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"ANÁLISIS ESTADÍSTICOS PARA LA EVALUACIÓN DEL PELIGRO VOLCÁNICO"

T E S I S

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DEDICO ESTA TESIS A MI FAMILIA

A la memoria de mi Padre:
Pedro Jesús Mendoza Pallares

A mi Madre:
Guillermina Rosas Espinosa

A mis Hermanas:
Claudia, Lina, Rosalba y Guillermina

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Capítulo 1.

INTRODUCCIÓN

La actividad volcánica es un fenómeno natural que puede bajo ciertas condiciones traducirse en un desastre, es decir, un desajuste en la estructura de la sociedad impidiendo su funcionamiento. A pesar de que las erupciones volcánicas pueden provocar catástrofes, también es cierto que acarrean efectos benéficos para las sociedades que habitan las tierras cercanas a los volcanes, debido a que los materiales emitidos contribuyen a la fertilidad del suelo y en algunos casos, los volcanes son generadores de importantes caudales de agua, por medio de corrientes y manantiales, provenientes de lluvia y/o deshielo. Los volcanes que presentan un peligro a plazo corto y que claramente amenazan la vida y la propiedad deben ser mantenidos bajo monitoreo, y deben establecerse restricciones al uso del suelo y a la ocupación permanente en las áreas de mayor peligro.

Generalmente las erupciones causantes de catástrofes, forman muestras muy pequeñas debido a baja frecuencia que exhibe la mayoría de los volcanes. El peligro en los volcanes que tienen una periodicidad de largo plazo es frecuentemente ignorado, no obstante, las erupciones de los volcanes considerados dormidos o inactivos han sido las responsables de grandes daños.

El riesgo volcánico es una medida compuesta, las primeras definiciones de este (Fournier d'Albe, 1979) han sido revisadas y modificadas a lo largo del tiempo (ver p.ej. Tilling y Punongbayan, 1993; De la Cruz - Reyna y Tilling, 2008). En particular, De la Cruz-Reyna y Tilling (2008) redefinen el riesgo volcánico como:

$R=H*(V-P)$

Donde H, el peligro, es la probabilidad de que una manifestación o fenómeno volcánico específico ocurra en un área dada en un intervalo de tiempo; V, la vulnerabilidad, es el porcentaje de la pérdida esperada del valor expuesto si ocurre dicha manifestación peligrosa (es decir, probabilidad de la pérdida). P representa la "preparación", refiriéndose a la serie de medidas para reducir la vulnerabilidad. *El riesgo es, por lo tanto, la probabilidad de perder un cierto porcentaje del valor de una región dada, sobre un intervalo de tiempo, causado por la posible ocurrencia de una manifestación volcánica particular*. La gestión del riesgo volcánico es un concepto que se está imponiendo en todo el mundo, así como diversos procedimientos de mitigación tales como: el establecimiento de sistemas de monitoreo y alertas, medidas de evacuación, medidas protectivas, programas de seguros, mapas de peligros, medidas de rehabilitación, etc. que han sido diseñadas como medidas de preparación basadas en las experiencias del impacto de las erupciones catastróficas. Las restricciones en el uso del suelo deben de ser planificadas con conocimiento de las consecuencias potenciales de futuras erupciones.

El peligro volcánico es una medida que no puede ser modificada debido a que representa la probabilidad de ocurrencia del fenómeno. El peligro volcánico contiene información del comportamiento del fenómeno volcánico y junto con la medida de la vulnerabilidad permite la toma de decisiones correctas para la protección civil y el diseño de medidas de prevención de algún desastre. El proceso de toma de decisiones proporciona una conclusión de un conjunto de consecuencias como resultado de la combinación de los diversos cursos de acción y de los diversos factores o estados de la naturaleza del fenómeno, por parte de la persona o personas que deciden, para alcanzar las metas y objetivos propuestos y fijados de antemano, a corto, mediano y largo plazo, mediante la recopilación de datos significativos, análisis, planeación y control administrativo. El planificador puede identificar las medidas potenciales de mitigación, comparando costos y beneficios potenciales con todos los otros elementos involucrados en el desarrollo del área de estudio.

Generalmente, las frecuencias de ocurrencia en los fenómenos naturales en general y de las erupciones volcánicas en particular, se relacionan inversamente con sus magnitudes. Esto hace que las erupciones mayores ocurran a tasas más bajas. La definición de magnitud en el caso de las erupciones es un problema que no ha sido del todo resuelto. La medida que más se utiliza en la actualidad es el valor VEI (*Índice de Explosividad Volcánica*), que es una escala compuesta para cuantificar las erupciones explosivas, definida por Newhall y Self en 1982, en la que se toman en cuenta diversas características de una erupción como son: el volumen de magma emitido, la energía térmica liberada, el alcance de los productos fragmentados, la altura de la columna eruptiva, la duración de la erupción, etc. De la Cruz-Reyna (1991) ha reportado una buena correlación entre el logaritmo de la tasa media de ocurrencia y la energía liberada por las erupciones, derivada del VEI, definiendo una escala de magnitudes basada en la relación entre el tamaño de las erupciones y su razón global de ocurrencia. Marzocchi y Zaccarelli (2006) proponen un modelo de la distribución tamaño-tiempo de erupciones donde utilizan una relación logarítmica similar a la publicada por De la Cruz- Reyna en 1991.

El conocimiento de la frecuencia y magnitud de erupciones para un volcán dado constituye un componente esencial en el cálculo del peligro volcánico. Una *serie o secuencia eruptiva* es un conjunto o sucesión de valores que indica la dimensión de las erupciones distribuidas en el tiempo. Las erupciones volcánicas son descritas como eventos cuyas magnitudes se caracterizan por sus índices de explosividad volcánica (VEI). Esta definición permite analizar a las secuencias eruptivas como procesos aleatorios y a los valores de las magnitudes VEI de las erupciones como variables aleatorias, susceptibles de ser analizadas por métodos estadísticos. El análisis de las secuencias de erupciones de magnitudes relativamente bajas, que se presentan con mayor frecuencia y de las que existen suficientes datos, generalmente se lleva a cabo utilizando métodos estadísticos convencionales, pero no todos los casos son fáciles de describir. Para esto, existen otras opciones que incluyen la búsqueda de distribuciones con mayor generalidad, como un proceso de renovación. Para los casos en que la secuencia eruptiva no sea estacionaria, o cuando se trata del análisis de eventos eruptivos de mediana y gran magnitud, que pueden ser considerados como eventos raros o extremos y de los cuales, por su naturaleza, se tienen muy pocos datos, es necesario aplicar metodologías específicas, como el análisis de procesos no-homogéneos y la teoría de los valores extremos. Además si consideramos que el sistema volcánico puede presentar largos periodos de reposo, o responder a determinadas secuencias de erupciones volcánicas con dependencia en el tiempo, en las que se pueden identificar regímenes o agrupamientos, entonces será adecuado aplicar un método estadístico específico basado en procesos de distribuciones mixtas. También hay que considerar la poca confiabilidad de la asignación del VEI en particular a las erupciones de magnitudes pequeñas, pues es una escala poco específica con criterios traslapados en la asignación de magnitud. Por ello, otro de los problemas de evaluación del peligro es la baja calidad de la base de datos, debido a la escasez de datos de erupciones mayores y la imprecisión en sus fechas y magnitudes. Por lo tanto, la elección del método estadístico para estimar el peligro volcánico y la calidad de la base de datos utilizada son factores críticos para la estimación cabal del peligro volcánico.

OBJETIVOS

El objetivo del presente trabajo es proponer y analizar diferentes metodologías estadísticas como la PPNHP (Proceso de Poisson No-Homogéneo Pareto generalizado) y la MEDE (Mezcla de Distribuciones Exponenciales) para cuantificar el peligro volcánico, aplicar a 5 volcanes poligenéticos mexicanos, comparar y evaluar los resultados.

La primera metodología estadística PPNHP (en idioma ingles, NHGPPP: Non-homogeneous generalized Pareto–Poisson process) se basa en la teoría de los valores extremos, enfocada a series eruptivas utilizando una serie combinada de datos de erupciones geológicas e históricas. La teoría de los valores extremos permite enfocarse en los datos que se concentran en la cola de la distribución de las series eruptivas, es decir las erupciones de mayor magnitud. Estas erupciones son de gran interés para la población, por el riesgo que estas implican, siendo posible analizarlas por medio de los datos geológicos que casi siempre corresponden a erupciones grandes, ya que depósitos de erupciones pequeñas generalmente no se preservan. Enfrentando así el problema de homologar datos de distinta naturaleza y escala en el tiempo, y la probable no-homogeneidad de la serie eruptiva, la incompletez y la escasez de los datos de la serie, permitiendo obtener una mejor estimación del peligro volcánico. Esta metodología es aplicada a volcanes poligenéticos mexicanos, obteniendo el peligro volcánico de erupciones de gran magnitud VEI para el Volcán de Colima, Popocatépetl, El Chichón, Citlaltépetl o Pico de Orizaba, y El Nevado de Toluca.

La segunda metodología estadística MEDE (en idioma ingles, MOED: Mixture of exponentials distribution), se basa en una suma pesada de distribuciones de exponenciales. Es de más sencilla aplicación y se enfoca a series eruptivas con regímenes identificados, es decir, secuencias eruptivas con dependencia en el tiempo, permitiendo agrupamientos de eventos con tasas eruptivas bien definidas que varían significativamente de una tasa media. Los regímenes pueden ser reconocidos directamente de la serie acumulativa de erupciones para diferentes categorías de valores VEI a partir de las historias eruptivas, pero requiere de un grado de completez de la base de datos utilizada para la estimación del peligro volcánico. Esta metodología se aplica a las series históricas eruptivas del Volcán de Colima y Popocatépetl.

Finalmente, las metodologías propuestas son analizadas y comparadas con métodos convencionales, como el proceso de Poisson y la distribución de Weibull, estimando el peligro volcánico para distintos periodos de tiempo durante el Holoceno para el volcán El Chichón, Chiapas, México.

ANTECEDENTES

Uno de los pioneros en la aplicación de métodos estadísticos a secuencias de erupciones volcánicas, tratando las erupciones como eventos aleatorios y la actividad volcánica como un proceso estocástico fue Wickman (1965a, 1965b, 1976). Wickman (1965a), analizó varios volcanes de distintas partes del mundo expresando la historia eruptiva como la distribución de períodos de reposo, por medio de la función de supervivencia. Sin embargo, Wickman en 1976 indica que tales modelos no fueron comparados contra datos u observaciones precisas. Wickman (1976), concluyó que la actividad de muchos volcanes puede ser descrita por un proceso de Poisson. Reyment (1969), encontró que el comportamiento de la actividad de algunos volcanes divergía considerablemente de un proceso de Poisson, pero era posible considerar algún otro tipo de proceso de renovación como un semiproceso de Markov o superposiciones de procesos puntuales, como es el caso en la actividad eruptiva del volcán Vesubio, que Carta et al. (1981) describió por cadenas de Markov. Para ello, dividió los periodos en una serie de ciclos eruptivos, modelando la duración de 4 estados: Reposo, actividad persistente, erupción intermedia y erupción final.

Para los volcanes Hawaianos, Klein (1982) utilizó pruebas estadísticas para los periodos de reposo entre erupciones y la secuencia de diferentes tipos de eventos, observando diferencias estadísticas entre grandes y pequeñas erupciones, erupciones en flancos y en la cumbre, y eventos extrusivos e intrusivos con consecuencias físicas. También observó que los periodos de reposo largos para el Kilauea y el Mauna Loa no se comportaban como fenómenos aleatorios. Posteriormente Miklius y Cervelli (2003) utilizaron simulaciones Monte Carlo, desde 2002, observando que la deformación desarrollada en el Mauna Loa tiene correlación con el alto volumen emitido de magma, en el volcán Kilauea en periodos cortos, los resultados, sugirieron que existe una interacción a nivel de la corteza entre los sistemas de magma, notando una contradicción debido a la diferencia en la química de magmas de estos dos volcanes. Otra posibilidad es la relajación de esfuerzos que ocurre en el flanco del Mauna Loa, que permite el ascenso de magma a niveles superficiales.

El volcán Etna, un volcán con erupciones predominantemente efusivas, ha sido estudiado por distintos autores como Casetti et al. (1981), quien utilizó distribuciones estadísticas de los eventos eruptivos, para identificar periodos con altas concentraciones de erupciones, concluyendo que la actividad eruptiva es influenciada por fluctuaciones estaciónales concentrándose las erupciones en la época de lluvia. La actividad eruptiva del volcán Etna fue estudiada ampliamente por Mulargia et al (1985), sin embargo, su prolongada y continua actividad hace difícil un análisis estadístico considerando las erupciones como procesos puntuales. Mulargia clasificó las erupciones en términos de su duración y el volumen de magma expulsado, considerando que la magnitud de una erupción es proporcional a su duración, mostrando que la teoría de los valores extremos puede para estimar la probabilidad de una erupción mayor. Mulargia et al. (1987), identificaron diferentes regímenes en la historia eruptiva del volcán Etna, utilizando series de tasas medias (volumen emitido/duración de la erupción) aplicadas a las erupciones laterales del volcán y a las erupciones en la cima. Este autor notó que en los puntos de cambio de régimen en la serie eruptiva del volcán Etna, los tiempos entre erupciones y el volumen total no coinciden, implicando que las erupciones son gobernadas por otros factores además del volumen de las intrusiones de magma. Además, el análisis de la actividad sísmica, no sugiere al campo de esfuerzos como el factor más importante para la detonación de una erupción. Por lo tanto, concluyó que la actividad eruptiva del Volcán Etna, tiene un comportamiento no estacionario y parece estar controlada por varios factores más. También se han caracterizado los patrones de precursores de erupciones laterales para el volcán Etna, usando métodos estadísticos y algoritmos computacionales (Mulargia et al, 1991) además por medio de análisis estadísticos de sismicidad y actividad eruptiva, utilizando un proceso generalizado de Poisson (Gasperini et al., 1990). En 2007, Bebbington aplicó una metodología basada en cadenas de Markov para identificar regímenes en los datos de erupciones de flanco para el volcán Etna, para un periodo de 1600-2006 considerado completo.

Ho (1990, 1991a y 1991b), concluyó que un modelo generalizado es preferible para modelar eventos volcánicos ya que la aplicación de un modelo simple de Poisson no es apropiado para todas las situaciones. Ho (1991a), utilizó un proceso no-homogéneo de Poisson, donde la función de intensidad se aproximó a una distribución de Weibull, estimando la tasa de recurrencia de actividad volcánica utilizando datos geológicos, para estimar el peligro asociado a una zona volcánica cercana a un depósito de deshechos nucleares en la región de la montaña Yucca al sur-centro de Nevada, E.U. Ho (1991a, 1991b, 1995) propone la distribución de Weibull, como un modelo general para los tiempos de reposo entre erupciones. En 1992, Ho propuso la distribución Binomial Negativa, resultado de una distribución de Poisson compuesta donde la función de intensidad es una función de densidad gamma, como otra alternativa para estimar la probabilidad de futuras erupciones.

Burt et al. (1994) evaluó el peligro volcánico utilizando un proceso de Weibull el cual fue aplicado a la historia eruptiva del volcán basáltico Nyamuragira, asociado al sistema del Rift Africano oriental. Por otro lado, Bebbington y Lai (1996a), utilizaron datos de ocurrencia de los volcanes de Nueva Zelanda Mt. Ruapehu y Mt. Ngauruhoe, sugiriendo un modelo de renovación de Weibull para describir los patrones eruptivos, y observaron que a pesar de la cercanía entre estos dos volcanes el comportamiento eruptivo es muy diferente. Bebbington y Lai (1996b) utilizaron la distribución de probabilidad de Weibull y la distribución de probabilidad Lognormal en un proceso no homogéneo.

Connor et al. (2000) realizaron un análisis estadístico espacial con información obtenida de depósitos geológicos para el depósito de deshechos nucleares en la región de la montaña Yucca, Nevada, E.U. Cronin et al. (2001) aplicaron un análisis espacio-temporal para la evaluación del peligro volcánico, combinando cadenas de Markov y un análisis de Weibull para obtener la intensidad de ocurrencia en el volcán Taveuni, de las islas Fiji, en el Pacífico Sur-Occidental.

Para identificar y apoyar de manera objetiva alertas volcánicas, donde la evidente incertidumbre vulcanológica es un problema, en la toma de decisiones, se han utilizado técnicas estadísticas como los árboles de eventos, análisis Bayesiano (Newhall y Hoblitt, 2002) y modelos de Markov de varios estados considerando diversos parámetros de datos eruptivos (Aspinall et al 2003; 2006). Marzocchi et al. (2004, 2008) también describen un árbol de eventos para estimar el peligro volcánico, a corto y largo plazo, haciendo uso de una aproximación Bayesiana utilizando la información disponible como modelos teóricos, datos geológicos e históricos y datos de monitoreo. También se ha realizado análisis de costo-beneficio para apoyar las medidas a tomar para mitigación del riesgo volcánico (Marzocchi y Woo, 2009). Estas metodologías difieren de las anteriores, en que se basan en mayor medida en el desarrollo de la actividad presente de un volcán que muestra actividad precursora.

Otras herramientas estadísticas han sido utilizadas por varios autores como: estadísticos de orden (Pyle, 1998), una mezcla de distribuciones de Weibull (Turner et al., 2007), y métodos geoestadísticos haciendo uso de un proceso de Cox para la estimación del peligro volcánico (Jaquet et al., 2000; Jaquet y Carniel, 2006).

Diversos métodos estadísticos han sido aplicados a los volcanes Mexicanos. De la Cruz - Reyna (1993, 1996), analizó datos a nivel global y para el volcán de Colima, concluyendo que para erupciones por encima de cierta magnitud VEI los patrones de ocurrencia tienen una distribución Poisson (De la Cruz-Reyna, 1991, 1993, 1996). Solow (2001) realizó una aproximación bayesiana para inferir la tasa eruptiva del volcán Colima. De la Cruz-Reyna y Carrasco-Núñez (2002) estimaron una distribución temporal y espacial para el Volcán Citlaltépetl utilizando datos históricos y geológicos, calculando las probabilidades de ocurrencia de erupciones. De la Cruz-Reyna y Tilling (2008), asumen un proceso de Bernoulli (Feller, 1973) para describir la secuencia de erupciones del volcán Popocatépetl, dividida en clases de magnitud VEI, obteniendo una estimación aproximada de la probabilidad de ocurrencia de futuras erupciones, utilizando la relación logarítmica entre las tasas de ocurrencia y las magnitudes VEI, corroborando la aplicación con datos históricos y geológicos del volcán Popocatépetl (De la Cruz-Reyna, 1991).

ALGUNOS VOLCÁNES ACTIVOS POLIGENÉTICOS DE MÉXICO

La mayor parte del vulcanismo en México se manifiesta principalmente en la Faja Volcánica Mexicana (FVM) una provincia Mioceno-Cuaternario (Ferrari et al., 1999), y está relacionado con las interacciones entre las placas tectónicas de Rivera y Cocos con la Placa de Norteamérica. La FVM está formado por aproximadamente 8000 estructuras volcánicas que se extienden desde las costas del Pacifico en los Estado de Nayarit y Jalisco hasta el Golfo de México en el Estado de Veracruz. Presenta una longitud cercana a 1000 km, con orientación aproximada Este-Oeste y una anchura que oscila entre 80 y 230 km. Otras regiones volcánicas en México además de la FVM se encuentran en el Noroeste (Baja California y Sonora), en las islas del Pacífico (principalmente las Revillagigedo), y en el Sureste (principalmente en Chiapas) (De la Cruz-Reyna, 2004; Macías, 2005; Mora et al., 2007) de los cuales al menos una docena de volcanes se consideran activos.

Los principales estratovolcanes mexicanos Colima, Popocatépetl, Citlaltépetl y Nevado de Toluca están ubicados en la Faja Volcánica Mexicana. El volcán de Colima es el de mayor actividad histórica en México y junto con el volcán Popocatépetl, actualmente tiene importantes manifestaciones de actividad eruptiva (Macias 2005; Martín Del Pozzo et al. 1995, 2002, 2008; Granados et al. 2008). Por otro lado, el Volcán Citlaltépetl, a pesar de ser un volcán activo, ha presentado sólo manifestaciones menores en tiempos recientes. El Citlaltépetl es considerado un volcán de alto riesgo debido a la extensa población que habita sus alrededores y a las erupciones mayores que ha producido en la escala de tiempo geológica. En forma análoga, El Nevado de Toluca es un volcán activo en estado de quietud, sin embargo se sabe que al menos en los últimos ~42 ka manifestó cuatro erupciones plinianas y por lo menos cinco eventos de destrucción de domo (Aceves et al., 2007). El volcán Chichón, localizado en la Arco volcánico chiapaneco en la porción noroeste del estado de Chiapas, ha causado el mayor desastre histórico registrado en la historia del país provocando la muerte de cerca de 2 000 personas y la destrucción de 9 poblados en su última erupción (1982) (Espíndola et al. 2000; De la Cruz-Reyna y Martin Del Pozzo, 2009).

VOLCÁN DE COLIMA

El Volcán de Colima, con una altitud de 3,850 m.s.n.m y coordenadas geográficas 19.51º N y 103.63º W situado en los límites de los Estados de Colima y Jalisco, es considerado el volcán más activo de México. Este estratovolcán es la manifestación más joven, de un complejo Volcánico alineado de N a S que incluye al Nevado de Colima y al Volcán el Cántaro. El Volcán de Colima ancestral comenzó a formarse en el flanco meridional del Nevado durante el pleistoceno tardío. Una erupción mayor causó un colapso masivo hacia el sur, que produjo una caldera en forma de herradura de 5 km de diámetro y una avalancha de escombros volcánicos que cubrió una superficie mayor a 1,500 km^2 y llegó hasta 70 km de la antigua cima del volcán. El actual volcán de Colima comenzó a crecer adentro de la caldera poco después de esta avalancha. Una de las características presentes del volcán de Colima ha sido la frecuente generación de flujos piroclásticos que han alcanzado hasta 15 km del cráter; los más notables son aquellos formados durante la erupción de 1913 (Martín Del Pozzo et al. 1995; Saucedo R., 1997; Saucedo et al., 2002.; 2006; Cortés et al. 2005; Macías, 2005). El volcán de Colima tiene el mayor número de erupciones registrado en la historia. En la tabla 1 se enlistan los datos geológicos e históricos eruptivos con magnitud VEI>1.

Datos Históricos				Datos Geológicos	
Año	VEI	Año	VEI	Año AP	VEI
1560	2	1872	3	2,300	>4
1576	3	1881	2	3,600	>4
1585	4	1886	3	7,040	>4
1590	3	1889	3		
1606	4	1890	4		
1611	3	1893	2		
1612	2	1903	3		
1622	4	1908	3		
1690	3	1909	2		
1749	2	1913	4		
1770	3	1994	2		
1795	2	1999	2		
1804	2	2003	2		
1818	4	2005	3		
1869	3				

Tabla 1. Datos Históricos del volcán de Colima (De la Cruz-Reyna, 1993; Global Volcanism Program (http://www.volcano.si.edu) y Observatorio del volcán de Colima (http://www.ucol.mx)); y Geológicos de depósitos de avalancha Holocenicos (Cortés et al. 2005).

