UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO

FACULTAD DE INGENIERÍA

Simulación numérica del flujo de fluidos bifásico mediante el modelo de aceite negro modificado para yacimientos naturalmente fracturados

TESIS

Que para obtener el título de

Ingeniero Petrolero

P R E S E N T A

José Armando Torres López

DIRECTOR DE TESIS

Dr. Víctor Leonardo Teja Juárez

Ciudad Universitaria, Cd. Mx., 2022

Agradecimientos

Agradezco a la vida, por brindarme la oportunidad de existir, por vivir tantas experiencias, por todos esos buenos y malos momentos, y por todo el camino que aún me falta por recorrer, agradezco por estar vivo en este momento, muchas gracias.

Agradezco a mis padres, a mi madre, por haber sido el principal apoyo durante todo el tiempo de vida que llevo, y que desde mucho tiempo atrás ha creído en mí, no hubiera llegado hasta este punto sin su apoyo, muchas gracias.

Agradezco a mis hermanos, por haber sido la principal motivación para continuar con mis estudios, muchas gracias.

Agradezco a la familia Sánchez, por haber sido tan amables conmigo, por su amistad y apoyo brindado, por ser tan buenas personas y por haber creído en mí, muchas gracias.

Agradezco a mis amigos más cercanos y de la carrera, por haberme brindado su amistad, por compartir conmigo esta parte importante de mi vida y que estoy seguro, por muchos años nuestra amistad seguirá existiendo, muchas gracias.

Agradezco al Dr. Víctor Leonardo Teja Juárez y al Dr. Bruno Armando López Jiménez, por haberme apoyado e interesarse en el desarrollo de este trabajo, por su tiempo, su asesoramiento, consejos y comentarios, realmente agradezco mucho su atención, interés y apoyo, de verdad son muy buenas personas, muchas gracias.

Agradezco a mis sinodales, por sus observaciones y comentarios para este trabajo, muchas gracias.

Agradezco a la Universidad Nacional Autónoma de México y a la Facultad de Ingeniería, por haberme brindado la oportunidad de cumplir este pequeño sueño, por todas las experiencias vividas y por todas las amistades que realicé, muchas gracias.

Agradezco al proyecto PAPIIT: IA106820 por facilitar los recursos computacionales para la realización de este trabajo, muchas gracias.

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Resumen

El siguiente trabajo presenta el modelo matemático y numérico para simular el flujo de fluidos en un medio poroso naturalmente fracturado, a través del modelo de aceite negro modificado (Galindo Nava, 1998). Siendo así una simulación de tipo pseudocomposicional, sugerida para yacimientos que presentan variación en su composición durante su vida productiva, como pueden ser los yacimientos de tipo gas y condensado, considerando que las fases de aceite y gas en el yacimiento están formadas por dos pseudocomponentes de hidrocarburo (aceite y gas) (Spivak & Dixon, 1973) y (Coats K. H., 1985).

Así mismo, se presenta un caso de estudio, el cual, emplea el modelo de aceite negro modificado para un yacimiento naturalmente fracturado, bifásico (aceite y gas), tridimensional, con efectos de fracturamiento hidráulico, y que contempla un pozo horizontal a lo largo de la dirección "x", desarrollado y codificado originalmente por (López Jiménez, 2017) en el lenguaje de programación Fortran 90, y que en este trabajo se decidió migrar al lenguaje de programación Python 3.8.

Para la representación de las fracturas hidráulicas se emplea la metodología "DK-LS-LGR" (es decir, doble permeabilidad, espaciamiento logarítmico y refinamiento local), donde el volumen estimulado del yacimiento (SRV por sus siglas en inglés) es representado por las celdas más refinadas del refinamiento local.

Respecto al modelado del pozo horizontal, se empleó el modelo corregido por (Peaceman D. , 1978) de un pozo vertical, el cual rotando los ejes del modelo se puede llegar a la representación de un pozo horizontal, con la finalidad de no aumentar la complejidad de la solución.

Adicionalmente, se comparan los resultados y rendimiento del simulador original realizado en el lenguaje de programación Fortran 90 por (López Jiménez, 2017), versus el mismo código migrado al lenguaje Python 3.8 e implementado la librería Numba para Python (Numba, 2022) como método de optimización.

Finalmente se obtiene el comportamiento de presiones y saturaciones del yacimiento para un periodo de 10 años de simulación, esto es a través del modelo 2D y 3D como una de las ventajas de haber migrado el código original, y con ayuda de la librería de Matplotlib para Python (Matplotlib, 2022).

1. Introducción

En un proyecto de recuperación de hidrocarburos, generalmente se involucran riesgos asociados con la estrategia de desarrollo y producción, los cuales pueden incluir factores tan importantes como la complejidad del yacimiento y los fluidos que lo conforman, la complejidad de los mecanismos de recuperación y la aplicabilidad de métodos predictivos (Chen, 2007). Es así que la simulación de yacimientos se ha convertido en una herramienta predictiva estándar en la industria petrolera, ya que estos riesgos pueden tomarse en cuenta en la simulación de yacimientos a través de datos de entrada confiables para el modelo, y a través de una buena práctica de simulación, con la que se pueden obtener predicciones de rendimiento precisas para un yacimiento de hidrocarburos en diferentes condiciones de operación.

Es así que la aplicación de la simulación de yacimientos se ha ido extendido para el trabajo de yacimientos naturalmente fracturados, viendo a principios de los años sesenta los primeros modelos desarrollados para simular formaciones fracturadas (Barenblatt, Zheltov, & Kochina, 1960), (Warren & Root, 1963), (Kazemi, 1969), (Pruess & Narasimhan, 1985), (Wu & Pruess, 1988), (Kazemi, Gilman, & Eisharkawy, 1992).

Sin embargo, el modelado del flujo de fluidos multifásico en medios porosos fracturados sigue siendo un desafío conceptual y matemático, con una amplia línea de investigación, con lo que en las últimas décadas, los modelos matemáticos desarrollados para el modelamiento de este tipo de yacimientos se basan, en general, en enfoques continuos e implican el desarrollo de modelos conceptuales de fractura, incorporando la información geométrica de un sistema dado de matriz-fractura, estableciendo ecuaciones de conservación de masa y energía para dominios de matriz-fractura y posteriormente resolver ecuaciones algebraicas no lineales discretas, que acoplan el flujo de fluido multifásico con otros procesos físicos que se resuelven numéricamente (Wu, 2016).

Al modelar el flujo multifásico en medios porosos fracturados, el tema más importante a tratar es en relación a cómo manejar el flujo interporoso en las conexiones matriz-fractura bajo flujo multifásico y condiciones isotérmicas o no isotérmicas (Wu, 2016). Los métodos matemáticos comúnmente utilizados para tratar esta interacción matriz-fractura incluyen:

a) Modelo explícito de fractura discreta: Este enfoque describe el flujo explícitamente a través de cada fractura con interacción matriz-fractura, sin embargo, requiere de un conocimiento detallado de las propiedades geométricas de la matriz y la fractura y sus distribuciones espaciales, que rara vez se conocen en un espacio determinado del yacimiento para las fracturas naturales. Aun así, el modelo de fractura discreta encontró una amplia aplicación para el manejo de fracturas hidráulicas en combinación con enfoques de simple o doble porosidad en yacimientos no convencionales (Wu, 2016).

b) Modelo de doble porosidad y doble permeabilidad: En este modelo el medio de flujo de un yacimiento naturalmente fracturado se compone de bloques de matriz con baja permeabilidad, incrustados en una red de fracturas interconectadas, donde el flujo global y el transporte en la formación ocurren solo a través del sistema de fracturas. Este modelo trata los bloques de matriz como fuentes o sumideros distribuidos espacialmente para el sistema de fracturas, donde la interacción matriz-fractura generalmente se aproxima usando una condición de flujo de estado pseudoestacionario, siendo así el enfoque principal para modelar el flujo de fluidos a través de yacimientos fracturados (Wu, 2016).

c) Modelo MINC (Interacción Continua Múltiple): El método MINC toma en cuenta los gradientes de presiones o fuerzas capilares, temperaturas y concentraciones al subdividir aún más los bloques de matriz individuales cerca de las fracturas y dentro de la matriz. El concepto MINC trata el flujo interporoso de una manera completamente transitoria al calcular los gradientes, que impulsan el flujo de interporosidad desde la interfaz matriz-fractura hacia o desde el bloque de matriz (Wu, 2016).

d) Modelo de triple porosidad: En yacimientos naturalmente fracturados puede existir una heterogeneidad significativa en las fracturas o dentro de la matriz, es por ello que, para investigar el efecto de estas heterogeneidades en las fracturas o en la matriz sobre el flujo a través de medios porosos fracturados, se ha extendido en la literatura el concepto de doble porosidad y doble permeabilidad para el desarrollo de modelos de triple porosidad para el efecto de matriz heterogénea. En general, estos modelos se han centrado en el manejo de diferentes niveles de heterogeneidad de las fracturas y matriz, por ejemplo, subdividiendo la matriz o fracturas en dos o más subdominios con diferentes propiedades para flujo monofásico y multifásico en yacimientos naturalmente fracturados (Wu, 2016).

e) Modelo ECM (Continuo Efectivo): Este modelo representa las fracturas y matriz como un único medio continuo, donde, si la permeabilidad entre la red de fracturas y la matriz no es muy grande o si se trata de un yacimiento muy fracturado con pequeños bloques de matriz, se espera que el modelo ECM proporcione una buena aproximación (Wu, 2016).

Considerando los modelos anteriormente mencionados, se decidió emplear el modelo de **doble porosidad y doble permeabilidad** para llevar a cabo la simulación del comportamiento del flujo de fluidos a través de un medio poroso fracturado, sin embargo, considerando que el flujo multifásico en este tipo de medios resulta bastante difícil de simular, sobre todo, en yacimientos en donde la composición varía de manera significativa durante su vida productiva, como pueden ser los yacimientos de tipo gas y condensado, el emplear un tratamiento de tipo composicional resulta bastante difícil y sobre todo costoso al consumir bastante memoria de cómputo y tiempo de simulación, se decidió emplear el modelo de aceite negro modificado en medios fracturados, presentado por (Galindo Nava, 1998)**,** siendo así una simulación de tipo pseudocomposicional, y que considera las fases de aceite y gas compuestas por dos pseudocomponentes de hidrocarburo (aceite y gas), la cual a su vez, es una extensión del modelo de aceite negro modificado (o beta modificado) presentado por (Spivak & Dixon, 1973) y (Coats K. H., 1985).

(López Jiménez, 2017) realizó un caso de estudio con un modelo de simulación que emplea esta metodología de aceite negro modificado, sin embargo, en su trabajo describe un modelo de quíntuple porosidad que considera fracturas hidráulicas, fracturas naturales, una porosidad orgánica en la matriz, una porosidad inorgánica en la matriz y una porosidad adsorbida en la matriz, para el tratamiento de yacimientos de tipo "Shale", presentado un caso específico en el que únicamente considera el modelo de doble porosidad y doble permeabilidad, efectos de fracturas hidráulicas, un sistema gas-aceite y con un pozo horizontal.

Es precisamente este caso específico el que se decidió utilizar como caso de estudio para este trabajo, ya que, con estas características se empela el modelo de aceite negro modificado para yacimientos naturalmente fracturados, sin embargo, el simulador desarrollado para este modelo se codificó originalmente en el lenguaje de programación Fortran 90, entonces, considerando tres rankings de comparación de los lenguajes de programación más demandados en el año 2021 (DistantJob, 2022) se decidió migrar al lenguaje de programación Python, específicamente la versión 3.8, ya que Python al ser un lenguaje de tipo interpretado (el programa se traduce mientras se ejecuta), multiparadigma (permite varios estilos de programación, como orientada a objetos, imperativa o funcional) y multiplataforma (puede ejecutarse en Windows, Linux o MacOS), además de considerar la amplia cantidad de librerías de código abierto que hay a disposición, se consideró como una buena herramienta para el desarrollo del trabajo presentado en este escrito.

Ranking	Primer lugar	Segundo lugar	Tercer lugar	Cuarto lugar	Quinto lugar
RedMonk	JavaScript	Python	Java	PHP	C++/C#
PYPL	Python	Java	JavaScript	C#	C/C++
TIOBE	C	Python	Java	C++	C#

Fig. 1. "Principales lenguajes de programación en el año 2021". (DistantJob, 2022).

Dicho esto, entonces los objetivos de este escrito son:

Objetivo General

Desarrollar una herramienta computacional portable y sencilla mediante un lenguaje de alto nivel aplicando la metodología de diferencias finitas y una discretización completamente implícita, con el objetivo de reproducir y predecir el comportamiento del flujo de fluidos bifásico mediante el modelo de aceite negro modificado para yacimientos naturalmente fracturados.

Objetivos Particulares

a) Implementar el modelo de aceite negro modificado (o beta modificado) para el tratamiento de yacimientos de tipo gas y condensado o aceite volátil (Galindo Nava, 1998), en la simulación del comportamiento del flujo de fluidos de yacimientos naturalmente fracturados.

b) Modelar un caso de estudio correspondiente a un modelo numérico tridimensional, isotérmico, bifásico (aceite y gas), naturalmente fracturado, y con efectos de fracturas hidráulicas, para un yacimiento de tipo gas y condensado, y que contempla un pozo horizontal a lo largo de la dirección "x", presentado por (López Jiménez, 2017).

c) Llevar a cabo la migración del simulador original del caso de estudio, codificado en el lenguaje de programación Fortran 90, al lenguaje Python 3.8, utilizando la librería de Numba de Python (Numba, 2022) como método de optimización e implementando diferentes métodos de solución del sistema de ecuaciones lineales para el modelo migrado, y así llevar a cabo comparativos de resultados y rendimiento entre ambos simuladores.

d) Visualizar el comportamiento de presiones y saturaciones del caso de estudio a través de sus modelos 2D y 3D, como resultado de la migración del simulador original al lenguaje Python 3.8 y con ayuda de la librería Matplotlib para Python (Matplotlib, 2022).

Respecto a la estructura del presente trabajo, a continuación, se mencionan los capítulos que lo conforman:

i) En el Capítulo 2 se presenta la formulación matemática del modelo de aceite negro modificado para medios fracturados.

ii) En el Capítulo 3 se lleva a cabo la discretización del modelo matemático planteado, para llevarlo a un problema de tipo numérico.

iii) En el Capítulo 4 se presenta la implementación del modelo discretizado aplicado a un caso de estudio y la migración de este, del lenguaje de programación Fortran 90 al lenguaje Python 3.8.

iv) En el Capítulo 5 se presentan los resultados de la migración del caso de estudio a través de un comparativo de resultados y rendimiento entre el simulador migrado y el original.

v) En el Capítulo 6 se muestran las conclusiones y recomendaciones del presente trabajo.

vi) Finalmente, en la sección de Anexo se presenta la formulación de las derivadas necesarias para el modelo presentado.

2. Formulación matemática del problema

En este capítulo se presenta el desarrollo de las ecuaciones que describen el comportamiento del flujo de fluidos en un medio poroso naturalmente fracturado a través del modelo de aceite negro modificado (o beta modificado), tomando como referencia la metodología empleada por (Galindo Nava, 1998), así mismo, se describen las condiciones iniciales y de frontera del modelo físico planteado, y de esta manera se lleva a cabo la formulación matemática que logra describir el comportamiento del flujo de fluidos en un medio poroso naturalmente fracturado, y que posteriormente se discretizará, para pasar de un problema matemático a un problema numérico computacional.

2.1. Modelo de aceite negro modificado

El estudio del comportamiento del flujo de fluidos en yacimientos en los que su composición varía de manera importante durante su vida y desarrollo, como es el caso de yacimientos de gas y condensado o aceite volátil, requiere de un tratamiento o simulación composicional, en el que se requiere de un gran número de ecuaciones e incógnitas, que crecen con el número de componentes a considerar (Rodríguez de la Garza & Galindo Nava, 2000).

Sin embargo, se ha demostrado en la literatura (Spivak & Dixon, 1973) y (Coats K. H., 1985), que el comportamiento de este tipo de yacimientos puede describirse a través de dos pseudocomponentes hidrocarburos y del uso de factores de volumen y relaciones de solubilidad, que dependen solo de la presión de las fases y suponiendo un equilibrio termodinámico de estas en todo momento (Galindo Nava, 1998) y (Rodríguez de la Garza & Galindo Nava, 2000), donde estos pseudocomponentes de aceite y gas únicamente se encuentran presentes en las fases de aceite y gas, es decir, solo existe intercambio másico entre el gas y el aceite, como se puede observar en la Fig. 2.

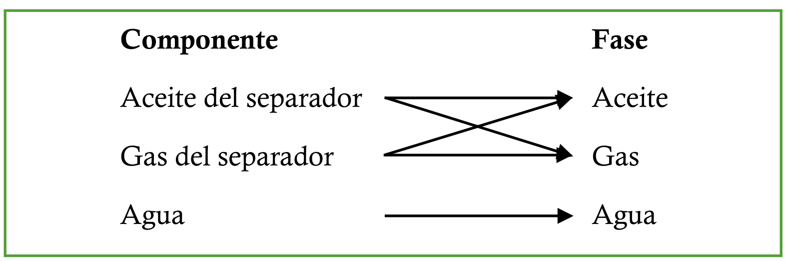

Fig. 2. "Distribución de los componentes de las fases". (Rodríguez de la Garza & Galindo Nava, 2000).

Estas ecuaciones que describen el comportamiento del flujo de fluidos a través de un medio poroso (ecuación de difusión) se determinan a través de la combinación de tres ecuaciones auxiliares:

1) Una ecuación de continuidad (o ecuación de conservación de masa).
2) Una ecuación de movimiento (generalmente la ley de Darcy).
3) Una ecuación de estado.

2.2. Desarrollo de las ecuaciones de flujo

Si las ecuaciones de continuidad para las tres fases en la fractura natural (Galindo Nava, 1998) son:

Para la fase de aceite:

$$-\frac{\partial}{\partial x}\left(C_{oo}\rho_o V_{o_x}+C_{og}\rho_g V_{g_x}+C_{ow}\rho_w V_{w_x}\right)_f-\frac{\partial}{\partial y}\left(C_{oo}\rho_o V_{o_y}+C_{og}\rho_g V_{g_y}+C_{ow}\rho_w V_{w_y}\right)_f-\frac{\partial}{\partial z}\left(C_{oo}\rho_o V_{o_z}+C_{og}\rho_g V_{g_z}+C_{ow}\rho_w V_{w_z}\right)_f$$
$$+C_{oo}\rho_o q_o^*+C_{og}\rho_g q_g^*+C_{ow}\rho_w q_w^*$$
$$+C_{oo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+C_{og}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)+C_{ow}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{oo}\rho_o S_{at\,o}+C_{og}\rho_g S_{at\,g}+C_{ow}\rho_w S_{at\,w}\right)\right)_f$$

Para la fase de gas:

$$-\frac{\partial}{\partial x}\left(C_{go}\rho_o V_{o_x}+C_{gg}\rho_g V_{g_x}+C_{gw}\rho_w V_{w_x}\right)_f-\frac{\partial}{\partial y}\left(C_{go}\rho_o V_{o_y}+C_{gg}\rho_g V_{g_y}+C_{gw}\rho_w V_{w_y}\right)_f-\frac{\partial}{\partial z}\left(C_{go}\rho_o V_{o_z}+C_{gg}\rho_g V_{g_z}+C_{gw}\rho_w V_{w_z}\right)_f$$
$$+C_{go}\rho_o q_o^*+C_{gg}\rho_g q_g^*+C_{gw}\rho_w q_w^*$$
$$+C_{go}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+C_{gg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)+C_{gw}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{go}\rho_o S_{at\,o}+C_{gg}\rho_g S_{at\,g}+C_{gw}\rho_w S_{at\,w}\right)\right)_f$$

Para la fase de agua:

$$-\frac{\partial}{\partial x}\left(C_{wo}\rho_o V_{o_x}+C_{wg}\rho_g V_{g_x}+C_{ww}\rho_w V_{w_x}\right)_f-\frac{\partial}{\partial y}\left(C_{wo}\rho_o V_{o_y}+C_{wg}\rho_g V_{g_y}+C_{ww}\rho_w V_{w_y}\right)_f-\frac{\partial}{\partial z}\left(C_{wo}\rho_o V_{o_z}+C_{wg}\rho_g V_{g_z}+C_{ww}\rho_w V_{w_z}\right)_f$$
$$+C_{wo}\rho_o q_o^*+C_{wg}\rho_g q_g^*+C_{ww}\rho_w q_w^*$$
$$+C_{wo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+C_{wg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)+C_{ww}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{wo}\rho_o S_{at\,o}+C_{wg}\rho_g S_{at\,g}+C_{ww}\rho_w S_{at\,w}\right)\right)_f$$

Si las ecuaciones de continuidad para las tres fases en la matriz son:

Para la fase de aceite:

$$-C_{oo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)-C_{og}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)-C_{ow}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{oo}\rho_o S_{at\,o}+C_{og}\rho_g S_{at\,g}+C_{ow}\rho_w S_{at\,w}\right)\right)_m$$

Para la fase de gas:

$$-C_{go}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)-C_{gg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)-C_{gw}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{go}\rho_o S_{at\,o}+C_{gg}\rho_g S_{at\,g}+C_{gw}\rho_w S_{at\,w}\right)\right)_f$$

Para la fase de agua:

$$-C_{wo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)-C_{wg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)-C_{ww}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right)$$
$$=\frac{\partial}{\partial t}\left(\phi\left(C_{wo}\rho_o S_{at\,o}+C_{wg}\rho_g S_{at\,g}+C_{ww}\rho_w S_{at\,w}\right)\right)_f$$

Con la ley de Darcy como ecuación de movimiento para las tres fases y considerando un sistema de coordenadas cartesiano:

$$V_{o_x} = -\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} \quad , \quad V_{g_x} = -\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x} \quad , \quad V_{w_x} = -\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}$$

$$V_{o_y} = -\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} \quad , \quad V_{g_y} = -\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y} \quad , \quad V_{w_y} = -\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}$$

$$V_{o_z} = -\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right) \quad , \quad V_{g_z} = -\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right) \quad , \quad V_{w_z} = -\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w\frac{\partial D}{\partial z}\right)$$

Se puede sustituir la ley de Darcy en las ecuaciones de continuidad, entonces:

En la fractura natural:

a) Para la fase de aceite:

$$
\begin{aligned}
&-\frac{\partial}{\partial x}\left(C_{oo}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right) + C_{og}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right) + C_{ow}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\
&-\frac{\partial}{\partial y}\left(C_{oo}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right) + C_{og}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right) + C_{ow}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f \\
&-\frac{\partial}{\partial z}\left(C_{oo}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right)\right) + C_{og}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right)\right) + C_{ow}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w\frac{\partial D}{\partial z}\right)\right)\right)_f \\
&+ C_{oo}\rho_o q_o^* + C_{og}\rho_g q_g^* + C_{ow}\rho_w q_w^* \\
&+ C_{oo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) + C_{og}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) + C_{ow}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right) \\
&= \frac{\partial}{\partial t}\left(\phi\left(C_{oo}\rho_o S_{at\,o} + C_{og}\rho_g S_{at\,g} + C_{ow}\rho_w S_{at\,w}\right)\right)_f
\end{aligned}
$$

b) Para la fase de gas:

$$
\begin{aligned}
&-\frac{\partial}{\partial x}\left(C_{go}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right) + C_{gg}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right) + C_{gw}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\
&-\frac{\partial}{\partial y}\left(C_{go}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right) + C_{gg}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right) + C_{gw}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f \\
&-\frac{\partial}{\partial z}\left(C_{go}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right)\right) + C_{gg}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right)\right) + C_{gw}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w\frac{\partial D}{\partial z}\right)\right)\right)_f \\
&+ C_{go}\rho_o q_o^* + C_{gg}\rho_g q_g^* + C_{gw}\rho_w q_w^* \\
&+ C_{go}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) + C_{gg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) + C_{gw}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right) \\
&= \frac{\partial}{\partial t}\left(\phi\left(C_{go}\rho_o S_{at\,o} + C_{gg}\rho_g S_{at\,g} + C_{gw}\rho_w S_{at\,w}\right)\right)_f
\end{aligned}
$$

c) Para la fase de agua:

$$
\begin{aligned}
&-\frac{\partial}{\partial x}\left(C_{wo}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right) + C_{wg}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right) + C_{ww}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\
&-\frac{\partial}{\partial y}\left(C_{wo}\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right) + C_{wg}\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right) + C_{ww}\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f
\end{aligned}
$$

$$- \frac{\partial}{\partial z}\left(C_{wo}\rho_o \left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o \frac{\partial D}{\partial z}\right)\right) + C_{wg}\rho_g \left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g \frac{\partial D}{\partial z}\right)\right) + C_{ww}\rho_w \left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w \frac{\partial D}{\partial z}\right)\right)\right)_f$$

$$+ C_{wo}\rho_o q_o^* + C_{wg}\rho_g q_g^* + C_{ww}\rho_w q_w^*$$

$$+ C_{wo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) + C_{wg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) + C_{ww}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right)$$

$$= \frac{\partial}{\partial t}\left(\emptyset\left(C_{wo}\rho_o S_{at\,o} + C_{wg}\rho_g S_{at\,g} + C_{ww}\rho_w S_{at\,w}\right)\right)_f$$

En la matriz:

a) Para la fase de aceite:

$$- C_{oo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) - C_{og}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) - C_{ow}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right)$$
$$= \frac{\partial}{\partial t}\left(\emptyset\left(C_{oo}\rho_o S_{at\,o} + C_{og}\rho_g S_{at\,g} + C_{ow}\rho_w S_{at\,w}\right)\right)_m$$

b) Para la fase de gas:

$$- C_{go}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) - C_{gg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) - C_{gw}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right)$$
$$= \frac{\partial}{\partial t}\left(\emptyset\left(C_{go}\rho_o S_{at\,o} + C_{gg}\rho_g S_{at\,g} + C_{gw}\rho_w S_{at\,w}\right)\right)_f$$

c) Para la fase de agua:

$$- C_{wo}\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) - C_{wg}\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) - C_{ww}\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right)$$
$$= \frac{\partial}{\partial t}\left(\emptyset\left(C_{wo}\rho_o S_{at\,o} + C_{wg}\rho_g S_{at\,g} + C_{ww}\rho_w S_{at\,w}\right)\right)_f$$

Ahora, considerando un sistema isotérmico, la ecuación de estado para cada fase se determina a partir de la fracción másica de los pseudocomponentes presentes en cada fase (Galindo Nava, 1998):

Para la fase de aceite:

a) Fracción másica de los pseudocomponentes de aceite presentes en la fase de aceite:

$$C_{oo} = \frac{m_o}{m_o + m_g}$$

Con la definición del factor de volumen de aceite:

$$B_o = \frac{Vol_o\ c.y.}{Vol_o\ c.s.}$$

Con la definición de densidad:

$$\rho = \frac{m}{Vol} \qquad Vol = \frac{m}{\rho}$$

Entonces:

$$B_o = \frac{Vol_o\ c.y.}{Vol_o\ c.s.} = \left(\frac{\frac{m_o + m_g}{\rho_o\ c.y.}}{\frac{m_o}{\rho_o\ c.s.}}\right) = \left(\frac{m_o + m_g}{m_o}\right)\left(\frac{\rho_o\ c.s.}{\rho_o\ c.y.}\right) = \left(\frac{1}{C_{oo}}\right)\left(\frac{\rho_o\ c.s.}{\rho_o\ c.y.}\right)$$

Despejando:

$$C_{oo} = \frac{\rho_o\ c.s.}{B_o\ \rho_o\ c.y.}$$

b) Fracción másica de los pseudocomponentes de aceite presentes en la fase de gas:

$$C_{og} = \frac{m_o}{m_g}$$

Con la definición del factor de volumen de gas y relación de solubilidad de la fase de aceite en la fase de gas:

$$B_g = \frac{Vol_g\ c.y.}{Vol_g\ c.s.} \qquad , \qquad r_s = \frac{Vol_o\ c.s.}{Vol_g\ c.s.}$$

Sustituyendo los términos de volumen con la definición de densidad:

$$B_g = \frac{Vol_g\ c.y.}{Vol_g\ c.s.} = \left(\frac{\frac{m_g}{\rho_g\ c.y.}}{\frac{m_g}{\rho_g\ c.s.}}\right) = \left(\frac{m_g}{m_g}\right)\left(\frac{\rho_g\ c.s.}{\rho_g\ c.y.}\right) = \frac{\rho_g\ c.s.}{\rho_g\ c.y.} \qquad , \qquad r_s = \frac{Vol_o\ c.s.}{Vol_g\ c.s.} = \left(\frac{\frac{m_o}{\rho_o\ c.s.}}{\frac{m_g}{\rho_g\ c.s.}}\right) = \left(\frac{m_o}{m_g}\right)\left(\frac{\rho_g\ c.s.}{\rho_o\ c.s.}\right)$$

Entonces:

$$\rho_g\ c.s. = B_g\ \rho_g\ c.y. \qquad , \qquad \frac{m_o}{m_g} = r_s\left(\frac{\rho_o\ c.s.}{\rho_g\ c.s.}\right)$$

Sustituyendo términos:

$$C_{og} = \frac{m_o}{m_g} = r_s\left(\frac{\rho_o\ c.s.}{\rho_g\ c.s.}\right) = r_s\left(\frac{\rho_o\ c.s.}{B_g\ \rho_g\ c.y.}\right) = \frac{\rho_o\ c.s.}{B_g\ \rho_g\ c.y.} r_s \qquad \boxed{C_{og} = \frac{\rho_o\ c.s.}{B_g\ \rho_g\ c.y.} r_s}$$

c) Fracción másica de los pseudocomponentes de aceite presentes en la fase de agua:

$$\boxed{C_{ow} = 0}$$

Esto es debido a que únicamente existe intercambio másico entre las fases de aceite y gas, como se muestra en la Fig. 2**.**

<u>Para la fase de gas:</u>

a) Fracción másica de los pseudocomponentes de gas presentes en la fase de aceite:

$$C_{go} = \frac{m_g}{m_o + m_g}$$

Con la definición del factor de volumen de aceite y relación de solubilidad de la fase de gas en la fase de aceite:

$$B_o = \frac{Vol_o\ c.y.}{Vol_o\ c.s.} \qquad , \qquad R_s = \frac{Vol_g\ c.s.}{Vol_o\ c.s.}$$

Sustituyendo los términos de volumen con la definición de densidad:

$$B_o = \frac{Vol_o\ c.y.}{Vol_o\ c.s.} = \left(\frac{\frac{m_o + m_g}{\rho_o\ c.y.}}{\frac{m_o}{\rho_o\ c.s.}}\right) = \left(\frac{m_o + m_g}{m_o}\right)\left(\frac{\rho_o\ c.s.}{\rho_o\ c.y.}\right) \qquad , \qquad R_s = \frac{Vol_g\ c.s.}{Vol_o\ c.s.} = \left(\frac{\frac{m_g}{\rho_g\ c.s.}}{\frac{m_o}{\rho_o\ c.s.}}\right) = \left(\frac{m_g}{m_o}\right)\left(\frac{\rho_o\ c.s.}{\rho_g\ c.s.}\right)$$

Entonces:

$$\frac{1}{B_o} = \left(\frac{m_o}{m_o + m_g}\right)\left(\frac{\rho_o\ c.y.}{\rho_o\ c.s.}\right) \qquad , \qquad R_s = \left(\frac{m_g}{m_o}\right)\left(\frac{\rho_o\ c.s.}{\rho_g\ c.s.}\right)$$

Si multiplicamos:

$$\frac{1}{B_o} R_s = \left(\frac{m_o}{m_o + m_g}\right)\left(\frac{\rho_o\ c.y.}{\rho_o\ c.s.}\right)\left(\frac{m_g}{m_o}\right)\left(\frac{\rho_o\ c.s.}{\rho_g\ c.s.}\right) = \left(\frac{m_g}{m_o + m_g}\right)\left(\frac{\rho_o\ c.y.}{\rho_g\ c.s.}\right)$$

Sustituyendo:

$$\frac{1}{B_o} R_s = \left(\frac{m_g}{m_o + m_g}\right)\left(\frac{\rho_o\ c.y.}{\rho_g\ c.s.}\right) = C_{go}\left(\frac{\rho_o\ c.y.}{\rho_g\ c.s.}\right) \qquad \boxed{C_{go} = \frac{\rho_g\ c.s.}{B_o\ \rho_o\ c.y.} R_s}$$

b) Fracción másica de los pseudocomponentes de gas presentes en la fase de gas:

$$C_{gg} = \frac{m_g}{m_g}$$

Con la definición del factor de volumen de gas:

$$B_g = \frac{Vol_g\ c.y.}{Vol_g\ c.s.}$$

Sustituyendo los términos de volumen con la definición de densidad:

$$B_g = \frac{Vol_g\ c.y.}{Vol_g\ c.s.} = \left(\frac{\frac{m_g}{\rho_g\ c.y.}}{\frac{m_g}{\rho_g\ c.s.}}\right) = \left(\frac{m_g}{m_g}\right)\left(\frac{\rho_g\ c.s.}{\rho_g\ c.y.}\right) = \left(\frac{1}{C_{gg}}\right)\left(\frac{\rho_g\ c.s.}{\rho_g\ c.y.}\right)$$

Despejando:

$$B_g = \left(\frac{1}{C_{gg}}\right)\left(\frac{\rho_g\ c.s.}{\rho_g\ c.y.}\right) \qquad \boxed{C_{gg} = \frac{\rho_g\ c.s.}{B_g\ \rho_g\ c.y.}}$$

c) Fracción másica de los pseudocomponentes de gas presentes en la fase de agua:

$$\boxed{C_{gw} = 0}$$

<u>**Para la fase de agua:**</u>

a) Fracción másica de los pseudocomponentes de agua presentes en la fase de aceite:

$$\boxed{C_{wo} = 0}$$

b) Fracción másica de los pseudocomponentes de agua presentes en la fase de gas:

$$\boxed{C_{wg} = 0}$$

c) Fracción másica de los pseudocomponentes de agua presentes en la fase de agua:

$$C_{ww} = \frac{m_w}{m_w}$$

Con la definición del factor de volumen de agua:

$$B_w = \frac{Vol_w\ c.y.}{Vol_w\ c.s.}$$

Sustituyendo los términos de volumen con la definición de densidad:

$$B_w = \frac{Vol_w\ c.y.}{Vol_w\ c.s.} = \left(\frac{\frac{m_w}{\rho_w\ c.y.}}{\frac{m_w}{\rho_w\ c.s.}}\right) = \left(\frac{m_w}{m_w}\right)\left(\frac{\rho_w\ c.s.}{\rho_w\ c.y.}\right) = \left(\frac{1}{C_{ww}}\right)\left(\frac{\rho_w\ c.s.}{\rho_w\ c.y.}\right)$$

