,\theta_{54})-z_{1312\ i}\ s\,\theta_{43})-$$
$$c\,\theta_{21\ i}\ s\,\theta_{21}\ (z_{1312\ i}\ c\,\theta_{43}-x_{1211\ i}\ c(\beta_{1413\ i}-\theta_{54})\ s\,\theta_{43}+x_{65}\ c\,\theta_{54}\ s\,\theta_{43})+$$
$$x_{1211\ i}\ s\,\theta_{21\ i}\ s(\beta_{1413\ i}-\theta_{54})+x_{65}\ s\,\theta_{21\ i}\ s\,\theta_{54})^2\ \tan(\theta_{76\ i})\Bigg)\Bigg)$$

Coeficientes de la ecuación (2.168)

$$G_{69}=(s(\beta_{1413\ i}-\theta_{54})\ (c\,\theta_{21\ i}\ c\,\theta_{65\ i}\ s\,\beta_{54\ i}-s\,\theta_{21\ i}\ s\,\theta_{65\ i})+$$
$$c\,\theta_{43}\ c(\beta_{1413\ i}-\theta_{54})\ (-c\,\beta_{54\ i}\ c\,\theta_{65\ i}\ s\,\theta_{21}+c\,\theta_{21}\ (c\,\theta_{65\ i}\ s\,\beta_{54\ i}\ s\,\theta_{21\ i}+c\,\theta_{21\ i}\ s\,\theta_{65\ i}))-$$
$$c(\beta_{1413\ i}-\theta_{54})\ s\,\theta_{43}\ (c\,\beta_{54\ i}\ c\,\theta_{21}\ c\,\theta_{65\ i}+s\,\theta_{21}\ (c\,\theta_{65\ i}\ s\,\beta_{54\ i}\ s\,\theta_{21\ i}+c\,\theta_{21\ i}\ s\,\theta_{65\ i})))^3$$

$$G_{70}=0$$

$$G_{71} = 0$$

$$G_{72} = 0$$

$$G_{73} = 0$$

$$G_{74} = 0$$

$$G_{75} = 0$$

$$G_{76} = -c\,\theta_{1110i}{}^{2} \Big(2\,(c(\beta_{1413i} - \theta_{54})\,(-c\,\theta_{21i}\,c\,\theta_{65i}\,s\,\beta_{54i} + s\,\theta_{21i}\,s\,\theta_{65i}) +$$
$$c\,\theta_{43}\,s(\beta_{1413i} - \theta_{54})\,(-c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} + c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) -$$
$$s\,\theta_{43}\,s(\beta_{1413i} - \theta_{54})\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})))$$
$$\Big(-s(\beta_{1413i} - \theta_{54})\,\big(c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big)\,s\,\theta_{21i} + c\,\theta_{21i}\,\big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{65i}\big) -$$
$$\dot\theta_{54}\,c(\beta_{1413i} - \theta_{54})\,(c\,\theta_{21i}\,c\,\theta_{65i}\,s\,\beta_{54i} - s\,\theta_{21i}\,s\,\theta_{65i}) +$$
$$\dot\theta_{43}\,c(\beta_{1413i} - \theta_{54})\,s\,\theta_{43}\,(c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} - c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) +$$
$$\dot\theta_{54}\,c\,\theta_{43}\,s(\beta_{1413i} - \theta_{54})\,(-c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} + c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) -$$
$$\dot\theta_{43}\,c\,\theta_{43}\,c(\beta_{1413i} - \theta_{54})\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) - \dot\theta_{54}\,s\,\theta_{43}$$
$$s(\beta_{1413i} - \theta_{54})\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) - c(\beta_{1413i} - \theta_{54})\,s\,\theta_{43}$$
$$\big(c\,\theta_{21i}\,\big(c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big)\,s\,\theta_{21} + \dot\theta_{21}\,c\,\theta_{21}\,s\,\theta_{65i}\big) - c\,\beta_{54i}\,\big(\dot\theta_{21}\,c\,\theta_{65i}\,s\,\theta_{21} + \dot\theta_{65i}\,c\,\theta_{21}\,s\,\theta_{65i}\big) +$$
$$s\,\theta_{21i}\,\big(\dot\theta_{21}\,c\,\theta_{21}\,c\,\theta_{65i}\,s\,\beta_{54i} - \big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{21}\,s\,\theta_{65i}\big)\big) + c\,\theta_{43}\,c(\beta_{1413i} - \theta_{54})$$
$$\big(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i}) + c\,\beta_{54i}\,\big(-\dot\theta_{21}\,c\,\theta_{21}\,c\,\theta_{65i} + \dot\theta_{65i}\,s\,\theta_{21}\,s\,\theta_{65i}\big) +$$
$$c\,\theta_{21}\,\big(c\,\theta_{21i}\,c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big) - \big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{21i}\,s\,\theta_{65i}\big)\big)\Big)^{2} -$$
$$2\,(s(\beta_{1413i} - \theta_{54})\,(c\,\theta_{21i}\,c\,\theta_{65i}\,s\,\beta_{54i} - s\,\theta_{21i}\,s\,\theta_{65i}) + c\,\theta_{43}\,c(\beta_{1413i} - \theta_{54})$$
$$(-c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} + c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) -$$
$$c(\beta_{1413i} - \theta_{54})\,s\,\theta_{43}\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})))$$
$$\Big(-s(\beta_{1413i} - \theta_{54})\,\big(c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big)\,s\,\theta_{21i} + c\,\theta_{21i}\,\big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{65i}\big) -$$
$$\dot\theta_{54}\,c(\beta_{1413i} - \theta_{54})\,(c\,\theta_{21i}\,c\,\theta_{65i}\,s\,\beta_{54i} - s\,\theta_{21i}\,s\,\theta_{65i}) +$$
$$\dot\theta_{43}\,c(\beta_{1413i} - \theta_{54})\,s\,\theta_{43}\,(c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} - c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) +$$
$$\dot\theta_{54}\,c\,\theta_{43}\,s(\beta_{1413i} - \theta_{54})\,(-c\,\beta_{54i}\,c\,\theta_{65i}\,s\,\theta_{21} + c\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) -$$
$$\dot\theta_{43}\,c\,\theta_{43}\,c(\beta_{1413i} - \theta_{54})\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) -$$
$$\dot\theta_{54}\,s\,\theta_{43}\,s(\beta_{1413i} - \theta_{54})\,(c\,\beta_{54i}\,c\,\theta_{21}\,c\,\theta_{65i} + s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i})) - c(\beta_{1413i} - \theta_{54})\,s\,\theta_{43}$$
$$\big(c\,\theta_{21i}\,\big(c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big)\,s\,\theta_{21} + \dot\theta_{21}\,c\,\theta_{21}\,s\,\theta_{65i}\big) - c\,\beta_{54i}\,\big(\dot\theta_{21}\,c\,\theta_{65i}\,s\,\theta_{21} + \dot\theta_{65i}\,c\,\theta_{21}\,s\,\theta_{65i}\big) +$$
$$s\,\theta_{21i}\,\big(\dot\theta_{21}\,c\,\theta_{21}\,c\,\theta_{65i}\,s\,\beta_{54i} - \big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{21}\,s\,\theta_{65i}\big)\big) + c\,\theta_{43}\,c(\beta_{1413i} - \theta_{54})$$
$$\big(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21i} + c\,\theta_{21i}\,s\,\theta_{65i}) + c\,\beta_{54i}\,\big(-\dot\theta_{21}\,c\,\theta_{21}\,c\,\theta_{65i} + \dot\theta_{65i}\,s\,\theta_{21}\,s\,\theta_{65i}\big) +$$
$$c\,\theta_{21}\,\big(c\,\theta_{21i}\,c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big) - \big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{21i}\,s\,\theta_{65i}\big)\big)\Big)\,\big(c(\beta_{1413i} - \theta_{54})$$
$$\big(c\,\theta_{65i}\,\big(\dot\theta_{65i} + \big(\dot\theta_{21i} - \dot\theta_{54}\,c(\theta_{21} + \theta_{43})\big)\,s\,\beta_{54i}\big)\,s\,\theta_{21i} + c\,\theta_{21i}\,\big(\dot\theta_{21i} - \dot\theta_{54}\,c(\theta_{21} + \theta_{43}) + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{65i}\big) +$$
$$s(\beta_{1413i} - \theta_{54})\,\big(-c\,\theta_{21i}\,\big(c\,\theta_{65i}\,\big(\dot\theta_{65i}\,s\,\theta_{21}\,s\,\theta_{43} + s\,\beta_{54i}\,\big(\dot\theta_{54} + \dot\theta_{21i}\,s\,\theta_{21}\,s\,\theta_{43}\big)\big) + \big(\dot\theta_{21} + \dot\theta_{43}\big)\,c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{65i}\big) +$$
$$s\,\theta_{21i}\,\big(\big(-\dot\theta_{21} - \dot\theta_{43}\big)\,c\,\theta_{43}\,c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{21} + \big(\dot\theta_{54} + \big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{21}\,s\,\theta_{43}\big)\,s\,\theta_{65i}\big) +$$
$$c\,\theta_{21}\,\big(-s\,\theta_{21i}\,\big(\big(\dot\theta_{21} + \dot\theta_{43}\big)\,c\,\theta_{65i}\,s\,\beta_{54i}\,s\,\theta_{43} + c\,\theta_{43}\,\big(\dot\theta_{21i} + \dot\theta_{65i}\,s\,\beta_{54i}\big)\,s\,\theta_{65i}\big) +$$
$$c\,\theta_{21i}\,\big(c\,\theta_{43}\,c\,\theta_{65i}\,\big(\dot\theta_{65i} + \dot\theta_{21i}\,s\,\beta_{54i}\big) - \big(\dot\theta_{21} + \dot\theta_{43}\big)\,s\,\theta_{43}\,s\,\theta_{65i}\big)\big)\big) +$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{c}\,\theta_{43}\left(\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}-\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}-\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{s}\,\theta_{43}\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{21}\left(\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}+\dot\theta_{65\,i}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{65\,i}\right)\right)\right)\Big)-$$

$$(\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})$$

$$(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))-$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})))$$

$$(\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))-$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})))$$

$$\Big(2\,\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)-$$

$$\left(\ddot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}{}^{2}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+$$

$$\left(-\mathrm{c}\,\theta_{43}\left(\left(\dot\theta_{43}{}^{2}+\dot\theta_{54}{}^{2}\right)\mathrm{c}(\beta_{1413\,i}-\theta_{54})-\ddot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)-\right.$$

$$\left.\mathrm{s}\,\theta_{43}\left(\ddot\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})+2\,\dot\theta_{43}\,\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)\right)$$

$$(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))+$$

$$\mathrm{s}\,\theta_{43}\left(\dot\theta_{54}{}^{2}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})-\ddot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))-$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$2\left(-\dot\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}+\dot\theta_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\right)$$

$$\left(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})+\mathrm{c}\,\beta_{54\,i}\left(-\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\left(\dot\theta_{21}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{c}\,\theta_{65\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{c}\,\theta_{21}\left(-\ddot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+2\,\dot\theta_{21}\,\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)-\right.$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(2\,\dot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\ddot\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-2\,\dot\theta_{21}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{c}\,\theta_{21}\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21}{}^{2}+\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot\theta_{21}{}^{2}+\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)\right)-$$

$$2\,\dot\theta_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\dot\theta_{43}\,\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))+\mathrm{s}\,\theta_{43}\right.$$

$$\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}+\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)-\mathrm{c}\,\beta_{54\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\left.\mathrm{s}\,\theta_{21\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\right)\right)-$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\left(\ddot\theta_{43}\,\mathrm{c}\,\theta_{43}-\dot\theta_{43}{}^{2}\,\mathrm{s}\,\theta_{43}\right)(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))+\right.$$

$$2\,\dot\theta_{43}\,\mathrm{c}\,\theta_{43}\left(\mathrm{c}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}+\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)-\mathrm{c}\,\beta_{54\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\right.\right.$$

$$\left.\left.\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21\,i}\left(\dot\theta_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{43}\left(-\mathrm{c}\,\beta_{54\,i}\left(\mathrm{c}\,\theta_{21}\left(\left(\dot\theta_{21}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{c}\,\theta_{65\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\mathrm{s}\,\theta_{21}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}-2\,\dot\theta_{21}\,\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)+\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\,i}\left(2\,\dot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)+\ddot\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\,i}\left(\ddot\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-2\,\dot\theta_{21}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(-\mathrm{s}\,\theta_{21\,i}\left(\mathrm{c}\,\theta_{65\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21}{}^{2}+\dot\theta_{21\,i}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{s}\,\beta_{54\,i}\right)+\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)+\right.$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\big(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)\Big)\Big)\Big)+$$

$$(\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})$$

$$(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})))^{2}$$

$$\Big(2\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+$$

$$\big(\dot{\theta}_{54}{}^{2}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})-\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\big)\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+$$

$$\Big(\mathrm{s}\,\theta_{43}\,\big(-2\,\dot{\theta}_{43}\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\ddot{\theta}_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\big)+$$

$$\mathrm{c}\,\theta_{43}\,\Big(\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\big(\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\big)\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big)\Big)$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}-\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))+$$

$$\big(-\ddot{\theta}_{43}\,\mathrm{c}\,\theta_{43}+\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\theta_{43}\big)\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))+$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\big(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\Big)+\big(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)+$$

$$2\,\big(\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\big)$$

$$\big(\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+$$

$$\mathrm{c}\,\theta_{21}\,\big(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\Big(\mathrm{s}\,\theta_{21}\,\Big(\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+\mathrm{c}\,\theta_{21}\,\big(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\Big)-$$

$$\mathrm{s}\,\theta_{21}\,\Big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-2\,\dot{\theta}_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\Big)+$$

$$\mathrm{c}\,\theta_{21}\,\Big(-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\Big)+\big(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)\Big)-$$

$$2\,\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\big(-\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+$$

$$\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\big)-\mathrm{s}\,\theta_{43}$$

$$\Big(\Big(-\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})-\dot{\theta}_{54}{}^{2}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\Big)\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+$$

$$\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\Big(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{21}\,\Big(\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+\mathrm{s}\,\theta_{21}\,\big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\Big)+$$

$$\mathrm{c}\,\theta_{21}\,\Big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)+$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-2\,\dot{\theta}_{21}\,\big(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\Big)+$$

$$\mathrm{s}\,\theta_{21}\,\Big(-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\Big(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\Big)+\big(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\Big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)-\big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\Big)\Big)\Big)\Big)\Big)+$$

$$2\,\dot{\theta}_{1110\,\mathrm{i}}{}^{2}\,\sec(\theta_{1110\,\mathrm{i}})^{2}\,(\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})+\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})$$

$$(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))-$$

$$\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})))^{3}\,\tan(\theta_{1110\,\mathrm{i}})\Big)$$

Coeficientes de la ecuación (2.170)

$$\mathrm{G}_{77} = (\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} + \mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))^3$$

$$\mathrm{G}_{78} = 0$$

$$\mathrm{G}_{79} = 0$$

$$\mathrm{G}_{80} = 0$$

$$\mathrm{G}_{81} = 0$$

$$\mathrm{G}_{82} = 0$$

$$\mathrm{G}_{83} = 0$$

$$\begin{aligned}
\mathrm{G}_{84} = {} & -\mathrm{c}\,\theta_{109\,\mathrm{i}}{}^2 \\
& \big(2\,\big(\big(-\dot\theta_{21}-\dot\theta_{43}\big)\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43}) - \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + {} \\
& \quad \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{s}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big) - \big(\dot\theta_{21\,\mathrm{i}} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)^2 \\
& \quad (-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}+\theta_{1110\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43}) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{1110\,\mathrm{i}} + {} \\
& \quad\quad \mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})) + {} \\
& \quad \mathrm{s}\,\theta_{21\,\mathrm{i}}\,(-\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54}) - \mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54}) + {} \\
& \quad\quad \mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + \mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - {} \\
& \quad\quad \mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43} - \mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))) - {} \\
& 2\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} + \mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})) \\
& \quad \big(\big(-\dot\theta_{21}-\dot\theta_{43}\big)\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43}) - \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + {} \\
& \quad\quad \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}) + {} \\
& \quad\quad \mathrm{s}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big) - \big(\dot\theta_{21\,\mathrm{i}} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big) \\
& \quad \big(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\big(\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}(\beta_{1413\,\mathrm{i}}+\theta_{1110\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}} - {} \\
& \quad\quad \mathrm{s}(\theta_{21}+\theta_{43})\,\big(\big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}+\theta_{1110\,\mathrm{i}}-\theta_{54}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}+\theta_{1110\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big) + {} \\
& \quad\quad \mathrm{c}\,\theta_{1110\,\mathrm{i}}\,\big(-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \big(\dot\theta_{21\,\mathrm{i}} + \big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54}) + \mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54}) \\
& \quad\quad\quad \big(\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43}) + \big(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - {} \\
& \quad\quad\quad\quad \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big) + \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\big(-\dot\theta_{21\,\mathrm{i}} - \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + {} \\
& \quad\quad\quad \mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big) + {} \\
& \quad\quad\quad \mathrm{c}(\theta_{21}+\theta_{43})\,\big(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big) + \big(-\dot\theta_{1110\,\mathrm{i}} + \dot\theta_{54}\big)\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\big) + {} \\
& \quad\quad \mathrm{s}\,\theta_{1110\,\mathrm{i}}\,\big(-\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \big(\dot\theta_{21\,\mathrm{i}} + \big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\big)\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{21\,\mathrm{i}} + {} \\
& \quad\quad\quad \mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\dot\theta_{21\,\mathrm{i}} + \big(\dot\theta_{1110\,\mathrm{i}}-\dot\theta_{54}\big)\,\mathrm{c}(\theta_{21}+\theta_{43}) + \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big) + {} \\
& \quad\quad\quad \mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\big(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\big(\big(-\dot\theta_{1110\,\mathrm{i}} + \dot\theta_{54}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}} - \mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\big(\dot\theta_{65\,\mathrm{i}} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\big) + {} \\
& \quad\quad\quad\quad \big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big) + \mathrm{s}\,\theta_{21\,\mathrm{i}}\,\big(\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{43} + {} \\
& \quad\quad\quad\quad \big(\dot\theta_{1110\,\mathrm{i}} - \dot\theta_{54} + \dot\theta_{21\,\mathrm{i}}\,\mathrm{c}(\theta_{21}+\theta_{43}) - \dot\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\big)\,\mathrm{s}\,\theta_{65\,\mathrm{i}} + {} \\
& \quad\quad\quad\quad \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\big(\big(\dot\theta_{21}+\dot\theta_{43}\big)\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21} + \dot\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\big)\big)\big)\big)\big) +
\end{aligned}$$

$$(\text{c}\,\beta_{54\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\,i}+\text{s}(\theta_{21}+\theta_{43})\,(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21\,i}+\text{c}\,\theta_{21\,i}\,\text{s}\,\theta_{65\,i}))$$

$$\Big(\Big(-\text{c}\,\beta_{54\,i}\,\text{c}\,\theta_{65\,i}\left(-\left(\dot\theta_{21}+\dot\theta_{43}\right)^2\text{c}(\theta_{21}+\theta_{43})-\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\text{s}(\theta_{21}+\theta_{43})\right)-$$

$$2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{65\,i}\,\text{c}\,\beta_{54\,i}\,\text{s}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i}+\text{c}\,\beta_{54\,i}\,\text{c}(\theta_{21}+\theta_{43})\left(\dot\theta_{65\,i}{}^2\,\text{c}\,\theta_{65\,i}+\ddot\theta_{65\,i}\,\text{s}\,\theta_{65\,i}\right)-$$

$$\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\text{c}(\theta_{21}+\theta_{43})-\left(\dot\theta_{21}+\dot\theta_{43}\right)^2\text{s}(\theta_{21}+\theta_{43})\right)(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21\,i}+\text{c}\,\theta_{21\,i}\,\text{s}\,\theta_{65\,i})-$$

$$2\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{c}(\theta_{21}+\theta_{43})\left(\text{c}\,\theta_{21\,i}\,\text{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,\text{s}\,\beta_{54\,i}\right)-\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\right)\text{s}\,\theta_{21\,i}\,\text{s}\,\theta_{65\,i}\right)+$$