VOLCÁN POPOCATÉPETL

El Popocatépetl (19.02º N, 98.62º W) es un volcán activo andesitico-dacítico con 5,452 metros de altura, localizado a 71 Km al sureste del centro de la Ciudad de México y a 40 Km al oeste de la Ciudad de Puebla, en la parte central de la FVM. Su edificio cubre un área de 500 km^2 abarcando parte de los estados de Puebla, México y Morelos. El Popocatépetl ha producido numerosas erupciones de diversas categorías de magnitudes (Tabla 2). De las categorías mayores persisten evidencias geológicas en forma de depósitos volcánicos, que permiten inferir muchos de los aspectos de la naturaleza del Popocatépetl y de sus erupciones (Macías et. al. 1995, Siebe et. al. 1995, Martin del Pozzo et al. 2008). Las erupciones plinianas del Popocatépetl desde tiempos prehispánicos han afectado los asentamientos humanos alrededor del volcán, con la generación de flujos incandescentes y el emplazamiento de gruesos espesores de material de caída de pómez y ceniza, o con la generación de lahares que inundaron extensas áreas situadas en la cuenca de Puebla (Siebe *et al.*, 1996). En la actualidad se estima que en un radio de 40 km alrededor del volcán habitan cerca de 1 millón de personas, y cerca de 23 millones en un radio de 100 km.

Datos		Datos Geológicos	
Año	VEI	Año A.P.	Erupción
1512	2	1,200	Pliniana
1519	3	1,700	Pliniana
1539-1540	2	2,150	Pliniana
1548	2	5,000	Pliniana
1571	2	7,100	Pliniana
1592	2	9,100	Pliniana
1642	2	10,700	Pliniana
1663	2	14,000	Pliniana
1664	3	23,000	Pliniana (Erupción tipo Sta. Elena)
1665	2		
1697	2		
1919-1920	2		
1921	2		
1925-1927	2		
1994-1997	2		
2000	3		
2001-presente	1-2		

Tabla 2. Actividad histórica del volcán Popocatépetl, basada en De la Cruz-Reyna y Tilling (2008); y actividad geológica del volcán Popocatépetl, basada en Siebe y Macías (2004).

VOLCÁN CITLALTÉPETL O PICO DE ORIZABA

El Citlaltépetl o Pico de Orizaba es un estratovolcán, andesítico, Cuaternario activo con coordenadas geográficas 19.03° N y 92.27° W. El volcán Citlaltépetl representa la cima más alta de México con una elevación de 5,675 m.s.n.m. (De la Cruz-Reyna y Carrasco-Núñez, 2002). Está situado en la parte Este del Cinturón Volcánico Transmexicano y en la frontera entre los Estados de Puebla y Veracruz, a menos de 100 Km del Golfo de México.

Las etapas de evolución del Pico de Orizaba fueron denominadas de la más antigua a la más reciente como Torrecillas, Espolón de Oro, domos silícicos periféricos y el cono Citlaltépetl (Carrasco-Núñez y Ban, 1994; Carrasco-Núñez, 2000). La actividad histórica y geológica registrada del Citlaltépetl (Tabla 3) indica que es un volcán con un potencial de riesgo elevado. Sistemas de monitoreo han sido recientemente instalados en este volcán para conocer el nivel de actividad y detectar oportunamente algún incremento que pudiera indicar un reactivamiento y poner en riesgo a la población.

Históricos	Año	VEI
	1867	2
	1846	2
	1687	2
	1569-89	2
	1545	2
	1533-39	2
Geológicos	Año AP	VEI
	4,100	≥4
	8,500-9,000	≥4
	13,000	≥4

Tabla 3. Actividad Histórica y geológica del volcán Citlaltépetl tomada de De la Cruz-Reyna y Carrasco-Núñez (2002).

EL VOLCÁN CHICHÓN

El volcán Chichón se localiza en Chiapas, en las coordenadas 17.36°N y 92.23°W, con una altitud de 1,260 msnm. A nivel regional se localiza dentro de la Provincia de Fallas Transcurrentes de Chiapas, la cual está caracterizada por una serie de bloques levantados y hundidos, delimitados por grandes fallas con movimiento lateral izquierdo. Localmente, el volcán se encuentra emplazado en rocas sedimentarias que varían en edad del Cretácico superior al Mioceno medio. Estudios estratigráficos recientes de los productos eruptivos del Volcán Chichón indican que durante los últimos 8,000 años ha tenido al menos 12 erupciones explosivas incluyendo la erupción de 1982 (Tabla 4). Los productos juveniles de estos eventos han mantenido una composición traquiandesitica con pocas variaciones importantes de composición (Espíndola et al. 2000). En marzo y abril de 1982, este volcán presentó actividad, con erupciones explosivas, abundante lluvia de cenizas, y flujos piroclásticos que destruyeron el domo del cráter, causando numerosas pérdidas humanas y miles de damnificados, daños a tierras cultivables, y pérdidas de ganado. Adicionalmente el Chichón inyectó a la atmósfera una enorme cantidad de material; partículas líquidas y sólidas formaron una densa nube que se extendió rápidamente hacia el oeste y en tres semanas ya formaba un cinturón alrededor del mundo (Carey y Sigurdsson, 1986). Por su historia eruptiva y la actividad presente, se considera indispensable mantener un monitoreo que permita reconocer en forma oportuna cualquier actividad anómala que represente un riesgo (De la Cruz Reyna y Martin Del Pozzo, 2009).

Años AP	VEI
25	5
550	4
900	3
1250	4 ~ 5
1500	3
1600	2 ~ 3
1900	2 ~ 3
2000	2 ~ 3
2500	2 ~ 3
3100	-
3700	4
7500/7700	3

Tabla 4. Actividad eruptiva del volcán El Chichón (Duffield et al 1984; Espíndola et al. 2000; Macías et al. 2003; 2007; 2008). Para la erupción de 1270 años AP se ha adaptado un VEI 5 basado en comunicación personal por J.L. Macías y J.M. Espíndola quienes han realizado trabajo de campo sobre los depósitos el Chichón y consideran que tal erupción es a lo menos igual de grande que la erupción de 1982.

EL VOLCÁN NEVADO DE TOLUCA

El Nevado de Toluca es un estratovolcán andesítico-dacítico del Pleistoceno-Holoceno ubicado en 19.01° N; 99.76° W; con 4,680 msnm, localizado a 80 km de la ciudad de México en la parte central del Cinturón Volcánico Trans-mexicano. El Nevado de Toluca es un volcán activo en estado de quietud desde hace aproximadamente 3,100 años. La actividad volcánica comenzó hace 2.5 Ma con el emplazamiento de flujos de lava andesiticos. Durante su historia ha tenido erupciones tanto de tipo efusivo como de tipo explosivo (Cantagrel et al. 1981). Una intensa actividad efusiva construyó el edificio volcánico primitivo hace aproximadamente 1.6 Ma. A partir de los últimos 42,000 años AP, el Nevado de Toluca cambió su estilo eruptivo dando origen a cuatro erupciones plinianas y sub-plinianas intercaladas al menos por cuatro eventos de tipo peleano con destrucción de domos dacíticos. El más reciente periodo de actividad intensa, pero no continua, duro 2,500 años desde 13 a 10.5 ka. Los depósitos de la erupción pliniana de mayor magnitud ocurrida hace 10,500 años (mejor conocida como la Pómez Toluca Superior), tuvieron su eje de dispersión principal hacia el NE. La erupción más reciente del volcán ocurrió hace 3,100 años y dejó depósitos de oleada y flujos piroclásticos hasta una distancia mínima de 10 km (Tabla 5). En el presente, la ocurrencia de una actividad pliniana representa el peligro mayor en el área debido a la magnitud y frecuencia de los eventos anteriores. Con base en las características de las erupciones pasadas Aceves et al. (2007) realizaron 5 mapas de peligro para el Nevado de Toluca.

Erupción	Años AP	VEI	Referencias
1	3,300		Macías et al. 1997
2	10,500	5	Arce et al. 2003
3	12,100	4	Cervantes 2001/Arce et al. 2005
4	13,000	4	Arce et al. 2006/ D´Antonio et al. 2008
5	21,700	4	Capra et al. 2006
6	28,000	4	García-Palomo et al. 2002/ D´Antonio 2008
7	36,000-39,000		García-Palomo et al. 2002
8	37,000		García-Palomo et al. 2002
9	42,000		Arce et al. 2003

Table 5. Actividad eruptiva para el volcán Nevado de Toluca.

ESTRUCTURA DE ESTA TESIS

El presente trabajo esta dividido en 5 capítulos el primero de ellos la introducción y los que a continuación se describen:

En el capítulo 2 se propone y describe una metodología estadística basada en la teoría de los Valores Extremos y los procesos puntuales de Poisson-no homogéneos para estimar el peligro volcánico. Primero, se realiza un análisis exploratorio de las series eruptivas haciendo uso de datos geológicos e históricos para reducir el problema de la escasez de datos eruptivos. La distribución Weibull es utilizada para analizar y ajustar los tiempos de espera entre ocurrencias eruptivas. Posteriormente se aplica un proceso de Poisson no-homogéneo con la Distribución Generalizada Pareto como función de intensidad. Se aplica esta metodología a los volcanes mexicanos: Colima, Popocatépetl, Citlaltépetl, El Chichón y El Nevado de Toluca.

En el capítulo 3 se propone y describe una metodología estadística alternativa para estimar el peligro volcánico basada en una función Mixta de Exponenciales, cuando la serie eruptiva evoluciona marcando regímenes en el tiempo. Esta metodología es aplicada a los volcanes Colima y Popocatépetl.

En el capítulo 4 se estima el peligro volcánico obtenido por las dos metodologías (PPNHP y MEDE) propuestas en los capítulos anteriores y se compara con metodologías convencionales como el Proceso de Poisson y la distribución de Weibull. La aplicación es llevada a cabo sobre diferentes periodos en la serie de tiempo eruptiva del volcán El Chichón.

En el capítulo 5 se exponen las conclusiones respecto a las metodologías estadísticas para la evaluación del peligro volcánico propuestas en el presente trabajo.

Capítulo 2.

Método estadístico para la estimación del Peligro Volcánico a partir de series de tiempo de erupciones geológicas e históricas: Aplicaciones a volcanes poligenéticos activos.

"A Statistical Method Linking Geological and Historical Eruption Time–Series for Volcanic Hazard Estimations: Applications to Active Polygenetic Volcanoes".

Ana Teresa Mendoza-Rosas, Servando De la Cruz-Reyna. JOURNAL OF VOLCANOLOGY AND GEOTHERMAL RESEARCH. 176, 277-290, 2008.

RESUMEN

Las secuencias o series de tiempo de las erupciones volcánicas, definidas como un conjunto o sucesión de valores que indican como se distribuye la dimensión de las erupciones en el tiempo se construyen a partir de la información geológica y de registros históricos. Esta información adecuadamente procesada permite estimar el peligro volcánico con alta confiabilidad. Las secuencias de erupciones volcánicas representan procesos de gran complejidad, a lo que se añade la escasez de datos de las erupciones, la incertidumbre en la asignación de magnitud VEI (*Índice de Explosividad Volcánica*) y la diferente naturaleza y escala de tiempo entre la información geológica e histórica. El peligro volcánico, probabilidad de ocurrencia de algún fenómeno volcánico en un intervalo de tiempo específico dentro de un área dada, es estimado por medio de métodos estadísticos basados en esta información. No obstante, aplicar metodologías convencionales a este tipo de secuencias eruptivas podría acarrear errores significativos en la estimación del peligro volcánico. La teoría de valores extremos y los procesos puntuales no homogéneos, posibilitan el tratamiento de series o secuencias eruptivas a partir de la combinación de la información geológica con los datos históricos.

La exploración de los datos es el primer paso para conocer el comportamiento de la serie eruptiva. Ésta consiste principalmente en analizar la dependencia en el tiempo y la independencia entre erupciones. La incertidumbre en la magnitud VEI es resuelta probando

diversos modelos de magnitud y utilizando una relación logarítmica entre la tasa de ocurrencia de erupciones y la magnitud VEI (Servando De la Cruz, 1991; 1993). La distribución de Weibull es utilizada para analizar y ajustar los tiempos de espera entre erupciones. La metodología propuesta para estimar el peligro volcánico consiste en un proceso de Poisson no-homogéneo con distribución generalizada Pareto como función de intensidad (NHGPPP: Non-homogeneous generalized Pareto–Poisson process) permitiendo enfatizar el efecto de la ocurrencia de valores extremos de magnitud de la serie eruptiva.

Esta metodología enfrenta el problema de homologar datos de distinta naturaleza y escala en el tiempo, la probable no-homogeneidad y escasez de los datos de la serie, permitiendo obtener una mejor estimación de peligro volcánico utilizando una base de datos combinada de las series histórica y geológica. Las erupciones de magnitudes altas son de gran interés para la población por el riesgo que estas implican, siendo posible analizarlas con los datos geológicos que por lo general corresponden a erupciones grandes ya que depósitos de pequeñas erupciones generalmente no se preservan.

La estimación del peligro volcánico es el primer paso para evaluar el riesgo volcánico e implementar medidas de prevención de un desastre, como mapas de peligro, rutas de evacuación, etc. Por lo tanto, la metodología propuesta es aplicada a 5 volcanes poligéneticos mexicanos, el Volcán de Colima, Popocatépetl. El Chichón, Citlaltépetl o Pico de Orizaba, y El nevado de Toluca.

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A statistical method linking geological and historical eruption time series for volcanic hazard estimations: Applications to active polygenetic volcanoes

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ABSTRACT

The probabilistic analysis of volcanic eruption time series is an essential step for the assessment of volcanic hazard and risk. Such series describe complex processes involving different types of eruptions over different time scales. A statistical method linking geological and historical eruption time series is proposed for calculating the probabilities of future eruptions. The first step of the analysis is to characterize the eruptions by their magnitudes. As is the case in most natural phenomena, lower magnitude events are more frequent, and the behavior of the eruption series may be biased by such events. On the other hand, eruptive series are commonly studied using conventional statistics and treated as homogeneous Poisson processes. However, time-dependent series, or sequences including rare or extreme events, represented by very few data of large eruptions require special methods of analysis, such as the extreme-value theory applied to non-homogeneous Poisson processes. Here we propose a general methodology for analyzing such processes attempting to obtain better estimates of the volcanic hazard. This is done in three steps: Firstly, the historical eruptive series is complemented with the available geological eruption data. The linking of these series is done assuming an inverse relationship between the eruption magnitudes and the occurrence rate of each magnitude class. Secondly, we perform a Weibull analysis of the distribution of repose time between successive eruptions. Thirdly, the linked eruption series are analyzed as a non-homogeneous Poisson process with a generalized Pareto distribution as intensity function. As an application, the method is tested on the eruption series of five active polygenetic Mexican volcanoes: Colima, Citlaltépetl, Nevado de Toluca, Popocatépetl and El Chichón, to obtain hazard estimates.

1. Introduction

Volcanic activity usually results from the interaction of many independent physical and geological processes acting over different time scales. The occurrence of volcanic eruptions may depend on the unknown nature of magma feeding from deeper sources, as well as the conditions of a previously resident magma, the nature of the magma mixing processes, the regional stresses, the local crustal composition and structure, the fluid distribution and composition under the volcano, the degree of fracturing, and even on some meteorological agents. These and other factors interact in complex ways introducing a random behavior on the time series of volcanic eruption occurrences.

On the other hand, volcanic eruptions may represent a serious threat on the people dwelling near a volcano, particularly when their perception of risk is negatively influenced by a large repose time, or by the lack of clear evidences of major past activity. Volcanic risk was first formally defined in UNDRO (1979) as a measure of the expected number of lives lost, persons injured, damage to property and disruption of economic activity as a result of a particular volcanic event. It was defined as the product of volcanic hazard, vulnerability and elements at risk (Fournier d'Albe, 1979). The volcanic hazard is consistently defined as the probability that a specific type of volcanic eruption occurs in a given area, within a given interval of time (De la Cruz-Reyna and Tilling, 2008) The volcanic risk is thus the probability of losing a certain percent of the value of a given exposed region over a given time interval caused by the possible occurrence of a particular volcanic eruption. Therefore, knowing the hazard allows designing adequate measures to reduce the risk through specific actions of vulnerability reduction.

Under the assumption that the past history of a volcano should reflect at least some relevant features of its expected future behavior, a careful analysis of the time series of past eruptions, that accounts for the scarcity of precise past eruption data, is essential to assess the hazard. The behavior of volcanic eruption time series of individual volcanoes shows a wide spectrum of possibilities. Some volcanoes show stationary patterns of activity, while others show time-dependent eruption rates. Nevertheless, combining the eruptions of large groups of volcanoes generates a definite homogeneous Poissonian behavior, as is the case of the overall global eruptive activity (De la Cruz-Reyna, 1991).

Early studies of volcanic time series were done by Wickman (1965, 1976) and Reyment (1969) used stochastic principles for the study of eruption patterns on specific volcanoes. However, the models presented

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by Wickman did not distinguish among eruption of different types, and as he stated in his 1976 paper, such models were not tested against observed records. Other studies, analyzed specific volcanic eruption series, as was the case of the Hawaiian volcanoes (Klein, 1982) or Colima (De la Cruz-Reyna, 1993; Solow, 2001). Bebbington and Lai (1996a,b) examined whether the Weibull renewal model was adequate to describe the patterns of two New Zealand volcanoes.

Subsequent studies became increasingly sophisticated including for instance transition probabilities of Markov chains (Carta et al., 1981; Aspinall et al., 2006; Bebbington, 2007), change-point detection techniques (Mulargia et al., 1987; Burt et al., 1994), Rank-order statistics (Pyle, 1998), Bayesian analysis of volcanic activity (Ho, 1990; Solow, 2001; Newhall and Hoblitt, 2002; Ho et al., 2006; Marzocchi et al., 2008), non-homogeneous models (Ho, 1991a; Bebbington and Lai, 1996b), a mixture of Weibull distributions (Turner et al., 2007), and geostatistical hazard-estimation methods (Jaquet et al., 2000; Jaquet and Carniel, 2006).

Different parameters have been used as random variables to characterize the eruptive time series. Among them, the most frequently used are: the duration of eruptions, the interval between eruptions, the effusion rate; the volume or mass released, and the intensity of eruptions.

The probabilities of occurrence of future eruptions, and thus the volcanic hazard, may be estimated analyzing the sequence of past eruptions in a volcano, characterizing the eruptions by a measure of size that reflects their destructive potential, and assuming that the impact and effects of an eruption are proportional to both, the total mass or energy release (magnitude) and the rate of mass or energy release (intensity). The Volcanic Explosivity Index VEI is the quantity that characterizes eruptions based on those parameters (Newhall and Self, 1982). Frequently, an eruption has been defined ambiguously as a sudden, violent discharge of volcanic material, as well as a gentle, protracted pouring of lava or fumes. For our purpose we shall consider here only significant explosive eruptions, which usually are short-duration events when compared with the time between eruptions (also referred as repose time, even if minor or gentle effusive activity occurs). The volcanic eruption sequences of polygenetic volcanoes are thus considered here as point processes developing in the time axis, and the distribution of eruptions and the repose times between them are analyzed in different VEI categories or classes.

On the other hand, merging historical (usually describing more frequent smaller eruptions) and geological (usually describing larger, infrequent eruptions) eruptive data has been pointed as an important factor for a proper estimation of the likelihood of more damaging events (Marzocchi et al. 2004).

In this paper we propose a statistical methodology for estimating the volcanic hazard of future explosive eruptions using VEI — characterized sequences linking historical and geological records to obtain robust volcanic eruption time series. We first test the independence between successive eruptions to detect possible memory effects, and the stationarity, or time dependence of the explosive eruption sequences to find a possible non-homogeneity of the process. We then use a Weibull analysis to study the distribution of repose times between successive eruptions, and a non-homogeneous generalized Pareto–Poisson process (NHGPPP, as defined below) to obtain volcanic hazard estimations. We apply this method to Colima, Citlaltépetl, El Chichón, Nevado de Toluca and Popocatépetl volcanoes in México. Finally, the hazard estimates obtained with this and other methods are discussed and compared.

2. Methodology

The first step is testing the eruptive time series for independence between successive events and for the time dependence or stationarity of the process. The independence test is simply made by means of a serial correlation scatterplot (Cox and Lewis, 1966). The latter test is performed examining the repose period series for each VEI category and using a moving average test that reveals the possible existence of significantly different eruption rates, not attributable to the local rate changes expected in a stationary random process (Klein, 1982; De la Cruz-Reyna, 1996). These tests should be performed on a portion of the time series that satisfies a criterion of completeness, i.e. a portion in which no significant eruption data are missing, which in most cases is the historical eruption data set of intermediate-to-high VEI magnitudes.

A second step is the Weibull analysis of the repose periods between eruptions, which allows a quantitative description of both, stationary and non-stationary time series through the distribution shape parameter. The time-independence tests applied on the portions of the series assumed to be complete do not guarantee that the whole of the series has been stationary over its whole length. Therefore, the third step involving the link between the historical, usually complete, and the geological, probably incomplete eruptive series requires of a method that makes the estimation of hazard less sensitive to such condition. We propose here as the best estimate of the volcanic hazard

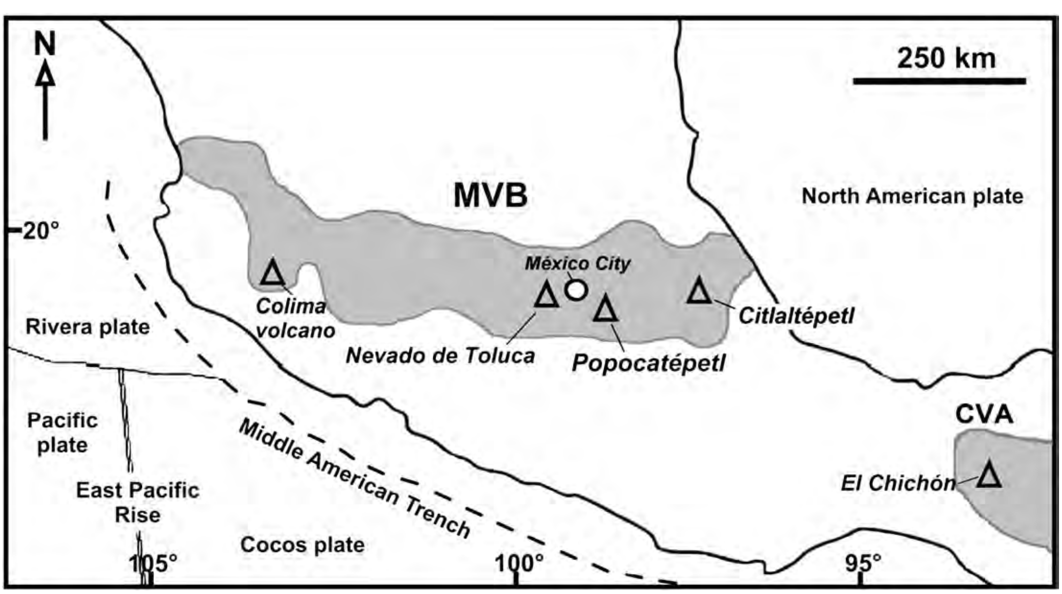

Fig. 1. Location of Colima, Nevado de Toluca, Popocatépetl, Citlaltépetl and El Chichón volcanoes.

Table 1
Historical eruptions of Colima volcano. (Adapted from De la Cruz- Reyna, 1993; Global Volcanism Program, http://www.volcano.si.edu; and Observatorio del Volcán de Colima; http://www.ucol.mx)

Year	VEI	Year	VEI
1560	2	1881	2
1576	3	1885	1
1585	4	1886	3
1590	3	1889	3
1606	4	1890	4
1611	3	1893	2
1612	2	1895	1
1622	4	1903	3
1690	3	1904	1
1749	2	1908	3
1770	3	1909	2
1795	2	1913	4
1804	2	1960	1
1818	4	1975	1
1869	3	1987	1
1872	3	1991	1
1873	1	1994	2
1874	1	1999	2
1877	1	2003	2
1879	1	2005	3
1880	1		

from the integrated time series a procedure based on the use of a non-homogenous Poisson process with a generalized Pareto distribution (GPD) as intensity function, referred herein as a non-homogeneous generalized Pareto–Poisson process (NHGPPP) (Coles, 2001). Such a process describes a series of independent, non-overlapping physical events occurring in a space A with an intensity density $\lambda(x_i)$, where x_i are the A-domain variables in which process develops. In our case, x_i are the coordinates t (time) and eruption size (VEI: magnitude–intensity) of a two-dimensional space, where the domain is limited by the historical and geological available eruption data. The process is homogeneous if λ is constant. If λ depends on either variable, the process is non-homogeneous.

