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MAESTRÍA EN ECONOMÍA

IMPACT OF RAW MATERIAL PRICES ON INFLATION IN MEXICO

TESIS

QUE PARA OPTAR POR EL GRADO DE:

MAESTRA EN ECONOMÍA

PRESENTA

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Chapter 1

Introduction

The Mexican economy has shown persistent inflation since the Covid-19 crisis, with a crucial impact on the purchasing power of the population and a reduction in output and employment, followed by a slow recovery that is nowadays still away from the situation before the pandemic. Inflation is the main variable remaining distant from the earliest value.

The food and the energy industries have been the industries with greater negative effects from the sanitary crisis. Both industries are characterized by highly concentrated markets where a couple of firms hoard most of sales (COFECE (2021), COFECE; 2020, and González Santana et al. (2015)). The prime source of this effects is the price increase in raw materials coming from the global supply chains cuts and, to some degree, from the Russo-Ukrainian War (BANXICO; 2022). We want to see how a positive cost-push shock affects the Mexican economy, particularly the food and energy industries (in order to account for the Covid-19 crisis), both industries characterized by concentrated markets.

For our purposes, there are two key results from the theoretical framework concerning imperfect markets that are essential to justify this study. The first one is that the cost pass-through is greater the more market power there is (Meyer and von Cramon-Taubadel; 2002), while the second one is the markup counter cyclical behavior, meaning that the markup tends to increase in times of lower economic activity (Chevalier and Scharfstein; 1996). We build upon both results to formulate our hypothesis, where we suggest that recent inflation in Mexico comes from a mixture of both price increases in raw materials and the market power in the most affected industries.

Even if inflation initially increased due to external conditions (Esquivel and Leal; 2022), the possibility of large cost pass-through in the affected national industries and the markup behavior on crisis seem to be fundamental causes for the huge inflation increase in Mexico since most of the rise in costs derived in a symmetrical increase in consumer prices, amplified by price increases coming from firm's lower expected markups and higher expected inflation. We will test the hypothesis that greater inflation in Mexico in the sanitary crisis is due to its impact on intermediate good industries with concentrated markets.

In this study, we will use two variations of a DSGE model to analyze recent inflation in Mexico and its fundamental sources, since we want to simulate the price increments from raw materials and the consequent increase in national consumer prices. We will estimate two DSGE models that includes a cost-push shock following an AR(1) stochastic process with zero mean and constant variance, the first one with an implicit mark-up equation and the second one with an explicit one. Market power will be modeled using firms that produce differentiated goods and that are restricted by some price stickiness when setting prices.

Our DSGE model consists of three sectors: households, firms and the monetary authority (Central Bank). To test our particular hypothesis about intermediate goods inflation we define a cost-push shock that simulates this inflation, to account for the market power and its positive impact on inflation, we set a continuous index of firms producing differentiated goods and facing some restrictions when setting their prices. In this case, we will not incorporate an explicit equation or parameter concerning global supply chains since we will model inflation shocks using the cost-push AR(1) shock (following the methodology framework).

We first implement a model with implicit sticky prices where the index of price stickiness has a positive impact on inflation and a negative impact on the implicit natural output. Since an increase in price stickiness reduces the natural level of output, it also decreases the markup gap (since the markup is counter cyclical and a reduction in the natural level of output tends to increase this markup, Zaleski; 1992), deriving in greater inflation.

In a second model, we define an explicit markup equation affected by the capital participation, and which increases when the output gap does, as Zaleski (1992) suggests. Price stickiness has the same impact on the New Keynesian Phillips Curve as in the first model. With this second model we plan to study the markup behavior during the Covid-19 crisis in Mexico and some differences with respect to the first model emerging from the output gap in this model.

For both models, we found that a cost-push shock has a negative effect on output, with a greater response and length for the second model, which is consistent with the Mexican data; in both models, we have an increase in price level and inflation. For model one, we have an increase in the output gap, and for model two, we have an increase in the markup. A key difference from both models is that, while in the first model we have a reduction in the interest rate after the cost-push shock with a subsequent increase and stabilization of the interest rate above the original rate, in the second model we see an initial increase on the interest rate followed by a reduction of the interest rate bellow the original rate converging asymptotically to the rate before the shock.

The reminder of this work is organized as follows. Chapter 2 has a brief introduction to current inflation in Mexico, as well as a study about market concentration in the main industries affected by recent inflation; Chapter 3 follows the theoretical framework about imperfect markets and markups; Chapter 4 summarizes some of the fundamental works and papers related to DSGE models; Chapter 5 breaks down the model used (based on Gali (2015)) with specific calibration for Mexico, all the mathematical derivation was done by us; Chapter 6 illustrates the pseudo-code latter implemented in Dynare; Chapter 7 shows the key results from applying the DSGE model for the Mexican economy based on the Impulse Response Functions obtained; Chapter 8 are the conclusions.

Chapter 2

Literature review

This work is related to New Keynesian Models applying some nominal rigidities such as Gali (2002), where they implement price rigidities for firms producing differentiated goods. Inflation follows a New Keynesian Phillips curve and this inflation will come from the impossibility from firms to set prices every period, causing a gap between the output and the natural level of output (defined as the output with flexible prices).

Some of the fundamental hypothesis from these type of models is that they define a dynamic general equilibrium model where the equilibrium conditions for the aggregate variables come from the individual optimization behavior of the economy sectors and assuming clearing for every market. Finally, these types of models put together Keynesian elements (such as imperfect markets and nominal rigidities) with the Real Business Cycle paradigm.

For the Gali (2002) model, some underlying hypothesis are that firms set their prices taking into account expectations on the future cost and demand conditions, so the inflation equation will also forward look and will be defined with a New Keynesian Phillips Curve, Another assumptions are a perfectly competitive labor market, a continuum of firms producing differentiated goods and a monetary policy looking to maximize households utility. This model focuses on a monetary shock given by an exogenous path fro the growth rate of money supply and the corresponding responses from output, inflation and the exchange rate.

The difference between this paper and our work is that while the author considers monetary and technology shocks on models with different characteristics such as sticky prices and different monetary policy rules, he does it by theoretical analyzing each situation and without applying the model to a particular economy. On the other side, we study cost shocks inside the inflation equation to evaluate the impact of a change in the firm's costs from an increase in intermediate good prices and we apply a model with nominal rigidities and imperfect competitions to the Mexican economy..

Another paper related to our work is Christiano et al. (2005), where they set price and wage rigidities to account for observed inflation dynamics and persistent effect on other variables from a monetary shock, such as the output level. They seek to account for inertial inflation and persistence output movements as a response to a monetary shock via nominal rigidities in the economy. Some keys assumptions of the model are nominal price and wage rigidities, adjustment costs in investment, and variable capital utilization.

They size the effect of a monetary policy shock, departing from a policy rule given as a function of the information available at the time, plus a stochastic shock. In contrast to our model, they separate the producing sector by setting perfectly competitive firms producing unique final goods and monopolistic firms producing intermediate goods indexed by $j \in (0, 1)$. This model also includes capital so the production function also depends on the capital stock and there is a capital formation equation, in contrast with our model, were we omit capital to simplify the analysis.

A key contribution for nominal rigidities in dynamic models comes from Calvo (1983). Most of the developments in models with nominal stickiness are derived from the model specified in this paper. This model introduces nominal rigidities by assuming that prices change probabilistically following a geometric distribution and independent of the last price change from the firm and independent across firms. In contrast with the model implemented in this work, prices will be updated by firms taking into account the average price and the excess demand (which depends on real monetary balances). In our model, instead, firms will update prices taking into account current and expected deviations on markups from the desired one (particularly, due to changes in costs and expected inflation) and expected periods until the next price update.

In Gali and Gertler (2000) they study inflation using a New Keynesian Phillips curve (NKPC) where real marginal costs play a fundamental role on inflation. They use marginal costs instead of output gap in the NKPC because , they explain, it allows to show productivity gains on inflation, in contrast to the output gap. Also, they introduce a "cost-push" term in the NKPC with the discounted expected future inflation. On the other hand, they incorporate two different characteristics for firm prices: the first one is some stickiness on prices in the style of Calvo (as have been introduced in the previous mentioned papers), the other one is allowing some fraction of firms to use a backward looking rule to set prices, this last parameter helps to account for the degree the NKPC fits the data. The main results are that real marginal costs is a better measure of inflation than the output gap and that the degree of firms using a forward looking rule goes from 60% to 80%.

The latter model uses US quarterly data from 1960 to 1997. A key difference from this model with respect to ours is that they define the NKPC using marginal cost instead of output gap, and this inflation equation has a forward looking component as weel as a backward looking component, while the NKPC defined in our model only has the forward looking component.

On the other hand, Smets and Wouters (2007) introduce several nominal rigidities, combined with seven types of shocks in a DSGE model for the US economy. There are demand and supply shocks, particularly, some rigidities come from price and wage stickiness, which allow to have a bigger general inflation. This model follows the classical DSGE model assumptions for the households, but there is some monopolistic power in the labor market. They introduce capital adjustment costs and a differentiated good market for the products in the economy. in this model, the price adjustment equation depends on current and future marginal costs, as well as the past inflation path, and the wage equation depends on the past and the future inflation and wages.

For the shocks introduced in the model, three of them come from a cost-push shock; two of those shocks are markup shock on goods and wages and the other one is a shock in the risk premium on capital. In contrast to this article, we use a forward looking price adjustment equation with no backward component, we omit capital, and we set perfectly competitive labor market with no rigidities, as we have the objective of showing the particular effect of marker power and a cost-push shock on inflation.

Andic et al. (2014) also study inflation dynamic for the Turkish economy. They focus on the role of costs in the consumer price index. In contrast to our model, they separate real marginal costs into domestic and foreign costs and focus on the first ones. With respect to the real marginal cost, they use the output gap as an approximate measure of it and they show that the output gap better fits the Turkish domestic marginal cost. Using this perspective, they are able to separate the impact from a foreign or domestic cost increase.

The last model uses an hybrid NKPC, meaning that the inflation equation has a forward looking component, as well as a backward looking component. Another interesting variation from our model is that the marginal costs in the NKPC add import prices and lagged import prices since it takes some quarters for the exchange rate pass-through to the CPI.

With respect to the Mexican case, Ramos-Francia and Torres (2008) estimate the short-run inflation dynamics for the country with an hybrid NKPC. With this approach, they found that the more backward looking firms there are, the inflation persistence is higher so, as inflation falls, there are less firms using a backward looking criteria to set prices and more firms focusing on the forward looking component. Unlike our model, the authors use a structural econometric approach to estimating the hybrid NKPC parameters and they focus on the study of this equation while we integrate the NKPC to a DSGE model for the Mexican economy.

While most of the literature related to our work use a hybrid NKPC, we will use a NKPC without the backward-looking component. Some of these models incorporate the exchange rate to study an open economy, and we do not integrate it since our NKPC uses the output gap instead of the marginal cost (the exchange rate is a component of this costs). on the other hand, unlike other models, we stick to a model that incorporates firms that produce differentiated goods for simplicity.

Chapter 3

The stylized facts on inflation and market power in Mexico

This work aims to develop a model able to explain recent inflation in Mexico and the transmission mechanism. For doing so, we first consider which factors or products have impacted to the greatest extended recent inflation in the country; as a second duty, we study the market structure in these particular products to define more realistic assumptions in our model. Finally, with the former research, we will simulate the impact of these product prices on inflation with a DSGE model characterizing the phenomenon for Mexico. In this section, we will characterize sectors and products influencing recent inflation so we can later define the particular market structure of these sectors or products.

Since march 2021, there has been an inflation increase in Mexico. At the beginning, the source of inflation was, on the one side, a rise in energetic prices due , initially, to the recovery from the lock down at the beginning of 2021 and, latter, to the Russian invasion later at 2022, on the other side, the cut of global supply chains due to the shut down of many firms because of the pandemic Celasun et al. (2022).

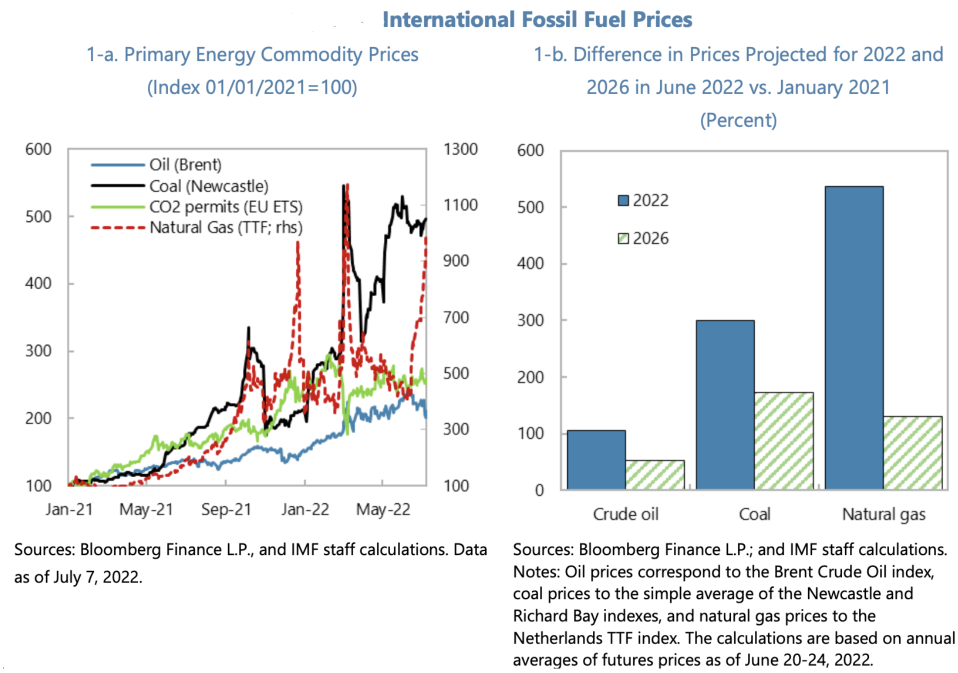

Figure 3.1: Source: Celasun et al. (2022)

Later, there has been a change in inflation source from energetic to merchandise (Index integrated by processed foods, beverages, tobacco and other merchandise) as can be shown in the next chart from Instituto Nacional de Estadistica y Geografia (INEGI).

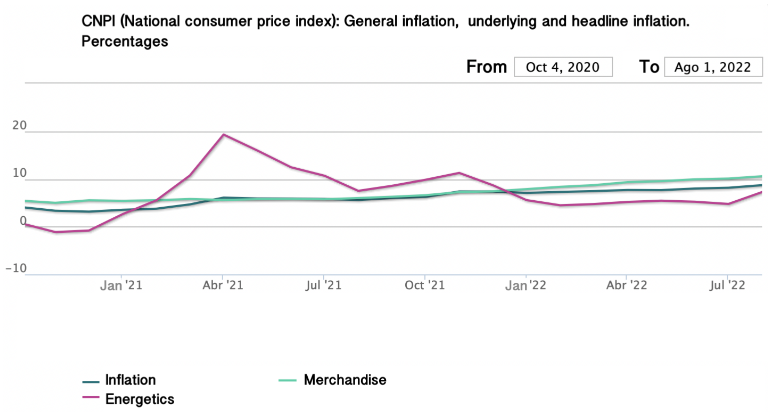

Figure 3.2: Source: Instituto Nacional de Estadistica y Geografia (INEGI), consulted on the web page: https://www.inegi.org.mx. Information calculated and published by INEGI from July 15, 2011, in accordance with articles 59, section III, First and Eleventh Transitory of the Law of the National Statistical and Geographic Information System (LSNIEG).

The change of inflation source from the energetic sector to the merchandise sector can be associated with a recent stabilization of energetic prices (particularly petrol for the Mexican case) joint with an increase in core inflation variation as a consequence of the service demand recovery and the continuous increase in merchandise prices. The increase and stabilization of both the energy sector and services can be associated with a phenomenon called the trend reversal (Bernoth and Gokhan; 2021), which refers to the recovery on some product prices to their trending levels, attributed to the reboot from some reduction in demand and prices of different products generated by the pandemic restrictions.

We apply Pearson's correlation coefficients between energetic price percent change over previous period (PNRG) and all commodity price percent change over previous period (PALLFNF) and between food and beverage price percent change over previous period (PFANDB) and all commodity price percent change over previous period using quarterly data from the World Economic Outlook publication of the International Monetary Fund (IMF) from 2020Q1 to Q1 2023Q1 world data to show how these sectors separately influence general inflation on this period of time.