$$\text{s}(\theta_{21}+\theta_{43})\left(\text{s}\,\theta_{21\,i}\left(\text{c}\,\theta_{65\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2\right)\text{s}\,\beta_{54\,i}\right)+\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\right)\text{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\text{c}\,\theta_{21\,i}\left(-\text{c}\,\theta_{65\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,\text{s}\,\beta_{54\,i}\right)+\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\right)\text{s}\,\theta_{65\,i}\right)\right)\Big)\Big)$$

$$(-\text{c}\,\beta_{54\,i}\,\text{c}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\,\text{c}\,\theta_{65\,i}\,\text{s}(\theta_{21}+\theta_{43})+\text{c}\,\theta_{21\,i}\,(\text{c}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}+\text{c}\,\theta_{1110\,i}$$

$$\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i})+\text{s}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}))+$$

$$\text{s}\,\theta_{21\,i}\,(-\text{s}\,\theta_{1110\,i}\,(\text{c}\,\theta_{21}\,\text{c}\,\theta_{43}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}(\beta_{1413\,i}-\theta_{54})-\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{43}\,\text{s}(\beta_{1413\,i}-\theta_{54})+$$

$$\text{c}(\beta_{1413\,i}-\theta_{54})\,\text{s}\,\theta_{65\,i})+\text{c}\,\theta_{1110\,i}\,(\text{c}\,\theta_{21}\,\text{c}\,\theta_{43}\,\text{c}(\beta_{1413\,i}-\theta_{54})\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-$$

$$\text{c}(\beta_{1413\,i}-\theta_{54})\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{43}-\text{s}(\beta_{1413\,i}-\theta_{54})\,\text{s}\,\theta_{65\,i})))+$$

$$(\text{c}\,\beta_{54\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\,i}+\text{s}(\theta_{21}+\theta_{43})\,(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21\,i}+\text{c}\,\theta_{21\,i}\,\text{s}\,\theta_{65\,i}))\Big(-\text{c}\,\beta_{54\,i}\,\text{c}\,\theta_{65\,i}$$

$$\text{s}(\theta_{21}+\theta_{43})\left(-\left(\dot\theta_{1110\,i}-\dot\theta_{54}\right)^2\text{c}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})+\left(-\ddot\theta_{1110\,i}+\ddot\theta_{54}\right)\text{s}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\right)+$$

$$2\left(\dot\theta_{1110\,i}-\dot\theta_{54}\right)\text{c}\,\beta_{54\,i}\,\text{s}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\,i}-\dot\theta_{65\,i}\,\text{s}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i}\right)-$$

$$\text{c}\,\beta_{54\,i}\,\text{c}(\beta_{1413\,i}+\theta_{1110\,i}-\theta_{54})\left(-\text{s}(\theta_{21}+\theta_{43})\left(\left(\dot\theta_{21}{}^2+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^2+\dot\theta_{65\,i}{}^2\right)\text{c}\,\theta_{65\,i}+\ddot\theta_{65\,i}\,\text{s}\,\theta_{65\,i}\right)+\right.$$

$$\left.\text{c}(\theta_{21}+\theta_{43})\left(\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\text{c}\,\theta_{65\,i}-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{65\,i}\,\text{s}\,\theta_{65\,i}\right)\right)+$$

$$\left(-\dot\theta_{21\,i}{}^2\,\text{c}\,\theta_{21\,i}-\ddot\theta_{21\,i}\,\text{s}\,\theta_{21\,i}\right)(\text{c}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}+\text{c}\,\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i})+$$

$$\text{s}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}))+$$

$$\left(\ddot\theta_{21\,i}\,\text{c}\,\theta_{21\,i}-\dot\theta_{21\,i}{}^2\,\text{s}\,\theta_{21\,i}\right)(-\text{s}\,\theta_{1110\,i}\,(\text{c}\,\theta_{21}\,\text{c}\,\theta_{43}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}(\beta_{1413\,i}-\theta_{54})-\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}$$

$$\text{s}\,\theta_{21}\,\text{s}\,\theta_{43}\,\text{s}(\beta_{1413\,i}-\theta_{54})+\text{c}(\beta_{1413\,i}-\theta_{54})\,\text{s}\,\theta_{65\,i})+\text{c}\,\theta_{1110\,i}\,(\text{c}\,\theta_{21}\,\text{c}\,\theta_{43}\,\text{c}(\beta_{1413\,i}-\theta_{54})$$

$$\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\text{c}(\beta_{1413\,i}-\theta_{54})\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{21}\,\text{s}\,\theta_{43}-\text{s}(\beta_{1413\,i}-\theta_{54})\,\text{s}\,\theta_{65\,i}))-$$

$$2\,\dot\theta_{21\,i}\,\text{s}\,\theta_{21\,i}\left(\dot\theta_{54}\,\text{s}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}+\text{c}\,\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i})-\right.$$

$$\dot\theta_{54}\,\text{c}(\beta_{1413\,i}-\theta_{54})\,(\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i})+$$

$$\text{c}(\beta_{1413\,i}-\theta_{54})\left(\dot\theta_{65\,i}\,\text{c}\,\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\,i}+\dot\theta_{1110\,i}\,\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\dot\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\right.$$

$$\left.\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}-\dot\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}-\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{c}\,\theta_{1110\,i}\,\text{s}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i}\right)+$$

$$\text{s}(\beta_{1413\,i}-\theta_{54})\left(-\dot\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}+\left(-\dot\theta_{65\,i}\,\text{c}\,\theta_{1110\,i}\,\text{s}\,\beta_{54\,i}+\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{s}\,\theta_{1110\,i}\right.\right.$$

$$\left.\left.\left.\text{s}(\theta_{21}+\theta_{43})\right)\text{s}\,\theta_{65\,i}-\text{c}(\theta_{21}+\theta_{43})\left(\dot\theta_{65\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\theta_{1110\,i}+\dot\theta_{1110\,i}\,\text{c}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}\right)\right)\right)+$$

$$\text{c}\,\theta_{21\,i}\left(\left(-\dot\theta_{54}{}^2\,\text{c}(\beta_{1413\,i}-\theta_{54})+\ddot\theta_{54}\,\text{s}(\beta_{1413\,i}-\theta_{54})\right)(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}+\text{c}\,\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{65\,i})-\right.$$

$$\left(\ddot\theta_{54}\,\text{c}(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}{}^2\,\text{s}(\beta_{1413\,i}-\theta_{54})\right)(\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i})+$$

$$2\,\dot\theta_{54}\,\text{s}(\beta_{1413\,i}-\theta_{54})\left(\dot\theta_{65\,i}\,\text{c}\,\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{c}\,\theta_{65\,i}+\dot\theta_{1110\,i}\,\text{c}\,\theta_{1110\,i}\,\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}-\dot\theta_{1110\,i}\right.$$

$$\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}-\dot\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{1110\,i}\,\text{s}\,\theta_{65\,i}-\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{c}\,\theta_{1110\,i}\,\text{s}(\theta_{21}+\theta_{43})$$

$$\left.\text{s}\,\theta_{65\,i}\right)+\text{c}(\beta_{1413\,i}-\theta_{54})\left(\text{c}\,\theta_{65\,i}\,\text{s}\,\beta_{54\,i}\left(\ddot\theta_{1110\,i}\,\text{c}\,\theta_{1110\,i}-\dot\theta_{1110\,i}{}^2\,\text{s}\,\theta_{1110\,i}\right)+2\,\dot\theta_{65\,i}\,\text{c}\,\theta_{65\,i}\right.$$

$$\left(-\dot\theta_{1110\,i}\,\text{c}(\theta_{21}+\theta_{43})\,\text{s}\,\theta_{1110\,i}-\left(\dot\theta_{21}+\dot\theta_{43}\right)\text{c}\,\theta_{1110\,i}\,\text{s}(\theta_{21}+\theta_{43})\right)-2\,\dot\theta_{1110\,i}\,\dot\theta_{65\,i}\,\text{c}\,\theta_{1110\,i}$$

$$\text{s}\,\beta_{54\,i}\,\text{s}\,\theta_{65\,i}+\left(-\text{c}\,\theta_{1110\,i}\left(\left(\dot\theta_{1110\,i}{}^2+\left(\dot\theta_{21}+\dot\theta_{43}\right)^2\right)\text{c}(\theta_{21}+\theta_{43})+\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\text{s}(\theta_{21}+\theta_{43})\right)+\right.$$

$$\mathrm{s}\,\theta_{1110\,i}\left(-\ddot{\theta}_{1110\,i}\,\mathrm{c}(\theta_{21}+\theta_{43})+2\,\dot{\theta}_{1110\,i}\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{s}(\theta_{21}+\theta_{43})\right)))\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}$$

$$\mathrm{s}\,\theta_{1110\,i}\,(\dot{\theta}_{65\,i}{}^{2}\,\mathrm{c}\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i})+\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}(\theta_{21}+\theta_{43})\,(\ddot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}-\dot{\theta}_{65\,i}{}^{2}\,\mathrm{s}\,\theta_{65\,i}))-$$

$$2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(-\dot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{1110\,i}+(-\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{s}\,\beta_{54\,i}+(\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{s}\,\theta_{1110\,i}$$

$$\mathrm{s}(\theta_{21}+\theta_{43}))\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}(\theta_{21}+\theta_{43})\,(\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{1110\,i}+\dot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{65\,i}))+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{s}\,\theta_{1110\,i}\,(((\dot{\theta}_{21}+\dot{\theta}_{43})^{2}\,\mathrm{c}(\theta_{21}+\theta_{43})+(\ddot{\theta}_{21}+\ddot{\theta}_{43})\,\mathrm{s}(\theta_{21}+\theta_{43}))\,\mathrm{s}\,\theta_{65\,i}+$$

$$2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{s}(\theta_{21}+\theta_{43})\,(\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{1110\,i}+\dot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{s}\,\theta_{65\,i})-$$

$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{1110\,i}\,(((\dot{\theta}_{1110\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2})\,\mathrm{c}\,\theta_{65\,i}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i})+\mathrm{s}\,\theta_{1110\,i}\,(\ddot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{65\,i}-2\,\dot{\theta}_{1110\,i}$$

$$\dot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{65\,i}))+\mathrm{c}(\theta_{21}+\theta_{43})\,(-\mathrm{c}\,\theta_{1110\,i}\,(2\,\dot{\theta}_{1110\,i}\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}+\ddot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{s}\,\theta_{1110\,i}\,(-\ddot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}+(\dot{\theta}_{1110\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2})\,\mathrm{s}\,\theta_{65\,i})))))+2\,\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}$$

$$(-\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,(\mathrm{s}\,\theta_{1110\,i}\,(\dot{\theta}_{65\,i}+(\dot{\theta}_{1110\,i}-\dot{\theta}_{54})\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}-(\dot{\theta}_{1110\,i}-\dot{\theta}_{54})$$

$$\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})+(\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{c}\,\theta_{1110\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(\theta_{21}+\theta_{43}))+\mathrm{c}\,\theta_{1110\,i}$$

$$(\dot{\theta}_{1110\,i}-\dot{\theta}_{54}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}-\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i})+\mathrm{s}(\beta_{1413\,i}-\theta_{54})$$

$$(-\mathrm{c}\,\theta_{1110\,i}\,\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{65\,i}+(\dot{\theta}_{1110\,i}-\dot{\theta}_{54})\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}-(\dot{\theta}_{1110\,i}-\dot{\theta}_{54})\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})+$$

$$\mathrm{s}\,\theta_{1110\,i}\,((\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{43}+(\dot{\theta}_{1110\,i}-\dot{\theta}_{54}-\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i}+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,((\dot{\theta}_{21}+\dot{\theta}_{43})\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}))))+$$

$$\mathrm{s}\,\theta_{21\,i}\,((-\ddot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}+\dot{\theta}_{1110\,i}{}^{2}\,\mathrm{s}\,\theta_{1110\,i})\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})-$$

$$\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})+\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{65\,i})-$$

$$(\dot{\theta}_{1110\,i}{}^{2}\,\mathrm{c}\,\theta_{1110\,i}+\ddot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i})\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}-$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{c}\,\theta_{1110\,i}\,(-\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,(\ddot{\theta}_{65\,i}+2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+$$

$$\ddot{\theta}_{54}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\,(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{43}-2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}))-$$

$$(\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,i}{}^{2}-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}+2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\theta_{65\,i}\,(-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}+(\ddot{\theta}_{21}+\ddot{\theta}_{43})\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}-$$

$$\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{65\,i}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\,((\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,i}{}^{2})\,\mathrm{c}\,\theta_{43}+(\ddot{\theta}_{21}+\ddot{\theta}_{43})\,\mathrm{s}\,\theta_{43}))+$$

$$(\ddot{\theta}_{54}+2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\,(\ddot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{43}-2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\theta_{43}))\,\mathrm{s}\,\theta_{65\,i}))-$$

$$\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,(\ddot{\theta}_{65\,i}+2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+$$

$$\ddot{\theta}_{54}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\,(\ddot{\theta}_{54}\,\mathrm{c}\,\theta_{43}-2\,(\dot{\theta}_{21}+\dot{\theta}_{43})\,\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}))-$$

$$(\dot{\theta}_{54}{}^{2}+\dot{\theta}_{65\,i}{}^{2}-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}+2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43})\,\mathrm{s}\,\theta_{65\,i})+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(-\mathrm{c}\,\theta_{65\,i}\,(-2\,\dot{\theta}_{54}\,\dot{\theta}_{65\,i}+(\ddot{\theta}_{21}+\ddot{\theta}_{43})\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}-$$

$$\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{54}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-\dot{\theta}_{65\,i}{}^{2}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}+$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\left(\left(\dot\theta_{21}{}^{2}+2\,\dot\theta_{21}\,\dot\theta_{43}+\dot\theta_{43}{}^{2}+\dot\theta_{54}{}^{2}+\dot\theta_{65\,i}{}^{2}\right)\mathrm{c}\,\theta_{43}+\left(\ddot\theta_{21}+\ddot\theta_{43}\right)\mathrm{s}\,\theta_{43}\right)\Big)+$$

$$\Big(\ddot\theta_{54}+2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}-$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,i}\left(\ddot\theta_{65\,i}\,\mathrm{c}\,\theta_{43}-2\left(\dot\theta_{21}+\dot\theta_{43}\right)\dot\theta_{65\,i}\,\mathrm{s}\,\theta_{43}\right)\Big)\,\mathrm{s}\,\theta_{65\,i}\Big)\Big)+$$

$$2\,\dot\theta_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\Big(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{43}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})+\dot\theta_{54}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})-\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{65\,i}-$$

$$\dot\theta_{65\,i}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\Big(-\dot\theta_{54}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\Big)\Big)-$$

$$2\,\dot\theta_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\Big(\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{c}\,\theta_{65\,i}\left(\dot\theta_{65\,i}-\dot\theta_{54}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}+\dot\theta_{54}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)-$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\Big(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{43}-\left(\dot\theta_{54}+\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}\,\theta_{65\,i}+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\beta_{54\,i}\left(\left(\dot\theta_{21}+\dot\theta_{43}\right)\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21}+\dot\theta_{65\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\Big)\Big)\Big)\Big)+$$

$$2\,\dot\theta_{109\,i}{}^{2}\,\sec(\theta_{109\,i})^{2}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}(\theta_{21}+\theta_{43})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))^{3}$$

$$\tan($$

$$\theta_{109\,i})\Big)$$

Coeficientes de la ecuación (2.172)

$$G_{85}=(\mathrm{s}\,\theta_{109\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}(\theta_{21}+\theta_{43})-$$

$$\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{c}\,\theta_{109\,i}\,(\mathrm{c}\,\theta_{1110\,i}\,(-\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}))+\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-$$

$$\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))))-$$

$$\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))+$$

$$\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})+$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}))))))^{3}$$

$G_{86}=0$

$G_{87}=0$

$G_{88}=0$

$G_{89}=0$

$G_{90}=0$

$G_{91}=0$

$G_{92}=$

$$c\,\theta_{98\,i}{}^{2}\Big(-2\,(s\,\theta_{109\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21}+\theta_{43}) - s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} + c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} -$$

$$c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i}) - c\,\beta_{54\,i}$$

$$(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} + c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{109\,i}\,(c\,\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} - c\,\theta_{21}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})))) -$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))))$$

$$\Big(\dot\theta_{109\,i}\,c\,\theta_{109\,i}\,(-s\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) + (c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} -$$

$$s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) + c\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) +$$

$$s\,\theta_{109\,i}\,\Big(-\dot\theta_{43}\,c\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) + (c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} +$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$\dot\theta_{43}\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\,\Big(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,\Big(\dot\theta_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} +$$

$$\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) - \dot\theta_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} -$$

$$\Big(c\,\theta_{65\,i}\,\Big(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i}\Big)\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,\Big(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,s\,\beta_{54\,i}\Big)\,s\,\theta_{65\,i}\Big)\,s\,\theta_{76\,i}\Big) -$$

$$\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}\,\Big(-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\,\Big(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\Big)\Big)\Big) -$$

$$s\,\theta_{43}\,\Big(-s\,\theta_{21}\,\Big(\dot\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i} + s\,\beta_{54\,i}\,\Big(\dot\theta_{76\,i} - \dot\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\Big)\Big)\,s\,\theta_{76\,i} +$$

$$c\,\theta_{21}\,\Big(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,\Big(\dot\theta_{21} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\Big) - c\,\theta_{65\,i}\,\Big(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i}\Big)\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} +$$

$$c\,\theta_{21\,i}\,\Big(\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - \Big(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,s\,\beta_{54\,i}\Big)\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\Big)\Big) +$$

$$c\,\beta_{54\,i}\,\Big(s\,\theta_{21}\,\Big(c\,\theta_{76\,i}\,\Big(-\dot\theta_{21}\,s\,\theta_{21\,i} + \dot\theta_{76\,i}\,s\,\theta_{65\,i}\Big) + \dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\Big) +$$

$$c\,\theta_{21}\,\Big(\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + \Big(-\dot\theta_{76\,i}\,s\,\theta_{21\,i} + \dot\theta_{21}\,s\,\theta_{65\,i}\Big)\,s\,\theta_{76\,i}\Big)\Big)\Big)\Big) -$$

$$\dot\theta_{109\,i}\,s\,\theta_{109\,i}\,(c\,\theta_{1110\,i}\,(s(\beta_{1413\,i}-\theta_{54})\,(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) -$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} - (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))) +$$

$$c\,\theta_{109\,i}\,\Big(-\dot\theta_{1110\,i}\,s\,\theta_{1110\,i}\,(s(\beta_{1413\,i}-\theta_{54})\,(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))-$$
$$\dot{\theta}_{1110\,i}\,c\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})-$$
$$c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$
$$s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))+$$
$$c\,\theta_{1110\,i}\,\big(\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$
$$s(\beta_{1413\,i}-\theta_{54})\,\big(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})+\dot{\theta}_{76\,i}$$
$$c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}+\big(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)-\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\big)$$
$$s\,\theta_{76\,i}\big)+\dot{\theta}_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$
$$\dot{\theta}_{54}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$
$$\dot{\theta}_{43}\,c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))-$$
$$\dot{\theta}_{54}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))-$$
$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,\big(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-s\,\theta_{21}\,\big(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}$$
$$c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}-$$
$$\big(c\,\theta_{65\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{65\,i}\big)\,s\,\theta_{76\,i}\big)-$$
$$\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$
$$c\,\theta_{21}\,\big(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)\big)\big)-$$
$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,\big(-s\,\theta_{21}\,\big(\dot{\theta}_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,\big(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\big)\big)\,s\,\theta_{76\,i}+$$
$$c\,\theta_{21}\,\big(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,\big(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\big)-c\,\theta_{65\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}+$$
$$c\,\theta_{21\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)\big)+$$
$$c\,\beta_{54\,i}\,\big(s\,\theta_{21}\,\big(c\,\theta_{76\,i}\,\big(-\dot{\theta}_{21}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\big)+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\big)+$$
$$c\,\theta_{21}\,\big(\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+\big(-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{21}\,s\,\theta_{65\,i}\big)\,s\,\theta_{76\,i}\big)\big)\big)\big)-$$
$$s\,\theta_{1110\,i}\,\big(\dot{\theta}_{54}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$
$$\dot{\theta}_{54}\,c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})+$$
$$\dot{\theta}_{43}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})+$$
$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})-$$
$$c(\beta_{1413\,i}-\theta_{54})\,\big(c\,\beta_{54\,i}\,\big(\dot{\theta}_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)+$$
$$c\,\theta_{65\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,\big(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{76\,i}\big)+$$
$$s\,\theta_{65\,i}\,\big(\dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}-\big(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i}\big)\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\big)\big)+$$
$$\dot{\theta}_{54}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))-$$
$$\dot{\theta}_{43}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-$$
$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))-$$
$$s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,\big(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-s\,\theta_{21}\,\big(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}$$
$$c\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}-$$