Hazard and risk estimates based on catalogues assumed to be complete may include a very few or not include at all "rare" large events, with very low or unknown occurrence rates. Hazard estimates from such databases may be underrated. On the other hand, dealing with extreme values means using a set containing very few, probably incomplete data. These few data, most likely extracted from the geological record, are represented by the right tail of the repose-time distribution. Although they may have a little influence on the distribution itself, they certainly should have a significant influence on the hazard estimation. The GPD is a robust tool which allows modeling extreme values, such as the "rare", very high-magnitude eruptions, allowing for a better fit of the whole distribution. Additionally, it is less sensitive to the possible time dependence of the large-magnitude eruption sequence, since it only considers the number of exceedances over a threshold of a series that may be stationary or not.

2.1. Historical and geological time series of volcanic magnitudes

The occurrence behavior of explosive eruptions is similar to that of earthquakes, (and many other natural phenomena) in the sense that the frequency of the events decreases as their size or magnitude increases. In the case of earthquakes, the distribution of magnitudes in a region can be described by the frequency–magnitude distribution of earthquakes based on the Ishimoto and Iida (1939) and Gutenberg-Richter (1944) law

$$\log\ N = A - BM \tag{1}$$

where N is the cumulative number of earthquakes with magnitude $\geq M$, and A and B are constants that describe the power law decay of occurrences with increasing magnitude over a given time interval.

The analysis of historical data classified by VEI (Newhall and Self, 1982) shows that the VEI magnitude M_{vei} of N events may be represented as a random variable with a distribution function $N=a10^{-bM_{\text{vei}}}$. When the eruption data are analyzed in terms of the number of eruptions occurring over time intervals, this relation may be more clearly expressed in terms of the eruption occurrence rate of each class magnitude $\lambda_{M_{\text{vei}}}$ as

$$\log\lambda_{M_{\text{vie}}} = a - bM_{\text{vei}}. \tag{2}$$

Notice that Eq. (2) relates the eruption size M_{vei} with the eruption rate (number of eruptions per unit time in the magnitude class M_{vei}) unlike Eq. (1), which relates the cumulative number of earthquakes exceeding a certain magnitude. Using the cumulative number of eruptions can be used equally well, since in both cases the linearity of the relations is maintained, and only the values of the coefficients are different (see for instance Palumbo, 1997). In the present work, we prefer to use the non-cumulative occurrence rates of the VEI categories since they directly provide a more intuitive perception of the probabilities of occurrence of eruptions in each magnitude class.

Table 2
Dates and magnitudes of major Holocenic eruptions of Colima volcano inferred from eruption deposits. (Cortés et al., 2005)

Year BP	VEI
2300	>4
3600	>4
7040	>4

Table 3
The historical and geological eruptive activity of Citlaltépetl volcano. (De la Cruz-Reyna and Carrasco-Nuñez,2002)

	Year	VEI
Historical	1867	2
	1846	2
	1687	2
	1569–89	2
	1545	2
	1533–39	2
	Year BP	VEI
Geological	4100	≥4
	8500–9000	≥4
	13000	≥4

Table 4
Volcanic Explosivity Indexes of known eruptions of Popocatépetl volcano reported since the 16th century. (De la Cruz-Reyna and Tilling, 2008)

Year	VEI
1512	2
1519	3
1539–1540	2
1548	2
1571	2
1592	2
1642	2
1663	2
1664	3
1665	2
1697	2
1720	1
1804	1
1919–1920	2
1921	2
1925–1927	2
1994–1997	2
2000	3
2001–present	1–2

Table 5
The geological activity record of Popocatépetl volcano (Siebe and Macías, 2004)

Year B.P.	Eruption type
1200	Plinian
1700	Plinian
2150	Plinian
5000	Plinian
7100	Plinian
9100	Plinian
10,700	Plinian
14,000	Plinian
23,000	Plinian (sector collapse and massive debris flows)

De la Cruz-Reyna (1991) estimated the values a=3.494 and b=0.789 for the global volcanic activity based on the historical eruption data of Newhall and Self (1982) in the VEI range 3–6. Later, Simkin and Siebert (1994, 2000), integrated eruption data for various time intervals: 20, 200, 1000 and 2000 yr. Based on their Fig. 10 in volcanoes of the World (1994), the best fit for the eruption data in the VEI range 2–6 yields was graphically determined to be a=5.8 and b=0.785. Although a strongly depends on the length of the sampled period, the slope b seems to be a constant Gusev et al. (2003) obtain b=0.75 with the same graphical method on the Simkin and Siebert (1994) plot.

We can conclude that since the VEI is a composite estimate of mass magnitude and/or mass rate intensity, depending on which data are available, and considering that the VEI of many of the explosive eruptions listed are based on intensity related parameters (such as eruptive column height), the VEI is an appropriate parameter to characterize the eruption size for hazard calculation purposes.

The above analysis has been used on individual volcanoes to estimate the eruption rates for different VEI magnitudes (De la Cruz-Reyna, 1991, 1993; De la Cruz-Reyna and Carrasco-Núñez, 2002; De la Cruz-Reyna and Tilling, 2008) using both, the historical and the geological eruption records to obtain self-consistent series.

Here, we use Eq. (2) to link the historical and geological eruptive series.

We construct different models of the distribution of large events using the available geological information, and selecting the model which best fits the eruption rates obtained from Eq. (2). From the best estimates of the large-eruption rates, we may infer the number of eruptions that have exceeded a threshold. To calculate the probability of occurrence of more large-magnitude eruptions exceeding a threshold, we use the NHGPPP.

2.2. Repose period distribution

To analyze the characteristics of the repose periods of successive volcanic eruptions produced by a specific volcano in a given magnitude class, and particularly, the stationarity of the process, we use the Weibull distribution on the complete portion of the catalogue. This distribution has been widely applied in statistical quality control, reliability analysis of system components, earthquake hazard assessment, and many other applications (see for instance Johnson, 1966; Ferráes, 2003). It has also been used to model volcanic eruption sequences (Ho, 1991b, 1995; Bebbington and Lai, 1996b).

The 2-parameter cumulative Weibull distribution function is

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^k}, \quad (3)$$

where α is a scale parameter, and k is a shape parameter.

The shape parameter is of particular interest because it characterizes the failure rate trends, i.e., reflects the stationary or non-stationary character of the time series (Yang and Xie, 2003). There are different methods to estimate the Weibull parameters (Johnson and Kotz, 1953; Ho, 1991b, 1995). In the present paper, we obtain the distribution parameters using a fairly simple graphical method (Bebbington and Lai, 1996a). The probability of having a repose period of duration greater than t has been thus obtained form the survival function $1-F(t)$.

Table 6
The geological eruptive activity of Nevado de Toluca volcano

Eruption (name)	Years BP	VEI	References
1	3300		Macías et al. (1997b)
2 (UTP)	10,500	5	Arce et al. (2003)
3 (MTP)	12,100	4	Cervantes (2001)/Arce et al. (2005)
4	13,000	4	Arce et al. (2006)/D'Antonio et al. (2008)
5 (LTP)	21,700	4	Capra et al. (2006)
6	28,000	4	García-Palomo et al. (2002)/D'Antonio (2008)
7 (OPF)	36,000–39,000		García-Palomo et al. (2002)
8	37,000		García-Palomo et al. (2002)
9 (Pink)	42,000		Arce et al. (2003)

Table 7
Volcanic Explosivity Indexes of known eruptions of El Chichón volcano (Macías et al., 2007)

Years BP	VEI
25	5
550	4
900	3
1250	4
1500	3
1600	
1900	
2000	2–3
2500	2–3
3100	
3700	4
7500/7700	

2.3. Estimation of volcanic hazard using extreme-event statistics

In this section, the recent history, the geological record and the extreme-value techniques are used to obtain estimates of the probability of intermediate- to high-magnitude eruption events.

2.3.1. Extreme-value theory

Two methods are normally used to sample the original data of extreme events: the annual (or any other adequate time interval) maximum (AM), and the peaks or exceedances over a threshold (EOT) series. AM series are composed by the largest event occurring in a given sample time interval, so the series length equals the number of recording intervals. According to some theorems originally due to Fisher and Tippett (1928) and Gnedenko (1943), a series of sample maxima like an AM series can be described by a generalized extreme-value distribution, which includes the Gumbel, Fréchet and Weibull distributions (Gumbel, 1958). In the EOT analysis, the samples are not collected at fixed intervals and it has several important advantages over the AM approach, as it adapts better to heavy-tailed distributions and makes a more efficient use of information since it permits to include more cases through the choice of the threshold (Beguería, 2005). The EOT analysis includes all the values of the variable that exceed an a-priori determined threshold, u, defined by the transformed variable

$$Y = X - u, \quad (4)$$

for every case where $X>u$. The EOT considers all the excesses, i.e., the events above a certain level u. This level is fixed according to the model needs and provides a physically based definition of what must be considered an extreme event. Lang et al. (1999) reviewed different systematic methods for the choice of the threshold value, since it usually has a strong subjective component. Pickands (1975) demonstrated that if X is an identically independent distributed variable, a threshold value u can be found that makes the process converge to a generalized Pareto distribution (GPD). The GPD is described by a shape parameter β, a scale parameter θ, and a location parameter u (threshold), and has the following cumulative distribution function:

$$\begin{aligned} G_{\beta,\theta}(y) &= 1-\left(1-\frac{\beta y}{\theta}\right)^{1/\beta} && \text{for } \beta\neq 0, \\ G_{\beta,\theta}(y) &= 1-e^{-y/\theta} && \text{for } \beta=0 \end{aligned} \tag{5}$$

where $y=x-u$ is a realization of an excess. The generalized Pareto distribution has a relation with the generalized extreme-value (GEV) distribution Coles (2001). The Weibull distribution, which is a particular case of GEV to estimate extremes, needs a discrete time series generated sequentially at equidistant time intervals unlike the GPD. The GPD distribution contains the exponential distribution as a special case, when $\beta=0$ (second expression in Eq. (5)). For $\beta<0$ the distribution is long-tailed, and for $\beta>0$ it becomes upper-bounded with endpoint at $-\theta/\beta$. This condition should be used with caution unless there is physical evidence of upper bounding.

The estimated parameters of the GPD can characterize the mean value of the excesses. There are many methods (maximum likelihood, (ML), Coles, 2001; Reiss and Thomas, 2001; moments (MM), probability weighted moments (PWM), generalized probability weighted moments (GPWM), Hosking and Wallis, 1987; and others) to estimate the parameters. The fitting method should be chosen carefully because it may produce upper bound estimations which can be inconsistent with the observed data. For instance, Dupuis (1996) found inconsistencies in the GPD upper bound estimated parameters obtained with the MM and PWM fitting methods. This inconsistency occurs when one or more sampled observations exceed the estimated upper bound. The problem of inconsistency of a fitting method with the observed data requires appropriate attention (Simiu, 1995; Ashkar and Nwentsa Tatsambon, 2007).

2.3.2. The non-homogeneous generalized Pareto–Poisson process

The original development of this characterization is due to Pickands (1971); however, Smith (1989) was the first to convert the model into a tool for inference. The Generalized Pareto–Poisson Process consists of two components (Davison and Smith, 1990; Coles, 2001; Reiss and Thomas, 2001): (i) the occurrences of exceedances of some high threshold u (i.e., $X_i>u$, for some value of i) may be described as a Poisson process (with rate parameter λ_e) and (ii), the excesses over threshold u (i.e., X_i-u, for some i) have a GP distribution (with scale and shape parameters, θ and β). In the case of volcanic eruptions, the magnitude of the eruptions and the time of their occurrence, are viewed as points in a two-dimensional space, which formally is the realization of a point process (Cox and Isham, 1980). The intensity measure of this two-dimensional Poisson process on $B=[t_1,t_2]x(u,\infty)$ with $[t_1,t_2]\subset[0,1]$ is given by

$$\Lambda(B)=(t_2-t_1)\left[1-\frac{\beta(x-u)}{\theta}\right]^{1/\beta}, \tag{6}$$

where β, and θ are the parameters of the GPD (Eq. (5)) (Brabson and Palutikof, 2000; Lin, 2003). An important property of the GPD is the threshold stability (Hosking and Wallis, 1987). If $Y=X-u$ is a variable distributed like a GPD with a shape parameter β, it continues distributed like GPD with an identical shape parameter β for any higher truncation value $u+q$. Another related property of the GPD (Davison and Smith, 1990) refers to the mean excess: if $Y=X-u$ is a GP-distributed variable, then the mean excess over threshold u is

$$E(x-u|x>u)=\frac{\theta-\beta u}{1+\beta} \tag{7}$$

for $\beta>-1$, $u>0$ and $\theta-u\beta>0$. This implies that the conditional mean exceedance over a threshold, u, is a linear function of u. Furthermore, $E(x-u|x>u)$ is the mean of excesses of the threshold u, for which the sample mean of the excesses of u provides an empirical estimate. The sample mean excess is the sum of excesses over the threshold u divided by the number of data points which exceed u. The sample mean excess is an empirical estimate of conditional mean exceedances and β and θ of GPD can be determined by slope and intercept of sample mean excess plot. Hence, $E(x-u|x>u)$ is linear in u with slope $\frac{-\beta}{1+\beta}$ and intercept $\frac{\theta}{1+\beta}$. Davison and Smith 1990; Díaz, 2003; Lin, 2003; Beguería, 2005). The real data series at different threshold values can be tested by the mean excess plot, i.e. the plot of the average excess over a threshold against

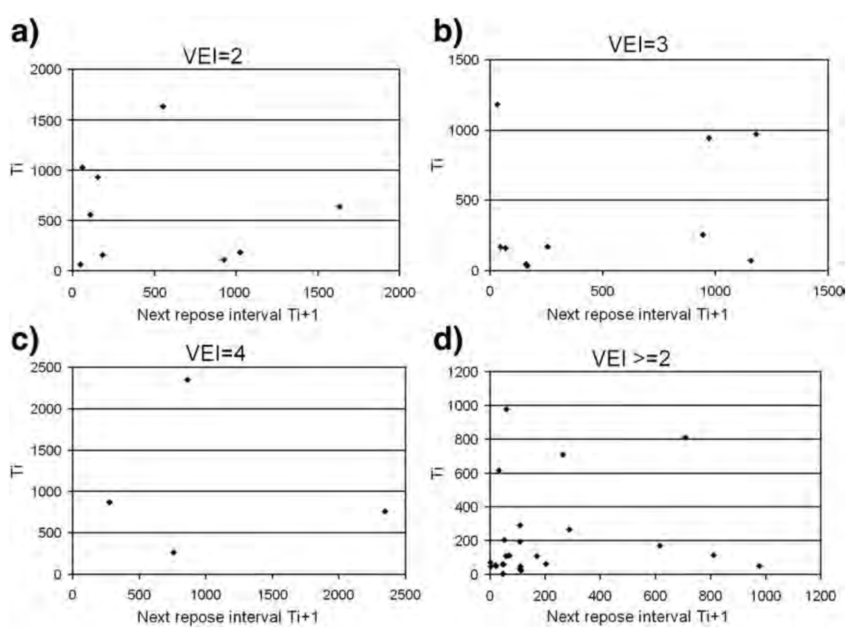

Fig. 2. Serial correlation diagram of successive repose intervals T_i and T_{i+1} (from Table 1, measured in months) between eruptions of Colima volcano with a) VEI = 2, b) VEI = 3, c) VEI = 4 and d) VEI ≥ 2. The highly scattered pattern indicates independent adjacent repose intervals.

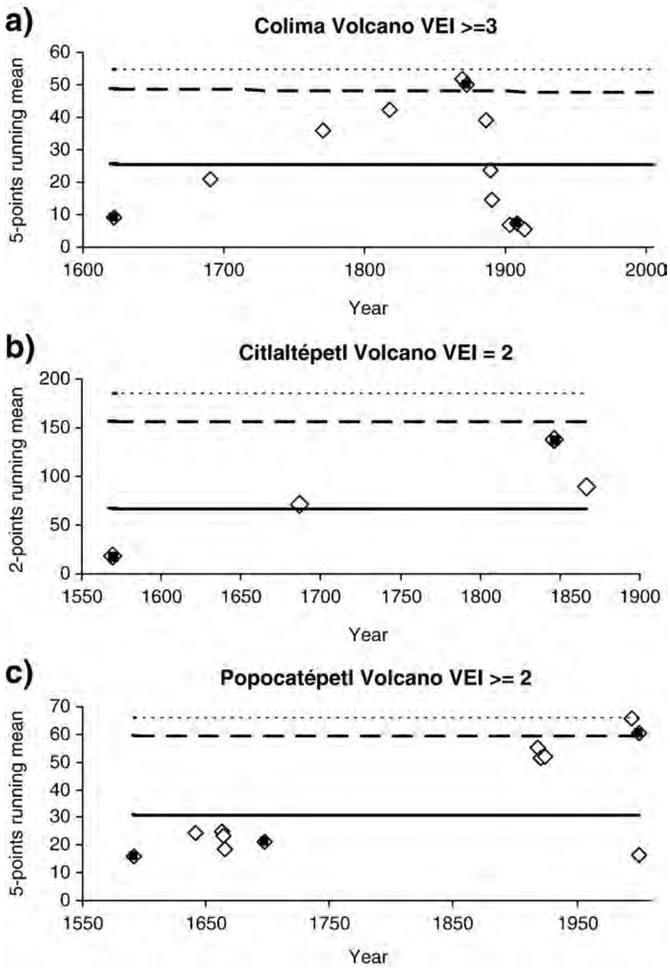

Fig. 3. Moving averages of consecutive reposes of a) Colima, b) Citlaltépetl, and c) Popocatépetl volcanoes. The diamonds represent averages of five (two for Citlaltépetl volcano) consecutive repose plotted at the date of eruption ending the fifth (second for Citlaltépetl volcano) repose interval making every fifth (second for Citlaltépetl volcano) point (filled square) independent (Klein, 1982; De la Cruz-Reyna, 1996). The solid horizontal line is the mean of all the repose periods; the thin dotted line represents the 95% upper confidence level, and the thick dashed line represents the 90% upper confidence level.

the value of the threshold, for a given set of threshold values. The sample mean excess plot is given by:

$$\overline{x}_u = \frac{\sum\limits_{i:x_i>u}(x_i - u)}{N_u} \tag{8}$$

Where N_u is the number of excess x_i over a threshold u (McNeil and Saladin,1997; Martínez, 2003; Lin, 2003).

The mean excess plot is also a diagnostic plot that should be drawn before fitting any model, providing guidance about what threshold to use. The key feature is that if Y is GPD then the mean excess over a threshold u, for any $u>0$, is a linear function (Eq. (7)) of u. Therefore, if the variable follows a GPD over a threshold value u, the mean excess plot should appear approximately linear at those points.

Table 8
Two possible models of the VEI magnitude distributions for the major Holocenic eruptions of Colima volcano

Years BP	Colima 1	Colima 2
2300	5	5
3600	5	5
7040	6	5

Table 9
Two possible models of the VEI magnitude distributions for the major geologic eruptions of Citlaltépetl volcano

Years BP	Citlaltépetl 1	Citlaltépetl 2
4100	4	4
8500	4	4
13,000	4	5

The generalized Pareto–Poisson process may be seen as a limiting form of the joint point process of exceedance times and excess values over the threshold.

3. Applications to active volcanoes

3.1. Case studies and data sets

We present here five case studies for the estimation of volcanic hazard with the proposed method. Colima, Nevado de Toluca, Citlaltépetl, Popocatépetl and El Chichón volcanoes (Fig. 1), are among the most active in México and they represent a significant threat to a large population dwelling in their neighborhoods. Except for the last, all of them are located in the Mexican Volcanic Belt, a Miocene–Quaternary province (Ferrari et al., 1999) that crosses the central part of México. El Chichón is located in the NW end of the Chiapas Volcanic Arc (CVA), which is associated with the subduction of the Cocos plate under the North American plate, but complicated by the geometry of the plate boundary fault system (Damon and Montesinos, 1978; Mora et al., 2007).

Colima volcano (19.512° N, 103.617° W) is the active volcano in México with the highest eruption rate, with a historical record (Table 1) of 41 eruptions in the past 500 years. For the present study, the records (De la Cruz-Reyna, 1993; Bretón et al., 2002) have been completed with recent data published in www.ucol.mx (Observatorio del Volcán de Colima) and www.volcano.si.edu (Global Volcanism Program, Smithsonian Institution).

The ancestral Colima volcano was formed in the late Pleistocene on the southern flank of Nevado de Colima, an older volcano located to the North of the current Colima volcano. About 10,700 B.P., (Cortés et al., 2005), this andesitic volcano rose to a presumed height of 4100 m. During a Bezimyanny–St Helens type eruption, the ancestral Colima volcano collapsed southwards, forming a 5-km-wide horseshoe-shaped caldera, and a massive volcanic debris avalanche deposit. This avalanche blanketed an area of about 1500 km^2, reaching up to 70 km from the former summit. The deposit has a volume estimated in 10 km^3. Soon after this avalanche, the currently active cone of Colima began to grow within the caldera. It is assumed that its mean magma production rate is about 0.3 km^3/1,000 yr, (Luhr and Carmichael, 1990). Table 2 lists the major holocenic eruptions of Colima volcano.

Citlaltépetl or Pico de Orizaba volcano (19.03°N, 92.27°W) is an ice-capped, andesitic, currently dormant, active stratovolcano. With an elevation of 5675 m a.s.l., it is one of the highest active volcanoes in North America. The Citlaltépetl record of historical activity, its high relief,

Table 10
Four possible models of the VEI magnitude distributions for the major geologic eruptions of Popocatépetl volcano

Years BP	Popocatépetl 1	Popocatépetl 2	Popocatépetl 3	Popocatépetl 4
1100	4	4	4	4
1700	4	4	4	4
2150	4	4	4	4
5000	4	4	4	4
7100	4	4	4	4
9100	4	4	4	4
10,700	4	4	5	5
14,000	4	5	5	6
23,000	5	5	5	5

Table 11
Three possible models of the VEI magnitude distributions for the historical and geologic eruptions of El Chichón volcano

	Reported	Chichón 1	Chichón 2	Chichón 3
Years BP	VEI	VEI	VEI	VEI
25	5	5	5	5
550	4	4	4	4
900	3	3	3	3
1250	4	4	4	4
1500	3	3	3	3
1600		3	3	3
1900		3	3	4
2000	3	3	3	3
2500	3	3	3	3
3100		4	3	5
3700	4	4	4	4

and the widespread distribution of its products are clear indications that it is a potentially high-risk volcano. In addition to this, over 750,000 people live within a radius of 40 km. As in many other cases, population growth around the volcano resulted from fertility of soils and a relatively low eruption rate. The evolution of Citlaltépetl can be divided into three distinct constructional stages (Carrasco-Núñez, 2000): (1) Growth of an ancestral large stratovolcano (Torrecillas cone), resulting from a predominantly effusive phase; (2) Construction of a superimposed cone (Espolón de Oro), and extrusion of various peripheral silicic domes; (3) Building of the present cone (Citlaltépetl) and emplacement of several overlapping dacitic domes. The historical and geological activity of Citlaltépetl is summarized in Table 3 (De la Cruz-Reyna and Carrasco-Núñez, 2002).