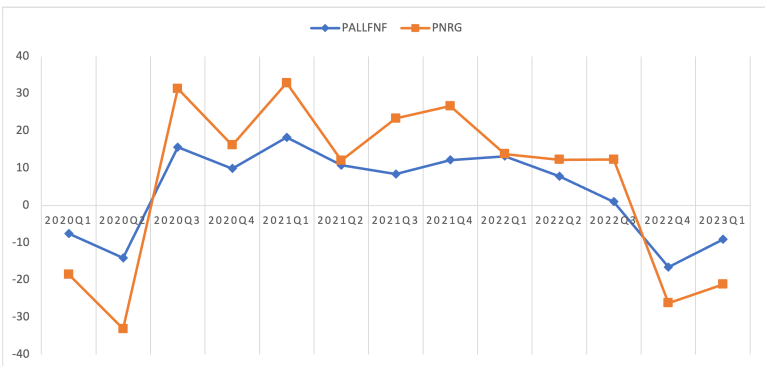

Figure 3.3: Data from International Monetary Fund (IMF), consulted on the web page: https://www.imf.org/en/Research/commodity-prices on April 17, 2023

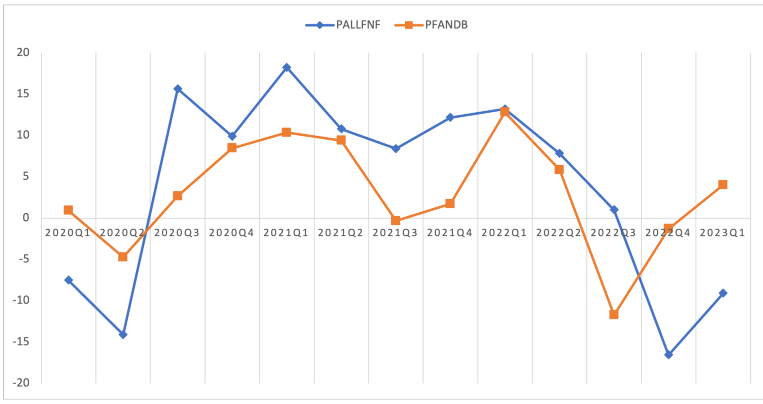

Figure 3.4: Data from International Monetary Fund (IMF), consulted on the web page: https://www.imf.org/en/Research/commodity-prices on April 17, 2023

We apply the Pearson correlation coefficient for both pairs and we found that the correlation coefficient between PALLFNF and PNRG is 0.9610801427404376, with a p-value of 1.7400789692967406e-07, while the Pearson correlation coefficient between PALLFNF and PFANDB is 0.5632882771672514 with a p-value of 0.04500910088926913.

The Pearson's correlation for both energy prices (PNRG) and inflation (PALLFNF) and food and beverage (PFANDB) and inflation shows a strong positive correlation and statistically significant relationship between the two variables for each correlation.

Inflation can be separated into core and non-core inflation. Core inflation is made from all the products considered in the National Consumer Price Index, excluding those that vary with factors different from the market conditions, like external conditions or supply shocks, while non-core inflation measures the inflation from the products excluded in the core inflation. Energetic sector is measured by the non-core inflation, while merchandise and services are associated with the core inflation.
With this in mind, we can measure the particular effect of energetics and merchandise on inflation in order to characterize the roots of this phenomenon in Mexico. We show the participation of core and non-core inflation on the consumer price index (CPI) with a simple linear regression to look for energetic and merchandise specific contribution on consumer price index and we conclude that core inflation contributes with 75% of consumer inflation, while non-core inflation contributes with 25% of consumer inflation.

Source	SS	df	MS			
Model	4.63875409	2	2.31937704	Number of obs	=	31
Residual	.00100075	28	.000035741	F(2, 28)	=	64893.87
				Prob > F	=	0.0000
				R-squared	=	0.9998
				Adj R-squared	=	0.9998
Total	4.63975484	30	.154658495	Root MSE	=	.00598

CPI	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
CoreInflation	.7494797	.0054777	136.82	0.000	.7382591	.7607003
Noncoreinflation	.2491646	.0007408	336.34	0.000	.2476471	.2506821
_cons	-.0023706	.0029252	-0.81	0.425	-.0083627	.0036215

Figure 3.5: Source: Prepared by the author on the basis of data supplied by Nacional de Estadistica y Geografia (INEGI), consulted on the web page: https://www.inegi.org.mx. Information calculated and published by INEGI from March 2020

With respect to the core inflation (services and merchandise), the particular behavior of services inflation has made us choose to focus on merchandise, leaving services aside. This is because, while merchandise have continuously increased it prices since the beginning of the pandemic (due to a change in consumption preferences and the cut of global supply chains), we have seen an initial decrease in service prices followed by an increase in service prices (in response to a demand recovery) and, lastly, a partial stabilization of services prices. The decrease in service prices can be explained by the consumption changes in the pandemic, since demand on services (particularly those with direct contact) fell at the beginning (Esquivel and Leal; 2022). If we want to focus on the effect of core inflation price increase on headline inflation, we have to leave services aside due to its particular behavior in the period of time evaluated.

Although production is know increasing with the recovery from the pandemic, it seems to be the case that producers are still not able to meet the demand requirements (primarily as a consequence of supply delays and material shortages), so this supply shock continues to contribute to inflation in the world (Bernoth and Gokhan; 2021). The former affects Mexican prices through the intermediate goods imported by firms to make their production.

In accordance to Esquivel and Leal (2022), the increase in merchandise prices associated with raw material external price shocks in the food industry has contributed to Mexican inflation with 67%, while the contribution to inflation from input supply shocks has been 16%. Inside the food industry, the sectors that have contributed the most are the corn tortilla, the edible oil industry, small food businesses, the soda and beverage industry and the milk industry.

According to Hellegers (2022), the cereal and edible oil global industries have been two of the most affected sectors with to Russian invasion to Ukraine, because both countries are among the largest producers in these sectors. This could be one of the external shocks affecting oil and corn tortilla prices. On the other hand, Russian invasion has give rise to an increase in the energy sector prices, affecting food industry and livestock industry input prices, inducing price increases for the entire agro-industry.

With this preliminary overview of inflation and its sources in Mexico, we want to test the assumption in the DSGE model concerning market concentration in the agro-industry and the energetic sector so we can describe how the increase in firms costs in this sectors have been passed to the consumers, in order to evaluate the contribution of raw material (including energetic) on inflation with a model that incorporates oligopolistic markets, following the market structure in the Mexican industry. The study of market concentration in Mexico will focus on those industries mainly affected by inflation following the Covid-19 crisis.

Globally, in the last decades there has been a continuous industrialization of the agriculture and an expansion of agro-industrial complexes, in view of the economies of scale present in that particular sector. Vázquez (2015)

Following Ibarra (2016), this tendency is consistent with the agro-industrial national sector for 2016. He divides this industry in 7 sectors (as in Urzúa (2009)): corn tortillas, processed meat, chicken and egg, milk, soda and beverages, beer and medicine; we are going to focus on the first 6 of them.

For the first sector, corn tortillas, we can split it into corn flour and nixtamal dough, with 36% and 64% of production respectively. Grupo Industrial Maseca (GIMSA), from Grupo Maseca (GRUMA), has a 70% market share in the corn flour side, while Minsa has 24% of market share. With respect to the nixtamal dough, we can see a market with over 80000 small firms, so no oligopoly is present.

Processed meat sector can be split into beef and pork. The first one has 3 big firms, SuKarne (from Grupo Viz), Grupo Gusi and Praderas Huasteca, with 16%, 6% and 5% of market share, respectively. The second one has 5 big firms, Gruupo Porcicola Mexicano, Grupo Kowi, Norson, Sonora Agropecuaria and Grupo Bafar, with 10%, 8%, 7%, 6% and 5% of market share, respectively. The rest of the market is divided with atomistic firms.

The chicken and egg sector can be divided as well in two sectors: chicken and egg. The chicken sector is dominated by 3 big firms: Bachoco, Prim's Pride and Tyson, with 38%, 14% and 12% of market share, respectively, the rest of the market is divided by 29 medium size firms and 150 small firms. For de egg market we have 4 big firms, Proan, Industrias Bachoco, El Calvario and Empresas Guadalupe, with 13%, 8%, 6% and 5% of market share, respectively, the rest are 37 medium size firms and 150 small firms.

For the milk sector we have 4 big firms, Lala, Alpura, Nestle and Santa

Clara (owned by Coca-cola), with a market share of 34%, 22%, 9% and 3%, respectively. The rest of the market is divided in a lot of small firms.

The soda and beverage sector con be separated into soda, juice and bottled water. We have a dominant big firm in the soda sector, Coca-cola, with 68% of market share, and the rest is divided by other 6 medium size firms. The juice sector is shaped by 3 big firms, Jugos del Valle, Grupo Jumex and Grupo Industrial Lala, with 29%, 27% and 16% of market share respectively, and the rest of the market is shared by 5 medium size firms. For the bottled water we have 4 big brands, Danone, Coca-cola, Pepsi and Nestle with 38%, 26%, 18% and 5% of market share each one, the rest of the market is hared by more than 8000 small brands.

At last, we have the beer sector, dominated by a duopoly with Anheuser Busch InBev and Heineken, with 52% and 34% of market share each one.

Taking into account the participation of different food industry products to inflation in Mexico (Esquivel and Leal; 2022), we still need to study the case of the oil industry in the country. Even though this industry has an important participation of total production in the agro-industry it has to import 95% of seeds and a large percentage of crude oils used in production.

For the energy sector, we have that Russian energy exports was around 11% of global exports for 2020. In the case of Mexico, this is the fourteenth country with the most energy imports worldwide, covering about 2% of global imports; Mexico consumes more energy than it produces, even if this tendency has decreased because of the pandemic, it is still a considerable amount SENER (2020). For 2020, crude oil represented more than 56% of total energy production, with 96.76% of national crude oil production from Petroleos Mexicanos (PEMEX), followed by natural gas production with 25.67%.

The trade balance with respect to the energy sector is negative, where the largest surplus contribution is from crude oil and the largest deficit contributor is from gas and gasoline. We have that, for 2020, 45.59% of total national energy was from imports, with the main representative of these imports being the USA.(SENER; 2020)

Incorporating the market structure for the agro-industry, we can confirm that there are oligopolistic markets in the sectors pushing recent inflation in Mexico. With respect to the energy sector, the previous analysis let us confirm that we have a significant share of imports in total national consumption, but also, most of national production is made by a few firms managed by the state. Whit this in mind, we are going to test our hypothesis with a DSGE model that assumes oligopolistic markets on intermediate goods to analyze the contribution of these sector prices on inflation in Mexico.

Chapter 4

Theoretical framework

4.1 Inflation and market concentration

Literature has described market power as the capacity of firms to set prices above marginal costs (Syverson; 2019). Following Structure-Conduct- Performance theory, market structure conditions firm's decisions on a specific market, where market structure consists on market characteristics such as market concentration, product differentiation, barriers to entry and the rate of market demand (Boru and Kuhil; 2018). All these factors determine the level of competition in a market.

The market concentration is related to the number of firms in an industry, the fewer firms there are the closest the market is to a monopoly and market concentration is greater. In the case of raw materials in Mexico, we have showed that these markets are characterized by a high market concentration.

Product differentiation can lead to market power following different frameworks. If we consider a Bertrand oligopoly where firms set their own prices, products are homogeneous and there is a single marginal cost, firms have no market power and there is an unique Nash equilibrium equal to the marginal cost, but if we introduce product differentiation we reduce the rate of competition and increase prices above marginal cost.

The economic intuition about the product differentiation leading to market power has been explained by Anderson et al. (1992) who considers that consumers are wiling to pay more to product variants that are closest to their tastes and this premium yields to market power for each firm.

Another aspect that determines market power is the residual demand curve and it incorporate the elasticity and cross-elasticity of demand, which depends on the substitutability of a product and the product differentiation in the supply side and on consumer preferences in the demand side. (Massey; 2000)

Scheffman and Spiller (1987) estimated the residual demand curve with a cost-push shock. The more a cost-push shock is passed through prices, the more the demand is inelastic and there is market power, the less a cost-push shock affects prices, the more elastic the demand is and the less market power a firm has. Massey (2000) accounts that market power can be determined with the slope of the residual demand curve, the steeper it is the greatest the market power of a firm.

When we mix inflation and market power we can think of the New Keynesian Phillips Curve developed by Gali and Gertler (2000). While the traditional Phillips Curve focuses on inflation dynamics based on backward looking agents were inflation depends on lagged values of itself, the New Keynesian Phillips Curve states that inflation is a forward looking phenomena consequence of some nominal stickiness (Roeger and Herz; 2012).

In Gali and Gertler (2000) New Keynesian Phillips Curve we have nominal stickiness via each firm adjusts its price at period t (p_t^*) with some probability $1 - \theta$ and keeps the last price with probability θ, so the aggregate price level is given by

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^*. \tag{4.1}$$

Each firm will adjust its price considering expectations on future marginal cost (mc) path and the probability θ of the price remaining the same for multiple periods, all affected by a discount factor β, so it follows

$$p_t^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ mc_{t+k}^n \right\}. \tag{4.2}$$

Since firms in this model set their prices according to some markup price from marginal cost, they have to consider expected future marginal cost to set a price at period t. Combining (2.1) and (2.2) and letting $\pi_t = p_t - p_{t-1}$ be the inflation rate at period t we get the following inflation equation

$$\pi_t = \lambda mc_t + \beta E_t \left\{ \pi_{t+1} \right\} \tag{4.3}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ depends on the probability of changing the price and the discount factor.

The previous model allows us to have a first approximation to incorporating market power in the inflation equation, stating that inflation depends on market power via the expected marginal cost path a firm has when setting its price for an uncertain number of periods.

4.2 Demand-pull and Cost-push inflation

When considering inflation itself, we have two canonical representations of the inflation causes: demand-pull and cost-push inflation, and each has different theoretical frameworks. The first one states that monetary policy is active on determining the inflation rate of the economy, driven by an increase in money demand or by a demand shock. The later states that inflation is due to market power, which makes possible to transfer costs shocks to prices, so monetary policy just increase money supply according to these price changes and it is passive on determining inflation. (Barth and Bennett; 1975)

The demand-pull inflation was conceived by neoclassical economists, this model assumes that real variables are independent from level prices, so if we have a doubling in all prices, we can keep relative prices and real variables levels if we double money. The main problem of neoclassical view of demand-pull inflation is that it is not able to explain price rises when we have unemployment or under capacity production, since wages and prices should not increase in this situations following their assumptions. Samuelson and Solow (1960)

Later, Keynes came back with demand-pull inflation but using different assumptions than the neoclassical theory. In the limit case, prices and wages had downward inflexibility, so any change in money spent will be translated into changes in the real variables. This point of view has some similarities with the neoclassical model, but it uses the total money spent instead of the stock of money. As well as the neoclassical model, keynesian demand-pull inflation is not able to explain price rises in scenarios with unemployment and under capacity production, so we need to extend the demand-pull inflation to study more general cases and market structures.

For the cost-push inflation, we focus on imperfect competition as the basis for the explanation. A first insight on this matter is that, given oligopolistic sellers or some other non-competitive market structure with more than one producer, they rise prices as an attempt to capture a bigger national income share, giving rise to "*seller's inflation*". Another explanation for the cost-push is the demand-shift theory of inflation, which states that we may have a sector with strong demand, which makes prices rise, and other sector without a strong demand but, with it's market power, it can imitate in some degree the price-wage trajectory from the strong demand sector, giving rise to a general price increase.

Gardiner Means is considered the father of "administered prices" since the 1930s when he described these prices as those established by some producer and kept for a number of periods unchanged, contrary to "market prices" that depend on supply and demand forces in the market. He recognized that "administered prices" in concentrated industries should be monitored since the greater the market concentration the less restrictions on pricing discretion and the greater possibility to abuse from this discretion. (Auerbach; 1962)

In a market power environment, following "administered price" theory, firms will pursue a target rate of return instead of a profit maximizing price. As demand becomes more inelastic, firms will adopt greater cost pass-through strategies when setting prices and this amplifies inflation. With this scenario, if there is a cost shock, it will full pass to consumers and inflation will increase (Zaleski; 1992).

Pricing controls involve some degree of planning from the producers or any economic agent setting the price, the way decisions are modeled corresponds to the formulation of expectations by the agents.

4.3 Rational and adaptive expectations

Literature concerning how agents formulate their expectations focus on two types of models: full-information rational expectations (FIRE) and models regarding constraints on agents capability to acquire information Kose et al. (2019). The first type of expectations was adopted by Lucas (1976) and has been used as a main assumption on most macroeconomic theory. The rational expectations assumption has the property concerning that the expected value of the agents is the same as the conditional expected value in the model used by the researcher, so it is consistent with the model; the agents know the construction and solution of the model Milani (2012). Rational expectations has been typically used in New Keynesian models, but there are critics on it's closeness to reality and they have not been able to model some phenomena, like sustained inflation.

For the models with constraints on information obtained by agents, there are mainly two types: those models considering sticky-information and those using noisy-information or inattention models.

The sticky-information models assume that agents have costly information, so they update forecast slowly, while the noisy-information and inattention models suppose agents continuously update their forecast but the information received is noisy of they do not pay attention to all signals.