$$(c\,\theta_{65\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})-$$

$$\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})))-$$

$$c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(-s\,\theta_{21}\,(\dot\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot\theta_{76\,i}-\dot\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}+$$

$$c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,(\dot\theta_{21}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-c\,\theta_{65\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}+$$

$$c\,\theta_{21\,i}\,(\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}))+$$

$$c\,\beta_{54\,i}\,(s\,\theta_{21}\,(c\,\theta_{76\,i}\,(-\dot\theta_{21}\,s\,\theta_{21\,i}+\dot\theta_{76\,i}\,s\,\theta_{65\,i})+\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+(-\dot\theta_{76\,i}\,s\,\theta_{21\,i}+\dot\theta_{21}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))))$$

$$(\dot\theta_{109\,i}\,c\,\theta_{109\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21}+\theta_{43})-s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}+$$

$$c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i}+s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i})-c\,\beta_{54\,i}$$

$$(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i}+c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}+c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))+$$

$$s\,\theta_{109\,i}\,((\dot\theta_{21}+\dot\theta_{43})\,c\,\theta_{21\,i}\,c(\theta_{21}+\theta_{43})\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s(\theta_{21}+\theta_{43})-$$

$$\dot\theta_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{65\,i}-$$

$$\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s(\theta_{21}+\theta_{43})\,s\,\theta_{76\,i}-$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,(s\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-(\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{65\,i})+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+((\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))+$$

$$s\,\theta_{21}\,(c\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-(\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{65\,i})+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})-$$

$$s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+((\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))-$$

$$s\,\beta_{54\,i}\,(s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\theta_{43}\,(\dot\theta_{76\,i}-(\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})+c\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+$$

$$\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))+$$

$$c\,\theta_{21}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,(-\dot\theta_{76\,i}+(\dot\theta_{21}+\dot\theta_{43})\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})+s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+$$

$$\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+(\dot\theta_{21}+\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))-$$

$$\dot\theta_{109\,i}\,s\,\theta_{109\,i}\,(c\,\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))))-$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))))+$$

$$c\,\theta_{109\,i}\,(-\dot\theta_{1110\,i}\,s\,\theta_{1110\,i}\,(-s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))))-$$

$$\dot\theta_{1110\,i}\,c\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+$$

$$s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+$$

$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))+$$

$$
\begin{aligned}
&c\,\theta_{1110\,i}\,(\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad\dot\theta_{43}\,c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\\
&\qquad\quad(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-c\,\theta_{21}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))+\\
&\quad\dot\theta_{54}\,s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}-\\
&\qquad\qquad c\,\theta_{21}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))-\\
&\quad s(\beta_{1413\,i}-\theta_{54})\,(-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}+\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\theta_{21\,i}\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}))-\\
&\quad\dot\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+\\
&\quad\dot\theta_{54}\,c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})))+\\
&\quad c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i})-\\
&\qquad\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))-\\
&\qquad c\,\theta_{21}\,(c\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad\quad c\,\theta_{65\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad\quad s\,\theta_{65\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}))-\\
&\qquad s\,\theta_{21}\,(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,(-\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot\theta_{76\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})))+\\
&\quad c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21}\,c\,\theta_{76\,i}\,(\dot\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot\theta_{76\,i}-\dot\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))+\\
&\qquad s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot\theta_{21}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}+\\
&\qquad\quad c\,\theta_{65\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}))+\\
&\qquad c\,\beta_{54\,i}\,(s\,\theta_{21}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-\dot\theta_{21}\,s\,\theta_{65\,i})+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad\quad c\,\theta_{21}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+(\dot\theta_{21}\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))-\\
&s\,\theta_{1110\,i}\,(\dot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad c(\beta_{1413\,i}-\theta_{54})\,(-s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}+\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad c\,\theta_{21\,i}\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}))-\\
&\quad\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\\
&\qquad\qquad(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\qquad s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+\\
&\qquad\quad c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))+\\
&\quad s(\beta_{1413\,i}-\theta_{54})\,(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+\\
&\qquad\qquad c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\qquad\dot\theta_{21}\,c\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+\\
&\qquad\quad c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))+\\
&\qquad c\,\theta_{21}\,(-c\,\theta_{76\,i}\,s\,\theta_{43}\,(\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot\theta_{76\,i}-\dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))-\\
&\qquad\quad c\,\theta_{43}\,(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}+\\
&\qquad\qquad c\,\theta_{65\,i}\,(c\,\theta_{76\,i}\,(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}))+\\
&\qquad\quad c\,\beta_{54\,i}\,(-c\,\theta_{43}\,(c\,\theta_{76\,i}\,(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-\dot\theta_{43}\,s\,\theta_{65\,i})+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i})+\\
&\qquad\qquad s\,\theta_{43}\,(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+(\dot\theta_{43}\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))+\\
&\qquad s\,\theta_{21}\,(-\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}+
\end{aligned}
$$

$$\dot{\theta}_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} + \dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i} +$$

$$c\,\beta_{54\,i}\left(s\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i} - \dot{\theta}_{43}\,s\,\theta_{65\,i}\right) + \dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right) +\right.$$

$$\left.c\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \left(\dot{\theta}_{43}\,s\,\theta_{21\,i} - \dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right) +$$

$$s\,\beta_{54\,i}\left(c\,\theta_{43}\,c\,\theta_{76\,i}\left(-\dot{\theta}_{76\,i} + \dot{\theta}_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right) + s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +\right.\right.$$

$$\left.\left.\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \left(\dot{\theta}_{43} - \dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)))))) +$$

$$2\,(s\,\theta_{109\,i}\,(-s\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) + (c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) +$$

$$c\,\theta_{109\,i}\,(c\,\theta_{1110\,i}\,(s(\beta_{1413\,i} - \theta_{54})\,(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,c(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$c(\beta_{1413\,i} - \theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) -$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i} - \theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} - (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{43}\,s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))))$$

$$\left(\dot{\theta}_{109\,i}\,c\,\theta_{109\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21} + \theta_{43}) - s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} + c\,\theta_{21}\,c\,\theta_{76\,i}\right.$$

$$s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i}) - c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}$$

$$s\,\theta_{65\,i} - c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} + c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{109\,i}\left(\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)c\,\theta_{21\,i}\,c(\theta_{21} + \theta_{43})\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - \dot{\theta}_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s(\theta_{21} + \theta_{43}) -\right.$$

$$\dot{\theta}_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s(\theta_{21} + \theta_{43})\,s\,\theta_{65\,i} - \dot{\theta}_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s(\theta_{21} + \theta_{43})\,s\,\theta_{76\,i} -$$

$$c\,\beta_{54\,i}\left(c\,\theta_{21}\left(s\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i} - \left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{65\,i}\right) + \dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right) +\right.\right.$$

$$\left.c\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \left(\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{21\,i} - \dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right) + s\,\theta_{21}$$

$$\left(c\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot{\theta}_{76\,i}\,s\,\theta_{21\,i} - \left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{65\,i}\right) + \dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right) -\right.$$

$$\left.\left.s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \left(\left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{21\,i} - \dot{\theta}_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right) -$$

$$s\,\beta_{54\,i}\left(s\,\theta_{21}\left(c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot{\theta}_{76\,i} - \left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right) + c\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +\right.\right.\right.$$

$$\left.\left.\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right) + c\,\theta_{21}$$

$$\left(c\,\theta_{43}\,c\,\theta_{76\,i}\left(-\dot{\theta}_{76\,i} + \left(\dot{\theta}_{21} + \dot{\theta}_{43}\right)s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right) + s\,\theta_{43}\left(\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +\right.\right.$$

$$\left.\left.\left.\left.\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \left(\dot{\theta}_{21} + \dot{\theta}_{43} - \dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)\right) - \dot{\theta}_{109\,i}\,s\,\theta_{109\,i}$$

$$(c\,\theta_{1110\,i}\,(-s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$c(\beta_{1413\,i} - \theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} -$$

$$c\,\theta_{21}\,s\,\theta_{76\,i}) + c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\,c(\beta_{1413\,i} - \theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})))) -$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} +$$

$$c\,\beta_{54\,i}\,s\,\theta_{76\,i}) + s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))) + c\,\theta_{109\,i}$$

$$\left(-\dot{\theta}_{1110\,i}\,s\,\theta_{1110\,i}\,(-s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +\right.$$

$$c(\beta_{1413\ i}-\theta_{54})\, s\theta_{43}\,(-c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21}+s\beta_{54\ i}\,(c\theta_{76\ i}\, s\theta_{21}\, s\theta_{21\ i}\, s\theta_{65\ i}-$$
$$c\theta_{21}\, s\theta_{76\ i})+c\beta_{54\ i}\,(c\theta_{21}\, c\theta_{76\ i}\, s\theta_{65\ i}+s\theta_{21}\, s\theta_{21\ i}\, s\theta_{76\ i}))+$$
$$c\theta_{43}\, c(\beta_{1413\ i}-\theta_{54})\,(s\theta_{21}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i})+$$
$$c\theta_{21}\,(c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}-s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i}))))-$$
$$\dot{\theta}_{1110\ i}\, c\theta_{1110\ i}\,(c(\beta_{1413\ i}-\theta_{54})\,(c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21\ i}+c\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i}))+$$
$$s(\beta_{1413\ i}-\theta_{54})\,(c\theta_{21}\,(c\theta_{21\ i}\, c\theta_{43}\, c\theta_{65\ i}\, c\theta_{76\ i}-c\theta_{43}\, s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+$$
$$c\beta_{54\ i}\, s\theta_{76\ i})+s\theta_{43}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i}))+$$
$$s\theta_{21}\,(-c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{43}+s\beta_{54\ i}\,(c\theta_{76\ i}\, s\theta_{21\ i}\, s\theta_{43}\, s\theta_{65\ i}-c\theta_{43}\, s\theta_{76\ i})+$$
$$c\beta_{54\ i}\,(c\theta_{43}\, c\theta_{76\ i}\, s\theta_{65\ i}+s\theta_{21\ i}\, s\theta_{43}\, s\theta_{76\ i}))))+c\theta_{1110\ i}$$
$$\Big(\dot{\theta}_{54}\, c(\beta_{1413\ i}-\theta_{54})\,(c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21\ i}+c\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i}))+\dot{\theta}_{43}\, c\theta_{43}$$
$$c(\beta_{1413\ i}-\theta_{54})\,(-c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21}+s\beta_{54\ i}\,(c\theta_{76\ i}\, s\theta_{21}\, s\theta_{21\ i}\, s\theta_{65\ i}-c\theta_{21}\, s\theta_{76\ i})+$$
$$c\beta_{54\ i}\,(c\theta_{21}\, c\theta_{76\ i}\, s\theta_{65\ i}+s\theta_{21}\, s\theta_{21\ i}\, s\theta_{76\ i}))+\dot{\theta}_{54}\, s\theta_{43}\, s(\beta_{1413\ i}-\theta_{54})$$
$$(-c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21}+s\beta_{54\ i}\,(c\theta_{76\ i}\, s\theta_{21}\, s\theta_{21\ i}\, s\theta_{65\ i}-c\theta_{21}\, s\theta_{76\ i})+$$
$$c\beta_{54\ i}\,(c\theta_{21}\, c\theta_{76\ i}\, s\theta_{65\ i}+s\theta_{21}\, s\theta_{21\ i}\, s\theta_{76\ i}))-s(\beta_{1413\ i}-\theta_{54})$$
$$\Big(-s\theta_{21\ i}\Big(c\theta_{76\ i}\Big(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\, s\beta_{54\ i}\Big)s\theta_{65\ i}+\dot{\theta}_{76\ i}\, c\theta_{65\ i}\, s\theta_{76\ i}\Big)+$$
$$c\beta_{54\ i}\Big(\dot{\theta}_{76\ i}\, c\theta_{21\ i}\, c\theta_{76\ i}-\dot{\theta}_{21\ i}\, s\theta_{21\ i}\, s\theta_{76\ i}\Big)+$$
$$c\theta_{21\ i}\Big(c\theta_{65\ i}\, c\theta_{76\ i}\Big(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\, s\beta_{54\ i}\Big)-\dot{\theta}_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}\, s\theta_{76\ i}\Big)\Big)-\dot{\theta}_{43}$$
$$c(\beta_{1413\ i}-\theta_{54})\, s\theta_{43}\,(s\theta_{21}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i})+$$
$$c\theta_{21}\,(c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}-s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i})))+\dot{\theta}_{54}$$
$$c\theta_{43}\, s(\beta_{1413\ i}-\theta_{54})\,(s\theta_{21}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i})+$$
$$c\theta_{21}\,(c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}-s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i})))+$$
$$c\theta_{43}\, c(\beta_{1413\ i}-\theta_{54})\Big(\dot{\theta}_{21}\, c\theta_{21}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i})-$$
$$\dot{\theta}_{21}\, s\theta_{21}\,(c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}-s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i}))-$$
$$c\theta_{21}\Big(c\beta_{54\ i}\Big(\dot{\theta}_{76\ i}\, c\theta_{76\ i}\, s\theta_{21\ i}+\dot{\theta}_{21\ i}\, c\theta_{21\ i}\, s\theta_{76\ i}\Big)+$$
$$c\theta_{65\ i}\Big(c\theta_{76\ i}\Big(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\, s\beta_{54\ i}\Big)s\theta_{21\ i}+\dot{\theta}_{76\ i}\, c\theta_{21\ i}\, s\theta_{76\ i}\Big)+$$
$$s\theta_{65\ i}\Big(c\theta_{21\ i}\, c\theta_{76\ i}\Big(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\, s\beta_{54\ i}\Big)-\dot{\theta}_{76\ i}\, s\beta_{54\ i}\, s\theta_{21\ i}\, s\theta_{76\ i}\Big)\Big)-$$
$$s\theta_{21}\Big(\dot{\theta}_{76\ i}\, c\theta_{76\ i}\, s\beta_{54\ i}+c\beta_{54\ i}\Big(-\dot{\theta}_{65\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}+\dot{\theta}_{76\ i}\, s\theta_{65\ i}\, s\theta_{76\ i}\Big)\Big)\Big)+$$
$$c(\beta_{1413\ i}-\theta_{54})\, s\theta_{43}\Big(-c\theta_{21}\, c\theta_{76\ i}\Big(\dot{\theta}_{21}\, c\theta_{21\ i}\, c\theta_{65\ i}+s\beta_{54\ i}\Big(\dot{\theta}_{76\ i}-\dot{\theta}_{21}\, s\theta_{21\ i}\, s\theta_{65\ i}\Big)\Big)+$$
$$s\theta_{21}\Big(c\theta_{21\ i}\, c\theta_{76\ i}\Big(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\, s\beta_{54\ i}\Big)s\theta_{65\ i}+s\beta_{54\ i}\Big(\dot{\theta}_{21}-\dot{\theta}_{76\ i}\, s\theta_{21\ i}\, s\theta_{65\ i}\Big)s\theta_{76\ i}+$$
$$c\theta_{65\ i}\Big(c\theta_{76\ i}\Big(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\, s\beta_{54\ i}\Big)s\theta_{21\ i}+\dot{\theta}_{76\ i}\, c\theta_{21\ i}\, s\theta_{76\ i}\Big)\Big)+$$
$$c\beta_{54\ i}\Big(s\theta_{21}\Big(c\theta_{76\ i}\Big(\dot{\theta}_{76\ i}\, s\theta_{21\ i}-\dot{\theta}_{21}\, s\theta_{65\ i}\Big)+\dot{\theta}_{21\ i}\, c\theta_{21\ i}\, s\theta_{76\ i}\Big)+$$
$$c\theta_{21}\Big(\dot{\theta}_{65\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}+\Big(\dot{\theta}_{21}\, s\theta_{21\ i}-\dot{\theta}_{76\ i}\, s\theta_{65\ i}\Big)s\theta_{76\ i}\Big)\Big)\Big)\Big)-$$
$$s\theta_{1110\ i}\Big(\dot{\theta}_{54}\, s(\beta_{1413\ i}-\theta_{54})\,(c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{21\ i}+c\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i}))+$$
$$c(\beta_{1413\ i}-\theta_{54})\Big(-s\theta_{21\ i}\Big(c\theta_{76\ i}\Big(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\, s\beta_{54\ i}\Big)s\theta_{65\ i}+\dot{\theta}_{76\ i}\, c\theta_{65\ i}\, s\theta_{76\ i}\Big)+$$
$$c\beta_{54\ i}\Big(\dot{\theta}_{76\ i}\, c\theta_{21\ i}\, c\theta_{76\ i}-\dot{\theta}_{21\ i}\, s\theta_{21\ i}\, s\theta_{76\ i}\Big)+$$
$$c\theta_{21\ i}\Big(c\theta_{65\ i}\, c\theta_{76\ i}\Big(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\, s\beta_{54\ i}\Big)-\dot{\theta}_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}\, s\theta_{76\ i}\Big)\Big)-\dot{\theta}_{54}\, c(\beta_{1413\ i}-\theta_{54})$$
$$(c\theta_{21}\,(c\theta_{21\ i}\, c\theta_{43}\, c\theta_{65\ i}\, c\theta_{76\ i}-c\theta_{43}\, s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i})+$$
$$s\theta_{43}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i}))+$$
$$s\theta_{21}\,(-c\theta_{21\ i}\, c\theta_{65\ i}\, c\theta_{76\ i}\, s\theta_{43}+s\beta_{54\ i}\,(c\theta_{76\ i}\, s\theta_{21\ i}\, s\theta_{43}\, s\theta_{65\ i}-c\theta_{43}\, s\theta_{76\ i})+$$
$$c\beta_{54\ i}\,(c\theta_{43}\, c\theta_{76\ i}\, s\theta_{65\ i}+s\theta_{21\ i}\, s\theta_{43}\, s\theta_{76\ i})))+s(\beta_{1413\ i}-\theta_{54})$$
$$\Big(-\dot{\theta}_{21}\, s\theta_{21}\,(c\theta_{21\ i}\, c\theta_{43}\, c\theta_{65\ i}\, c\theta_{76\ i}-c\theta_{43}\, s\theta_{21\ i}\,(c\theta_{76\ i}\, s\beta_{54\ i}\, s\theta_{65\ i}+c\beta_{54\ i}\, s\theta_{76\ i})+$$
$$s\theta_{43}\,(c\beta_{54\ i}\, c\theta_{76\ i}\, s\theta_{65\ i}-s\beta_{54\ i}\, s\theta_{76\ i}))+\dot{\theta}_{21}\, c\theta_{21}$$

$$(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{21}\left(-c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i} + s\,\beta_{54\,i}\left(\dot\theta_{76\,i} - \dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right) -\right.$$

$$c\,\theta_{43}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i} + s\,\beta_{54\,i}\left(\dot\theta_{43} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right.$$

$$\left.s\,\theta_{76\,i} + c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i} + \dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\right) +$$

$$c\,\beta_{54\,i}\left(-c\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,s\,\theta_{21\,i} - \dot\theta_{43}\,s\,\theta_{65\,i}\right) + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right) +\right.$$

$$\left.\left.s\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \left(\dot\theta_{43}\,s\,\theta_{21\,i} - \dot\theta_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right) +$$

$$s\,\theta_{21}\left(-\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \dot\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43} +\right.$$

$$\dot\theta_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} + \dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i} +$$

$$c\,\beta_{54\,i}\left(s\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,s\,\theta_{21\,i} - \dot\theta_{43}\,s\,\theta_{65\,i}\right) + \dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right) +\right.$$

$$\left.c\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} + \left(\dot\theta_{43}\,s\,\theta_{21\,i} - \dot\theta_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right) +$$