Popocatépetl volcano (19.02°N, 98.62°W) is located within a densely populated region about 70 km southeast of downtown Mexico City and 40 km west of the city of Puebla, with over 20 million people vulnerable to direct hazards associated with a major explosive eruption. Because of the large exposed population, this volcano is arguably the most risky in the country. This 5454 m a.s.l. volcano geologic past clearly indicates that it is capable of producing major catastrophic eruptions: three Plinian events have occurred within the past 5000 yr B.P., well within the period of human settlement in central Mexico (Siebe et al., 1996; Siebe and Macías, 2004). The modern cone of Popocatépetl consists of interlayered andesitic to dacitic lava flows and pyroclastic deposits (Robin, 1984), all erupted in the last 23,000 years following a cataclysmic eruption that included edifice collapse and debris avalanche emplacement (Siebe and Macías, 2004). Fortunately, the current eruptive episode – beginning in December 1994 after being dormant for nearly seven decades – has consisted to date of relatively minor activity which has characterized Popocatépetl's activity since the 14th century (De la Cruz-Reyna et al., 1995). The historical activity (De la Cruz-Reyna and Tilling, 2008) is shown in Table 4, and the geological record is summarized in Table 5 (Siebe and Macías, 2004).

Nevado de Toluca volcano (19.108°N, 99.758°W) is the fourth highest peak in Mexico with an altitude of 4680 m a.s.l (Fig. 1). It is located 21 and 80 km southwest of the cities of Toluca and Mexico, respectively, in the central sector of the Trans-Mexican volcanic belt. The volcano, which has no historical eruptions, has a complex geological record. Studies on the stratigraphy (Macías et al., 1997; Arce, 1999; Cervantes, 2001), the flank collapses (Capra and Macías, 2000), the structural geology (García-Palomo et al., 2000), and the overall geology (García-Palomo et al., 2002) have permitted to reconstruct an eruptive history, of which five major geological events listed in Table 6 have been widely recognized. The volcano has been quiet in the last 3.3 ka, but, as shown by their stratigraphic record, Plinian eruptions may occur after long periods of inactivity. Should this happen, it would represent a significant hazard at least for the entire Lerma basin, where more than 1 million people live (Capra et al., 2006).

El Chichón volcano (17.36°N, 92.23°W) has an elevation of 1100 m a.s.l. The volcano has in its summit a 1 km wide crater with gently dipping outward slopes. Its average depth is 140 m, and contains a central lake. The 1982 eruption of El Chichón volcano in southeastern Mexico had a strong social and environmental impact (Luhr 1991; Rampino and Self 1984; Schönwiese 1988). Recent studies on the stratigraphy of the volcano and new radiocarbon ages show that at least 11 eruptions prior to the 1892 eruption have occurred at El Chichón in the past 8000 years. Explosive events, most of them producing block-and-ash flow and surge deposits, occurred around 550, 900, 1250, 1500, 600, 1900, 2000, 2500, 3100, 3700 and 7700 yr BP. (Espíndola et al., 2000, Macías et al., 2007). Table 7 summarizes the historical and geological records.

3.2. Analysis of historical eruption series

3.2.1. Independence of repose interval durations between historical eruptions

In each of the cases, the independence of the repose intervals between eruptions of VEI categories high enough to assure completeness of the catalogs has been tested, by means of serial correlation scatterplots, in which the duration of each interval T_i+1 between two successive eruptions for each VEI magnitude category is plotted against the previous repose interval durations T_i. Diagrams showing large dispersion, a relatively high concentration of points near the axes, and very low correlation coefficients may be characterized as random process of independent events (Cox and Lewis, 1966).

For Colima volcano, the repose times in months between successive historical eruptions calculated from Table 1 are shown in Fig. 2. The correlation coefficients between set T_i and set T_i+1 for each VEI category 2, 3, 4, and ≥2 are: −0.096, 0.256, −0.156, and 0.152, respectively, indicating a very low serial correlation.

Similar scatterplots were calculated for historical data (complete portion) of the other volcanoes. The scatter diagrams also show large dispersions and higher concentrations of points near the axes. The correlation coefficient between the sets T_i and T_i+1 for Citlaltépetl volcano for VEI 2 is 0.00165 and for Popocatépetl volcano for VEI 2 and VEI≥2 categories is −0.174 and −0.097, respectively.

In all cases no significant correlation was found and thus independence may be assumed.

3.2.2. Test for stationarity of historical eruption sequences

To verify a possible non-stationary character of the eruption histories, and test for the existence of regimes with significantly different eruption rates, we performed a moving average test (Klein, 1982) on each of the historical series. Fig. 3a plots the moving averages of five consecutive reposes of Colima for the VEI≥3 eruptions. One point is plotted for each possible sequence of five successive reposes at the date of the last eruption of each sequence. The overall mean repose of 25.24 years is represented by the solid horizontal line. The probability

Table 12
Estimated magnitudes of the geologic eruptions of Colima, Popocatépetl, Citlaltépetl and El Chichón volcanoes based on the power law (2)

Colima 1		Popocatépetl 2		Citlaltépetl 1		El Chichón 2	
Year BP	VEI	Year BP	VEI	Year BP	VEI	Year BP	VEI
2300	5	1100	4	4100	4	25	5
3600	5	1700	4	8500–9000	4	550	4
7040	6	2150	4	13,000	4	900	3
		5000	4			1250	4
		7100	4			1500	3
		9100	4			1600	4
		10,700	4			1900	3
		14,000	5			2000	3
		23,000	5			2500	3
						1900	3
						2000	3
						2500	3

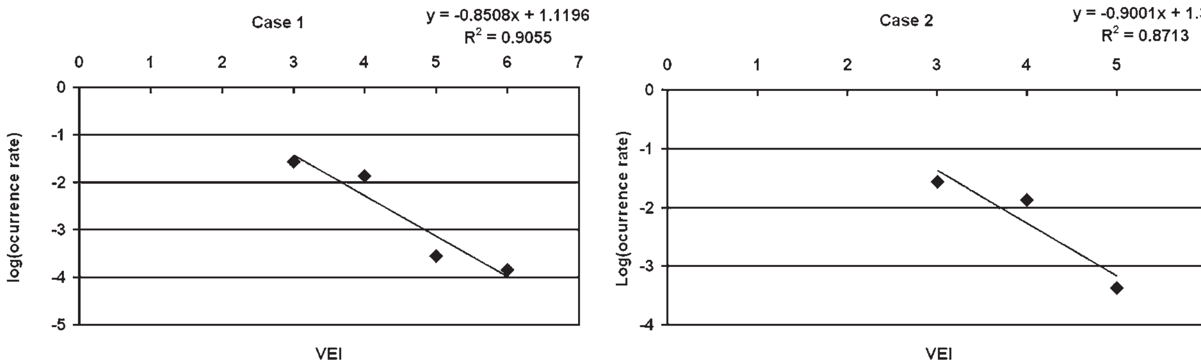

Fig. 4. Best linear fits of Eq. (2) for all the geological activity models of Colima volcano listed in Table 8. The lower magnitude eruption rates correspond to the historical record (Table 1). The highest regression coefficient corresponds to the case 1 (model "Colima 1").

that a given set of successive repose intervals departs from a certain amount from the mean by chance is represented by the thin dotted line (95% confidence level) and the thick (90% confidence level) dashed line. If a certain average departs from the average more than the amount indicated by the broken lines, one may say that the eruption sequence is non-stationary for the corresponding confidence level.

Error limits were computed from the combined chi-square and binomial distributions (Klein, 1982). Fig. 3a shows a weak non-stationary behavior for Colima volcano, because one of the points exceeds the 90% level, but not the 95% confidence level. Therefore, we cannot reject the hypothesis that Colima may have a non-stationary behavior, with at least two alternating eruption regimes (De la Cruz-Reyna, 1996).

A similar analysis of the sequence of available historical volcanic eruptions for Citlaltépetl volcano shows no evidence that the eruptive process for the VEI=2 magnitude eruptions may be non-stationary (Fig. 3b). On the other hand the analysis of Popocatépetl volcano for the VEI≥2 eruptions shows a weakly non-stationary eruptive series similar to the case of Colima volcano (Fig. 3c).

3.3. Analysis eruption series

The available information, from historical records (Tables 1, 3 and 4), and from the deposits of major eruptions at Colima (Table 2), Citlaltépetl (Table 3), Popocatépetl (Table 5), Nevado de Toluca (Table 6) and El Chichón (Table 7) volcanoes is not sufficient to assign precise VEI values to all the eruptions, although some constrictions may be set on their relative sizes. Tables 8–11 show sets of likely values (models) of the VEI of those eruptions. For Nevado de Toluca, we use a single model based on only five eruptions occurred between 28,000 and 10,500 yr B.P., using published VEI data, or estimating them from erupted volume values reported in the references cited in Table 6. In the other cases, the VEI's of eruptions in which no volume or intensity data were available were estimated probing the best fit to the VEI values of other eruptions based on the power law described by Eq. (2).

Fig. 4, illustrates the loglinear relationship between the VEI magnitudes and occurrence rates from Eq. (2) for the Colima volcano eruptive history models. The regression coefficients indicate that the best fit is for the case "Colima 1" of Table 8. Applying the same procedure to Popocatépetl, Citlaltépetl, and El Chichón, we conclude that the best estimations of magnitudes for the geologic records are, "Citlaltépetl 1", "Chichón 2" and "Popocatépetl 2" (Table 12).

3.3.1. Analysis of repose-time series

In this part of the study, we attempt to find the survival Weibull functions (Eq. (3)) that best fit the repose-time distributions of the volcanoes referred in Section 3.1. To do this we only use the VEI

Table 13
The Weibull distribution parameters for the indicated volcanoes

VEI	Shape parameter	Scale parameter
Colima		
2	0.52	3.03
3	0.85	3.89
4	0.41	2.48
>2	0.37	1.02
Citlaltépetl		
2	0.7	4.9
Popocatépetl		
2	1.01	2.75
3	1.47	19.26
Nevado de Toluca		
>3	1	1.44
El Chichón		
>2	1.19	3.97

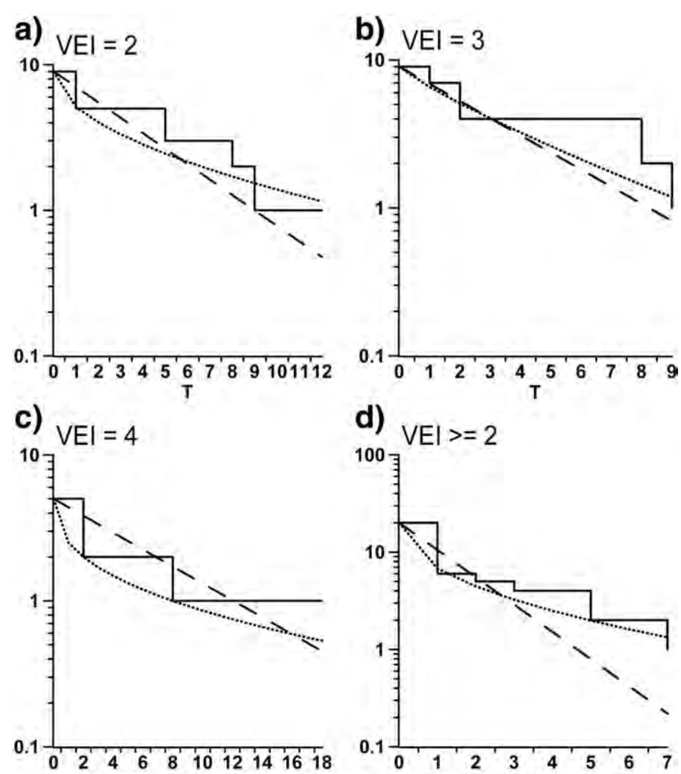

Fig. 5. Distribution of observed repose intervals for a) VEI=2, b) VEI=3, c) VEI=4, and d) VEI≥2 with duration greater than *T* decades (steps) for eruptions at Colima volcano in the period 1560 to present. The survival Weibull distribution (dotted line) better fits the data than the exponential distribution (dashed line).

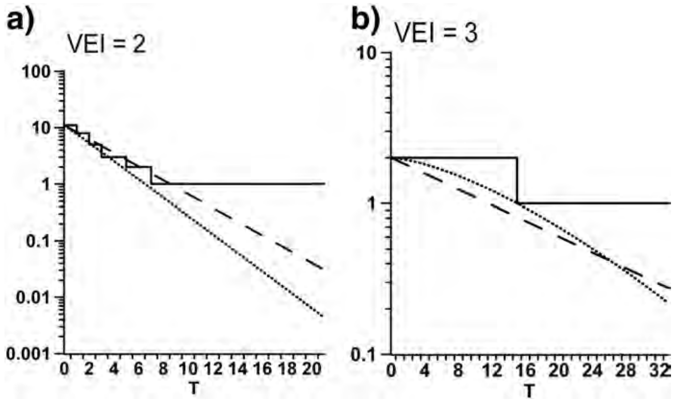

Fig. 6. Distribution of observed repose intervals with duration greater than *T* decades (steps) for a) VEI 2 and b) VEI 3 eruptions at Popocatépetl volcano in the period 1510 to present. The survival Weibull distribution (dotted line) shows a slightly better fitting than the exponential distribution (dashed line).

categories that made possible to consider the eruptive series as complete. Such eruptive series are the historical data listed in Tables 1, 3 and 4, and the geological series of Nevado de Toluca and El Chichón. Although In the last two cases it is difficult to sustain completeness, we are including the portions of Table 6 in which date and VEI data have been published for Nevado de Toluca, and El Chichón 2 model of Table 11 as an example of the Weibull representation of available data. The resulting Weibull distribution parameters are summarized in Table 13.

The comparison between the exponential and Weibull distributions is shown in Figs. 5–9. In various cases, the Weibull survival function provided better fits to the repose-time data than the exponential function, because the shape parameter accounts for the non-stationary character of some of the series. Stationary repose-time series may be equally well described by both distributions.

3.4. Assessment of volcanic hazard from geological and historical eruption series.

The volcanic hazards for Colima, Nevado de Toluca, Popocatépetl, Citlaltépetl and El Chichón, volcanoes were estimated using the *non-homogeneous generalized Pareto–Poisson* process described in a previous section. We use the number of excesses (Eq. (4)) inferred from the eruptive rates (Eq. (2)) of the geological and historical data (Tables 1, 3, 4, 6 and 12) to calculate the probabilities of occurrence of eruptions in the different magnitude classes.

First we use a graphical method to estimate the parameters from the linear regression (using Eq. (7)) of the plot of the mean of the excesses (obtained with Eq. (8)) vs their thresholds (Davison and Smith, 1990).

The linear fittings of the means of the excesses and the means of the exceedances vs their thresholds, obtained as described in Section 2.3, are illustrated in Fig. 10. The good fittings of the mean exceedances and the fair fittings of the mean excesses indicates that the method is satisfactory. The problem of the fair fitting of the mean excesses may be addressed considering not one, but two regression lines, one for the lower threshold values, and other for threshold 4 and above, and calculating the NHGPPP parameters for each line. However, in this case we have used the single mean excess lines for the

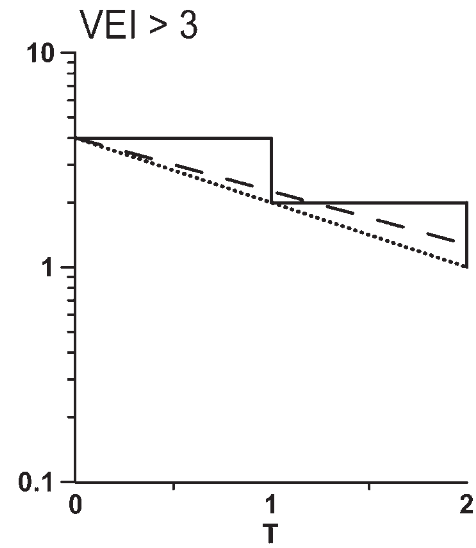

Fig. 8. Distribution of observed repose intervals (steps) with duration greater than *T* (in units of 4,000 years) for eruptions at Nevado de Toluca volcano in the last 28,000 years. The survival Weibull distribution (dotted line) fits the data better than the exponential distribution (dashed line).

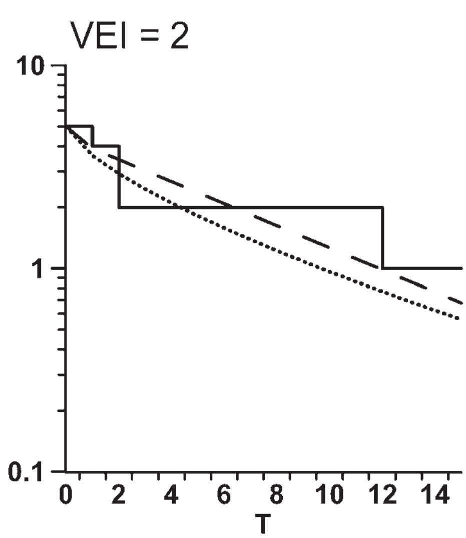

Fig. 7. Distribution of observed repose intervals with duration greater than *T* decades (steps) for VEI=2 eruptions at Citlaltépetl volcano in the period 1530 to present. The survival Weibull distribution (dotted line) and the exponential distribution (dashed line), show similar degrees of fitting.

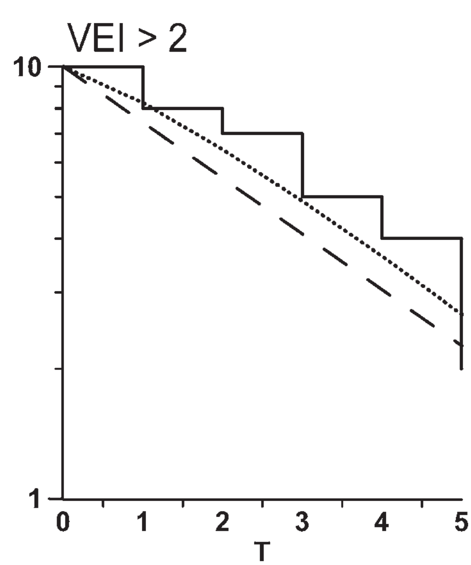

Fig. 9. Distribution of observed repose intervals (steps) with duration greater than *T* (in units of 100 years) for eruptions at El Chichón volcano in the last 3700 years. The survival Weibull distribution (dotted line) fits the data better than the exponential distribution (dashed line).

probability calculations. The Pareto generalized parameters for each volcano are summarized in Table 14.

We used Eq. (6) to calculate the intensity of the NHGPPP and to obtain the probabilities of eruption occurrences. Since the approach of exceedances do implicitly assume that the scale measuring the phenomena is open, and considering that the VEI scale ends in 8, we subtracted the probabilities of eruptions exceeding that magnitude from the probabilities of exceeding VEI's lower than 8. Table 15 and Fig. 11 show the probabilities of at least one eruption exceeding a given VEI occurring in the stipulated time intervals for each activity model. We also compare the results obtained with the NHGPPP with volcanic hazard estimates obtained from direct application of the Binomial and simple Poisson distributions for the same eruption series.

Inspection of these results shows that the probabilities of occurrence of eruptions in the lower magnitude classes, calculated with the method proposed here differ very little from the standard Binomial–Poisson methods. However, the probabilities of occurrence of eruptions exceeding moderate magnitudes are significantly increased. This difference arises from the added information that the GPD (Eq. (5)) introduces in the NHGPPP when the estimated eruption rates of large-magnitude eruptions are introduced.

4. Discussion and conclusions

The relatively simple methodology proposed in this paper allows the use of historical and geological eruption time series to obtain more precise estimates of the volcanic hazard. The method considers the limitations inherent to each of those series: short sample time, probable absence of large events and incomplete reporting of very small magnitudes for the historical series; incomplete reporting of small and intermediate magnitudes and uncertainties in the age and magnitude of major eruptions for the geological series. It also considers the

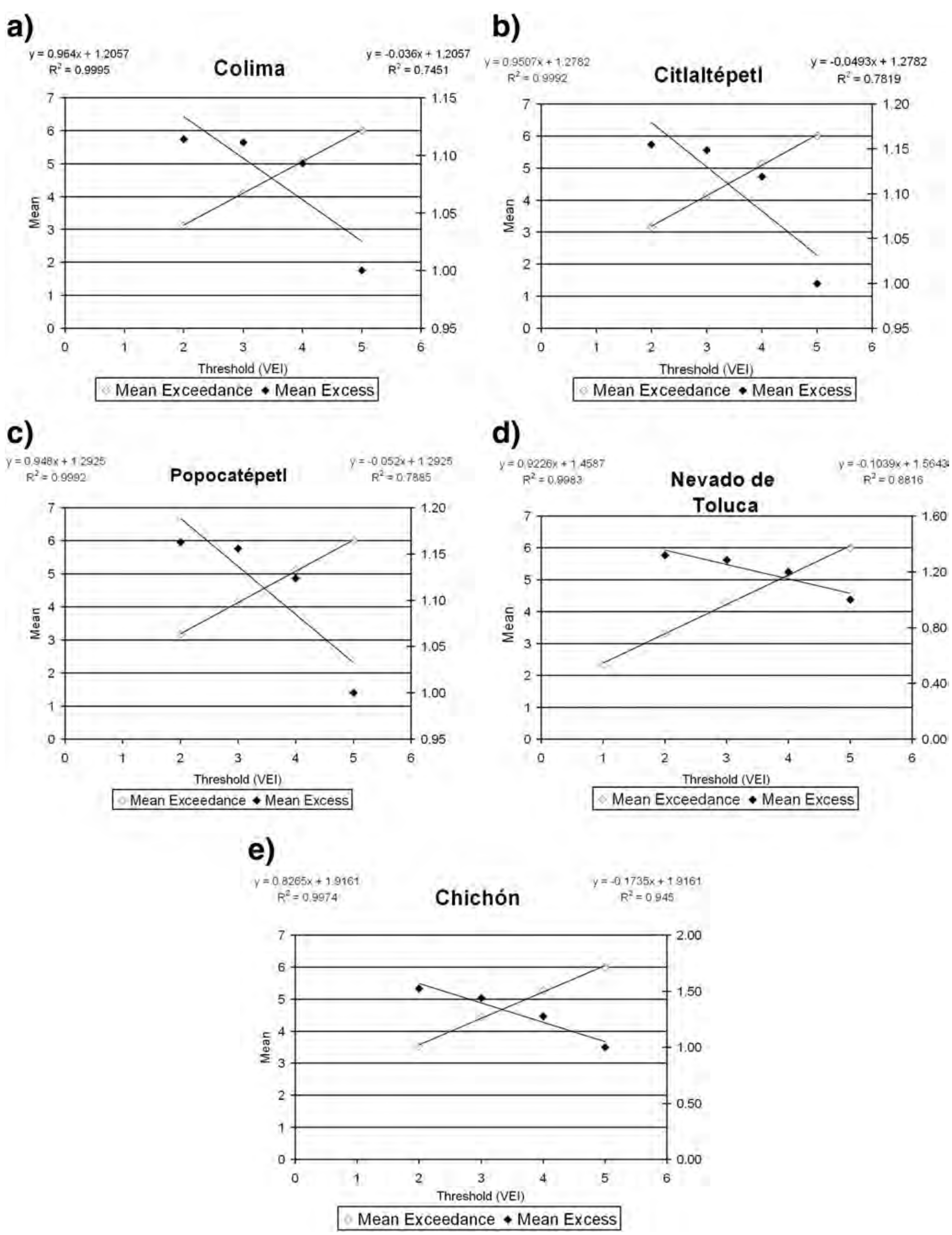

Fig. 10. Plots of exceedance and excess mean vs. threshold for a) Colima, b) Citlaltépetl, c) Popocatépetl, d) Nevado de Toluca and e) El Chichón volcanoes.

possibility of time-dependent eruption rates (non-stationary eruption series). These problems are addressed assuming a characteristic behavior of most natural phenomena: the inverse relationship between occurrence rate and magnitude. (De la Cruz-Reyna, 1991, 1996; De la Cruz- Reyna and Carrasco-Núñez, 2002). The use of relation (Eq. (2)), describing this behavior, permits linking the historical and geological time series, greatly expanding the data base, both in duration and eruption magnitude range.