If we combine rational agents with agents that have information constraints, we get the hybrid New Keynesian Phillips curve. Models using both rational and adaptive expectations are empirically justified because, as there are normally three ways to measure expected inflation (market-implied measure, professional forecaster's measure and consumer survey measure) it has been shown that surveys to professional economic forecasters from the private sector tend to better capture the true expected value of these type of agents and they normally incorporate broader economic conditions than other type of agents, so we can model these expectations using rational expectations, while for the expected inflation of typical consumers we can model them using adaptive expectations. Adeney et al. (2017)

We only have one way of modeling rational expectations while there are a lot of ways to restrict the rational expectation model to get a more realistic expectation formulation of agents. When we try to conciliate reality with agent's beliefs, using empirical evidence, we get the adaptative expectations model where agents do not have full information so they base their forecast on some parameters that get updated every time agents have new information See (Özden; 2021). This models allows for some deviations from the rational expectation model while keeping agents formulations close to reality.

Adaptative expectations models first focused on agents having the correct model specification but without knowing the magnitudes involved in the model, but they have recently evolved to allow for misspecification models where agents use some learning rule to update the model parameters. Different algorithms have been used for the learning rule, with AR(1) being one of the most used.

4.4 Imperfect competition and price setting

Regarding the behavior of the agents, we need to consider producer's choice model. Since we are concerned about the particular case of market power in Mexico, we are going to present some important characteristics about oligopolies and the way prices are chosen in this kind of models.

Rotemberg and Saloner (1984) show that prices in a concentrated market tend to be counter cyclical, since a large shock in demand makes the profits from cheating on the commitment made by all the firms in the oligopoly greater than the punishment received from cheating. Since producers will not higher their prices at times with more demand because they would lose a lot of sales, they will compete by lowering their prices, this means that there will be price wars at times of high demand. Thus, if there is a big commitment to agreed prices or if there are price adjustment costs, it is more likely to cheat on the agreement if the demand shock is large. The oligopoly theory proposed by Rotemberg and Saloner suggests that distortions in the economy are bigger in recessions than in booms, so it is desirable to have a high output equilibrium.

On the other hand, Afrouzi Khosroshahi (2018) introduce a dynamic general equilibrium model with oligopolistic sectors, pointing out that this type of markets implies bigger monetary impact on real variables compared to the monopolistic competition model or to the perfect competition model. Oligopolies tend to have less information about external shocks because, the more concentrated the market is, the less the firms spend on acquiring information of the market conditions and the more they base their prices to changes in the rest of the firm's prices. Since more concentrated markets have less information about market conditions, oligopoly firms will react less and more persistent to external shocks, making monetary shocks impact greater and larger in time.

Markups in monopolistic competition depends on consumers elasticity of subtitution and it is constant, so any change in market concentration or the producers behavior impact these markups and, ultimately, monetary policy. Monopolistic competition and CES preferences imply constant markups, incomplete cost pass-through and no strategic complementarities (a change in competitors prices leads to a change on a firm own price). (Wang and Werning; 2022)

Moreover, contrary to monopolistic competition with constant elasticity of substitution, markup is not constant in oligopolies and depends on relative prices from the rest of the firms in the particular industry. The level of optimal markups increase with larger market share from each firm, and converges to the monopolistic competitive markup as the firm's share goes to zero. (Afrouzi Khosroshahi; 2018)

Finally, the later model shows that large strategic complementarities (which are positively correlated with the market concentration of a given industry) implies that prices for a particular firm will dependent more on the rest of the firm's prices than on aggregate demand; at the end, inflation in this model is driven from expectations on prices from competitors and not from changes in aggregate demand. The higher the strategic complementarities, there will be bigger markups and it will give rise to greater output responses to monetary shocks. (Wang and Werning; 2022)

Amiti et al. (2019) develop a theoretical framework and create an empirical approach to calculate how strategic complementarities around firms impact markups and cost pass-through. While the monopolistic competition model considers constant markups and a 100% cost pass-through, the results from Amiti et al. (2019) show that strategic complementarities have a positive impact on price change and cost pass-through is less than 100%.

Despite results from the later paper show this behavior, the authors found heterogeneity around firm responses based on their size. Small firms have no strategic complementarities and have a 100% cost pass-through (so they can be modeled using monopolistic competition), while large firms exhibit strong strategic complementarities and incomplete cost pass-through. When there is a cost shock, since small firms have a minor markup, they can only reduce it to some point so they appeal to a cost pass-through in order to offset the cost shock, while large firms have higher markups they can adjust to maintain their market share (markup elasticity is increasing as the market share is bigger). (Amiti et al.; 2019)

The advantage of using monopolistic competition models is that we can model imperfect competitive markets with a framework easier than oligopolistic competition and taking into account general equilibrium. (Parenti et al.; 2017)

Chapter 5

Methodology framework

5.1 RBC and MBC models

The first step for constructing our model is to study the fundamental model that tried to use dynamic stochastic general equilibrium (DSGE) models to evaluate the impact some specific shock in different macroeconomic variables by establishing a micro-founded model for different sectors in the economy. We will begin with a Standard Real Business Cycle (RBC) model, which follows the basic assumptions of a neoclassical framework and exacerbates the role played by a productivity shock on the economy with no role for monetary fluctuations on real variables. We have two fundamental models, the classical RBC model with high substitution, and the monetary one, which adds a Taylor rule and a Phillips curve to the RBC model (Nakamura; 2009).

Real business cycle models link the classical growth theory with business cycles to generate a model capable of simulating responses on macroeconomic variables due to some economic shock (usually they emphasize the productivity shocks). The outstanding feature about the RBC models is that they can approximate the fluctuations on official data given by different events very precisely, so they are able to evaluate a multi-dimensional problem by just locking a particular variation. Also, the microfundamentation of these type of models allows to use different sources that examine the behavior of individual agents when justifying the quantitative definition of the model.

Moreover, the parameters in these models refer to concepts in economic literature (such as labor participation or depreciation) so we can rely on empirical knowledge to evaluate the result or calibrate the model, and not only adjust the parameters to have statistical fit. (Rotemberg and Woodford; 1997).

After this first look of the classical RBC model, we will introduce Monetary Business Cycles (MBC) to establish a central bank in our model, based on Rotemberg and Woodford (1997). The approach of MBC models have been applied mainly for evaluating monetary policy and its impact on macroeconomic variables, by using an utility-based measure of the dead weight loss, instead of defining a loss function to measure this.

Then, we focus on a combined expectations model, that establishes different types of households with distinct expectations respectively. This model was motivated by the 2007 crisis, as standard DSGE models were not able to reproduce this phenomenon without the implementation of arbitrary enormous preference shocks, and their performance on reproducing the crisis has been very satisfying.

The final goal is to use the latter model to construct a framework with a law of motion for inflation as a random walk plus some noise, like in the case of shocks implemented in the adaptive expectations model explained in the last part of this section.

5.1.1 Standard RBC

RBC models where first introduced by Kydland and Prescott (1982) in order to integrate business cycles with the equilibrium growth model. They take the household's consumption problem in the same way as conventional growth theory, which takes a representative infinity-lived household, but also aggregates inter temporal substitutability between past and future leisure. On the other hand, the production function assumes constant returns to scale and this function will be affected by some production shock, also it is required some constant period to build capital. Investment is split into investment on projects and inventories; elasticity of substitution between capital and inventory is less than one. Capital in the model reflects all tangible goods, including housing. Consumption does not consider purchases of durable goods but income includes services from consumer's stock of them. In addition, different types of capital have different periods of construction.

Classical RBC literature select productivity shocks to evaluate fluctuations and responses on the economy due to their capacity to simulate empirical responses on almost every industrialized country. They were preferred to monetary shocks because it was a common belief that monetary shocks have minimal or no effect on real variables. The issue with a standard productivity shock and production function is that they require a large shock to simulate actual responses of the variables in the data. To simulate this large responses on the economy, RBC models make use of high subtitutability in production factors via two assumptions: labor supply is highly responsive to changes in wages, and the capital services are highly responsive to changes in hours of labor, hence, when calibrating the model, the parameters representing both phenomena are going to be close to 1. (See King and Rebelo; 2000)

The microfundamentation of the consumption function on this model says that leisure is used in household production. They will allocate leisure on projects with the greatest returns per unit time, thus, when they have allocated in past more hours in household projects, they will allocate less hours to projects with less returns per unit time, so the marginal utility of leisure will be smaller. There will be a variable that reflects past leisure choices effects on current and future preferences (Kydland and Prescott; 1982).

Stating the information structure of the model, there will be a productivity shock with different persistence components affecting the production function and it reflects the fact that nobody has a precise measure of the productivity at period t, but at the beginning of the period, consumers have some noisy measure of it, so investment and consumption will be chosen taking into account this measure. The noise could represent some lack of full information in the model. Decisions on labor and investment will be made without knowing the precise productivity, just taking into consideration past realizations on this variable, while decisions on inventories will be made later in that period, when firms can estimate productivity by observing output and inputs.

Finally, authors calculate the steady state for the model (no shocks added) and they approximate the problem by a quadratic function in the neighborhood of the steady state. The model has average good adjustment with US data, but inaccurate reflects dynamics in hours of labor; authors suggest that this dynamics in hours worked (that does not have relation with changes in output) might be due to heterogeneity in labor force and on the way productivity shocks are introduced in the model. The influence of past leisure on current consumption makes possible intertemporal substitution of leisure, which helps making consistent the observation of employment fluctuating substantially more than productivity. (Kydland and Prescott; 1982).

The general assumptions of the classical Kyndland and Prescott model are:

a) Perfect competition.

b) Fully flexible prices.

c) The role of money is just to serve as a unit of account.

d) Households are firm's owners.

e) Produced goods are only consumed (they do not serve as investments, government purchases or net exports); hence $Y_t = C_t$.

f) Money does not affect real variables like output, consumption or real wage, but inflation and nominal interest rate are affected by money.

g) Output and real wage are positively affected by a productivity shock.

h) Employment will be affected positively or negatively depending on the curvature of the utility of consumption, the substitution effect of labor supply due to an increase in the real wage can dominate over the reduction in marginal utility of consumption, depending on the later factor.

They model the infinite life representative household maximizing problem with an utility function where there will be decisions on time t of consumption and labor, and this function will be affected by a preference shock which summarizes the effect of past elections of leisure and labor on current and future preferences, so an increase in the shock increases the marginal utility of consumption. Also, there is a discount factor meaning that decisions today get less affected by future elections the farthest you are in time. Consumption has a positive effect on utility, but the effect is decelerating, while hours of labor have a negative effect on consumption and are non increasing.

The household obtains income from labor and dividends from firms (households are the owners) and they spend their income in the unique consumption good and in a one-period risk-less bond paying a monetary unit at the next period, this means that households only save for future consumption in the form of bonds. In this particular model, the only tradable asset are these bonds, and the clearing condition for markets tells us that there is no asset trade in equilibrium. Finally, we add the transversality condition, otherwise, we could have a time t where it is possible to consume more and increase utility without giving up to future consumption.

In the case of firms, we have a large number of identical firms with a Cobb-Douglas production function with labor as the only input, so we have positive but diminishing returns to scale on labor. Additionally, the state of technology will behave as an AR(1) process, so we will have the productivity shock there. These firms will maximize their profits according to the income from sales (which is equal to consumption, due to the assumption $Y_t = C_t$) minus the payment on labor.

Since we do not have any equation about prices or any monetary policy, money in this type of models is neutral in the sense that in is considered an intermediate good that rises or fall depending on the output level and have no effect on the economy. This is a very simple model and can be extended to add other aspects of the economy, as the monetary policy rule, and a price equation, which is fundamental for our duty given that we try to use this type of models to evaluate the impact of a shock in raw material prices on inflation. Next section explores an extension of this model to include a monetary sector.

5.1.2 MBC

The motivation of this type of models is the introduction of explicit monetary influence on the economy, as well as the inclusion of an inflation-target equation and some monetary variables to the standard RBC model, by presenting a bank sector, the monetary rule that it follows and some other equations that reflects the effect of monetary variables in the economy. One significant difference between this model and the RBC one is that prices have no longer free adjustment costs. This is made by introducing a stochastic equation to the price adjustment of the firm where they can change the prices only when this stochastic moment arises, otherwise, they have to keep prices constant. We follow Ireland (2004) to represent the general idea of a MBC model in this section.

To see how money aggregates can affect real variables, we will split produced goods in two sections; the first one is a group of intermediate goods, where firms producing these type of goods behave as monopolistic competitors facing a price adjustment cost, while the second group of goods is of final goods, where we have a large number of firms in a competitive market. Furthermore, we have a household sector with a representative household and a monetary authority.

First we have to establish the monetary rule followed by the central bank, in this case we followed Taylor (1993) to achieve our goal. The rule will be defined as a reaction function that sets interest rate levels based on past and present output and inflation. One of the most important contributions of this model is the insertion of a shock in the monetary policy, the monetary shock, to estimate the reaction on macroeconomic variables due to changes in monetary policy. Money and nominal fluctuations are no longer neutral to output and real wages, instead, they can affect the real economy at different levels.

The monetary rule is described as follows:

$$r_t = r^* + \sum_{k=1}^{n_r} \boldsymbol{\mu}_k \left(r_{t-k} - r^*\right) + \sum_{k=0}^{n_\pi} \phi_k \left(\pi_{t-k} - \pi^*\right) + \sum_{k=0}^{n_y} \theta_k y_{t-k} + \epsilon_t$$

where r_t and π_t are the central bank interest rate at period t and the inflation rate between period $t-1$ and t, respectively; y_t is the deviation of product from its trend at t; r^* and π^* are long run target values for the interest rate and the inflation, respectively; ϵ_t is the monetary stochastic shock, it follows an AR(1) process. $\boldsymbol{\mu}_k, \phi_k, \theta_k$ are parameters describing specific preferences of the monetary rule.

A first implication on output and inflation is that interest rate on period t does not affect output or inflation at period t, this can be due to lack of information that agents have in this model, so they do not know the current money-market conditions when making their decisions. (Bernanke and Rotemberg; 1997)

Another feasible monetary policy, described in Ireland (2004), is as follows:

$$\hat{r}_t = \rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t + \rho_{\boldsymbol{\mu}} \hat{\boldsymbol{\mu}}_t + \epsilon_{rt}$$

where $\hat{r}_t, \hat{y}_t, \hat{\pi}_t$ and $\hat{\boldsymbol{\mu}}_t$ are deviations of the respective variable from the steady state value at period t. $\boldsymbol{\mu}_t = \frac{M_t}{M_{t-1}}$ is the money growth.

We will have an infinitive live representative household maximizing problem with an utility function that is going to be positively affected by consumption, negatively affected by hours of labor, but also positively affected by real money balances. The household receives nominal income from work, nominal transfers from the monetary authority, nominal dividend payments (households are owners of firms), nominal value of bonds maturing at period t, and also nominal money balance carried to period t. The household spend on consumption, bonds maturing in the next period and nominal money balances carried to the next period.

We have two shocks for households, both following an AR(1) process, one concerning general preferences on consumption and the other one representing a shock in money preferences. So, one is going to impact our IS demand curve, while the other one is going to impact the money demand curve. This means that the money balances increase with deviations of GDP from steady state and from the shock on money preferences, and are reduce with deviations of interest rate from the steady state value

In the case of the final-good producer, each firm uses $y_t(i)$, with i $\in [0, 1]$, units of intermediate good i to produce a unit of final good y_t with a constant-returns-to-scale technology. This firms are going to maximize profits but, as we have competitive markets for the final good, in equilibrium, profits are going to be equal to zero.

For the intermediate producing firms, they are continuously indexed by $[0, 1]$, each firm produces a differentiated intermediate good, so we can also index intermediate goods with $[0, 1]$. Each firm uses $h_t(i)$ units of labor in a constant-returns-to-scale-technology affected by a technology shock following an AR(1) process. In equilibrium, this supply-side disturbances act as a shock in the Phillips curve (Ireland; 2004). As each intermediate good is different and enters in a different way in the final-good production function, the intermediate firms are in a monopolistic competitive market, so every single firm settles it's own price, but it most satisfy the demand from final-good firms at that price.

There will be a quadratic price adjustment cost for intermediate goods, inflation will increase this cost. The profits from the production of intermediate goods are distributed as dividends to households. At equilibrium, with symmetric intermediate firms, each firm will produce the same amount of goods and they will sell them at the same price, paying the same dividends to each firm.