$$s\,\beta_{54\,i}\left(c\,\theta_{43}\,c\,\theta_{76\,i}\left(-\dot\theta_{76\,i} + \dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right) + s\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +\right.\right.$$

$$\left.\left.\left.\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \left(\dot\theta_{43} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\right)\right)\Big)\Big)\Big)\Big)^2 +$$

$$(s\,\theta_{109\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21} + \theta_{43}) - s\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} + c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}$$

$$s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i}) -$$

$$c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}$$

$$s\,\theta_{76\,i} + c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{109\,i}\,(c\,\theta_{1110\,i}\,(-s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$c(\beta_{1413\,i} - \theta_{54})\,s\,\theta_{43}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} -$$

$$c\,\theta_{21}\,s\,\theta_{76\,i}) + c\,\beta_{54\,i}\,(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\,c(\beta_{1413\,i} - \theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})))) -$$

$$s\,\theta_{1110\,i}\,(c(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s(\beta_{1413\,i} - \theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - c\,\theta_{43}\,s\,\theta_{21\,i}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} +$$

$$c\,\beta_{54\,i}\,s\,\theta_{76\,i}) + s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))))^2$$

$$\left(\left(\ddot\theta_{109\,i}\,c\,\theta_{109\,i} - \dot\theta_{109\,i}{}^2\,s\,\theta_{109\,i}\right)(-s\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +\right.$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) +$$

$$2\,\dot\theta_{109\,i}\,c\,\theta_{109\,i}\left(-\dot\theta_{43}\,c\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) + (c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} +\right.$$

$$c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$\dot\theta_{43}\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\left(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}\left(\dot\theta_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} +\right.\right.$$

$$\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}) - \dot\theta_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} -$$

$$\left.\left(c\,\theta_{65\,i}\left(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i} + c\,\theta_{21\,i}\left(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right) -$$

$$\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) +$$

$$\left.c\,\theta_{21}\left(-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i} + c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + \dot\theta_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\right)\right) -$$

$$s\,\theta_{43}\left(-s\,\theta_{21}\left(\dot\theta_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i} + s\,\beta_{54\,i}\left(\dot\theta_{76\,i} - \dot\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)s\,\theta_{76\,i} +\right.$$

$$c\,\theta_{21}\left(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\left(\dot\theta_{21} - \dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right) - c\,\theta_{65\,i}\left(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}\,s\,\theta_{76\,i} +\right.$$

$$
\begin{aligned}
&c\,\theta_{21\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}))\,+\\
&c\,\beta_{54\,i}\,(s\,\theta_{21}\,(c\,\theta_{76\,i}\,(-\dot{\theta}_{21}\,s\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,s\,\theta_{65\,i})+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})+c\,\theta_{21}\\
&(\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+(-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}+\dot{\theta}_{21}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))+(-\dot{\theta}_{109\,i}{}^{2}\,c\,\theta_{109\,i}-\ddot{\theta}_{109\,i}\,s\,\theta_{109\,i})\\
&(c\,\theta_{1110\,i}\,(s(\beta_{1413\,i}-\theta_{54})\,(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}+(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})\,-\\
&c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,+\\
&(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})\,-\\
&c(\beta_{1413\,i}-\theta_{54})\,s\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})\,-\\
&s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))\,-\\
&s\,\theta_{1110\,i}\,(c(\beta_{1413\,i}-\theta_{54})\,(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-(c\,\theta_{65\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})\,-\\
&c\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,+\\
&(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})\,-\\
&s\,\theta_{43}\,s(\beta_{1413\,i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})\,-\\
&s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))))\,+\\
&s\,\theta_{109\,i}\,((-\ddot{\theta}_{43}\,c\,\theta_{43}+\dot{\theta}_{43}{}^{2}\,s\,\theta_{43})\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,+\\
&(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i})\,-\\
&s\,\theta_{43}\,(c\,\theta_{76\,i}\,(s\,\beta_{54\,i}\,(\ddot{\theta}_{21}\,c\,\theta_{21}-\dot{\theta}_{21}{}^{2}\,s\,\theta_{21})+c\,\beta_{54\,i}\,(-s\,\theta_{21}\,(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\,i}\,c\,\theta_{21\,i}+\ddot{\theta}_{21}\,s\,\theta_{21\,i})\,+\\
&c\,\theta_{21}\,(\ddot{\theta}_{21\,i}\,c\,\theta_{21\,i}-(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,i}{}^{2})\,s\,\theta_{21\,i})))+2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\\
&(\dot{\theta}_{65\,i}\,c\,\beta_{54\,i}\,c\,\theta_{65\,i}\,s\,\theta_{21}+\dot{\theta}_{21}\,c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{65\,i}-c\,\theta_{21}\,(c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}\,+\\
&c\,\theta_{21\,i}\,(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,-\\
&2\,\dot{\theta}_{76\,i}\,(\dot{\theta}_{21}\,c\,\theta_{21}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,(\dot{\theta}_{21\,i}\,c\,\theta_{21}\,c\,\theta_{21\,i}-\dot{\theta}_{21}\,s\,\theta_{21}\,s\,\theta_{21\,i}))\,s\,\theta_{76\,i}\,+\\
&(c\,\beta_{54\,i}\,(c\,\theta_{21}\,(2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}+\ddot{\theta}_{21}\,s\,\theta_{65\,i})+s\,\theta_{21}\,(\ddot{\theta}_{65\,i}\,c\,\theta_{65\,i}-(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{65\,i}{}^{2})\,s\,\theta_{65\,i}))\,+\\
&s\,\theta_{21}\,(s\,\theta_{21\,i}\,(2\,\dot{\theta}_{21}\,c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})+\ddot{\theta}_{21}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,+\\
&c\,\theta_{21\,i}\,(-\ddot{\theta}_{21}\,c\,\theta_{65\,i}+2\,\dot{\theta}_{21}\,(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i}))\,-\\
&c\,\theta_{21}\,(c\,\theta_{21\,i}\,(c\,\theta_{65\,i}\,(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2}+2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,+\\
&(\ddot{\theta}_{65\,i}+\ddot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})+s\,\theta_{21\,i}\,(c\,\theta_{65\,i}\,(\ddot{\theta}_{21\,i}+\ddot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,-\\
&(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}+(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,i}{}^{2}+\dot{\theta}_{65\,i}{}^{2})\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})))\\
&s\,\theta_{76\,i}-(s\,\beta_{54\,i}\,s\,\theta_{21}+c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i})\,(\dot{\theta}_{76\,i}{}^{2}\,c\,\theta_{76\,i}+\ddot{\theta}_{76\,i}\,s\,\theta_{76\,i})\,+\\
&(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i}+c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}-\dot{\theta}_{76\,i}{}^{2}\,s\,\theta_{76\,i}))\,-\\
&(\dot{\theta}_{43}{}^{2}\,c\,\theta_{43}+\ddot{\theta}_{43}\,s\,\theta_{43})\,(c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})\,-\\
&s\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))\,-\\
&2\,\dot{\theta}_{43}\,s\,\theta_{43}\,(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}+c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i})-s\,\theta_{21}\,(\dot{\theta}_{21\,i}\,c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,+\\
&\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,c\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\,-\\
&(c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\,(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})\,-\\
&\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+(c\,\theta_{21\,i}\,c\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})\,+\\
&c\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}+c\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i})))\,-\\
&2\,\dot{\theta}_{43}\,c\,\theta_{43}\,(-s\,\theta_{21}\,(\dot{\theta}_{21}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}\,+\\
&c\,\theta_{21}\,(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})-c\,\theta_{65\,i}\,(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\,+
\end{aligned}
$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\dot\theta_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot\theta_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) +$$

$$\mathrm{c}\,\theta_{43}\,((-\dot\theta_{21}{}^{2}\,\mathrm{c}\,\theta_{21} - \ddot\theta_{21}\,\mathrm{s}\,\theta_{21})\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) - 2\,\dot\theta_{21}\,\mathrm{c}\,\theta_{21}$$

$$(\dot\theta_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot\theta_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}$$

$$\mathrm{s}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$(-\ddot\theta_{21}\,\mathrm{c}\,\theta_{21} + \dot\theta_{21}{}^{2}\,\mathrm{s}\,\theta_{21})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$2\,\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\,(-\dot\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot\theta_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{21}\,(-\mathrm{s}\,\beta_{54\,i}\,(\dot\theta_{76\,i}{}^{2}\,\mathrm{c}\,\theta_{76\,i} + \ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{65\,i}\,(2\,\dot\theta_{65\,i}\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i} + \ddot\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{65\,i}\,(\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{65\,i}{}^{2} + \dot\theta_{76\,i}{}^{2})\,\mathrm{s}\,\theta_{76\,i})))) - \mathrm{s}\,\theta_{21}$$

$$(\mathrm{c}\,\beta_{54\,i}\,(-\mathrm{s}\,\theta_{21\,i}\,((\dot\theta_{21\,i}{}^{2} + \dot\theta_{76\,i}{}^{2})\,\mathrm{c}\,\theta_{76\,i} + \ddot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\theta_{21\,i}\,(\ddot\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - 2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{21\,i}\,(-\mathrm{s}\,\theta_{65\,i}\,(2\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}) + (\ddot\theta_{65\,i} + \ddot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{65\,i}\,(\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot\theta_{21\,i}{}^{2} + \dot\theta_{65\,i}{}^{2} + \dot\theta_{76\,i}{}^{2} + 2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}\,\theta_{21\,i}\,(-\mathrm{c}\,\theta_{65\,i}\,(2\,\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) + (\ddot\theta_{21\,i} + \ddot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{65\,i}\,(-\ddot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + (2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i} + (\dot\theta_{21\,i}{}^{2} + \dot\theta_{65\,i}{}^{2} + \dot\theta_{76\,i}{}^{2})\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{76\,i}))))) -$$

$$2\,\dot\theta_{109\,i}\,\mathrm{s}\,\theta_{109\,i}\,(-\dot\theta_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})$$

$$\mathrm{s}\,\theta_{76\,i}) - \mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) -$$

$$\dot\theta_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) +$$

$$\mathrm{c}\,\theta_{1110\,i}\,(\dot\theta_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\dot\theta_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} +$$

$$\dot\theta_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot\theta_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +$$

$$(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,(\dot\theta_{21\,i} + \dot\theta_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) - (\dot\theta_{65\,i} + \dot\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\dot\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\dot\theta_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\dot\theta_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\dot\theta_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\dot\theta_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) - \mathrm{s}\,\theta_{21}\,(\dot\theta_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}$$

$$\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} -$$

$$\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\Big) -$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\Big) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\left(-\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)\mathrm{s}\,\theta_{76\,i} +\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{21} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right) - \mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +\right.$$

$$\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \left(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\Big) +$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right) + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right) +\right.$$

$$\mathrm{c}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\Big)\Big)\Big) -$$

$$\mathrm{s}\,\theta_{1110\,i}\left(\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +\right.$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right) +\right.$$

$$\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{76\,i}\right) +$$

$$\mathrm{s}\,\theta_{65\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} - \left(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\Big) +$$

$$\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) - \mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\right.\right.$$

$$\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} -$$

$$\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\Big) -$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\Big) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\left(-\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)\mathrm{s}\,\theta_{76\,i} +\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{21} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right) - \mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +\right.$$

$$\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \left(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\Big) +$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right) + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right) +\right.$$

$$\mathrm{c}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\Big)\Big)\Big)\Big) +$$

$$\mathrm{c}\,\theta_{109\,i}\left(\left(-\dot{\theta}_{1110\,i}^{\;2}\,\mathrm{c}\,\theta_{1110\,i} - \ddot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\right)(\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} +\right.$$

$$(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))) +$$

$$\left(-\ddot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i} + \dot{\theta}_{1110\,i}^{\;2}\,\mathrm{s}\,\theta_{1110\,i}\right)(\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} -$$

$$(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))-$$

$$2\,\dot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\left(\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}-(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})+\right.$$

$$\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\right.$$

$$\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})+\dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}+$$

$$\left.\left(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)-\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)+$$

$$\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i})+$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i})-$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i})+$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i})-$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))-$$

$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))-$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\right.\right.$$

$$\mathrm{c}\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}-$$

$$\left.\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})+$$

$$\left.\mathrm{c}\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}+\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\right)-$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(-\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}+\mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)\right)\mathrm{s}\,\theta_{76\,i}+\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\left(\dot{\theta}_{21}-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}\right)-\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}+\right.$$

$$\left.\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}-\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\mathrm{c}\,\beta_{54\,i}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\,i}\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}\right)+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\left.\left.\left.\mathrm{c}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}+\left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)\right)\right)\right)-$$

$$2\,\dot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\left(\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}-(\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})+\right.$$

$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i})+$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i})+$$

$$\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i})+$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{c}(\beta_{1413\,i}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)+\right.$$

$$\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)+$$

$$\left.\mathrm{s}\,\theta_{65\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}-\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)+$$

$$\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))-$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))-$$

$$\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i}-\theta_{54})\left(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}+\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})-\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\right.\right.$$

$$\mathrm{c}\,\theta_{76\,i}+\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})-\dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}-$$

$$\left.\left(\mathrm{c}\,\theta_{65\,i}\left(\dot{\theta}_{21\,i}+\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{21\,i}+\mathrm{c}\,\theta_{21\,i}\left(\dot{\theta}_{65\,i}+\dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i}\right)\mathrm{s}\,\theta_{65\,i}\right)\mathrm{s}\,\theta_{76\,i}\right)-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}+(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}-\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})+$$

$$\left.\mathrm{c}\,\theta_{21}\left(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}+\mathrm{c}\,\beta_{54\,i}\left(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i}+\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}\right)\right)\right)-$$

$$c\,\theta_{43}\,s(\beta_{1413\ i}-\theta_{54})\,(-s\,\theta_{21}\,(\dot\theta_{21}\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\,(\dot\theta_{76\ i}-\dot\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}))\,s\,\theta_{76\ i}+$$

$$c\,\theta_{21}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,(\dot\theta_{21}-\dot\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})-c\,\theta_{65\ i}\,(\dot\theta_{21\ i}+\dot\theta_{65\ i}\,s\,\beta_{54\ i})\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}+$$

$$c\,\theta_{21\ i}\,(\dot\theta_{76\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-(\dot\theta_{65\ i}+\dot\theta_{21\ i}\,s\,\beta_{54\ i})\,s\,\theta_{65\ i}\,s\,\theta_{76\ i}))+$$

$$c\,\beta_{54\ i}\,(s\,\theta_{21}\,(c\,\theta_{76\ i}\,(-\dot\theta_{21}\,s\,\theta_{21\ i}+\dot\theta_{76\ i}\,s\,\theta_{65\ i})+\dot\theta_{65\ i}\,c\,\theta_{65\ i}\,s\,\theta_{76\ i})+$$

$$c\,\theta_{21}\,(\dot\theta_{21\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}+(-\dot\theta_{76\ i}\,s\,\theta_{21\ i}+\dot\theta_{21}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i}))))+$$

$$c\,\theta_{1110\ i}\,((\ddot\theta_{54}\,c(\beta_{1413\ i}-\theta_{54})+\dot\theta_{54}{}^{2}\,s(\beta_{1413\ i}-\theta_{54}))\,(c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}-$$

$$(c\,\theta_{65\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i})-2\,\dot\theta_{54}\,c(\beta_{1413\ i}-\theta_{54})\,(\dot\theta_{21\ i}\,c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+$$

$$\dot\theta_{76\ i}\,c\,\theta_{76\ i}\,(c\,\theta_{65\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i})+\dot\theta_{76\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}+$$

$$(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,(\dot\theta_{21\ i}+\dot\theta_{65\ i}\,s\,\beta_{54\ i})-(\dot\theta_{65\ i}+\dot\theta_{21\ i}\,s\,\beta_{54\ i})\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i})+$$

$$c(\beta_{1413\ i}-\theta_{54})\,(\dot\theta_{43}{}^{2}\,c\,\theta_{43}+\ddot\theta_{43}\,s\,\theta_{43})\,(c\,\theta_{76\ i}\,(s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})+$$

$$(c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{65\ i}+c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}))\,s\,\theta_{76\ i})+$$

$$s\,\theta_{43}\,(\dot\theta_{54}{}^{2}\,c(\beta_{1413\ i}-\theta_{54})-\ddot\theta_{54}\,s(\beta_{1413\ i}-\theta_{54}))\,(c\,\theta_{21}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}+c\,\beta_{54\ i}\,s\,\theta_{65\ i}\,s\,\theta_{76\ i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i}))+s(\beta_{1413\ i}-\theta_{54})$$

$$(c\,\beta_{54\ i}\,(c\,\theta_{21\ i}\,((\dot\theta_{21\ i}{}^{2}+\dot\theta_{76\ i}{}^{2})\,c\,\theta_{76\ i}+\ddot\theta_{76\ i}\,s\,\theta_{76\ i})+s\,\theta_{21\ i}\,(\ddot\theta_{21\ i}\,c\,\theta_{76\ i}-2\,\dot\theta_{21\ i}\,\dot\theta_{76\ i}\,s\,\theta_{76\ i}))+$$

$$s\,\theta_{21\ i}\,(-s\,\theta_{65\ i}\,(2\,\dot\theta_{76\ i}\,c\,\theta_{76\ i}\,(\dot\theta_{65\ i}+\dot\theta_{21\ i}\,s\,\beta_{54\ i})+(\ddot\theta_{65\ i}+\ddot\theta_{21\ i}\,s\,\beta_{54\ i})\,s\,\theta_{76\ i})+$$

$$c\,\theta_{65\ i}\,(\ddot\theta_{76\ i}\,c\,\theta_{76\ i}-(\dot\theta_{21\ i}{}^{2}+\dot\theta_{65\ i}{}^{2}+\dot\theta_{76\ i}{}^{2}+2\,\dot\theta_{21\ i}\,\dot\theta_{65\ i}\,s\,\beta_{54\ i})\,s\,\theta_{76\ i}))+$$

$$c\,\theta_{21\ i}\,(c\,\theta_{65\ i}\,(2\,\dot\theta_{76\ i}\,c\,\theta_{76\ i}\,(\dot\theta_{21\ i}+\dot\theta_{65\ i}\,s\,\beta_{54\ i})+(\ddot\theta_{21\ i}+\ddot\theta_{65\ i}\,s\,\beta_{54\ i})\,s\,\theta_{76\ i})+$$

$$s\,\theta_{65\ i}\,(\ddot\theta_{76\ i}\,c\,\theta_{76\ i}\,s\,\beta_{54\ i}-(2\,\dot\theta_{21\ i}\,\dot\theta_{65\ i}+(\dot\theta_{21\ i}{}^{2}+\dot\theta_{65\ i}{}^{2}+\dot\theta_{76\ i}{}^{2})\,s\,\beta_{54\ i})\,s\,\theta_{76\ i})))-$$

$$2\,\dot\theta_{54}\,s(\beta_{1413\ i}-\theta_{54})\,(\dot\theta_{43}\,c\,\theta_{43}\,(c\,\theta_{21}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}+c\,\beta_{54\ i}\,s\,\theta_{65\ i}\,s\,\theta_{76\ i})-$$

$$s\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i}))+$$

$$s\,\theta_{43}\,(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}+c\,\beta_{54\ i}\,s\,\theta_{65\ i}\,s\,\theta_{76\ i})-s\,\theta_{21}\,(\dot\theta_{21\ i}\,c\,\beta_{54\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}+$$

$$\dot\theta_{76\ i}\,c\,\theta_{76\ i}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})-\dot\theta_{76\ i}\,c\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}-$$

$$(c\,\theta_{65\ i}\,(\dot\theta_{21\ i}+\dot\theta_{65\ i}\,s\,\beta_{54\ i})\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,(\dot\theta_{65\ i}+\dot\theta_{21\ i}\,s\,\beta_{54\ i})\,s\,\theta_{65\ i})\,s\,\theta_{76\ i})-$$

$$\dot\theta_{21}\,c\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i})+$$

$$c\,\theta_{21}\,(-\dot\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{76\ i}+c\,\beta_{54\ i}\,(\dot\theta_{76\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}+\dot\theta_{65\ i}\,c\,\theta_{65\ i}\,s\,\theta_{76\ i}))))+$$