Despejando:

$$B_w = \left(\frac{1}{C_{ww}}\right)\left(\frac{\rho_w\ c.s.}{\rho_w\ c.y.}\right) \qquad \boxed{C_{ww} = \frac{\rho_w\ c.s.}{B_w\ \rho_w\ c.y.}}$$

<u>En la fractura natural:</u>

a) <u>Para la fase de aceite:</u> Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de aceite a condiciones estándar.

$$
\left(\frac{1}{\rho_o\,c.s.}\right)\left(
\begin{array}{c}
-\frac{\partial}{\partial x}\left(\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right)+\left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)+(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\
-\frac{\partial}{\partial y}\left(\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right)+\left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)+(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f \\
-\frac{\partial}{\partial z}\left(\begin{array}{c}\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z}-\gamma_o\frac{\partial D}{\partial z}\right)\right)+\left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z}-\gamma_g\frac{\partial D}{\partial z}\right)\right) \\ +(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z}-\gamma_w\frac{\partial D}{\partial z}\right)\right)\end{array}\right)_f \\
+\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\rho_o q_o^* + \left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\rho_g q_g^* + (0)\rho_w q_w^* \\
+\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+\left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)+(0)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right) \\
=\frac{\partial}{\partial t}\left(\emptyset\left(\left(\frac{\rho_o\,c.s.}{B_o\;\rho_o\,c.y.}\right)\rho_o S_{at\,o}+\left(\frac{\rho_o\,c.s.}{B_g\,\rho_g\,c.y.}r_s\right)\rho_g S_{at\,g}+(0)\rho_w S_{at\,w}\right)\right)_f
\end{array}\right)
$$

Si definimos:

$$
b_o=\frac{1}{B_o}\quad,\quad b_g=\frac{1}{B_g}\quad,\quad b_w=\frac{1}{B_w}
$$

Entonces:

$$
\begin{array}{c}
\frac{\partial}{\partial x}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}+b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f+\frac{\partial}{\partial y}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}+b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f+\frac{\partial}{\partial z}\left(b_o\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z}-\gamma_o\frac{\partial D}{\partial z}\right)+b_g r_s\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z}-\gamma_g\frac{\partial D}{\partial z}\right)\right)_f \\
+b_{o_f}q_o^*+b_{g_f}r_{s_f}q_g^*+\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+\left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)r_{s_m}=\frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o}+b_g r_s S_{at\,g}\right)\right)_f
\end{array}
$$

b) <u>Para la fase de gas:</u> Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de gas a condiciones estándar.

$$
\left(\frac{1}{\rho_g\,c.s.}\right)\left(
\begin{array}{c}
-\frac{\partial}{\partial x}\left(\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right)+\left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)+(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\
-\frac{\partial}{\partial y}\left(\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right)+\left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)+(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f \\
-\frac{\partial}{\partial z}\left(\begin{array}{c}\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z}-\gamma_o\frac{\partial D}{\partial z}\right)\right)+\left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z}-\gamma_g\frac{\partial D}{\partial z}\right)\right) \\ +(0)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z}-\gamma_w\frac{\partial D}{\partial z}\right)\right)\end{array}\right)_f \\
+\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\rho_o q_o^* + \left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\rho_g q_g^* + (0)\rho_w q_w^* \\
+\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)+\left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)+(0)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right) \\
=\frac{\partial}{\partial t}\left(\emptyset\left(\left(\frac{\rho_g\,c.s.}{B_o\,\rho_o\,c.y.}R_s\right)\rho_o S_{at\,o}+\left(\frac{\rho_g\,c.s.}{B_g\,\rho_g\,c.y.}\right)\rho_g S_{at\,g}+(0)\rho_w S_{at\,w}\right)\right)_f
\end{array}\right)
$$

Entonces:

$$\frac{\partial}{\partial x}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o \frac{\partial D}{\partial z}\right) + b_g \frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g \frac{\partial D}{\partial z}\right)\right)_f + b_{o_f} R_{s_f} q_o^* + b_{g_f} q_g^* + \left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) R_{s_m} + \left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_f$$

c) <u>Para la fase de agua:</u> Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de agua a condiciones estándar.

$$\left(\frac{1}{\rho_w\, c.s.}\right)\left(\begin{array}{c} -\frac{\partial}{\partial x}\left((0)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}\right) + (0)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right) + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)\right)_f \\ -\frac{\partial}{\partial y}\left((0)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}\right) + (0)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right) + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)\right)_f \\ -\frac{\partial}{\partial z}\left(\begin{array}{c}(0)\rho_o\left(-\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right)\right) + (0)\rho_g\left(-\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right)\right) \\ + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\rho_w\left(-\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w\frac{\partial D}{\partial z}\right)\right)\end{array}\right)_f \\ + (0)\rho_o q_o^* + (0)\rho_g q_g^* + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\rho_w q_w^* \\ + (0)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) + (0)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right) \\ = \frac{\partial}{\partial t}\left(\emptyset\left((0)\rho_o S_{at\,o} + (0)\rho_g S_{at\,g} + \left(\frac{\rho_w\, c.s.}{B_w\, \rho_w\, c.y.}\right)\rho_w S_{at\,w}\right)\right)_f \end{array}\right)$$

Entonces:

$$\frac{\partial}{\partial x}\left(b_w \frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_w \frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_w \frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w \frac{\partial D}{\partial z}\right)\right)_f + b_{w_f} q_w^* + \left(\frac{b_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right) = \frac{\partial}{\partial t}\left(\emptyset b_w S_{at\,w}\right)_f$$

<u>En la matriz:</u>

a) <u>Para la fase de aceite:</u> Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de aceite a condiciones estándar.

$$\left(\frac{1}{\rho_o\, c.s.}\right)\left(\begin{array}{c} -\left(\frac{\rho_o\, c.s.}{B_o\, \rho_o\, c.y.}\right)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) - \left(\frac{\rho_o\, c.s.}{B_g\, \rho_g\, c.y.} r_s\right)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) - (0)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m} - P_{w_f}\right) \\ = \frac{\partial}{\partial t}\left(\emptyset\left(\left(\frac{\rho_o\, c.s.}{B_o\, \rho_o\, c.y.}\right)\rho_o S_{at\,o} + \left(\frac{\rho_o\, c.s.}{B_g\, \rho_g\, c.y.} r_s\right)\rho_g S_{at\,g} + (0)\rho_w S_{at\,w}\right)\right)_m \end{array}\right)$$

Entonces:

$$-\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right) - \left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right) r_{s_m} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right)\right)_m$$

b) Para la fase de gas: Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de gas a condiciones estándar.

$$\left(\frac{1}{\rho_g\ c.s.}\right)\left(\begin{array}{c} -\left(\frac{\rho_g\ c.s.}{B_o\,\rho_o\ c.y.}R_s\right)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right) - \left(\frac{\rho_g\ c.s.}{B_g\,\rho_g\ c.y.}\right)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right) - (0)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right) \\ = \frac{\partial}{\partial t}\left(\emptyset\left(\left(\frac{\rho_g\ c.s.}{B_o\,\rho_o\ c.y.}R_s\right)\rho_o S_{at\,o} + \left(\frac{\rho_g\ c.s.}{B_g\,\rho_g\ c.y.}\right)\rho_g S_{at\,g} + (0)\rho_w S_{at\,w}\right)\right)_m \end{array}\right)$$

Entonces:

$$-\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)R_{s_m} - \left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right) = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_m$$

c) Para la fase de agua: Se sustituyen los valores de los pseudocomponentes y divide entre la densidad de agua a condiciones estándar.

$$\left(\frac{1}{\rho_w\ c.s.}\right)\left(\begin{array}{c} +(0)\left(\frac{\rho_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right) + (0)\left(\frac{\rho_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right) + \left(\frac{\rho_w\ c.s.}{B_w\,\rho_w\ c.y.}\right)\left(\frac{\rho_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right) \\ = \frac{\partial}{\partial t}\left(\emptyset\left((0)\rho_o S_{at\,o} + (0)\rho_g S_{at\,g} + \left(\frac{\rho_w\ c.s.}{B_w\,\rho_w\ c.y.}\right)\rho_w S_{at\,w}\right)\right)_m \end{array}\right)$$

Entonces:

$$-\left(\frac{b_{wm}\sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)\left(P_{w_m}-P_{w_f}\right) = \frac{\partial}{\partial t}\left(\emptyset b_w S_{at\,w}\right)_m$$

Obteniendo así, 3 ecuaciones de flujo (considerando un modelo trifásico) para el medio continuo (la fractura natural) y 3 ecuaciones de flujo para el medio discontinuo (la matriz), por ejemplo, para el modelo fracturado de la Fig. 3:

En la fractura natural:

a) Para la fase de aceite:

$$\frac{\partial}{\partial x}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right) + b_g r_s\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right)\right)_f$$
$$+ b_{o_f}q_o^* + b_{g_f}r_{s_f}q_g^* + \tau_{omf}^* + \tau_{gmf}^* r_{sm} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right)\right)_f$$

b) Para la fase de gas:

$$\frac{\partial}{\partial x}\left(b_o R_s\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o R_s\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o R_s\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o\frac{\partial D}{\partial z}\right) + b_g\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g\frac{\partial D}{\partial z}\right)\right)_f$$
$$+ b_{o_f}R_{s_f}q_o^* + b_{g_f}q_g^* + \tau_{omf}^* R_{sm} + \tau_{gmf}^* = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_f$$

c) Para la fase de agua:

$$\frac{\partial}{\partial x}\left(b_w\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_w\frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_w\frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w\frac{\partial D}{\partial z}\right)\right)_f + b_{w_f}q_w^* + \tau_{wmf}^* = \frac{\partial}{\partial t}\left(\emptyset b_w S_{at\,w}\right)_f$$

En la matriz:

a) Para la fase de aceite:

$$-\tau^*_{o_{mf}} - \tau^*_{g_{mf}} r_{s_m} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right) \right)_m$$

b) Para la fase de gas:

$$-\tau^*_{o_{mf}} R_{s_m} - \tau^*_{g_{mf}} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right) \right)_m$$

c) Para la fase de agua:

$$-\tau^*_{w_{mf}} = \frac{\partial}{\partial t}\left(\emptyset b_w S_{at\,w} \right)_m$$

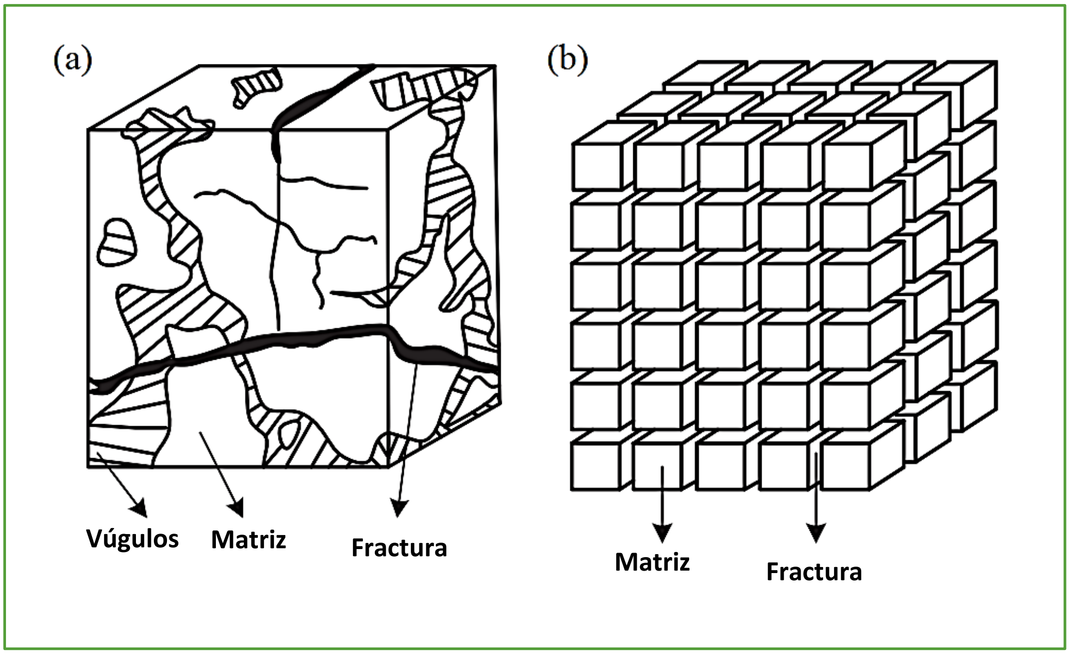

Fig. 3. "Idealización del medio fracturado". (Warren & Root, 1963).

Donde las funciones de transferencia matriz-fractura (Thomas, 1983) se definen como:

$$\tau^*_{o_{mf}} = \left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right)$$

$$\tau^*_{g_{mf}} = \left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right)$$

Y σ es el factor de forma, o "shape factor" (Kazemi, 1976) : Que considera el área de contacto entre los bloques de la matriz y la fractura natural durante el intercambio de fluidos matriz-fractura por unidad de volumen y considerando una geometría rectangular o de paralelepípedos para los bloques de la matriz.

$$\sigma = 4\left(\frac{1}{h_x^2} + \frac{1}{h_y^2} + \frac{1}{h_z^2}\right)$$

Así mismo, estas ecuaciones de flujo requieren del empleo de ecuaciones auxiliares para poder resolverse. Estas ecuaciones auxiliares son:

Relaciones de presiones capilares:

$$P_{cap\ go}(S_{at\ g}) = P_g - P_o$$
$$P_{cap\ wo}(S_{at\ w}) = P_o - P_w$$

Relación de saturaciones:

$$S_{at\ o} + S_{at\ g} + S_{at\ w} = 1$$

Permeabilidades relativas en función de las saturaciones.

$$k_{ro} = k_{ro}(S_{at\ g}, S_{at\ w})$$
$$k_{rg} = k_{rg}(S_{at\ g})$$
$$k_{rw} = k_{rw}(S_{at\ w})$$

Estas seis ecuaciones de flujo, junto con las ecuaciones auxiliares mencionadas, será el modelo matemático que nos ayude a describir el comportamiento del flujo de fluidos en un medio poroso naturalmente fracturado, para yacimientos de tipo gas y condensado o gas retrógrado.

2.3. Condiciones iniciales

La inicialización de un modelo de simulación se obtiene suponiendo condiciones de equilibrio gravitacional/capilar inicial, lo que implica cero flujo de las fases a t=0 (Cortes Rubio, 2008). Considerando estas condiciones iniciales de equilibrio, solo existirá variación de presión y saturación respecto a la profundidad, definiendo zonas de una fase y zonas de transición donde tenemos más de una fase, sin embargo, primero es necesario contar con cierta información sobre la geometría del yacimiento, propiedades de la roca y de los fluidos (Aranda Ramírez, 2006) :

Datos necesarios para la inicialización de un modelo de simulación
Dimensiones del yacimiento (Longitud en "x", "y" y "z") y cima del yacimiento.
Una profundidad y presión de referencia (z_{ref} y P_{ref}), necesarios para determinar el equilibrio vertical del yacimiento.
Profundidad del contacto agua-aceite (z_{woc}) y gas-aceite (z_{goc}).
Curvas de presiones capilares agua-aceite y gas-aceite, necesarias para determinar la distribución de saturaciones.
Fases en la que se encuentra el fluido inicialmente (líquido o gas).
Saturación de agua irreductible (S_{wi}).
Saturación de aceite residual (S_{or}).
Saturación de gas crítica (S_{gc}).

Tabla 1. "Datos necesarios para la inicialización de un modelo de simulación". (Aranda Ramírez, 2006).

Inicialización de presiones

Suponiendo equilibrio gravitacional y capilar sin flujo de las fases al tiempo cero, los gradientes de potencial de las fases en cualquier punto y en cualquier dirección son cero. Por ejemplo, considerando la ley de Darcy (Reséndiz Torres & Peña Chaparro, 2005) para la fase de aceite:

$$\frac{k\, k_{r_o}}{\mu_o}(\nabla P_o - \gamma_o \nabla \mathrm{D}) = 0$$

Para un plano horizontal ubicado en una posición cualquiera en "z":

$$\frac{\partial P_o}{\partial x} = 0 \quad , \quad \frac{\partial P_o}{\partial y} = 0$$

Lo que indica que las presiones en el plano horizontal permanecen constantes, entonces, para la dirección "z":

$$\frac{\partial P_o}{\partial z} - \gamma_o = 0$$

Entonces, en condiciones de equilibrio, la distribución vertical de presiones para la fase de aceite está dado por el peso de la columna de los fluidos. Con una presión y profundidad de referencia conocida $(P_{ref}\,, z_{ref})$ podemos calcular la presión de aceite en cualquier profundidad del yacimiento:

$$\frac{\partial P_o}{\partial z} - \gamma_o = 0$$

$$\partial P_o = \gamma_o\, \partial z$$

$$\int_{P_{ref}}^{P_o} dP_o = \int_{z_{ref}}^{z} \bar{\gamma}_o(P_o)\, dz$$

$$(P_o - P_{ref}) = \bar{\gamma}_o(\bar{P}_o)[z - z_{ref}]$$

$$\boxed{P_o = P_{ref} + \bar{\gamma}_o(\bar{P}_o)[z - z_{ref}]}$$

Donde $\bar{P}_o$ es la presión promedio de P_o y P_{ref}:

$$\boxed{\bar{P}_o = \frac{P_o + P_{ref}}{2}}$$

Y $\bar{\gamma}_o(\bar{P}_o)$ se define como:

$$\boxed{\bar{\gamma}_o(\bar{P}_o) = \rho_o(\bar{P}_o)\, g}$$

Debido a que $\bar{\gamma}_o$ depende de la presión promedio $(\bar{P}_o)$ y esta presión promedio de la presión incógnita (P_o), el problema se convierte en uno de tipo no lineal, con lo cual es necesario emplear el método de Newton-Raphson para determinar el valor de P_o.

Estas ecuaciones serán las mismas para determinar la distribución de presión vertical para las fases de agua y gas, únicamente se cambian las propiedades del fluido según corresponda.

Determinando factores de conversión

De la ecuación original (para la fase de aceite), P_o debe de estar en [Psi]:

$$P_o = P_{ref} + \bar{\gamma}_o(\bar{P}_o)[z - z_{ref}]$$

Sustituyendo $\bar{\gamma}_o(\bar{P}_o)$:

$$P_o = P_{ref} + \rho_o(\bar{P}_o)\, g[z - z_{ref}]$$

Como P_{ref} ya está en [Psi], el término $\rho_o(\bar{P}_o)\, g[z - z_{ref}]$ debe de quedar en [Psi], entonces, si se tienen las siguientes unidades:

$$\rho_o(\bar{P}_o) = \left[\frac{lbm}{ft^3}\right] \quad , \quad g = 32.174\left[\frac{ft}{s^2}\right] \quad , \quad [z - z_{ref}] = [\,ft\,]$$

Multiplicando, las unidades serían:

$$\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \left(\rho_o(\bar{P}_o)\left[\frac{lbm}{ft^3}\right]\right)\left(32.174\left[\frac{ft}{s^2}\right]\right)\left([z - z_{ref}]\,[\,ft\,]\right)$$

$$\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \rho_o(\bar{P}_o)\,[z - z_{ref}]\left[32.174\left[\frac{lbm\,ft}{s^2}\;\frac{1}{ft^2}\right]\right]$$

Si utilizamos libras fuerzas [lbf]:

$$\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \rho_o(\bar{P}_o)\,[z - z_{ref}]\left(32.174\left[\frac{lbm\,ft}{s^2}\;\frac{1}{ft^2}\right]\right)\left(\frac{1\,[\,lbf\,]}{32.174\left[\frac{lbm\,ft}{s^2}\right]}\right)$$

Entonces:

$$\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \rho_o(\bar{P}_o)\,[z - z_{ref}]\left[\frac{lbf}{ft^2}\right]$$

Convirtiendo a [Psi]:

$$\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \rho_o(\bar{P}_o)\,[z - z_{ref}]\left[\frac{lbf}{ft^2}\right]\left[\frac{1\,[Psi]}{144\left[\frac{lbf}{ft^2}\right]}\right]$$

Entonces, el factor de conversión para que esta multiplicación quede en términos de [Psi] es:

$$\boxed{\rho_o(\bar{P}_o)\, g[z - z_{ref}] = \rho_o(\bar{P}_o)\,[z - z_{ref}]\left[\frac{1}{144}[\,Psi\,]\right]}$$

Sustituyendo en la ecuación original:

$$P_o = P_{ref} + \bar{\gamma}_o(\bar{P}_o)\left[z - z_{ref}\right]$$

$$P_o = P_{ref} + \rho_o(\bar{P}_o)\ g\left[z - z_{ref}\right]$$

$$\boxed{P_o = P_{ref} + \left[\frac{1}{144}\right]\rho_o(\bar{P}_o)\left[z - z_{ref}\right]}$$

Donde:

$$\begin{aligned}
&P_o\ [Psi] && = Presión\ incógnita\\
&P_{ref}\ [Psi] && = Presión\ de\ referencia\\
&\rho_o\left[\frac{lbm}{ft^3}\right] && = Densidad\\
&z[ft] && = Profundidad\ de\ la\ presión\ incógnita\\
&z_{ref}[ft] && = Profundidad\ de\ referencia
\end{aligned}$$

<u>Utilizando el método de Newton-Raphson</u>

Para utilizar el método de Newton-Raphson, primero determinamos la función de residuos, esto es igualando la ecuación a cero:

$$P_o = P_{ref} + \bar{\gamma}_o(\bar{P}_o)\left[z - z_{ref}\right] \quad \rightarrow \quad \boxed{F(P_o) = P_o - P_{ref} - \bar{\gamma}_o(\bar{P}_o)\left[z - z_{ref}\right] = 0}$$

Ahora, para determinar el nuevo valor de presión de aceite (P_o^{v+1}), la función de residuos se aproxima a través de una serie de Taylor truncada en la primera derivada, que en esencia es el método de Newton-Raphson:

$$F(P_o^{v+1}) = F(P_o^v) + F'(P_o^v)[P_o^{v+1} - P_o^v] = 0$$

$$F(P_o^{v+1}) = F(P_o^v) + F'(P_o^v)[P_o^{v+1}] - F'(P_o^v)[P_o^v] = 0$$

Despejando P_o^{v+1} :

$$P_o^{v+1} = \frac{-F(P_o^v) + F'(P_o^v)[P_o^v]}{F'(P_o^v)}$$

$$\boxed{P_o^{v+1} = \frac{-F(P_o^v)}{F'(P_o^v)} + P_o^v}$$

<u>Definiendo la derivada de la función de residuos</u>

Si nuestra función de residuos se define como:

$$F(P_o) = P_o - P_{ref} - \bar{\gamma}_o(\bar{P}_o)\left[z - z_{ref}\right]$$

$$F(P_o) = P_o - P_{ref} - \left[\frac{1}{144}\right]\rho_o(\bar{P}_o)\left[z - z_{ref}\right]$$

Derivando respecto a la presión incógnita (P_o):

$$\frac{dF(P_o)}{dP_o} = \frac{d}{dP_o}\left(P_o - P_{ref} - \left[\frac{1}{144}\right]\rho_o(\bar{P}_o)[z - z_{ref}]\right)$$

$$\frac{dF(P_o)}{dP_o} = 1 - 0 - \left[\frac{z - z_{ref}}{144}\right]\frac{d\rho_o(\bar{P}_o)}{dP_o}$$

Para la derivada de la densidad, es necesario aplicar la regla de la cadena, ya que esta densidad depende de la presión promedio ($\bar{P}_o$) y esta presión promedio, a su vez, depende de la presión incógnita (P_o):

$$\frac{d\rho_o(\bar{P}_o)}{dP_o} = \frac{d\rho_o(\bar{P}_o)}{d\bar{P}_o}\frac{d\bar{P}_o}{dP_o}$$

Si la presión promedio se define como:

$$\bar{P}_o = \frac{P_o + P_{ref}}{2}$$

Entonces:

$$\frac{d\bar{P}_o}{dP_o} = \frac{1}{2}$$

Sustituyendo en la derivada de la densidad:

$$\frac{d\rho_o(\bar{P}_o)}{dP_o} = \frac{d\rho_o(\bar{P}_o)}{d\bar{P}_o}\left[\frac{1}{2}\right]$$

Sustituyendo en la derivada principal:

$$\frac{dF(P_o)}{dP_o} = 1 - 0 - \left[\frac{z - z_{ref}}{144}\right]\frac{d\rho_o(\bar{P}_o)}{dP_o}$$

$$\frac{dF(P_o)}{dP_o} = 1 - 0 - \left[\frac{z - z_{ref}}{144}\right]\frac{d\rho_o(\bar{P}_o)}{d\bar{P}_o}\frac{d\bar{P}_o}{dP_o}$$

$$\boxed{\frac{dF(P_o)}{dP_o} = 1 - \left[\frac{z - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_o(\bar{P}_o)}{d\bar{P}_o}}$$

Ahora, la derivada de la densidad (evaluada en la presión promedio) respecto a la presión promedio, se puede determinar de manera numérica, a través de tablas PVT, por ejemplo, algunas maneras de calcular derivadas de manera numérica (Chapra, 2018) son:

Primera derivada en diferencias finitas progresivas:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$$

Primera derivada en diferencias finitas regresivas:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$
$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$$

Primera derivada en diferencias finitas centrales:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$
$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

Entonces, para determinar la distribución vertical de presiones para la fase de aceite

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Asignamos una presión inicial de aceite en esta profundidad (P_{o1}) para entrar al ciclo iterativo de Newton-Raphson. Podemos asignar el valor de esta presión inicial como la presión de referencia $\left(P_{ref}\right)$:

$$P_{o_1}^{v} = P_{ref}$$

c) Se inicia el ciclo iterativo de Newton-Raphson, para determinar el valor de la presión de aceite correcta en esta profundidad (P_{o1}^{v+1}):

$$P_{o_1}^{v+1} = \frac{-F\left(P_{o_1}^{v}\right)}{F'\left(P_{o_1}^{v}\right)} + P_{o_1}^{v}$$

Donde:

$$F\left(P_{o_1}^{v}\right) = P_{o_1}^{v} - P_{ref} - \left[\frac{1}{144}\right]\rho_o\left(\overline{P^v}_{o_1}\right)\left[z_1 - z_{ref}\right]$$
$$F'\left(P_{o_1}^{v}\right) = 1 - \left[\frac{z_1 - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_o\left(\overline{P^v}_{o_1}\right)}{d\overline{P^v}_{o_1}}$$
$$\overline{P^v}_{o_1} = \frac{P_{o_1}^{v} + P_{ref}}{2}$$

Recordando que la derivada de la densidad (evaluada en la presión promedio) respecto a la presión promedio, se calcula de manera numérica, a través de tablas PVT.

d) Dentro del ciclo iterativo, se va actualizando el valor de $P_{o_1}^{v}$ en cada iteración:

$$P_{o_1}^{v} = P_{o_1}^{v+1}$$

e) El valor converge, una vez que el porcentaje de error entre el valor de presión a una iteración anterior $\left(P_{o_1}^{v}\right)$ y la presión calculada en la iteración actual $\left(P_{o_1}^{v+1}\right)$ sea menor a una tolerancia, la cual uno es libre de definir:

$$\%\,Error = \left|\frac{P_{o_1}^{v+1} - P_{o_1}^{v}}{P_{o_1}^{v+1}}\right| \times 100$$

$$\begin{array}{lll} Si: & \%\,Error < Tolerancia & \rightarrow \quad Converge\ y\ se\ obtiene\ el\ valor\ correcto\ de\ P_{o_1} \\ En\ caso\ contrario: & \%\,Error > Tolerancia & \rightarrow \quad Continuamos\ con\ el\ ciclo\ iterativo \end{array}$$

Una vez que converge, se puede decir que se determinó la presión de aceite correcta (P_{o1}) en esta profundidad (z_1):

$$P_{o_1} = P_{o_1}^{v+1}$$

f) Este proceso, desde **a)** hasta **e),** se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, calculando así todas las presiones de aceite en estas mismas profundidades $\left(P_{o_2}, P_{o_3}, P_{o_4}, P_{o_5}, \ldots\right)$.

<u>Distribución de presiones para la fase de gas:</u>

Para determinar la distribución vertical de presiones para la fase de gas, primero debemos de determinar la presión de aceite en el contacto gas-aceite $\left(z_{goc}\right)$, ya que, en este punto, la presión de gas será igual a la presión de aceite, esto debido a que la presión capilar gas-aceite $\left(P_{cap\ go}\right)$ es igual a cero en esta profundidad:

$$P_{cap\ go} = P_g - P_o$$

En z_{goc} :

$$0 = P_g - P_o \quad \rightarrow \quad P_g = P_o$$

Entonces para calcular la presión de aceite en el contacto gas-aceite $\left(z_{goc}\right)$ se realiza el mismo proceso iterativo que se comentó anteriormente, solo que ahora cambiamos la profundidad z_1 por la profundidad del contacto gas-aceite z_{goc} :

$$P_{o_{zgoc}}^{v} = P_{ref}$$

$$P_{o_{zgoc}}^{v+1} = \frac{-F\left(P_{o_{zgoc}}^{v}\right)}{F'\left(P_{o_{zgoc}}^{v}\right)} + P_{o_{zgoc}}^{v}$$

Donde:

$$F\left(P_{o_{zgoc}}^{v}\right) = P_{o_{zgoc}}^{v} - P_{ref} - \left[\frac{1}{144}\right]\rho_o\left(\overline{P^{v}}_{o_{zgoc}}\right)\left[z_{goc} - z_{ref}\right]$$

$$F'\left(P^v_{o_{zgoc}}\right) = 1 - \left[\frac{z_{goc} - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_o\left(\overline{P^v}_{o_{zgoc}}\right)}{d\overline{P^v}_{o_{zgoc}}}$$

$$\overline{P^v}_{o_{zgoc}} = \frac{P^v_{o_{zgoc}} + P_{ref}}{2}$$

Una vez que convergió el proceso iterativo, y obtuvimos la presión de aceite en el contacto gas-aceite, esta presión de aceite será igual a la presión de gas, ya que ha esta profundidad, como se dijo, la presión capilar gas-aceite es igual a cero:

$$P_{g_{zgoc}} = P_{o_{zgoc}}$$

Esta presión de gas en el contacto gas-aceite $\left(P_{g_{zgoc}}\right)$ será ahora nuestro nuevo valor de presión de referencia $\left(P_{ref}\right)$, y por ello, la profundidad del contacto gas-aceite $\left(z_{goc}\right)$ será nuestra nueva profundidad de referencia $\left(z_{ref}\right)$:

$$P_{ref} = P_{g_{zgoc}}$$
$$z_{ref} = z_{goc}$$

Con esta nueva presión y profundidad de referencia, podemos ahora realizar el mismo procedimiento iterativo para determinar la distribución vertical de presiones que se realizó para la fase de aceite, pero ahora, considerando las propiedades del gas:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Asignamos una presión inicial de gas en esta profundidad $\left(P_{g1}\right)$ para entrar al ciclo iterativo de Newton-Raphson. Podemos asignar el valor de esta presión inicial como la presión de referencia $\left(P_{ref}\right)$:

$$P^v_{g_1} = P_{ref}$$

c) Se inicia el ciclo iterativo de Newton-Raphson, para determinar el valor de la presión de gas correcta en esta profundidad $\left(P^{v+1}_{g1}\right)$:

$$P^{v+1}_{g_1} = \frac{-F\left(P^v_{g_1}\right)}{F'\left(P^v_{g_1}\right)} + P^v_{g_1}$$

Donde:

$$F\left(P^v_{g_1}\right) = P^v_{g_1} - P_{ref} - \left[\frac{1}{144}\right]\rho_g\left(\overline{P^v}_{g_1}\right)\left[z_1 - z_{ref}\right]$$

$$F'\left(P^v_{g_1}\right) = 1 - \left[\frac{z_1 - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_g\left(\overline{P^v}_{g_1}\right)}{d\overline{P^v}_{g_1}}$$

$$\overline{P^v}_{g_1} = \frac{P^v_{g_1} + P_{ref}}{2}$$

d) Dentro del ciclo iterativo, se va actualizando el valor de $P_{g_1}^{v}$ en cada iteración:

$$P_{g_1}^{v} = P_{g_1}^{v+1}$$

e) El valor converge, una vez que el porcentaje de error entre el valor de presión a una iteración anterior $\left(P_{g_1}^{v}\right)$ y la presión calculada en la iteración actual $\left(P_{g_1}^{v+1}\right)$ sea menor a una tolerancia, la cual uno es libre de definir:

$$\%\,Error = \left|\frac{P_{g_1}^{v+1} - P_{g_1}^{v}}{P_{g_1}^{v+1}}\right| \times 100$$

$$\begin{array}{lll} Si: & \%\,Error < Tolerancia & \rightarrow \quad Converge\ y\ se\ obtiene\ el\ valor\ correcto\ de\ P_{g_1} \\ En\ caso\ contrario: & \%\,Error > Tolerancia & \rightarrow \quad Continuamos\ con\ el\ ciclo\ iterativo \end{array}$$

Una vez que converge, se puede decir que se determinó la presión de gas correcta $\left(P_{g1}\right)$ en esta profundidad (z_1):

$$P_{g_1} = P_{g_1}^{v+1}$$

f) Este proceso, desde **a)** hasta **e),** se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, calculando así todas las presiones de gas en estas mismas profundidades $\left(P_{g_2}, P_{g_3}, P_{g_4}, P_{g_5}, \ldots\right)$.

Distribución de presiones para la fase de agua:

Para determinar la distribución vertical de presiones para la fase de agua se repite exactamente el mismo procedimiento que se realizó para la fase de gas, pero ahora, considerando las propiedades del agua.