If an eruptive series is non-stationary, the hazard estimate depends on the time it is made. For instance, the hazard evaluation of a volcano which shows a succession of high and low occurrence rate regimes may be inaccurate if only the current regime is taken into account. Thus determining the degree of time dependence of the occurrence rate is an essential step. The method proposed here provides a criterion to determine such a condition, and how to deal with it. The degree and nature of the non-stationarity is revealed by the Weibull analysis on the repose-time distribution between successive eruptions. The adjustable shape parameter of this distribution can describe variable eruptive rates, allowing a better description of the time-dependent distribution of reposes.

To deal with the difficulties derived from the lack of catalogue completeness of very low and very high eruption magnitudes in the historical series, and of low and intermediate magnitudes in the geological series, the hazard estimates for the full series is done linking both time scales using the power law distribution (Eq. (2)). Although the validity of such relationship for groups of volcanoes has proven effective (De la Cruz-Reyna, 1991), its use on individual volcanoes may be questioned (Marzocchi and Zaccarelli, 2006), particularly in the case of strongly time-dependent eruptive series, its applications to individual volcanoes showing a stationary or a quasi-stationary behavior using mean eruption rates calculated over periods long enough to include the representative rate variations, render satisfactory results as shown in the examples presented here. The method also allows testing different models of geologic past behavior, when the uncertainties of the date and magnitude of older eruptions are high, and search for the most likely combination of date-magnitude that is consistent with the more recent and reliable data, even for weakly time-dependent (quasi-stationary) series. In this respect, no problem raises form the historical and the geological records having overlapping VEI categories. For example, in the case of El Chichón we used the VEI 4 data in both subsets, with good fittings in both the mean exceedances and the mean excesses vs their thresholds.

Once a representative eruption rate has been determined, the volcanic hazard estimations based on extreme values are obtained using a NHGPPP. This method renders eruption probabilities of exceedance for each VEI magnitude category, i.e. hazard estimates that takes into account all the above mentioned factors, since it gives the appropriate weights to the more reliable (yet incomplete) historical data, and to the scarce large-magnitude geological data.

In general terms, the application of a NHGPPP in the final stage of the method emphasizes the effect of large magnitudes in the hazard estimation.

The application of this method to the eruption sequences of the Popocatépetl, Citlaltépetl and Colima volcanoes was compared with published results of other hazard estimates (De la Cruz-Reyna, 1993; De la Cruz-Reyna and Carrasco-Núñez, 2002; De la Cruz-Reyna and Tilling, 2008). Although the results were similar, a characteristic difference when comparing with the Poisson and Binomial distribution based hazard estimates was that the exceedance probabilities calculated with the NHGPPP were increasingly larger for VEI magnitudes greater than 3.

These results should thus be taken into account in the assessment of volcanic risk and in the design of prevention and response measures, particularly for major eruptions to which larger areas may be 100% vulnerable.

Table 14
The Pareto distribution parameters to calculate the NHGPPP for the indicated volcanoes

	Colima 1	Citlaltépetl 1	Popocatépetl 2	Nevado de Toluca	El Chichón 2
Shape parameter	0.037	0.052	0.055	0.116	0.143
Scale parameter	1.251	1.344	1.363	1.746	2.191

Table 15
Volcanic eruption hazards of Colima, Citlaltépetl, Nevado de Toluca, Popocatépetl and El Chichón volcanoes as probabilities of occurrence of at least one eruption exceeding a VEI magnitude over different time periods, for different models of the past activity. The probabilities were calculated using the NHGPP

Years	Colima 1	Popocatépetl 2	Citlaltépetl 1	Nevado de Toluca	El Chichón 2
VEI>2					
20	0.63290	0.10311	0.03851	0.01487	0.05835
50	0.90840	0.23792	0.09348	0.03675	0.13913
100	0.96840	0.41839	0.17810	0.07211	0.25758
500	0.87936	0.91781	0.62160	0.31095	0.74175
VEI>3					
20	0.35806	0.04982	0.01810	0.00816	0.03611
50	0.66361	0.11980	0.04461	0.02028	0.08759
100	0.86989	0.22482	0.08718	0.04012	0.16666
500	0.87935	0.70881	0.36435	0.18444	0.57378
VEI>4					
20	0.17236	0.02276	0.00812	0.00424	0.02118
50	0.37367	0.05587	0.02018	0.01056	0.05195
100	0.59816	0.10843	0.03992	0.02100	0.10071
500	0.87180	0.43014	0.18338	0.10030	0.39558
VEI>5					
20	0.07391	0.00974	0.00344	0.00204	0.01153
50	0.17328	0.02415	0.00857	0.00509	0.02848
100	0.31210	0.04763	0.01705	0.01015	0.05589
500	0.75173	0.21337	0.08197	0.04955	0.24023
VEI>6					
20	0.02802	0.00375	0.00131	0.00087	0.00554
50	0.06806	0.00934	0.00328	0.00217	0.01375
100	0.12975	0.01857	0.00654	0.00433	0.02718
500	0.44890	0.08817	0.03213	0.02138	0.12381

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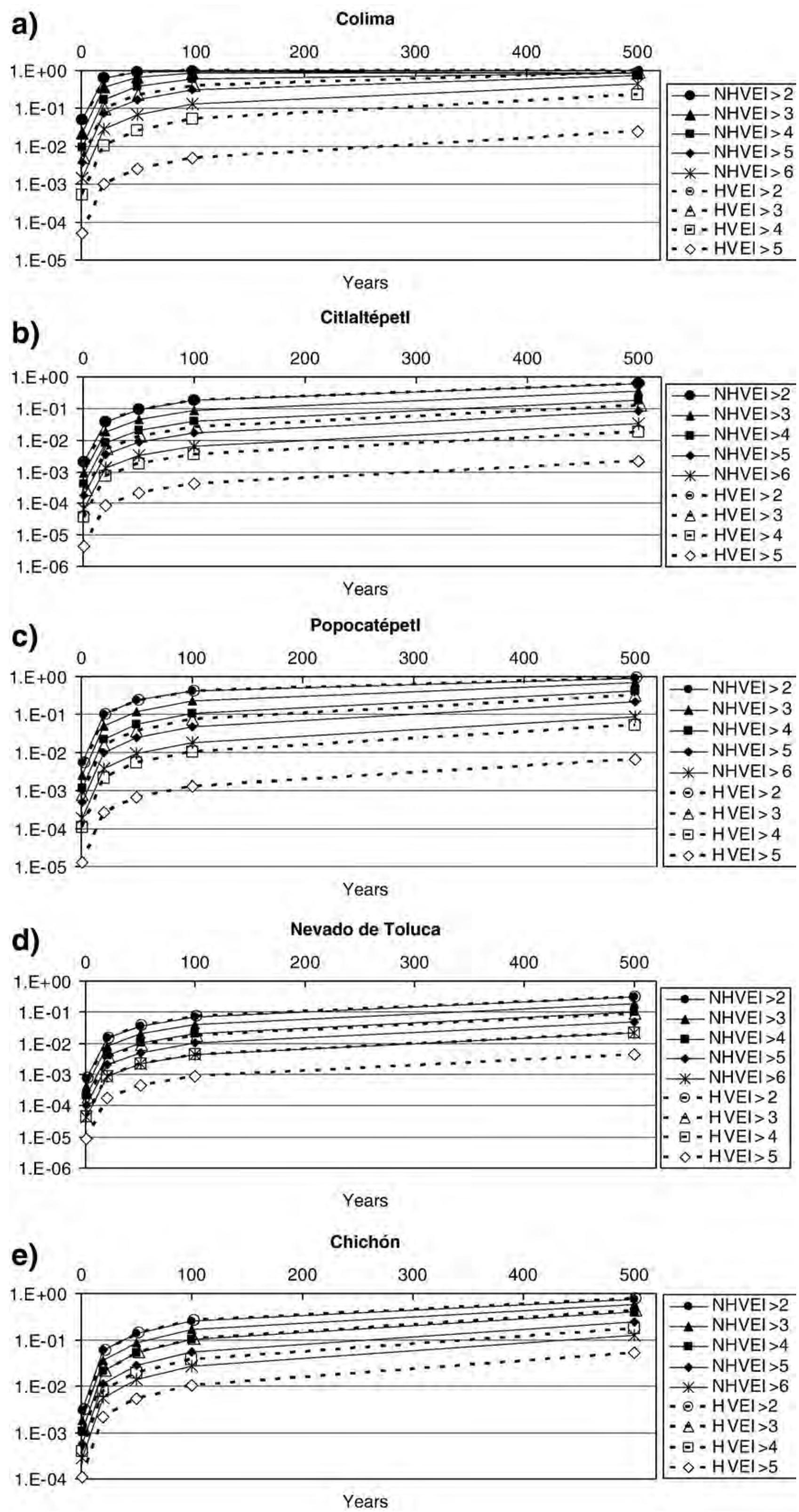

Fig. 11. Probabilities calculated with NHGPPP (filled symbols marked NH) and Homogeneous Poisson distribution (open symbols marked H) of at least one eruption, with a VEI magnitude greater than a given VEI threshold for a) "Colima 1", b) "Citlaltépetl 1" c) "Popocatépetl 2", d) Nevado de Toluca and e) "El Chichón 2" eruptive series and models.

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Capítulo 3.

Una distribución de mezcla de exponenciales para la evaluación simple y precisa del peligro volcánico.

"A mixture of exponentials distribution for a simple and precise assessment of the volcanic hazard".

Ana Teresa Mendoza-Rosas, Servando De la Cruz-Reyna.
NATURAL HAZARDS AND EARTH SYSTEM SCIENCES. 9, 425-431, 2009.

RESUMEN

El comportamiento de series de erupciones volcánicas puede mostrar agrupamientos de eventos, o regímenes en el tiempo mostrando una dependencia temporal de la tasa eruptiva. La aplicación de métodos convencionales para evaluar el peligro volcánico, como el proceso de Poisson simple que utiliza una tasa media de ocurrencia eruptiva, provocaría una subestimación o sobreestimación dependiendo del régimen en que nos encontramos debido a que el régimen alto considera una tasa de ocurrencia eruptiva más alta y el régimen bajo una tasa de ocurrencia más baja que la media respectivamente. Para evitar una evaluación errónea de peligro volcánico se propone una distribución mixta de exponenciales (MOED: Mixture of exponentials distribution), esto es, una suma pesada de distribuciones exponenciales donde cada sumando considera los diferentes regímenes observados. Los regímenes y parámetros de cada distribución exponencial, componentes de la distribución mixta de exponenciales, son identificados y evaluados a priori desde la distribución acumulativa de erupciones para diferentes categorías de valores VEI a partir de las historias eruptivas con tasas eruptivas que pueden variar en el tiempo. Esta metodología proporciona una evaluación de peligro volcánico simple y confiable que considera la dependencia en el tiempo o no estacionaridad. Es importante aclara que esta metodología debe ser aplicada a series de erupciones volcánicas que cumplan con la condición de completez para tener una evaluación de peligro volcánico correcta.

La distribución de mezclas de exponenciales es aplicada a las series eruptivas de los volcanes mexicanos, el volcán de Colima y el Popocatépetl para evaluar el peligro volcánico.

El comportamiento de la serie eruptiva del volcán de Colima muestra cambios de régimen donde las tasas eruptivas de cada régimen oscilan alrededor de la tasa media eruptiva confirmando la no estacionaridad de las erupciones históricas registradas. La estimación de peligro volcánico refleja la dependencia en el tiempo a comparación de otros métodos estadísticos como la distribución de Weibull y el proceso de Poisson.

La estimación de peligro volcánico para el volcán Popocatépetl muestra similitud con otros métodos debido a la débil dependencia del tiempo de la tasa eruptiva de este volcán confirmando un comportamiento más cercano a la estacionaridad.

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Natural Hazards and Earth System Sciences

A mixture of exponentials distribution for a simple and precise assessment of the volcanic hazard

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Abstract. The assessment of volcanic hazard is the first step for disaster mitigation. The distribution of repose periods between eruptions provides important information about the probability of new eruptions occurring within given time intervals. The quality of the probability estimate, i.e., of the hazard assessment, depends on the capacity of the chosen statistical model to describe the actual distribution of the repose times. In this work, we use a mixture of exponentials distribution, namely the sum of exponential distributions characterized by the different eruption occurrence rates that may be recognized inspecting the cumulative number of eruptions with time in specific VEI (Volcanic Explosivity Index) categories. The most striking property of an exponential mixture density is that the shape of the density function is flexible in a way similar to the frequently used Weibull distribution, matching long-tailed distributions and allowing clustering and time dependence of the eruption sequence, with distribution parameters that can be readily obtained from the observed occurrence rates. Thus, the mixture of exponentials turns out to be more precise and much easier to apply than the Weibull distribution. We recommended the use of a mixture of exponentials distribution when regimes with well-defined eruption rates can be identified in the cumulative series of events. As an example, we apply the mixture of exponential distributions to the repose-time sequences between explosive eruptions of the Colima and Popocatépetl volcanoes, México, and compare the results obtained with the Weibull and other distributions.

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1 Introduction

Hazardous processes associated with volcanic eruptions may represent a serious threat to the lives, the property, and the economy of people dwelling near volcanoes. This threat may be quantified in terms of the volcano risk, which is the product of the hazard, the probability of occurrence of a potentially destructive eruption, and the vulnerability, the probability of damage to the exposed population and property associated with such an eruption. The knowledge of the hazard may help to reduce the vulnerability through the preparation, the series of measures specifically designed for the most probable eruption scenarios (De la Cruz-Reyna and Tilling, 2008). Assessing the hazard is thus an essential step of risk reduction that requires a broad understanding of the eruption occurrence patterns.

Most volcanoes have complex and irregular patterns of activity. This complexity is derived from the extent of the interaction between concurrent geophysical, geological and geochemical processes involved in the buildup of the mass and energy to be erupted, introducing a random behavior in the volcanic eruption time series.

Studies of volcanic eruption time series have been carried out by many authors using several statistical techniques on specific volcanoes. Some of the earliest, (Wickman, 1965, 1976; Reyment, 1969; Klein, 1982) employed stochastic principles to analyze eruption patterns. Further studies included transition probabilities of Markov chains (Carta et al., 1981; Aspinall et al., 2006; Bebbington, 2007), Bayesian analysis of volcanic activity (Ho, 1990; Solow, 2001; Newhall and Hoblitt, 2002; Ho et al., 2006; Marzocchi et al., 2008), homogeneous and non-homogeneous Poisson process applied to volcanic series (De la Cruz-Reyna, 1991; Ho, 1991), a Weibull renewal model (Bebbington and Lai, 1996a, b), geostatistical hazard-estimation methods (Jaquet et al., 2000; Jaquet and Carniel, 2006), a mixture of Weibull distributions (Turner et al., 2008) and non-homogeneous statistics to link geological and historical eruption time series (Mendoza-Rosas and De la Cruz-Reyna, 2008). An exhaustive list of the available literature on this subject is beyond the scope of this paper, and the above references only attempt to illustrate the diversity of methods that have been applied to the volcanic eruption sequences.

Different parameters have been used as random variables to characterize the eruptive time series, among them, the time of onset of eruptions, the interval between eruptions, the volume or mass released, and the intensity of eruptions (mass ejection rate). The probabilities of occurrence of future eruptions, i.e. the volcanic hazard, may be estimated analyzing the sequence of past eruptions at a volcano, characterizing the eruptions by a measure of their size that reflects their destructive potential, and assuming that the impact and effects of an eruption are proportional to both, the total mass or energy release (magnitude) and the rate of mass or energy release (intensity). The Volcanic Explosivity Index VEI is the semi-quantitative ranking of the eruption "size" based on those parameters (Newhall and Self, 1982).

The random variable considered in this study is the repose period, i.e., the length of the time intervals between successive eruption onsets according to specific VEI categories. The volcanic eruption sequences of polygenetic volcanoes are thus considered here as point processes developing along the time axis, and the distribution of the repose times between them is analyzed for each magnitude category. For our purpose, we shall consider here only significant explosive eruptions (i.e., larger VEI's), which usually are short duration events when compared with the time between eruptions. In this sort of point processes, volcanic activity regimes characterized by well-defined eruption rates may be readily identified. We thus propose a mixture of exponentials distribution (MOED) to study the distribution of repose times between successive eruptions for estimating the volcanic hazard of future explosive eruptions using VEI – characterized sequences of records.

The MOED is a sum of exponential distributions that may be quickly evaluated and interpreted. This method is particularly useful when the eruptive time series is non-stationary i.e. the distribution of the number of eruptions varies upon translation in a fixed interval (Cox and Lewis, 1966) and develops as a succession of eruptive regimes, each having a characteristic eruption rate. In such case, a single exponential distribution may not fit the observed distribution of repose times. Although a Weibull distribution (Johnson and Kotz, 1953; Ho, 1995; Bebbington and Lai, 1996b) may indeed fit such a distribution, the MOED permits an equally good fitting using a priori calculated distribution parameters, directly obtained from the identified occurrence rates of the regimes, whereas the Weibull distribution requires more complex calculations. In the present paper, we apply a MOED to the eruptive time series for Colima and Popocatépetl volcanoes, two of the most active polygenetic volcanoes in Mexico.

The hazard estimates are discussed and compared with the results obtained from the Weibull distribution and from other methods.

2 Mixtures of distributions

Mixtures of distributions occur frequently in the theory and applications of probability and statistics. Generally, an arbitrary distribution can be defined as a weighted sum of components distribution

$$f(t\,|\Lambda) = \sum_{j=1}^{m} w_j f_j(t\,|\lambda_j) \tag{1}$$

where t is the time between eruptions (or failure of any component in the context of quality control), and the parameters $\Lambda=(w_1,\ldots w_m, \lambda_1,\ldots \lambda_m)$ are such that $w_i>0$ for $(j=1,\ldots,m)$, and $\sum_{i=1}^{m} w_i = 1$. w_j is a weighting factor, and f_j a component density function parameterized by λ_j. Generally, a mixture distribution can be composed of m component distributions f_j, each of a different type. Significant simplification can be achieved if all f_j are of the same type, and only the parameters differ. Further simplification is attained if the chosen distributions depend on a single parameter. Since there is no restriction on the form of the distributions f_j, a mixture of exponentials distribution was chosen for its simplicity, and because, considering that sequences of explosive eruptions follow Poissonian patterns (De la Cruz-Reyna, 1991, 1996), it is the distribution that describes the repose times of a Poisson process. On the other hand, Feldmann and Whitt (1998) showed that any monotone probability distribution function can be approximated by a finite mixture of exponentials. They also showed that a MOED is especially useful in modeling long-tailed data without some of the mathematical complications of other distributions such as the Pareto (Johnson and Kotz, 1953) and Weibull probability distributions. Therefore, we choose a mixture of m exponential distributions, also called a hyperexponential distribution, to describe the repose time distribution of explosive eruptions in Poissonian volcanoes with non-stationary behavior. The hyperexponential cumulative distribution function has the form:

$$F(t\,|\Lambda) = \sum_{j=1}^{m} w_j (1 - e^{-\lambda_j t}) \tag{2}$$

with a survival distribution of probability given by:

$$S(t\,|\Lambda) = 1 - F(t\,|\Lambda) \tag{3}$$

The probability density function is:

$$f(t\,|\Lambda) = \sum_{j=1}^{m} w_j \lambda_j e^{-\lambda_j t} \quad , \tag{4}$$

where λ_j, $w_j>0$ for $(j=1,\ldots,m)$, and $\sum_{i=1}^{m} w_i=1$.

Table 1. Observed eruptive regimes and calculated parameters of the MOED exponential distributions for Colima volcano (VEI>2).

Regime	Time period	Number of eruptions	Duration of regime (years)	Annual rate λ	Weighting factor w
1	1560–1622	6	63	0.095238	0.286563
2	1623–1868	3	246	0.012195	0.150705
3	1869–1913	8	45	0.177778	0.299926
4	1914–2008	1	95	0.010526	0.262806

The parameters $\lambda_j{}'$s are the rates of the single exponential distributions; namely the number of occurring events per duration of each regime j. The weighting factors $w_j{}'$s are calculated as the normalized complement of the corresponding proportions of the duration of regimes, considering that regimes of shorter duration regimes tend to have higher eruption rates.

This model is used here to estimate the likelihood of at least one eruption in a given VEI category at a specified time in the future, i.e., the volcanic hazard. Supposing that the most recent event occurred s years ago, and assuming a duration t in years, the probability of no event in the next t years is $P(T>s+t|T>s)$. We then obtain the probability of at least one eruption occurring within the next t years as

$$P(T \leq s+t\,|T > s) = 1 - \frac{1-F(s+t)}{1-F(s)} \tag{5}$$

3 Applications

To illustrate the method, we apply a MOED to Colima and Popocatépetl volcanoes, intending to obtain a simply calculated estimation of the volcanic hazard. Both these volcanoes represent a significant threat to large populations dwelling around them. Colima volcano (19.51° N, 103.62° W) is the active volcano in México with the highest eruption rate, and a historical record of 41 eruptive events of varied magnitudes in the past 500 years (De la Cruz-Reyna, 1993; Mendoza-Rosas and De la Cruz-Reyna, 2008). Popocatépetl volcano (19.02° N, 98.62° W) is located within a densely populated region, about 70 km southeast of downtown Mexico City and 40 km west of the city of Puebla, which with other nearby cities add up to over 20 million people vulnerable to direct hazards associated with a major explosive eruption (De la Cruz-Reyna and Tilling, 2008). To calculate the eruption rates and the weighting factors, we use here the compilation of historical eruptive time series obtained by Mendoza-Rosas and De la Cruz-Reyna (2008) for those volcanoes.

Colima and Popocatépetl volcanoes show a weak non-stationary component in the behavior of their eruption time sequences (De la Cruz-Reyna 1993; Mendoza-Rosas and De la Cruz-Reyna, 2008). This component appears as a succession of regimes, each one having a characteristic eruption rate, as can be recognized in Figs. 1 and 3. High and low eruption rate regimes alternate about a mean rate in such a way that the clustering of high regimes may not be attributed to chance, according to running mean tests (Klein, 1982; De la Cruz-Reyna, 1996; Mendoza-Rosas and De la Cruz-Reyna, 2008).

Figure 1 shows the four regimes that can be recognized from the historical activity of Colima volcano, based on reports from 1560 to the present for events with eruptive magnitudes greater than VEI 2. The rates of the eruption regimes are listed in Table 1. The time sequence of Colima is then fitted to a MOED, using the sum of four exponential distributions, each one having the corresponding observed eruption rate λ_j. We use Eq. (2) with m=4. The weighting factors are calculated as the normalized complement of the duration in years of each regime,

$$w_i = \frac{D_t - D_i}{\sum_{i=1}^{m} (D_t - D_i)} \tag{6}$$

where D_t is the duration of the sampled interval (449 years for Colima, and 497 years for Popocatépetl), and D_i is the duration of the identified regime. Tables 1 and 3 summarize the MOED parameters for both volcanoes.

To compare the results obtained for Colima volcano using the MOED with the results from a Weibull distribution, we calculated the shape and scale parameters of the Weibull distribution by the graphical method described by Bebbington and Lai (1996b). The resulting parameters were 0.7767 and 19.2699, respectively. The comparison between the MOED and the Weibull distribution is shown in Fig. 2. Both the MOED and the Weibull distributions show good fits with the repose time data.

To obtain an objective measure of the quality of the fits, we performed a Kolmgorov-Smirnov (K-S) test as described in texts on nonparametric statistics (e.g., Gibbons, 1976), calculating the maximum absolute differences between both cumulative distributions, and the observed cumulative frequencies, obtaining 0.13182 and 0.15861, respectively and thus indicating a better fit of the MOED. These difference values are smaller than the critical K-S differences for the 0.01 level of significance. The MOED and Weibull probabilities of occurrence of at least one significant eruption at Colima volcano in the next t years, calculated from Eq. (5), are listed in Table 2.