$$2\,\dot\theta_{43}\,s\,\theta_{43}\,(\dot\theta_{54}\,s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{76\ i}\,(s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})+$$

$$(c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{65\ i}+c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}))\,s\,\theta_{76\ i})+$$

$$c(\beta_{1413\ i}-\theta_{54})\,(-s\,\theta_{21}\,(\dot\theta_{21}\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\,(\dot\theta_{76\ i}-\dot\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}))\,s\,\theta_{76\ i}+$$

$$c\,\theta_{21}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,(\dot\theta_{21}-\dot\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i})-c\,\theta_{65\ i}\,(\dot\theta_{21\ i}+\dot\theta_{65\ i}\,s\,\beta_{54\ i})\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}+$$

$$c\,\theta_{21\ i}\,(\dot\theta_{76\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-(\dot\theta_{65\ i}+\dot\theta_{21\ i}\,s\,\beta_{54\ i})\,s\,\theta_{65\ i}\,s\,\theta_{76\ i}))+$$

$$c\,\beta_{54\ i}\,(s\,\theta_{21}\,(c\,\theta_{76\ i}\,(-\dot\theta_{21}\,s\,\theta_{21\ i}+\dot\theta_{76\ i}\,s\,\theta_{65\ i})+\dot\theta_{65\ i}\,c\,\theta_{65\ i}\,s\,\theta_{76\ i})+$$

$$c\,\theta_{21}\,(\dot\theta_{21\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}+(-\dot\theta_{76\ i}\,s\,\theta_{21\ i}+\dot\theta_{21}\,s\,\theta_{65\ i})\,s\,\theta_{76\ i}))))-$$

$$c\,\theta_{43}\,((-\dot\theta_{54}{}^{2}\,c(\beta_{1413\ i}-\theta_{54})+\ddot\theta_{54}\,s(\beta_{1413\ i}-\theta_{54}))\,(c\,\theta_{76\ i}\,(s\,\beta_{54\ i}\,s\,\theta_{21}+c\,\beta_{54\ i}\,c\,\theta_{21}\,s\,\theta_{21\ i})+$$

$$(c\,\beta_{54\ i}\,s\,\theta_{21}\,s\,\theta_{65\ i}+c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}))\,s\,\theta_{76\ i})+$$

$$c(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{76\ i}\,(s\,\beta_{54\ i}\,(\ddot\theta_{21}\,c\,\theta_{21}-\dot\theta_{21}{}^{2}\,s\,\theta_{21})+c\,\beta_{54\ i}\,(-s\,\theta_{21}\,(2\,\dot\theta_{21}\,\dot\theta_{21\ i}\,c\,\theta_{21\ i}+$$

$$\ddot\theta_{21}\,s\,\theta_{21\ i})+c\,\theta_{21}\,(\ddot\theta_{21\ i}\,c\,\theta_{21\ i}-(\dot\theta_{21}{}^{2}+\dot\theta_{21\ i}{}^{2})\,s\,\theta_{21\ i})))+$$

$$2\dot\theta_{76 i}\, c\theta_{76 i}\, (\dot\theta_{65 i}\, c\beta_{54 i}\, c\theta_{65 i}\, s\theta_{21} + \dot\theta_{21}\, c\beta_{54 i}\, c\theta_{21}\, s\theta_{65 i} -$$

$$c\theta_{21}\, (c\theta_{65 i}\, (\dot\theta_{21 i} + \dot\theta_{65 i}\, s\beta_{54 i})\, s\theta_{21 i} + c\theta_{21 i}\, (\dot\theta_{65 i} + \dot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i}) -$$

$$\dot\theta_{21}\, s\theta_{21}\, (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i})) - 2\dot\theta_{76 i}\, (\dot\theta_{21}\, c\theta_{21}\, s\beta_{54 i} +$$

$$c\beta_{54 i}\, (\dot\theta_{21 i}\, c\theta_{21}\, c\theta_{21 i} - \dot\theta_{21}\, s\theta_{21}\, s\theta_{21 i}))\, s\theta_{76 i} + (c\beta_{54 i}\, (c\theta_{21}\, (2\dot\theta_{21}\, \dot\theta_{65 i}$$

$$c\theta_{65 i} + \ddot\theta_{21}\, s\theta_{65 i}) + s\theta_{21}\, (\ddot\theta_{65 i}\, c\theta_{65 i} - (\dot\theta_{21}^{2} + \dot\theta_{65 i}^{2})\, s\theta_{65 i})) +$$

$$s\theta_{21}\, (s\theta_{21 i}\, (2\dot\theta_{21}\, c\theta_{65 i}\, (\dot\theta_{21 i} + \dot\theta_{65 i}\, s\beta_{54 i}) + \ddot\theta_{21}\, s\beta_{54 i}\, s\theta_{65 i}) +$$

$$c\theta_{21 i}\, (-\ddot\theta_{21}\, c\theta_{65 i} + 2\dot\theta_{21}\, (\dot\theta_{65 i} + \dot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i})) -$$

$$c\theta_{21}\, (c\theta_{21 i}\, (c\theta_{65 i}\, (\dot\theta_{21}^{2} + \dot\theta_{21 i}^{2} + \dot\theta_{65 i}^{2} + 2\dot\theta_{21 i}\, \dot\theta_{65 i}\, s\beta_{54 i}) +$$

$$(\ddot\theta_{65 i} + \ddot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i}) + s\theta_{21 i}\, (c\theta_{65 i}\, (\ddot\theta_{21 i} + \ddot\theta_{65 i}\, s\beta_{54 i}) -$$

$$(2\dot\theta_{21 i}\, \dot\theta_{65 i} + (\dot\theta_{21}^{2} + \dot\theta_{21 i}^{2} + \dot\theta_{65 i}^{2})\, s\beta_{54 i})\, s\theta_{65 i})))\, s\theta_{76 i} -$$

$$(s\beta_{54 i}\, s\theta_{21} + c\beta_{54 i}\, c\theta_{21}\, s\theta_{21 i})\, (\dot\theta_{76 i}^{2}\, c\theta_{76 i} + \ddot\theta_{76 i}\, s\theta_{76 i}) + (c\beta_{54 i}\, s\theta_{21}\, s\theta_{65 i} +$$

$$c\theta_{21}\, (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i}))\, (\ddot\theta_{76 i}\, c\theta_{76 i} - \dot\theta_{76 i}^{2}\, s\theta_{76 i})) +$$

$$2\dot\theta_{54}\, s(\beta_{1413 i} - \theta_{54})\, (-s\theta_{21}\, (\dot\theta_{21}\, c\theta_{21 i}\, c\theta_{65 i} + s\beta_{54 i}\, (\dot\theta_{76 i} - \dot\theta_{21}\, s\theta_{21 i}\, s\theta_{65 i}))\, s\theta_{76 i} +$$

$$c\theta_{21}\, (c\theta_{76 i}\, s\beta_{54 i}\, (\dot\theta_{21} - \dot\theta_{76 i}\, s\theta_{21 i}\, s\theta_{65 i}) - c\theta_{65 i}\, (\dot\theta_{21 i} + \dot\theta_{65 i}\, s\beta_{54 i})\, s\theta_{21 i}\, s\theta_{76 i} +$$

$$c\theta_{21 i}\, (\dot\theta_{76 i}\, c\theta_{65 i}\, c\theta_{76 i} - (\dot\theta_{65 i} + \dot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i}\, s\theta_{76 i})) +$$

$$c\beta_{54 i}\, (s\theta_{21}\, (c\theta_{76 i}\, (-\dot\theta_{21}\, s\theta_{21 i} + \dot\theta_{76 i}\, s\theta_{65 i}) + \dot\theta_{65 i}\, c\theta_{65 i}\, s\theta_{76 i}) +$$

$$c\theta_{21}\, (\dot\theta_{21 i}\, c\theta_{21 i}\, c\theta_{76 i} + (-\dot\theta_{76 i}\, s\theta_{21 i} + \dot\theta_{21}\, s\theta_{65 i})\, s\theta_{76 i})))) -$$

$$c(\beta_{1413 i} - \theta_{54})\, ((\ddot\theta_{43}\, c\theta_{43} - \dot\theta_{43}^{2}\, s\theta_{43})\, (c\theta_{21}\, (c\theta_{76 i}\, s\beta_{54 i} + c\beta_{54 i}\, s\theta_{65 i}\, s\theta_{76 i}) -$$

$$s\theta_{21}\, (c\beta_{54 i}\, c\theta_{76 i}\, s\theta_{21 i} + (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i})\, s\theta_{76 i})) +$$

$$2\dot\theta_{43}\, c\theta_{43}\, (-\dot\theta_{21}\, s\theta_{21}\, (c\theta_{76 i}\, s\beta_{54 i} + c\beta_{54 i}\, s\theta_{65 i}\, s\theta_{76 i}) - s\theta_{21}\, (\dot\theta_{21 i}\, c\beta_{54 i}\, c\theta_{21 i}\, c\theta_{76 i} +$$

$$\dot\theta_{76 i}\, c\theta_{76 i}\, (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i}) - \dot\theta_{76 i}\, c\beta_{54 i}\, s\theta_{21 i}\, s\theta_{76 i} -$$

$$(c\theta_{65 i}\, (\dot\theta_{21 i} + \dot\theta_{65 i}\, s\beta_{54 i})\, s\theta_{21 i} + c\theta_{21 i}\, (\dot\theta_{65 i} + \dot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i})\, s\theta_{76 i}) -$$

$$\dot\theta_{21}\, c\theta_{21}\, (c\beta_{54 i}\, c\theta_{76 i}\, s\theta_{21 i} + (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i})\, s\theta_{76 i}) +$$

$$c\theta_{21}\, (-\dot\theta_{76 i}\, s\beta_{54 i}\, s\theta_{76 i} + c\beta_{54 i}\, (\dot\theta_{76 i}\, c\theta_{76 i}\, s\theta_{65 i} + \dot\theta_{65 i}\, c\theta_{65 i}\, s\theta_{76 i}))) +$$

$$s\theta_{43}\, ((-\dot\theta_{21}^{2}\, c\theta_{21} - \ddot\theta_{21}\, s\theta_{21})\, (c\theta_{76 i}\, s\beta_{54 i} + c\beta_{54 i}\, s\theta_{65 i}\, s\theta_{76 i}) -$$

$$2\dot\theta_{21}\, c\theta_{21}\, (\dot\theta_{21 i}\, c\beta_{54 i}\, c\theta_{21 i}\, c\theta_{76 i} + \dot\theta_{76 i}\, c\theta_{76 i}\, (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i}) -$$

$$\dot\theta_{76 i}\, c\beta_{54 i}\, s\theta_{21 i}\, s\theta_{76 i} - (c\theta_{65 i}\, (\dot\theta_{21 i} + \dot\theta_{65 i}\, s\beta_{54 i})\, s\theta_{21 i} +$$

$$c\theta_{21 i}\, (\dot\theta_{65 i} + \dot\theta_{21 i}\, s\beta_{54 i})\, s\theta_{65 i})\, s\theta_{76 i}) + (-\ddot\theta_{21}\, c\theta_{21} + \dot\theta_{21}^{2}\, s\theta_{21})$$

$$(c\beta_{54 i}\, c\theta_{76 i}\, s\theta_{21 i} + (c\theta_{21 i}\, c\theta_{65 i} - s\beta_{54 i}\, s\theta_{21 i}\, s\theta_{65 i})\, s\theta_{76 i}) -$$

$$2\dot\theta_{21}\, s\theta_{21}\, (-\dot\theta_{76 i}\, s\beta_{54 i}\, s\theta_{76 i} + c\beta_{54 i}\, (\dot\theta_{76 i}\, c\theta_{76 i}\, s\theta_{65 i} + \dot\theta_{65 i}\, c\theta_{65 i}\, s\theta_{76 i})) +$$

$$c\theta_{21}\, (-s\beta_{54 i}\, (\dot\theta_{76 i}^{2}\, c\theta_{76 i} + \ddot\theta_{76 i}\, s\theta_{76 i}) + c\beta_{54 i}\, (c\theta_{65 i}\, (2\dot\theta_{65 i}\, \dot\theta_{76 i}\, c\theta_{76 i} +$$

$$\ddot\theta_{65 i}\, s\theta_{76 i}) + s\theta_{65 i}\, (\ddot\theta_{76 i}\, c\theta_{76 i} - (\dot\theta_{65 i}^{2} + \dot\theta_{76 i}^{2})\, s\theta_{76 i}))) -$$

$$s\theta_{21}\, (c\beta_{54 i}\, (-s\theta_{21 i}\, ((\dot\theta_{21 i}^{2} + \dot\theta_{76 i}^{2})\, c\theta_{76 i} + \ddot\theta_{76 i}\, s\theta_{76 i}) + c\theta_{21 i}$$

$$(\ddot\theta_{21 i}\, c\theta_{76 i} - 2\dot\theta_{21 i}\, \dot\theta_{76 i}\, s\theta_{76 i})) + c\theta_{21 i}$$

$$\left(-\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\left(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\right.$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\right.$$

$$\left.\left.\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\right.\right.$$

$$\left.\left(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(-\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\right.$$

$$\left.\left(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}\right)\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)))))-$$

$$\mathrm{s}\,\theta_{1110\,\mathrm{i}}\left(\left(-\dot{\theta}_{54}{}^{2}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})+\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\right)(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\right.$$

$$(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\left(\dot{\theta}_{43}{}^{2}\,\mathrm{c}\,\theta_{43}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$2\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\left(\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\right.$$

$$\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+$$

$$\left.\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}-\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)+$$

$$\left(-\ddot{\theta}_{43}\,\mathrm{c}\,\theta_{43}+\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\theta_{43}\right)\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})$$

$$\left(-\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)+\right.$$

$$\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\left(\ddot{\theta}_{65\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(-\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)+$$

$$\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(-\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)+\left(\ddot{\theta}_{21\,\mathrm{i}}+\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\right.$$

$$\left.\left.\left.\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(-\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\left(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}+\left(\dot{\theta}_{21\,\mathrm{i}}{}^{2}+\dot{\theta}_{65\,\mathrm{i}}{}^{2}+\dot{\theta}_{76\,\mathrm{i}}{}^{2}\right)\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)\right)\right)-$$

$$2\,\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,(-\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{s}\,\theta_{21}$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})$$

$$(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+$$

$$\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}-$$

$$\left(\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}})-$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\left(-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)))-$$

$$\mathrm{c}\,\theta_{43}\left(\left(-\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})-\dot{\theta}_{54}{}^{2}\,\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\right)(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,(\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}})+\right.$$

$$(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}))\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+$$

$$\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\left(\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{21}-\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\theta_{21}\right)+\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(-\mathrm{s}\,\theta_{21}\left(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\right)+\mathrm{c}\,\theta_{21}\left(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\left(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\,\mathrm{i}}{}^{2}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}\right)\right)\right)+$$

$$2\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\right.$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}+\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}+\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\right)\mathrm{s}\,\theta_{65\,\mathrm{i}}\right)-$$

$$\left.\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}})\right)-2\,\dot{\theta}_{76\,\mathrm{i}}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}+\right.$$

$$\left.\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\right)\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}+\left(\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\mathrm{c}\,\theta_{21}\left(2\,\dot{\theta}_{21}\,\dot{\theta}_{65\,\mathrm{i}}\right.\right.\right.$$

$$\mathrm{c}\,\theta_{65\,i} + \ddot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i}) + \mathrm{s}\,\theta_{21}\,(\ddot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i} - (\dot{\theta}_{21}^{\;2} + \dot{\theta}_{65\,i}^{\;2})\,\mathrm{s}\,\theta_{65\,i})) +$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{s}\,\theta_{21\,i}\,(2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) + \ddot{\theta}_{21}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}) +$$

$$\mathrm{c}\,\theta_{21\,i}\,(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{65\,i} + 2\,\dot{\theta}_{21}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})) -$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21}^{\;2} + \dot{\theta}_{21\,i}^{\;2} + \dot{\theta}_{65\,i}^{\;2} + 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) +$$

$$(\ddot{\theta}_{65\,i} + \ddot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}) + \mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{65\,i}\,(\ddot{\theta}_{21\,i} + \ddot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) -$$

$$(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i} + (\dot{\theta}_{21}^{\;2} + \dot{\theta}_{21\,i}^{\;2} + \dot{\theta}_{65\,i}^{\;2})\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})))\,\mathrm{s}\,\theta_{76\,i} -$$

$$(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i})\,(\dot{\theta}_{76\,i}^{\;2}\,\mathrm{c}\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))$$

$$(\ddot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i} - \dot{\theta}_{76\,i}^{\;2}\,\mathrm{s}\,\theta_{76\,i})) - 2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})$$

$$(-\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{21} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) +$$

$$2\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\,(-\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{76\,i}\,(\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}) +$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} + \mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i} - \dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}))\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{21} - \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - (\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{65\,i}) + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + (-\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})))) -$$

$$\mathrm{s}\,\theta_{43}\,((-\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54}) - \dot{\theta}_{54}^{\;2}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54}))\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\mathrm{s}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}$$

$$(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) - \dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} -$$

$$(\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}))) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,((-\dot{\theta}_{21}^{\;2}\,\mathrm{c}\,\theta_{21} - \ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21})\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) -$$

$$2\,\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\dot{\theta}_{21\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) -$$

$$\dot{\theta}_{76\,i}\,\mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i} - (\mathrm{c}\,\theta_{65\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{21\,i} +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) + (-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{21} + \dot{\theta}_{21}^{\;2}\,\mathrm{s}\,\theta_{21})$$

$$(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + (\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}) -$$

$$2\,\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(-\dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i} + \mathrm{c}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{21}\,(-\mathrm{s}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}^{\;2}\,\mathrm{c}\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{65\,i}\,(2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{76\,i} +$$

$$\ddot{\theta}_{65\,i}\,s\,\theta_{76\,i}) + s\,\theta_{65\,i}(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i} - (\dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,s\,\theta_{76\,i}))) -$$

$$s\,\theta_{21}(c\,\beta_{54\,i}(-s\,\theta_{21\,i}(((\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,c\,\theta_{76\,i} + \ddot{\theta}_{76\,i}\,s\,\theta_{76\,i}) + c\,\theta_{21\,i}$$

$$(\ddot{\theta}_{21\,i}\,c\,\theta_{76\,i} - 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,s\,\theta_{76\,i})) + c\,\theta_{21\,i}$$

$$(-s\,\theta_{65\,i}(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,s\,\beta_{54\,i}) + (\ddot{\theta}_{65\,i} + \ddot{\theta}_{21\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{65\,i}(\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i} - (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2} + 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,s\,\beta_{54\,i})$$

$$s\,\theta_{76\,i})) + s\,\theta_{21\,i}(-c\,\theta_{65\,i}(2\,\dot{\theta}_{76\,i}\,c\,\theta_{76\,i}(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,s\,\beta_{54\,i}) +$$

$$(\ddot{\theta}_{21\,i} + \ddot{\theta}_{65\,i}\,s\,\beta_{54\,i})\,s\,\theta_{76\,i}) + s\,\theta_{65\,i}(-\ddot{\theta}_{76\,i}\,c\,\theta_{76\,i}\,s\,\beta_{54\,i} +$$

$$(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i} + (\dot{\theta}_{21\,i}{}^{2} + \dot{\theta}_{65\,i}{}^{2} + \dot{\theta}_{76\,i}{}^{2})\,s\,\beta_{54\,i})\,s\,\theta_{76\,i})))))))) -$$

$$(s\,\theta_{109\,i}(-s\,\theta_{43}(c\,\theta_{76\,i}(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) + (c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}(c\,\theta_{21\,i}\,c\,\theta_{65\,i} -$$

$$s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{43}(c\,\theta_{21}(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) - s\,\theta_{21}(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} +$$

$$(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) +$$

$$c\,\theta_{109\,i}(c\,\theta_{1110\,i}(s(\beta_{1413\,i} - \theta_{54})(-c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} + (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,c(\beta_{1413\,i} - \theta_{54})(c\,\theta_{76\,i}(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$c(\beta_{1413\,i} - \theta_{54})\,s\,\theta_{43}(c\,\theta_{21}(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}))) -$$