Primero determinamos la presión de aceite en el contacto agua-aceite (z_{woc}), ya que, en este punto, la presión de agua será igual a la presión de aceite, esto debido a que la presión capilar agua-aceite $\left(P_{cap\ wo}\right)$ es igual a cero en esta profundidad:

$$P_{cap\ wo} = P_o - P_w$$

En z_{woc} :

$$0 = P_o - P_w \quad \rightarrow \quad P_w = P_o$$

Calculamos ahora la presión de aceite en el contacto agua-aceite (z_{woc}):

$$P_{o_{zwoc}}^{v} = P_{ref}$$

$$P_{o_{zwoc}}^{v+1} = \frac{-F\left(P_{o_{zwoc}}^{v}\right)}{F'\left(P_{o_{zwoc}}^{v}\right)} + P_{o_{zwoc}}^{v}$$

Donde:

$$F\left(P_{o_{zwoc}}^{v}\right) = P_{o_{zwoc}}^{v} - P_{ref} - \left[\frac{1}{144}\right]\rho_o\left(\overline{P^v}_{o_{zwoc}}\right)\left[z_{woc} - z_{ref}\right]$$

$$F'\left(P_{o_{zwoc}}^{v}\right) = 1 - \left[\frac{z_{woc} - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_o\left(\overline{P^v}_{o_{zwoc}}\right)}{d\overline{P^v}_{o_{zwoc}}}$$

$$\overline{P^v}_{o_{zwoc}} = \frac{P_{o_{zwoc}}^{v} + P_{ref}}{2}$$

Obtenida la presión de aceite en el contacto agua-aceite, una vez que convergió el proceso iterativo, esta presión de aceite será igual a la presión de agua, ya que ha esta profundidad, como se dijo, la presión capilar agua-aceite es igual a cero:

$$P_{w_{zwoc}} = P_{o_{zwoc}}$$

Esta presión de agua en el contacto agua-aceite $\left(P_{w_{zwoc}}\right)$ será ahora nuestro nuevo valor de presión de referencia $\left(P_{ref}\right)$, y por ello, la profundidad del contacto agua-aceite (z_{woc}) será nuestra nueva profundidad de referencia $\left(z_{ref}\right)$:

$$P_{ref} = P_{w_{zwoc}}$$
$$z_{ref} = z_{woc}$$

Con esta nueva presión y profundidad de referencia, podemos ahora realizar el procedimiento iterativo para cada una de las profundidades de la malla, obteniendo así la distribución vertical de presiones del agua:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Asignamos una presión inicial de agua en esta profundidad (P_{w1}) para entrar al ciclo iterativo de Newton-Raphson. Podemos asignar el valor de esta presión inicial como la presión de referencia $\left(P_{ref}\right)$:

$$P_{w_1}^{v} = P_{ref}$$

c) Se inicia el ciclo iterativo de Newton-Raphson, para determinar el valor de la presión de agua correcta en esta profundidad $\left(P_{w_1}^{v+1}\right)$:

$$P_{w_1}^{v+1} = \frac{-F\left(P_{w_1}^{v}\right)}{F'\left(P_{w_1}^{v}\right)} + P_{w_1}^{v}$$

Donde:

$$F\left(P_{w_1}^{v}\right) = P_{w_1}^{v} - P_{ref} - \left[\frac{1}{144}\right]\rho_w\left(\overline{P^v}_{w_1}\right)\left[z_1 - z_{ref}\right]$$

$$F'\left(P_{w_1}^{v}\right) = 1 - \left[\frac{z_1 - z_{ref}}{144}\right]\left[\frac{1}{2}\right]\frac{d\rho_w\left(\overline{P^v}_{w_1}\right)}{d\overline{P^v}_{w_1}}$$

$$\overline{P^v}_{w_1} = \frac{P_{w_1}^{v} + P_{ref}}{2}$$

d) Dentro del ciclo iterativo, se va actualizando el valor de $P_{w_1}^{v}$ en cada iteración:

$$P_{w_1}^{v} = P_{w_1}^{v+1}$$

e) El valor converge, una vez que el porcentaje de error entre el valor de presión a una iteración anterior $\left(P_{w_1}^{v}\right)$ y la presión calculada en la iteración actual $\left(P_{w_1}^{v+1}\right)$ sea menor a una tolerancia, la cual uno es libre de definir:

$$\%\,Error = \left|\frac{P_{w_1}^{v+1} - P_{w_1}^{v}}{P_{w_1}^{v+1}}\right| \times 100$$

Si:	$\%\,Error < Tolerancia$	$\rightarrow$	$Converge\ y\ se\ obtiene\ el\ valor\ correcto\ de\ P_{w_1}$
$En\ caso\ contrario$:	$\%\,Error > Tolerancia$	$\rightarrow$	$Continuamos\ con\ el\ ciclo\ iterativo$

Una vez que converge, se puede decir que se determinó la presión de agua correcta (P_{w1}) en esta profundidad (z_1):

$$P_{w_1} = P_{w_1}^{v+1}$$

f) Este proceso, desde **a)** hasta **e),** se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \dots)$, calculando así todas las presiones de gas en estas mismas profundidades $(P_{w_2}, P_{w_3}, P_{w_4}, P_{w_5}, \dots)$.

El resultado de la inicialización de las presiones, para las tres fases, nos daría como resultado lo siguiente:

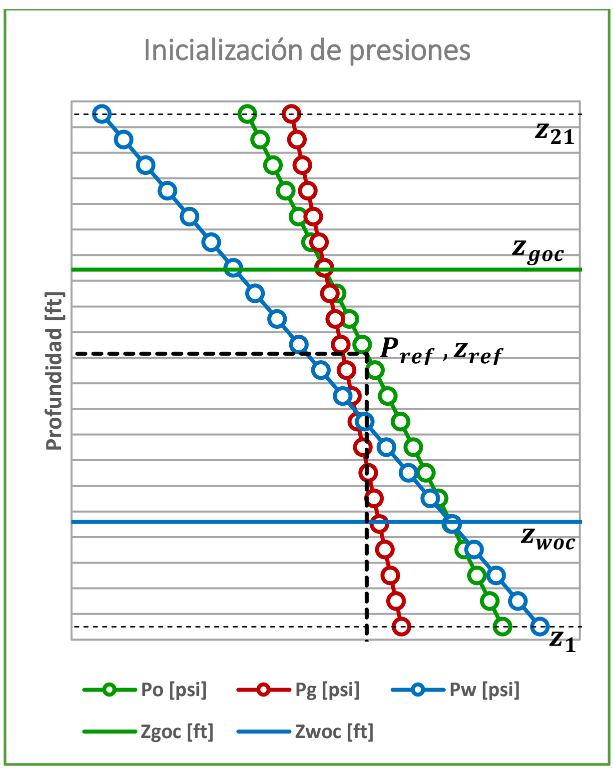

Fig. 4. "Inicialización de presiones".

Corrección de presiones

Ahora bien, los resultados de la inicialización anterior se realizan suponiendo que únicamente se tiene un único fluido en el yacimiento, es decir, la distribución vertical de presiones de aceite se obtuvo suponiendo que únicamente se tiene aceite en todo el yacimiento (lo mismo para las fases de gas y agua), sin embargo, esta suposición es incorrecta, ya que en un yacimiento pueden existir hasta cinco diferentes zonas verticalmente, desde la cima hasta la base (Reséndiz Torres & Peña Chaparro, 2005):

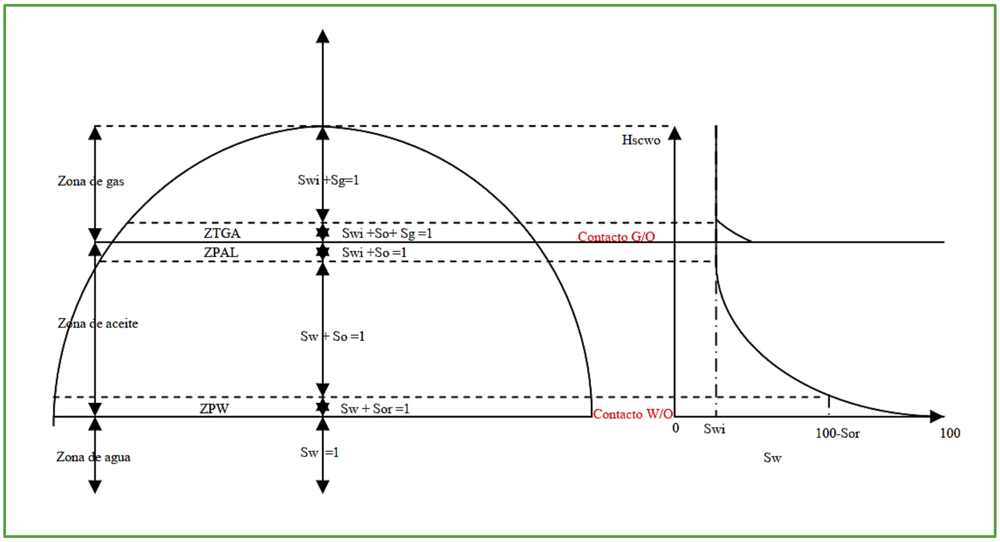

Fig. 5. "Distribución de zonas de fluidos en un yacimiento". (Reséndiz Torres & Peña Chaparro, 2005).

Con lo cual, las presiones anteriormente calculadas deben de ser corregidas, y esta corrección se realiza a través la distribución vertical de presiones capilares, pero para obtenerla primero debemos determinar las presiones capilares máximas $(P_{cap\ go\ max}\ ,\ P_{cap\ wo\ max})$ y mínimas $(P_{cap\ go\ min}\ ,\ P_{cap\ wo\ min})$ que se pueden tener, estos valores se obtienen a partir de las curvas de presiones capilares gas-aceite y agua-aceite:

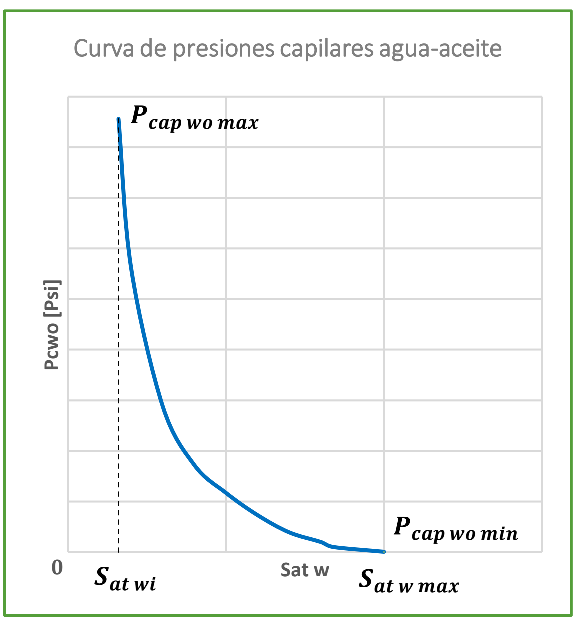

Fig. 6. "Identificación de $P_{cap\ wo\ max}$ y $P_{cap\ wo\ min}$".

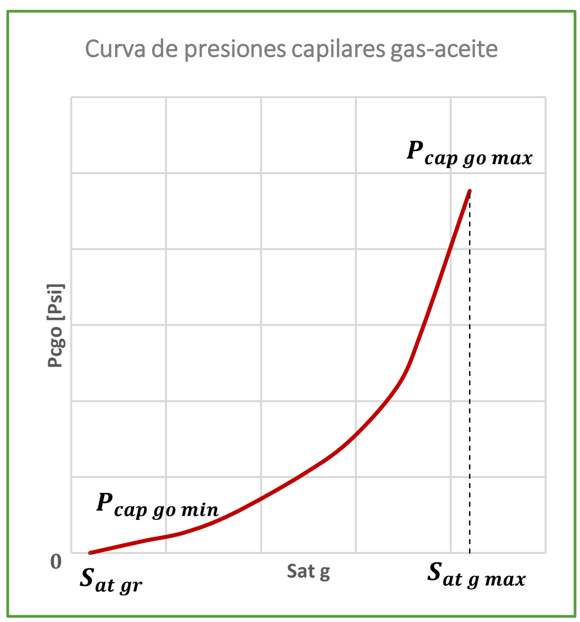

Fig. 7. "Identificación de $P_{cap\ go\ max}$ y $P_{cap\ go\ min}$".

Una vez determinadas estas presiones capilares máximas y mínimas, se puede determinar la distribución vertical de presiones capilares de la siguiente forma:

Para la presión capilar gas-aceite:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Calculamos la presión capilar gas-aceite a esta profundidad de la siguiente forma:

$$P_{cap\ go\ z1} = P_{g\ z1} - P_{o\ z1}$$

c) Evaluamos la presión capilar gas-aceite calculada según los siguientes casos:

$Si\ \ P_{cap\ go\ z1} \geq P_{cap\ go\ max}$ $\quad Entonces$: $\ P_{cap\ go\ z1} = P_{cap\ go\ max}$

$Si\ \ P_{cap\ go\ z1} \leq P_{cap\ go\ min}$ $\quad Entonces$: $\ P_{cap\ go\ z1} = P_{cap\ go\ min}$

$Si\ \ P_{cap\ go\ min} < P_{cap\ go\ z1} < P_{cap\ go\ max}$ $\quad Entonces$: $\ P_{cap\ go\ z1} = P_{cap\ go\ z1}$

f) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \dots)$, calculando así todas las presiones capilares gas-aceite en estas mismas profundidades $(P_{cap\ go\ z2}, P_{cap\ go\ z3}, P_{cap\ go\ z4}, P_{cap\ go\ z5}, \dots)$.

Para la presión capilar agua-aceite:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Calculamos la presión capilar agua-aceite a esta profundidad de la siguiente forma:

$$P_{cap\ wo\ z1} = P_{o\ z1} - P_{w\ z1}$$

c) Evaluamos la presión capilar agua-aceite calculada a esta profundidad según los siguientes casos:

$Si\ \ P_{cap\ wo\ z1} \geq P_{cap\ wo\ max}$ $\quad Entonces$: $\ P_{cap\ wo\ z1} = P_{cap\ wo\ max}$

$Si\ \ P_{cap\ wo\ z1} \leq P_{cap\ wo\ min}$ $\quad Entonces$: $\ P_{cap\ wo\ z1} = P_{cap\ wo\ min}$

$Si\ \ P_{cap\ wo\ min} < P_{cap\ wo\ z1} < P_{cap\ wo\ max}$ $\quad Entonces$: $\ P_{cap\ wo\ z1} = P_{cap\ wo\ z1}$

f) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \dots)$, calculando así todas las presiones capilares agua-aceite en estas mismas profundidades $(P_{cap\ wo\ z2}, P_{cap\ wo\ z3}, P_{cap\ wo\ z4}, P_{cap\ wo\ z5}, \dots)$.

Al determinar esta distribución vertical de presiones capilares, lo que estamos haciendo es identificar las cinco zonas de distribución de fluidos, por ejemplo, graficando las presiones capilares respecto a la profundidad:

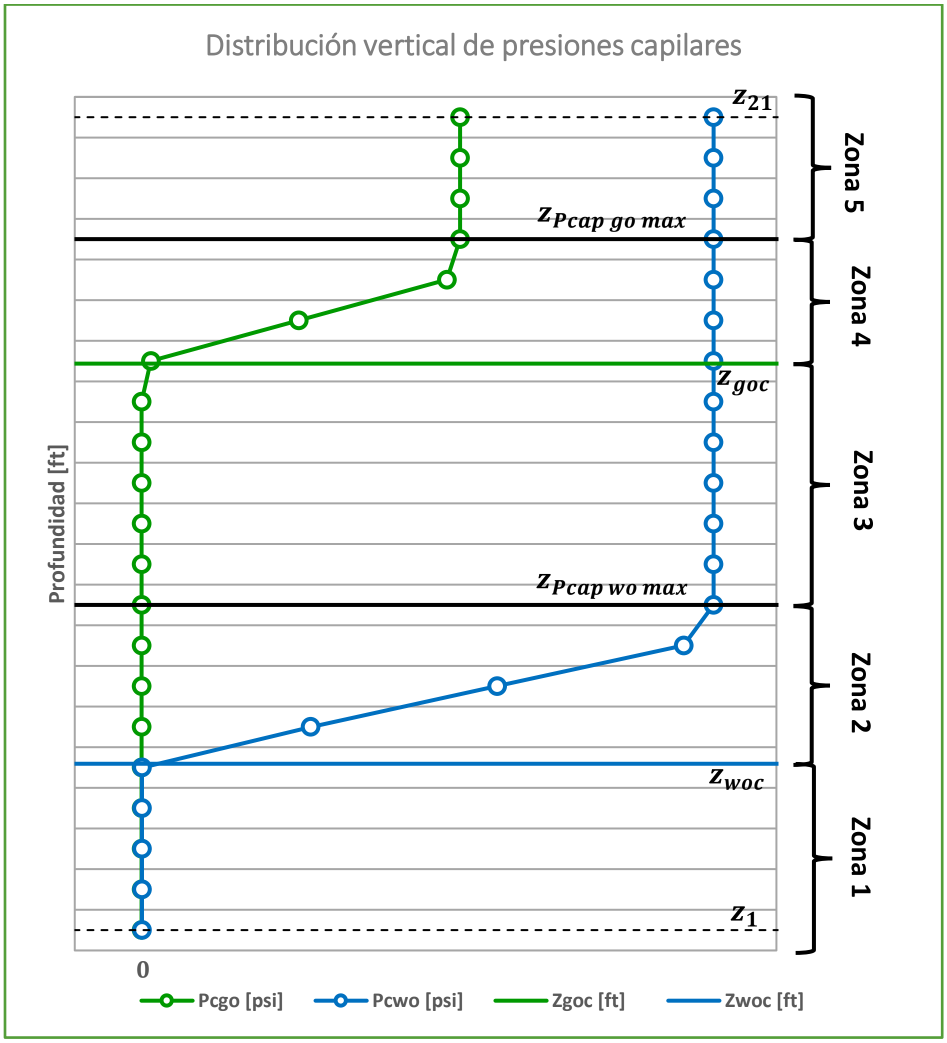

Fig. 8. "Corrección de presiones capilares".

Donde:

a) En la zona 1 (conocida como zona de agua, debajo del contacto agua-aceite), únicamente se tendrá presencia de agua, por ello la presión capilar sea igual a cero, ya que hay presencia de una única fase:

Zona 1
$S_{at\ w} = 1$
$S_{at\ g} = 0$
$S_{at\ o} = 0$
$P_{cap\ wo} = P_{cap\ wo\ min} = 0$

Tabla 2. "Propiedades de la Zona 1".

b) En la zona 2, (conocida como zona de transición entre el agua y aceite) se tendrá flujo de las fases de aceite y agua, hasta el punto en que la saturación de agua se vuelva irreductible $(S_{at\,wi})$, que justamente será la profundidad $(z_{Pcap\,wo\,max})$ a la que nuestra presión capilar agua-aceite sea el máximo posible $(P_{cap\,wo\,max})$. Como se puede observar en la curva de presiones capilares agua-aceite (Fig. 6) cuando la saturación de agua se vuelve irreductible $(S_{at\,wi})$ la presión capilar agua-aceite $(P_{cap\,wo})$ que tendremos en ese punto será la máxima posible $(P_{cap\,wo\,max})$ y por ello que la distribución vertical de presiones capilares que se realizó anteriormente nos ayude a determinar las zonas de transición:

Zona 2
$S_{at\,w} = S_{at\,w}$
$S_{at\,g} = 0$
$S_{at\,o} = 1 - S_{at\,g} - S_{at\,w}$
$P_{cap\,wo} = P_o - P_w$

Tabla 3. "Propiedades de la Zona 2".

c) En la zona 3, (conocida como zona de aceite) se tendrá flujo únicamente de la fase de aceite, ya que en este punto la saturación de agua se vuelve irreductible $(S_{at\,wi})$, lo que indica que la fase de agua no puede moverse:

Zona 3
$S_{at\,w} = S_{at\,wi}$
$S_{at\,g} = 0$
$S_{at\,o} = 1 - S_{at\,g} - S_{at\,w}$
$P_{cap\,wo} = P_o - P_w$

Tabla 4. "Propiedades de la Zona 3".

d) En la zona 4, (conocida como zona de transición entre el gas y aceite) se tendrá flujo de las fases de gas y aceite, hasta el punto en que la saturación de aceite se vuelva residual $(S_{at\,or})$, este punto será igual al punto en que la saturación de gas sea la máxima posible $(S_{at\,g\,max})$ basado en la curva de presiones capilares gas-aceite (Fig. 7), y este punto donde la saturación de gas es la máxima posible es justamente la profundidad $(z_{Pcap\,go\,max})$ a la que nuestra presión capilar gas-aceite sea el máximo posible $(P_{cap\,go\,max})$. Como se puede observar en la curva de presiones capilares gas-aceite (Fig. 7) cuando la saturación de gas se vuelve la máxima posible $(S_{at\,g\,max})$ la presión capilar gas-aceite $(P_{cap\,go})$ que tendremos en ese punto será la máxima posible $(P_{cap\,go\,max})$:

Zona 4
$S_{at\,w} = S_{at\,wi}$
$S_{at\,g} = S_{at\,g}$
$S_{at\,o} = 1 - S_{at\,g} - S_{at\,w}$
$P_{cap\,go} = P_g - P_o$

Tabla 5. "Propiedades de la Zona 4".

e) En la zona 5, (conocida como zona de gas) se tendrá flujo únicamente de la fase de gas, es decir, la saturación máxima de gas que se puede tener $(S_{at\ g\ max})$, ya que, en este punto, y hasta la cima del yacimiento, la saturación de aceite será residual $(S_{at\ or})$ y la de agua será irreductible $(S_{at\ wi})$, por lo que no existe movimiento de estas dos fases:

Zona 5
$S_{at\ w} = S_{at\ wi}$
$S_{at\ g} = S_{at\ g\ max}$
$S_{at\ o} = S_{at\ or} = 1 - S_{at\ g} - S_{at\ w}$
$P_{cap\ go} = P_g - P_o$

Tabla 6. "Propiedades de la Zona 5".

Una vez calculadas las presiones capilares, y definidas nuestras cinco zonas de distribución de fluidos, podemos ahora realizar la corrección de presiones de las fases de aceite, gas y agua, de la siguiente forma:

Para la fase de aceite:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Corregimos la presión de la fase de aceite, según los siguientes casos:

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 5\ \ Entonces: \quad P_o = P_g - P_{cap\ go} = P_g - P_{cap\ go\ max}$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 1\ \ Entonces: \quad P_o = P_{cap\ wo} + P_w = P_{cap\ wo\ min} + P_w = 0 + P_w = P_w$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 2, 3\ o\ 4\ \ Entonces: \quad P_o = P_o$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2 , z_3 , z_4 , z_5 , \ldots)$, corrigiendo así las presiones de la fase de aceite en estas mismas profundidades $\left(P_{o_2} , P_{o_3} , P_{o_4} , P_{o_5} , \ldots \right)$.

Para la fase de gas:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Corregimos la presión de la fase de gas, según los siguientes casos:

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 5 \qquad Entonces: \quad P_g = P_g$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 4 \qquad Entonces: \quad P_g = P_o + P_{cap\ go}$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 1, 2\ o\ 3\ \ Entonces: \quad P_g = P_o + P_{cap\ go} = P_o + P_{cap\ go\ min} = P_o + 0 = P_o$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2 , z_3 , z_4 , z_5 , \ldots)$, corrigiendo así las presiones de la fase de gas en estas mismas profundidades $\left(P_{g_2} , P_{g_3} , P_{g_4} , P_{g_5} , \ldots \right)$.

Para la fase de agua:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Corregimos la presión de la fase de agua, según los siguientes casos:

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 1$ $\quad Entonces:\ P_w = P_w$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 2$ $\quad Entonces:\ P_w = P_o - P_{cap\ wo}$

$Si\ z_1 se\ encuentra\ dentro\ de\ la\ zona\ 3, 4\ o\ 5$ $\quad Entonces:\ P_w = P_o - P_{cap\ wo} = P_o - P_{cap\ wo\ max}$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, corrigiendo así las presiones de la fase de agua en estas mismas profundidades $(P_{w_2}, P_{w_3}, P_{w_4}, P_{w_5}, \ldots)$.

El resultado de la corrección de las presiones, para las tres fases, nos daría como resultado lo siguiente:

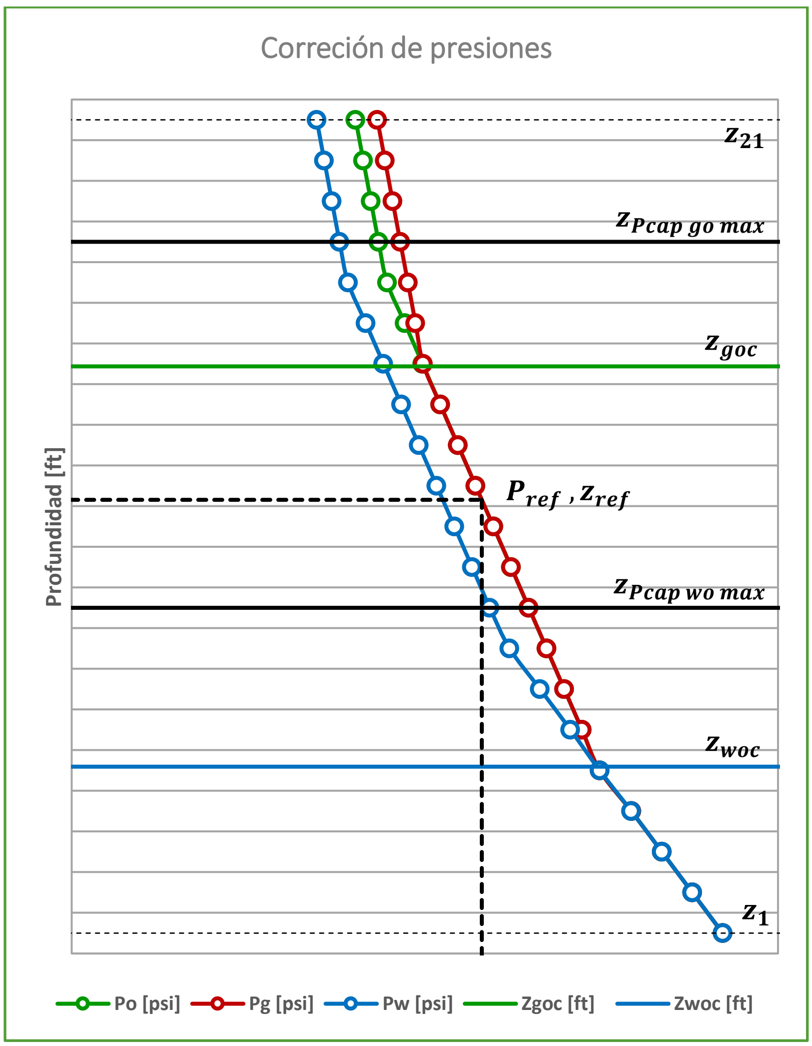

Fig. 9. "Corrección de distribución vertical de presiones".

Inicialización de saturaciones

La corrección anterior de las presiones de las fases de aceite, gas y agua, como se mencionó, se realizó gracias a que realizamos una distribución vertical de presiones capilares, que serán las que utilizaremos ahora para realizar la distribución inicial de saturaciones, esto es, a través de una interpolación inversas de las curvas de presiones capilares gas-aceite (Fig. 7) y agua-aceite (Fig. 6) de la siguiente forma:

Para la saturación de gas:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Seleccionamos nuestra presión capilar gas-aceite a esta profundidad $\left(P_{cap\ go\ z1}\right)$ y con ayuda de los datos de curvas de presión capilar gas-aceite determinamos la saturación de gas a través de una interpolación inversa:

$$S_{at\ g\ z1} = interpol\ inversa\ \left(P_{cap\ go\ z1}\right)$$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, obteniendo así las saturaciones de gas en estas mismas profundidades $\left(S_{at\ g\ z2}, S_{at\ g\ z3}, S_{at\ g\ z4}, S_{at\ g\ z5}, \ldots\right)$.

Para la saturación de agua:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Seleccionamos nuestra presión capilar agua-aceite a esta profundidad $\left(P_{cap\ wo\ z1}\right)$ y con ayuda de los datos de curvas de presión capilar agua-aceite determinamos la saturación de agua a través de una interpolación inversa:

$$S_{at\ w\ z1} = interpol\ inversa\ \left(P_{cap\ wo\ z1}\right)$$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, obteniendo así las saturaciones de agua en estas mismas profundidades $(S_{at\ w\ z2}, S_{at\ w\ z3}, S_{at\ w\ z4}, S_{at\ w\ z5}, \ldots)$.

Para la saturación de aceite:

a) Seleccionamos la primera profundidad de nuestra malla (z_1):

$$z = z_1$$

b) Seleccionamos nuestras saturaciones de gas $\left(S_{at\ g\ z1}\right)$ y agua ($S_{at\ w\ z1}$) en estas profundidades y determinamos la saturación de aceite ($S_{at\ o\ z1}$) en esta profundidad con la siguiente fórmula:

$$S_{at\ o\ z1} = 1 - S_{at\ g\ z1} - S_{at\ w\ z1}$$

c) Este proceso se repite pasando por cada una de las profundidades de la malla $(z_2, z_3, z_4, z_5, \ldots)$, obteniendo así las saturaciones de aceite en estas mismas profundidades $(S_{at\ o\ z2}, S_{at\ o\ z3}, S_{at\ o\ z4}, S_{at\ o\ z5}, \ldots)$.

El resultado de este procedimiento nos da como resultado la distribución vertical inicial de saturaciones en el yacimiento:

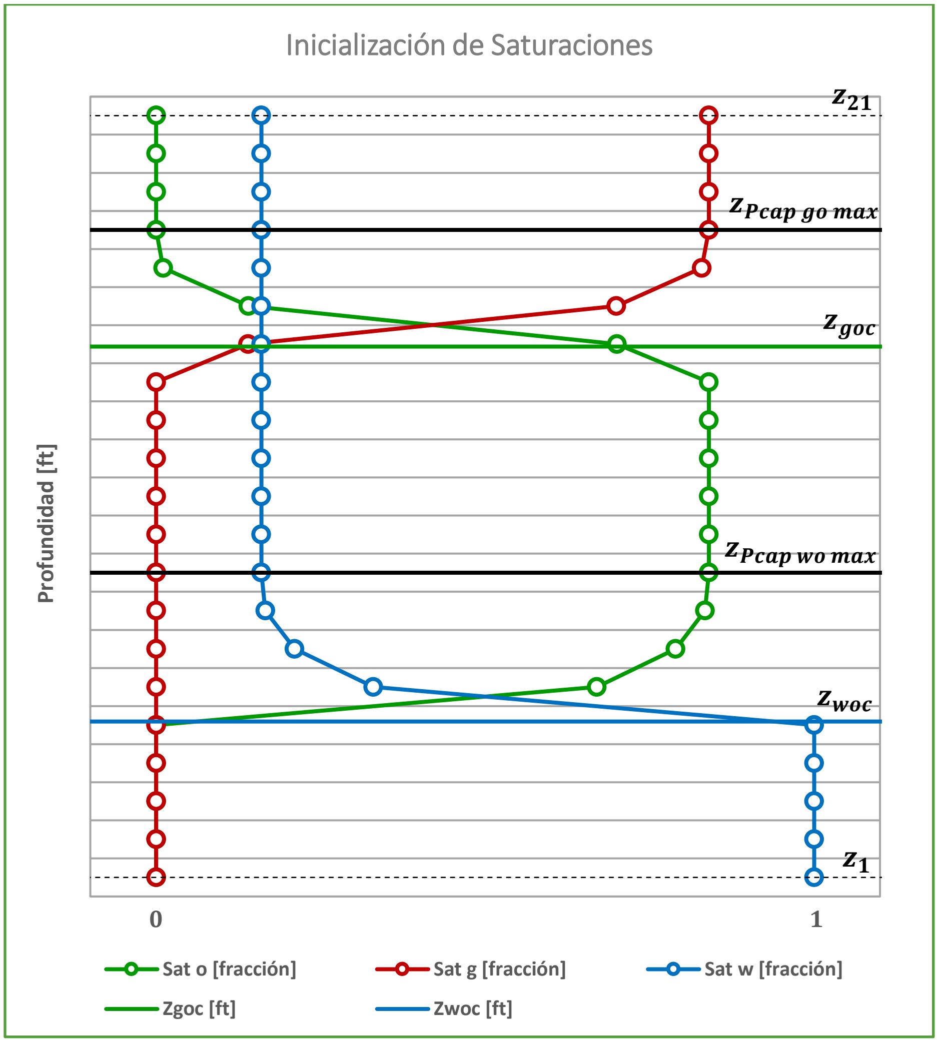

Fig. 10. "Inicialización de saturaciones".

Una vez obtenida la distribución vertical inicial de saturaciones es necesario hacer una corrección de estas saturaciones en las celdas donde se encuentren los contactos gas-aceite y agua-aceite:

<u>Corrección de la celda que contiene el contacto gas-aceite</u>

a) Para calcular la saturación de gas en esta celda se debe de realizar un promedio ponderado por la altura de las saturaciones que existen en las dos zonas que se encuentran en esta celda (Reséndiz Torres & Peña Chaparro, 2005). La primera zona será la que está arriba del contacto gas-aceite (z_{goc}) hasta la primera frontera de nuestra celda (z_{front1}), la segunda zona será la que está debajo del contacto gas-aceite hasta la segunda frontera de nuestra celda (z_{front2}).