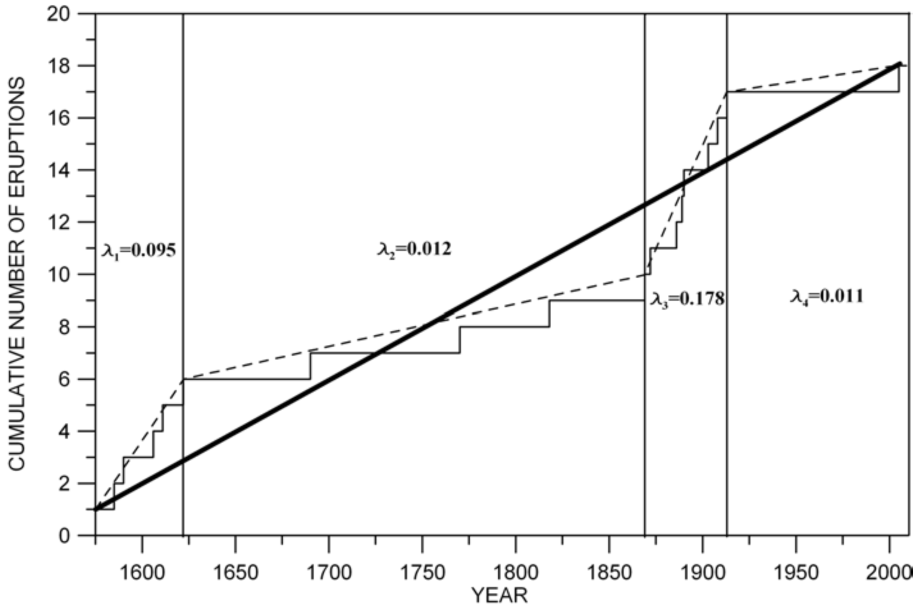

Fig. 1. Cumulative number of eruptions reported in the period 1560-present in the magnitude class VEI>2 for Colima volcano and the four regimes that can be recognized (separated by solid vertical lines). The slopes of the dashed lines represent the eruption rate λ of each regime and the thick line slope represents the eruption rate of whole series (λ_{global}=0.040089).

Table 2. Eruption hazards of Colima volcano expressed as probabilities of occurrence of at least one eruption with VEI>2 over different time periods.

Colima Volcano (VEI>2)		
t years	Mixture of exponentials distribution	Weibull distribution
20	0.548519	0.598076
50	0.707716	0.858913
100	0.833418	0.967949
500	0.997920	0.999996

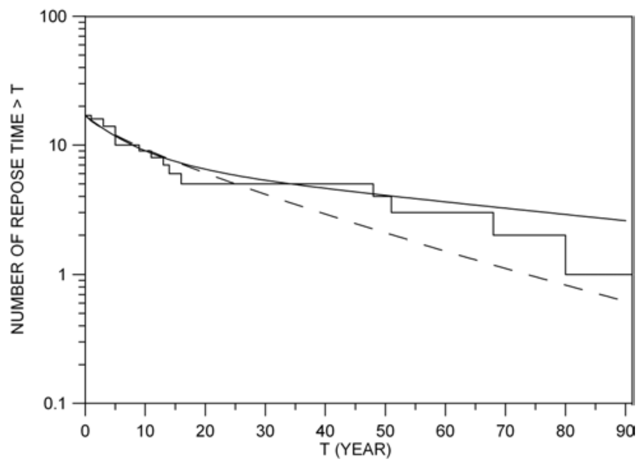

Fig. 2. Distribution of observed repose intervals with duration greater than *t* years (steps) for eruptions at Colima volcano with VEI>2 in the period 1560 to the present. The dashed line is the survival Weibull distribution and the solid line is the MOED.

We applied the same procedure to the historical time series records of Popocatépetl volcano for eruptions with VEI≥2. Figure 3 shows a cumulative distribution of repose periods, in which two regimes may be recognized. Table 3 lists the 2-term MOED parameters for these regimes. The shape and scale parameters of the corresponding Weibull distribution were calculated with the same method used with Colima volcano as 0.6207 and 16.3137, respectively. Figure 4 shows the fits of the survival distributions obtained from the MOED and the Weibull distributions with the observed data of the eruptive time series of Popocatépetl volcano. The Kolmogorov-Smirnov test at the significance level 0.01 yields 0.15355 and 0.16687 for the MOED and the Weibull distribution, respectively, indicating again a better fit of the MOED. The MOED and Weibull probabilities of occurrence of at least one eruption in different time periods to Popocatépetl volcano are shown in Table 4.

In this case we selected the regimes inspecting the cumulative plot of the number of eruptions, and fixed the vertical lines at the points showing a clear slope change. In other cases the change of slope may be smooth, and selecting the point of regime change in the cumulative curves could be more difficult. To test how sensitive is the quality of the MOED fit to the choice of the change of regime points, we tried neighbor points around the intuitively sharp and clear slope changes of Colima and Popocatépetl cumulative curves

Table 3. Observed eruptive regimes and calculated parameters of the MOED exponential distributions for Popocatépetl volcano (VEI≥2).

Regime	Time period	Number of eruptions	Duration of regime (years)	Annual rate λ	Weighting factor w
1	1512–1665	10	154	0.064935	0.690141
2	1666–2008	7	343	0.020408	0.309859

Table 4. Eruption hazards of Popocatépetl volcano expressed as probabilities of occurrence of at least one eruption with VEI≥2 over different time periods.

	Popocatépetl Volcano (VEI≥2)	
t years	Mixture of exponentials distribution	Weibull distribution
20	0.578118	0.539616
50	0.838877	0.7945279
100	0.949677	0.9273610
500	0.999986	0.9996099

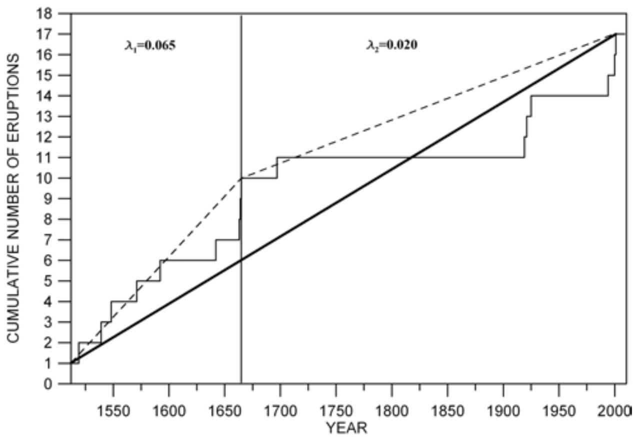

Fig. 3. Cumulative number of eruptions reported in the period 1512-present in the magnitude class VEI≥2 for Popocatépetl volcano, and the two regimes that can be recognized (separated by the vertical lines). The dashed lines represent the eruption rate λ of each regime, and the thick line slope represents the eruption rate of whole series ($\lambda_{\mathrm{global}}$=0.034205).

and K-S tested the corresponding survival distributions. To cancel the advantage of the MOED over the Weibull distribution, the chosen change point should be moved at least 2 and 8 positions away from the graphically evident slope-change point for Popocatépetl and Colima volcanoes, respectively, indicating a good stability of the method. To deal with the case of smoothly changing regimes we suggest a simple procedure to decompose the MOED using the method of moments (Rider, 1961; Everitt and Hand, 1981; Sum and Oommen, 1995). Assuming that one can tell the number m

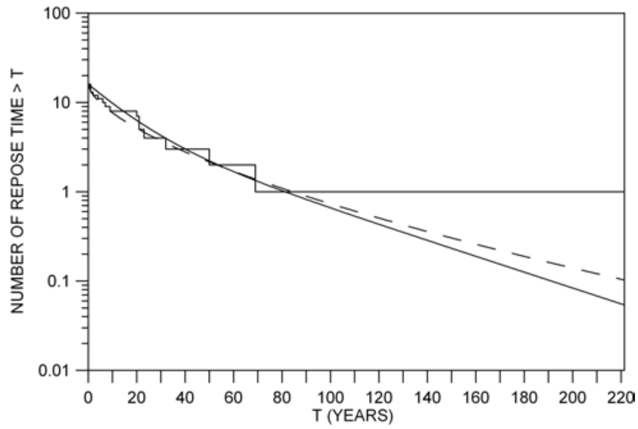

Fig. 4. Distribution of observed repose intervals with duration greater than t years (steps) for eruptions at Popocatépetl volcano with VEI ≥2 in the period 1512 to the present. The dashed line is the survival Weibull distribution and the solid line is the MOED.

of significantly different regimes composing the eruptive sequence, the MOED may be written as a sum of that many exponential distributions with unknown parameters. The MOED n-th moment about zero may be expressed as a function of the distribution parameters $\Lambda=(w_1,\dots w_m,\lambda_1,\dots\lambda_m)$ (see Eq. 1) as

$$n!\sum_{j=1}^{m} w_j \Big/ \lambda_j^n \qquad (7)$$

The $2m-1$ unknown parameters may then be estimated equating the distribution moments (7) to the k-data sample moments $\sum_{i=1}^{k} x_i^n \big/ k$, and from them the regime changes may be identified.

4 Discussion and conclusions

The simple MOED method discussed in this paper allows a simple assessment of the volcanic hazard from a straightforward analysis of the eruption time series. Apart from its simplicity, the flexibility of its shape allows good fits, even for long-tail distributions, and for non-stationary processes.

Perhaps the most important feature of the MOED is that, unlike the Weibull distribution, the MOED parameters contain direct information of the physical process involved

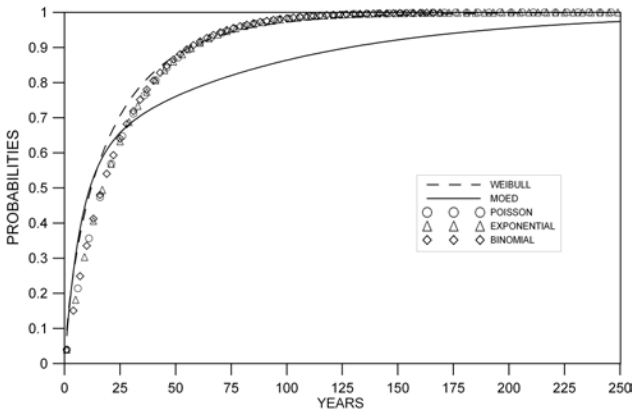

Fig. 5. Probabilities of occurrence of at least one eruption in a period t (years) calculated with a MOED (solid line), a Weibull, (dashed line) an exponential (triangles), a Binomial (diamonds), and a Poisson (circles) cumulative distribution functions for the eruptive time series of Colima volcano with VEI>2 in the period 1560 to the present.

in the eruption time series, namely the eruption rates characterizing the successive regimes of a non-stationary process, and their durations. In the stationary case, defined by a single eruption rate invariant under translations along the time axis, both the MOED and the Weibull distribution reduce to the simple exponential distribution. This tends to confirm the Poissionian character of the eruption sequences, which is not lost when the eruption series shows a succession of eruptive episodes. An underlying assumption involved in this argument is that the eruption sequence may be perceived as a time-dependent process with a known interval distribution, namely the succession of high and low regimes alternating in such a way to maintain a steady long-term mean. This requires that the high-regimes are in the average systematically shorter (or longer) than the low-regimes. For example, in the case of Colima volcano the duration of the low regime (246 yr) is about five times the mean (54 yr) of the high regimes. Assuming the regimes maintain an approximately uniform distribution such that keeps the long-term eruption rate constant (Fig. 1), and noting that the current regime, initiated after the eruption of 1913, has produced only one VEI 3 eruption in the year 2005, we are inclined to believe that this eruption is a normal part of a low-regime, rather than signaling the onset of a high-regime. To emphasize the effects of the time dependence of the eruptive process characterized by a succession of low and high regimes, we compare the probabilities of at least one eruption occurring in given times intervals using the MOED, with the probabilities obtained from the Weibull distribution and from the stationary Exponential, Binomial and Poisson cumulative distribution functions characterized by the overall eruption rate of the whole sequence. The result of this comparison is summarized in Fig. 5. There, we observe that the Poisson, Binomial and Exponential distributions render the same results, as expected from a stationary eruption sequence having the mean eruption rate of the whole series, concealing in this way the effects of the non-stationary behavior. In contrast, the MOED and the Weibull distributions properly describe the increased probability of an eruption occurring in shorter periods (up to about 20 years), derived from the contribution of the high regimes. However, at longer repose periods, the Weibull probabilities approach the stationary exponential results, "saturating" at periods of about 120 years, and revealing some inability of those distributions to deal with the long-tailed part of the observed distribution. On the other hand, the MOED allows estimation of the probabilities of eruptions occurring after long repose periods, accounting for the low regimes.

One important point that must be emphasized is that this method should be performed on a portion of the time series that satisfies a criterion of completeness, i.e. a portion in which no significant eruption data are missing, which in most cases is the historical eruption data. A single missing event may importantly distort the observed distribution of repose times. We therefore use only the VEI categories that made possible to consider the eruptive series as complete over a period with reliable reporting, i.e., the last five centuries. A chief issue derived from this is that the relatively short historical series may not include low-rate major eruptions, that the geologic records indicate the volcano is capable to produce. How the estimates of volcanic hazard may be adjusted for such cases is a problem that has been addressed elsewhere (Mendoza-Rosas and De la Cruz-Reyna, 2008). The MOED method is thus recommended to estimate the volcanic hazard of volcanoes showing high rates of eruptive activity and significant evidence of a distribution of high and low eruptive regimes.

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Capítulo 4.

Estimaciones de peligro volcánico para el volcán El Chichón, Chiapas, México: Una aproximación estadística para historias eruptivas complejas.

"Hazard estimates for El Chichón volcano, Chiapas, México: A statistical approach for complex eruptive histories".

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RESUMEN

En 1982, la erupción volcánica de magnitud VEI 5 generada por el volcán El Chichón causó el mayor desastre registrado en la historia de México resultando al menos 2000 perdidas humanas. El volcán El Chichón, antes de este desastre, no era considerado como una amenaza volcánica debido a la baja frecuencia de erupciones en el pasado. La correcta evaluación del peligro volcánico es el primer paso para evitar un desastre. En este capítulo analizamos la historia eruptiva reportada en el Holoceno del volcán El Chichón, caracterizando la tasa eruptiva en diferentes categorías VEI, analizando y probando la posible dependencia entre erupciones, y la dependencia en el tiempo, es decir, la no - estacionaridad. El peligro volcánico para el volcán El Chichón es evaluado con diferentes métodos estadísticos: El Proceso Poisson no - homogéneo con función de intensidad generalizado Pareto (The Non Homogeneous Generalized Pareto - Poisson Process: NHGPPP) y la Distribución de Mezcla de Exponenciales (The Mixture of Exponentials Distribution: MOED). Los resultados son comparados con las probabilidades obtenidas con la distribución de Weibull y la distribución de Poisson. La distribución de Weibull y la MOED producen resultados similares aun cuando también se aplican a una base de datos extendida e incompleta de la historia eruptiva del volcán El Chichón, en contraste la NHGPPP es altamente influenciada al extender el periodo de la historia eruptiva. Nosotros recomendamos las estimaciones obtenidas desde la MOED para intervalos de tiempo cortos (menos de 40 décadas) y las de la NHGPPP para intervalos de tiempo largos.

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Natural Hazards and Earth System Sciences

Hazard estimates for El Chichón volcano, Chiapas, México: a statistical approach for complex eruptive histories

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Abstract. The El Chichón volcano (Chiapas, México) most recent eruption occurred in 1982 causing the worst volcanic disaster in the recorded history of Mexico. Prior to the eruption, El Chichón volcano was not considered a very hazardous volcano, a perception mostly caused by the low eruption rate of the past eruptions. The correct assessment of volcanic hazard is the first step to prevent a disaster. In this paper, we analyze two periods of the reported eruptive history of El Chichón volcano during the Holocene, searching for the eruption rates of different VEI magnitude categories and testing their time dependence. One period accounting the eruptions of the last 3707 years before the last eruption (BLE) is assumed to be complete, with no missing relevant events. More scarce information of a period extending to 7772 years BLE is then added. We then apply the Non-Homogeneous Generalized Pareto-Poisson Process (NHGPPP), and the Mixture of Exponentials Distribution (MOED) methods to estimate the volcanic hazard of El Chichón considering both periods. The results are compared with the probabilities obtained from the homogeneous Poisson and Weibull distributions. In this case the MOED and the Weibull distribution are rather insensitive to the inclusion of the extended period. In contrast, the NHGPPP is strongly influenced by the extended period.

1 Introduction

Currently, millions of people worldwide are at risk by volcanic eruptions. The average annual death quota remains high due to more people living in close proximity to active volcanoes. The increasing exposure of a larger population is in some cases derived from an inadequate assessment of the volcanic risk, allowing the land use of vulnerable areas. A correct assessment thus represents an essential component for the management of risk and disaster prevention. The statistical methods to evaluate the volcanic risk are an emergent and rapidly growing area aimed to provide more objective land use criteria, and to support decision making that may help preventing a disaster. The first step in this assessment is the appropriate estimation of volcanic hazard, i.e., the probability that a specific type of volcanic eruption occurs in a given area within a given interval of time. The quality of this estimate depends mainly on the capacity of the chosen statistical model and the quality of the available data base.

In this paper, we compare different methods to calculate the hazard of El Chichón volcano (Chiapas, México). One, the Non-Homogeneous Generalized Pareto-Poisson process (NHGPPP), is a relatively complex method that permits precise hazard estimates even on non-stationary and incomplete data bases of volcanoes, having a short list of major eruptions separated by long repose periods with little information on smaller activity during those intervals, as is the case of El Chichón volcano. This method is discussed in detail elsewhere (Mendoza-Rosas and De la Cruz-Reyna, 2008). Other method uses the Mixture of Exponentials Distribution (MOED), also called the hyperexponential distribution (Mendoza-Rosas and De la Cruz-Reyna, 2009). This method is much simpler and needs less calculation. The MOED has been applied to other volcanoes of Mexico (Mendoza-Rosas and De la Cruz-Reyna, 2009) and Chile (Dzierma and Wehrmann, 2010), and compared with other distributions such as the Poisson, Weibull, exponential, NHGPPP and log-logistic distributions, showing satisfactory estimates. Although in principle the MOED requires completeness of the data set, the results obtained here show that the MOED also provides acceptable estimates for extended and probably incomplete data set such as the Holocenic El Chichón eruptive series. To increase the comparison perspective, we again weigh these methods against

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other well known and commonly used distributions, mostly to analyze the effects of completeness and non-homogeneity, i.e. non-stationarity. The quality of the volcanic hazard estimates obtained from the different methods is evaluated using the Anderson-Darling, Cramer-von-Mises and Kolmogorov-Smirnov goodness of fit tests, as well as the AIC (Akaike Information Criterion), against the distribution of repose times obtained from the reported eruptive history.

2 El Chichón Volcano

El Chichón volcano (17.36° N, 92.23° W) is located in the Chiapas Volcanic Arc (CVA), a structure associated to the subduction of the Cocos plate under the North American plate in a complex way due to the changing subduction angle and the close interaction with the Caribbean plate (Damon and Montesinos, 1978; Mora et al., 2007; Layer et al., 2009). Tephrostratigraphy, ^{14}C-dating and palynology to estimate the timing and magnitude of past volcanic eruptions of El Chichón volcano have been used by Nooren et al. (2009). El Chichón volcano has an altitude of 1100 m a.s.l., and a 1 km wide, 140 m deep crater in its summit formed during the most recent eruption, beginning on 28 March 1982 (Espíndola et al., 2000; Macías et al., 2007, 2008). This week-long eruption (VEI 5) produced planetary scale volcanic gas clouds (Kruger, 1983), extensive ash fall, and pyroclastic surges and flows that resulted in the worst volcanic disaster in the recorded history of Mexico devastating a radius of about 10 km around the volcano and covering southeastern Mexico with ash fall (Macías et al., 2008). It caused about 2000 fatalities, displaced thousands, and produced severe economic loss (De la Cruz-Reyna and Martin-Del Pozzo, 2009).

Recent studies on the stratigraphy of the volcano and new radiocarbon ages show that at least other 11 major eruptions prior to the 1982 eruption have occurred at El Chichón in the Holocene, and more precisely, in the past 8000 years, with most of the repose intervals lasting between 100 to 600 years (Tilling et al., 1984; Espíndola et al., 2000; Macías et al., 2007, 2008; Layer et al., 2009). Table 1 summarizes the historical and Holocenic geological records. Given the limited available geological data of recognizable deposits with approximate geochronological dating of the eruptive history of El Chichón, the completeness, i.e., the certainty that all the eruptions regardless of their magnitudes have been accounted in the eruptive data base cannot be fully sustained, especially for the older events. We thus analyze the eruptive records using a scaling logarithmic relationship between the magnitude VEI (Volcanic Explosivity Index; Newhall and Self, 1982) and the eruption occurrence rate for each magnitude category VEI. This relationship (De la Cruz-Reyna, 1991, 1996; De la Cruz-Reyna and Carrasco-Núñez, 2002) provides a criterion to estimate the most probable magnitude of eruptions to which no VEI has been assigned. Although this criterion is non-unique, different models of the eruption rate-VEI relationship may be constructed and the best model may be chosen by optimizing the fit with the assumed complete catalogue of higher magnitudes (Mendoza-Rosas and De la Cruz-Reyna, 2008). To analyze the eruptive history of El Chichón volcano, we used the available data for the last 7772 years before the last eruption (BLE) and searched for the best fit of the scaling law to estimate the eruption rates of the probably incomplete lower-range magnitudes.

Table 1. Volcanic Explosivity Indexes and ages of known eruptions with magnitude VEI≥3 of El Chichón volcano from different sources (Duffield et al., 1984; Tilling et al., 1984; Espíndola et al., 2000; Macías et al., 1993, 2003, 2007, 2008). The dates are averages of the reported radiometric age in years before the last (1982) eruption. The dash indicates an unknown VEI value. For the eruption of 1270 years BLE we adopted a VEI 5 based on personal communications by J. L. Macías and J. M. Espíndola who are carrying out most of the field work on El Chichón deposits, and consider that such eruption was at least as large as the 1982 one. For the eruptions in the range 2∼3, we adopted the higher value on the premise that deposits of eruptions older than 1000 years that remain recognizable in a tropical humid climate more probably correspond to the upper end of the range.

Years BLE		VEI
0		5
635	±75	4
905	±83	3
1270	±89	4∼5
1524	±75	3
1657	±128	2∼3
1857	±83	2∼3
2065	+107/–102	2∼3
2590	±53	2∼3
3107	±89	–
3707	+80/–75	4
7772	±50	3

3 Statistical methods

To make a comparative analysis of the MOED and the NHGPPP methods aimed to the search of precise and simple estimates of the volcanic hazard of El Chichón volcano, we consider the eruption sequence as a time-dependent point processes of independent events developing along the time axis, and study the distributions of the eruptive occurrences and repose times between eruptions for each VEI magnitude category.

The NHGPPP is a procedure based on the use of a non-homogenous Poisson process on the eruptive time series, with a Generalized Pareto Distribution (GPD) as intensity function (Coles, 2001). The GPD is described by a shape parameter β, a scale parameter θ, and a location parameter u (threshold), and has the following cumulative distribution function:

$$G_{\beta,\theta}(y) = 1 - \left(1 - \frac{\beta y}{\theta}\right)^{1/\beta} \quad \text{for } \beta \neq 0$$
$$G_{\beta,\theta}(y) = 1 - e^{-y/\theta} \qquad \text{for } \beta = 0,$$

where $y = x - u$ is a realization of an excess (Brabson and Palutikof, 2000; Lin, 2003). Since we are assuming that the completeness (i.e. a portion in which no significant eruption data are missing) of the eruptive time series improves as the magnitude of the eruptions increases, this approach is strongly influenced by the few data that are represented by the right tail of the repose-time distribution. The GPD intensity function of the NHGPPP permits modeling extreme values, such as the very high-magnitude eruptions, allowing for a better fit of the whole distribution. Additionally, it is less sensitive to the incompleteness of the data base and possible time dependence of the large-magnitude eruption sequence, since it only considers the number of exceedances over a threshold (the exceedances are the events with a magnitude higher than a reference threshold magnitude), of a series that may be homogeneous or not. A more detailed description of this statistical method and its applications to other four volcanoes may be found in Mendoza-Rosas and De la Cruz-Reyna (2008).