$$s\,\theta_{1110\,i}(c(\beta_{1413\,i} - \theta_{54})(c\,\beta_{54\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i} - (c\,\theta_{65\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i}) -$$

$$c\,\theta_{43}\,s(\beta_{1413\,i} - \theta_{54})(c\,\theta_{76\,i}(s\,\beta_{54\,i}\,s\,\theta_{21} + c\,\beta_{54\,i}\,c\,\theta_{21}\,s\,\theta_{21\,i}) +$$

$$(c\,\beta_{54\,i}\,s\,\theta_{21}\,s\,\theta_{65\,i} + c\,\theta_{21}(c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}))\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{43}\,s(\beta_{1413\,i} - \theta_{54})(c\,\theta_{21}(c\,\theta_{76\,i}\,s\,\beta_{54\,i} + c\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}) -$$

$$s\,\theta_{21}(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + (c\,\theta_{21\,i}\,c\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i})\,s\,\theta_{76\,i})))))$$

$$(s\,\theta_{109\,i}(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21} + \theta_{43}) - s\,\beta_{54\,i}(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} +$$

$$c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i}) -$$

$$c\,\beta_{54\,i}(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} +$$

$$c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{109\,i}(c\,\theta_{1110\,i}(-s(\beta_{1413\,i} - \theta_{54})(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$c(\beta_{1413\,i} - \theta_{54})\,s\,\theta_{43}(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21} + s\,\beta_{54\,i}(c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} - c\,\theta_{21}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}(c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i})) +$$

$$c\,\theta_{43}\,c(\beta_{1413\,i} - \theta_{54})(s\,\theta_{21}(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$c\,\theta_{21}(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - s\,\theta_{21\,i}(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})))) -$$

$$s\,\theta_{1110\,i}(c(\beta_{1413\,i} - \theta_{54})(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i} + c\,\theta_{21\,i}(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s(\beta_{1413\,i} - \theta_{54})(c\,\theta_{21}(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - c\,\theta_{43}\,s\,\theta_{21\,i}(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i} + c\,\beta_{54\,i}\,s\,\theta_{76\,i}) +$$

$$s\,\theta_{43}(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - s\,\beta_{54\,i}\,s\,\theta_{76\,i})) +$$

$$s\,\theta_{21}(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43} + s\,\beta_{54\,i}(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{43}\,s\,\theta_{76\,i}) +$$

$$c\,\beta_{54\,i}(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} + s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))))))$$

$$((\ddot{\theta}_{109\,i}\,c\,\theta_{109\,i} - \dot{\theta}_{109\,i}{}^{2}\,s\,\theta_{109\,i})(c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s(\theta_{21} + \theta_{43}) - s\,\beta_{54\,i}(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i} +$$

$$c\,\theta_{21}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i} - c\,\theta_{21}\,c\,\theta_{43}\,s\,\theta_{76\,i} + s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{76\,i}) -$$

$$c\,\beta_{54\,i}(c\,\theta_{21}\,c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i} - c\,\theta_{76\,i}\,s\,\theta_{21}\,s\,\theta_{43}\,s\,\theta_{65\,i} + c\,\theta_{43}\,s\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i} +$$

$$c\,\theta_{21}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})) +$$

$$2\,\dot{\theta}_{109\,i}\,c\,\theta_{109\,i}((\dot{\theta}_{21} + \dot{\theta}_{43})\,c\,\theta_{21\,i}\,c(\theta_{21} + \theta_{43})\,c\,\theta_{65\,i}\,c\,\theta_{76\,i} - \dot{\theta}_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}$$

$$
\begin{aligned}
&\mathrm{s}(\theta_{21}+\theta_{43})-\dot{\theta}_{65\ \mathrm{i}}\\
&\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\\
&\mathrm{s}\,\theta_{65\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\\
&\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{76\ \mathrm{i}}-\\
&\mathrm{c}\,\beta_{54\ \mathrm{i}}\Big(\mathrm{c}\,\theta_{21}\Big(\mathrm{s}\,\theta_{43}\Big(\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad\mathrm{c}\,\theta_{43}\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\Big(\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)+\\
&\quad\mathrm{s}\,\theta_{21}\Big(\mathrm{c}\,\theta_{43}\Big(\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)-\\
&\quad\mathrm{s}\,\theta_{43}\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\Big(\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)\Big)-\\
&\mathrm{s}\,\beta_{54\ \mathrm{i}}\Big(\mathrm{s}\,\theta_{21}\Big(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}\Big(\dot{\theta}_{76\ \mathrm{i}}-\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\mathrm{c}\,\theta_{43}\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\\
&\quad\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\Big(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)+\\
&\quad\mathrm{c}\,\theta_{21}\Big(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(-\dot{\theta}_{76\ \mathrm{i}}+\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\mathrm{s}\,\theta_{43}\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\\
&\quad\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\Big(\dot{\theta}_{21}+\dot{\theta}_{43}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)\Big)\Big)+\\
&\mathrm{s}\,\theta_{109\ \mathrm{i}}\Big(-\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\Big(\mathrm{c}\,\theta_{21\ \mathrm{i}}\Big(\Big(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\mathrm{s}\,\theta_{21\ \mathrm{i}}\\
&\quad\Big(\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}-2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\Big)+\\
&2\Big(-\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\Big(\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\mathrm{c}(\theta_{21}+\theta_{43})\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\\
&\Big(-\mathrm{s}(\theta_{21}+\theta_{43})\Big(\Big(\dot{\theta}_{21}{}^{2}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}+\dot{\theta}_{43}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad\mathrm{c}(\theta_{21}+\theta_{43})\Big(\Big(\ddot{\theta}_{21}+\ddot{\theta}_{43}\Big)\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-2\,\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)-\\
&\mathrm{c}\,\beta_{54\ \mathrm{i}}\Big(-\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(\mathrm{c}\,\theta_{21}\Big(\Big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{43}{}^{2}\Big)\,\mathrm{c}\,\theta_{43}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\Big)+\mathrm{s}\,\theta_{21}\Big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{43}-2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\Big)\Big)\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\\
&\quad\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\Big(-\mathrm{c}\,\theta_{43}\Big(2\,\dot{\theta}_{43}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)+\mathrm{s}\,\theta_{43}\Big(-\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\Big(\dot{\theta}_{43}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\Big)-\\
&\quad\Big(\mathrm{s}\,\theta_{21}\Big(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big)+\mathrm{c}\,\theta_{21}\Big(-\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}+\Big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\ \mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big)\Big)\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}+\\
&\quad\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big(-\mathrm{s}\,\theta_{21}\Big(\Big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{43}{}^{2}\Big)\,\mathrm{c}\,\theta_{43}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\Big)+\mathrm{c}\,\theta_{21}\Big(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{43}-2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\Big)\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}+\\
&\quad 2\Big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}-\dot{\theta}_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)\Big(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad 2\Big(\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big)\Big(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}+\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad 2\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}+\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\Big(-\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)-\\
&\quad 2\Big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{43}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\Big)\Big(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big(\mathrm{s}\,\theta_{21}\Big(\Big(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\mathrm{c}\,\theta_{21}\Big(-\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+2\,\dot{\theta}_{21}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)+\\
&\quad\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\Big(-\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big(\Big(\dot{\theta}_{65\ \mathrm{i}}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\Big)\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\mathrm{c}\,\theta_{65\ \mathrm{i}}\Big(\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-2\,\dot{\theta}_{65\ \mathrm{i}}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)+\\
&\quad\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\Big(\mathrm{c}\,\theta_{21\ \mathrm{i}}\Big(2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big(\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\Big(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)+\\
&\quad\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\Big(\mathrm{c}\,\theta_{43}\Big(2\,\dot{\theta}_{43}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{43}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\mathrm{s}\,\theta_{43}\Big(\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\Big(\dot{\theta}_{43}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)\Big)\Big)-\\
&\mathrm{s}\,\beta_{54\ \mathrm{i}}\Big(\mathrm{c}\,\theta_{21}\Big(-\mathrm{s}\,\theta_{43}\Big(\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(-2\,\dot{\theta}_{21}\,\dot{\theta}_{76\ \mathrm{i}}-2\,\dot{\theta}_{43}\,\dot{\theta}_{76\ \mathrm{i}}-\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\\
&\quad\mathrm{s}\,\theta_{65\ \mathrm{i}}+2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\\
&\quad\dot{\theta}_{76\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{c}\,\theta_{21\ \mathrm{i}}\Big(2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\Big)-\Big(\ddot{\theta}_{21}+\ddot{\theta}_{43}-\\
&\quad 2\,\dot{\theta}_{65\ \mathrm{i}}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\Big)\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\Big)+\\
&\quad\mathrm{c}\,\theta_{43}\Big(\mathrm{c}\,\theta_{76\ \mathrm{i}}\Big(-\ddot{\theta}_{76\ \mathrm{i}}+2\,\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+2\,\dot{\theta}_{21\ \mathrm{i}}\big(\dot{\theta}_{21}+\dot{\theta}_{43}\big)
\end{aligned}
$$

$$\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \ddot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) +$$

$$(\dot{\theta}_{21}{}^{2} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}{}^{2} + \dot{\theta}_{76\,i}{}^{2} - 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(-2\,\dot{\theta}_{21}\,\dot{\theta}_{76\,i} - 2\,\dot{\theta}_{43}\,\dot{\theta}_{76\,i} - \ddot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} + \dot{\theta}_{21}{}^{2}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} +$$

$$\dot{\theta}_{21\,i}{}^{2}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{43}{}^{2}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{65\,i}{}^{2}\,\mathrm{s}\,\theta_{21\,i}$$

$$\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{76\,i}{}^{2}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{21\,i}\,(2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i} + \ddot{\theta}_{21\,i}\,\mathrm{s}\,\theta_{65\,i})) -$$

$$(\ddot{\theta}_{21} + \ddot{\theta}_{43} - 2\,\dot{\theta}_{65\,i}\,\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} - 2\,\dot{\theta}_{21\,i}\,\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} - \ddot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})$$

$$\mathrm{s}\,\theta_{76\,i}) + \mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\theta_{76\,i}\,(-\ddot{\theta}_{76\,i} + 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{65\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{21\,i} +$$

$$2\,\dot{\theta}_{21\,i}\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} + \ddot{\theta}_{43}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i}) +$$

$$(\dot{\theta}_{21}{}^{2} + 2\,\dot{\theta}_{21}\,\dot{\theta}_{43} + \dot{\theta}_{43}{}^{2} + \dot{\theta}_{76\,i}{}^{2} - 2\,(\dot{\theta}_{21} + \dot{\theta}_{43})\,\dot{\theta}_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i})\,\mathrm{s}\,\theta_{76\,i}))))) +$$

$$(-\dot{\theta}_{109\,i}{}^{2}\,\mathrm{c}\,\theta_{109\,i} - \ddot{\theta}_{109\,i}\,\mathrm{s}\,\theta_{109\,i})\,(\mathrm{c}\,\theta_{1110\,i}\,(-\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}$$

$$(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) + \mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}$$

$$(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})))) -$$

$$\mathrm{s}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43} + \mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}))))) - 2\,\dot{\theta}_{109\,i}\,\mathrm{s}\,\theta_{109\,i}$$

$$(-\dot{\theta}_{1110\,i}\,\mathrm{s}\,\theta_{1110\,i}\,(-\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})))) -$$

$$\dot{\theta}_{1110\,i}\,\mathrm{c}\,\theta_{1110\,i}\,(\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{43} + \mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,i})))) +$$

$$\mathrm{c}\,\theta_{1110\,i}\,(\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21\,i} + \mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{s}\,\beta_{54\,i}$$

$$(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) +$$

$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21} + \mathrm{s}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{65\,i} -$$

$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,i}) + \mathrm{c}\,\beta_{54\,i}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\mathrm{s}(\beta_{1413\,i} - \theta_{54})\,(-\mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{65\,i} + \dot{\theta}_{21\,i}\,\mathrm{s}\,\beta_{54\,i})\,\mathrm{s}\,\theta_{65\,i} + \dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\beta_{54\,i}\,(\dot{\theta}_{76\,i}\,\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{76\,i} - \dot{\theta}_{21\,i}\,\mathrm{s}\,\theta_{21\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i}\,(\dot{\theta}_{21\,i} + \dot{\theta}_{65\,i}\,\mathrm{s}\,\beta_{54\,i}) - \dot{\theta}_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i}\,\mathrm{s}\,\theta_{76\,i})) -$$

$$\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\,i} - \theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,i}\,\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\theta_{65\,i} - \mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}) +$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,i}\,\mathrm{c}\,\theta_{65\,i}\,\mathrm{c}\,\theta_{76\,i} - \mathrm{s}\,\theta_{21\,i}\,(\mathrm{c}\,\theta_{76\,i}\,\mathrm{s}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{65\,i} + \mathrm{c}\,\beta_{54\,i}\,\mathrm{s}\,\theta_{76\,i}))) +$$

$$\dot{\theta}_{54}\,c\,\theta_{43}\,s(\beta_{1413\ i}-\theta_{54})\,(s\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{76\ i})+$$
$$c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-s\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+c\,\beta_{54\ i}\,s\,\theta_{76\ i})))+$$
$$c\,\theta_{43}\,c(\beta_{1413\ i}-\theta_{54})\left(\dot{\theta}_{21}\,c\,\theta_{21}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{76\ i})-\right.$$
$$\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-s\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+c\,\beta_{54\ i}\,s\,\theta_{76\ i}))-$$
$$c\,\theta_{21}\left(c\,\beta_{54\ i}\left(\dot{\theta}_{76\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+\dot{\theta}_{21\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+\right.$$
$$c\,\theta_{65\ i}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{21\ i}+\dot{\theta}_{76\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+$$
$$\left.s\,\theta_{65\ i}\left(c\,\theta_{21\ i}\,c\,\theta_{76\ i}\left(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\,s\,\beta_{54\ i}\right)-\dot{\theta}_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)\right)-$$
$$\left.s\,\theta_{21}\left(\dot{\theta}_{76\ i}\,c\,\theta_{76\ i}\,s\,\beta_{54\ i}+c\,\beta_{54\ i}\left(-\dot{\theta}_{65\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}+\dot{\theta}_{76\ i}\,s\,\theta_{65\ i}\,s\,\theta_{76\ i}\right)\right)\right)+$$
$$c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}\left(-c\,\theta_{21}\,c\,\theta_{76\ i}\left(\dot{\theta}_{21}\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\left(\dot{\theta}_{76\ i}-\dot{\theta}_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)\right)+\right.$$
$$s\,\theta_{21}\left(c\,\theta_{21\ i}\,c\,\theta_{76\ i}\left(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{65\ i}+s\,\beta_{54\ i}\left(\dot{\theta}_{21}-\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}+\right.$$
$$\left.c\,\theta_{65\ i}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{21\ i}+\dot{\theta}_{76\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)\right)+$$
$$c\,\beta_{54\ i}\left(s\,\theta_{21}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}-\dot{\theta}_{21}\,s\,\theta_{65\ i}\right)+\dot{\theta}_{21\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+\right.$$
$$\left.\left.\left.c\,\theta_{21}\left(\dot{\theta}_{65\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}+\left(\dot{\theta}_{21}\,s\,\theta_{21\ i}-\dot{\theta}_{76\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}\right)\right)\right)\right)-$$
$$s\,\theta_{1110\ i}\left(\dot{\theta}_{54}\,s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+c\,\beta_{54\ i}\,s\,\theta_{76\ i}))+\right.$$
$$c(\beta_{1413\ i}-\theta_{54})\left(-s\,\theta_{21\ i}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{65\ i}+\dot{\theta}_{76\ i}\,c\,\theta_{65\ i}\,s\,\theta_{76\ i}\right)+\right.$$
$$c\,\beta_{54\ i}\left(\dot{\theta}_{76\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}-\dot{\theta}_{21\ i}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+$$
$$\left.c\,\theta_{21\ i}\left(c\,\theta_{65\ i}\,c\,\theta_{76\ i}\left(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\,s\,\beta_{54\ i}\right)-\dot{\theta}_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}\,s\,\theta_{76\ i}\right)\right)-$$
$$\dot{\theta}_{54}\,c(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{43}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-c\,\theta_{43}\,s\,\theta_{21\ i}$$
$$(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+c\,\beta_{54\ i}\,s\,\theta_{76\ i})+s\,\theta_{43}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{76\ i}))+$$
$$s\,\theta_{21}\,(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{43}+s\,\beta_{54\ i}\,(c\,\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{65\ i}-c\,\theta_{43}\,s\,\theta_{76\ i})+$$
$$c\,\beta_{54\ i}\,(c\,\theta_{43}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}+s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{76\ i})))+$$
$$s(\beta_{1413\ i}-\theta_{54})\left(-\dot{\theta}_{21}\,s\,\theta_{21}\,(c\,\theta_{21\ i}\,c\,\theta_{43}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}-c\,\theta_{43}\,s\,\theta_{21\ i}\,(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+\right.$$
$$c\,\beta_{54\ i}\,s\,\theta_{76\ i})+s\,\theta_{43}\,(c\,\beta_{54\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}-s\,\beta_{54\ i}\,s\,\theta_{76\ i}))+$$
$$\dot{\theta}_{21}\,c\,\theta_{21}\,(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{43}+s\,\beta_{54\ i}\,(c\,\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{65\ i}-c\,\theta_{43}\,s\,\theta_{76\ i})+$$
$$c\,\beta_{54\ i}\,(c\,\theta_{43}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}+s\,\theta_{21\ i}\,s\,\theta_{43}\,s\,\theta_{76\ i}))+$$
$$c\,\theta_{21}\left(-c\,\theta_{76\ i}\,s\,\theta_{43}\left(\dot{\theta}_{43}\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}+s\,\beta_{54\ i}\left(\dot{\theta}_{76\ i}-\dot{\theta}_{43}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)\right)-\right.$$
$$c\,\theta_{43}\left(c\,\theta_{21\ i}\,c\,\theta_{76\ i}\left(\dot{\theta}_{65\ i}+\dot{\theta}_{21\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{65\ i}+s\,\beta_{54\ i}\left(\dot{\theta}_{43}-\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}+\right.$$
$$\left.c\,\theta_{65\ i}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{21\ i}+\dot{\theta}_{65\ i}\,s\,\beta_{54\ i}\right)s\,\theta_{21\ i}+\dot{\theta}_{76\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)\right)+$$
$$c\,\beta_{54\ i}\left(-c\,\theta_{43}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}-\dot{\theta}_{43}\,s\,\theta_{65\ i}\right)+\dot{\theta}_{21\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+\right.$$
$$\left.\left.s\,\theta_{43}\left(\dot{\theta}_{65\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}+\left(\dot{\theta}_{43}\,s\,\theta_{21\ i}-\dot{\theta}_{76\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}\right)\right)\right)+$$
$$s\,\theta_{21}\left(-\dot{\theta}_{43}\,c\,\theta_{21\ i}\,c\,\theta_{43}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}+\dot{\theta}_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{43}+\right.$$
$$\dot{\theta}_{65\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}\,s\,\theta_{43}\,s\,\theta_{65\ i}+\dot{\theta}_{76\ i}\,c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,s\,\theta_{43}\,s\,\theta_{76\ i}+$$
$$c\,\beta_{54\ i}\left(s\,\theta_{43}\left(c\,\theta_{76\ i}\left(\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}-\dot{\theta}_{43}\,s\,\theta_{65\ i}\right)+\dot{\theta}_{21\ i}\,c\,\theta_{21\ i}\,s\,\theta_{76\ i}\right)+\right.$$
$$\left.c\,\theta_{43}\left(\dot{\theta}_{65\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}+\left(\dot{\theta}_{43}\,s\,\theta_{21\ i}-\dot{\theta}_{76\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}\right)\right)+$$
$$s\,\beta_{54\ i}\left(c\,\theta_{43}\,c\,\theta_{76\ i}\left(-\dot{\theta}_{76\ i}+\dot{\theta}_{43}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)+s\,\theta_{43}\left(\dot{\theta}_{65\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+\right.\right.$$
$$\left.\left.\left.\left.\left.\dot{\theta}_{21\ i}\,c\,\theta_{21\ i}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}+\left(\dot{\theta}_{43}-\dot{\theta}_{76\ i}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}\right)s\,\theta_{76\ i}\right)\right)\right)\right)\right)\right)+$$
$$c\,\theta_{109\ i}\left(\left(-\dot{\theta}_{1110\ i}{}^{2}\,c\,\theta_{1110\ i}-\ddot{\theta}_{1110\ i}\,s\,\theta_{1110\ i}\right)(-s(\beta_{1413\ i}-\theta_{54})\,(c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21\ i}+c\,\theta_{21\ i}\right.$$
$$(c\,\theta_{76\ i}\,s\,\beta_{54\ i}\,s\,\theta_{65\ i}+c\,\beta_{54\ i}\,s\,\theta_{76\ i}))+c(\beta_{1413\ i}-\theta_{54})\,s\,\theta_{43}$$
$$(-c\,\theta_{21\ i}\,c\,\theta_{65\ i}\,c\,\theta_{76\ i}\,s\,\theta_{21}+s\,\beta_{54\ i}\,(c\,\theta_{76\ i}\,s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{65\ i}-c\,\theta_{21}\,s\,\theta_{76\ i})+$$
$$c\,\beta_{54\ i}\,(c\,\theta_{21}\,c\,\theta_{76\ i}\,s\,\theta_{65\ i}+s\,\theta_{21}\,s\,\theta_{21\ i}\,s\,\theta_{76\ i}))+$$