From the first order conditions for the intermediate good, we can derive the Phillips curve and the IS demand curve that optimizes every maximization problem in this model, given by

$$\hat{\pi}_t = \frac{\pi}{r} E_t \hat{\pi}_{t+1} + \psi \left[\frac{1}{w_1} \hat{y}_t - \frac{w_2}{w_1} (\hat{m}_t - \hat{e}_t) - \hat{z}_t \right] \tag{5.1}$$

$$\begin{aligned} \hat{y}_t =& E_t \hat{y}_{t+1} - w_1 (\hat{r}_t - E_t \hat{\pi}_{t+1}) + \\ & w_2 [(\hat{m}_t - \hat{e}_t) - (E_t \hat{m}_{t+1} - E_t \hat{e}_{t+1})] + w_1 (\hat{a}_t - E_t \hat{a}_{t+1}) \end{aligned} \tag{5.2}$$

every variable with a hat refers to the deviation of the variable at period t from the steady state variable (the steady state is the variable without hat or sub index). The parameters are given by different preferences and technologies from the households and firms.

For the IS curve, we can see that changes in monetary balances and in preference on these balances actually affect the output. Also, the interest rate affects negatively the output and the inflation affects it positively. In the case of the Phillips curve, we have that nominal changes on inflation at period t are affected by expected changes in next period inflation, by deviations of GDP from the steady state, by changes in money balances and by the productivity shock.

5.2 DSGE with adaptive expectations

This type of models will integrate adaptive expectations to households; they were first motivated by the necessity to address the excessive volatility in house prices, because standard DSGE models needed enormous and persistent preference shocks to adjust to the phenomenon seen in 2007. The problem of the standard DSGE is that there is no explanation on why preferences changed that much and even on the basis that the changes in house prices were truly driven by changes in preferences. (Gelain and Lansing; 2014)

This adaptive expectations are modeled with moving-average forecast rules which give great importance to recent values of the variables. The moving-average rule uses a unit root in a random walk to shape the increasing volatility. They are combined with rational expectations in weighted average to produce the amount of volatility consistent with empirical house price, house debt and output.

The model has two types of household, the first one is a patient one, which are owners of the entire capital and operate monopolistic firms, while the second type are impatient households, who only receive income from labor and their credit constraint is given by their housing stock. This particular way of dividing households aims to simplify the difference in income distribution in an economy, which has been one of the main criticisms of standard RBC models, which ignore the distribution of income.

Combining these two types of household with their respective expectations, we can get a model that reflects the 2007 crisis, where this moving-average households could be thought of as this "new homebuyers" with simplified expectations just taking into account the recent values to forecast future house prices and income.

5.2.1 The model with hybrid expectations

We have two types of households, patient households and impatient households. The discount factor from the utility function is lower for impatient households, so impatient households tend to borrow more. To focus on the house pricing problem, we set nominal stick prices for consumption goods. We also have a monetary policy following a Taylor rule. We will follow the model by Gelain, et al (2013).

Households

The general maximization problem involves a function where households generate utility from a flow of consumption and services from their house stock, and disutility from labor . We have a parameter that represents the habit formation in the utility function, concerning a referential consumption level. In addition, we have three parameters showing the importance of housing services and labor on utility, and the elasticity of labor supply. House stock is normalized so the combination from both types of households sums one at each period of time.

The impatient household (borrower), receives income from labor and debt acquired at that period of time and have expenses on consumption, house stock and the payment of the debt from the past period; in this case, debt lasts one period. Moreover, we must consider the constraint on the quantity impatient household can borrow. They can only borrow a fraction of the subjective expected value lenders have about their house stock at time $t + 1$.

In the case of the patient household (lender), they receive income from the one-period loans to impatient households, plus the real wage from labor, plus payments from capital services and profits for being the owners of capital and firms. The patient household spends on consumption, investment, increment in house stock and loans to impatient households, which are equal to the borrowing by the impatient households in each period. Finally, the law of capital accumulation will be affected by depreciation of capital and some price adjustment cost.

Firms

We have two types of firms: final-good firms and intermediate-good firms. Final-good firms are assumed to be in a competitive market and are owned by patient households, so this firms also have rational expectations. There is going to be an unique final good made with continuum intermediary goods , where the production of the final good has a constant returns to scale function.

For the intermediate-good firms, each one of them produces one of the intermediate goods, and they are owned by patient households. In this case, we have a monopolistic market, with a constant-returns-to-scale function and a stochastic shock following an AR(1) process.

Ultimately, we are going to model inflation following Calvo (1983), with some stickiness in prices given by a probability of changing prices for intermediate goods. Thus, we will get a New Keynesian Phillips curve (NKPC).

5.3 New Keynesian model

There is a step further regarding DSGE models, this is a combination of RBC model structure with some assumptions departing from the classical monetary models (Gali; 2015). We will present the particular characteristics taken from the RBC model followed with assumptions added to the classical DSGE in order to create a New Keynesian DSGE model framework.

RBC assumptions

1. An infinitely lived representative household, seeking to maximize utility by selecting a path for consumption, leisure and work, subject to an inter temporal budget constraint.

2. A large number of forms with access to the same technology and affected by external shocks.

3. Equilibrium takes the form of stochastic processes for endogenous variables, consistent with optimal decisions from firms and households and implying market clearance.

New assumptions departing from the classical model

1. Monopolistic competition. Differentiated products produced by firms that choose the market price of their own product.

2. Nominal rigidities. There are price stickiness since just some proportion of firms are able to change prices at time t. Alternatively, the face price adjustment costs.

3. Non-neutrality of money in the short run. Since there are nominal rigidities, a change in nominal interest rate will not be followed by the exact same change in prices, which leads to changes in real interest rates.

With the previous framework, we obtain the sticky price Phillips curve, which sets that prices will be adjusted following current output gap and the expected next period price change, so it is a forward-looking inflation equation. (Arslan; 2008)

Chapter 6

Model specification

We will use a New Keynesian DSGE model with monopolistic competition and price stickiness following Gali (2015), there will be consumers, producing firms and a monetary policy rule. Due to its simplicity advantage when handling with imperfect competition and market power, we opted to use a model with monopolistic competition instead of one with oligopolistic market structure.

Regarding the cost-push shock, it will be introduced in the NKPC by adding an stochastic term to the forward-looking inflation equation.

6.1 Households

We have a representative household that maximizes a lifetime utility function U_t depending on its election on consumption C_t, hours worked N_t and an exogenous preference shifter Z_t subject to a budget constraint.

$$\text{Max.} \quad E_0 \sum_{t=0}^{\infty} \beta^k U_t = U(C_t, N_t; Z_t) \qquad s.t. \qquad P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{6.1}$$

where the utility function takes the form:

$$U_t = \begin{cases} (\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}) Z_t, & \text{if } \sigma = 1 \\ (\frac{C_t^{1-\sigma}-1}{1-\sigma}) - \frac{N_t^{1+\varphi}}{1+\varphi}) Z_t, & \text{if } \sigma \neq 1 \end{cases} \tag{6.2}$$

β is the discount factor for future expected utility, $\sigma \geq 0$ is the curvature of the utility of consumption and $\varphi \geq 0$ is the disutility of labor.

The utility function satisfies the following conditions: $U_{c,t} = \frac{\partial U_t}{\partial C_t} > 0$, $U_{cc,t} = \frac{\partial^2 U_t}{\partial C_t^2} \leq 0$, $U_{n,t} = \frac{\partial U_t}{\partial N_t} \leq 0$, $U_{nn,t} = \frac{\partial^2 U_t}{\partial N_t^2} \leq 0$ and $U_{cz,t} = \frac{\partial^2 U_t}{\partial N C_t \partial Z_t} > 0$, which means that an increase in consumption will have a positive impact on utility but with diminishing consumption marginal utility, leisure will have a positive impact on utility but with diminishing leisure marginal utility and an external increase in the preference shifter Z will amplify the impact on the utility from an increase on consumption.

The consumption index will be given by:

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}} \tag{6.3}$$

where ϵ is the elasticity of substitution between goods, $C_t(i)$ is consumption of good i at time t, and the firms are assumed to be continuous and indexed on the interval [0,1]

From the consumption index, we define $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ as the price index for all goods.

Adding the consumption index definition to the budget constraint, we have that the household can spend his income in one-period riskless bonds B_t, the price of which is Q_t, and in consumption, where $P_t(i)$ is the good i price, where $i \in [0, 1]$. The household receives income from the bonds from period *t-1* that gives one unit of money at maturity, a wage W_t from the hours worked N_t, and dividends D_t since, as a model assumption, households are owners of firms.

Substituting C_t and P_t, we get the following budget constraint:

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t \ + D_t \tag{6.4}$$

Since total consumption must be equal to the aggregate of consumption of every good, we have

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t \tag{6.5}$$

There is a transversality condition that comes from an extra assumption for households to avoid Ponzi schemes and it states that any current consumption reduces future consumption and it is established as follows:

$$lim_{T\to\infty} E_t \left\{ \beta^{T-t} \frac{U_{c,T}}{U_{c,t}} \frac{B_T}{P_T} \right\} \geq 0 \tag{6.6}$$

The solution for the consumption allocation on goods must follow (For details, see Gali; 2015, appendix 3.1):

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \tag{6.7}$$

Substituting (6.5) in (6.4) we get

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t \ + D_t \tag{6.8}$$

Optimality conditions

Solving (6.1) we can derivate the optimality conditions (For further detail, see Gali; 2015, Chapter 2):

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{6.9}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\} \tag{6.10}$$

In log therms (See Gali; 2015, appendix 2.1 for (6.12)):

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{6.11}$$

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t \{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \tag{6.12}$$

where $i_t = -\log Q$ is the *nominal interest rate* and $\rho = -\log \beta$ is the household's discount rate.

Finally, we have the following money demand equation (in log terms):

$$m_i - p_t = y_t - \eta i_t \tag{6.13}$$

where m_t is the log nominal money holding, $p_t = \log P_t$, $y_t = \log Y_t$ and η is the semi-elasticity of money demand.

Parameters:

- The discount factor β
- The demand elasticity between goods ϵ
- The curvature of the utility of consumption σ
- The disutility of labor φ
- The semi-elasticity of money demand η

6.2 Firms

We have a continuous number of firms indexed by [0,1] producing differentiated goods with the same technology (for simplicity, production only depends on labor) given by the following Cobb-Douglass production function:

$$Y_t(i) = A_t N_t^{1-\alpha} \tag{6.14}$$

where $0 \leq \alpha \leq 1$ is the capital share and A_t is the level of technology evolving exogenous in log therms as

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{6.15}$$

were $a = \log A$, we have here our first shock ϵ_t^a, which is a productivity shock, and $\rho_a \in [0, 1]$.

Each firm faces the same demand given by (5.7). The price stickiness index is given by θ, we have that in each period there is a proportion $1 - \theta$ that can change its price while the proportion θ keeps the last price and it is independent of the last time a firm changed its price. Given this, the average duration of a price is given by $\frac{1}{1-\theta}$.

A firm will maximize its profits, given by:

$$P_t Y_t - W_t N_t \tag{6.16}$$

subject to (5.14), with price and wage taken as given.

Let us use the Lagrangian method to solve this:

Max $P_t Y_t - W_t N_t$ s.t. $Y_t(i) = A_t N_t^{1-\alpha}$

$$L = P_t Y_t - W_t N_t - \Lambda(A_t N_t^{1-\alpha} - Y_t(i))$$

F.O.C.

$$\frac{\delta L}{\delta Y_t} = P_t + \Lambda = 0$$

$$\frac{\delta L}{\delta N_t} = -W_t - \Lambda A_t (1-\alpha) N_t^{-\alpha} = 0$$

$$\frac{\delta L}{\Lambda} = -A_t N_t^{1-\alpha} + Y_t(i)) = 0$$

$\therefore P_t = -\Lambda$ and then $-W_t + (1-\alpha) P_t A_t N^{-\alpha} = 0$

Then

Optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha} \tag{6.17}$$

Parameters

- The capital share α
- The productivity shock ρ_a

6.3 New Keynesian Phillips Curve

We have price rigidities, which means that only a fraction $1-\theta$ of total firms are able to adjust their prices at period t. We define prices at period t by

$$P_t = \theta P_{t-1} + (1-\theta)P_t^* \tag{6.18}$$

where P_t^* is the price optimized by the proportion $1-\theta$ of firms at period t (it is the same for all of these firms), and P_t-1 comes from the fact that the price from firms not changing its prices at period t is the same as the price at period t-1.

Using this definition and the price index for all goods, we have

$$\begin{aligned} P_t &= \left[\int_0^1 \theta P_{t-1}(i)^{1-\epsilon} di + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\ &= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \end{aligned} \tag{6.19}$$

Dividing by P_{t-1} both sides, and defining $\Pi_t = \frac{P_t}{P_{t-1}}$ we get the following inflation equation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{6.20}$$

where Π_t is the inflation rate from period *t-1* to *t*. In the steady state $\Pi_t = 1$ so $P_t^* = P_{t-1} = P_t$.

Inflation of period t in log therms is given by

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{6.21}$$

The adjustment on prices at period t (P_t^*) is forward-looking, so it depends on a discount factor from future and expected future marginal cost, as the Calvo formulation (Calvo; 1983):

$$P_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k}^n\right\} \tag{6.22}$$

Following Galí and Gertler (1999), we define a New Phillip's Curve using marginal costs instead of the output gap,

$$\pi_t = \lambda mc_t + \beta E_t \{\pi_{t+1}\}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, and it implies that we have market power, so we can set a markup on price that will depend on expected future inflation, the fraction of firms able to change prices and a discount factor.

If we have no price rigidities, the price that firms update at period t is given by:

$$P_T^* = \mathbb{M}\Psi_t \tag{6.23}$$

where $\mathbb{M}$ is the optimal markup without price stickiness and it is often referred as the natural or desired markup and Ψ_t is the nominal marginal cost at period t.

Rearranging therms and using the optimality conditions for the firms (See McAdam and Willman; 2007, Appendix B), we get that firms will update their prices at period t following the next equation (in log therms):

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k|t}\right\} \tag{6.24}$$

where $\mu = \log \mathbb{M}$ and $\psi = \log \Psi$. The previous price equation tells us that firms will set prices at period t taking into consideration the desired markup and the expected present and future nominal marginal costs with weights being proportional to the probability of the price remaining the same at each period k, θ^k and the discount factor β^k.

Finally, we have the market clearing condition $Y_t = C_t$.

Parameters:

- The index of price stickiness θ

6.4 Marginal cost

A key variable for the marginal cost is employment N_t given by:

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)di \\
&= \int_0^1 \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \int_0^1 \left(\frac{C_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \int_0^1 \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \int_0^1 \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di
\end{aligned}
\tag{6.25}
$$

where the second equality is obtained by replacing the production function inside the integral, the third equality comes from using the clearing condition and the forth equality comes from applying (5.7).

If we apply logarithms to both sides we get

$$
(1-\alpha)n_t = y_t - a_t + (1-\alpha)\log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{6.26}
$$

where lowercase letter n_t is the logarithm for $N(t)$.

Let p_t and $p_t(i)$ be the logarithm of P_t and $P_t(i)$ respectively. Defining $\hat{p}_t(i) = p_t(i) - p_t$, we have that

$$
\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} = e^{(1-\epsilon)\hat{p}_t(i)} \tag{6.27}
$$

Using the first three therms of the Euler expansion $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ with $x = (1-\epsilon)\hat{p}_t(i)$ as an approximation of the equation, we have

$$
\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} = 1 + (1-\epsilon)\hat{p}_t(i) + \frac{(1-\epsilon)^2}{2}\hat{p}_t(i)^2 \tag{6.28}
$$

Since $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di = 1$, using a second-order approximation from the Taylor expansion around the zero inflation steady state, we get

$$\begin{aligned}
1 &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di \\
&= \int_0^1 e^{(1-\epsilon)\hat{p}_t(i)} di \\
&\approx 1 + (1-\epsilon)\int_0^1 \hat{p}_t(i)di + \frac{(1-\epsilon)^2}{2}\int_0^1 \hat{p}_t(i)^2 di \qquad (6.29)\\
&\approx 1 + (1-\epsilon)\int_0^1 (p_t(i)-p_t)di + \frac{(1-\epsilon)^2}{2}\int_0^1 (p_t(i)-p_t)^2 di \\
&\approx \int_0^1 p_t(i)di - p_t + \frac{(1-\epsilon)}{2}\int_0^1 (p_t(i)-p_t)^2 di
\end{aligned}$$

and then

$$p_t \approx E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2}\int_0^1 (p_t(i)-p_t)^2 di \qquad (6.30)$$

$$E_i\{\hat{p}_t(i)\} \approx \frac{(\epsilon-1)}{2}E_i(\hat{p}_t(i))^2 di \qquad (6.31)$$

where $E_i\{p_t(i)\} = \int_0^1 p_t(i)di$ is the expected cross-sectional mean of the log prices, and $E_i\{\hat{p}_t(i)\} = \int_0^1 \hat{p}_t(i)di$ is the expected cross-sectional mean of the log price differences.