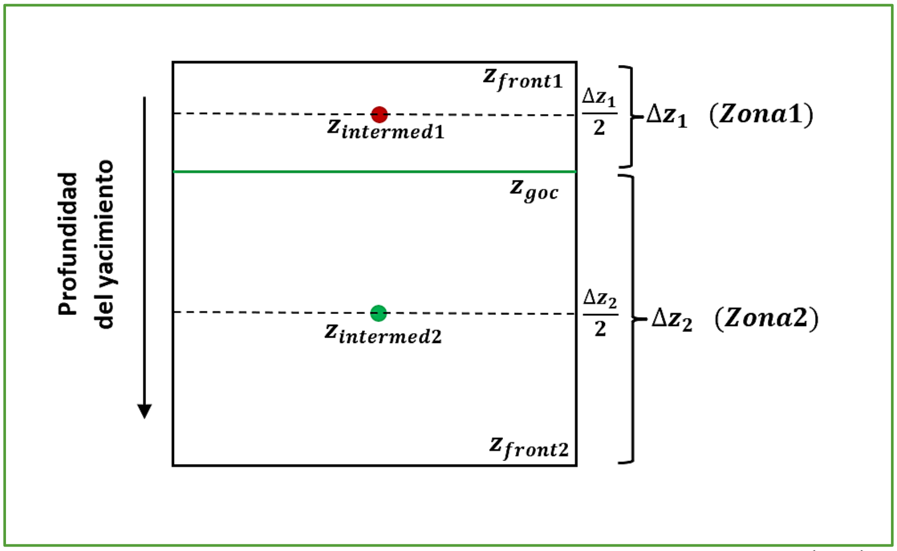

Fig. 11. "Esquematización de la celda donde se encuentra el contacto gas-aceite (z_{goc})".

b) Calculamos el espesor de estas dos zonas de la siguiente forma:

$$\Delta z_1 = z_{goc} - z_{front1}$$
$$\Delta z_2 = z_{front2} - z_{goc}$$

c) Conocidos estos espesores, determinamos la profundidad intermedia de cada zona:

$$Profundidad\ intermedia\ de\ la\ zona\ 1\ (z_{intermed1}) = z_{front1} + \frac{\Delta z_1}{2}$$
$$Profundidad\ intermedia\ de\ la\ zona\ 2\ (z_{intermed2}) = z_{goc} + \frac{\Delta z_2}{2}$$

d) Con las profundidades intermedias de las dos zonas, calculamos las presiones de aceite y gas en estas profundidades a través de la distribución de presiones verticales de aceite y gas que ya tenemos, esto se puede realizar con una interpolación:

$$P_{o\ z_{intermed1}} = interpolación\ de\ P_o\ evaluada\ en\ z_{intermed1}$$
$$P_{o\ z_{intermed2}} = interpolación\ de\ P_o\ evaluada\ en\ z_{intermed2}$$
$$P_{g\ z_{intermed1}} = interpolación\ de\ P_g\ evaluada\ en\ z_{intermed1}$$
$$P_{g\ z_{intermed2}} = interpolación\ de\ P_g\ evaluada\ en\ z_{intermed2}$$

e) Con estas presiones, podemos calcular la presión capilar gas-aceite en las profundidades intermedias de las dos zonas:

$$P_{cap\ go\ \ z_{intermed1}} = (P_g - P_o)_{z_{intermed1}}$$
$$P_{cap\ go\ \ z_{intermed2}} = (P_g - P_o)_{z_{intermed2}}$$

f) Conocidas las presiones capilares gas-aceite en estas zonas intermedias, podemos determinar las saturaciones de gas en estas zonas, esto sería a través de una interpolación inversa de la curva de presiones capilares gas-aceite (Fig. 7):

$$S_{at\ g\ \ z_{intermed1}} = interpolación\ inversa\ de\ S_{at\ g}\ evaluada\ en\ \left(P_{cap\ go}\right)_{z_{intermed1}}$$
$$S_{at\ g\ \ z_{intermed2}} = interpolación\ inversa\ de\ S_{at\ g}\ evaluada\ en\ \left(P_{cap\ go}\right)_{z_{intermed2}}$$

g) En este punto, ya tenemos determinadas las saturaciones de gas en las profundidades intermedias de cada zona, entonces, ahora si podemos realizar el promedio ponderado por profundidades de la siguiente forma (basados en la definición de saturación de gas):

$$S_{at\ g\ celda} = \frac{V_g}{V_p} = \frac{\Delta x\ \Delta y\ \left[\left(\Delta z\ S_{at\ g}\right)_{zona\ 1} + \left(\Delta z\ S_{at\ g}\right)_{zona\ 2}\right] \emptyset}{\Delta x\ \Delta y\ \Delta z\ \emptyset} = \frac{\left(\Delta z\ S_{at\ g}\right)_{zona\ 1} + \left(\Delta z\ S_{at\ g}\right)_{zona\ 2}}{\Delta z}$$

Sustituyendo términos:

$$S_{at\ g\ celda} = \frac{\Delta z_1\ S_{at\ g\ \ z_{intermed1}} + \Delta z_2\ S_{at\ g\ \ z_{intermed2}}}{(\Delta z_1 + \Delta z_2)}$$

h) Calculada la saturación de gas promedio de la celda, finalmente calculamos la saturación de aceite promedio de la celda:

$$S_{at\ o\ celda} = 1 - S_{at\ g\ celda} - S_{at\ wi}$$

Recordando que la saturación de agua en esta sección es igual a la saturación de agua irreductible ($S_{at\ wi}$), ya que nos encontramos por arriba de la zona de transición agua-aceite. Este proceso se repite para la celda donde se encuentra el contacto agua-aceite.

<u>Corrección de la celda que contiene el contacto agua-aceite</u>

a) Definiendo las zonas de la celda donde se encuentra el contacto agua-aceite (z_{woc}):

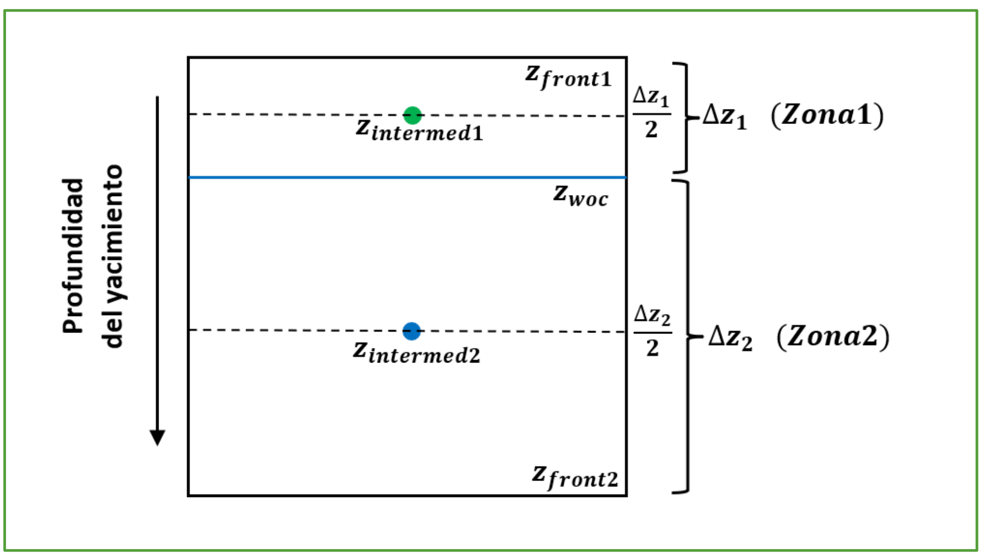

Fig. 12. "Esquematización de la celda donde se encuentra el contacto agua-aceite (z_{woc})".

b) Calculamos el espesor de estas dos zonas de la siguiente forma:

$$\Delta z_1 = z_{woc} - z_{front1}$$
$$\Delta z_2 = z_{front2} - z_{woc}$$

c) Determinando la profundidad intermedia de cada zona:

$$Profundidad\ intermedia\ de\ la\ zona\ 1\ (z_{intermed1}) = z_{front1} + \frac{\Delta z_1}{2}$$
$$Profundidad\ intermedia\ de\ la\ zona\ 2\ (z_{intermed2}) = z_{woc} + \frac{\Delta z_2}{2}$$

d) Con las profundidades intermedias de las dos zonas, calculamos las presiones de aceite y agua en estas profundidades a través de la distribución de presiones verticales de aceite y agua que ya tenemos, esto se puede realizar con una interpolación:

$$P_{o\ z_{intermed1}} = interpolación\ de\ P_o\ evaluada\ en\ z_{intermed1}$$
$$P_{o\ z_{intermed2}} = interpolación\ de\ P_o\ evaluada\ en\ z_{intermed2}$$
$$P_{w\ z_{intermed1}} = interpolación\ de\ P_w\ evaluada\ en\ z_{intermed1}$$
$$P_{w\ z_{intermed2}} = interpolación\ de\ P_w\ evaluada\ en\ z_{intermed2}$$

e) Con estas presiones, podemos calcular la presión capilar agua-aceite en las profundidades intermedias de las dos zonas:

$$P_{cap\ wo\ \ z_{intermed1}} = (P_o - P_w)_{z_{intermed1}}$$
$$P_{cap\ wo\ \ z_{intermed2}} = (P_o - P_w)_{z_{intermed2}}$$

f) Conocidas las presiones capilares agua-aceite en estas zonas intermedias, podemos determinar las saturaciones de agua en estas zonas, esto sería a través de una interpolación inversa de la curva de presiones capilares agua-aceite (Fig. 6):

$$S_{at\ w\ \ z_{intermed1}} = interpolación\ inversa\ de\ S_{at\ w}\ evaluada\ en\ \left(P_{cap\ wo}\right)_{z_{intermed1}}$$
$$S_{at\ w\ \ z_{intermed2}} = interpolación\ inversa\ de\ S_{at\ w}\ evaluada\ en\ \left(P_{cap\ wo}\right)_{z_{intermed2}}$$

g) En este punto, ya tenemos determinadas las saturaciones de agua en las profundidades intermedias de cada zona, entonces, ahora si podemos realizar el promedio ponderado por profundidades de la siguiente forma (basados en la definición de saturación de agua):

$$S_{at\ w\ celda} = \frac{V_w}{V_p} = \frac{\Delta x\ \Delta y\ [(\Delta z\ S_{at\ w})_{zona\ 1} + (\Delta z\ S_{at\ w})_{zona\ 2}]\emptyset}{\Delta x\ \Delta y\ \Delta z\ \emptyset} = \frac{(\Delta z\ S_{at\ w})_{zona\ 1} + (\Delta z\ S_{at\ w})_{zona\ 2}}{\Delta z}$$

Sustituyendo términos:

$$S_{at\ w\ celda} = \frac{\Delta z_1\ S_{at\ w\ \ z_{intermed1}} + \Delta z_2\ S_{at\ w\ \ z_{intermed2}}}{(\Delta z_1 + \Delta z_2)}$$

h) Calculada la saturación de agua promedio de la celda, finalmente calculamos la saturación de aceite promedio de la celda:

$$S_{at\ o\ celda} = 1 - S_{at\ w\ celda} - S_{at\ gr} = 1 - S_{at\ w\ celda}$$

Recordando que la saturación de gas en esta sección es igual a la saturación de gas residual $(S_{at\ gr})$, que a su vez es igual a cero, ya que nos encontramos por debajo de la zona de transición gas-aceite.

Realizando la corrección de estas zonas, finalmente tenemos nuestra distribución vertical de saturaciones correcta:

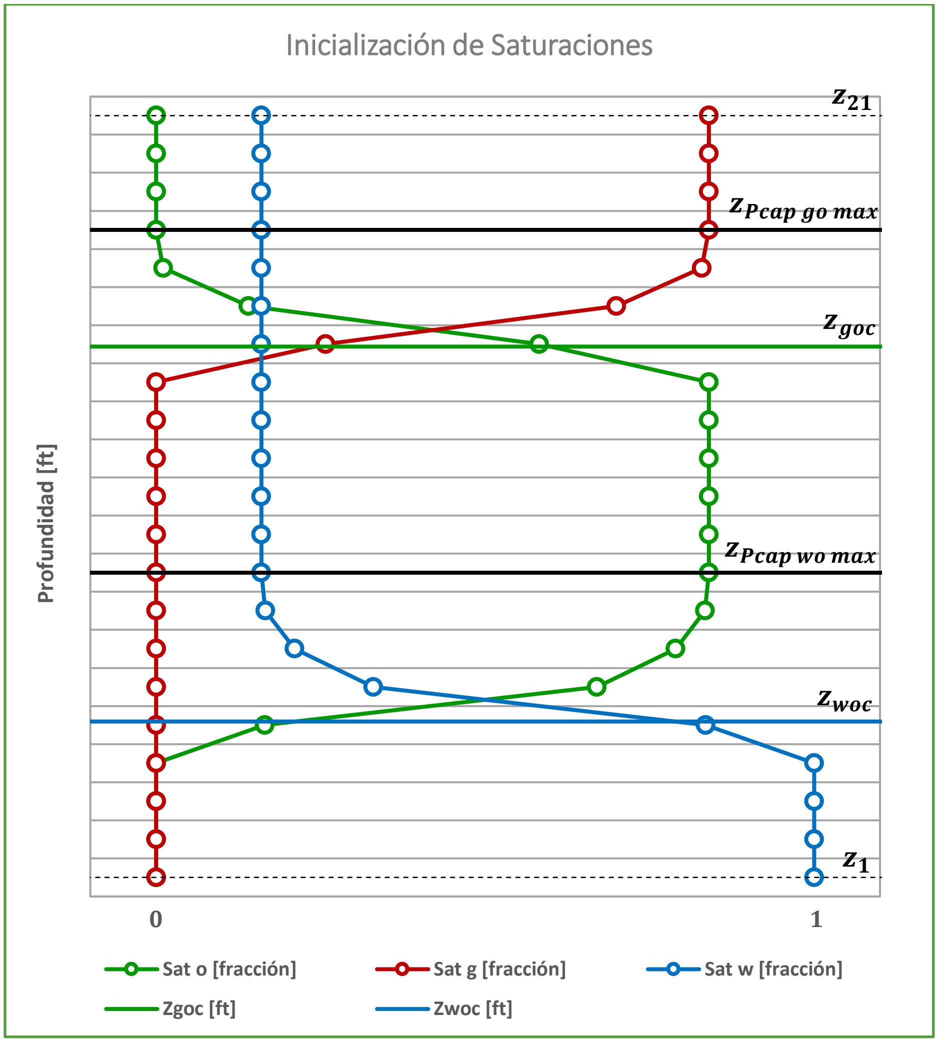

Fig. 13. "Inicialización de Saturaciones corregidas".

2.4. Condiciones de frontera

Las ecuaciones diferenciales que representan el comportamiento del flujo de fluidos dentro de un medio poroso (y con efectos de fracturas naturales) requieren que se especifique sus condiciones de fronteras, tanto internas o externas (de acuerdo al lugar donde se establece su valor).

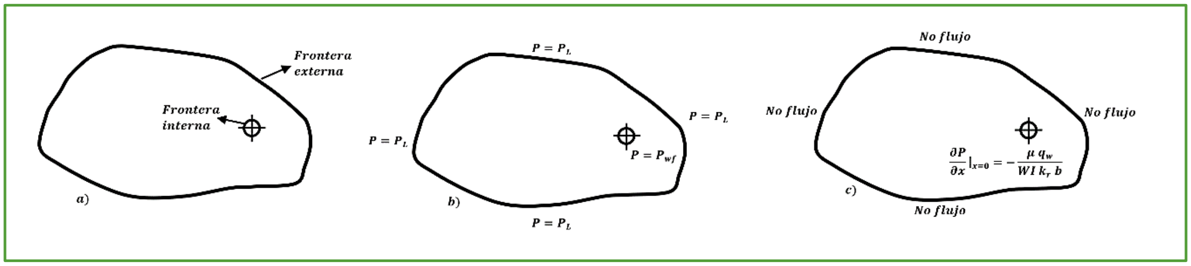

Fig. 14. "Esquematización de fronteras a) internas y externas, b) Dirichlet y c) Neumann". Modificado de (Ertekin, Jamal H., & R. King, 2001).

Condiciones tipo Dirichlet:

Las condiciones tipo Dirichlet describen el comportamiento de una variable en un punto definido (Gallardo Ferrera & Becerra Zepeda, 2016). Debido a que el modelo de simulación representa al yacimiento a través de una malla de puntos discretos, donde el pozo se encuentra circunscrito dentro del mismo, este se describe como una condición de frontera interna (CFI), mientras que en L $(Límite\ del\ yacimiento)$ se tienen condiciones de frontera externa (CFE).

Por ejemplo, **en un modelo de simulación con un modelo de pozo que produce a presión de fondo fluyendo constante** se puede representar como:

$$\boldsymbol{P|_{x=0} = P_{wf}}$$

Mientras que el mantenimiento de presión en el límite del yacimiento (L) se representa como:

$$P|_{x=L} = P_L$$

Condiciones tipo Neumann:

Las condiciones tipo Neumann especifican el comportamiento del gradiente de una variable en un punto cualquiera (Gallardo Ferrera & Becerra Zepeda, 2016). Por ejemplo, aplicando la ley de Darcy a través del modelo de pozo para un modelo de simulación de (Peaceman D. , 1978) en coordenadas cartesianas:

$$q = -WI\left(\frac{k_r b}{\mu}\right)\frac{\partial P}{\partial x}$$

$$q = -WI\left(\frac{k_r b}{\mu}\right)\left(P_{bloque} - P_{wf}\right)$$

$$WI = \frac{2\,\pi\,k\,h}{\ln\left(\frac{r_e}{r_w}\right)}$$

$$r_e = \frac{\Delta x}{\sqrt{\pi}}\,e^{\left(-\frac{1}{2}\right)}$$

Entonces, **en un modelo de simulación con un modelo de pozo que produce a gasto constante** q_w puede representarse como:

$$\frac{\partial P}{\partial x}|_{x=0} = (P_{bloque} - P_{wf})|_{x=0} = -\frac{\mu\, q_w}{WI\; k_r\; b}$$

Ahora, en el límite del yacimiento (L), generalmente se supone que no existe flujo en la frontera, donde $q_L = 0$:

$$\frac{\partial P}{\partial x}|_{x=0} = (P_{bloque} - P_{wf})|_{x=L} = -\frac{\mu\, q_L}{WI\; k_r\; b} = 0$$

3. Formulación numérica del problema

En este capítulo se presenta la formulación numérica de las seis ecuaciones de flujo (tres correspondientes a la fractura natural y tres correspondientes a la matriz), esto es, a través de la discretización de éstas, pasando así de un modelo matemático continuo a un modelo algebraico no lineal (Ertekin, Jamal H., & R. King, 2001) empleando el método de diferencias finitas, donde las variables primarias $(P_o, S_{at\,g}, S_{at\,w})$ para la fractura y la matriz se calculan en los puntos discretos de la malla, en este caso, considerando una malla cartesiana.

Además, se hace una pequeña descripción de la formulación totalmente implícita y del método de Newton-Raphson para la solución del sistema de ecuaciones no lineales, como resultado de la discretización del modelo. Así mismo, se describen los métodos de espaciamiento logarítmico, refinamiento local, y el modelado de un pozo horizontal, como una pequeña descripción para el caso de estudio, que se presentará en el capítulo siguiente.

3.1. Discretización del modelo

Dado que las ecuaciones desarrolladas son de tipo diferenciales no lineales, y por ello carecen de soluciones analíticas (Ertekin, Jamal H., & R. King, 2001), es necesario emplear métodos numéricos para darles solución, es así que, a través del método de diferencias finitas, se lleva a cabo la discretización de estas ecuaciones para las fases de aceite, gas y agua. Donde los términos de flujo se aproximan mediante diferencias finitas centrales en el espacio, y los términos de acumulación a través de diferencias finitas regresivas en el tiempo (Arana Ortiz, 1996).

Este método de diferencias finitas se implementa superponiendo una cuadrícula o malla de diferencias finitas sobre el yacimiento que se va a modelar, utilizado para aproximar las derivadas espaciales en las ecuaciones continuas, dando como resultado ecuaciones algebraicas llamadas ecuaciones en diferencias finitas, que solo se pueden obtener en los puntos discretos definidos por el sistema de cuadrícula o malla (Ertekin, Jamal H., & R. King, 2001), y por ello las variables primarias $(P_o\,,\ S_{at\,g}\,,\ S_{at\,w})$ calculadas por el simulador se conocerán únicamente en estos puntos.

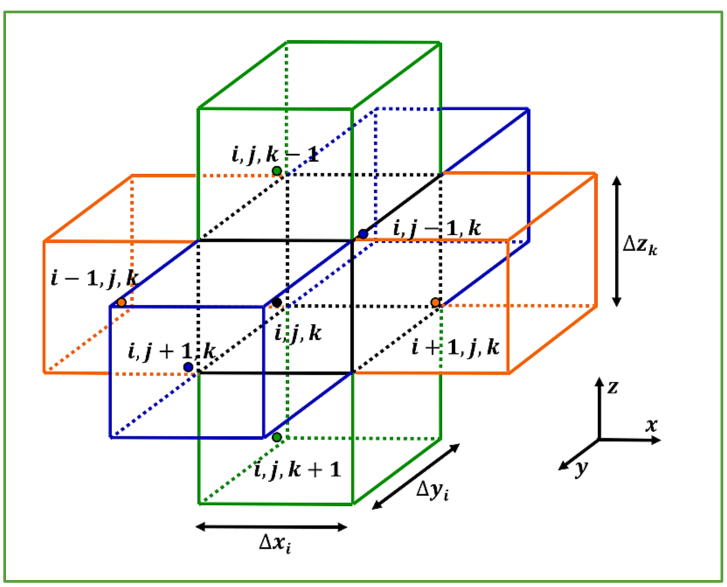

Fig. 15. "Esquema de celdas tridimensional en coordenadas cartesianas". Modificado de (Rodríguez de la Garza & Galindo Nava, 2000).

a) Ecuación diferencial para la fase de aceite en la fractura natural:

$$\frac{\partial}{\partial x}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x}+b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f+\frac{\partial}{\partial y}\left(b_o\frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y}+b_g r_s\frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f+\frac{\partial}{\partial z}\left(b_o\frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z}-\gamma_o\frac{\partial D}{\partial z}\right)+b_g r_s\frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z}-\gamma_g\frac{\partial D}{\partial z}\right)\right)_f$$
$$+b_{o_f}q_o^*+b_{g_f}r_{s_f}q_g^*+\tau_{o_{mf}}^*+\tau_{g_{mf}}^*r_{s_m}=\frac{\partial}{\partial t}\left(\emptyset\left(b_oS_{at\,o}+b_gr_sS_{at\,g}\right)\right)_f$$

Donde las funciones de transferencia matriz-fractura (Thomas, 1983) se definen como:

$$\tau_{o_{mf}}^*=\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m}-P_{o_f}\right)$$

$$\tau_{g_{mf}}^*=\left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m}-P_{g_f}\right)$$

Y σ es el factor de forma, o "shape factor" (Kazemi, 1976):

$$\sigma=4\left(\frac{1}{h_x^2}+\frac{1}{h_y^2}+\frac{1}{h_z^2}\right)$$

La aproximación de la primera derivada a través del método de diferencias finitas (Rodríguez de la Garza & Galindo Nava, 2000) se puede definir como:

Diferencias progresivas:

$$\frac{df}{dx}|_{x_i}=\frac{f_{i+1}-f_i}{\Delta x}+O_p(\Delta x)$$

Diferencias regresivas:

$$\frac{df}{dx}|_{x_i}=\frac{f_i-f_{i-1}}{\Delta x}+O_r(\Delta x)$$

Diferencias centrales:

$$\frac{df}{dx}|_{x_i}=\frac{f_{i+1}-f_{i-1}}{2\Delta x}+O_c(\Delta x^2)$$

De manera similar se definen las aproximaciones en diferencias finitas en el tiempo como:

Diferencias progresivas en el tiempo:

$$\frac{\partial p}{\partial t}|_i^n=\frac{p_i^{n+1}-p_i^n}{\Delta t}+O(\Delta t)$$

Diferencias regresivas en el tiempo:

$$\frac{\partial p}{\partial t}|_i^{n+1}=\frac{p_i^{n+1}-p_i^n}{\Delta t}+O(\Delta t)$$

Diferencias centrales en el tiempo:

$$\frac{\partial p}{\partial t}|_i^{n+\frac{1}{2}}=\frac{p_i^{n+1}-p_i^n}{\Delta t}+O(\Delta t^2)$$

Ahora, aproximando términos de la forma $\frac{\partial}{\partial x}\left(\lambda\frac{\partial p}{\partial x}\right)$:

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial p}{\partial x}\right)|_i\approx\frac{1}{\Delta x_i}\left[\left(\frac{\lambda}{\Delta x}\right)_{i+\frac{1}{2}}(p_{i+1}-p_i)-\left(\frac{\lambda}{\Delta x}\right)_{i-\frac{1}{2}}(p_i-p_{i-1})\right]$$

Con estas definiciones, se puede llevar a cabo la discretización de nuestra ecuación diferencial (multiplicada por el volumen de roca):

$$
\left[
\begin{array}{l}
\frac{1}{\Delta x_{i,j,k}}
\left[
\begin{array}{l}
\left(\frac{b_o k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_o k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)^{n+1} \\
+\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{g_{i+1,j,k}} - P_{g_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{g_{i,j,k}} - P_{g_{i-1,j,k}}\right)^{n+1}
\end{array}
\right]_f
+ \frac{1}{\Delta y_{i,j,k}}
\left[
\begin{array}{l}
\left(\frac{b_o k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{i,j+\frac{1}{2},k} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_o k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{i,j-\frac{1}{2},k} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)^{n+1} \\
+\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{i,j+\frac{1}{2},k} \left(P_{g_{i,j+1,k}} - P_{g_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{i,j-\frac{1}{2},k} \left(P_{g_{i,j,k}} - P_{g_{i,j-1,k}}\right)^{n+1}
\end{array}
\right]_f \\
+ \frac{1}{\Delta z_{i,j,k}}
\left[
\begin{array}{l}
\left(\frac{b_o k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{i,j,k+\frac{1}{2}} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1} \\
-\left(\frac{b_o k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{i,j,k-\frac{1}{2}} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1} \\
+\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{i,j,k+\frac{1}{2}} \left(P_{g_{i,j,k+1}} - P_{g_{i,j,k}} - (\gamma_g \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1} \\
-\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{i,j,k-\frac{1}{2}} \left(P_{g_{i,j,k}} - P_{g_{i,j,k-1}} - (\gamma_g \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1}
\end{array}
\right]_f
+ \left(b_{o_f} q_o^*\right)^{n+1}_{i,j,k} + \left(b_{g_f} r_{s_f} q_g^*\right)^{n+1}_{i,j,k} \\
+ \left[
\begin{array}{l}
\left(\frac{b_{om} \sigma k_m k_{r_{om}}}{\mu_{om}}\right)^{n+1}_{i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\
+\left(\frac{b_{gm} r_{s_m} \sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)^{n+1}_{i,j,k} \left(P_{g_m} - P_{g_f}\right)^{n+1}_{i,j,k}
\end{array}
\right] \\
= \frac{1}{\Delta t}\left[\left((\emptyset b_o S_{at\,o})^{n+1} - \left(\emptyset b_o S_{at\,o}\right)^{n}\right)_{i,j,k} + \left(\left(\emptyset b_g r_s S_{at\,g}\right)^{n+1} - \left(\emptyset b_g r_s S_{at\,g}\right)^{n}\right)_{i,j,k}\right]_f
\end{array}
\right] [\,Vr\,]_{i,j,k}
$$

Entonces:

$$
\begin{aligned}
&\left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_o k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{f\ i+\frac{1}{2},j,k} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{\ n+1} \\
&- \left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_o k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{f\ i-\frac{1}{2},j,k} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{\ n+1} \\
&+ \left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{f\ i+\frac{1}{2},j,k} \left(P_{g_{i+1,j,k}} - P_{g_{i,j,k}}\right)_f^{\ n+1} \\
&- \left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{f\ i-\frac{1}{2},j,k} \left(P_{g_{i,j,k}} - P_{g_{i-1,j,k}}\right)_f^{\ n+1} \\
&+ \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_o k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{f\ i,j+\frac{1}{2},k} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{\ n+1} \\
&- \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_o k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{\ n+1} \\
&+ \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{f\ i,j+\frac{1}{2},k} \left(P_{g_{i,j+1,k}} - P_{g_{i,j,k}}\right)_f^{\ n+1} \\
&- \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(P_{g_{i,j,k}} - P_{g_{i,j-1,k}}\right)_f^{\ n+1}
\end{aligned}
$$

$$+\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_o k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}}\left(P_{o_{i,j,k+1}}-P_{o_{i,j,k}}-(\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1}_f$$
$$-\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_o k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}}\left(P_{o_{i,j,k}}-P_{o_{i,j,k-1}}-(\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1}_f$$
$$+\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}}\left(P_{g_{i,j,k+1}}-P_{g_{i,j,k}}-(\gamma_g \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1}_f$$
$$-\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_g r_s k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}}\left(P_{g_{i,j,k}}-P_{g_{i,j,k-1}}-(\gamma_g \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1}_f$$
$$+\left(b_{o_f} q_o\right)^{n+1}_{i,j,k}$$
$$+\left(b_{g_f} r_{s_f} q_g\right)^{n+1}_{i,j,k}$$
$$+Vr_{i,j,k}\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)^{n+1}_{i,j,k}\left(P_{o_m}-P_{o_f}\right)^{n+1}_{i,j,k}$$
$$+Vr_{i,j,k}\left(\frac{b_{gm} r_{s_m}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)^{n+1}_{i,j,k}\left(P_{g_m}-P_{g_f}\right)^{n+1}_{i,j,k}$$
$$=\frac{Vr_{i,j,k}}{\Delta t}\left[\left((\emptyset b_o S_{at\ o})^{n+1}-(\emptyset b_o S_{at\ o})^{n}\right)_{i,j,k}+\left((\emptyset b_g r_s S_{at\ g})^{n+1}-(\emptyset b_g r_s S_{at\ g})^{n}\right)_{i,j,k}\right]_f$$

Con la ecuación de presión capilar gas-aceite:

$$P_{cap\ go}(S_{at\ g}) = P_g - P_o$$

Podemos despejar la presión de gas:

$$P_{g_f} = P_{cap_f\ go} + P_{o_f}$$
$$P_{g_m} = P_{cap_m\ go} + P_{o_m}$$

Con la definición de saturación de aceite:

$$S_{at\ o_f} = 1 - S_{at\ g_f} - S_{at\ w_f}$$
$$S_{at\ o_m} = 1 - S_{at\ g_m} - S_{at\ w_m}$$

Definiendo el volumen de roca:

En el eje x:
$Vr = A_x \Delta x$
$A_x = \Delta y \Delta z$

En el eje y:
$Vr = A_y \Delta y$
$A_y = \Delta x \Delta z$

En el eje z:
$Vr = A_z \Delta z$
$A_z = \Delta x \Delta y$

Donde:

$A_x = \text{Área de flujo transversal al eje } x$
$A_y = \text{Área de flujo transversal al eje } y$
$A_z = \text{Área de flujo transversal al eje } z$

Si la movilidad de las fases de aceite y gas se definen como (López Jiménez, 2017):

$$\lambda_o = \left(\frac{b_o k_{r_o}}{\mu_o}\right) \quad , \quad \lambda_g = \left(\frac{b_g k_{r_g}}{\mu_g}\right)$$

Empleando el promedio harmónico de permeabilidad (por ejemplo, en el eje x), de manera similar a la presentada por (Galindo Nava, 1998):

$$k_{i\pm\frac{1}{2},j,k} = \Delta x_{i\pm\frac{1}{2},j,k} \left[\frac{2\, k_{i,j,k}\, k_{i\pm1.j.k}}{k_{i\pm1,j,k}\, \Delta x_{i\,,j,k} + k_{i,j,k}\, \Delta x_{i\pm1,j,k}}\right]$$

Este mismo concepto se repite para los ejes *"y"* y *"z"*

Con la definición de porosidad en función de la compresibilidad (Chen, 2007):

$$\emptyset^{n+1} = \emptyset^{n}\left(1 + C_r\left(P_{i,j,k}^{n+1} - P_{i,j,k}^{n}\right)\right)$$

Y definiendo el volumen poroso como:

$$Vp_{i,j,k} = Vr_{i,j,k}\, \emptyset_{i,j,k}^{n}$$

Podemos sustituir los valores de presiones de gas por presiones capilares gas-aceite, saturaciones de aceite, volúmenes de roca, movilidades y porosidad:

$$
\begin{aligned}
&\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta x}\right)_{f\, i+\frac{1}{2},j,k}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
&- \left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta x}\right)_{f\, i-\frac{1}{2},j,k}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
&+ \left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta x}\right)_{f\, i+\frac{1}{2},j,k}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
&- \left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta x}\right)_{f\, i-\frac{1}{2},j,k}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
&+ \left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta y}\right)_{f\, i,j+\frac{1}{2},k}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
&- \left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta y}\right)_{f\, i,j-\frac{1}{2},k}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
&+ \left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta y}\right)_{f\, i,j+\frac{1}{2},k}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
&- \left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta y}\right)_{f\, i,j-\frac{1}{2},k}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
&+ \left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta z}\right)_{f\, i,j,k+\frac{1}{2}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - \left(\gamma_o\Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
&- \left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta z}\right)_{f\, i,j,k-\frac{1}{2}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - \left(\gamma_o\Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
&+ \left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta z}\right)_{f\, i,j,k+\frac{1}{2}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g\Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
&- \left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{r_s k\lambda_g}{\Delta z}\right)_{f\, i,j,k-\frac{1}{2}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g\Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
&+ \left(b_{o_f} q_o\right)_{i,j,k}^{n+1} \\
&+ \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1} \\
&+ Vr_{i,j,k}\ \left(\sigma k\lambda_o\right)_{m\, i,j,k}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1}
\end{aligned}
$$

$$+ Vr_{i,j,k}\ (\sigma r_s k\lambda_g)_{m\,i,j,k}^{n+1}\left(\left(P_{cap\,go}+P_o\right)_{m_{i,j,k}}-\left(P_{cap\,go}+P_o\right)_{f_{i,j,k}}\right)_{i,j,k}^{n+1}$$

$$= \frac{Vp_{i,j,k}}{\Delta t}\left[\begin{array}{c}\left(\ [1+C_r(P_o^{n+1}-P_o^n)]\left[b_o\left(1-S_{at\,g}-S_{at\,w}\right)\right]^{n+1}-\left[b_o\left(1-S_{at\,g}-S_{at\,w}\right)\right]^n\ \right)_{i,j,k}\\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1}-\left[b_g r_s S_{at\,g}\right]^n\ \right)_{i,j,k}\end{array}\right]_f$$

Ahora, definiendo los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o *"upstream weighting"* (Peaceman D. W., 1977) y (Aziz, 1979):