A second, and simpler, method to assess the volcanic hazard is based on a mixture of exponentials distribution (MOED), also called a hyperexponential distribution (Mendoza-Rosas and De la Cruz-Reyna, 2009). The MOED uses a sum of exponential distributions that may be quickly evaluated and interpreted. This method is particularly useful when the eruptive time series develops as a succession of eruptive regimes, each having a characteristic eruption rate. Usually, these regimes may be readily identified in the cumulative series of events. The MOED permits a good fitting to the eruption data using an a priori calculated distribution parameters, directly obtained from the identified occurrence rates of the regimes. The involved parameters are the rates of the individual exponential distributions, namely the number of occurring events per duration of each regime; and the coefficients or weighting factors, calculated as the normalized complement of the corresponding proportions of the duration of regimes (Eq. 6 in Mendoza-Rosas and De la Cruz-Reyna, 2009). The MOED's weighting factors are defined in an interval [0, 1] and their sum should equal 1. The MOED is assuming that the eruptive process is non-stationary, and that the time dependence is expressed as a succession of regimes with high and low eruption rates. Therefore, there is a mean eruption rate representing an average of the eruption rate regimes. This requires that the duration of the high regimes (many eruptions in a shorter time) is shorter than the duration of the slow regimes (few eruptions over a longer time). The shorter periods containing more eruptions (thus representing a higher hazard) should then be weighted by the complements of their durations. MOED is also useful in modeling long-tailed data without some of the mathematical complications of other distributions such as the Pareto and Weibull probability distributions (Johnson and Kotz, 1953; Bebbington and Lai, 1996).

Both methods, NHGPPP and MOED are used here to estimate the likelihood of at least one eruption of El Chichón volcano in a given VEI category at a specified time in the future, i.e., the volcanic hazard, and compared with the results of standard distributions such as Poisson and Weibull. The quality of the estimates is evaluated through the goodness of fit with the available eruptive history data: occurrence rates and distribution of repose times in specific magnitude categories.

3.1 Goodness of fit tests

Suppose that $x_1, \ldots, x_n$ are identical independent distributed observations from an unknown distribution F. We wish to use $x_1, \ldots, x_n$ to test whether F coincides with a fully-specified distribution F^*. The goodness-of-fit approach to this problem consists of testing under the null hypothesis $H_0 :$ $F = F^*$ against $H_1 : F \neq F^*$, and a number of distribution-free test procedures are available. In order to have a broader criterion to test the quality of the distributions treated here, we use the Cramer-von Mises, the Kolmogorov-Smirnov and the Anderson-Darling tests.

To test the goodness-of-fit for a hypothesized distribution F^*, we can use the discrepancy between the empirical F and the hypothesized F^* as a statistical test. The Kolmogorov-Smirnov test (Gibbons, 1976), uses a maximum distance or separation criterion based on the statistic $D_{\mathrm{KS}} = \sup_x |F(x) - F^*(x)|$. In some cases, the Kolmogorov-Smirnov test is perceived as providing a poor estimate of the quality of fit, particularly when the significant separation criterion occurs only at a single point of the data set (we use in this work the central point of each repose period to measure the separation). The null hypothesis may thus be rejected if F and F^* significantly differs at one single data point, even if the overall fit is reasonably good. Therefore, some alternative tests have been proposed in the literature (Jesse, 2009), such as the Cramér-von-Mises and the Anderson-Darling tests (Anderson and Darling, 1952, 1954; Anderson, 1962).

A test which does not involve a subjective grouping of the data is the Cramér-von-Mises criterion, which is based on a "quadratic distance" to judge the goodness of fit of a probability distribution F^* compared to a given empirical distribution function F. It is defined as

$$W^2 = n w^2 = \int_{-\infty}^{\infty} \left[F(x) - F^*(x)\right]^2 dF^*(x),$$

where $x_1, \ldots, x_n$ are the observed values, in increasing order.

The statistic is expressed as

$$\mathrm{CvM} = \frac{1}{12n} + \sum_{i=1}^{n} \left(\frac{2i-1}{2n} - F^*(x_i) \right)^2,$$

(Anderson and Darling, 1954; Anderson, 1962). If a measure CvM is adopted, the hypothesis is rejected for those samples for which CvM>z. The number z, namely the asymptotic significance point is chosen in such a way that the probability of rejection of a true hypothesis is some specified number (for example, 0.01 or 0.05). The asymptotic distribution of the statistic CvM can be found in Anderson and Darling (1952).

To consider a more convenient measure of the separation or "distance" between two distribution functions, Anderson and Darling (1952) incorporate a weighting function to allow more flexibility in the Kolmogorov-Smirnov and Cramer-von-Mises tests. Let $F_n(x) = \frac{i}{n}$, if i observations are $\leq x$ for $i = 0, 1, \ldots, n$.

$$K_n = \sqrt{n} \sup_x \left| F_n(x) - F^*(x) \right| \sqrt{\psi(F^*(x))},$$

and

$$W_n^2 = n \int_{-\infty}^{\infty} \left[F_n(x) - F^*(x) \right]^2 \psi\left(F^*(x)\right) dF^*(x),$$

where $\psi(u), 0 \leq u \leq 1$, and $u = F^*(x)$, is a weighting function, which is chosen by the statistician so as to weight the deviations according to the importance attached to various portions of the distribution function. The selection of $\psi(u) = 1$ yields nw^2, the criterion of von-Mises for W_n^2, and K_n for the criterion of Kolmogorov-Smirnov. The criterion $W_n^2$is an average of the squared discrepancy $\left[F_n(x) - F^*(x) \right]^2$, weighted by $\psi(F^*(x))$ and normalized by n. For a given value of x, $F_n(x)$ is a binomial variable; it is distributed in the same way as the proportion of successes in n trials, where the probability of success is $H(x)$. Thus, $E[F_n(x)] = H(x)$, and under the null hypothesis $(H(x) = F^*(x))$, the variance is $F^*(x)(1 - F^*(x))$ (Anderson and Darling, 1954). This is equivalent to dispersing the sampling error over the entire range of x by weighting the deviation with the reciprocal of the standard deviation under the null hypothesis, i.e., using $\psi(u) = \frac{1}{u(1-u)}$ as a weighting function, with $u = F^*(x)$. This weighting function strongly emphasizes the effect of the distribution tails. Then, letting $x_1 \leq x_2 \leq \ldots \leq x_n$ be the n ordered observations in the sample, the Anderson-Darling statistics is given by $A_n^2 = -n - s_n$, where

$$s_n = \sum_{i=1}^{n} \frac{2i-1}{n} \left[\ln F^*(x_i) + \ln\left(1 - F^*(x_{n+1-i})\right) \right].$$

The asymptotic distribution of this statistics, and approximate values of the significance points $z's$ can be found in Anderson and Darling (1954). If the data produce a value of A_n^2 larger than the asymptotic significance point z, the hypothesis may be rejected. This test uses the actual observations without grouping, and is more sensitive to discrepancies at the tails of the distribution rather than near the mean.

An alternative method is the Akaike Information Criterion (AIC) proposed by Akaike (1973), which is not an absolute hypothesis test; it rather provides a relative comparison between different models, in which the lowest AIC value indicates the best fit. If all the models in the set assume normally distributed errors with a constant variance, the AIC can be computed as

$$\mathrm{AIC} = n \cdot \ln \frac{\sum \left(y_i^* - y_i \right)^2}{n} + 2k,$$

where n is the number of data points, k the number of free parameters, and y_i^* are the points of the model used to fit the data points y_i. In the case of a small number of data points n, a correction needs to be applied (Burnham and Anderson, 1998),

$$\mathrm{AIC_C} = \mathrm{AIC} + \frac{2k(k+1)}{n-k-1}.$$

The Akaike Information Criterion penalises the misfit and the number of parameters used in the tested distributions (Akaike, 1973; Bebbington, 2007; Turner et al., 2008; Dzierma and Wehrmann, 2010).

3.2 Stationarity tests

The choice of the method to assess the volcanic hazard also depends on the time dependence of the process. Such dependence or non-stationarity may adopt diverse forms. Of special relevance is the existence of regimes with significantly different eruption rates in the eruptive time-series. This means that the process may keep a constant eruption rate for some time, and abruptly change it. If these changes form a succession of regimes around a mean overall regime characterizing the whole eruptive series, the calculated probabilities of eruptions based on the overall regime may be underestimated if the current process has a rate higher than the mean, and overestimated otherwise. Precise estimates of those probabilities thus require knowing if the fluctuations of the eruption rates around a mean are caused by the actual existence of eruptive regimes, or are just random variations of a natural stationary process.

There are different tests to observe the homogeneity or stationary character of the data, as for example the moving average test (Klein, 1982) or the dispersion test (Cox and Lewis, 1966). However, these tests require a subjective grouping of the data. The results may thus be affected by the choice of the number of groups or the intervals length. Additionally, if completeness of the eruptive record cannot be assured, application of these methods requires great caution.

When the presence of regimes is suspected from the inspection of the cumulative number of eruptions versus time

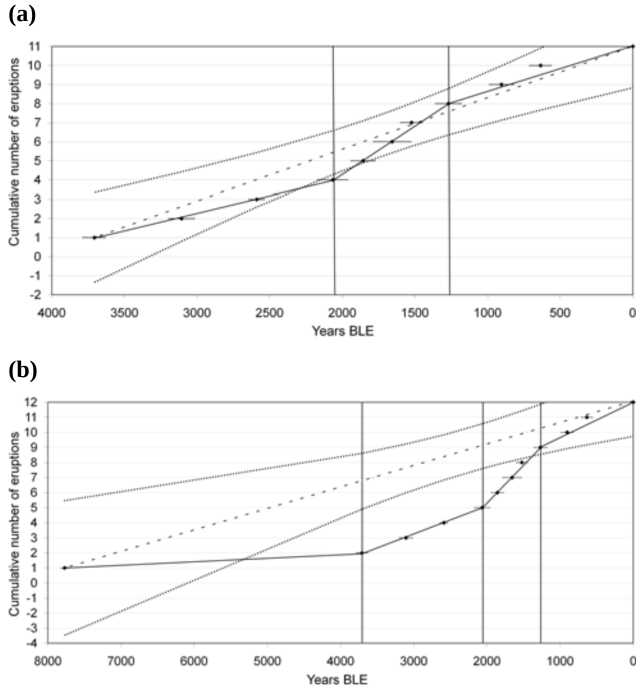

Fig. 1. Cumulative number of eruptions of El Chichón volcano reported for the periods **(a)** 3707 and **(b)** 7772 years before of the last 1982 eruption in the magnitude class VEI≥3, and the regimes that can be recognized. The slopes of the solid lines represent the eruption rates of each regime, the slope of the dashed line represents the eruption rate of the whole series and the dotted lines are their 95% confidence bounds (λ_{global}=0.026976 eruptions per decade in 3707 years BLE, and λ_{global}=0.014153 eruptions per decade in 7772 years BLE). The vertical lines mark the points of regime change.

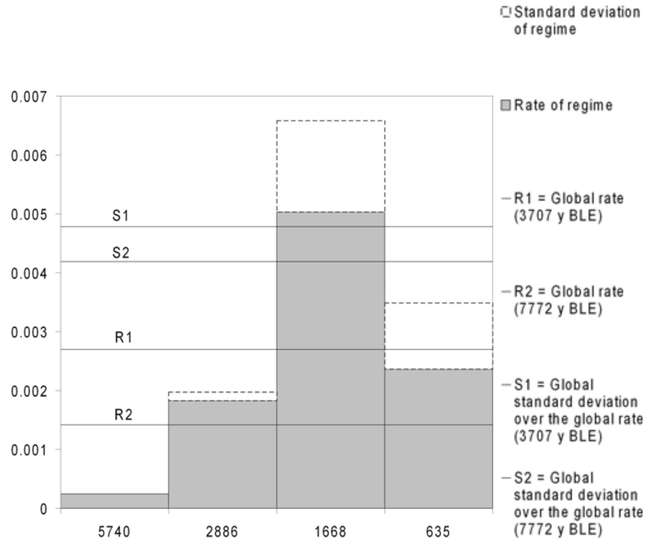

Fig. 2. Rates and standard deviation for each regimen (vertical axis) versus the time of the central point of each regimen (horizontal axis).

(as is the case in Fig. 1), one may verify their existence using a test similar to the procedure used by Mulargia et al. (1987). We thus apply a simple t student test on the regime's and the global eruption rates to test the hypothesis of whether the regime's rates and the global rate belong to the same population. Call λ_i the rate of each regime i, i.e., the number of eruptions in the given magnitude category occurring during that regime, and λ_{global} the global rate (total number of eruptions in the given magnitude category during the whole sampled period). The value of each λ_i may be represented by the slope of the best-fitting line to the cumulative number of eruptions for each regime, if the rate remains constant along the regime. Figure 1 shows the regime (solid lines) and global (dashes line) lines of different periods. The dotted lines represent 95% confidence bounds for the global rate lines. We then compare the slopes defined by pairs of successive eruptions with the λ_i of the assumed regime to obtain a standard deviation for each regime. Finally, we apply a t student test to the null hypothesis that the λ_i and λ_{global} belong to the same population with the following criterion:

$$t=\frac{\left(\lambda_i-\lambda_{\text{global}}\right)}{\sigma\sqrt{\frac{1}{n_i}+\frac{1}{n_{\text{global}}}}},$$

where

$$\sigma=\sqrt{\frac{(n_i-1)\sigma_i^2+\left(n_{\text{global}}-1\right)\sigma_{\text{global}}^2}{n_i+n_{\text{global}}-2}}.$$

4 Applications to El Chichón Volcano

The eruptive sequence of El Chichón volcano includes 12 distinct events with VEI greater or equal than 3 occurring in the last 7772 years BLE, as shown in Table 1. Figure 1a shows the cumulative eruptive number of the last 3707 years BLE, that include a sample of 11 eruptions assumed to be complete in the specified magnitude range. Figure 1b shows the cumulative eruptive number including the oldest dated Holocenic eruption of El Chichón volcano. Completeness of the above VEI range of this longer period is not assumed. The eruptive rates can be recognized as the slopes of the lines in Fig. 1. The mean rates λ_i and their standard deviations over the 7772 and 3707 years BLE period are plotted in Fig. 2, showing the significant difference between the eruptive rates. The plots in Fig. 1 suggest that the eruptive activity of El Chichón volcano for the magnitude range VEI ≥3 in both periods 3707 and 7772 years BLE had different degrees of non-stationary behavior characterized by well defined regimes with different eruption rates. The rate of

Table 2. Two possible models of VEI magnitude distributions for the VEI≥3 eruptions of El Chichón volcano in 3707 years BLE. The second column reproduces the published VEI values listed in Table 1.

Years BLE	Reported VEI	Chichón 1	Chichón 2
0	5	5	5
635	4	4	4
905	3	3	3
1270	4∼5	5	5
1524	3	3	3
1657	2∼3	3	3
1857	2∼3	3	3
2065	2∼3	3	3
2590	2∼3	3	3
3107	*?*	*3*	*4*
3707	4	4	4

Table 3. Two possible models of the VEI magnitude distributions for eruptions with magnitude VEI≥3 of El Chichón volcano during a period of 7772 years BLE. The second column reproduces the reported VEI values listed in Table 1.

Years BLE	Reported VEI	Chichón A	Chichón B
0	5	5	5
635	4	4	4
905	3	3	3
1270	4∼5	5	5
1524	3	3	3
1657	2∼3	3	3
1857	2∼3	3	3
2065	2∼3	3	3
2590	2∼3	3	3
3107	*?*	*3*	*4*
3707	4	4	4
7772	3	3	3

the regime 2064–1271 years BLE (0.005031 eruptions/year) strongly differs from the others; it is in fact 350% larger than the global rate for 7772 years BLE. To objectively test this, following Mulargia et al. (1987), we include the 95% confidence bounds, and we see that most of the observed points fall out of the bounds in the 7772 years BLE extended period; on the contrary, for the 3707 y BLE period only one single point falls out of the bounds at the change point between 2064–1271 and 3707–2066 years BLE regimes. To further test the stationarity of these periods, we also apply two additional tests. First the Kolmogorov-Smirnov criterion was applied to the global rate for the periods 3707 and 7772 years BLE: the null hypothesis (the eruptive history is stationary) is rejected at the 95% level of confidence for both periods. Secondly, we apply a t student test on the regime's and global rates ($\lambda_{\mathrm{global}}$=0.002698 and $\lambda_{\mathrm{global}}$=0.001415 eruptions per year for the time periods 3707–0 and 7772–0 years BLE, respectively). The t-statistics from the 2064–1271 years BLE period compared with the global rates of the periods 3707 and 7772 years BLE are 2.4345 and 2.0086 (with 12 and 13 degrees of freedom), respectively, and the null hypothesis, that all rates belong to the same population, should be rejected for a two-tailed test at the 90% level of confidence. We thus take for granted the non-stationary character of the eruptive sequence, and mark the transitions between regimes by the vertical lines shown in Fig. 1.

4.1 Non-Homogeneous Generalized Pareto-Poisson Process (NHGPPP) method

The available information on the past activity of El Chichón volcano is not sufficient to assign precise VEI values to all of the eruptions. We thus estimate the most likely VEI values of the past eruptions in terms of the eruption occurrence rate of each class magnitude λ_{Mvei} using the scaling relationship

$$\log\lambda_{\mathrm{Mvei}} = a - bM_{\mathrm{vei}}\,. \tag{1}$$

This relation has been used on groups of volcanoes and on individual volcanoes to estimate the eruption rates for different VEI magnitudes (De la Cruz-Reyna, 1991, 1993; De la Cruz-Reyna and Carrasco-Núñez, 2002; De la Cruz-Reyna and Tilling, 2008; Mendoza-Rosas and De la Cruz-Reyna, 2008) using the available eruption records to obtain self-consistent series. We constructed two eruptive history models, shown in Table 2, for the eruptive series of El Chichón volcano with the data available for the last 3707 years BLE. Similarly, other two models based on the volcanic data for the last 7772 years BLE are listed in Table 3. These models assign possible VEI values to the eruptions with unknown magnitudes using the implicit condition expressed in Eq. (1) that eruptive rates and magnitudes are inversely related. The order of the assignment of VEI values for each eruption unknown does not affect the eruption rate values.

The VEI of the eruption of 3107 years BLE in which no volume or intensity data were available, was estimated testing the best fit to the VEI values of the other eruptions based on Eq. (1), and then selecting the model which best fitted the eruption rates determined by the scaling law. Tables 4 and 5 show the eruption rates λ_{Mvei} for each VEI class, the slope $-b$ from the loglinear relationships (1), and the regression coefficients for each of the models listed in Tables 2 and 3. The regression coefficients indicate that the best fits are for the models "Chichón 2" in Table 4 and "Chichón B" in Table 5. Although the volcanic hazard of El Chichón volcano has been previously estimated (Mendoza-Rosas and De la Cruz-Reyna, 2008), we have recalculated it here using the direct radiometric datings and errors, listed in Table 1, rather than the published "rounded" values (Espíndola et al.,

Table 4. Eruption rates λ_{Mvei} for each VEI class. The slope-b of the loglinear relationships given by Eq. (1), and the regression coefficients for each model of El Chichón volcano listed in the Table 2 for the period 3707 years BLE are included in the lower rows.

	Eruption annual rate λ_{Mvei}	
	Chichón 1	Chichón 2
VEI 3	0.001888	0.001619
VEI 4	0.000540	0.000809
VEI 5	0.000540	0.000540
Slope-b	–0.2720	–0.2386
R2	0.7500	0.9777

Table 5. Eruption rates λ_{Mvei} for each VEI class, the slope-b of the loglinear relationships from Eq. (1), and the regression coefficients for the two models listed in the Table 3 for El Chichón volcano in the period 7772 years BLE.

	Eruption annual rate λ_{Mvei}	
	Chichón A	Chichón B
VEI 3	0.001029	0.000901
VEI 4	0.000257	0.000386
VEI 5	0.000257	0.000257
Slope-b	–0.3010	–0.2720
R2	0.7500	0.9602

2000). The VEI values have also been revised using a different correction on the fact that the VEI magnitude scale is not open, as explained below. Additionally, some magnitudes have been reassigned on the basis of recent field work providing strong evidence that the eruption of 1270 years BLE was at least as large as the 1982 eruption (J. L. Macías, J. M. Espíndola, personal communications, 2010). For that reason, we are using VEI 5 rather that the previously published VEI 4 for that event as indicated in Table 1.

Using the "Chichón 2" model (Table 2), Eq. (1) may be written as $\log\lambda_{\mathrm{Mvei}}=-0.239M_{\mathrm{vei}}-2.096$. We now infer the number of eruptions that have exceeded a threshold (VEI=2), and calculate the excess and exceedance means (the exceedances are the events with magnitude higher than a threshold magnitude u, and an excess is the difference between the magnitude of the exceedance and the threshold u) to obtain the shape and scale parameters of the GPD, which result to be 0.290 and 3.462, respectively. The same procedure applied to the "Chichón B" model yield $\log\lambda_{\mathrm{Mvei}}=-0.272M_{\mathrm{vei}}-2.261$, and 0.247 and 3.121 as the respective GPD shape and scale parameters.

Table 6. Eruptive regimes and calculated parameters of the MOED for the eruptive sequence of El Chichón volcano (VEI≥3). The global rates for the 3707 and 7772 years BLE periods are 0.026976 and 0.014153 eruptions per decade respectively. The regime rates are also in eruptions per decade.

Regime	Time periods (BLE)	Number of eruptions	Duration of regime (decades)	Rate λ	Weighting factor w
Sum of three exponentials distribution parameters (3707 years BLE)					
1	3707–2066	3	164.2	0.018270	0.278527
2	2065–1271	4	79.5	0.050314	0.392770
3	1270–0	3	127	0.023622	0.328702
Sum of four exponentials distribution parameters (7772 years BLE)					
1	7772–3708	1	406.5	0.024600	0.158990
2	3707–2066	3	164.2	0.018270	0.262910
3	2065–1271	4	79.5	0.050314	0.299237
4	1270–0	3	127	0.023622	0.278864

Since the VEI scale is not defined for values greater than 8, and the exceedances method assumes that the scale measuring the phenomena is open, we subtract the probability of exceeding the VEI 8 magnitude from the probabilities of exceeding VEI's lower than 8 obtained with the GPD intensity function, and not from the occurrence probabilities as in the previous work (Mendoza-Rosas and De la Cruz-Reyna, 2008).

4.2 The Mixture of Exponentials (MOED) method

The first question to address in the application of the MOED method refers to the completeness of the eruption data base. Figure 1 shows that three and four regimes may be recognized from the cumulative plot of El Chichón eruptions VEI≥3, considering the eruptive history to 3707 and 7772 years BLE, respectively. The regimes are distinctly differentiated from the overall mean regime; the difference among them is larger in the case of the extended eruptive history shown in Fig. 1b. In fact, the global rate eruption for the 7772 years BLE period is not even similar to the eruption rates of the regimes (Table 6). We speculate that this may be a reflection of the lack of completeness of the extended eruptive history in the specified VEI range, rather than a very different eruption rate of the longer period. Although the MOED method requires completeness, we nevertheless estimate the volcanic hazard using both, the eruptive history to 3707 years BLE, and the extended history to 7772 years BLE (Fig. 3), and compare the results with those obtained form the NHGPPP, Poisson and Weibull distributions for the same periods. The MOED parameters calculated from the eruptive history of El Chichón volcano are shown in Table 6. The resulting probabilities of future eruptions are discussed next.