$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))))+$$
$$\left(-\ddot{\theta}_{1110\ \mathrm{i}}\,\mathrm{c}\,\theta_{1110\ \mathrm{i}}+\dot{\theta}_{1110\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{1110\ \mathrm{i}}\right)(\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+$$
$$\mathrm{c}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+$$
$$\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+$$
$$\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))))-2\,\dot{\theta}_{1110\ \mathrm{i}}\,\mathrm{s}\,\theta_{1110\ \mathrm{i}}$$
$$\left(\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\mathrm{c}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+\right.$$
$$\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(-\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\ \mathrm{i}}$$
$$(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+$$
$$\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(-\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-$$
$$\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))-$$
$$\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\left(-\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$
$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+$$
$$\left.\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)-$$
$$\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})))+$$
$$\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})))+$$
$$\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})-\right.$$
$$\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))-$$
$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$
$$\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+$$
$$\left.\mathrm{s}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)-$$
$$\left.\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(-\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\right)+$$
$$\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\left(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\right)+\right.$$
$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{21}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}+\right.$$
$$\left.\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)+$$
$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$
$$\left.\left.\left.\mathrm{c}\,\theta_{21}\left(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\left(\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\right)\right)+$$
$$\mathrm{c}\,\theta_{1110\ \mathrm{i}}\left(\left(\ddot{\theta}_{54}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\dot{\theta}_{54}{}^{2}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)(\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\right.$$
$$\mathrm{c}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+$$
$$\left(-\mathrm{s}\,\theta_{43}\left(\left(\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})-\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)+\right.$$
$$\left.\mathrm{c}\,\theta_{43}\left(\ddot{\theta}_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+2\,\dot{\theta}_{43}\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)\right)$$
$$(-\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$
$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))+$$
$$2\,\dot{\theta}_{54}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\left(-\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$
$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+$$
$$\left.\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)+$$

$$\Big(-\mathrm{c}\,\theta_{43}\left(\left(\dot{\theta}_{43}{}^{2}+\dot{\theta}_{54}{}^{2}\right)\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})-\ddot{\theta}_{54}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)-\mathrm{s}\,\theta_{43}\left(\ddot{\theta}_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\right.$$

$$\left.\left.2\,\dot{\theta}_{43}\,\dot{\theta}_{54}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)\Big)\right(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})+$$

$$\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})))-\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})$$

$$\Big(\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(-\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\left(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)-$$

$$\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}+2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\ddot{\theta}_{65\ \mathrm{i}}+\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-2\,\dot{\theta}_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)+$$

$$\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(-\mathrm{s}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}+\left(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\right)\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$

$$\left.\left.\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\ddot{\theta}_{21\ \mathrm{i}}+\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-2\,\dot{\theta}_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\Big)+$$

$$2\left(-\dot{\theta}_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}+\dot{\theta}_{54}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)$$

$$\Big(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}-\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}})-$$

$$\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\mathrm{s}\,\theta_{21\ \mathrm{i}}\,(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}))-$$

$$\mathrm{c}\,\theta_{21}\left(\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$

$$\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+$$

$$\left.\mathrm{s}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)-$$

$$\mathrm{s}\,\theta_{21}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}+\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(-\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\Big)+$$

$$2\left(\dot{\theta}_{43}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})+\dot{\theta}_{54}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}(\beta_{1413\ \mathrm{i}}-\theta_{54})\right)$$

$$\Big(-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\right)+$$

$$\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{65\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}+\mathrm{s}\,\beta_{54\ \mathrm{i}}\left(\dot{\theta}_{21}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}+\right.$$

$$\left.\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\beta_{54\ \mathrm{i}}\right)\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)+$$

$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\left(\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{21}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\right.$$

$$\left.\mathrm{c}\,\theta_{21}\left(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\left(\dot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\Big)+$$

$$\mathrm{c}(\beta_{1413\ \mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\left(\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(\left(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{65\ \mathrm{i}}+\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)+\right.\right.$$

$$\left.\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}-2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\right)+$$

$$2\left(\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)-\mathrm{c}\,\theta_{21\ \mathrm{i}}$$

$$\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(-\mathrm{s}\,\theta_{21}\left(\left(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\mathrm{c}\,\theta_{21}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-2\,\dot{\theta}_{21}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)+$$

$$\mathrm{s}\,\beta_{54\ \mathrm{i}}\Big(2\,\dot{\theta}_{21}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\left(\mathrm{c}\,\theta_{21\ \mathrm{i}}\left(2\,\dot{\theta}_{21\ \mathrm{i}}\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}+\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}-\left(\dot{\theta}_{21\ \mathrm{i}}{}^{2}+\dot{\theta}_{65\ \mathrm{i}}{}^{2}\right)\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)\right)+\left(\dot{\theta}_{21}{}^{2}\,\mathrm{c}\,\theta_{21}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21}\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}+$$

$$\mathrm{c}\,\theta_{21}\left(-\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+2\left(\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}+\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)$$

$$\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\mathrm{s}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}$$

$$\left.\left(-\mathrm{s}\,\theta_{21}\left(\left(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{76\ \mathrm{i}}{}^{2}\right)\mathrm{c}\,\theta_{76\ \mathrm{i}}+\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)+\mathrm{c}\,\theta_{21}\left(\ddot{\theta}_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-2\,\dot{\theta}_{21}\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)\right)\Big)\right)+$$

$$\mathrm{c}\,\beta_{54\ \mathrm{i}}\Big(2\,\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\left(\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{21}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\right)+\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}$$

$$\left(\ddot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}-\dot{\theta}_{65\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{65\ \mathrm{i}}\right)+\left(\mathrm{c}\,\theta_{21}\left(2\,\dot{\theta}_{21}\,\dot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}+\ddot{\theta}_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\right)+\right.$$

$$\left.\mathrm{s}\,\theta_{21}\left(\ddot{\theta}_{21\ \mathrm{i}}\,\mathrm{c}\,\theta_{21\ \mathrm{i}}-\left(\dot{\theta}_{21}{}^{2}+\dot{\theta}_{21\ \mathrm{i}}{}^{2}\right)\mathrm{s}\,\theta_{21\ \mathrm{i}}\right)\right)\mathrm{s}\,\theta_{76\ \mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\ \mathrm{i}}\left(\ddot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}-\right.$$

$$\left.\dot{\theta}_{76\ \mathrm{i}}{}^{2}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)-2\,\dot{\theta}_{65\ \mathrm{i}}\,\mathrm{c}\,\theta_{65\ \mathrm{i}}\left(\dot{\theta}_{21}\,\mathrm{c}\,\theta_{76\ \mathrm{i}}\,\mathrm{s}\,\theta_{21}+\dot{\theta}_{76\ \mathrm{i}}\,\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\ \mathrm{i}}\right)-\mathrm{s}\,\theta_{65\ \mathrm{i}}$$

$$(c\,\theta_{21}((\dot\theta_{21}^2+\dot\theta_{76\,i}^2)c\,\theta_{76\,i}+\ddot\theta_{76\,i}s\,\theta_{76\,i})+s\,\theta_{21}(\ddot\theta_{21}c\,\theta_{76\,i}-2\dot\theta_{21}\dot\theta_{76\,i}s\,\theta_{76\,i})))))+$$

$$c\,\theta_{43}\,c(\beta_{1413\,i}-\theta_{54})((\ddot\theta_{21}c\,\theta_{21}-\dot\theta_{21}^2 s\,\theta_{21})(c\,\beta_{54\,i}c\,\theta_{76\,i}s\,\theta_{65\,i}-s\,\beta_{54\,i}s\,\theta_{76\,i})-$$

$$(\dot\theta_{21}^2 c\,\theta_{21}+\ddot\theta_{21}s\,\theta_{21})(c\,\theta_{21\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}-s\,\theta_{21\,i}(c\,\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{65\,i}+c\,\beta_{54\,i}s\,\theta_{76\,i}))+$$

$$2\dot\theta_{21}s\,\theta_{21}(c\,\beta_{54\,i}(\dot\theta_{76\,i}c\,\theta_{76\,i}s\,\theta_{21\,i}+\dot\theta_{21\,i}c\,\theta_{21\,i}s\,\theta_{76\,i})+$$

$$c\,\theta_{65\,i}(c\,\theta_{76\,i}(\dot\theta_{21\,i}+\dot\theta_{65\,i}s\,\beta_{54\,i})s\,\theta_{21\,i}+\dot\theta_{76\,i}c\,\theta_{21\,i}s\,\theta_{76\,i})+$$

$$s\,\theta_{65\,i}(c\,\theta_{21\,i}c\,\theta_{76\,i}(\dot\theta_{65\,i}+\dot\theta_{21\,i}s\,\beta_{54\,i})-\dot\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{21\,i}s\,\theta_{76\,i}))-$$

$$2\dot\theta_{21}c\,\theta_{21}(\dot\theta_{76\,i}c\,\theta_{76\,i}s\,\beta_{54\,i}+c\,\beta_{54\,i}(-\dot\theta_{65\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}+\dot\theta_{76\,i}s\,\theta_{65\,i}s\,\theta_{76\,i}))+$$

$$s\,\theta_{21}(s\,\beta_{54\,i}(-\ddot\theta_{76\,i}c\,\theta_{76\,i}+\dot\theta_{76\,i}^2 s\,\theta_{76\,i})+c\,\beta_{54\,i}(-s\,\theta_{65\,i}((\dot\theta_{65\,i}^2+\dot\theta_{76\,i}^2)c\,\theta_{76\,i}+$$

$$\ddot\theta_{76\,i}s\,\theta_{76\,i})+c\,\theta_{65\,i}(\ddot\theta_{65\,i}c\,\theta_{76\,i}-2\dot\theta_{65\,i}\dot\theta_{76\,i}s\,\theta_{76\,i})))+$$

$$c\,\theta_{21}(c\,\beta_{54\,i}(-c\,\theta_{21\,i}(2\dot\theta_{21\,i}\dot\theta_{76\,i}c\,\theta_{76\,i}+\ddot\theta_{21\,i}s\,\theta_{76\,i})+s\,\theta_{21\,i}$$

$$(-\ddot\theta_{76\,i}c\,\theta_{76\,i}+(\dot\theta_{21\,i}^2+\dot\theta_{76\,i}^2)s\,\theta_{76\,i}))-$$

$$c\,\theta_{21\,i}(c\,\theta_{65\,i}(c\,\theta_{76\,i}(\dot\theta_{21\,i}^2+\dot\theta_{65\,i}^2+\dot\theta_{76\,i}^2+2\dot\theta_{21\,i}\dot\theta_{65\,i}s\,\beta_{54\,i})+\ddot\theta_{76\,i}s\,\theta_{76\,i})+$$

$$s\,\theta_{65\,i}(c\,\theta_{76\,i}(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}s\,\beta_{54\,i})-2\dot\theta_{76\,i}(\dot\theta_{65\,i}+\dot\theta_{21\,i}s\,\beta_{54\,i})s\,\theta_{76\,i}))+s\,\theta_{21\,i}$$

$$(s\,\theta_{65\,i}(c\,\theta_{76\,i}(2\dot\theta_{21\,i}\dot\theta_{65\,i}+(\dot\theta_{21\,i}^2+\dot\theta_{65\,i}^2+\dot\theta_{76\,i}^2)s\,\beta_{54\,i})+\ddot\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{76\,i})-$$

$$c\,\theta_{65\,i}(c\,\theta_{76\,i}(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}s\,\beta_{54\,i})-2\dot\theta_{76\,i}(\dot\theta_{21\,i}+\dot\theta_{65\,i}s\,\beta_{54\,i})s\,\theta_{76\,i})))))-$$

$$2\dot\theta_{1110\,i}c\,\theta_{1110\,i}(\dot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})(c\,\theta_{65\,i}c\,\theta_{76\,i}s\,\theta_{21\,i}+c\,\theta_{21\,i}(c\,\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{65\,i}+c\,\beta_{54\,i}s\,\theta_{76\,i}))+$$

$$c(\beta_{1413\,i}-\theta_{54})(-s\,\theta_{21\,i}(c\,\theta_{76\,i}(\dot\theta_{65\,i}+\dot\theta_{21\,i}s\,\beta_{54\,i})s\,\theta_{65\,i}+\dot\theta_{76\,i}c\,\theta_{65\,i}s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}(\dot\theta_{76\,i}c\,\theta_{21\,i}c\,\theta_{76\,i}-\dot\theta_{21\,i}s\,\theta_{21\,i}s\,\theta_{76\,i})+$$

$$c\,\theta_{21\,i}(c\,\theta_{65\,i}c\,\theta_{76\,i}(\dot\theta_{21\,i}+\dot\theta_{65\,i}s\,\beta_{54\,i})-\dot\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{65\,i}s\,\theta_{76\,i}))-$$

$$\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})(c\,\theta_{21}(c\,\theta_{21\,i}c\,\theta_{43}c\,\theta_{65\,i}c\,\theta_{76\,i}-c\,\theta_{43}s\,\theta_{21\,i}$$

$$(c\,\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{65\,i}+c\,\beta_{54\,i}s\,\theta_{76\,i})+s\,\theta_{43}(c\,\beta_{54\,i}c\,\theta_{76\,i}s\,\theta_{65\,i}-s\,\beta_{54\,i}s\,\theta_{76\,i}))+$$

$$s\,\theta_{21}(-c\,\theta_{21\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}s\,\theta_{43}+s\,\beta_{54\,i}(c\,\theta_{76\,i}s\,\theta_{21\,i}s\,\theta_{43}s\,\theta_{65\,i}-c\,\theta_{43}s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}(c\,\theta_{43}c\,\theta_{76\,i}s\,\theta_{65\,i}+s\,\theta_{21\,i}s\,\theta_{43}s\,\theta_{76\,i})))+$$

$$s(\beta_{1413\,i}-\theta_{54})(-\dot\theta_{21}s\,\theta_{21}(c\,\theta_{21\,i}c\,\theta_{43}c\,\theta_{65\,i}c\,\theta_{76\,i}-c\,\theta_{43}s\,\theta_{21\,i}(c\,\theta_{76\,i}s\,\beta_{54\,i}s\,\theta_{65\,i}+$$

$$c\,\beta_{54\,i}s\,\theta_{76\,i})+s\,\theta_{43}(c\,\beta_{54\,i}c\,\theta_{76\,i}s\,\theta_{65\,i}-s\,\beta_{54\,i}s\,\theta_{76\,i}))+$$

$$\dot\theta_{21}c\,\theta_{21}(-c\,\theta_{21\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}s\,\theta_{43}+s\,\beta_{54\,i}(c\,\theta_{76\,i}s\,\theta_{21\,i}s\,\theta_{43}s\,\theta_{65\,i}-c\,\theta_{43}s\,\theta_{76\,i})+$$

$$c\,\beta_{54\,i}(c\,\theta_{43}c\,\theta_{76\,i}s\,\theta_{65\,i}+s\,\theta_{21\,i}s\,\theta_{43}s\,\theta_{76\,i}))+$$

$$c\,\theta_{21}(-c\,\theta_{76\,i}s\,\theta_{43}(\dot\theta_{43}c\,\theta_{21\,i}c\,\theta_{65\,i}+s\,\beta_{54\,i}(\dot\theta_{76\,i}-\dot\theta_{43}s\,\theta_{21\,i}s\,\theta_{65\,i}))-$$

$$c\,\theta_{43}(c\,\theta_{21\,i}c\,\theta_{76\,i}(\dot\theta_{65\,i}+\dot\theta_{21\,i}s\,\beta_{54\,i})s\,\theta_{65\,i}+s\,\beta_{54\,i}(\dot\theta_{43}-\dot\theta_{76\,i}s\,\theta_{21\,i}s\,\theta_{65\,i})s\,\theta_{76\,i}+$$

$$c\,\theta_{65\,i}(c\,\theta_{76\,i}(\dot\theta_{21\,i}+\dot\theta_{65\,i}s\,\beta_{54\,i})s\,\theta_{21\,i}+\dot\theta_{76\,i}c\,\theta_{21\,i}s\,\theta_{76\,i}))+$$

$$c\,\beta_{54\,i}(-c\,\theta_{43}(c\,\theta_{76\,i}(\dot\theta_{76\,i}s\,\theta_{21\,i}-\dot\theta_{43}s\,\theta_{65\,i})+\dot\theta_{21\,i}c\,\theta_{21\,i}s\,\theta_{76\,i})+$$

$$s\,\theta_{43}(\dot\theta_{65\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}+(\dot\theta_{43}s\,\theta_{21\,i}-\dot\theta_{76\,i}s\,\theta_{65\,i})s\,\theta_{76\,i})))+$$

$$s\,\theta_{21}(-\dot\theta_{43}c\,\theta_{21\,i}c\,\theta_{43}c\,\theta_{65\,i}c\,\theta_{76\,i}+\dot\theta_{21\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}s\,\theta_{21\,i}s\,\theta_{43}+$$

$$\dot\theta_{65\,i}c\,\theta_{21\,i}c\,\theta_{76\,i}s\,\theta_{43}s\,\theta_{65\,i}+\dot\theta_{76\,i}c\,\theta_{21\,i}c\,\theta_{65\,i}s\,\theta_{43}s\,\theta_{76\,i}+$$

$$c\,\beta_{54\,i}(s\,\theta_{43}(c\,\theta_{76\,i}(\dot\theta_{76\,i}s\,\theta_{21\,i}-\dot\theta_{43}s\,\theta_{65\,i})+\dot\theta_{21\,i}c\,\theta_{21\,i}s\,\theta_{76\,i})+$$

$$c\,\theta_{43}(\dot\theta_{65\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}+(\dot\theta_{43}s\,\theta_{21\,i}-\dot\theta_{76\,i}s\,\theta_{65\,i})s\,\theta_{76\,i}))+$$

$$s\,\beta_{54\,i}(c\,\theta_{43}c\,\theta_{76\,i}(-\dot\theta_{76\,i}+\dot\theta_{43}s\,\theta_{21\,i}s\,\theta_{65\,i})+s\,\theta_{43}(\dot\theta_{65\,i}c\,\theta_{65\,i}c\,\theta_{76\,i}s\,\theta_{21\,i}+$$