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i\pm1,j,k}}}\right)_{\boldsymbol{f}}\geq \boldsymbol{0}$$

$$T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}=\left(\frac{A_x k\lambda_o}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}=\left(\frac{A_x k}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\quad;\quad R_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1}=R_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1}$$

$$T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}=\left(\frac{A_x k\lambda_o}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}=\left(\frac{A_x k}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\quad;\quad R_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1}=R_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i\pm1,j,k}}}\right)_{\boldsymbol{f}}< \boldsymbol{0}$$

$$T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta x}\right)_{f\ i+1,j,k}^{n+1}=\left(\frac{A_x k\lambda_o}{\Delta x}\right)_{f\ i+1,j,k}^{n+1}=\left(\frac{A_x k}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i+1,j,k}^{n+1}\quad;\quad R_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1}=R_{s_{f_{i+1,j,k}}}^{n+1}$$

$$T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta x}\right)_{f\ i-1,j,k}^{n+1}=\left(\frac{A_x k\lambda_o}{\Delta x}\right)_{f\ i-1,j,k}^{n+1}=\left(\frac{A_x k}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i-1,j,k}^{n+1}\quad;\quad R_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1}=R_{s_{f_{i-1,j,k}}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j\pm1,k}}}\right)_{\boldsymbol{f}}\geq \boldsymbol{0}$$

$$T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}=\left(\frac{A_y k\lambda_o}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}=\left(\frac{A_y k}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\quad;\quad R_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1}=R_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1}$$

$$T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}=\left(\frac{A_y k\lambda_o}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}=\left(\frac{A_y k}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\quad;\quad R_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1}=R_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j\pm1,k}}}\right)_{\boldsymbol{f}}< \boldsymbol{0}$$

$$T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta y}\right)_{f\ i,j+1,k}^{n+1}=\left(\frac{A_y k\lambda_o}{\Delta y}\right)_{f\ i,j+1,k}^{n+1}=\left(\frac{A_y k}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j+1,k}^{n+1}\quad;\quad R_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1}=R_{s_{f_{i,j+1,k}}}^{n+1}$$

$$T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta y}\right)_{f\ i,j-1,k}^{n+1}=\left(\frac{A_y k\lambda_o}{\Delta y}\right)_{f\ i,j-1,k}^{n+1}=\left(\frac{A_y k}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j-1,k}^{n+1}\quad;\quad R_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1}=R_{s_{f_{i,j-1,k}}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k\pm1}}}\right)_{\boldsymbol{f}}-\left(\frac{\left(\boldsymbol{\gamma}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}+\boldsymbol{\gamma}_{\boldsymbol{o}_{\boldsymbol{i,j,k\pm1}}}\right)\left(\boldsymbol{D}_{\boldsymbol{i,j,k}}-\boldsymbol{D}_{\boldsymbol{i,j,k\pm1}}\right)}{\boldsymbol{2}}\right)\geq \boldsymbol{0}$$

$$T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}=\left(\frac{A_z k\lambda_o}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}=\left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\quad;\quad R_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1}=R_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1}$$

$$T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\ \left(\frac{k\lambda_o}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}=\left(\frac{A_z k\lambda_o}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}=\left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\quad;\quad R_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1}=R_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}-\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{i,j,k\pm1}}}\right)_{\boldsymbol{f}}-\left(\frac{\left(\boldsymbol{\gamma}_{\boldsymbol{o}_{\boldsymbol{i,j,k}}}+\boldsymbol{\gamma}_{\boldsymbol{o}_{\boldsymbol{i,j,k\pm1}}}\right)\left(\boldsymbol{D}_{\boldsymbol{i,j,k}}-\boldsymbol{D}_{\boldsymbol{i,j,k\pm1}}\right)}{\boldsymbol{2}}\right)< \boldsymbol{0}$$

$$T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta z}\right)_{f\ i,j,k+1}^{n+1} = \left(\frac{A_z k\lambda_o}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} \left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j,k+1}^{n+1} \quad ; \quad R_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = R_{s_{f_{i,j,k+1}}}^{n+1}$$

$$T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_o}{\Delta z}\right)_{f\ i,j,k-1}^{n+1} = \left(\frac{A_z k\lambda_o}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} \left(\frac{b_o k_{ro}}{\mu_o}\right)_{f\ i,j,k-1}^{n+1} \quad ; \quad R_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = R_{s_{f_{i,j,k-1}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i\pm1,j,k}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i\pm1,j,k}}}\right)_{\boldsymbol{f}} \geq \boldsymbol{0}$$

$$T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} = \left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1} = \left(\frac{A_x k\lambda_g}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1} = \left(\frac{A_x k}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i+\frac{1}{2},j,k}^{n+1} \quad ; \quad r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} = r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1}$$

$$T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} = \left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1} = \left(\frac{A_x k\lambda_g}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1} = \left(\frac{A_x k}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i-\frac{1}{2},j,k}^{n+1} \quad ; \quad r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} = r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i\pm1,j,k}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i\pm1,j,k}}}\right)_{\boldsymbol{f}} < \boldsymbol{0}$$

$$T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} = \left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i+1,j,k}^{n+1} = \left(\frac{A_x k\lambda_g}{\Delta x}\right)_{f\ i+1,j,k}^{n+1} = \left(\frac{A_x k}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i+1,j,k}^{n+1} \quad ; \quad r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} = r_{s_{f_{i+1,j,k}}}^{n+1}$$

$$T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} = \left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i-1,j,k}^{n+1} = \left(\frac{A_x k\lambda_g}{\Delta x}\right)_{f\ i-1,j,k}^{n+1} = \left(\frac{A_x k}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i-1,j,k}^{n+1} \quad ; \quad r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} = r_{s_{f_{i-1,j,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i,j\pm1,k}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i,j\pm1,k}}}\right)_{\boldsymbol{f}} \geq \boldsymbol{0}$$

$$T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} = \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1} = \left(\frac{A_y k\lambda_g}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1} = \left(\frac{A_y k}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j+\frac{1}{2},k}^{n+1} \quad ; \quad r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} = r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1}$$

$$T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} = \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1} = \left(\frac{A_y k\lambda_g}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1} = \left(\frac{A_y k}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j-\frac{1}{2},k}^{n+1} \quad ; \quad r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} = r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i,j\pm1,k}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i,j\pm1,k}}}\right)_{\boldsymbol{f}} < \boldsymbol{0}$$

$$T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} = \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)_{f\ i,j+1,k}^{n+1} = \left(\frac{A_y k\lambda_g}{\Delta y}\right)_{f\ i,j+1,k}^{n+1} = \left(\frac{A_y k}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j+1,k}^{n+1} \quad ; \quad r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} = r_{s_{f_{i,j+1,k}}}^{n+1}$$

$$T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} = \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)_{f\ i,j-1,k}^{n+1} = \left(\frac{A_y k\lambda_g}{\Delta y}\right)_{f\ i,j-1,k}^{n+1} = \left(\frac{A_y k}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j-1,k}^{n+1} \quad ; \quad r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} = r_{s_{f_{i,j-1,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i,j,k\pm1}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i,j,k\pm1}}}\right)_{\boldsymbol{f}} - \left(\frac{\left(\boldsymbol{\gamma_{g_{i,j,k}}} + \boldsymbol{\gamma_{g_{i,j,k\pm1}}}\right)\left(\boldsymbol{D_{i,j,k}} - \boldsymbol{D_{i,j,k\pm1}}\right)}{\boldsymbol{2}}\right) \geq \boldsymbol{0}$$

$$T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} = \left(\frac{A_z k\lambda_g}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} \quad ; \quad r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1}$$

$$T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} = \left(\frac{A_z k\lambda_g}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} \quad ; \quad r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{i,j,k}}} - \boldsymbol{P_{o_{i,j,k\pm1}}}\right)_{\boldsymbol{f}} + \left(\boldsymbol{P_{cap\ go_{i,j,k}}} - \boldsymbol{P_{cap\ go_{i,j,k\pm1}}}\right)_{\boldsymbol{f}} - \left(\frac{\left(\boldsymbol{\gamma_{g_{i,j,k}}} + \boldsymbol{\gamma_{g_{i,j,k\pm1}}}\right)\left(\boldsymbol{D_{i,j,k}} - \boldsymbol{D_{i,j,k\pm1}}\right)}{\boldsymbol{2}}\right) < \boldsymbol{0}$$

$$T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)_{f\ i,j,k+1}^{n+1} = \left(\frac{A_z k\lambda_g}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j,k+1}^{n+1} \quad ; \quad r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} = r_{s_{f_{i,j,k+1}}}^{n+1}$$

$$T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)_{f\ i,j,k-1}^{n+1} = \left(\frac{A_z k\lambda_g}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} = \left(\frac{A_z k}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g}\right)_{f\ i,j,k-1}^{n+1} \quad ; \quad r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} = r_{s_{f_{i,j,k-1}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_o)_{m\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\ i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{m_{i,j,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) < \boldsymbol{0}$$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_o)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\ i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{f_{i,j,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) + \left(\boldsymbol{P_{cap\ go_{m_{i,j,k}}}} - \boldsymbol{P_{cap\ go_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_g)_{m\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{m_{i,j,k}}}^{n+1}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) + \left(\boldsymbol{P_{cap\ go_{m_{i,j,k}}}} - \boldsymbol{P_{cap\ go_{f_{i,j,k}}}}\right) < \boldsymbol{0}$$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_g)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{f_{i,j,k}}}^{n+1}$$

Entonces, sustituyendo los términos de transmisibilidades, la ecuación discretizada para la fase de aceite en la fractura natural es:

$$\begin{aligned}
& T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; & T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
+ \; & T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i+1,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; & T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
+ \; & T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; & T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
+ \; & T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i,j+1,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; & T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
+ \; & T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; & T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \; & T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k+1} - \left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; & T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ & \left(b_{o_f} q_o\right)_{i,j,k}^{n+1} \\
+ & \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1}
\end{aligned}$$

$$
\begin{aligned}
&+ \; T^{n+1}_{omf_{i,j,k}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\
&+ \; T^{n+1}_{gmf_{i,j,k}} \; r^{n+1}_{s_{m i,j,k}} \; \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{\,i,j,k}} \right)^{n+1} \\
&= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)][b_o(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o(1 - S_{at\,g} - S_{at\,w})]^n \right)_{i,j,k} \\ + \left(\; [1 + C_r(P_o^{n+1} - P_o^n)][b_g r_s S_{at\,g}]^{n+1} - [b_g r_s S_{at\,g}]^n \right)_{i,j,k} \end{array} \right]_f
\end{aligned}
$$

b) Ecuación diferencial para la fase de gas en la fractura natural:

$$
\frac{\partial}{\partial x}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o \frac{\partial D}{\partial z}\right) + b_g \frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g \frac{\partial D}{\partial z}\right)\right)_f
$$

$$
+ \, b_{o_f} R_{s_f} q_o^* + b_{g_f} q_g^* + \tau^*_{o_{mf}} \, R_{s_m} + \tau^*_{g_{mf}} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_f
$$

Llevando a cabo el proceso de discretización (de manera similar a la fase de aceite):

$$
\left[
\begin{array}{l}
\frac{1}{\Delta x_{i,j,k}} \left[\begin{array}{l}
\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta x}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)^{n+1} \\
+\left(\frac{b_g k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{g_{i+1,j,k}} - P_{g_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_g k k_{r_g}}{\mu_g \Delta x}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{g_{i,j,k}} - P_{g_{i-1,j,k}}\right)^{n+1}
\end{array} \right]_f
+ \frac{1}{\Delta y_{i,j,k}} \left[\begin{array}{l}
\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{i,j+\frac{1}{2},k} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta y}\right)^{n+1}_{i,j-\frac{1}{2},k} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)^{n+1} \\
+\left(\frac{b_g k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{i,j+\frac{1}{2},k} \left(P_{g_{i,j+1,k}} - P_{g_{i,j,k}}\right)^{n+1} \\
-\left(\frac{b_g k k_{r_g}}{\mu_g \Delta y}\right)^{n+1}_{i,j-\frac{1}{2},k} \left(P_{g_{i,j,k}} - P_{g_{i,j-1,k}}\right)^{n+1}
\end{array} \right]_f \\
+ \frac{1}{\Delta z_{i,j,k}} \left[\begin{array}{l}
\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{i,j,k+\frac{1}{2}} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1} \\
-\left(\frac{b_o R_s k k_{r_o}}{\mu_o \Delta z}\right)^{n+1}_{i,j,k-\frac{1}{2}} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1} \\
+\left(\frac{b_g k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{i,j,k+\frac{1}{2}} \left(P_{g_{i,j,k+1}} - P_{g_{i,j,k}} - (\gamma_g \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1} \\
-\left(\frac{b_g k k_{r_g}}{\mu_g \Delta z}\right)^{n+1}_{i,j,k-\frac{1}{2}} \left(P_{g_{i,j,k}} - P_{g_{i,j,k-1}} - (\gamma_g \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1}
\end{array} \right]_f + \left(b_{o_f} R_{s_f} q_o^*\right)^{n+1}_{i,j,k} + \left(b_{g_f} q_g^*\right)^{n+1}_{i,j,k} \\
\qquad + \left[\begin{array}{l}
\left(\frac{b_{om} R_{s_m} \sigma k_m k_{r_{om}}}{\mu_{om}}\right)^{n+1}_{i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\
+\left(\frac{b_{gm} \sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)^{n+1}_{i,j,k} \left(P_{g_m} - P_{g_f}\right)^{n+1}_{i,j,k}
\end{array} \right] \\
= \frac{1}{\Delta t}\left[\left((\emptyset b_o R_s S_{at\,o})^{n+1} - \left(\emptyset b_o R_s S_{at\,o}\right)^n\right)_{i,j,k} + \left(\left(\emptyset b_g S_{at\,g}\right)^{n+1} - \left(\emptyset b_g S_{at\,g}\right)^n\right)_{i,j,k}\right]_f
\end{array}
\right] [\,Vr\,]_{i,j,k}
$$

Entonces:

$$
\begin{aligned}
&\left(\frac{Vr}{\Delta x}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(P_{o_{i+1,j,k}}-P_{o_{i,j,k}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta x}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(P_{o_{i,j,k}}-P_{o_{i-1,j,k}}\right)_f^{n+1}\\
&+\left(\frac{Vr}{\Delta x}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(P_{g_{i+1,j,k}}-P_{g_{i,j,k}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta x}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(P_{g_{i,j,k}}-P_{g_{i-1,j,k}}\right)_f^{n+1}\\
&+\left(\frac{Vr}{\Delta y}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(P_{o_{i,j+1,k}}-P_{o_{i,j,k}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta y}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(P_{o_{i,j,k}}-P_{o_{i,j-1,k}}\right)_f^{n+1}\\
&+\left(\frac{Vr}{\Delta y}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(P_{g_{i,j+1,k}}-P_{g_{i,j,k}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta y}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(P_{g_{i,j,k}}-P_{g_{i,j-1,k}}\right)_f^{n+1}\\
&+\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\left(P_{o_{i,j,k+1}}-P_{o_{i,j,k}}-(\gamma_o\Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_oR_skk_{r_o}}{\mu_o\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\left(P_{o_{i,j,k}}-P_{o_{i,j,k-1}}-(\gamma_o\Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}\\
&+\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\left(P_{g_{i,j,k+1}}-P_{g_{i,j,k}}-(\gamma_g\Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}\\
&-\left(\frac{Vr}{\Delta z}\right)_{i,j,k}\left(\frac{b_gkk_{r_g}}{\mu_g\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\left(P_{g_{i,j,k}}-P_{g_{i,j,k-1}}-(\gamma_g\Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}\\
&+\left(b_{o_f}R_{s_f}q_o\right)_{i,j,k}^{n+1}\\
&+\left(b_{g_f}q_g\right)_{i,j,k}^{n+1}\\
&+Vr_{i,j,k}\left(\frac{b_{om}R_{s_m}\sigma k_mk_{r_{om}}}{\mu_{om}}\right)_{i,j,k}^{n+1}\left(P_{o_m}-P_{o_f}\right)_{i,j,k}^{n+1}\\
&+Vr_{i,j,k}\left(\frac{b_{gm}\sigma k_mk_{r_{gm}}}{\mu_{gm}}\right)_{i,j,k}^{n+1}\left(P_{g_m}-P_{g_f}\right)_{i,j,k}^{n+1}\\
&=\frac{Vr_{i,j,k}}{\Delta t}\left[\left((\emptyset b_oR_sS_{at\,o})^{n+1}-\left(\emptyset b_oR_sS_{at\,o}\right)^n\right)_{i,j,k}+\left(\left(\emptyset b_gS_{at\,g}\right)^{n+1}-\left(\emptyset b_gS_{at\,g}\right)^n\right)_{i,j,k}\right]_f
\end{aligned}
$$

Ahora, sustituyendo valores de presiones de gas por presiones capilares gas-aceite, saturaciones de aceite, volúmenes de roca, movilidades y porosidad (de manera similar a la fase de aceite):

$$
\begin{aligned}
&\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{kR_s\lambda_o}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(P_{o_{i+1,j,k}}-P_{o_{i,j,k}}\right)_f^{n+1}\\
&-\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{kR_s\lambda_o}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(P_{o_{i,j,k}}-P_{o_{i-1,j,k}}\right)_f^{n+1}\\
&+\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(\left(P_{cap\,go}+P_o\right)_{i+1,j,k}-\left(P_{cap\,go}+P_o\right)_{i,j,k}\right)_f^{n+1}\\
&-\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_g}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(\left(P_{cap\,go}+P_o\right)_{i,j,k}-\left(P_{cap\,go}+P_o\right)_{i-1,j,k}\right)_f^{n+1}\\
&+\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\left(\frac{kR_s\lambda_o}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(P_{o_{i,j+1,k}}-P_{o_{i,j,k}}\right)_f^{n+1}
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{kR_s\lambda_o}{\Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
&+ \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)^{n+1}_{f\ i,j+\frac{1}{2},k} \left(\left(P_{cap\ go} + P_o\right)_{i,j+1,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
&- \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
&+ \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{kR_s\lambda_o}{\Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
&- \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{kR_s\lambda_o}{\Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
&+ \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k+1} - \left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
&- \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_g}{\Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
&+ \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_{i,j,k} \\
&+ \left(b_{g_f} q_g\right)^{n+1}_{i,j,k} \\
&+ Vr_{i,j,k} \ (\sigma k R_s \lambda_o)^{n+1}_{m\ i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\
&+ Vr_{i,j,k} \ (\sigma k \lambda_g)^{n+1}_{m\ i,j,k} \left(\left(P_{cap\ go} + P_o\right)_{m\ i,j,k} - \left(P_{cap\ go} + P_o\right)_{f\ i,j,k}\right)^{n+1}_{i,j,k} \\
&= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\ g} - S_{at\ w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\ g} - S_{at\ w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\ g}\right]^{n+1} - \left[b_g S_{at\ g}\right]^n \right)_{i,j,k} \end{array} \right]_f
\end{aligned}
$$

Sustituyendo los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), la ecuación discretizada para la fase de gas en la fractura natural es:

$$
\begin{aligned}
&\quad T^{n+1}_{o_{f_{i+\frac{1}{2},j,k}}} \ R^{n+1}_{s_{f_{i+\frac{1}{2},j,k}}} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
&- \ T^{n+1}_{o_{f_{i-\frac{1}{2},j,k}}} \ R^{n+1}_{s_{f_{i-\frac{1}{2},j,k}}} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
&+ \ T^{n+1}_{g_{f_{i+\frac{1}{2},j,k}}} \left(\left(P_{cap\ go} + P_o\right)_{i+1,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
&- \ T^{n+1}_{g_{f_{i-\frac{1}{2},j,k}}} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
&+ \ T^{n+1}_{o_{f_{i,j+\frac{1}{2},k}}} \ R^{n+1}_{s_{f_{i,j+\frac{1}{2},k}}} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
&- \ T^{n+1}_{o_{f_{i,j-\frac{1}{2},k}}} \ R^{n+1}_{s_{f_{i,j-\frac{1}{2},k}}} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
&+ \ T^{n+1}_{g_{f_{i,j+\frac{1}{2},k}}} \left(\left(P_{cap\ go} + P_o\right)_{i,j+1,k} - \left(P_{cap\ go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
&- \ T^{n+1}_{g_{f_{i,j-\frac{1}{2},k}}} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(P_{cap\ go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
&+ \ T^{n+1}_{o_{f_{i,j,k+\frac{1}{2}}}} \ R^{n+1}_{s_{f_{i,j,k+\frac{1}{2}}}} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
&- \ T^{n+1}_{o_{f_{i,j,k-\frac{1}{2}}}} \ R^{n+1}_{s_{f_{i,j,k-\frac{1}{2}}}} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
&+ \ T^{n+1}_{g_{f_{i,j,k+\frac{1}{2}}}} \left(\left(P_{cap\ go} + P_o\right)_{i,j,k+1} - \left(P_{cap\ go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}
\end{aligned}
$$

$$- \; T^{n+1}_{g_{f_{i,j,k-\frac{1}{2}}}} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}} \right)^{n+1}_f$$

$$+ \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_{i,j,k}$$

$$+ \left(b_{g_f} q_g\right)^{n+1}_{i,j,k}$$

$$+ \; T^{n+1}_{o_{mf_{i,j,k}}} \; R^{n+1}_{s_{m_{i,j,k}}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k}$$

$$+ \; T^{n+1}_{g_{mf_{i,j,k}}} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}} \right)^{n+1}$$

$$= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_{i,j,k} \end{array} \right]_f$$

c) Ecuación diferencial para la fase de agua en la fractura natural:

$$\frac{\partial}{\partial x}\left(b_w \frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_w \frac{kk_{r_w}}{\mu_w}\frac{\partial P_w}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_w \frac{kk_{r_w}}{\mu_w}\left(\frac{\partial P_w}{\partial z} - \gamma_w \frac{\partial D}{\partial z}\right)\right)_f + b_{w_f} q_w^* + \tau^*_{w_{mf}} = \frac{\partial}{\partial t}\left(\emptyset b_w S_{at\,w}\right)_f$$

Llevando a cabo el proceso de discretización (de manera similar a la fase de aceite y gas):

$$\left[\begin{array}{c} \frac{1}{\Delta x_{i,j,k}} \left[\begin{array}{l} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta x}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{w_{i+1,j,k}} - P_{w_{i,j,k}}\right)^{n+1} \\ - \left(\frac{b_w k k_{r_w}}{\mu_w \Delta x}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{w_{i,j,k}} - P_{w_{i-1,j,k}}\right)^{n+1} \end{array} \right]_f + \frac{1}{\Delta y_{i,j,k}} \left[\begin{array}{l} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta y}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{w_{i,j+1,k}} - P_{w_{i,j,k}}\right)^{n+1} \\ - \left(\frac{b_w k k_{r_w}}{\mu_w \Delta y}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{w_{i,j,k}} - P_{w_{i,j+1,k}}\right)^{n+1} \end{array} \right]_f \\ + \frac{1}{\Delta z_{i,j,k}} \left[\begin{array}{l} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta z}\right)^{n+1}_{i+\frac{1}{2},j,k} \left(P_{w_{i,j,k+1}} - P_{w_{i,j,k}} - \left(\gamma_w \Delta D\right)_{i,j,k+\frac{1}{2}}\right)^{n+1} \\ - \left(\frac{b_w k k_{r_w}}{\mu_w \Delta z}\right)^{n+1}_{i-\frac{1}{2},j,k} \left(P_{w_{i,j,k}} - P_{w_{i,j,k-1}} - \left(\gamma_w \Delta D\right)_{i,j,k-\frac{1}{2}}\right)^{n+1} \end{array} \right]_f + \left(b_{w_f} q_w^*\right)^{n+1}_{i,j,k} \\ + \left[\left(\frac{b_{wm} \sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)^{n+1}_{i,j,k} \left(P_{w_m} - P_{w_f}\right)^{n+1}_{i,j,k} \right] \\ = \frac{1}{\Delta t}\left[\left(\emptyset b_w S_{at\,w}\right)^{n+1}_{i,j,k} - \left(\emptyset b_w S_{at\,w}\right)^{n}_{i,j,k} \right]_f \end{array} \right] [\,Vr\,]_{i,j,k}$$

Entonces:

$$\left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta x}\right)^{n+1}_{f\; i+\frac{1}{2},j,k} \left(P_{w_{i+1,j,k}} - P_{w_{i,j,k}}\right)^{n+1}_f$$

$$- \left(\frac{Vr}{\Delta x}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta x}\right)^{n+1}_{f\; i-\frac{1}{2},j,k} \left(P_{w_{i,j,k}} - P_{w_{i-1,j,k}}\right)^{n+1}_f$$

$$+ \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta y}\right)^{n+1}_{f\; i,j+\frac{1}{2},k} \left(P_{w_{i,j+1,k}} - P_{w_{i,j,k}}\right)^{n+1}_f$$

$$- \left(\frac{Vr}{\Delta y}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(P_{w_{i,j,k}} - P_{w_{i,j+1,k}}\right)_f^{n+1}$$
$$+ \left(\frac{Vr}{\Delta z}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}} \left(P_{w_{i,j,k+1}} - P_{w_{i,j,k}} - (\gamma_w \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$
$$- \left(\frac{Vr}{\Delta z}\right)_{i,j,k} \left(\frac{b_w k k_{r_w}}{\mu_w \Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}} \left(P_{w_{i,j,k}} - P_{w_{i,j,k-1}} - (\gamma_w \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$
$$+ \left(b_{w_f} q_w\right)^{n+1}_{i,j,k}$$
$$+ Vr_{i,j,k} \left(\frac{b_{wm} \sigma k_m k_{r_{wm}}}{\mu_{wm}}\right)^{n+1}_{i,j,k} \left(P_{w_m} - P_{w_f}\right)^{n+1}_{i,j,k}$$
$$= \frac{Vr_{i,j,k}}{\Delta t} \left[(\emptyset b_w S_{at\,w})^{n+1}_{i,j,k} - (\emptyset b_w S_{at\,w})^{n}_{i,j,k}\right]_f$$

Con la ecuación de presión capilar agua-aceite:

$$P_{cap\ wo}\ (S_{at\ w}) = P_o - P_w$$

Podemos despejar la presión de agua:

$$P_{w_f} = P_{o_f} - P_{cap_f\ wo}$$
$$P_{w_m} = P_{o_m} - P_{cap_m\ wo}$$

Si la movilidad de la fase de agua se define como (López Jiménez, 2017):

$$\lambda_w = \left(\frac{b_w k_{r_w}}{\mu_w}\right)$$

Podemos sustituir los valores de presiones de agua por presiones capilares agua-aceite, volúmenes de roca, movilidades y porosidad (de manera similar a la fase de aceite y gas):

$$\left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta x}\right)^{n+1}_{f\ i+\frac{1}{2},j,k} \left(\left(P_o - P_{cap\ wo}\right)_{i+1,j,k} - \left(P_o - P_{cap\ wo}\right)_{i,j,k}\right)_f^{n+1}$$
$$- \left(\frac{A_x \Delta x}{\Delta x}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta x}\right)^{n+1}_{f\ i-\frac{1}{2},j,k} \left(\left(P_o - P_{cap\ wo}\right)_{i,j,k} - \left(P_o - P_{cap\ wo}\right)_{i-1,j,k}\right)_f^{n+1}$$
$$+ \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta y}\right)^{n+1}_{f\ i,j+\frac{1}{2},k} \left(\left(P_o - P_{cap\ wo}\right)_{i,j+1,k} - \left(P_o - P_{cap\ wo}\right)_{i,j,k}\right)_f^{n+1}$$
$$- \left(\frac{A_y \Delta y}{\Delta y}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta y}\right)^{n+1}_{f\ i,j-\frac{1}{2},k} \left(\left(P_o - P_{cap\ wo}\right)_{i,j,k} - \left(P_o - P_{cap\ wo}\right)_{i,j-1,k}\right)_f^{n+1}$$
$$+ \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta z}\right)^{n+1}_{f\ i,j,k+\frac{1}{2}} \left(\left(P_o - P_{cap\ wo}\right)_{i,j,k+1} - \left(P_o - P_{cap\ wo}\right)_{i,j,k} - (\gamma_w \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$
$$- \left(\frac{A_z \Delta z}{\Delta z}\right)_{i,j,k} \left(\frac{k\lambda_w}{\Delta z}\right)^{n+1}_{f\ i,j,k-\frac{1}{2}} \left(\left(P_o - P_{cap\ wo}\right)_{i,j,k} - \left(P_o - P_{cap\ wo}\right)_{i,j,k-1} - (\gamma_w \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$
$$+ \left(b_{w_f} q_w\right)^{n+1}_{i,j,k}$$
$$+ Vr_{i,j,k}\ (\sigma k \lambda_w)^{n+1}_{m\ i,j,k} \left(\left(P_o - P_{cap\ wo}\right)_{m_{i,j,k}} - \left(P_o - P_{cap\ wo}\right)_{f_{i,j,k}}\right)^{n+1}$$
$$= \frac{Vp_{i,j,k}}{\Delta t} \left[\ (\ [1 + C_r(P_o^{n+1} - P_o^{n})][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^{n}\ )_{i,j,k}\ \right]_f$$

Ahora, definiendo los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979):

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i\pm1,j,k}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i\pm1,j,k}}\right)_{\boldsymbol{f}}\geq\mathbf{0}$$

$$T_{w_{f_{i+\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}=\left(\frac{A_xk\lambda_w}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}=\left(\frac{A_xk}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}$$

$$T_{w_{f_{i-\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}=\left(\frac{A_xk\lambda_w}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}=\left(\frac{A_xk}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i\pm1,j,k}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i\pm1,j,k}}\right)_{\boldsymbol{f}}<\mathbf{0}$$

$$T_{w_{f_{i+\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta x}\right)_{f\ i+1,j,k}^{n+1}=\left(\frac{A_xk\lambda_w}{\Delta x}\right)_{f\ i+1,j,k}^{n+1}=\left(\frac{A_xk}{\Delta x}\right)_{f\ i+\frac{1}{2},j,k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i+1,j,k}^{n+1}$$

$$T_{w_{f_{i-\frac{1}{2},j,k}}}^{n+1}=\left(\frac{A_x\Delta x}{\Delta x}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta x}\right)_{f\ i-1,j,k}^{n+1}=\left(\frac{A_xk\lambda_w}{\Delta x}\right)_{f\ i-1,j,k}^{n+1}=\left(\frac{A_xk}{\Delta x}\right)_{f\ i-\frac{1}{2},j,k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i-1,j,k}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i,j\pm1,k}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j\pm1,k}}\right)_{\boldsymbol{f}}\geq\mathbf{0}$$

$$T_{w_{f_{i,j+\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}=\left(\frac{A_yk\lambda_w}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}=\left(\frac{A_yk}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}$$

$$T_{w_{f_{i,j-\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}=\left(\frac{A_yk\lambda_w}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}=\left(\frac{A_yk}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i,j\pm1,k}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j\pm1,k}}\right)_{\boldsymbol{f}}<\mathbf{0}$$

$$T_{w_{f_{i,j+\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta y}\right)_{f\ i,j+1,k}^{n+1}=\left(\frac{A_yk\lambda_w}{\Delta y}\right)_{f\ i,j+1,k}^{n+1}=\left(\frac{A_yk}{\Delta y}\right)_{f\ i,j+\frac{1}{2},k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j+1,k}^{n+1}$$

$$T_{w_{f_{i,j-\frac{1}{2},k}}}^{n+1}=\left(\frac{A_y\Delta y}{\Delta y}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta y}\right)_{f\ i,j-1,k}^{n+1}=\left(\frac{A_yk\lambda_w}{\Delta y}\right)_{f\ i,j-1,k}^{n+1}=\left(\frac{A_yk}{\Delta y}\right)_{f\ i,j-\frac{1}{2},k}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j-1,k}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i,j,k\pm1}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k\pm1}}\right)_{\boldsymbol{f}}-\left(\frac{\left(\boldsymbol{\gamma}_{\boldsymbol{w}_{i,j,k}}+\boldsymbol{\gamma}_{\boldsymbol{w}_{i,j,k\pm1}}\right)\left(\boldsymbol{D}_{i,j,k}-\boldsymbol{D}_{i,j,k\pm1}\right)}{\mathbf{2}}\right)\geq\mathbf{0}$$

$$T_{w_{f_{i,j,k+\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}=\left(\frac{A_zk\lambda_w}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}=\left(\frac{A_zk}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}$$

$$T_{w_{f_{i,j,k-\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}=\left(\frac{A_zk\lambda_w}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}=\left(\frac{A_zk}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{o}_{i,j,k\pm1}}\right)_{\boldsymbol{f}}-\left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k}}-\boldsymbol{P}_{\boldsymbol{cap\,wo}_{i,j,k\pm1}}\right)_{\boldsymbol{f}}-\left(\frac{\left(\boldsymbol{\gamma}_{\boldsymbol{w}_{i,j,k}}+\boldsymbol{\gamma}_{\boldsymbol{w}_{i,j,k\pm1}}\right)\left(\boldsymbol{D}_{i,j,k}-\boldsymbol{D}_{i,j,k\pm1}\right)}{\mathbf{2}}\right)<\mathbf{0}$$

$$T_{w_{f_{i,j,k+\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta z}\right)_{f\ i,j,k+1}^{n+1}=\left(\frac{A_zk\lambda_w}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}=\left(\frac{A_zk}{\Delta z}\right)_{f\ i,j,k+\frac{1}{2}}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j,k+1}^{n+1}$$

$$T_{w_{f_{i,j,k-\frac{1}{2}}}}^{n+1}=\left(\frac{A_z\Delta z}{\Delta z}\right)_{i,j,k}\left(\frac{k\lambda_w}{\Delta z}\right)_{f\ i,j,k-1}^{n+1}=\left(\frac{A_zk\lambda_w}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}=\left(\frac{A_zk}{\Delta z}\right)_{f\ i,j,k-\frac{1}{2}}^{n+1}\left(\frac{b_wk_{rw}}{\mu_w}\right)_{f\ i,j,k-1}^{n+1}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) - \left(\boldsymbol{P_{cap\,wo_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,wo_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$

$$T_{wmf_{i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_w)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)_{m\,i,j,k}^{n+1}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) - \left(\boldsymbol{P_{cap\,wo_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,wo_{f_{i,j,k}}}}\right) < \boldsymbol{0}$

$$T_{wmf_{i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_w)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)_{f\,i,j,k}^{n+1}$$

Entonces, sustituyendo los términos de transmisibilidades, la ecuación discretizada para la fase de agua en la fractura natural es:

$$\begin{aligned}
& T_{w f_{i+\frac{1}{2},j,k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i+1,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k}\right)_f^{n+1} \\
- \; & T_{w f_{i-\frac{1}{2},j,k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i-1,j,k}\right)_f^{n+1} \\
+ \; & T_{w f_{i,j+\frac{1}{2},k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j+1,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k}\right)_f^{n+1} \\
- \; & T_{w f_{i,j-\frac{1}{2},k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j-1,k}\right)_f^{n+1} \\
+ \; & T_{w f_{i,j,k+\frac{1}{2}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k+1} - \left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(\gamma_w \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; & T_{w f_{i,j,k-\frac{1}{2}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k-1} - \left(\gamma_w \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ & \left(b_{w_f} q_w\right)_{i,j,k}^{n+1} \\
+ \; & T_{wmf_{i,j,k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_{i,j,k}} - \left(P_o - P_{cap\,wo}\right)_{f_{i,j,k}}\right)^{n+1} \\
= \; & \frac{Vp_{i,j,k}}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_{i,j,k} \right]_f
\end{aligned}$$

d) Ecuación diferencial para la fase de aceite en la matriz:

$$-\tau_{o_{mf}}^* - \tau_{g_{mf}}^* r_{s_m} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right)\right)_m$$