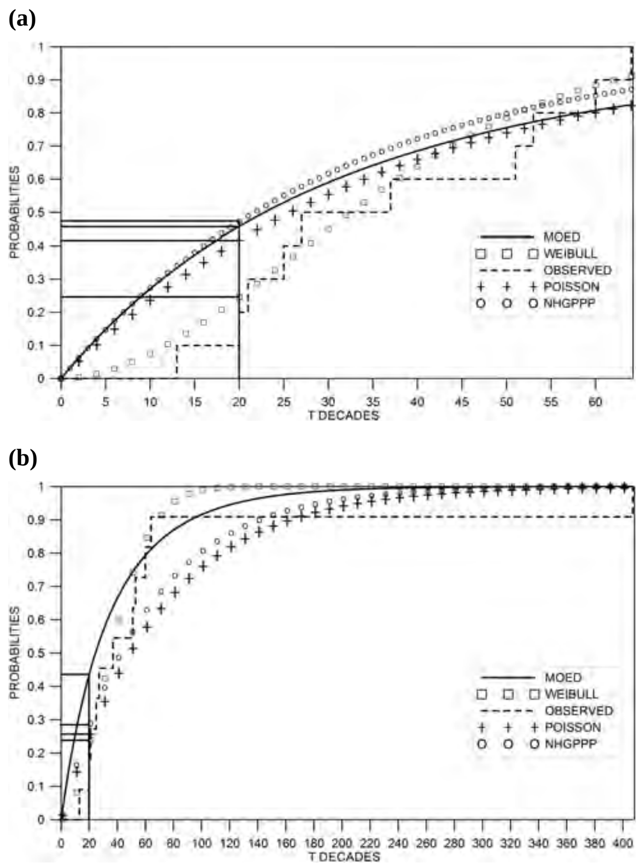

Fig. 3. Probabilities calculated by NHGPPP, MOED, Poisson and Weibull distributions of at least one eruption, with VEI≥3 in a given time interval from **(a)** "El Chichón 2" over 3707 years BLE. The horizontal and vertical lines show the different probabilities of at least one eruption in 20 decades with VEI≥3 (NHGPPP: 0.4724, MOED: 0.4582, the Weibull distribution: 0.2460 and Poisson distribution: 0.4170), and **(b)** "Chichón B" over 7772 years BLE eruptive models. This is equivalent to the probability of observing a repose time less or equal than T decades. The horizontal and vertical lines show the different probabilities of at least one eruption in 20 decades with VEI≥3 (NHGPPP: 0.2777, MOED: 0.4371, the Weibull distribution: 0.2219 and Poisson distribution: 0.2465).

5 Discussion

The probabilities of occurrence of at least one eruption exceeding a VEI magnitude in a given time interval by the MOED, NHGPPP, Poisson and Weibull distributions from the "Chichón 2" and "Chichón B" are plotted in Fig. 3.

To obtain an objective measure of the quality of the fit between each distribution and the eruptive history data, we performed three non-parametric goodness-of-fit tests (Cramer-von-Mises, Kolmogorov-Smirnov and Anderson-Darling), and the AIC method described in a previous section.

In the 3707 years BLE period (Fig. 3a), the NHGPPP, MOED and Poisson distributions behave similarly. The MOED shows probabilities between the NHGPPP and the Poisson probabilities, because the weighting criterion emphasizes the importance of the short-duration high-rate regimes not considered by the Poisson distribution. The NHGPPP's probabilities yield slightly higher values in the intermediate-high time range i.e. somewhat higher occurrence probabilities for longer intervals. The fact that the NHGPPP values are not very different from the other distributions, mainly for the short periods, is consistent with the assumed completeness of the 3707 years BLE eruptive period. The best fit in the shorter periods is obtained by the Weibull distribution for its ductility resulting from the empirical adjustment of the shape and scale parameters and the AIC method confirms the best fit to the Weibull distribution. Despite those differences, all of the distributions pass the goodness of fit tests with the eruption data, and each of them may be accepted at a 0.05 significance level (Anderson and darling, 1952, 1954). The tests results are listed in Table 7. The acceptable results of the Poisson distribution are a consequence of a mean rate that provides a good average of the high and low regimes. However, it yields slightly lower probabilities, reflecting the influence of the longer duration of the low regimes (Fig. 3a). This may underestimate the probability that an eruption ended a low regime and the next may correspond to a high regime, an effect that is accounted by the MOED. Although the MOED probabilities also are the averages of a renewal process, they result to be more sensitive to the existence of previous regimes than the Poisson estimates. It must be remarked that this does not mean that the MOED estimates are "regime variables" as those obtained by Bebbington (2007), who uses Hidden Markov (HM) models to identify regime changes, and thus estimate the probability of being in a regime. The renewal processes are series of events in which the times between events are independently and identically distributed (Cox and Lewis, 1966) as is the case of the models used here. In contrast, the HM models may require some degree of periodicity or structure of the eruptive record, or at least of some of its sections.

Figure 3b shows the calculated probabilities and the observed distribution for the period of 7772 years BLE. Now, the Poisson probabilities separate significantly, as may be expected from the inaccuracy of the global rate of eruption as a single parameter describing the whole eruptive process (Fig. 1b). Although the Weibull distribution shows a good fit in the short repose periods, it "saturates" at periods of about 100 decades revealing some inability to deal with the long-tail of the distribution.

Unlike the previous case of Fig. 3a, now the NHGPPP yields probabilities significantly lower than the MOED. This is a consequence of decreased eruption annual rates when using an extended period containing only one eruption (Tables 4 and 5). Contrastingly, the weighting criterion of the MOED emphasizes the importance of the short-duration

Table 7. Statistics of goodness of fit tests for the different statistical methods.

	Poisson Distribution	MOED	Weibull Distribution	NHGPPP
3707 years BLE		(Three regimes)		
Cramer – Von-Mises	0.2325	0.3200	0.0570	0.3653
Anderson – Darling	1.2832	1.6516	0.3904	1.8308
K-S	0.1878	0.2001	0.1320	0.2100
7772 years BLE		(Four regimes)		
Cramer – Von-Mises	0.1963	0.3273	0.0596	0.1407
Anderson – Darling	1.1706	2.0391	2.0856	1.0382
K-S	0.3249	0.1984	0.1233	0.2738

Table 8. Serial correlation coefficients of different sets of eruptions. The sets are formed with different combinations of the average from the 5th to the 8th eruption (Table 1).

{Eruptions}	Correlation Coefficient
{0, 635, 905, 1270, 1524, 1657, 1857, 2065, 2590, 3107, 3707}	0.4037
{0, 635, 905, 1270, *1591*, 1857, 2065, 2590, 3107, 3707}	0.1657
{0, 635, 905, 1270, 1524, *1757*, 2065, 2590, 3107, 3707}	0.2908
{0, 635, 905, 1270, 1524, 1657, *1961*, 2590, 3107, 3707}	0.3037
{0, 635, 905, 1270, 1524, 1657, 1857, 2065, 2590, 3107, 3707, 7772}	0.4832
{0, 635, 905, 1270, *1591*, 1857, 2065, 2590, 3107, 3707, 7772}	0.4571
{0, 635, 905, 1270, 1524, *1757*, 2065, 2590, 3107, 3707, 7772}	0.4727
{0, 635, 905, 1270, 1524, 1657, *1961*, 2590, 3107, 3707, 7772}	0.4116

high-rate regimes at the expense of the low rate regimes. Therefore, its probability estimates do not change much between the 3707 and the 7772 periods (Fig. 3). Although one may expect that the MOED probabilities would be affected by the possible incompleteness of the longer period data, the probabilities for both periods are very similar. Despite the differences among the probabilities calculated from the distributions, all of them pass the goodness of fit tests and each of them may be accepted at a 0.05 significance level (Table 7) (Anderson and darling, 1952, 1954).

How sensitive are these methods to possible errors in the sampling of the eruptive history? We may illustrate the answer to this question through an example that involves an inherent difficulty in the construction of eruptive records: the identification of pairs of past eruptions as single events. The point process hypothesis stated in Sect. 3 requires that the events are independent. The independence of successive repose periods may be difficult to determine due to the uncertainty in the recognition and dating of geological deposits. A measure of the independence among events is the serial correlation between successive repose periods, T_i vs. T_{i+1} (Cox and Lewis, 1966). In the present case, the serial correlation coefficients between eruptions over the periods 3707 and 7772 years BLE are 0.40 and 0.48, respectively (Table 8). Independence requires serial correlation coefficients near zero. Although the above values are not statistically significant, they point to a possible weak serial correlation of the repose times. Assuming that some events of the eruptive series of El Chichón volcano may be correlated, we addressed the possibility that two successive, near in time events may be a single eruption, considering the overlapping of the dating error ranges (Table 1). This possibility is reinforced by the absence of paleosoils between the deposits of assumedly different eruptions (Espíndola et al., 2000), as is the case of the 5th to the 8th eruption (1524, 1657, 1857, and 2065 years BLE). Table 8 shows the serial correlation coefficients of different possible cases. The eruptive sequence with the lowest serial correlation coefficient (0.1657), is $\{0, 635, 905, 1270, \mathit{1591}, 1857, 2065, 2590, 3107, 3707\}$. We consider that the drop of the serial correlation coefficient by a factor of 2.5 after merging those adjacent eruptions is a relevant indicator for selecting them. The date 1591 years BLE is the average between the reported 1524 and 1657 years BLE events. It is thus possible that the reported eruptions of 1524 and 1657 years BLE may be a single event occurring near 1591. Assuming the above eruption sequence as true, the occurrence rate of regime (2064–1271 years BLE) decrease and the eruptive series would approach a stationary

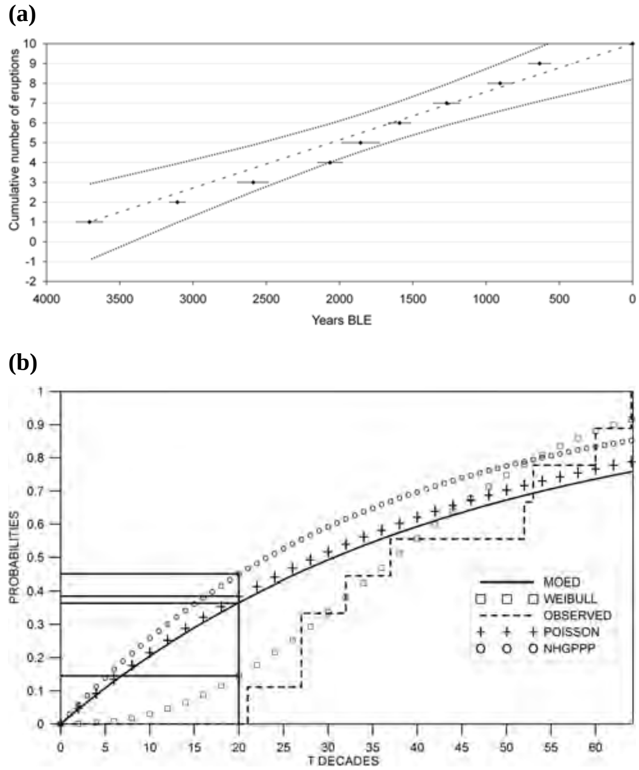

Fig. 4. **(a)** Cumulative number of eruptions of El Chichón volcano for the period 3707 years BLE in the magnitude class VEI≥3, assuming that the eruptions 1524 and 1657 years BLE are a single event occurred near 1591. The slopes of the dashed line represent the eruption rate of the whole series. (λ_{global}=0.0243 eruptions per decade). **(b)** Probabilities calculated by NHGPPP, Poisson and Weibull distributions of at least one eruption, with VEI≥3 in a given time interval from "El Chichón 2C" over 3707 years BLE. The horizontal and vertical lines show the different probabilities of at least one eruption in 20 decades with VEI≥3 (NHGPPP: 0.4499, MOED: 0.3657, the Weibull distribution: 0.1452 and Poisson distribution: 0.3847).

behavior (Fig. 4a). In this case, the best model would be the Chichón 2C in Table 9. Notwithstanding, the probabilities obtained with the NHGPPP, Poisson, MOED and Weibull distributions from this model are not so different from the estimates of "Chichón 2" model (Figs. 3a and 4b). The MOED yields almost the same probabilities that Poisson distribution due to the stationary behavior. The AIC proves the best fit to the Weibull distribution and all of the distributions may be accepted at a 0.01 significance level with different hypothesis tests. It is important to emphasize that if the interevent times or repose periods are not independent, the methods applied in these paper are not appropriate because they would not satisfy the renewal process definition.

Table 9. Models and Eruption rates λ_{Mvei} for each VEI class, the slope *-b* of the loglinear relationships from Eq. (1), and the regression coefficients for the two models for El Chichón volcano in the period 3707 years BLE assuming the eruptive sequence $\{0, 635, 905, 1270, \mathit{1591}, 1857, 2065, 2590, 3107, 3707\}$ (Table 8).

YearsBLE	Reported VEI	Chichón 1C VEI	Chichón 2C VEI
0	5	5	5
635	4	4	4
905	3	3	3
1270	4∼5	5	5
1591	3	*3*	*3*
1857	2∼3	3	3
2065	2∼3	3	3
2590	2∼3	3	3
3107	*?*	*3*	*4*
3707	4	4	4
Eruption annual rate λ_{Mvei}			
VEI 3		0.001619	0.001349
VEI 4		0.000540	0.000809
VEI 5		0.000540	0.000540
Slope-*b*		–0.2386	–0.1990
R2		0.7500	0.9956

6 Conclusions

In the present case, all of the distributions pass the goodness of fit tests with the El Chichón eruption data. There are, however differences among the results that provide important information about the reaches of the distributions. The homogeneous Poisson distribution gives results similar to the other distributions in the 3707 years period. However, in the extended period has a poorer performance, compared with the other distributions. This result confirms the difficulties of the Poisson distribution to describe strongly non-stationary, incomplete series, yet it stills provides acceptable results for weak non-stationarities using the mean rates. The Weibull distribution has an overall good performance, also passing all the goodness of fit tests. However, in the present case the fitting is best only with the probabilities for short waiting times (Fig. 3). At longer waiting times it saturates before any of the other distributions, and no information of the long term behavior may be obtained. In contrast, the NHGPPP distribution shows a better fit in the tail of the distribution, and a significant difference between the probabilities calculated for both periods of study. This is a consequence of the ability of this distribution to comprise the different characteristic of the extended period: a higher degree of non-stationarity and a probable incompleteness of eruptions with VEI below 4.

The MOED proves to be a straightforward and simple method that provides reliable hazard estimates for non-stationary eruptive histories, yielding similar results to the other methods. The MOED parameters can be simply and directly calculated from the inspection of the cumulative distribution of eruption occurrences in specific VEI classes and thus contain information of the process involved in the non-stationarity of the eruption time series, reflected as a succession of eruptive regimes. In the present case, the MOED shows little change between the probabilities calculated for both periods. This is a consequence of the way the MOED weights the contribution of the regimes: inversely to their duration. Therefore, using extended periods containing incomplete eruptive histories may produce results that must be interpreted with caution, because it may underestimate or overestimate the probabilities depending on the length and eruptive rate of the extended regimes.

The NHGPPP probabilities are also affected by the duration of the extended period in a different way. Unlike the MOED, the NHGPPP probabilities are calculated from the number of VEI values exceeding a threshold for the whole period, independently of the regimes. Therefore, in the present case, the rate of excesses is drastically reduced after adding only one eruption (7772 years BLE) exceeding the VEI 3 threshold. The NHGPPP distribution thus provides the best estimates when the low-rate extended period is included.

The above arguments are supported by the goodness of fit tests. MOED and NHGPPP fit almost equally well the data of the 3707 years BLE period. On the contrary, in the extended period NHGPPP fits better than MOED (Table 7). The Akaike Information Criterion produces similar results yielding essentially the same AIC value for MOED and NHGPPP in the 3707 years BLE period, and a much lower AIC value for NHGPPP that for MOED in the extended period. However, an important issue regarding the AIC – and other parametric tests – must be considered here. When parametric tests are used, the relatively large number of parameters required by the MOED modifies the degrees of freedom and hence the significance levels of the tests, penalizing the number of distribution parameters. To account for this, in the present case we have also relied on non-parametric tests.

Except for the Weibull distribution, all of the other distributions tend to produce higher probabilities of at least one eruption of El Chichón occurring in the short-period range (less than 40 decades). This reflects the nature of the distributions. In the present case, Weibull shows a remarkable capacity to adapt its shape to the observed data in the short-period range, but it does not fit well the data of El Chichón long tail. Attempts to adjust the distribution parameters to fit the tail data would reduce the quality of fit at the short periods. This was observed by Dzierma and Wehrmann (2010), in their study of different eruptive histories of Chilean volcanoes. They noted that varying the scale parameter of the Weibull distribution improved the fit to longer repose times, but at the expense of the quality of fit to the short repose times.

Increased reliability in the assessment of the volcanic hazard from long, non-stationary and probably incomplete eruptive histories may be gained testing the data with different statistical distributions. For the eruptive series of El Chichón volcano we may conclude that the probabilities of at least one eruption occurring in the relatively short time range estimated from most of the tested distributions (except Weibull) yield greater values than expected from the observed distribution. This is a consequence of the absence of short repose times (less than 50 decades) in the Holocenic eruptive history of El Chichón, a fact that may be related to the apparent absence of low-magnitude eruptions. Although the Weibull distribution fits well the eruptive record in the short-time range, we believe that the lower Weibull probabilities resulting from the absence of short repose periods in the small population of Holocenic eruptions may lead to underrating the volcanic hazard in the short-period range. We thus prefer the seemingly overestimated probabilities of the other distributions, rather than the low Weibull probabilities.

For the long-term probabilities the NHGPPP shows an additional ability to incorporate the effect of scarce data of major past eruptions. If an extended eruptive history is available, as in the present case, using simplicity as a ranking criterion, we would recommend the MOED for estimating the probabilities of future eruptions in the short time range and the NHGPPP for the long time range.

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Capítulo 5.

CONCLUSIONES

La complejidad en la evolución de la historia eruptiva de un volcán y la escasez de datos precisos de las fechas y magnitudes de los eventos pasados dificulta la estimación del peligro volcánico, y consecuentemente reduce la confiabilidad de la evaluación del riesgo volcánico.

Las metodologías propuestas en el presente trabajo buscan analizar los factores involucrados en esa complejidad, y permiten cuantificar el peligro volcánico para volcanes poligenéticos cuyos datos se obtienen de las historias eruptivas compuestas de datos geológicos e históricos; sin importar las diferencias en las escalas de tiempo y en la naturaleza de los datos disponibles. Estos métodos buscan extraer información de series en las que puede haber ausencia de datos principalmente de eventos mayores, generalmente de edades grandes; cuyos depósitos no permiten determinar con precisión sus edades ni sus magnitudes. Asimismo, estos métodos incorporan características de inhomogeneidad de las series, tales como la dependencia en el tiempo (no-estacionaridad) de las historias eruptivas.

Un problema central lo representa la ausencia de datos de magnitud de muchas de las erupciones de las publicaciones y catálogos disponibles. Para abordar este problema, el análisis estadístico realizado asume una relación inversa entre la tasa de ocurrencia y su magnitud (De la Cruz-Reyna S. 1991, 1996), obteniendo diferentes modelos de los valores más probables de las magnitudes VEI y así incorporar al análisis aquellas erupciones con incertidumbre en su magnitud.

La primera metodología estadística propuesta es el PPNHP (NHGPPP: Non-homogeneous generalized Pareto–Poisson process), que permite enfatizar la importancia de los datos que se encuentran en la cola de la distribución de las series eruptivas, los que en general corresponden a las erupciones de mayor magnitud. Estas grandes erupciones son de gran importancia por el impacto en el entorno, y pueden no ser consideradas en la medida adecuada por otras distribuciones debido al número generalmente muy pequeño de eventos que las representan. Generalmente, este tipo de eventos mayores se encuentran en las porciones de las series correspondientes a los datos geológicos. Esta metodología, a pesar de que su aplicación es compleja, tiene las ventajas de enfrentar los problemas de: escasez de datos, incertidumbre en la asignación de la magnitud VEI, describir la posible dependencia en el tiempo y homologar datos de distinta naturaleza (geológicos e históricos) y escala en el tiempo.

La segunda metodología estadística propuesta, MEDE (MOED: Mixture of exponentials distribution) se basa en una suma pesada de distribuciones de exponenciales. Está enfocada a series eruptivas completas (esto es, en las que se presupone un catálogo completo de erupciones; como es el caso del catálogo de erupciones históricas del Volcán de Colima y del Volcán Popocatépetl) con comportamiento no estacionario, donde varios regímenes de actividad pueden ser identificados como periodos con tasa eruptiva alta intercalados con periodos de tasa eruptiva baja. Esta metodología además de tener la ventaja de ser simple en su aplicación y considerar la no-estacionariedad de la serie eruptiva, proporciona un mejor ajuste de los datos extremos que las distribuciones de Weibull y Poisson.

La MEDE tiene la ventaja de ser más sencilla que el PPNHP y permite obtener los parámetros de la distribución directamente de la serie acumulativa de erupciones. Eso significa que a los parámetros de la distribución se les puede asignar a priori una propiedad física importante del proceso eruptivo, que es la secuencia de regímenes. Esta propiedad refleja la capacidad de los sistemas volcánicos de equilibrar las tasas de captación de fuentes profundas de magma con la capacidad de liberación de masa y energía hacia la superficie a distintas tasas; dado que una condición de equilibrio se refleja en la alternancia entre las tasas bajas y altas. Sin embargo, es importante recalcar que esta metodología está limitada a ser aplicada a bases de datos completas en comparación del PPNHP que proporciona buenos y confiables resultados aún con bases de datos incompletas.

El desempeño y alcance de estas metodologías se evaluó por medio de la comparación entre el PPNHP y la MEDE con métodos convencionales como el proceso de Poisson y la distribución de Weibull. También se consideraron varias pruebas de bondad de ajuste para evaluar los resultados. Las metodologías utilizadas requieren que los procesos sean de Bernoulli (procesos sin memoria), esto es, que no exista dependencia entre los eventos. También pueden ser aplicadas a diversos volcanes en el mundo en los que se cuente con una historia eruptiva mínima, permitiendo obtener una evaluación confiable del peligro volcánico como un sistema de caja negra donde no se tiene control de las entradas al sistema, y el análisis se realiza con la respuesta del sistema que son las series eruptivas.

En el presente análisis, se evalúa el peligro para distintos volcanes mexicanos, en los que destaca la complejidad de sus series eruptivas (la respuesta del sistema). Por lo que se concluye que el volcán de Colima tiene el más alto nivel de peligro entre los casos estudiados, debido a la alta probabilidad de que ocurra una erupción en un intervalo de tiempo específico, consecuencia de la mayor frecuencia eruptiva reflejada en las tasas de ocurrencia, además de ser el que tiene un comportamiento más inhomogéneo, esto por una dependencia en el tiempo más fuerte mostrada en la alternancia de las tasas bajas y altas de ocurrencia eruptiva, es decir, la ocurrencia de una erupción volcánica depende de la posición en el tiempo. El volcán Popocatépetl, que muestra un comportamiento cuasiestacionario es el segundo volcán de mayor peligro. El volcán El Chichón con una ligera no-estacionariedad (ligera dependencia en el tiempo) muestra menores valores relativos de peligro volcánico comparado con los anteriores. Por otro lado, el Citlaltépetl o Pico de Orizaba y el Nevado de Toluca muestran una menor peligrosidad relativa entre los volcanes estudiados en este trabajo. Los resultados obtenidos permiten comparar cuantitativamente la peligrosidad entre varios volcanes facilitando la toma de decisiones objetivas en el reparto de recursos y en el uso de suelo. Esta contribución intenta lograr una evaluación del peligro volcánico lo más objetiva posible que contribuya a la prevención de un desastre.

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