With the two previous results, we can see that

$$\begin{aligned}
\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= \int_0^1 e^{-\frac{\epsilon}{1-\alpha}(p_t(i)-p_t)} di \\
&\approx 1 - \frac{\epsilon}{1-\alpha}\int_0^1 (p_t(i)-p_t)di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2\int_0^1 (p_t(i)-p_t)^2 di \\
&\approx 1 + \frac{1}{2}\frac{\epsilon(1-\epsilon)}{1-\alpha}\int_0^1 (p_t(i)-p_t)^2 di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2\int_0^1 (p_t(i)-p_t)^2 di \\
&= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\left((1-\epsilon) + \frac{\epsilon}{1-\alpha}\right)\int_0^1 (p_t(i)-p_t)^2 di \\
&= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\left(\frac{1-\alpha+\epsilon\alpha}{1-\alpha}\right)\int_0^1 (p_t(i)-p_t)^2 di
\end{aligned} \qquad (6.32)$$

Since variance can be expressed as $Var(X) = E[(X - E[X])^2]$, setting $X = p_t$ and $E[X] = p_t(i)$, we have that

$$\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di = 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}var_i\{p_t(i)\} \qquad (6.33)$$

with $\Theta = \frac{1-\alpha}{1-\alpha+\epsilon\alpha}$.

And then,

$$(1-\alpha)log\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}di \approx (1-\alpha)\left(1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}var_i\left\{p_t(i)\right\}\right)$$
$$\approx \frac{\epsilon}{2\Theta}var_i\left\{p_t(i)\right\} \tag{6.34}$$

Finally, with the previous result, an approximately relation between employment, production and technology would be:

$$n_t = \frac{1}{1-\alpha}(y_t - a_t) \tag{6.35}$$

The marginal product of labor = MP_N is the same for all firms (they use the same technology) and it is derived from the production function as

$$MP_N = \frac{\partial Y_t}{\partial N_t} = (1-\alpha)A_t N_T^{-\alpha} \tag{6.36}$$

in logarithmic therms:

$$\begin{aligned}\log MP_N = mp_n &= \log{(1-\alpha)A_t} - \alpha\log N_t\\ &= \log{(1-\alpha)} + \log A_t - \alpha\log N_t\\ &= \log{(1-\alpha)} + a_t - \alpha n_t\end{aligned} \tag{6.37}$$

Since marginal cost is equal to the labor needed to produce an additional unit of output times the wage (the production function in the model only requires work), we have

$$\Psi_t = \frac{W_t}{(MP_N)_t} \tag{6.38}$$

Now, if we consider the marginal cost for an individual firm at period $t+k$ where its last price update was at period t, we get

$$\begin{aligned}\psi_{t+k|t} &= w_{t+k} - (mp_n)_{t+k|t}\\ &= w_{t+k} - (\log{(1-\alpha)} + a_{t+k} - \alpha n_{t+k|t})\end{aligned} \tag{6.39}$$

and the average marginal cost $\psi_t = \int_0^1 \psi_t(i)di$ is given by

$$\psi_t = w_t - (\log(1-\alpha) + a_t - \alpha n_t) \tag{6.40}$$

Using the previous results, we get

$$\begin{aligned}
\psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\
&= \psi_{t+k} + \alpha\left(\frac{1}{1-\alpha}(y_{t+k|t} - a_{t+k} - (y_{t+k} - a_{t+k}))\right) \\
&= \psi_{t+k} - \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) \\
&= \psi_{t+k} + \frac{\alpha}{1-\alpha}\left((-\epsilon)\left((p_t^* - p_{t+k} - \log C_{t+k}) - (p_{t+k} - p_{t+k} - \log C_{t+k})\right)\right) \\
&= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}((p_t^* - p_{t+k})
\end{aligned} \tag{6.41}$$

where the second equality follows from (5.26) and the fact that $a_{t+k} = a_{t+k|t}$ since technology is the same for all firms and the forth equality follows from (5.3), (5.7) and the clearing condition $Y_t = C_t$.

6.5 Expectations and the inflation equation

Substituting (6.41) in (6.24) we get

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k})\right\} \tag{6.42}$$

Remember that, if $r \in (0,1)$, $\sum_{k=0}^{\infty} r^k$ is a geometric series converging to $\frac{1}{1-r}$. Using the fact that the expected value E is a linear function and $\beta\theta \in (0,1)$ we have that $\sum_{k=0}^{\infty}(\beta\theta)^k$ is a geometric series that converges to $\frac{1}{1-\beta\theta}$, so we get

$$\begin{aligned}
p_t^* &= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\left(-E_t\left\{\frac{\alpha\epsilon}{1-\alpha}p_t^*\right\} + E_t\left\{\psi_{t+k} + \frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\right) \\
&= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\left(-\left(\frac{1}{1-\beta\theta}\right)\frac{\alpha\epsilon}{1-\alpha}p_t^* + E_t\left\{\psi_{t+k} + \frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\right)
\end{aligned} \tag{6.43}$$

Since $-\frac{1}{1-\beta\theta}\frac{\alpha\epsilon}{1-\alpha}p_t^*$ does not depend on k, we have that $\sum_{k=0}^{\infty}(\beta\theta)^k\left(-\left(\frac{1}{1-\beta\theta}\right)\frac{\alpha\epsilon}{1-\alpha}p_t^*\right) = -\left(\frac{1}{1-\beta\theta}\right)\frac{\alpha\epsilon}{1-\alpha}p_t^*$. Assuming that $\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}$ converges, we can split the sum to get

$$
\begin{aligned}
p_t^* &= \mu - \frac{\alpha\epsilon}{1-\alpha}p_t^* + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\\
(1+\frac{\alpha\epsilon}{1-\alpha})p_t^* &= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\\
\left(\frac{1-\alpha+\alpha\epsilon}{1-\alpha}\right)p_t^* &= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}
\end{aligned}
\tag{6.44}
$$

Using the previous definition of $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ we get

$$
\begin{aligned}
p_t^* &= \Theta\mu + \Theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\\
&= \Theta\mu\left(\frac{1-\beta\theta}{1-\beta\theta}\right) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\Theta E_t\left\{\psi_{t+k}+\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\\
&= (1-\beta\theta)\left(\frac{\Theta\mu}{1-\beta\theta}+\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta\psi_{t+k}+\Theta\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\right)\\
&= (1-\beta\theta)\left(\sum_{k=0}^{\infty}(\beta\theta)^k\Theta\mu+\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta\psi_{t+k}+\Theta\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\right)\\
&= (1-\beta\theta)\left(\sum_{k=0}^{\infty}(\beta\theta)^k\left(\Theta\mu+E_t\left\{\Theta\psi_{t+k}+\Theta\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\right)\right)\\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta\mu+\Theta\psi_{t+k}+\left(\frac{1-\alpha}{1-\alpha+\alpha\epsilon}\right)\frac{\alpha\epsilon}{1-\alpha}p_{t+k}\right\}\\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta(\mu+\psi_{t+k})+\frac{\alpha\epsilon}{1-\alpha+\alpha\epsilon}p_{t+k}\right\}
\end{aligned}
\tag{6.45}
$$

Since $\alpha\epsilon > 0$ and $1-\alpha > 0$, then $0 < \Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon} < 1$ so $\Theta = 1-\Lambda$ with $\Lambda = \frac{\alpha\epsilon}{1-\alpha+\alpha\epsilon}$, which implies that $\frac{\alpha\epsilon}{1-\alpha+\alpha\epsilon} = 1-\Theta$, using this we have

$$\begin{aligned}
p_t^* &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta(\mu+\psi_{t+k})+(1-\Theta)p_{t+k}\right\} \\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta(\mu+\psi_{t+k})+(p_{t+k}-\Theta p_{t+k})\right\} \qquad (6.46)\\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k}-\Theta(p_{t+k}-(\mu+\psi_{t+k})\right\}
\end{aligned}$$

If we set $\mu_t = p_t - \psi_t$ as the markup at period t and $\hat{\mu}_t = \mu_t - \mu = p_t - (\psi_t + \mu)$ the difference between the markup at period t and the desired markup, then

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k}-\Theta\hat{\mu}_{t+k}\right\} \qquad (6.47)$$

We can manipulate the previous equation to get a recursive expression of it:

$$\begin{aligned}
p_t^* &= (1-\beta\theta)E_t\left\{p_t-\Theta\hat{\mu}_t\right\}+(1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k}-\Theta\hat{\mu}_{t+k}\right\} \\
&= (1-\beta\theta)E_t\left\{p_t-\Theta\hat{\mu}_t\right\}+(1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^{k-1}(\beta\theta)E_t\left\{p_{t+k}-\Theta\hat{\mu}_{t+k}\right\} \\
&= (1-\beta\theta)E_t\left\{p_t-\Theta\hat{\mu}_t\right\}+(1-\beta\theta)(\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^{k-1}E_t\left\{p_{t+1+(k-1)}-\Theta\hat{\mu}_{t+1+(k-1)}\right\} \\
&= (1-\beta\theta)E_t\left\{p_t-\Theta\hat{\mu}_t\right\}+(1-\beta\theta)(\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^{k}E_t\left\{p_{t+1+k}-\Theta\hat{\mu}_{t+1+k}\right\}
\end{aligned} \qquad (6.48)$$

P.D. $E_t\left\{p_{t+1}^*\right\} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+1+k}-\Theta\hat{\mu}_{t+1+k}\right\}$

Proof 1

$$\begin{aligned}
E_t\left\{p_{t+1}^*\right\} &= E_t\left((1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\left\{p_{t+1+k}-\Theta\hat{\mu}_{t+1+k}\right\}\right) \\
&= (1-\beta\theta)\sum_{k=0}^{\infty}E_t\left((\beta\theta)^k E_{t+1}\left\{p_{t+1+k}-\Theta\hat{\mu}_{t+1+k}\right\}\right) \qquad (6.49)\\
&= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(E_{t+1}\left\{p_{t+1+k}-\Theta\hat{\mu}_{t+1+k}\right\}\right)
\end{aligned}$$

Applying the tower property of conditional expectation (Kallenberg; 2021, , Theorem 6.1 (VII)) we get

$$E_t\left\{p^*_{t+1}\right\} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+1+k} - \Theta\hat{\mu}_{t+1+k}\right\} \tag{6.50}$$

□

Substituting (6.50) in (6.48) we get the following recursive equation for p^*_t:

$$p^*_t = (1-\beta\theta)E_t\left\{p_t - \Theta\hat{\mu}_t\right\} + (\beta\theta)E_t\left\{p^*_{t+1}\right\} \tag{6.51}$$

Combining (6.51) with (6.21) we get the following inflation equation:

$$\begin{aligned}
\pi_t &= (1-\theta)\left[(1-\beta\theta)E_t\left\{p_t - \Theta\hat{\mu}_t\right\} + (\beta\theta)E_t\left\{p^*_{t+1}\right\} - p_{t-1}\right] \\
&= (1-\theta)\left[(1-\beta\theta)(p_t - \Theta\hat{\mu}_t) + (\beta\theta)E_t\left\{p^*_{t+1}\right\} - p_{t-1}\right] \\
&= (1-\theta)\left[p_t - \beta\theta p_t - \Theta\hat{\mu}_t + \beta\theta\Theta\hat{\mu}_t + (\beta\theta)E_t\left\{p^*_{t+1}\right\} - p_{t-1}\right] \\
&= (1-\theta)\left[\pi_t - \beta\theta p_t - \Theta\hat{\mu}_t + \beta\theta\Theta\hat{\mu}_t + (\beta\theta)E_t\left\{p^*_{t+1}\right\}\right]
\end{aligned} \tag{6.52}$$

then

$$\begin{aligned}
\pi_t - (1-\theta)\pi_t &= (1-\theta)\left[-\beta\theta p_t - \Theta\hat{\mu}_t + \beta\theta\Theta\hat{\mu}_t + (\beta\theta)E_t\left\{p^*_{t+1}\right\}\right] \\
(\theta)\pi_t &= (1-\theta)\left[-\beta\theta p_t - \Theta\hat{\mu}_t + \beta\theta\Theta\hat{\mu}_t + (\beta\theta)E_t\left\{p^*_{t+1}\right\}\right] \\
\pi_t &= \frac{1-\theta}{\theta}\left[-\beta\theta p_t - \Theta\hat{\mu}_t + \beta\theta\Theta\hat{\mu}_t + (\beta\theta)E_t\left\{p^*_{t+1}\right\}\right] \\
&= (1-\theta)\left[-\beta p_t - \frac{\Theta}{\theta}\hat{\mu}_t + \beta\Theta\hat{\mu}_t + \beta E_t\left\{p^*_{t+1}\right\}\right] \\
&= (1-\theta)\left[-\beta p_t - \frac{1-\beta\theta}{\theta}\Theta\hat{\mu}_t + \beta E_t\left\{p^*_{t+1}\right\}\right]
\end{aligned} \tag{6.53}$$

Defining $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$, using the linearity of the expected value and (5.20), we get the NKPC expressed in one step forward expected inflation and the output gap

$$\begin{aligned}
\pi_t &= (1-\theta)\beta E_t\left\{p^*_{t+1}\right\} - (1-\theta)\beta p_t - \lambda\hat{\mu}_t \\
&= E_t\left\{\beta(1-\theta)(p^*_{t+1} - p_t)\right\} - \lambda\hat{\mu}_t \\
&= E_t\left\{\beta\pi_{t+1}\right\} - \lambda\hat{\mu}_t \\
&= \beta E_t\left\{\pi_{t+1}\right\} - \lambda\hat{\mu}_t
\end{aligned} \tag{6.54}$$

We have that inflation is forward-looking since it depends on the expectation of the future inflation, also, it is increasing with the index of price stickiness θ which means that higher price stickiness will increase inflation. It is also decreasing with the demand elasticity ϵ, meaning that greater demand elasticity will decrease inflation (as stated in the previously examined market power theory).

P.D. $\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t \{\hat{\mu}_{t+k}\}$

Proof 2

$$\begin{aligned} \pi_t &= \beta E_t \{\pi_{t+1}\} - \lambda \hat{\mu}_t \\ &= -\lambda \hat{\mu}_t + \beta E_t \{\beta E_{t+1} \{\pi_{t+2}\} - \lambda \hat{\mu}_{t+1}\} \end{aligned} \tag{6.55}$$

Applying the tower property of conditional expectation

$$\begin{aligned} \pi_t &= -\lambda \hat{\mu}_t - \lambda \beta E_t \{\hat{\mu}_{t+1}\} + \beta^2 E_t \{\pi_{t+2}\} \\ &= -\lambda \sum_{k=0}^{1} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^2 E_t \{\pi_{t+2}\} \\ &= -\lambda \sum_{k=0}^{1} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^2 E_t \{\beta E_{t+2} \{\pi_{t+3}\} - \lambda \hat{\mu}_{t+2}\} \\ &= -\lambda \sum_{k=0}^{1} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^3 E_t \{\pi_{t+3}\} - \lambda \beta^2 E_t \{\hat{\mu}_{t+2}\} \\ &= -\lambda \sum_{k=0}^{2} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^3 E_t \{\pi_{t+3}\} \\ &\vdots \\ &= -\lambda \sum_{k=0}^{n} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^{n+1} E_t \left\{\pi_{t+(n+1)}\right\} \end{aligned} \tag{6.56}$$

Applying a limit to both sides we get

$$\begin{aligned} \lim_{n \leftarrow \infty} \pi_t &= \lim_{n \leftarrow \infty} \left[-\lambda \sum_{k=0}^{n} \beta^k E_t \{\hat{\mu}_{t+k}\} + \beta^{n+1} E_t \left\{\pi_{t+(n+1)}\right\} \right] \\ \pi_t &= -\lim_{n \leftarrow \infty} \lambda \sum_{k=0}^{n} \beta^k E_t \{\hat{\mu}_{t+k}\} + \lim_{n \leftarrow \infty} \beta^{n+1} E_t \left\{\pi_{t+(n+1)}\right\} \\ &= -\lambda \sum_{k=0}^{\infty} \beta^k E_t \{\hat{\mu}_{t+k}\} + \lim_{n \leftarrow \infty} \beta^{n+1} \lim_{n \leftarrow \infty} E_t \left\{\pi_{t+(n+1)}\right\} \end{aligned} \tag{6.57}$$

since $\beta \in (0,1)$, $\lim_{n \leftarrow \infty} \beta^{n+1} = 0$,

$$\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t \{\hat{\mu}_{t+k}\} \tag{6.58}$$

□

With this recursive equation we can see that inflation depends on current and future expected markup deviations from the desired markup, which is consistent with the fact that firms with the possibility to change prices will set a greater price if they expect future markups to be lower than the desired one.

In this context, contrary to the classical monetary model, inflation comes from market power that allows firms to set their prices following their expectations.