Llevando a cabo el proceso de discretización:

$$\left[\begin{array}{c}
-\left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)_{i,j,k}^{n+1}\left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
-\left(\frac{b_{gm} r_{s_m} \sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)_{i,j,k}^{n+1}\left(P_{g_m} - P_{g_f}\right)_{i,j,k}^{n+1} \\
= \frac{1}{\Delta t}\left[\left((\emptyset b_o S_{at\,o})^{n+1} - \left(\emptyset b_o S_{at\,o}\right)^n\right)_{i,j,k} + \left(\left(\emptyset b_g r_s S_{at\,g}\right)^{n+1} - \left(\emptyset b_g r_s S_{at\,g}\right)^n\right)_{i,j,k}\right]_m
\end{array}\right] [\,Vr\,]_{i,j,k}$$

Entonces:

$$- Vr_{i,j,k} \left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)_{i,j,k}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1}$$
$$- Vr_{i,j,k} \left(\frac{b_{gm} r_{s_m}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)_{i,j,k}^{n+1} \left(P_{g_m} - P_{g_f}\right)_{i,j,k}^{n+1}$$
$$= \frac{Vr_{i,j,k}}{\Delta t} \left[\left((\emptyset b_o S_{at\,o})^{n+1} - (\emptyset b_o S_{at\,o})^{n}\right)_{i,j,k} + \left((\emptyset b_g r_s S_{at\,g})^{n+1} - (\emptyset b_g r_s S_{at\,g})^{n}\right)_{i,j,k}\right]_m$$

Con la ecuación de presión capilar gas-aceite:

$$P_{cap\,go}(S_{at\,g}) = P_g - P_o$$

Podemos despejar la presión de gas:

$$P_{g_f} = P_{cap_f\,go} + P_{o_f}$$
$$P_{g_m} = P_{cap_m\,go} + P_{o_m}$$

Con la definición de saturación de aceite:

$$S_{at\,o_m} = 1 - S_{at\,g_m} - S_{at\,w_m}$$

Definiendo el volumen de roca:

En el eje x:
$Vr = A_x\,\Delta x$
$A_x = \Delta y \Delta z$

En el eje y:
$Vr = A_y\,\Delta y$
$A_y = \Delta x \Delta z$

En el eje z:
$Vr = A_z\,\Delta z$
$A_z = \Delta x \Delta y$

Donde:

$A_x = \acute{A}rea\ de\ flujo\ transversal\ al\ eje\ x$
$A_y = \acute{A}rea\ de\ flujo\ transversal\ al\ eje\ y$
$A_z = \acute{A}rea\ de\ flujo\ transversal\ al\ eje\ z$

Si la movilidad de las fases de aceite y gas se definen como (López Jiménez, 2017):

$$\lambda_o = \left(\frac{b_o k_{r_o}}{\mu_o}\right) \quad , \quad \lambda_g = \left(\frac{b_g k_{r_g}}{\mu_g}\right)$$

Empleando el promedio harmónico de permeabilidad (por ejemplo, en el eje x), de manera similar a la presentada por (Galindo Nava, 1998):

$$k_{i\pm\frac{1}{2},j,k} = \Delta x_{i\pm\frac{1}{2},j,k} \left[\frac{2\,k_{i,j,k}\,k_{i\pm1.j.k}}{k_{i\pm1,j,k}\,\Delta x_{i\,,j,k} + k_{i,j,k}\,\Delta x_{i\pm1,j,k}}\right]$$

Este mismo concepto se repite para los ejes *"y"* y *"z"*

Con la definición de porosidad en función de la compresibilidad (Chen, 2007):

$$\emptyset^{n+1} = \emptyset^{n}\left(1 + C_r\left(P_{i,j,k}^{n+1} - P_{i,j,k}^{n}\right)\right)$$

Y definiendo el volumen poroso como:

$$Vp_{i,j,k} = Vr_{i,j,k}\, \emptyset_{i,j,k}^{n}$$

Podemos sustituir los valores de presiones de gas por presiones capilares gas-aceite, saturaciones de aceite, volúmenes de roca, movilidades y porosidad:

$$\begin{aligned}
&- Vr_{i,j,k}\ (\sigma k \lambda_o)_{m\,i,j,k}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
&- Vr_{i,j,k}\ (\sigma r_s k \lambda_g)_{m\,i,j,k}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m\,i,j,k} - \left(P_{cap\,go} + P_o\right)_{f\,i,j,k}\right)^{n+1} \\
&= \frac{Vp_{i,j,k}}{\Delta t} \begin{bmatrix} \left([1 + C_r(P_o^{n+1} - P_o^{n})][b_o(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o(1 - S_{at\,g} - S_{at\,w})]^{n} \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^{n})][b_g r_s S_{at\,g}]^{n+1} - [b_g r_s S_{at\,g}]^{n} \right)_{i,j,k} \end{bmatrix}_m
\end{aligned}$$

Ahora, definiendo los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979):

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m\,i,j,k}}} - \boldsymbol{P_{o_{f\,i,j,k}}}\right) \geq \boldsymbol{0}$

$$T_{o_{mf\,i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\,i,j,k}^{n+1} \quad ; \quad R_{s_{m\,i,j,k}}^{n+1} = R_{s_{m\,i,j,k}}^{n+1}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m\,i,j,k}}} - \boldsymbol{P_{o_{f\,i,j,k}}}\right) < \boldsymbol{0}$

$$T_{o_{mf\,i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\,i,j,k}^{n+1} \quad ; \quad R_{s_{m\,i,j,k}}^{n+1} = R_{s_{f\,i,j,k}}^{n+1}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m\,i,j,k}}} - \boldsymbol{P_{o_{f\,i,j,k}}}\right) + \left(\boldsymbol{P_{cap\,go_{m\,i,j,k}}} - \boldsymbol{P_{cap\,go_{f\,i,j,k}}}\right) \geq \boldsymbol{0}$

$$T_{g_{mf\,i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_g)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\,i,j,k}^{n+1} \quad ; \quad r_{s_{m\,i,j,k}}^{n+1} = r_{s_{m\,i,j,k}}^{n+1}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m\,i,j,k}}} - \boldsymbol{P_{o_{f\,i,j,k}}}\right) + \left(\boldsymbol{P_{cap\,go_{m\,i,j,k}}} - \boldsymbol{P_{cap\,go_{f\,i,j,k}}}\right) < \boldsymbol{0}$

$$T_{g_{mf\,i,j,k}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_g)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\,i,j,k}^{n+1} \quad ; \quad r_{s_{m\,i,j,k}}^{n+1} = r_{s_{f\,i,j,k}}^{n+1}$$

Entonces, sustituyendo los términos de transmisibilidades, la ecuación discretizada para la fase de aceite en la matriz es:

$$\begin{aligned}
&- T_{o_{mf\,i,j,k}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
&- T_{g_{mf\,i,j,k}}^{n+1}\ r_{s_{m\,i,j,k}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m\,i,j,k} - \left(P_{cap\,go} + P_o\right)_{f\,i,j,k}\right)^{n+1} \\
&= \frac{Vp_{i,j,k}}{\Delta t} \begin{bmatrix} \left([1 + C_r(P_o^{n+1} - P_o^{n})][b_o(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o(1 - S_{at\,g} - S_{at\,w})]^{n} \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^{n})][b_g r_s S_{at\,g}]^{n+1} - [b_g r_s S_{at\,g}]^{n} \right)_{i,j,k} \end{bmatrix}_m
\end{aligned}$$

e) Ecuación diferencial para la fase de gas en la matriz:

$$- \tau^*_{o_{mf}} R_{s_m} - \tau^*_{g_{mf}} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_m$$

Llevando a cabo el proceso de discretización:

$$\left[\begin{array}{c} -\left(\frac{b_{om} R_{s_m} \sigma k_m k_{r_{om}}}{\mu_{om}}\right)^{n+1}_{i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\ -\left(\frac{b_{gm} \sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)^{n+1}_{i,j,k} \left(P_{g_m} - P_{g_f}\right)^{n+1}_{i,j,k} \\ = \frac{1}{\Delta t}\left[\left((\emptyset b_o R_s S_{at\,o})^{n+1} - \left(\emptyset b_o R_s S_{at\,o}\right)^n\right)_{i,j,k} + \left(\left(\emptyset b_g S_{at\,g}\right)^{n+1} - \left(\emptyset b_g S_{at\,g}\right)^n\right)_{i,j,k}\right]_m \end{array}\right] [\,Vr\,]_{i,j,k}$$

Entonces:

$$\begin{aligned} &- Vr_{i,j,k} \left(\frac{b_{om} R_{s_m} \sigma k_m k_{r_{om}}}{\mu_{om}}\right)^{n+1}_{i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\ &- Vr_{i,j,k} \left(\frac{b_{gm} \sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)^{n+1}_{i,j,k} \left(P_{g_m} - P_{g_f}\right)^{n+1}_{i,j,k} \\ &= \frac{Vr_{i,j,k}}{\Delta t} \left[\left((\emptyset b_o R_s S_{at\,o})^{n+1} - \left(\emptyset b_o R_s S_{at\,o}\right)^n\right)_{i,j,k} + \left(\left(\emptyset b_g S_{at\,g}\right)^{n+1} - \left(\emptyset b_g S_{at\,g}\right)^n\right)_{i,j,k}\right]_m \end{aligned}$$

Ahora, sustituyendo valores de presiones de gas por presiones capilares gas-aceite, saturaciones de aceite, volúmenes de roca, movilidades y porosidad (de manera similar a la fase de aceite):

$$\begin{aligned} &- Vr_{i,j,k} \ (\sigma R_s k \lambda_o)^{n+1}_{m\,i,j,k} \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\ &- Vr_{i,j,k} \ \left(\sigma k \lambda_g\right)^{n+1}_{m\,i,j,k} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{\,i,j,k}}\right)^{n+1} \\ &= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_{i,j,k} \end{array}\right]_m \end{aligned}$$

Sustituyendo los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), la ecuación discretizada para la fase de gas en la matriz es:

$$\begin{aligned} &- \ T^{n+1}_{o_{mf_{i,j,k}}} \ R^{n+1}_{s_{m_{i,j,k}}} \ \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\ &- \ T^{n+1}_{g_{mf_{i,j,k}}} \ \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{\,i,j,k}}\right)^{n+1} \\ &= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_{i,j,k} \end{array}\right]_m \end{aligned}$$

f) Ecuación diferencial para la fase de agua en la matriz:

$$- \tau^{*}_{w_{mf}} = \frac{\partial}{\partial t}(\emptyset b_w S_{at\,w})_m$$

Llevando a cabo el proceso de discretización:

$$\left[\begin{array}{c} -\left(\frac{b_{w_m}\sigma k_m k_{r_{wm}}}{\mu_{w_m}}\right)^{n+1}_{i,j,k}\left(P_{w_m} - P_{w_f}\right)^{n+1}_{i,j,k} \\ \\ = \frac{1}{\Delta t}\left[(\emptyset b_w S_{at\,w})^{n+1}_{i,j,k} - (\emptyset b_w S_{at\,w})^{n}_{i,j,k}\right]_m \end{array} \right] [\, Vr\,]_{i,j,k}$$

Entonces:

$$- Vr_{i,j,k}\left(\frac{b_{w_m}\sigma k_m k_{r_{wm}}}{\mu_{w_m}}\right)^{n+1}_{i,j,k}\left(P_{w_m} - P_{w_f}\right)^{n+1}_{i,j,k} = \frac{Vr_{i,j,k}}{\Delta t}\left[(\emptyset b_w S_{at\,w})^{n+1}_{i,j,k} - (\emptyset b_w S_{at\,w})^{n}_{i,j,k}\right]_m$$

Con la ecuación de presión capilar agua-aceite:

$$P_{cap\,wo}\,(S_{at\,w}) = P_o - P_w$$

Podemos despejar la presión de agua:

$$P_{w_f} = P_{o_f} - P_{cap_f\,wo}$$
$$P_{w_m} = P_{o_m} - P_{cap_m\,wo}$$

Si la movilidad de la fase de agua se define como (López Jiménez, 2017):

$$\lambda_w = \left(\frac{b_w k_{r_w}}{\mu_w}\right)$$

Ahora, sustituyendo los valores de presiones de agua por presiones capilares agua-aceite, volúmenes de roca, movilidades y porosidad:

$$- Vr_{i,j,k}\,(\sigma k \lambda_w)^{n+1}_{m\,i,j,k}\left(\left(P_o - P_{cap\,wo}\right)_{m\,i,j,k} - \left(P_o - P_{cap\,wo}\right)_{f\,i,j,k}\right)^{n+1} = \frac{Vp_{i,j,k}}{\Delta t}\left[\,(\,[1 + C_r(P_o^{n+1} - P_o^{n})][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^{n}\,)_{i,j,k}\,\right]_m$$

De manera similar a las fases de aceite y gas, se definen los términos de transmisibilidades, siendo evaluados en el nodo con mayor potencial, conocido como corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979):

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) - \left(\boldsymbol{P_{cap\,wo_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,wo_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$$

$$T^{n+1}_{w_{mf_{i,j,k}}} = Vr_{i,j,k}(\sigma k \lambda_w)^{n+1}_{m\,i,j,k} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)^{n+1}_{m\,i,j,k}$$

$$\boldsymbol{Si}: \left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) - \left(\boldsymbol{P_{cap\,wo_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,wo_{f_{i,j,k}}}}\right) < \boldsymbol{0}$$

$$T^{n+1}_{w_{mf_{i,j,k}}} = Vr_{i,j,k}(\sigma k \lambda_w)^{n+1}_{f\,i,j,k} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)^{n+1}_{f\,i,j,k}$$

Entonces, sustituyendo los términos de transmisibilidades, la ecuación discretizada para la fase de agua en la matriz es:

$$-\;T^{n+1}_{w_{mf_{i,j,k}}}\left(\left(P_o - P_{cap\,wo}\right)_{m_{i,j,k}} - \left(P_o - P_{cap\,wo}\right)_{f_{i,j,k}}\right)^{n+1} = \frac{Vp_{i,j,k}}{\Delta t}\left[\left(\left[1 + C_r(P_o^{n+1} - P_o^n)\right][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n\right)_{i,j,k}\right]_m$$

3.2. Formulación totalmente implícita

La formulación totalmente implícita del problema hace referencia a que los términos de transmisibilidad, propiedades de los fluidos, presiones y saturaciones serán incógnitas para cada paso de tiempo en nuestro sistema de ecuaciones, es decir, sabemos que nuestras variables primarías para una formulación trifásica son $P_o\,, S_{at\,g}\;y\;S_{at\,w}$, entonces, para obtener los valores de estas en un paso de tiempo (en $n+1$), estas variables primarias, así como las transmisibilidades y propiedades, serán dependientes de estas mismas al tiempo $n+1$, en otras palabras, para determinar el valor de $P_o^{n+1}\,,\,S_{at\,g}^{n+1}\;y\;S_{at\,w}^{n+1}$ así como las transmisibilidades $T_o^{n+1}\,, T_g^{n+1}\;y\;T_w^{n+1}$ es necesario saber los valores de nuestras variables primarias en este mismo paso de tiempo $\left(P_o^{n+1}\,,\,S_{at\,g}^{n+1}\;y\;S_{at\,w}^{n+1}\right)$, lo cual, convierte a todo el sistema en un sistema de ecuaciones altamente no lineal.

Si bien, existen métodos tales como el método IMPES (implícito en presiones y explícito en saturaciones), los cuales, ayudan a reducir la complejidad del sistema de ecuaciones, para este trabajo se decidió implementar una formulación totalmente implícita, ya que, de esta manera, se obtiene una solución más rigurosa del problema.

Así mismo, para resolver el problema de la alta no linealidad que implica trabajar con esta formulación, se decidió emplear el método de Newton-Raphson, el cual, ayuda linealizar nuestro sistema de ecuaciones, a través de la implementación una función de residuos expandida en una serie de Taylor y truncada en su primera derivada para la solución del sistema.

3.3. Método de Newton-Raphson

Una vez discretizadas las ecuaciones de flujo para la fractura y la matriz, se genera un conjunto de ecuaciones en diferencias finitas que describen el comportamiento del flujo de fluidos en el yacimiento, que constituyen un sistema algebraico de ecuaciones no lineales, donde su solución se obtiene a través del método de Newton-Raphson, generando así un sistema lineal de ecuaciones en cada iteración (Cortes Rubio, 2008).

Este método es el más utilizado para la solución del flujo de fluidos en medios porosos, debido a su estabilidad numérica en comparación con los métodos que emplean una discretización explícita en el tiempo, aunque una de sus principales desventajas es su alto costo computacional (Teja Juárez, 2018).

La aplicación del método de Newton-Raphson requiere la definición de una función de residuos, por ejemplo, la función de residuos para la fase de aceite en la fractura natural es la ecuación discretizada igualada a cero:

$$
F_{o_{f_{i,j,k}}}^{n+1} = \left[\begin{gathered}
T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \; \left(b_{o_f} q_o\right)_{i,j,k}^{n+1} \\
+ \; \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1} \\
+ \; T_{o_{mf_{i,j,k}}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
+ \; T_{g_{mf_{i,j,k}}}^{n+1} \, r_{s_{m_{i,j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
- \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{gathered} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_{i,j,k} \end{gathered} \right]_f
\end{gathered} \right] = 0
$$

La función de residuos se expande en series de Taylor (Teja Juárez, 2018) y se trunca hasta los términos de primer orden como sigue:

$$F(\,p + \delta p\,) \;\approx\; F(\,p\,) \;+\; \nabla F(\,p\,)\,\delta p$$

Esta ecuación puede escribirse de la siguiente forma:

$$[\,J\,]^{v} \;\; \overline{\delta p}^{\,v+1} \;=\; -\,\bar{F}^{v}(\,p\,)$$

Donde J es la matriz jacobiana cuyos elementos son submatrices que contienen las derivadas de la función de residuos respecto a cada una de las variables primarias $\left(P_o \,, S_{at\,g} \,, S_{at\,w} \right)$, $\overline{\delta p}^{v+1}$ es el vector de incógnitas y $\bar{F}^v$ es el vector que contiene la función de residuos (López Jiménez, 2017).

O escrito de forma matricial:

$$\begin{bmatrix} \left(\dfrac{\partial F_{o\,i,j,k}}{\partial P_{o\,i,j,j}}\right) & \left(\dfrac{\partial F_{o\,i,j,k}}{\partial S_{at\,g\,i,j,j}}\right) & \left(\dfrac{\partial F_{o\,i,j,k}}{\partial S_{at\,w\,i,j,j}}\right) \\ \left(\dfrac{\partial F_{g\,i,j,k}}{\partial P_{o\,i,j,j}}\right) & \left(\dfrac{\partial F_{g\,i,j,k}}{\partial S_{at\,g\,i,j,j}}\right) & \left(\dfrac{\partial F_{g\,i,j,k}}{\partial S_{at\,w\,i,j,j}}\right) \\ \left(\dfrac{\partial F_{w\,i,j,k}}{\partial P_{o\,i,j,j}}\right) & \left(\dfrac{\partial F_{w\,i,j,k}}{\partial S_{at\,g\,i,j,j}}\right) & \left(\dfrac{\partial F_{w\,i,j,k}}{\partial S_{at\,w\,i,j,j}}\right) \end{bmatrix}^v \begin{bmatrix} \delta P_o \\ \delta S_{at\,g} \\ \delta S_{at\,w} \end{bmatrix}^{v+1} = -\begin{bmatrix} F_o \\ F_g \\ F_w \end{bmatrix}^v$$

El proceso iterativo converge a la solución una vez que el valor absoluto de los cambios iterativos de las variables primarias en cada uno de los medios es menor que una tolerancia de convergencia establecida ε (López Jiménez, 2017)**.**

$$|\,\delta P_o^{v+1}\,| < \varepsilon_{P_o} \quad , \quad \left|\,\delta S_{at\,g}^{v+1}\,\right| < \varepsilon_{S_{at\,g}} \quad , \quad |\,\delta S_{at\,w}^{v+1}\,| < \varepsilon_{S_{at\,w}}$$

Obteniendo así, el resultado de nuestra variable primaria de la siguiente manera:

$$P_o^{v+1} = \delta P_o^{v+1} + P_o^v \quad , \quad S_{at\,g}^{v+1} = \delta S_{at\,g}^{v+1} + S_{at\,g}^v \quad , \quad S_{at\,w}^{v+1} = \delta S_{at\,w}^{v+1} + S_{at\,w}^v$$

Entonces el sistema de ecuaciones quedaría de la siguiente forma:

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & & \boldsymbol{D} & & & & \boldsymbol{F} & & & & \\ \boldsymbol{C} & \ddots & \ddots & & \ddots & & & & \ddots & & & \\ & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & & \\ \boldsymbol{E} & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & \\ & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots \\ & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & \\ & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \\ \boldsymbol{G} & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & \\ & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots \\ & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & \\ & & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots \\ & & & & \ddots & & & & \ddots & & \ddots & \ddots \end{bmatrix}^v \begin{bmatrix} \boldsymbol{\delta P_{o_f}} \\ \boldsymbol{\delta S_{at\,g_f}} \\ \boldsymbol{\delta S_{at\,w_f}} \\ \boldsymbol{\delta P_{o_m}} \\ \boldsymbol{\delta S_{at\,g_m}} \\ \boldsymbol{\delta S_{at\,w_m}} \end{bmatrix}^{v+1} = -\begin{bmatrix} \boldsymbol{F_{o_f}} \\ \boldsymbol{F_{g_f}} \\ \boldsymbol{F_{w_f}} \\ \boldsymbol{F_{o_m}} \\ \boldsymbol{F_{g_m}} \\ \boldsymbol{F_{w_m}} \end{bmatrix}^v$$

Como puede observarse, la matriz jacobiana está compuesta por siete bandas principales (A, B, C, D, E, F, G) correspondientes a las siete direcciones del proceso de discretización $(\ i,j,k\ ,\ i+1,j,k\ ,\ i-1,j,k\ ,\ i,j+1,k\ ,\ i,j-1,k\ ,\ i,j,k+1\ ,\ i,j,k-1\ )$, las cuales, cada una de ellas contiene cuatro submatrices de 3x3, correspondientes a:

1) Las derivadas de las funciones de residuos de las tres fases en la fractura natural $(F_o, F_g, F_w)_f$ respecto a las variables primarias de la fractura natural $(P_o,\ S_{at\,g},\ S_{at\,w})_f$

2) Las derivadas de las funciones de residuos de las tres fases en la fractura natural $(F_o, F_g, F_w)_f$ respecto a las variables primarias de la matriz $(P_o,\ S_{at\,g},\ S_{at\,w})_m$

3) Las derivadas de las funciones de residuos de las tres fases en la matriz $(F_o, F_g, F_w)_m$ respecto a las variables primarias de la fractura natural $(P_o,\ S_{at\,g},\ S_{at\,w})_f$

4) Las derivadas de las funciones de residuos de las tres fases en la matriz $(F_o, F_g, F_w)_m$ respecto a las variables primarias de la matriz $(P_o,\ S_{at\,g},\ S_{at\,w})_m$

Por ejemplo, la banda central A sería:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1)} & \mathbf{2)} \\ \mathbf{3)} & \mathbf{4)} \end{bmatrix}$$

Entonces:

$$\boldsymbol{A} = \begin{bmatrix}
\left(\frac{\partial F_{o\,i,j,k\,f}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,f}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{o\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{o\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,m}}\right) \\
\left(\frac{\partial F_{g\,i,j,k\,f}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,f}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{g\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{g\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,m}}\right) \\
\left(\frac{\partial F_{w\,i,j,k\,f}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,f}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{w\,i,j,k\,f}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{w\,i,j,k\,f}}{\partial S_{at\,w\,i,j,k\,m}}\right) \\
\left(\frac{\partial F_{o\,i,j,k\,m}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{o\,i,j,k\,m}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{o\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{o\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,m}}\right) \\
\left(\frac{\partial F_{g\,i,j,k\,m}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{g\,i,j,k\,m}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{g\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{g\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,m}}\right) \\
\left(\frac{\partial F_{w\,i,j,k\,m}}{\partial P_{o\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,f}}\right) & \left(\frac{\partial F_{w\,i,j,k\,m}}{\partial P_{o\,i,j,k\,m}}\right) & \left(\frac{\partial F_{w\,i,j,k\,m}}{\partial S_{at\,g\,i,j,k\,m}}\right) & \left(\frac{\partial F_{w\,i,j,k\,m}}{\partial S_{at\,w\,i,j,k\,m}}\right)
\end{bmatrix}$$

La definición de las seis bandas restantes que conforman la matriz jacobiana sería exactamente la misma a la banda central **A**, únicamente cambiarían los subíndices (i, j, k).

Sin embargo, podemos observar que el sistema de ecuaciones sería bastante extenso de resolver, con lo cual, se pueden llevar a cabo procesos de reducción, por ejemplo, aplicando el complemento de Schur para reducir el sistema de ecuaciones, dejando las variables principales de la matriz en función de las variables principales de la fractura natural (Reséndiz Torres & Peña Chaparro, 2005).

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} & & \boldsymbol{D} & & & & \boldsymbol{F} & & & & \\ \boldsymbol{C} & \ddots & \ddots & & \ddots & & & & \ddots & & & \\ & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & & \\ \boldsymbol{E} & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & \\ & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots \\ & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & \\ & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \\ \boldsymbol{G} & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & \\ & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots \\ & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & \\ & & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots \\ & & & & \ddots & & & & \ddots & & \ddots & \ddots \end{bmatrix}^{v} \begin{bmatrix} \delta \boldsymbol{P}_{o_f} \\ \delta \boldsymbol{S}_{at\, g_f} \\ \delta \boldsymbol{S}_{at\, w_f} \end{bmatrix}^{v+1} = - \begin{bmatrix} \boldsymbol{F}^{*}_{o_f} \\ \boldsymbol{F}^{*}_{g_f} \\ \boldsymbol{F}^{*}_{w_f} \end{bmatrix}^{v}$$

Donde:

$$\boldsymbol{A} = \begin{bmatrix} \left(\dfrac{\partial \boldsymbol{F}_{o\, i,j,k\, f}}{\partial \boldsymbol{P}_{o\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{o\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, g\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{o\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, w\, i,j,k\, f}}\right) \\ \left(\dfrac{\partial \boldsymbol{F}_{g\, i,j,k\, f}}{\partial \boldsymbol{P}_{o\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{g\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, g\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{g\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, w\, i,j,k\, f}}\right) \\ \left(\dfrac{\partial \boldsymbol{F}_{w\, i,j,k\, f}}{\partial \boldsymbol{P}_{o\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{w\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, g\, i,j,k\, f}}\right) & \left(\dfrac{\partial \boldsymbol{F}_{w\, i,j,k\, f}}{\partial \boldsymbol{S}_{at\, w\, i,j,k\, f}}\right) \end{bmatrix}$$

3.4. Espaciamiento logarítmico

El método de espaciamiento logarítmico, generalmente, es utilizado en modelos de simulación desarrollados en coordenadas cilíndricas, los cuales, emplean una malla logarítmica convencional de nodos distribuidos o de nodos centrados en la dirección radial (Rodríguez de la Garza & Galindo Nava, 2000).

Sin embargo, se ha demostrado que, el empleo del método de espaciamiento logarítmico puede ser implementado en coordenadas cartesianas, principalmente, para modelar de manera explícita efectos de fracturas hidráulicas (Cipolla, 2009) y (Rubin, 2010), presentando resultados bastantes satisfactorios.

Es por ello que, a continuación, se describe el método de espaciamiento logarítmico, basado de su implementación en coordenadas cilíndricas y extendido a su aplicación en el modelamiento de fracturas hidráulicas en coordenadas cartesianas, ya que, más adelante en la sección de "caso de estudio" de este escrito, se empleará para el modelado de fracturas hidráulicas.

Malla radial de nodos distribuidos

La distribución de los radios de los nodos, para una malla de nodos distribuidos (Rodríguez de la Garza & Galindo Nava, 2000) está dada como:

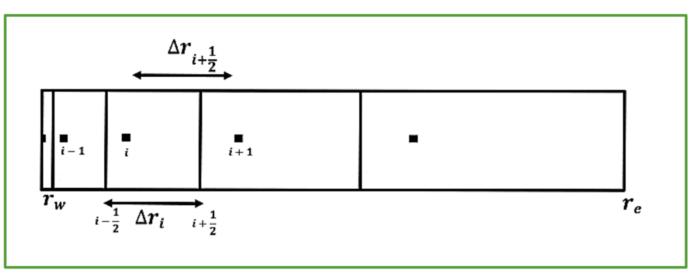

Fig. 16. "Malla radial logarítmica de nodos distribuidos". (Rodríguez de la Garza & Galindo Nava, 2000).

Donde los radios en cada nodo se calculan como:

$$\Delta u = \frac{1}{imax - 1} \ln\left(\frac{r_e}{r_w}\right)$$
$$r_i = r_w e^{\left(i-\frac{1}{2}\right)\Delta u}$$
$$r_{i+\frac{1}{2}} = r_w e^{(i)\Delta u}$$
$$r_{i-\frac{1}{2}} = r_w e^{(i-1)\Delta u}$$

$Donde$:

$r_w = radio\ del\ pozo$
$r_e = radio\ de\ drene$
$imax = número\ total\ de\ nodos$

Espaciamiento logarítmico en una malla cartesiana en "x"

Extendiendo estas ecuaciones, a un sistema cartesiano, para el modelamiento de fracturas hidráulicas:

$$\Delta u = \frac{1}{imax\_x} \ln\left(\frac{a}{b}\right)$$
$$x_i = b\ e^{\left(i-\frac{1}{2}\right)\Delta u}$$
$$x_{i+\frac{1}{2}} = b\ e^{(i)\Delta u}$$
$$x_{i-\frac{1}{2}} = b\ e^{(i-1)\Delta u}$$

$Donde$:

$$a = \frac{L_x}{2}$$
$$b = \frac{whf}{2}$$
$$imax\_x = \frac{imax - 1}{2}$$

$whf =$
$espesor\ de\ la\ fractura\ hidráulica\ simulada$
$L_x = Longitud\ del\ yacimiento\ en\ "x"$
$imax = número\ total\ de\ nodos$

Como se puede observar, para el modelo cartesiano algunos términos se encuentran divididos entre dos, esto se debe a que el espaciamiento se está haciendo del centro del espesor de la fractura hidráulica simulada (que a su vez será la mitad de la longitud total en "x"), hacia su frontera en "x" (L_x), es decir, solo se representa la mitad de la longitud total del yacimiento en "x", ya que la otra mitad sería exactamente el mismo, con lo que el espaciamiento para esta otra mitad sería el mismo que se calcula con las ecuaciones anteriormente descritas.

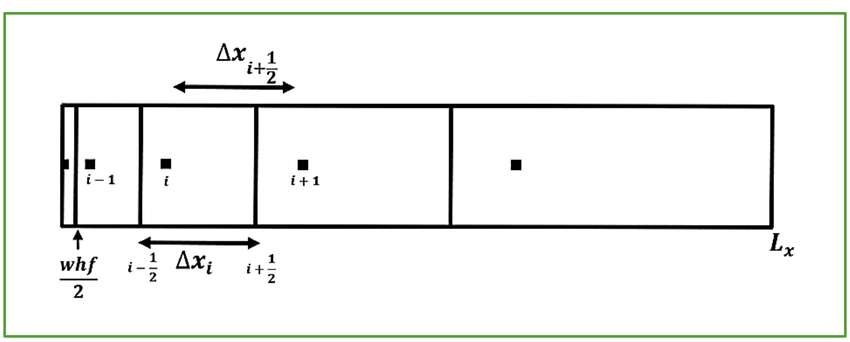

Fig. 17. "Espaciamiento logarítmico en dirección x".

3.5. Refinamiento local

Al llevar a cabo la simulación del comportamiento de un yacimiento, pueden existir áreas donde sea necesario tener una mejor aproximación para las incógnitas a resolver, como pueden ser áreas cercanas a los pozos, zonas de contactos de fluidos o zonas a disparase, donde una forma práctica para realizar dichas aproximaciones es llevar a cabo subdivisiones en el bloque de interés, es decir, se puede refinar la malla de simulación (Cedillo Trejo, 2014).

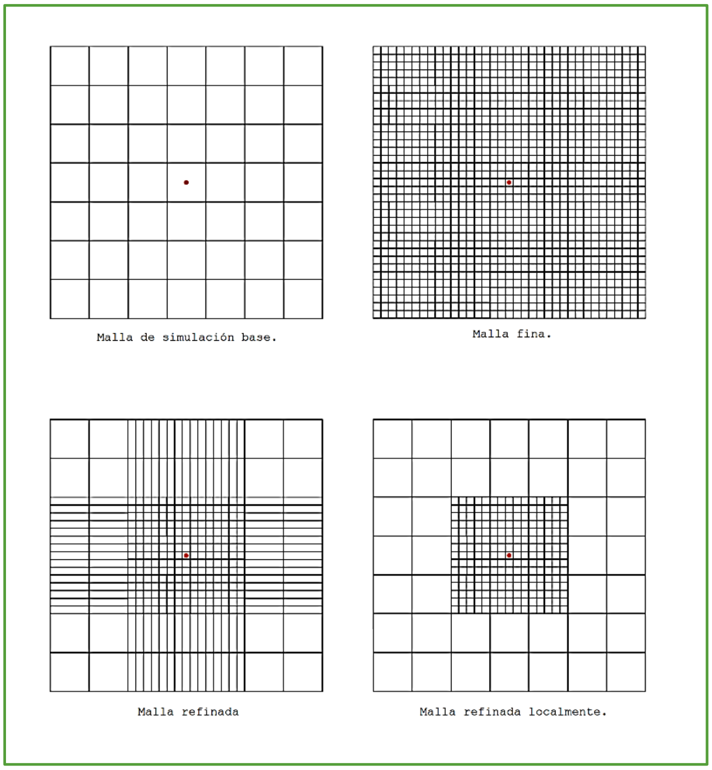

Fig. 18. "Ejemplos de mallas y refinamiento local" (Cedillo Trejo, 2014).

Es así que, el método de refinamiento local pudo extenderse para llevar a cabo la representación de fracturas hidráulicas en un yacimiento, demostrando resultados satisfactorios (Cipolla, 2009) y (Rubin, 2010). Dicha metodología emplea el concepto de doble permeabilidad para representar la red de fracturas naturales y la red de las fracturas hidráulicas correspondientes al volumen estimulado del yacimiento (SRV por sus siglas en inglés). Esta metodología es de tipo explícita y lleva por nombre método "DK-LS-LGR" (es decir, doble permeabilidad, espaciamiento logarítmico y refinamiento local).