$$
\begin{aligned}
&\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\left(\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\Big)\Big)\Big)\Big)\Big)-\\
&s\,\theta_{1110\,i}\Big(\Big(-\dot\theta_{54}{}^2\,c(\beta_{1413\,i}-\theta_{54})+\ddot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\Big)(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+c\,\theta_{21\,i}\\
&\quad(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&2\,\dot\theta_{54}\,s(\beta_{1413\,i}-\theta_{54})\left(-s\,\theta_{21\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+\dot\theta_{76\,i}\,c\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\quad c\,\beta_{54\,i}\left(\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}-\dot\theta_{21\,i}\,s\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\\
&\quad c\,\theta_{21\,i}\left(c\,\theta_{65\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)-\dot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}\,s\,\theta_{76\,i}\right)\Big)+c(\beta_{1413\,i}-\theta_{54})\\
&\Big(c\,\beta_{54\,i}\Big(-s\,\theta_{21\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{76\,i}\,c\,\theta_{76\,i}+\ddot\theta_{21\,i}\,s\,\theta_{76\,i}\right)+c\,\theta_{21\,i}\left(\ddot\theta_{76\,i}\,c\,\theta_{76\,i}-\left(\dot\theta_{21\,i}{}^2+\dot\theta_{76\,i}{}^2\right)s\,\theta_{76\,i}\right)\Big)-\\
&\quad s\,\theta_{21\,i}\left(c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2+\dot\theta_{76\,i}{}^2+2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot\theta_{65\,i}+\ddot\theta_{21\,i}\,s\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\Big)+\\
&\quad c\,\theta_{21\,i}\left(-s\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(2\,\dot\theta_{21\,i}\,\dot\theta_{65\,i}+\left(\dot\theta_{21\,i}{}^2+\dot\theta_{65\,i}{}^2+\dot\theta_{76\,i}{}^2\right)s\,\beta_{54\,i}\right)+\ddot\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\ddot\theta_{21\,i}+\ddot\theta_{65\,i}\,s\,\beta_{54\,i}\right)-2\,\dot\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{76\,i}\right)\Big)\Big)+\\
&\Big(-\ddot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})-\dot\theta_{54}{}^2\,s(\beta_{1413\,i}-\theta_{54})\Big)(c\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\\
&\qquad(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad s\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+\\
&\qquad c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i})))-\\
&2\,\dot\theta_{54}\,c(\beta_{1413\,i}-\theta_{54})\left(-\dot\theta_{21}\,s\,\theta_{21}\,(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\right.\\
&\qquad(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad\dot\theta_{21}\,c\,\theta_{21}\,(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-c\,\theta_{43}\,s\,\theta_{76\,i})+\\
&\qquad c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))+\\
&\quad c\,\theta_{21}\left(-c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)-\right.\\
&\qquad c\,\theta_{43}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}+\right.\\
&\qquad\quad c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\Big)+\\
&\qquad c\,\beta_{54\,i}\left(-c\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-\dot\theta_{43}\,s\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad\quad s\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot\theta_{43}\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\Big)\Big)+\\
&\quad s\,\theta_{21}\,(-\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\dot\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}+\\
&\qquad\dot\theta_{65\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}+\\
&\qquad c\,\beta_{54\,i}\left(s\,\theta_{43}\left(c\,\theta_{76\,i}\left(\dot\theta_{76\,i}\,s\,\theta_{21\,i}-\dot\theta_{43}\,s\,\theta_{65\,i}\right)+\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)+\right.\\
&\qquad\quad c\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}+\left(\dot\theta_{43}\,s\,\theta_{21\,i}-\dot\theta_{76\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\right)\Big)+\\
&\qquad s\,\beta_{54\,i}\left(c\,\theta_{43}\,c\,\theta_{76\,i}\left(-\dot\theta_{76\,i}+\dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)+s\,\theta_{43}\left(\dot\theta_{65\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{21\,i}+\right.\right.\\
&\qquad\quad\dot\theta_{21\,i}\,c\,\theta_{21\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+\left(\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}\Big)\Big)\Big)\Big)+\\
&s(\beta_{1413\,i}-\theta_{54})\Big(\Big(-\dot\theta_{21}{}^2\,c\,\theta_{21}-\ddot\theta_{21}\,s\,\theta_{21}\Big)(c\,\theta_{21\,i}\,c\,\theta_{43}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}-c\,\theta_{43}\,s\,\theta_{21\,i}\\
&\qquad(c\,\theta_{76\,i}\,s\,\beta_{54\,i}\,s\,\theta_{65\,i}+c\,\beta_{54\,i}\,s\,\theta_{76\,i})+s\,\theta_{43}\,(c\,\beta_{54\,i}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}-s\,\beta_{54\,i}\,s\,\theta_{76\,i}))+\\
&\quad\Big(\ddot\theta_{21}\,c\,\theta_{21}-\dot\theta_{21}{}^2\,s\,\theta_{21}\Big)(-c\,\theta_{21\,i}\,c\,\theta_{65\,i}\,c\,\theta_{76\,i}\,s\,\theta_{43}+s\,\beta_{54\,i}\,(c\,\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{65\,i}-\\
&\qquad\quad c\,\theta_{43}\,s\,\theta_{76\,i})+c\,\beta_{54\,i}\,(c\,\theta_{43}\,c\,\theta_{76\,i}\,s\,\theta_{65\,i}+s\,\theta_{21\,i}\,s\,\theta_{43}\,s\,\theta_{76\,i}))-\\
&\quad 2\,\dot\theta_{21}\,s\,\theta_{21}\left(-c\,\theta_{76\,i}\,s\,\theta_{43}\left(\dot\theta_{43}\,c\,\theta_{21\,i}\,c\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{76\,i}-\dot\theta_{43}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)\right)-\right.\\
&\qquad c\,\theta_{43}\left(c\,\theta_{21\,i}\,c\,\theta_{76\,i}\left(\dot\theta_{65\,i}+\dot\theta_{21\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{65\,i}+s\,\beta_{54\,i}\left(\dot\theta_{43}-\dot\theta_{76\,i}\,s\,\theta_{21\,i}\,s\,\theta_{65\,i}\right)s\,\theta_{76\,i}+\right.\\
&\qquad\quad c\,\theta_{65\,i}\left(c\,\theta_{76\,i}\left(\dot\theta_{21\,i}+\dot\theta_{65\,i}\,s\,\beta_{54\,i}\right)s\,\theta_{21\,i}+\dot\theta_{76\,i}\,c\,\theta_{21\,i}\,s\,\theta_{76\,i}\right)\Big)+
\end{aligned}
$$

$$c\,\beta_{54i}\,(-c\,\theta_{43}\,(c\,\theta_{76i}\,(\dot\theta_{76i}\,s\,\theta_{21i}-\dot\theta_{43}\,s\,\theta_{65i})+\dot\theta_{21i}\,c\,\theta_{21i}\,s\,\theta_{76i})+$$

$$s\,\theta_{43}\,(\dot\theta_{65i}\,c\,\theta_{65i}\,c\,\theta_{76i}+(\dot\theta_{43}\,s\,\theta_{21i}-\dot\theta_{76i}\,s\,\theta_{65i})\,s\,\theta_{76i})))+$$

$$2\,\dot\theta_{21}\,c\,\theta_{21}\,(-\dot\theta_{43}\,c\,\theta_{21i}\,c\,\theta_{43}\,c\,\theta_{65i}\,c\,\theta_{76i}+\dot\theta_{21i}\,c\,\theta_{65i}\,c\,\theta_{76i}\,s\,\theta_{21i}\,s\,\theta_{43}+$$

$$\dot\theta_{65i}\,c\,\theta_{21i}\,c\,\theta_{76i}\,s\,\theta_{43}\,s\,\theta_{65i}+\dot\theta_{76i}\,c\,\theta_{21i}\,c\,\theta_{65i}\,s\,\theta_{43}\,s\,\theta_{76i}+$$

$$c\,\beta_{54i}\,(s\,\theta_{43}\,(c\,\theta_{76i}\,(\dot\theta_{76i}\,s\,\theta_{21i}-\dot\theta_{43}\,s\,\theta_{65i})+\dot\theta_{21i}\,c\,\theta_{21i}\,s\,\theta_{76i})+$$

$$c\,\theta_{43}\,(\dot\theta_{65i}\,c\,\theta_{65i}\,c\,\theta_{76i}+(\dot\theta_{43}\,s\,\theta_{21i}-\dot\theta_{76i}\,s\,\theta_{65i})\,s\,\theta_{76i}))+$$

$$s\,\beta_{54i}\,(c\,\theta_{43}\,c\,\theta_{76i}\,(-\dot\theta_{76i}+\dot\theta_{43}\,s\,\theta_{21i}\,s\,\theta_{65i})+s\,\theta_{43}\,(\dot\theta_{65i}\,c\,\theta_{65i}\,c\,\theta_{76i}\,s\,\theta_{21i}+$$

$$\dot\theta_{21i}\,c\,\theta_{21i}\,c\,\theta_{76i}\,s\,\theta_{65i}+(\dot\theta_{43}-\dot\theta_{76i}\,s\,\theta_{21i}\,s\,\theta_{65i})\,s\,\theta_{76i})))+$$

$$s\,\theta_{21}\,(c\,\theta_{76i}\,s\,\theta_{43}\,(c\,\theta_{21i}\,((\dot\theta_{21i}{}^2+\dot\theta_{65i}{}^2)\,c\,\theta_{65i}+\ddot\theta_{65i}\,s\,\theta_{65i})+s\,\theta_{21i}$$

$$(\ddot\theta_{21i}\,c\,\theta_{65i}-2\,\dot\theta_{21i}\,\dot\theta_{65i}\,s\,\theta_{65i}))+2\,(\dot\theta_{21i}\,c\,\theta_{65i}\,s\,\theta_{21i}+\dot\theta_{65i}\,c\,\theta_{21i}\,s\,\theta_{65i})$$

$$(\dot\theta_{43}\,c\,\theta_{43}\,c\,\theta_{76i}-\dot\theta_{76i}\,s\,\theta_{43}\,s\,\theta_{76i})-c\,\theta_{21i}\,c\,\theta_{65i}$$

$$(-s\,\theta_{43}\,((\dot\theta_{43}{}^2+\dot\theta_{76i}{}^2)\,c\,\theta_{76i}+\ddot\theta_{76i}\,s\,\theta_{76i})+c\,\theta_{43}\,(\ddot\theta_{43}\,c\,\theta_{76i}-2\,\dot\theta_{43}\,\dot\theta_{76i}\,s\,\theta_{76i}))+$$

$$s\,\beta_{54i}\,(2\,\dot\theta_{43}\,\dot\theta_{76i}\,c\,\theta_{76i}\,s\,\theta_{43}+c\,\theta_{76i}\,s\,\theta_{21i}\,(c\,\theta_{43}\,(2\,\dot\theta_{43}\,\dot\theta_{65i}\,c\,\theta_{65i}+\ddot\theta_{43}\,s\,\theta_{65i})+$$

$$s\,\theta_{43}\,(\ddot\theta_{65i}\,c\,\theta_{65i}-(\dot\theta_{43}{}^2+\dot\theta_{65i}{}^2)\,s\,\theta_{65i}))+$$

$$(\dot\theta_{43}{}^2\,c\,\theta_{43}+\ddot\theta_{43}\,s\,\theta_{43})\,s\,\theta_{76i}+c\,\theta_{43}\,(-\ddot\theta_{76i}\,c\,\theta_{76i}+\dot\theta_{76i}{}^2\,s\,\theta_{76i})+$$

$$2\,(\dot\theta_{65i}\,c\,\theta_{65i}\,s\,\theta_{43}+\dot\theta_{43}\,c\,\theta_{43}\,s\,\theta_{65i})\,(\dot\theta_{21i}\,c\,\theta_{21i}\,c\,\theta_{76i}-\dot\theta_{76i}\,s\,\theta_{21i}\,s\,\theta_{76i})+$$

$$s\,\theta_{43}\,s\,\theta_{65i}\,(-s\,\theta_{21i}\,((\dot\theta_{21i}{}^2+\dot\theta_{76i}{}^2)\,c\,\theta_{76i}+\ddot\theta_{76i}\,s\,\theta_{76i})+$$

$$c\,\theta_{21i}\,(\ddot\theta_{21i}\,c\,\theta_{76i}-2\,\dot\theta_{21i}\,\dot\theta_{76i}\,s\,\theta_{76i})))+c\,\beta_{54i}\,(2\,\dot\theta_{76i}\,c\,\theta_{76i}$$

$$(\dot\theta_{43}\,c\,\theta_{43}\,s\,\theta_{21i}+\dot\theta_{21i}\,c\,\theta_{21i}\,s\,\theta_{43})+c\,\theta_{43}\,c\,\theta_{76i}\,(\ddot\theta_{65i}\,c\,\theta_{65i}-\dot\theta_{65i}{}^2\,s\,\theta_{65i})+$$

$$(c\,\theta_{21i}\,(2\,\dot\theta_{21i}\,\dot\theta_{43}\,c\,\theta_{43}+\ddot\theta_{21i}\,s\,\theta_{43})+s\,\theta_{21i}\,(\ddot\theta_{43}\,c\,\theta_{43}-(\dot\theta_{21i}{}^2+\dot\theta_{43}{}^2)\,s\,\theta_{43}))$$

$$s\,\theta_{76i}+s\,\theta_{21i}\,s\,\theta_{43}\,(\ddot\theta_{76i}\,c\,\theta_{76i}-\dot\theta_{76i}{}^2\,s\,\theta_{76i})-2\,\dot\theta_{65i}\,c\,\theta_{65i}$$

$$(\dot\theta_{43}\,c\,\theta_{76i}\,s\,\theta_{43}+\dot\theta_{76i}\,c\,\theta_{43}\,s\,\theta_{76i})-s\,\theta_{65i}\,(c\,\theta_{43}\,((\dot\theta_{43}{}^2+\dot\theta_{76i}{}^2)\,c\,\theta_{76i}+$$

$$\ddot\theta_{76i}\,s\,\theta_{76i})+s\,\theta_{43}\,(\ddot\theta_{43}\,c\,\theta_{76i}-2\,\dot\theta_{43}\,\dot\theta_{76i}\,s\,\theta_{76i})))))+$$

$$c\,\theta_{21}\,(-c\,\theta_{65i}\,c\,\theta_{76i}\,(c\,\theta_{21i}\,((\dot\theta_{21i}{}^2+\dot\theta_{43}{}^2)\,c\,\theta_{43}+\ddot\theta_{43}\,s\,\theta_{43})+s\,\theta_{21i}$$

$$(\ddot\theta_{21i}\,c\,\theta_{43}-2\,\dot\theta_{21i}\,\dot\theta_{43}\,s\,\theta_{43}))+$$

$$s\,\theta_{21i}\,(\dot\theta_{43}{}^2\,c\,\theta_{43}+\ddot\theta_{43}\,s\,\theta_{43})\,(c\,\theta_{76i}\,s\,\beta_{54i}\,s\,\theta_{65i}+c\,\beta_{54i}\,s\,\theta_{76i})+$$

$$2\,(\dot\theta_{21i}\,c\,\theta_{43}\,s\,\theta_{21i}+\dot\theta_{43}\,c\,\theta_{21i}\,s\,\theta_{43})\,(\dot\theta_{65i}\,c\,\theta_{76i}\,s\,\theta_{65i}+\dot\theta_{76i}\,c\,\theta_{65i}\,s\,\theta_{76i})+$$

$$(\ddot\theta_{43}\,c\,\theta_{43}-\dot\theta_{43}{}^2\,s\,\theta_{43})\,(c\,\beta_{54i}\,c\,\theta_{76i}\,s\,\theta_{65i}-s\,\beta_{54i}\,s\,\theta_{76i})-$$

$$c\,\theta_{21i}\,c\,\theta_{43}\,(c\,\theta_{65i}\,((\dot\theta_{65i}{}^2+\dot\theta_{76i}{}^2)\,c\,\theta_{76i}+\ddot\theta_{76i}\,s\,\theta_{76i})+$$

$$s\,\theta_{65i}\,(\ddot\theta_{65i}\,c\,\theta_{76i}-2\,\dot\theta_{65i}\,\dot\theta_{76i}\,s\,\theta_{76i}))-2\,\dot\theta_{43}\,c\,\theta_{43}$$

$$(\dot\theta_{76i}\,c\,\theta_{76i}\,s\,\beta_{54i}+c\,\beta_{54i}\,(-\dot\theta_{65i}\,c\,\theta_{65i}\,c\,\theta_{76i}+\dot\theta_{76i}\,s\,\theta_{65i}\,s\,\theta_{76i}))+$$

$$s\,\theta_{43}\,(s\,\beta_{54i}\,(-\ddot\theta_{76i}\,c\,\theta_{76i}+\dot\theta_{76i}{}^2\,s\,\theta_{76i})+c\,\beta_{54i}\,(-s\,\theta_{65i}\,((\dot\theta_{65i}{}^2+\dot\theta_{76i}{}^2)\,c\,\theta_{76i}+$$

$$\ddot\theta_{76i}\,s\,\theta_{76i})+c\,\theta_{65i}\,(\ddot\theta_{65i}\,c\,\theta_{76i}-2\,\dot\theta_{65i}\,\dot\theta_{76i}\,s\,\theta_{76i})))+$$

$$
\begin{aligned}
&2\,\dot{\theta}_{43}\,\mathrm{s}\,\theta_{43}\left(\dot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{s}\,\theta_{21\,\mathrm{i}}\right.\\
&\quad\left(\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\left(\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)\Big)-\\
&\mathrm{c}\,\theta_{43}\left(\mathrm{c}\,\beta_{54\,\mathrm{i}}\left(\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\mathrm{s}\,\theta_{21\,\mathrm{i}}\right.\right.\\
&\quad\left(\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\left(\dot{\theta}_{21\,\mathrm{i}}^{\,2}+\dot{\theta}_{76\,\mathrm{i}}^{\,2}\right)\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\Big)-\mathrm{s}\,\beta_{54\,\mathrm{i}}\\
&\quad\left(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\left(2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(\ddot{\theta}_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-2\,\dot{\theta}_{21\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)+\right.\\
&\quad\mathrm{s}\,\theta_{21\,\mathrm{i}}\left(\mathrm{s}\,\theta_{65\,\mathrm{i}}\left(\left(\dot{\theta}_{21\,\mathrm{i}}^{\,2}+\dot{\theta}_{65\,\mathrm{i}}^{\,2}+\dot{\theta}_{76\,\mathrm{i}}^{\,2}\right)\mathrm{c}\,\theta_{76\,\mathrm{i}}+\ddot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)+\right.\\
&\quad\left.\left.\mathrm{c}\,\theta_{65\,\mathrm{i}}\left(-\ddot{\theta}_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}+2\,\dot{\theta}_{65\,\mathrm{i}}\,\dot{\theta}_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}\right)\right)\right)\Big)\Big)\Big)\Big)\Big)\Big)-\\
&2\,\dot{\theta}_{98\,\mathrm{i}}^{\,2}\,\sec(\theta_{98\,\mathrm{i}})^2\,(\mathrm{s}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}(\theta_{21}+\theta_{43})-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\\
&\quad\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})-\\
&\quad\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\\
&\quad\mathrm{s}\,\theta_{76\,\mathrm{i}}+\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\\
&\quad\mathrm{c}\,\theta_{109\,\mathrm{i}}\,(\mathrm{c}\,\theta_{1110\,\mathrm{i}}\,(-\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\\
&\quad\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,\mathrm{s}\,\theta_{43}\,(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\\
&\quad\mathrm{c}\,\theta_{21}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{21}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\\
&\quad\mathrm{c}\,\theta_{43}\,\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{s}\,\theta_{21}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\\
&\quad\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))))-\\
&\quad\mathrm{s}\,\theta_{1110\,\mathrm{i}}\,(\mathrm{c}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}+\mathrm{c}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\\
&\quad\mathrm{s}(\beta_{1413\,\mathrm{i}}-\theta_{54})\,(\mathrm{c}\,\theta_{21}\,(\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\\
&\quad\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\mathrm{s}\,\theta_{43}\,(\mathrm{c}\,\beta_{54\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{s}\,\beta_{54\,\mathrm{i}}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))+\\
&\quad\mathrm{s}\,\theta_{21}\,(-\mathrm{c}\,\theta_{21\,\mathrm{i}}\,\mathrm{c}\,\theta_{65\,\mathrm{i}}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}+\mathrm{s}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}-\mathrm{c}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}})+\\
&\quad\mathrm{c}\,\beta_{54\,\mathrm{i}}\,(\mathrm{c}\,\theta_{43}\,\mathrm{c}\,\theta_{76\,\mathrm{i}}\,\mathrm{s}\,\theta_{65\,\mathrm{i}}+\mathrm{s}\,\theta_{21\,\mathrm{i}}\,\mathrm{s}\,\theta_{43}\,\mathrm{s}\,\theta_{76\,\mathrm{i}}))))))^3\,\tan(\theta_{98\,\mathrm{i}})\Big)
\end{aligned}
$$