6.6 Dynamic IS equation

Since we already defined the logarithm of price markup as

$$\mu_t = p_t - \psi_t \tag{6.59}$$

Combining the household optimality condition (6.11) with (6.35) and (6.40), and applying the clearing condition $Y_t = C_t$ yields:

$$\begin{aligned}
\mu_t &= (w_t - \sigma c_t - \varphi n_t) - \psi_t \\
&= (w_t - \sigma c_t - \varphi \frac{1}{1-\alpha}(y_t - a_t)) - \psi_t \\
&= w_t - \sigma c_t - \varphi \frac{1}{1-\alpha}(y_t - a_t) - w_t + \log(1-\alpha) + a_t - \alpha n_t) \\
&= -\sigma y_t - \varphi \frac{1}{1-\alpha}(y_t - a_t) + \log(1-\alpha) + a_t - \alpha n_t) \\
&= -\sigma y_t - \frac{\varphi}{1-\alpha}(y_t - a_t) + \log(1-\alpha) + a_t - \frac{\alpha}{1-\alpha}(y_t - a_t) \\
&= -\sigma y_t - \frac{\varphi}{1-\alpha} y_t + \frac{\varphi}{1-\alpha} a_t + \log(1-\alpha) + a_t - \frac{\alpha}{1-\alpha} y_t + \frac{\alpha}{1-\alpha} a_t \\
&= -(\sigma + \frac{\varphi+\alpha}{1-\alpha}) y_t + a_t(\frac{\varphi+\alpha}{1-\alpha} + 1) + \log(1-\alpha) \\
&= -(\sigma + \frac{\varphi+\alpha}{1-\alpha}) y_t + a_t(\frac{\varphi+\alpha}{1-\alpha} + \frac{1-\alpha}{1-\alpha}) + \log(1-\alpha) \\
&= -(\sigma + \frac{\varphi+\alpha}{1-\alpha}) y_t + (\frac{\varphi+1}{1-\alpha}) a_t + \log(1-\alpha)
\end{aligned} \tag{6.60}$$

Let us examine the case where there are flexible prices, meaning $\theta = 0$. In this case, the average markup μ_t is equal to the desired markup μ. The output from this case is denominated *natural level of output*, and will be denoted as y_t^n. Replacing both the desired markup and the natural level of output on (6.60) we get:

$$\mu = -(\sigma + \frac{\varphi + \alpha}{1 - \alpha})y_t^n + (\frac{\varphi + 1}{1 - \alpha})a_t + \log(1 - \alpha) \tag{6.61}$$

clearing y_t^n yields

$$\begin{aligned} y_t^n &= -(\frac{1 - \alpha}{\sigma(1 - \alpha) + \varphi + \alpha})\mu + (\frac{1 - \alpha}{\sigma(1 - \alpha) + \varphi + \alpha})(\frac{\varphi + 1}{1 - \alpha})a_t \\ &+ (\frac{1 - \alpha}{\sigma(1 - \alpha) + \varphi + \alpha})\log(1 - \alpha) \\ &= -\frac{(1 - \alpha)(\mu - \log(1 - \alpha))}{\sigma(1 - \alpha) + \varphi + \alpha} + \frac{\varphi + 1}{\sigma(1 - \alpha) + \varphi + \alpha}a_t \\ &= \Psi_y + \Psi_{ya}a_t \end{aligned} \tag{6.62}$$

where $\Psi_y = -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\Psi_{ya} = \frac{\varphi+1}{\sigma(1-\alpha)+\varphi-\alpha}$. Since $(1 - \alpha) \in (0, 1)$, $\log(1 - \alpha) < 0$, also, we have that $M_t \in (0, 1)$ so $m_t < 0$ and then $\Psi_y = -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$.

We can see with this that the markup (that comes from market power) lowers the natural level of output, while it does not depend on the preference shock z_t or the monetary policy.

Since $\hat{\mu}_t = \mu_t - \mu$, when subtracting (6.61) from (6.60) we get

$$\begin{aligned} \hat{\mu}_t &= -(\sigma + \frac{\varphi + \alpha}{1 - \alpha})y_t + (\frac{\varphi + 1}{1 - \alpha})a_t + \log(1 - \alpha) + (\sigma + \frac{\varphi + \alpha}{1 - \alpha})y_t^n \\ &- (\frac{\varphi + 1}{1 - \alpha})a_t - \log(1 - \alpha) \\ &= -(\sigma + \frac{\varphi + \alpha}{1 - \alpha})(y_t - y_t^n) \end{aligned} \tag{6.63}$$

This equation shows that the markup gap is proportional to the output gap $y_t - y_t^n$ denoted as $\tilde{y}_t$. Substituting (6.63) in (6.54) we have:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \tag{6.64}$$

where $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$.

The last equation is the New Keynesian Phillips Curve (NKPC). It depends on expectations on future inflation and the output gap, so it is forward looking.

Using the clearing condition in the optimality condition (6.12) and rewriting all in therms of the output gap, we get:

$$
\begin{aligned}
y_t - y^n_{t+1} &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
y_t - (\Psi_y + \Psi_{ya}a_{t+1}) &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
y_t - (\Psi_y + \Psi_{ya}(\rho_a a_t + E_t\left\{\epsilon^a_{t+1}\right\})) &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
y_t - (\Psi_y + \Psi_{ya}\rho_a a_t) &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
y_t - (\Psi_y + \Psi_{ya}\rho_a a_t) + \Psi_{ya}a_t - \Psi_{ya}a_t &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
\tilde{y_t} - \Psi_{ya}\rho_a a_t + \Psi_{ya}a_t &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
\tilde{y_t} + (1-\rho_a)\Psi_{ya}a_t &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - \rho - (1-\rho_z)z_t) \\
\tilde{y_t} &= E_t\left\{\tilde{y_{t+1}}\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - r^n_t)
\end{aligned} \tag{6.65}
$$

where the third equality comes from (6.15), the technology process, and the forth equation comes from the fact that the expected error therm in the technology process is zero. Equation (6.65) is defined as the *Dynamic IS curve.*

We define

$$
r^n_t = \rho + (1-\rho_z)z_t - \sigma(1-\rho_a)\Psi_{ya}a_t \tag{6.66}
$$

as the natural interest rate.

Using the tower property for conditional expectation we can express (6.65) as

$$
\begin{aligned}
\tilde{y_t} &= E_t\left\{E_{t+1}\left\{\tilde{y_{t+2}}\right\} - \frac{1}{\sigma}(i_{t+1} - E_{t+1}\left\{\pi_{t+2}\right\} - r^n_{t+1})\right\} - \frac{1}{\sigma}(i_t - E_t\left\{\pi_{t+1}\right\} - r^n_t) \\
&= -\frac{1}{\sigma}E_t\left\{i_t - E_t\left\{\pi_{t+1}\right\} - r^n_t\right\} - \frac{1}{\sigma}E_t\left\{(i_{t+1} - E_{t+1}\left\{\pi_{t+2}\right\} - r^n_{t+1}\right\} + E_t\left\{\tilde{y_{t+2}}\right\} \\
&= -\frac{1}{\sigma}\sum_{k=0}^{1}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\} + E_t\left\{\tilde{y}_{t+2}\right\} \\
&= -\frac{1}{\sigma}\sum_{k=0}^{1}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\} + E_t\left\{\tilde{y}_{t+2}\right\} \\
&+ E_t\left\{E_{t+2}\left\{\tilde{y}_{t+3}\right\} - \frac{1}{\sigma}(i_{t+1} - E_{t+1}\left\{\pi_{t+2}\right\} - r^n_{t+1})\right\} \\
&= -\frac{1}{\sigma}\sum_{k=0}^{2}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\} + E_t\left\{\tilde{y}_{t+3}\right\} \\
&\vdots \\
&= -\frac{1}{\sigma}\sum_{k=0}^{m}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\} + E_t\left\{\tilde{y}_{k+m+1}\right\}
\end{aligned}
\tag{6.67}
$$

Applying limits to both sides and assuming that the effect from expected nominal rigidities vanishes after some time ($\lim_{m\to\infty} E_t\left\{\tilde{y}_{k+m+1}\right\} = 0$) we get

$$
\begin{aligned}
\tilde{y}_t &= -\frac{1}{\sigma}\sum_{k=0}^{\infty}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\} + \lim_{m\to\infty} E_t\left\{\tilde{y}_{k+m+1}\right\} \\
&= -\frac{1}{\sigma}\sum_{k=0}^{\infty}E_t\left\{i_{t+k} - E_{t+k}\left\{\pi_{t+1+k}\right\} - r^n_{t+k}\right\}
\end{aligned}
\tag{6.68}
$$

Finally, we can define the real interest rate as

$$r_t = i_t - E_t\left\{\pi_{t+1}\right\} \tag{6.69}$$

so we get

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}E_t\left\{r_{t+k} - r^n_{t+k}\right\} \tag{6.70}$$

it states that current deviation from output will be directly correlated to the actual and future deviations of the real interest rate from the natural interest rate.

6.7 Monetary policy rule

In the previous section about the model specification we where assuming that the natural level of output was equal to the efficient level of output ($y_t^n = y_t^e$) at every period t. In the present section, we are departing from that assumption and letting real imperfections to have an impact on the natural level of output. This particular instance of real imperfections produces a short-run deviation of the natural level of output y_t^n from the efficient output level y_t^e. We will assume that the steady states is the same as in a model without real imperfections, so in the long run we have that $y_t^n = y_t^e$.

Let us define $x_t = y_t - y_t^e$ as the welfare-relevant output gap. In this case, we have that $\tilde{y}_t = (y_t - y_t^e) + (y_t^e - y_t^n)$. Substituting the last equation into (5.55) we get

$$\begin{aligned} \pi_t &= \beta E_t \{\pi_{t+1}\} + \kappa ((y_t - y_t^e) + (y_t^e - y_t^n)) \\ &= \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t \end{aligned} \tag{6.71}$$

where $u_t = \kappa(y_t^e - y_t^n)$.

Since monetary authorities have no influence on the natural or the efficient level of output, we assume u_t comes from an exogenous process. Following Gali (2015) (Chapter 5), deviations in the short-run of the natural level of output from the efficient level of output will come from a cost-push shock u_t given by an exogenous variation of desired price markup following the stochastic process

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \tag{6.72}$$

with $\rho_u \in [0, 1)$ and ϵ_t^u have zero mean and constant variance σ_u^2.

From the consumer optimality condition definition (6.14), we have

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

applying the clearing condition $y_t = c_t$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t)$$

using the definition $x_t = y_t - y_t^e$

$$= E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t) + E_t\{y_{t+1}^e\}$$

$$= E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t - \sigma E_t\{y_{t+1}^e\}\right)$$

$$= E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t - \sigma E_t\{y_{t+1}^e\}\right)$$

hence

$$y_t - y_t^e = E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t - \sigma E_t\{y_{t+1}^e\}\right) - E_t\{y_t^e\}$$

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t - \sigma E_t\{y_{t+1}^e\} + \sigma E_t\{y_t^e\}\right)$$

$$= E_t\{x_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho - (1-\rho_z)z_t - \sigma(E_t\{y_{t+1}^e - y_t^e\})\right) \tag{6.73}$$

We have that the Dynamic IS equation from the assumption that there are deviations of the natural level of output from the efficient one is given by

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) \tag{6.74}$$

where $r_t^e = \rho + (1-\rho_z)z_t + \sigma E_t\{\Delta y_{t+1}^e\}$ is the real interest rate corresponding to y_t^e.

From (6.62), and assuming that the efficient level of output is proportional to the technology level, we have

$$y_t^e = \Psi_{ya} a_t \tag{6.75}$$

6.8 Calibration

The discount factor, which accounts for how much relevance future periods in time will have on the agent's current decisions, is set as $\beta = 0.987$, following Aguiar and Gopinath (2007), who estimates a business cycle for emerging countries.

For the Frisch elasticity of labor φ, we took the unemployment rate for Mexico from 2005 to 2021 from INEGI (INEGI; 2023) and the average wage per hour in USD from the OCDE (OCDE; 2023a, , OCDE (2023b)). Using a simple linear regression we found that there is a Frisch elasticity of labor of 0.14, according to this, we set $\varphi = 7$.

We use the Taylor rule as the monetary policy rule, since there is evidence suggesting that this is the underlying Mexican Central Bank monetary rule (Carlos and Galindo; 2003). The Taylor rule involves two parameters, the first one is the weight of the product gap on the interest rate, ϕ_y, and the other one is the weight of the inflation on the interest rate, ϕ_π. Following Sánchez Vargas (2020) we have that $\phi_y = 0.636$ and $\phi_\pi = 1.611$.

With respect to the capital share of production α, we first calculated the labor participation on production by dividing total salaries by total gross domestic product using data from the 2009 economic census (INEGI; 2009). We got that the labor participation in production is 0.1524, so we set $\alpha = 0.8476$.

For the Calvo parameter, we follow the estimation by Ramos-Francia and Torres (2008), who uses Generalized Methods of Moments, and is set as $\theta = 0.858$.

As we use the following characterization of household's utility (cf. Gali; 2015, Chapter 3 pp. 54):

$$U_t = \left\{ (\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}) Z_t \right\}$$

So we have that $\sigma = 1$.

In the case of the interest semi elasticity of money demand, we estimated an OLS on the next model:

$$m_d = \gamma_i i + \gamma_{gdp} GDP + \gamma_\pi \pi + u \tag{6.76}$$

where m_d is the money demand (we used the M1 monetary aggregate from the Mexican Central Bank to proximate this variable), i is the interest rate from the 28-days government bonds, GDP is the gross domestic product, and π is the CPI. We converted each variable to monthly data and took logarithms for each variable and the time horizon is from December 2000 to December 2017. All data was obtained from BANXICO (BANXICO; 2023). The γ's are the variables respectively contribution on money demand and u is the error therm.

The parameter of interest in the former equation is γ_i, since it is the interest semi elasticity on money demand. From the OLS regression we got that this elasticity is equal to 0.17, so in the model ge set $\eta = 0.17$.

With respect of the elasticity of substitution between goods, which inverse is given by ϵ, as this elasticity depends on the market concentration and we have market power (who's key point is the price elasticity of demand,) we keep these elasticity small to show the market concentration in the consumption goods. Following Gali (2015), we set $\epsilon = 9$.

Finally, we define a cost-push shock in the model, which allows us to introduce the raw material price increase and the following consumer price index (CPI) parallel increase due to market concentration. This cost-push shock is defined with an AR(1) process so we have to estimate a value for the parameter ρ_u reflecting the persistence of the shock.

Since this cost-push shock is directly introduced into the NKPC, this parameter will somehow reflect the persistence of a cost shock in inflation. Having said this, we follow Sánchez Vargas (2020) to set a value for this parameter. In their paper, they estimate the Phillips Curve for the Mexican economy estimating the following equation:

$$\pi_t = \lambda_\pi \pi_{t-1} - \lambda_u U_{t-1} + \lambda_d diesel\_t + \lambda_p pa\_t \tag{6.77}$$

They introduce energy prices $diesel\_t$ and international food prices pa_t to the standard Phillips curve to take into account the particular characteristics of the Mexican economy. Each λ_i (with $i \in \{\pi, u, d, p\}$) corresponds to the weigh each variable has on the Phillips Curve.

What interests us about this model is the parameter λ_π since it accounts for the inflation persistence as we need. The value found by the author is $\lambda_\pi = 0.528$, so we will set $\rho_u = 0.528$.