Cabe resaltar que este modelo considera al volumen estimulado del yacimiento (o SRV) como únicamente el espesor de la fractura hidráulica simulada, es decir, la celda central (y por ello más refinada) a partir donde comienza el espaciamiento logarítmico (visto en la página anterior), con lo cual, únicamente estas celdas de la malla de simulación contemplarán las propiedades de la fractura hidráulica, mientras que las demás celdas contemplarán las propiedades de la fractura natural. Se ha demostrado que dicha suposición del SRV presenta resultados satisfactorios (Rubin, 2010).

Además, si únicamente se está simulando una única fractura natural, el espaciamiento logarítmico puede realizarse a lo largo de todo el yacimiento, y con ello, evitar trabajar con una mayor cantidad de celdas (teniendo de ejemplo el modelo realizado por (Alharthy, 2015) y que posteriormente (López Jiménez, 2017) implementaría una metodología similar), ya que, al hacerse el espaciamiento a lo largo del yacimiento, la cantidad de celdas que se trabajan sería la misma que una malla cartesiana convencional, donde, la única diferencia sería el espesor de las celdas que componen la malla.

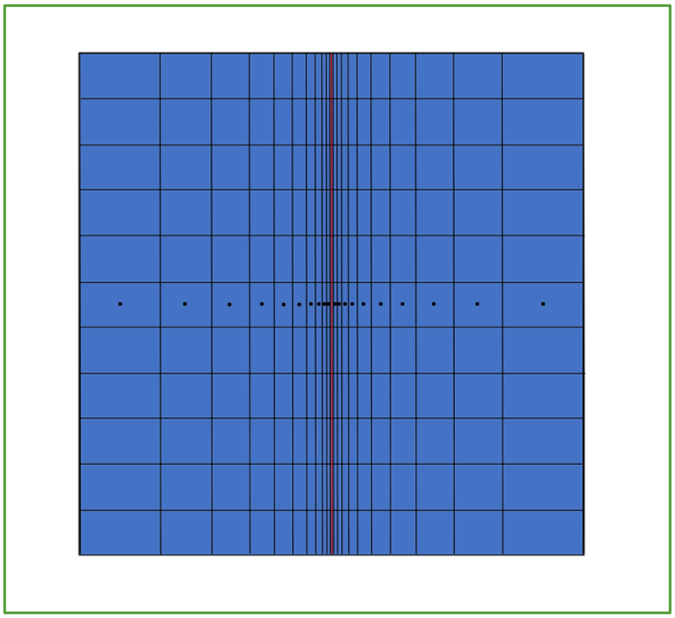

Fig. 19. "Ejemplo de malla con espaciamiento logarítmico". Modificado de (Alharthy, 2015).

3.6. <u>Modelado de un pozo horizontal</u>

En un modelo de simulación, es necesario contar con un término que nos ayude a representar el comportamiento de los pozos y la presión que se tiene de estos a la altura del intervalo disparado con el comportamiento del yacimiento (Reséndiz Torres & Peña Chaparro, 2005).

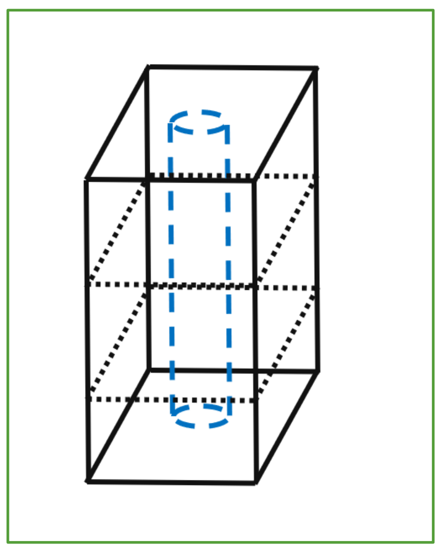

Fig. 20. "Representación esquemática de un pozo vertical en una celda de simulación". Modificado de (Ertekin, Jamal H., & R. King, 2001).

Es así que (Coats, George, & Marcum, 1974) emplearon la presión de fondo fluyendo (P_{wf}) como término de relación entre el comportamiento de un pozo vertical y el yacimiento, esto es, a través de la definición del índice de productividad (o well index en inglés):

$$q = -WI\left(\frac{k_r b}{\mu}\right)\left(P_{bloque} - P_{wf}\right)$$

$$WI = \frac{2\,\pi\,k\,h}{\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + S} \quad ; \quad r_e = \sqrt{\frac{\Delta x\,\Delta y}{\pi}}$$

Donde q es la producción diaria, k_r es la permeabilidad relativa, $b = 1/B$, B es el factor de volumen, μ es la viscosidad, P_{bloque} es la presión del bloque de simulación donde se encuentra representada la zona disparada del pozo, P_{wf} es la presión de fondo fluyendo del pozo, WI es el índice de productividad, h es el espesor del bloque de simulación donde se encuentra representada la zona disparada del pozo, r_w es el radio del pozo, S es el daño y r_e es el radio equivalente del bloque donde se encuentra representado el pozo.

Sin embargo, (Peaceman D. , 1978) indicó que cuando se modela el comportamiento del yacimiento mediante métodos numéricos, inevitablemente las dimensiones horizontales de cualquier bloque de la malla que contenga un pozo (como el caso anterior), serán mucho mayores que el radio del pozo, y con ello la presión calculada para este bloque de pozo será muy diferente de la presión de fondo fluyendo del pozo modelado.

Con lo cual (Peaceman D. , 1978) presentó la siguiente corrección para el cálculo del r_e sin considerar el daño, suponiendo que el bloque de la malla donde se está representando el pozo es completamente cuadrado, es decir, $\Delta x = \Delta y$:

$$q = -WI\left(\frac{k_r b}{\mu}\right)\left(P_{bloque} - P_{wf}\right)$$

$$WI = \frac{2\,\pi\,k\,h}{\ln\left(\frac{r_e}{r_w}\right)}$$

$$r_e = \frac{\Delta x}{\sqrt{\pi}}\;e^{\left(-\frac{1}{2}\right)}$$

Aplicando la corrección de (Peaceman D. , 1978), considerando que Δx y Δy son de diferentes tamaños, es decir, el bloque de la malla donde se representa el pozo ya no tiene forma cuadrada (considerando los efectos del daño):

$$q = -WI\left(\frac{k_r b}{\mu}\right)\left(P_{bloque} - P_{wf}\right)$$

$$WI = \frac{2\,\pi\,k\,h}{\ln\left(\frac{r_e}{r_w}\right) + S}$$

$$r_e = \sqrt{\frac{\Delta x\,\Delta y}{\pi}}\;e^{\left(-\frac{1}{2}\right)}$$

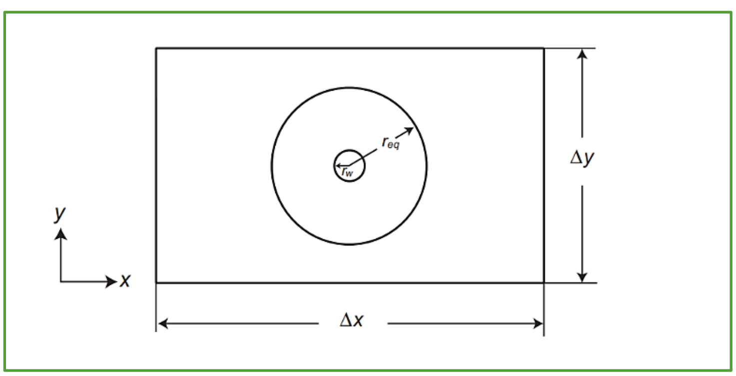

Fig. 21. "Representación esquemática del radio equivalente (r_e)".
(Abou Kaseem, Rafiqul Islam, & Farouq Ali, 2020).

Ahora, para representar el modelo de un pozo horizontal sin aumentar la complejidad de la solución, (Ertekin, Jamal H., & R. King, 2001) indican que puede realizarse a través del modelo de pozo vertical de Peaceman, esto es rotando los ejes del modelo.

Por ejemplo, para el modelo de un pozo horizontal paralelo al eje "x" en coordenadas cartesianas, el índice de productividad y el radio equivalente serían:

$$WI = \frac{2\,\pi\,k\,L_{ong\ pozo\ x}}{\ln\left(\frac{r_e}{r_w}\right) + S}$$

$$r_e = \sqrt{\frac{\Delta y\,\Delta z}{\pi}}\;e^{\left(-\frac{1}{2}\right)}$$

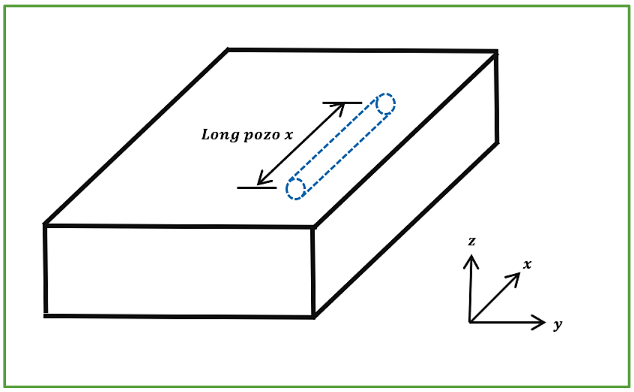

Fig. 22. "Representación esquemática de un pozo horizontal paralelo al eje x". Modificado de (Ertekin, Jamal H., & R. King, 2001).

Y para el modelo de un pozo horizontal paralelo al eje "y" en coordenadas cartesianas:

$$WI = \frac{2\,\pi\,k\,L_{ong\ pozo\ y}}{\ln\left(\frac{r_e}{r_w}\right) + S}$$

$$r_e = \sqrt{\frac{\Delta x\,\Delta z}{\pi}}\;e^{\left(-\frac{1}{2}\right)}$$

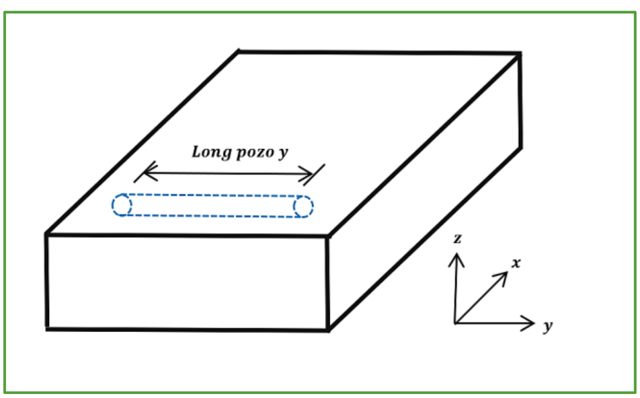

Fig. 23. "Representación esquemática de un pozo horizontal paralelo al eje y". Modificado de (Ertekin, Jamal H., & R. King, 2001).

4. Caso de estudio

En este capítulo se presenta un caso de estudio, el cual, emplea el modelo de aceite negro modificado para un yacimiento naturalmente fracturado, bifásico (aceite y gas), tridimensional, con efectos de fracturamiento hidráulico, y que contempla un pozo horizontal a lo largo de la dirección "x". Dicho modelo fue desarrollado y codificado originalmente por (López Jiménez, 2017) en el lenguaje de programación Fortran 90, es así que, se llevó a cabo la migración de este simulador al lenguaje de programación Python 3.8, con la finalidad de demostrar la utilidad de este modelo de aceite negro modificado en yacimientos naturalmente fracturados y de transcribir dicho modelo a un lenguaje de programación de alto nivel, junto con las ventajas que esto conlleva.

4.1. Caracterización del caso de estudio

Para emplear el modelo de aceite negro modificado (descrito con anterioridad) enfocado a yacimientos naturalmente fracturados, se retomó y adecuó el modelo de simulación desarrollado por (López Jiménez, 2017), el cual consiste en un modelo numérico tridimensional, isotérmico, bifásico (aceite y gas), naturalmente fracturado, y con efectos de fracturas hidráulicas, para un yacimiento de tipo gas y condensado, y que contempla un pozo horizontal a lo largo de la dirección "x".

Parámetro	Valor	Unidades
Número de celdas en la dirección "x", nx	21	[celdas]
Número de celdas en la dirección "y", ny	11	[celdas]
Número de celdas en la dirección "z", nz	10	[celdas]
Longitud del yacimiento, lx	1050	[ft]
Ancho del yacimiento, ly	550	[ft]
Espesor del yacimiento, h	250	[ft]
Top de la formación	8250	[ft]
Contacto gas-aceite	8250	[ft]
Porosidad de la matriz	0.07	[fracción]
Permeabilidad de la matriz	0.001	[md]
Porosidad de la fractura natural	0.009	[fracción]
Permeabilidad de la fractura natural	0.9	[md]
Permeabilidad de la fractura hidráulica	2000	[md]
Ancho de la fractura hidráulica	0.01	[ft]
Ancho de la fractura hidráulica simulada	2	[ft]
Longitud de la fractura hidráulica	550	[ft]
Daño (skin factor)	0	[adim]
Tamaño de los bloques de matriz en dirección "x", "y" y "z", hx, hy, hz	10	[ft]
Posición del pozo en "x", nx	11	[celda]
Posición del pozo en "y", ny	6	[celda]
Posición del pozo en "z", nz	5	[celda]

Tabla 7. "Parámetros empleados en el modelo de simulación". (López Jiménez, 2017).

Al considerar una simulación bifásica, las ecuaciones de flujo del modelo de aceite negro modificado que se han estado trabajando a lo largo de este escrito, serán exactamente las mismas, únicamente considerando ahora, cuatro ecuaciones de flujo para los dos medios (en vez de seis ecuaciones considerando una formulación trifásica) y omitiendo la saturación de agua en los términos de acumulación. Entonces, el modelo matemático del problema sería:

<u>En la fractura natural:</u>

<u>Para la fase de aceite:</u>

$$\frac{\partial}{\partial x}\left(b_o \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g r_s \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g r_s \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o \frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o \frac{\partial D}{\partial z}\right) + b_g r_s \frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g \frac{\partial D}{\partial z}\right)\right)_f$$

$$+ b_{o_f} q_o^* + b_{g_f} r_{s_f} q_g^* + \tau_{o_{mf}}^* + \tau_{g_{mf}}^* r_{s_m} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right)\right)_f$$

<u>Para la fase de gas:</u>

$$\frac{\partial}{\partial x}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial x} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial x}\right)_f + \frac{\partial}{\partial y}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\frac{\partial P_o}{\partial y} + b_g \frac{kk_{r_g}}{\mu_g}\frac{\partial P_g}{\partial y}\right)_f + \frac{\partial}{\partial z}\left(b_o R_s \frac{kk_{r_o}}{\mu_o}\left(\frac{\partial P_o}{\partial z} - \gamma_o \frac{\partial D}{\partial z}\right) + b_g \frac{kk_{r_g}}{\mu_g}\left(\frac{\partial P_g}{\partial z} - \gamma_g \frac{\partial D}{\partial z}\right)\right)_f$$

$$+ b_{o_f} R_{s_f} q_o^* + b_{g_f} q_g^* + \tau_{o_{mf}}^* R_{s_m} + \tau_{g_{mf}}^* = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_f$$

<u>En la matriz:</u>

<u>Para la fase de aceite:</u>

$$-\tau_{o_{mf}}^* - \tau_{g_{mf}}^* r_{s_m} = \frac{\partial}{\partial t}\left(\emptyset\left(b_o S_{at\,o} + b_g r_s S_{at\,g}\right)\right)_m$$

<u>Para la fase de gas:</u>

$$-\tau_{o_{mf}}^* R_{s_m} - \tau_{g_{mf}}^* = \frac{\partial}{\partial t}\left(\emptyset\left(b_o R_s S_{at\,o} + b_g S_{at\,g}\right)\right)_m$$

Donde las funciones de transferencia matriz-fractura (Thomas, 1983) se definen como:

$$\tau_{o_{mf}}^* = \left(\frac{b_{om}\sigma k_m k_{r_{om}}}{\mu_{om}}\right)\left(P_{o_m} - P_{o_f}\right)$$

$$\tau_{g_{mf}}^* = \left(\frac{b_{gm}\sigma k_m k_{r_{gm}}}{\mu_{gm}}\right)\left(P_{g_m} - P_{g_f}\right)$$

Y σ es el factor de forma, o "shape factor" (Kazemi, 1976) :

$$\sigma = 4\left(\frac{1}{h_x^2} + \frac{1}{h_y^2} + \frac{1}{h_z^2}\right)$$

El modelo numérico, es decir, la discretización de las ecuaciones quedaría como:

<u>En la fractura natural:</u>

<u>Para la fase de aceite:</u>

$$T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1}\left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1}$$

$$- T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1}\left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1}$$

$$+ T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1}\left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1}$$

$$- T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1}\left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1}$$

$$+ T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1}\left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1}$$

$$- \; T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1}$$

$$+ \; T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} \; r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1}$$

$$- \; T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} \; r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1}$$

$$+ \; T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$

$$- \; T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$

$$+ \; T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$

$$- \; T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$

$$+ \left(b_{o_f} q_o\right)_{i,j,k}^{n+1}$$

$$+ \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1}$$

$$+ \; T_{om_{f_{i,j,k}}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1}$$

$$+ \; T_{gm_{f_{i,j,k}}}^{n+1} \; r_{s_{m_{i,j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}}\right)^{n+1}$$

$$= \frac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{l} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g}\right)\right]^n \right)_{i,j,k} \\ \quad + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_{i,j,k} \end{array} \right]_f$$

<u>Para la fase de gas:</u>

$$T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1} \; R_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1}$$

$$- \; T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1} \; R_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1}$$

$$+ \; T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1}$$

$$- \; T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1}$$

$$+ \; T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1} \; R_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1}$$

$$- \; T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \; R_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1}$$

$$+ \; T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1}$$

$$- \; T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1}$$

$$+ \; T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$

$$- \; T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$

$$+ \; T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1}$$

$$- \; T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1}$$

$$+ \left(b_{o_f} R_{s_f} q_o\right)_{i,j,k}^{n+1}$$

$$+ \left(b_{g_f} q_g\right)_{i,j,k}^{n+1}$$

$$+ \; T_{om_{f_{i,j,k}}}^{n+1} \; R_{s_{m_{i,j,k}}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1}$$

$$
\begin{aligned}
&+\ T_{gmf_{i,j,k}}^{n+1}\ \left(\left(P_{cap\ go}+P_o\right)_{m_{i,j,k}}-\left(P_{cap\ go}+P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
&=\frac{Vp_{i,j,k}}{\Delta t}\left[\begin{array}{c}\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_oR_s(1-S_{at\ g})]^{n+1}-[b_oR_s(1-S_{at\ g})]^n\ \right)_{i,j,k} \\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_gS_{at\ g}]^{n+1}-[b_gS_{at\ g}]^n\ \right)_{i,j,k}\end{array}\right]_f
\end{aligned}
$$

<u>En la matriz:</u>

<u>Para la fase de aceite:</u>

$$
\begin{aligned}
&-\ T_{omf_{i,j,k}}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_{i,j,k}^{n+1} \\
&-\ T_{gmf_{i,j,k}}^{n+1}\ r_{s_{m_{i,j,k}}}^{n+1}\ \left(\left(P_{cap\ go}+P_o\right)_{m_{i,j,k}}-\left(P_{cap\ go}+P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
&=\frac{Vp_{i,j,k}}{\Delta t}\left[\begin{array}{c}\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_o(1-S_{at\ g})]^{n+1}-[b_o(1-S_{at\ g})]^n\ \right)_{i,j,k} \\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_gr_sS_{at\ g}]^{n+1}-[b_gr_sS_{at\ g}]^n\ \right)_{i,j,k}\end{array}\right]_m
\end{aligned}
$$

<u>Para la fase de gas:</u>

$$
\begin{aligned}
&-\ T_{omf_{i,j,k}}^{n+1}\ R_{s_{m_{i,j,k}}}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_{i,j,k}^{n+1} \\
&-\ T_{gmf_{i,j,k}}^{n+1}\ \left(\left(P_{cap\ go}+P_o\right)_{m_{i,j,k}}-\left(P_{cap\ go}+P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
&=\frac{Vp_{i,j,k}}{\Delta t}\left[\begin{array}{c}\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_oR_s(1-S_{at\ g})]^{n+1}-[b_oR_s(1-S_{at\ g})]^n\ \right)_{i,j,k} \\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_gS_{at\ g}]^{n+1}-[b_gS_{at\ g}]^n\ \right)_{i,j,k}\end{array}\right]_m
\end{aligned}
$$

Ahora, la solución del sistema de ecuaciones no lineales se lleva a cabo a través del método iterativo de Newton-Raphson y considerando una formulación totalmente implícita (visto en el capítulo anterior). Por ejemplo, definiendo la función de residuos de aceite en la fractura natural:

<u>En la fractura natural:</u>

<u>Para la fase de aceite:</u>

$$
F_{o_{f_{i,j,k}}}^{n+1} = \begin{bmatrix}
T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - \left(\gamma_o \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - \left(\gamma_o \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \left(b_{o_f} q_o\right)_{i,j,k}^{n+1} \\
+ \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1} \\
+ \; T_{o_{mf_{i,j,k}}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
+ \; T_{g_{mf_{i,j,k}}}^{n+1} \, r_{s_{m_{i,j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
- \dfrac{Vp_{i,j,k}}{\Delta t} \begin{bmatrix} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_{i,j,k} \end{bmatrix}_f
\end{bmatrix} = 0
$$

Entonces el sistema de ecuaciones quedaría de la siguiente forma:

$$
\begin{bmatrix}
\boldsymbol{A} & \boldsymbol{B} & & \boldsymbol{D} & & & & \boldsymbol{F} & & & & \\
\boldsymbol{C} & \ddots & \ddots & & \ddots & & & & \ddots & & & \\
 & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & & \\
\boldsymbol{E} & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & \\
 & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots \\
 & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & \\
 & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \\
\boldsymbol{G} & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & \\
 & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots \\
 & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & \\
 & & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots \\
 & & & & \ddots & & & & \ddots & & \ddots & \ddots
\end{bmatrix}^{v}
\begin{bmatrix}
\boldsymbol{\delta P_{o_f}} \\ \boldsymbol{\delta S_{g_f}} \\ \boldsymbol{\delta P_{o_m}} \\ \boldsymbol{\delta S_{g_m}}
\end{bmatrix}^{v+1}
= -\begin{bmatrix}
\boldsymbol{F_{o_f}} \\ \boldsymbol{F_{g_f}} \\ \boldsymbol{F_{o_m}} \\ \boldsymbol{F_{g_m}}
\end{bmatrix}^{v}
$$

Donde la matriz jacobiana estaría compuesta por siete bandas principales (A, B, C, D, E, F, G) correspondientes a las siete direcciones del proceso de discretización (i, j, k , $i+1, j, k$, $i-1, j, k$, $i, j+1, k$, $i, j-1, k$, $i, j, k+1$, $i, j, k-1$), de manera similar al modelo trifásico, estas bandas estarán constituidas por cuatro submatrices, sin embargo, en vez de tener un tamaño de 3x3, para el modelo bifásico estas cuatro submatrices constarán de un tamaño de 2x2 correspondientes a:

1) Las derivadas de las funciones de residuos de las dos fases en la fractura natural $(F_o, F_g)_f$ respecto a las variables primarias de la fractura natural $(P_o,\ S_{at\ g})_f$

2) Las derivadas de las funciones de residuos de las dos fases en la fractura natural $(F_o, F_g)_f$ respecto a las variables primarias de la matriz $(P_o,\ S_{at\ g})_m$

3) Las derivadas de las funciones de residuos de las dos fases en la matriz $(F_o, F_g)_m$ respecto a las variables primarias de la fractura natural $(P_o,\ S_{at\ g})_f$

4) Las derivadas de las funciones de residuos de las dos fases en la matriz $(F_o, F_g)_m$ respecto a las variables primarias de la matriz $(P_o,\ S_{at\ g})_m$

Por ejemplo, la banda central A sería:

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1)} & \mathbf{2)} \\ \mathbf{3)} & \mathbf{4)} \end{bmatrix}$$

Entonces:

$$\boldsymbol{A} = \begin{bmatrix} \left(\frac{\partial F_{o\ i,j,k\ f}}{\partial P_{o\ i,j,k\ f}}\right) & \left(\frac{\partial F_{o\ i,j,k\ f}}{\partial S_{g\ i,j,k\ f}}\right) & \left(\frac{\partial F_{o\ i,j,k\ f}}{\partial P_{o\ i,j,k\ m}}\right) & \left(\frac{\partial F_{o\ i,j,k\ f}}{\partial S_{g\ i,j,k\ m}}\right) \\ \left(\frac{\partial F_{g\ i,j,k\ f}}{\partial P_{o\ i,j,k\ f}}\right) & \left(\frac{\partial F_{g\ i,j,k\ f}}{\partial S_{g\ i,j,k\ f}}\right) & \left(\frac{\partial F_{g\ i,j,k\ f}}{\partial P_{o\ i,j,k\ m}}\right) & \left(\frac{\partial F_{g\ i,j,k\ f}}{\partial S_{g\ i,j,k\ m}}\right) \\ \left(\frac{\partial F_{o\ i,j,k\ m}}{\partial P_{o\ i,j,k\ f}}\right) & \left(\frac{\partial F_{o\ i,j,k\ m}}{\partial S_{g\ i,j,k\ f}}\right) & \left(\frac{\partial F_{o\ i,j,k\ m}}{\partial P_{o\ i,j,k\ m}}\right) & \left(\frac{\partial F_{o\ i,j,k\ m}}{\partial S_{g\ i,j,k\ m}}\right) \\ \left(\frac{\partial F_{g\ i,j,k\ m}}{\partial P_{o\ i,j,k\ f}}\right) & \left(\frac{\partial F_{g\ i,j,k\ m}}{\partial S_{g\ i,j,k\ f}}\right) & \left(\frac{\partial F_{g\ i,j,k\ m}}{\partial P_{o\ i,j,k\ m}}\right) & \left(\frac{\partial F_{g\ i,j,k\ m}}{\partial S_{g\ i,j,k\ m}}\right) \end{bmatrix}$$

La definición de las seis bandas restantes que conforman la matriz jacobiana sería exactamente la misma a la banda central **A**, únicamente cambiarían los subíndices (i, j, k).

Aplicando el complemento de Schur para reducir el sistema de ecuaciones y dejar las variables de la matriz en función de las variables de la fractura (Reséndiz Torres & Peña Chaparro, 2005), nuestro sistema de ecuaciones quedaría como:

$$
\begin{bmatrix}
A & B & & D & & & & F & & & & \\
C & \ddots & \ddots & & \ddots & & & & \ddots & & & \\
& \ddots & \ddots & \ddots & & \ddots & & & & \ddots & & \\
E & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots & \\
& \ddots & & \ddots & \ddots & \ddots & & \ddots & & & & \ddots \\
& & \ddots & & \ddots & \ddots & \ddots & & \ddots & & & \\
& & & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \\
G & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots & \\
& \ddots & & & & \ddots & & \ddots & \ddots & \ddots & & \ddots \\
& & \ddots & & & & \ddots & & \ddots & \ddots & \ddots & \\
& & & \ddots & & & & \ddots & & \ddots & \ddots & \ddots \\
& & & & \ddots & & & & \ddots & & \ddots & \ddots
\end{bmatrix}^{v}
\begin{bmatrix}
\\ \\ \delta P_{o_f} \\ \delta S_{at\, g_f} \\ \\ \\
\end{bmatrix}^{v+1}
= -
\begin{bmatrix}
\\ \\ F^{*}_{o_f} \\ F^{*}_{g_f} \\ \\ \\
\end{bmatrix}^{v}
$$

Donde:

$$
A = \begin{bmatrix}
\left(\dfrac{\partial F_{o\, i,j,k\, f}}{\partial P_{o\, i,j,k\, f}}\right) & \left(\dfrac{\partial F_{o\, i,j,k\, f}}{\partial S_{g\, i,j,k\, f}}\right) \\
\left(\dfrac{\partial F_{g\, i,j,k\, f}}{\partial P_{o\, i,j,k\, f}}\right) & \left(\dfrac{\partial F_{g\, i,j,k\, f}}{\partial S_{g\, i,j,k\, f}}\right)
\end{bmatrix}
$$

Ahora, para el modelamiento de los efectos del fracturamiento hidráulico se implementó la metodología **DK-LS-LGR** (Dual Permeability, Logarithmically Spaced, and Locally Grid Refined, por sus siglas en inglés) misma utilizada por (López Jiménez, 2017), con la finalidad de reducir el número de celdas requeridas para modelar la fractura hidráulica. Así mismo, el pozo horizontal se contempla a lo largo de la dirección "x", en la celda número 6 en la dirección "y" y en la celda número 5 en la dirección "z", mientras que la fractura hidráulica se contempla a lo largo de la dirección "y", en la celda número 11 en la dirección "x" y en la celda número 5 en la dirección "z".

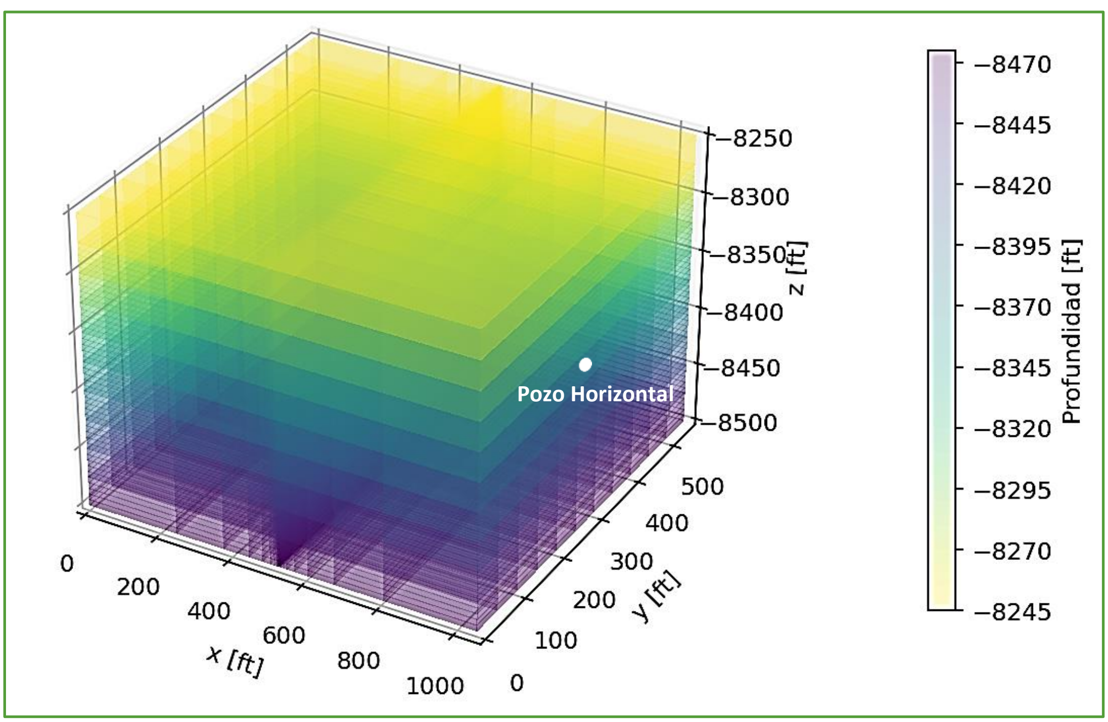

Fig. 24. "Modelo de simulación 3D realizado en Python". Modificado de (López Jiménez, 2017).

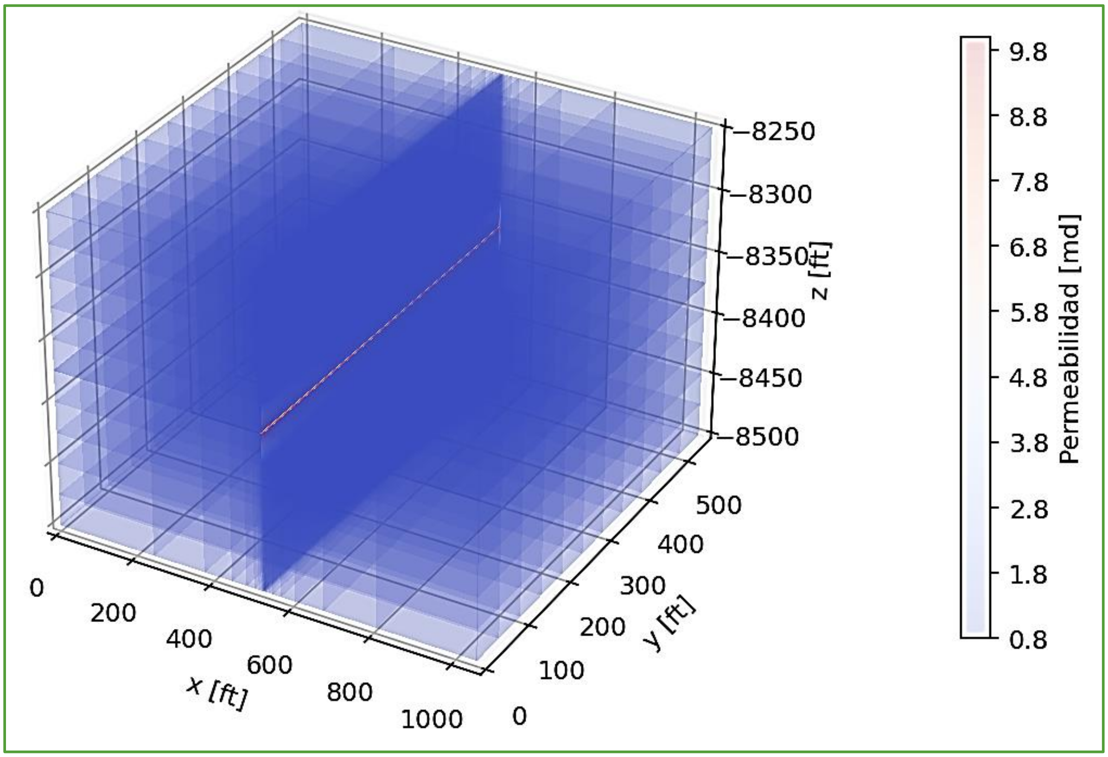

Fig. 25. “Distinción de las propiedades de la fractura hidráulica (color rojo) y la fractura natural (color azul) 3D en nx=11 y nz=5”. Modificado de (López Jiménez, 2017).