6.9 Model implementation

Variables

$$
\begin{aligned}
\pi &= \text{inflation} \\
\tilde{y} &= \text{output gap} \\
y^n &= \text{natural output} \\
y &= \text{output} \\
r^e &= \text{efficient interest rate} \\
y^e &= \text{efficient output} \\
x &= \text{welfare-relevant output gap} \\
r^r &= \text{real interest rate} \\
i &= \text{nominal interest rate} \\
n &= \text{hours worked} \\
\delta_m &= \text{money growth} \\
u &= \text{AR(1) cost-push shock process} \\
a &= \text{AR(1) technology shock process} \\
r^{r,ann} &= \text{annualized real interest rate} \\
i^{ann} &= \text{annualized nominal interest rate} \\
\pi^{ann} &= \text{annualized inflation rate} \\
p &= \text{price level} \\
z &= \text{AR(1) preference shock process}
\end{aligned} \tag{6.78}
$$

Shocks

$$
\begin{aligned}
\epsilon_a &= \text{technology shock} \\
\epsilon_u &= \text{monetary policy shock} \\
\epsilon_z &= \text{preference shock innovation}
\end{aligned} \tag{6.79}
$$

Parameters

$$
\begin{aligned}
\alpha &= \text{capital share} \\
\beta &= \text{discount factor} \\
\rho_a &= \text{autocorrelation technology shock} \\
\rho_u &= \text{autocorrelation cost-push shock} \\
\rho_z &= \text{autocorrelation preference shock} \\
\sigma &= \text{log utility} \\
\varphi &= \text{unitary Frisch elasticity} \\
\eta &= \text{semi-elasticity of money demand} \\
\epsilon &= \text{demand elasticity} \\
\theta &= \text{Calvo parameter} \\
\omega &= \text{Composite parameter Phillips curve} \\
\lambda &= \text{Composite parameter Phillips curve} \\
\kappa &= \text{Composite parameter Phillips curve} \\
\vartheta &= \text{weight of x in utility function}
\end{aligned}
\tag{6.80}
$$

Model equations

Composite parameter

$$\psi_{n_{ya}} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

Efficient interest rate:

$$r_e = \sigma(y^{e(+1)} - y_e) + (1-\rho_z)z$$

Efficient output:

$$y_e = \psi_{n_{ya(a)}}$$

Output gap:

$$y_{gap} = x + (y^e - y^n)$$

New Keynesian Phillips Curve:

$$\pi = \beta\pi(+1) + \kappa x + u$$

Dynamic IS Curve:

$$x = x(+1) - \frac{1}{\sigma}(i - \pi(+1) - r_e)$$

Real interest rate:

$$r_{real} = i - \pi(+1)$$

Implicit definition of natural output:

$$u = \kappa(y^e - y^n)$$

Output gap:

$$y_{gap} = y - y^n$$

Cost-push shock:

$$u = \rho_u u(-1) + \epsilon_u$$

Total Factor Productivity shock:

$$a = \rho_a a(-1) + \epsilon_a$$

Production function:

$$y = a + (1 - \alpha)n$$

Money growth:

$$m_{growth_{ann}} = 4(y - y(-1) - \eta(i - i(-1)) + \pi)$$

Annualized nominal interest rate:

$$i_{ann} = 4i$$

Annualized real interest rate:

$$r_{real_{ann}} = 4r_{real}$$

Annualized inflation:

$$\pi_{ann} = 4\pi$$

Price level:

$$\pi = p - p(-1)$$

Preference shock:

$$z = \rho_z z(-1) - \epsilon_z;$$

Chapter 7

Cost-push shock simulations for the Mexican case (in the presence of oligopolistic markets)

7.1 Impulse response functions

We show the effect of a cost-push shock with a calibrated DSGE model for the Mexican economy with the Impulse Response Functions corresponding to the effect of this shock on output **x**, price level **p** and inflation **pi**. We first assume that the cost-push shock u_t is transitory, which means that $\rho_u = 0$ in the stochastic process defining u_t, so the shock will only last one period.

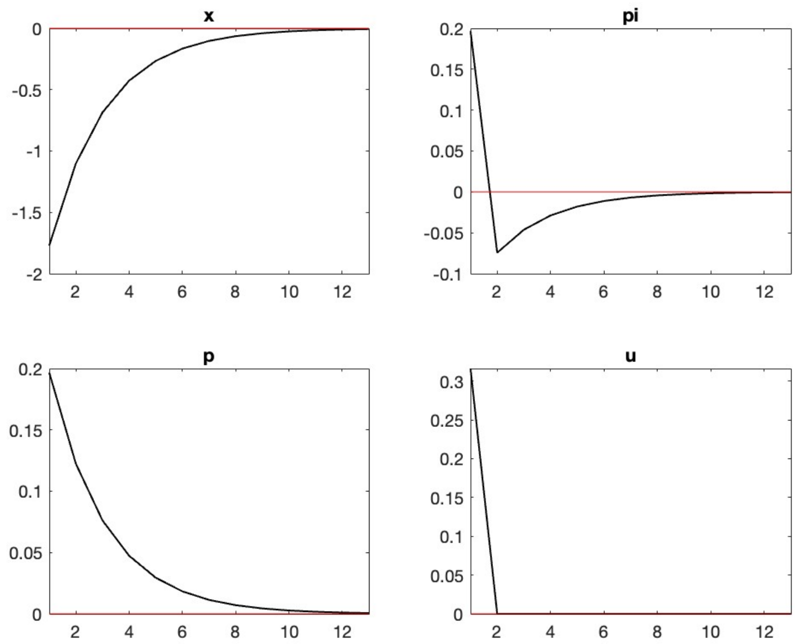

Figure 7.1: IRF from a transitory cost-push shock

Figure 7.1 shows that the cost-push shock given by **u** will affect negatively the output, but it will return to the initial level; it will first increase inflation followed by a decrease until it returns to the level before the shock (this behavior can be explained with the components of the NKPC, having a first positive impact via the shock and followed by a decrease on inflation due to the output reduction); the price level will increase with a cost-push shock with a gradual return to the initial level. All the variables considered here return to their former level within the 12 periods after the shock.

Based on Figure 7.1 and the analysis provided, we can observe the following effects of a cost-push shock:

- Output: The cost-push shock initially has a negative impact on output, causing a decline. However, over time, the output gradually returns to its initial level. This suggests a temporary disruption in production, followed by a recovery.

- Inflation: The cost-push shock leads to an increase in inflation initially. This can be attributed to the components of the New Keynesian Phillips Curve (NKPC), where the shock has a positive impact on inflation. However, as output decreases due to the shock, inflation subsequently decreases as well. Eventually, inflation returns to its level before the shock.

- Price Level: The cost-push shock causes the price level to increase. This increase is followed by a gradual return to the initial level. The behavior of the price level can be explained by the impact of the shock on production costs, leading to higher prices in the short term, but with a subsequent adjustment towards the pre-shock level.

Overall, the analysis indicates that the effects of a cost-push shock are temporary, and the economy tends to revert to its previous levels within 12 periods after the shock. The initial negative impact on output and the subsequent adjustments in inflation and price level highlight the dynamic nature of the economy and the interplay between different economic variables in response to shocks.

A persistent shock with different DSGE model specifications

Since inflation in Mexico from pandemic has been assiduous, we allow our model to have a persistent cost-push shock by setting $\rho_u = 0.5$ so it lasts further periods of time.

Model 1

This is the simplest model since we do not specify an explicit markup equation and the price stickiness θ is only reflected in the parameter κ that shows the weight of output changes in the New Keynesian Phillips Curve (NKPC). The NKPC is defined as

$$\pi_t = \beta\pi_{t+1} + \kappa x_t + u_t; \tag{7.1}$$

where $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$, with $\lambda = (1-\theta)\frac{1-\beta\theta}{\theta\omega}$. An increase in price stickiness θ will reduce the weight of the output on inflation (this reflects the fact that less firms can change their price after a change in production, so inflation gets less affected). Also, if the capital share α increases, it will generate an increase on inflation.

Applying Model 1 with a persistent cost-push shock, $\rho_u = 0.5$, we get the following IRF:

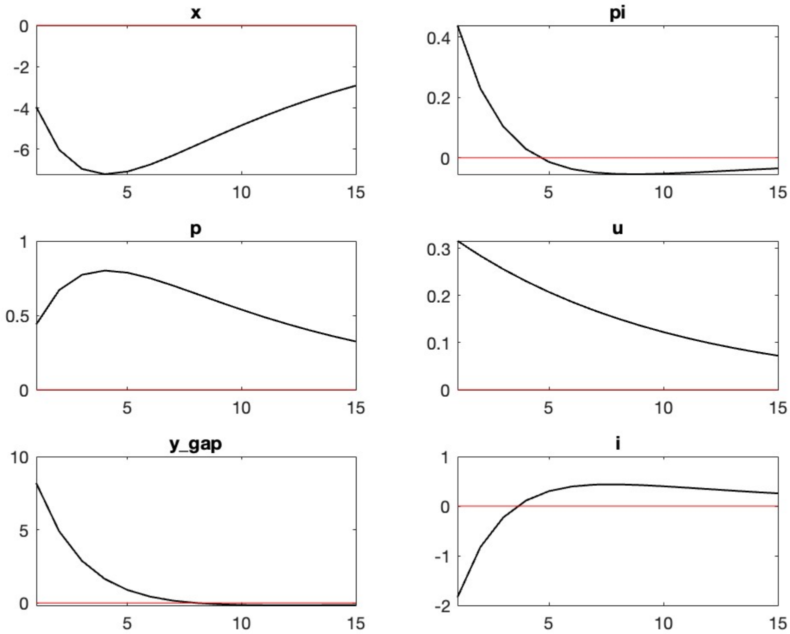

Figure 7.2: IRF from a persistent cost-push shock

From Figure 7.2, we have that the cost-push shock will reduce output by a larger amount that in the transitory case and recovery will be slower. For the inflation, the cost-push shock will increase inflation more than twice than in the transitory case and with a slower recovery that will eventually take inflation back to its initial level. The price level will increase as well more than in the transitory case followed by some recovery but keeping prices higher than the initial level.

The following observations can be made:

- Output: The persistent cost-push shock has a more significant impact on reducing output compared to the transitory case. This suggests that the shock has a more prolonged and severe effect on production, leading to a larger decline in output. Additionally, the recovery process for output is slower, indicating that it takes more time for the economy to recover from the shock and return to its initial level of output.

- Inflation: The persistent cost-push shock has a more pronounced effect on increasing inflation compared to the transitory case. Inflation rises by more than twice the amount observed in the transitory case. Furthermore, the recovery of inflation from the shock is slower, implying that it takes a longer time for inflation to return to its initial level. This indicates that the shock has a more persistent impact on inflation dynamics.

- Price Level: The persistence cost-push shock leads to a more substantial increase in the price level compared to the transitory case. The initial impact of the shock causes prices to rise more significantly. Although there is some recovery observed in the price level, it remains higher than the initial level. This suggests that the shock has a lasting effect on price levels, contributing to a higher overall price level in the economy.

In summary, Figure 7.2 demonstrates that a persistent cost-push shock has a more detrimental and prolonged impact on output, inflation, and price level compared to the transitory case. The slower recovery in all three variables implies that the effects of the shock persist for a longer duration, indicating a more challenging economic adjustment process.

Model 2

In this model we have an explicit log markup equation (from 6.52) given by

$$\mu_p = -(\sigma + \frac{\varphi + \alpha}{1 - \alpha})x \tag{7.2}$$

A first thing to notice in the previous equation is that an increase in the Frisch elasticity φ will reduce the markup and an increase in capital share α will reduce the markup if we have an output gap increase.

Even thought the cost-push shock enters in the NKPC as in model 1, this shock affects the markup via the output. Since we have a decrease in output from the cost-push shock, we would expect an increase in the markup from firms.

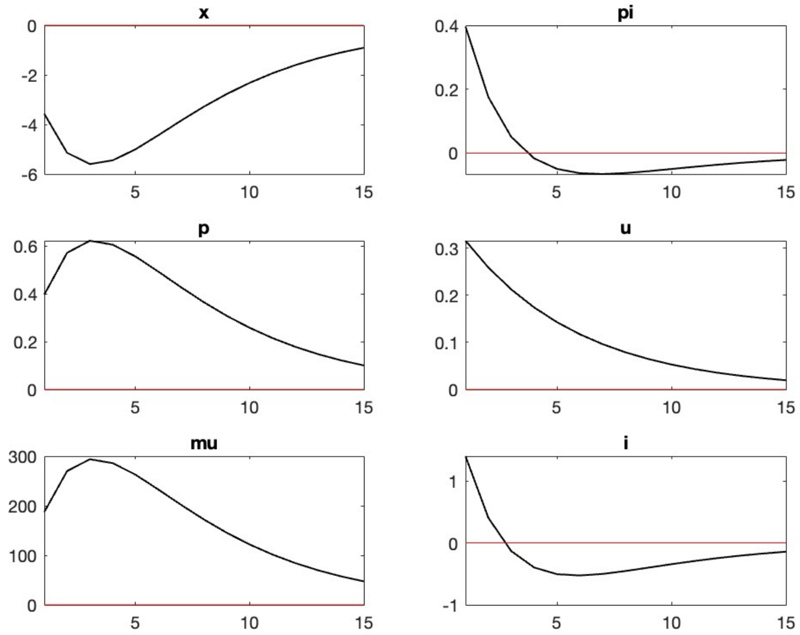

Figure 7.3: IRF from a persistent cost-push shock

From the previous IRFs we can see similar behavior for the output **x**, inflation **pi** and price level **p**, but in each case, the recovery is faster in the second model and the first impact from the shock is lower.

Another difference from Model 1 is the interest rate, in this case, it is reduced at the beginning and it recovers until exceeding the rate before the cost-push shock converging to a higher rate than the initial one. This can be interpreted as the first response from the Central Bank after a reduction in the output, followed by an increase in the interest rate after the new price level path (which stays above the original level).

Finally, we have the markup graph μ that shows a high increase following the output decrease and a slow recovery without returning to the markup before the shock. This is interesting since it is consistent with the theory presented before, where the markup is assumed to have a counter cyclical behavior. Another observation is that the markup does not return to its original rate and it stays above it; this last result is consistent with our hypothesis where the exceeded markup has contributed to the higher price level.

There are some point to be highlighted from the previous analysis:

- Markup Equation: The explicit log markup equation (Equation 6.52) reveals that an increase in the Frisch elasticity (φ) reduces the markup, while an increase in the capital share (α) reduces the markup when there is an increase in the output gap. This indicates that changes in these parameters affect the pricing behavior of firms and their ability to set prices.

- Impact of Cost-Push Shock: The cost-push shock affects the markup indirectly through its impact on output. Since the shock leads to a decrease in output, it is expected to result in an increase in the markup from firms. This aligns with the theory that a negative output shock would typically lead to firms exerting more pricing power, thereby increasing their markups.

Output

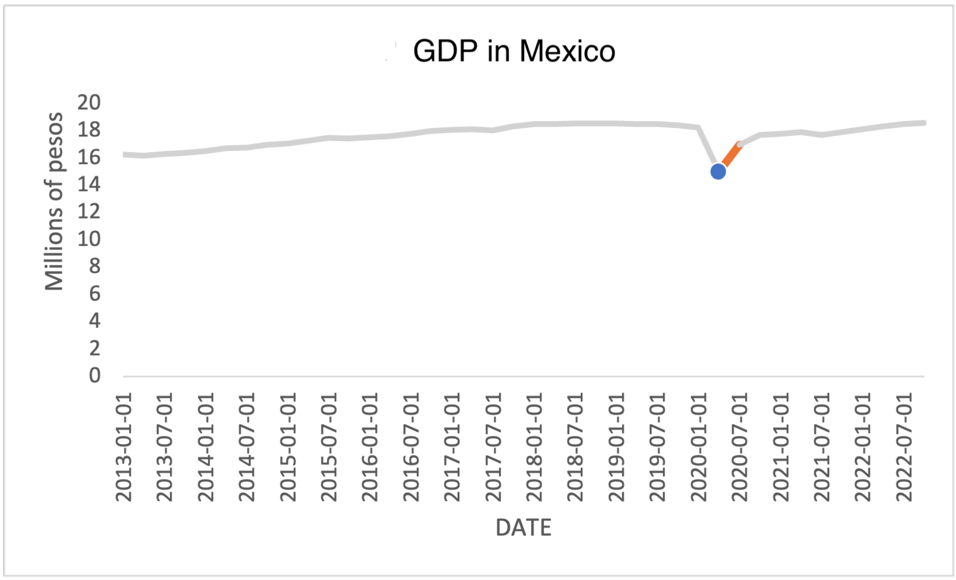

Figure 7.4: GDP in Mexico 2013-2022
Source: Sistema de información Económica from Banco de México at: *https : //www.banxico.org.mx/*

The GDP has a positive tendency in the period of time studied. We can see an outlier at date 2020-04-01, which is consistent with our analysis about the impact of the COVID-19 crisis. There is also a slow recovery from the outlier date, but without returning to the former tendency after several quarters.

Both Model 1 and Model 2 have accurately reproduced the reduction on GDP and the slow recovery without a current return to its former tendency, but Model 2 has been able to reproduced an accelerated recovery that seems to behave closer to the actual data.

Some highlights from the output behavior would be:

- Impact of COVID-19 Crisis: An outlier is observed at date 2020-04-01, which aligns with the analysis of the impact of the COVID-19 crisis. This suggests that the outlier corresponds to a significant downturn in economic activity, likely as a result of the pandemic and associated restrictions.

- Slow Recovery: Following the outlier date, there is a slow recovery in GDP. However, it is noted that the GDP does not return to its former tendency even after several quarters. This indicates a persistent impact or structural changes in the economy that have affected the growth trajectory.

- Model Replication: Both Model 1 and Model 2 have accurately reproduced the reduction in GDP and the slow recovery observed in the actual data. However, it is mentioned that Model 2 has been able to replicate an accelerated recovery that appears to be closer to the actual data. This suggests that Model 2 captures certain factors or dynamics that contribute to a faster rebound in GDP following a downturn, providing a better fit to the observed economic behavior.

Inflation

To compare the DSGE model results concerning inflation we used cumulative inflation in Mexico as a measure of inflation in this country.

Figure 7.5: Inflation in Mexico 2010-2023
Source: Sistema de información Económica from Banco de México at: *https : //www.banxico.org.mx/*

From the previous chart we can see that until 2017, inflation in Mexico was around 3%, which is consistent with Mexico's Central Bank main goal. In 2017 the country has an inflation increase mainly attributed to the depreciation of the Mexican peso as a consequence of the uncertainty rewarding the bilateral relationship with the EEUU and the international increase on energy prices and some agricultural products (BANXICO; 2017). We can see a slow recovery until the end of 2019, from where it has been a constant increase in inflation (that we attribute to the increase in raw material and energy costs) that has not returned to the 3% goal from the Central Bank until today.

In this case, Model 1 and Model 2 behave the same way with respect to inflation, showing an increase up to 4%, followed by a quick reduction below former inflation with an asymptotic stabilization below the initial level. The reason is that both models use the same NKPC and the shock impacts both the same way.

A first consideration about these is that probably the NKPC from the model is giving more weight to output than expected inflation so even if we see increases in price level, expectation about inflation need to be modified in order to better asses higher inflation expectations. One possible solution would be to use an hybrid Phillips Curve that allows to better account for past inflation when generating expectations. Even if the markup changes are not directly affecting the NKPC, these hybrid expectations with backward looking agents could help estimate higher and longer price levels and inflation due to firms expectations.

Another consideration is that the weight given to the output in the NKPC might be higher than the actual parameter. Each Model has its own weight parameter, being κ the parameter for Model 1 and $\aleph_y$ the parameter fro Model 2, were $\kappa = \lambda(\sigma + \frac{\varphi+\alpha}{1-\alpha})$ and $\aleph_y = \lambda_p(\sigma + \frac{\varphi+\alpha}{1-\alpha})$. One suggestion for a further analysis would be to estimate both parameters with bayesian methods instead of using the definitions given above to better asses the parameters for the Mexican economy.

We can see that inflation in Mexico has been fluctuating over the years and has not been consistent with the Central Bank's goal of maintaining inflation at around 3%. The inflation increase in 2017 was mainly due to the depreciation of the Mexican peso, uncertainty regarding the bilateral relationship with the United States, and an increase in energy and agricultural product prices. After a slow recovery until the end of 2019, inflation has been constantly increasing, mainly due to the rise in raw material and energy costs, and has not returned to the 3% goal set by the Central Bank.

Model 1 and Model 2 show a similar trend in inflation, with both exhibiting an increase up to 4% followed by a quick reduction below former inflation with an asymptotic stabilization below the initial level. However, one consideration is that the NKPC in the model might be giving more weight to output than expected inflation, which can lead to incorrect assessment of higher inflation expectations. Using a hybrid Phillips Curve that accounts for past inflation when generating expectations could be a possible solution. Another suggestion for further analysis would be to estimate the weight parameters for each model with Bayesian methods to better assess their suitability for the Mexican economy.

Price level

A first thing to mention is that the price level for the Mexican economy in the period evaluated has a positive trend, that is why before analyzing the short run behavior we first seasonally adjusted the series and the we detrended it with an HP-fliter with $\lambda = 1600$, to allow for a better adjustment to the real data. We took a period of 10 years to correctly estimate the cycle and the trend since it needs at least two business cycles to be calculated and the estimated length of a business cycle in Mexico is 4 years. The following graph shows the resulting series

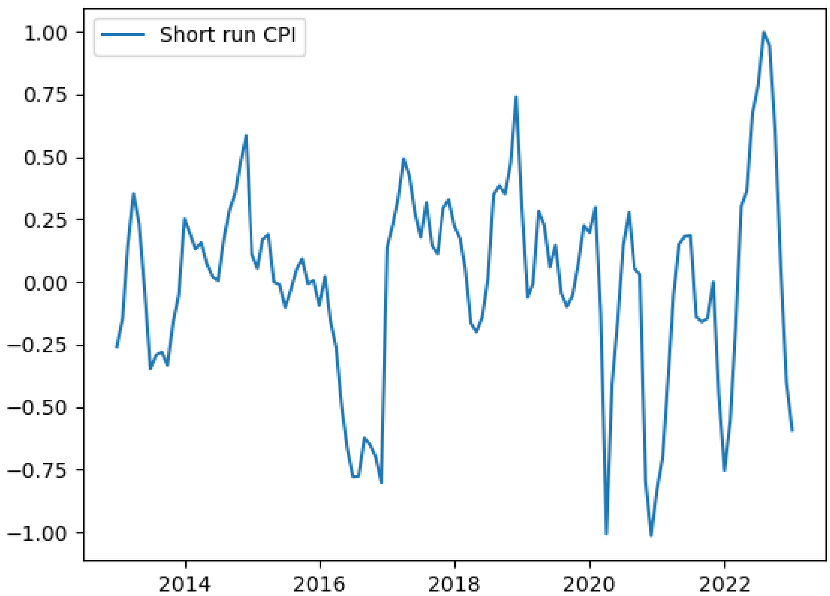

Figure 7.6: Price level variation 2013-2023
Source: Sistema de información Económica from Banco de México at: *https : //www.banxico.org.mx/*

The graph shows that there is a clear change in behavior from 2020 than continues until today. Variation seems to be higher and with higher maximums and minimums.

From the estimations made with both DSGE implemented in this investigation, we see a rapid increase in prices in both models and a slow recovery not returning to the initial price level but stabilizing at a higher price level. In this case, price increases are greater in Model 1 and more persistent than in Model 2, and prices stay at a higher level in Model 1 than in Model 2.

This first view from both models left us with some considerations about the implementation on both models. Contrary to expectations, Model 1 had greater response on prices from the cost-push shock than the model incorporating a markup equation. This could possibly by a consequence of the way the markup equation was introduced within the model, since it seems not to affect price level in the correct direction, which proposes us to modify the relationship of the markup with the rest of the equations of the model in a future study.

The analysis of the price level in the Mexican economy reveals a positive trend during the evaluated period. To better understand the short-run behavior, the series was first seasonally adjusted and then detrended using an HP-filter with a lambda value of 1600. This approach allows for a better adjustment to the real data. A 10-year period was chosen to estimate the cycle and trend, considering that at least two business cycles are needed for accurate calculation and the estimated length of a business cycle in Mexico is 4 years.

The resulting series, as depicted in the graph, shows a clear change in behavior starting from 2020, which continues until the present. The variations in the price level appear to be higher, with increased maximums and minimums.

Moving on to the estimations made with both DSGE models in this investigation, it is observed that prices rapidly increase in both models following a shock, but they recover slowly and do not return to the initial price level. Instead, they stabilize at a higher price level. Model 1 exhibits greater and more persistent price increases compared to Model 2. Furthermore, prices remain at a higher level in Model 1 compared to Model 2.

This initial analysis of both models raises some considerations regarding their implementation. Contrary to expectations, Model 1 shows a stronger response in terms of prices to the cost-push shock compared to the model incorporating a markup equation (Model 2). This unexpected outcome may be attributed to the way the markup equation was introduced within the model, as it appears to have an incorrect impact on the price level. Therefore, it is suggested that the relationship of the markup equation with the rest of the model's equations be modified in future studies to improve its alignment with the desired price dynamics.

Output Gap

For getting the output gap data for Mexico, we first got the quarterly GDP for the period 2013-2022, we calculated the potential output and finally got the output gap by subtracting the original GDP series from the estimated potential output. Since the original GDP series had an outlier observation at date 2020-04-01 (as a consequence of the COVID-19 crisis), to correctly estimate the trend and cycle for this period of time, we had to change the outlier data to avoid contamination in the estimation of both the trend and cycle.

To do this we fist changed the outlier value to the mean between the last data before the outlier and the data after the outlier observation. from here, we calculated the trend and cycle for the series with the HP-filter, setting $\lambda = 1600$. Once we got the potential output, we restored the outlier data to calculate the output gap for the entire period of time.

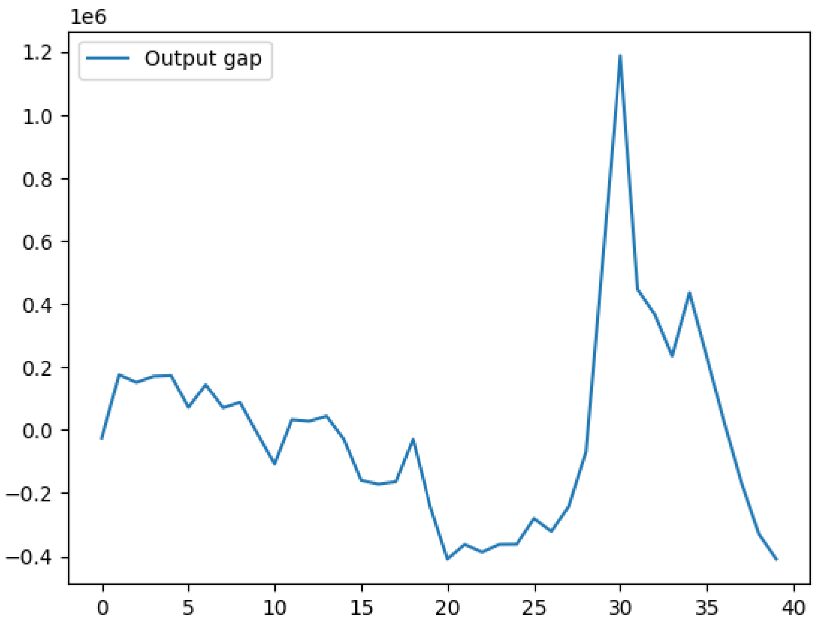

Figure 7.7: Output gap 2013-2022
Source: Sistema de información Económica from Banco de México at: $https://www.banxico.org.mx/$

We can see a big increase in the output gap as a first response to the COVID-19 crisis and the price and output changes consequence of this crisis, followed by a downward change in the trend of the series until its initial rate.

There was only measured output gap for Model 1 and it seems to correctly estimate the output gap for the Mexican economy, since it estimates a rapid increase in output gap followed by a quick return to the initial rate and stabilizing there. Since the output gap in Model 1 is given by $y_{gap} = y - y_{nat}$ (where y is production and y_{nat} is the natural level of production that is the product consistent with firms setting the desired markup), the quicker return of the output gap to its initial rate than the return from the output to its original level could be suggesting a change in the natural output in the economy, meaning that the crisis originated a lower natural output so the output gap returned faster to its former rate.

Interest rate

For the Central Bank interest rate we used the 90 days government bond interest since we are using quarterly frequency in our model and these bonds better adjust the data.

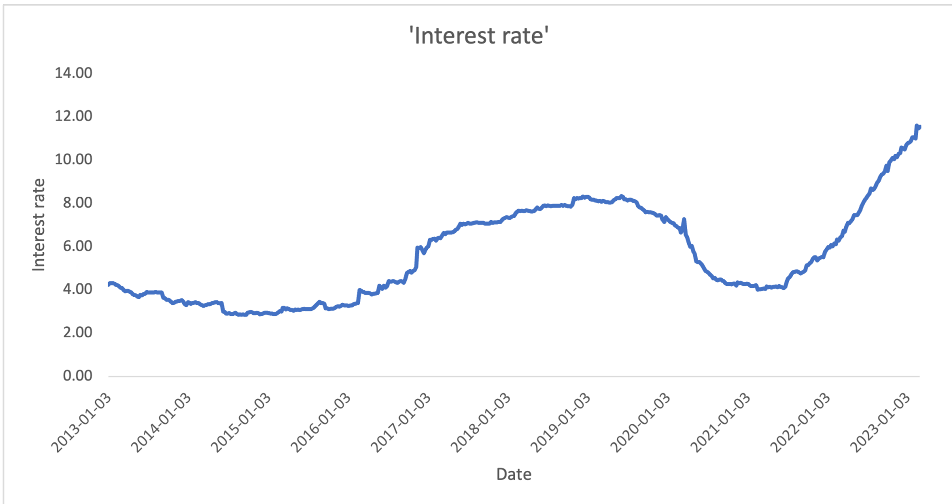

Figure 7.8: BANXICO's interest rate 2013-2023
Source: Sistema de información Económica from Banco de México at: *https : //www.banxico.org.mx/*

From the results of Model 1, we can see that at the beginning of the COVID-19 crisis we had a reduction in the interest rate due to the deceleration of the economy, followed by an increase in the interest rate in accordance with our hypothesis about the cost increases and the cost past-through from firms to consumers, leading to a considerable increase in inflation.

Interestingly, Model 2 shows an opposite behavior as the first response from the cost-push shock was an increase in the interest rate, followed by a reduction of this interest rate below the initial level and a slow asymptotic recovery to the initial level but staying below it. This has a possible explanation from the way the monetary policy is introduced in each model.

While in Model 1 we used the Ramsey policy command to introduce the monetary policy

$$i = \pi^2 + \vartheta x^2$$

for Model 2 we applied an explicit monetary policy given by:

$$i = 0.5(\sigma + \frac{\varphi + \alpha}{1 - \alpha} x^2 + \frac{\epsilon}{\lambda}\pi^2)$$

showing that the Central Bank has a reactive function with respect of the output gap and inflation in both cases but the weight associated to each variable are different in each model.

From these equations we can see inflation having a bigger impact on the Central Bank interest rate for Model 1, so, as long as inflation and price level stays high, interest rates tend to be above the initial level; inflation and the interest rate actually seems to have an inverse behavior. In Model 2 it seems to be the case that the output gap leads the interest rate and that is the fundamental reason why interest rates stay below the initial level as a strategy from the Central Bank to incentive output.

Chapter 8

Conclusions

In this analysis, we examined the effect of cost-push shocks on the Mexican economy using two different DSGE models, considering the presence of oligopolistic markets. We investigated the impulse response functions of output, price level, inflation, and interest rate to understand the short-run dynamics and the long-term effects of persistent shocks.

In the case of a transitory cost-push shock, we found that output initially decreased but returned to its initial level within 12 periods. Inflation increased due to the shock but gradually decreased and eventually returned to its pre-shock level. The price level showed a gradual increase before returning to its initial level.

When considering a persistent cost-push shock, we observed that the negative impact on output was more substantial compared to the transitory shock, and the recovery was slower. Inflation increased by more than twice the amount observed in the transitory case and took longer to return to its initial level. The price level also experienced a greater increase, followed by a slower recovery, although it did not return to its original level.

Comparing the two DSGE models, we found that both models captured the reduction in GDP and the slow recovery without a return to the pre-shock trend. However, Model 2 exhibited a faster recovery and better aligned with the actual data.

In terms of inflation, both models exhibited similar behavior, with an initial increase followed by a gradual decrease. However, the models did not fully capture the sustained increase in inflation observed in the real data, suggesting the need for further refinements, such as incorporating hybrid Phillips Curve models that account for backward-looking expectations or estimating the model parameters using Bayesian methods.

Regarding the price level, both models showed a rapid increase followed by a slow recovery, with Model 1 displaying higher and more persistent price increases compared to Model 2. This discrepancy suggests the need for adjustments in the way the markup equation is incorporated into the models to better capture the relationship between the markup and the rest of the model equations.

For the output gap, Model 1 accurately estimated the rapid increase and subsequent return to the initial rate, indicating a potential change in the natural output level caused by the crisis. However, no output gap data was available for Model 2.

Analyzing the Central Bank interest rate, Model 1 demonstrated a response consistent with expectations, with a reduction in the interest rate followed by an increase in line with the cost increases and inflationary pressures. In contrast, Model 2 exhibited an initial increase in the interest rate followed by a reduction below the initial level and a gradual recovery. These differences can be attributed to the differing weight assigned to inflation and the output gap in the monetary policy rules of each model.

The DSGE model calibrated for the Mexican economy showed that a cost-push shock is able to duplicate recent behavior for key variables in the Mexican economy, such as the output level and recent inflation. Particularly, a persistent cost-push shock better fits the data and it is consistent with recent events in the country.

These results are consistent with the theoretical framework where we suggested that recent inflation in Mexico came from a cost-push shock due to price increases in raw materials and the counter cyclical behavior of markups coming from an economy with imperfect markets where firms have some market power to set prices.

A first suggestion for further investigation is to define a model able to separate the effect from a cost-push shock on inflation from the effect on the markup increase on inflation, since the second phenomena seems to be a consequence from the cost-push shock but it also has an independent effect on inflation that would be interesting to study for the case of an economy with market power, as the Mexican one.

Another thing to take into account for future research is the behavior of inflation, which turned out to have a lower response and less persistence than in the data for Mexico, so it is suggested to review the NKPC to modify the weights of its components (using baesian methods to estimate the parameters), or even its structure, adding a component with lags from previous inflation (as is the case of the hybrid NKPC).

As a final suggestion, it would be pertinent to introduce the exchange rate and intermediate good prices to the NKPC to allow for a richer study on the roots of inflation in Mexico, since it is an open economy that seems to be directly affected by external conditions (both reflected in the exchange rate and in the intermediate good prices).

Overall, the analysis provides insights into the short-run dynamics and long-term effects of cost-push shocks in the Mexican economy, showing the importance of accurately capturing inflation dynamics and the interaction between different variables in DSGE models. Further research and refinements are necessary to improve the models' ability to replicate real-world outcomes and enhance their policy implications.

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