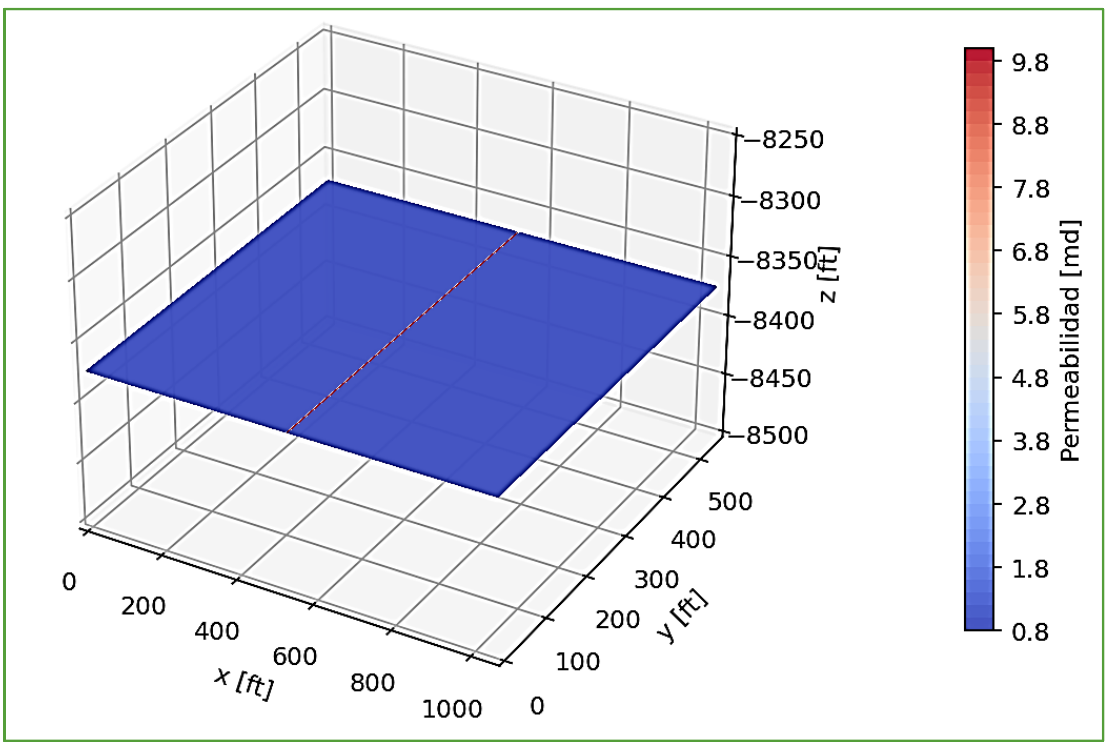

Fig. 26. “Distinción de las propiedades de la fractura hidráulica (color rojo) y la fractura natural (color azul) 2D en nx=11 y nz=5”. Modificado de (López Jiménez, 2017).

Donde, las propiedades de los fluidos para la fractura natural y la matriz (López Jiménez, 2017) son:

P [Psi]	Bo [ft3 c.y./ft3 c.s.]	Bg [ft3 c.y./ft3 c.s.]	μo [cP]	μg [cP]	ρo [lbm/ft3]	ρg [lbm/ft3]	Rs [ft3 c.s./ft3 c.s.]]
14.7	1.062	0.93583	1.04	0.008	46.243674	0.064691	0.1781
264.7	1.15	0.067902	0.975	0.0096	43.544131	0.891576	16.11876
514.7	1.207	0.035229	0.91	0.0112	42.287257	1.718494	32.05942
1014.7	1.295	0.017951	0.83	0.014	41.00389	3.372485	66.07804
2014.7	1.435	0.009063	0.695	0.0189	38.99457	6.680195	113.27664
2514.7	1.5	0.007266	0.641	0.0208	38.303916	8.332175	138.03365
3014.7	1.565	0.006064	0.594	0.0228	37.780862	9.98318	165.64038
4014.7	1.695	0.004554	0.51	0.0268	37.04593	13.294494	226.19708
5014.7	1.827	0.003644	0.449	0.0309	36.423047	16.612996	288.17864
9014.7	2.36	0.002167	0.203	0.047	34.437631	27.932213	531.47408

Tabla 8. “Propiedades de los fluidos empleados en el modelo de simulación”. (López Jiménez, 2017).

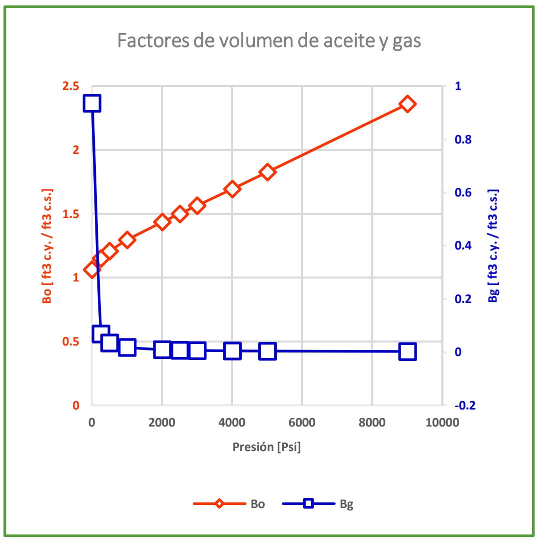

Fig. 27. “Factores de volumen de aceite y gas”.

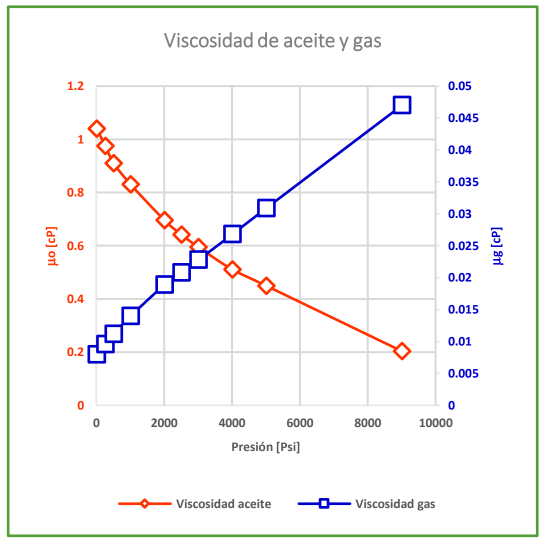

Fig. 28. “Viscosidad de aceite y gas”.

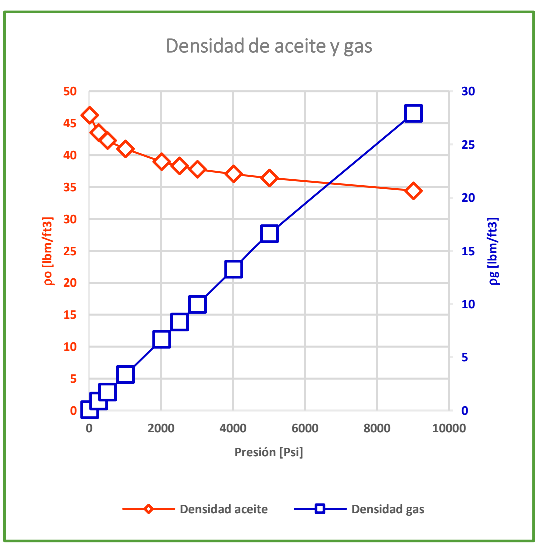

Fig. 29. “Densidad de aceite y gas”.

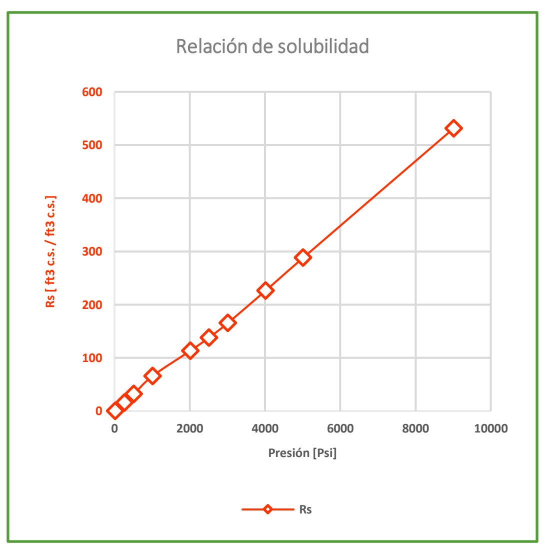

Fig. 30. Relaciones de solubilidad el aceite y gas.

Respecto a las curvas de permeabilidades relativas utilizadas:

Para la fractura natural:

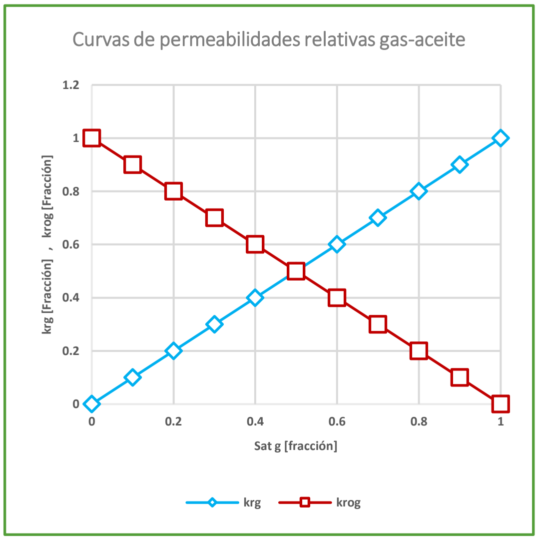

Fig. 31. "Curvas de permeabilidades relativas gas-aceite para la fractura natural".

Para la matriz:

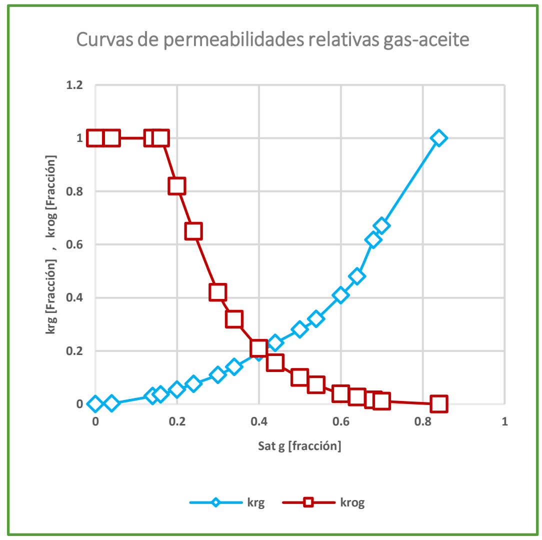

Fig. 32. "Curvas de permeabilidades relativas gas-aceite para la matriz".

Y, por último, las curvas de presiones capilares uilizadas:

Para la fractura natural: La presión capilar gas-aceite en la fractura natural es considerada como cero.

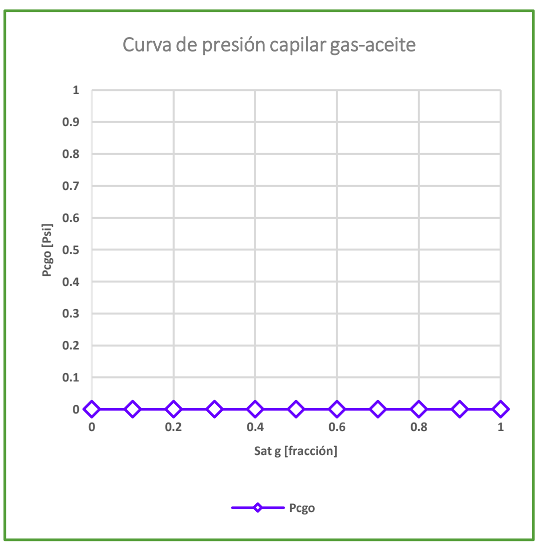

Fig. 33. "Curva de presión capilar gas -aceite nula para la fractura natural".

Para la matriz:

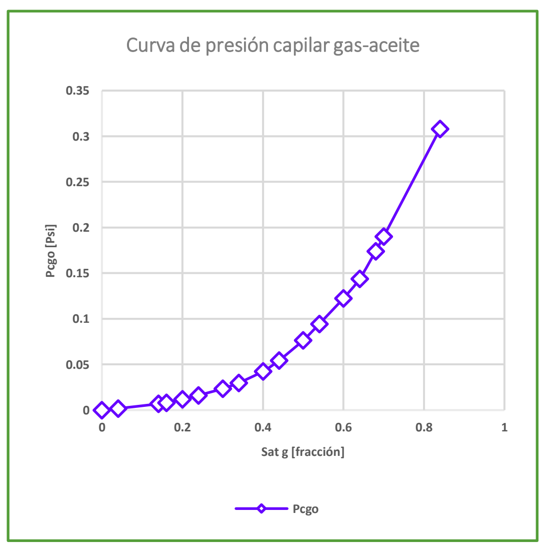

Fig. 34. "Curva de presión capilar gas-aceite para la matriz".

4.2. Migración del código

Una vez descrito y comprendido el caso de estudio, se decidió llevar a cabo la migración del modelo de simulación, codificado originalmente en el lenguaje de programación Fortran 90 por (López Jiménez, 2017), al lenguaje de programación Python 3.8, con la finalidad de demostrar la utilidad del modelo de aceite negro modificado en yacimientos naturalmente fracturados, y de desarrollarlo en un lenguaje de programación de alto nivel, empleando diferentes métodos de solución del sistema de ecuaciones lineales y técnicas de optimización, además de aprovechar las ventajas que proporciona Python para la visualización de datos y generación de gráficos, tanto 2D y 3D.

El código original desarrollado en Fortran 90 se codificó en el entorno de desarrollo integrado (IDE) de Microsoft Visual Studio (Microsoft, 2022), y corrido con el compilador Intel (R) Fortran Compiler for One API (Intel (R), 2022) compatible con el entorno de desarrollo de Microsoft Visual Studio.

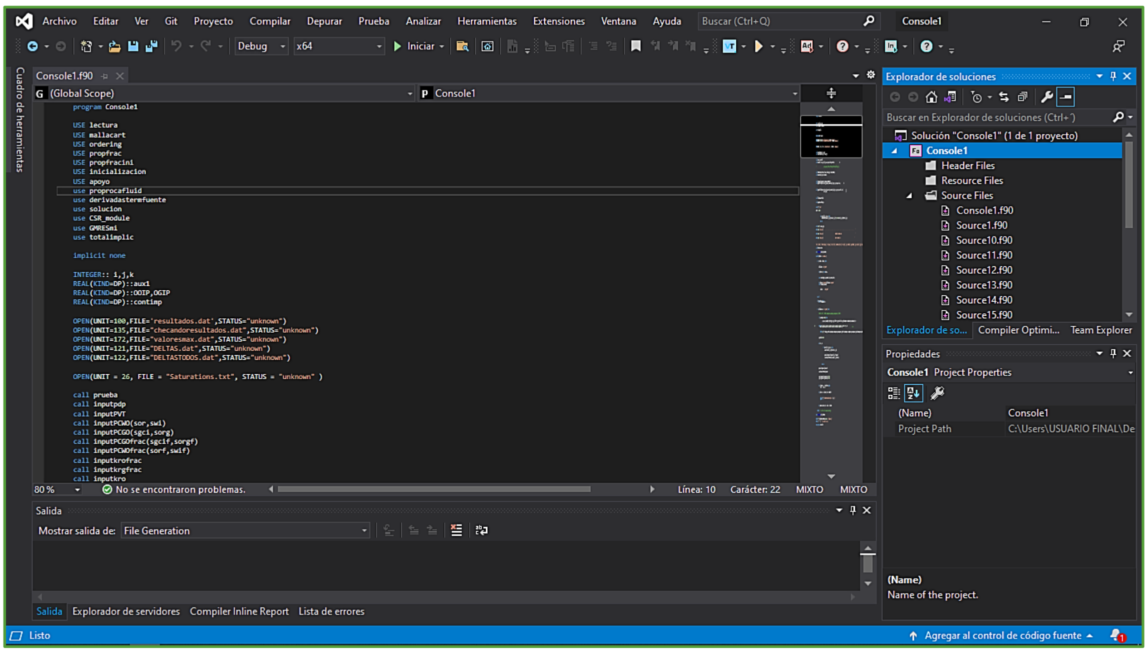

Fig. 35. "Entorno de desarrollo de Microsoft Visual Studio".

Para la migración del código, se decidió emplear el entorno de desarrollo integrado (IDE) de Spyder (Spyder, 2022), el cual, es un entorno científico gratuito y de código abierto escrito en Python, para Python y diseñado por y para científicos, ingenieros y analistas de datos (Spyder, 2022) .

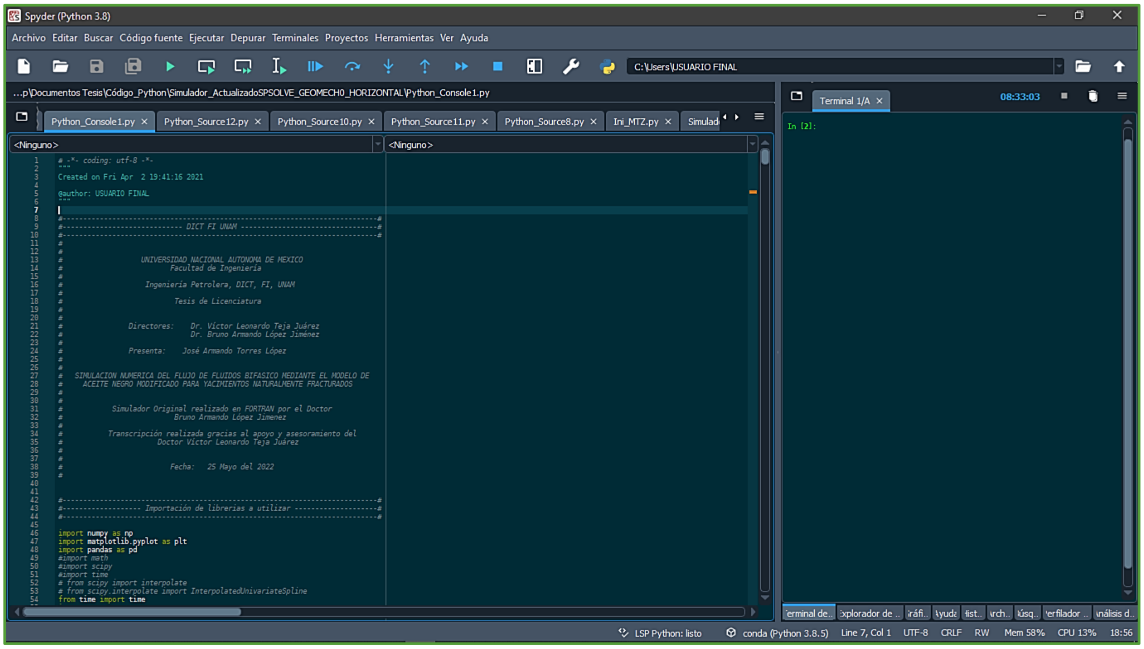

Fig. 36. “Entorno de desarrollo de Spyder”.

Es así que cada uno de los módulos que integran al simulador original fueron migrados uno por uno y de manera manual, llevando a cabo ciertas adecuaciones cuando fuese necesario, ya que la sintaxis entre ambos lenguajes es bastante diferente, además de que en términos de problemas numéricos, ambos lenguajes pueden abordarlos y tratarlos de manera diferente, por ejemplo, en Fortran al momento de declarar un vector, la numeración correspondiente a cada uno de los términos que integran al vector comienza con el número 1, en contraste con Python, donde su numeración comienza con el número 0. Problemas de este tipo y demás tuvieron que ser adecuados a lo largo de la migración.

4.3. <u>Numba como método de optimización</u>

Numba es una librería de Python que traduce las funciones de Python a un código de máquina optimizado en tiempo de ejecución. Donde los algoritmos numéricos compilados por Numba en Python indican que pueden acercarse a las velocidades de C o FORTRAN (Numba, 2022)**.**

A través del decorador **@jit()** se puede marcar una función en Python para que la optimice el compilador JIT de Numba, (Numba, 2022). Con lo cual se optó por emplear esta librería para optimizar los tiempos de cómputo relacionados a las funciones definidas durante la migración del código, ya que el simulador tiene una gran cantidad de funciones que constantemente están siendo utilizadas durante el tiempo de compilación y realización de cálculos.

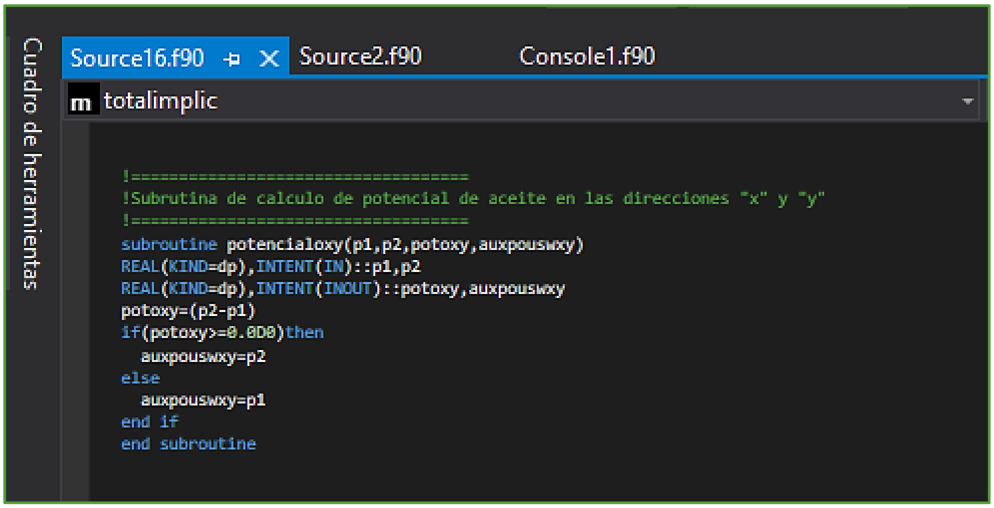

Fig. 37. "Ejemplo de una función definida en lenguaje Fortran 90".

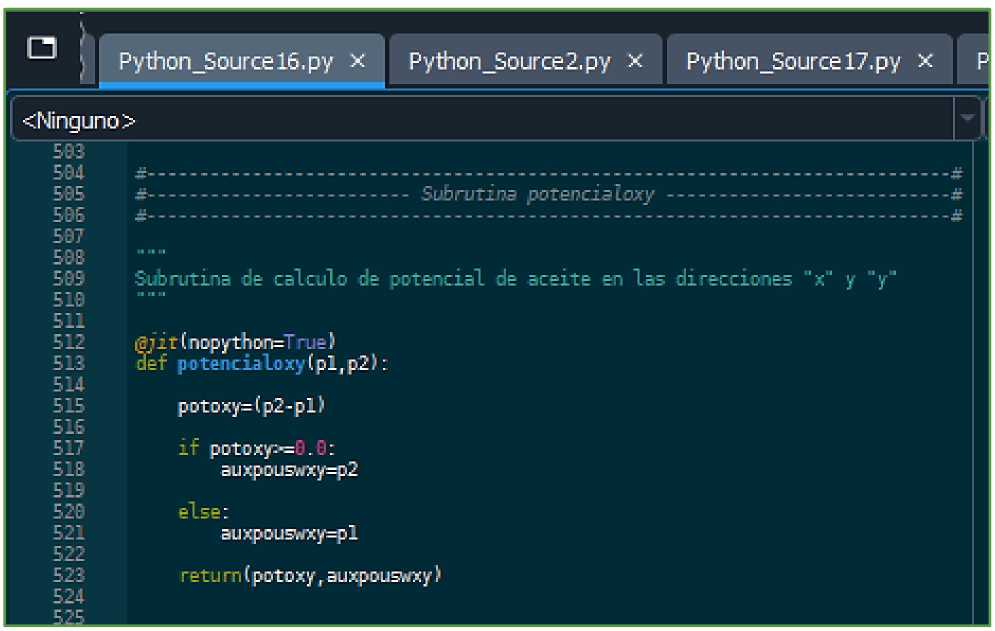

Fig. 38. "Ejemplo de la función anterior, migrada a lenguaje Python 3.8 y aplicando el decorador @jit() de Numba como método de optimización".

5. Resultados

En este capítulo se presentan los resultados obtenidos del caso de estudio para ambos simuladores, haciendo un comparativo de estos y determinando el porcentaje de error del simulador migrado respecto al original, así mismo se emplearon dos métodos de solución para el sistema de ecuaciones lineales del simulador migrado (el método GMRES y el método de descomposición LU optimizado, de la librería de Scipy (Scipy, 2022) para Python), comparando los tiempos de cómputo entre ambos simuladores y explicando el porqué de la diferencia de estos.

5.1. Comparación de resultados

Una vez migrado el código e implementando la librería de Numba como método de optimización, se llevó a cabo la simulación del caso de estudio para un periodo de 10 años en ambos simuladores, donde los gráficos generados entre el original y el migrado empalman bastante bien.

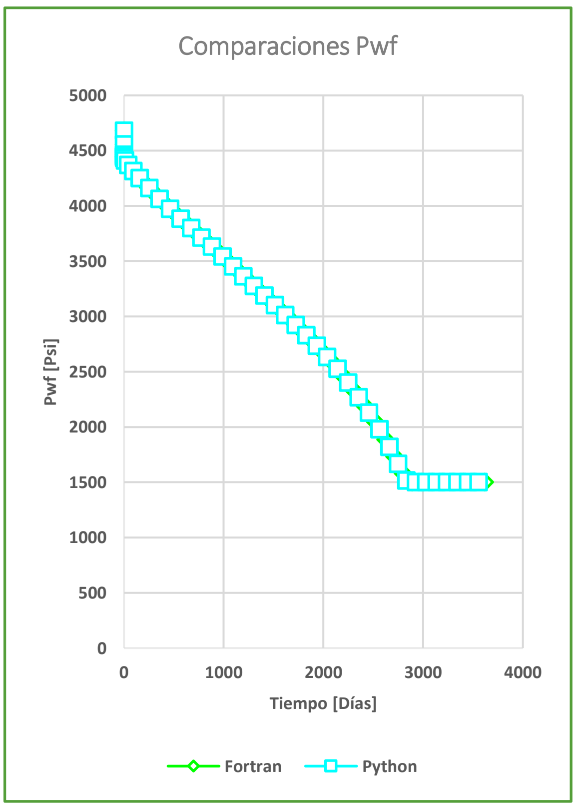

Fig. 41. "Comparaciones Pwf".

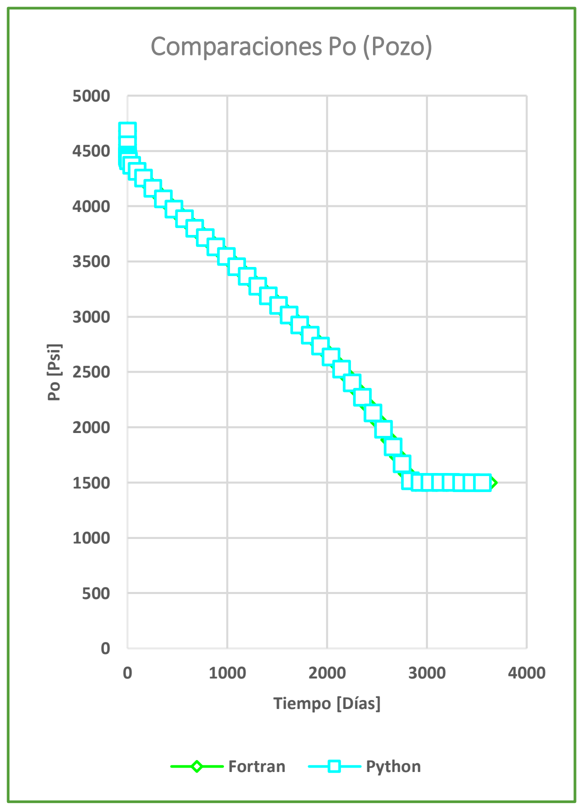

Fig. 40. "Comparaciones Po (Pozo)".

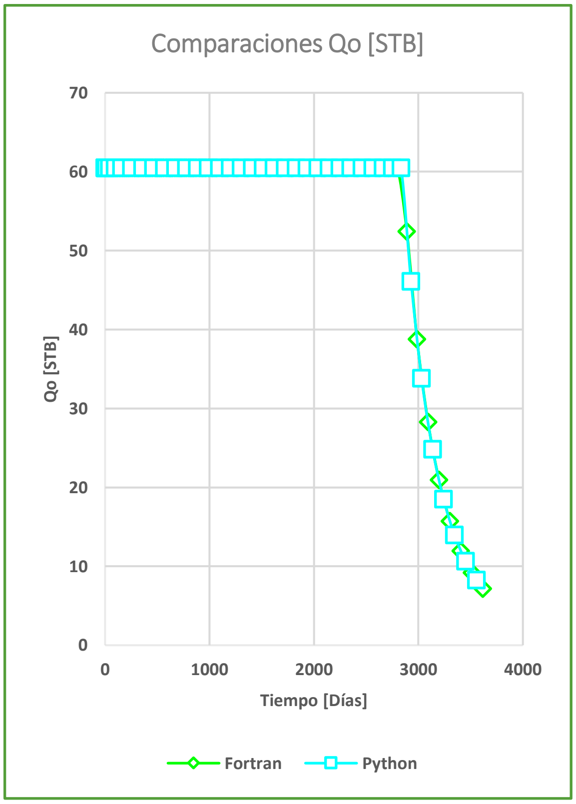

Fig. 43. “Comparaciones Qo [STB]”.

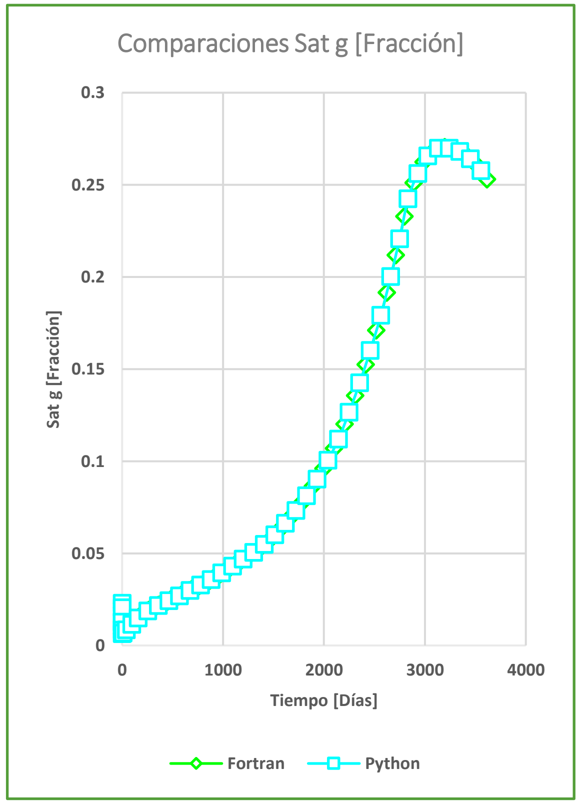

Fig. 42. “Comparaciones Sat g [Fracción]”.

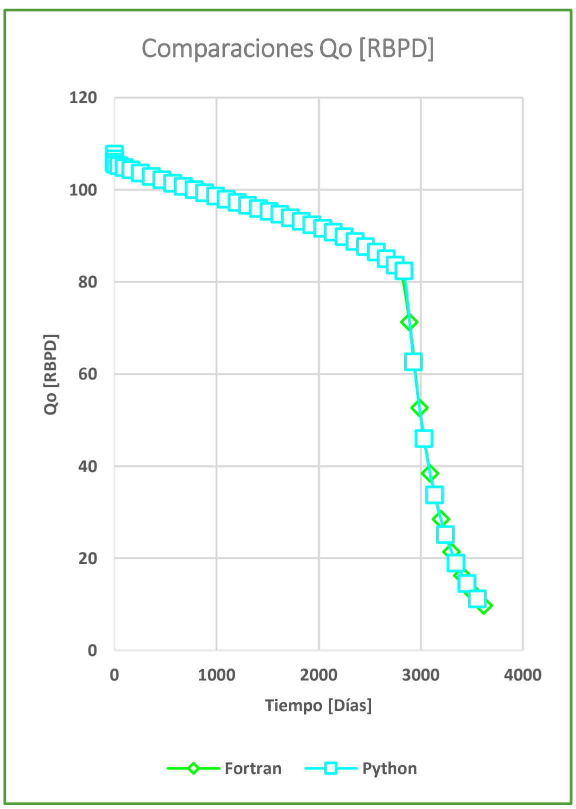

Fig. 45. “Comparaciones Qo [RBPD]”.

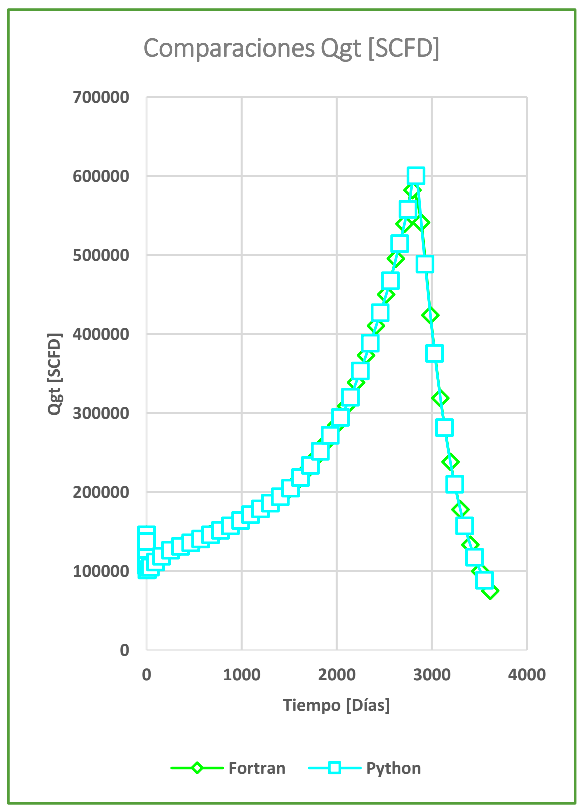

Fig. 44. “Comparaciones Qgt [SCFD]”.

Para la solución del sistema de ecuaciones lineales del simulador original, se empleó el método GMRES (residuo mínimo generalizado), mientras que, para el código migrado se empleó el solver SPSOLVE (que implementa el método descomposición LU optimizado) de la librería de Scipy para Python (Scipy, 2022).

Comparando los resultados obtenidos de la presión de fondo fluyendo $\left(P_{wf}\right)$ de ambos simuladores (Tabla 9 y Tabla 10), podemos observar que el simulador original codificado en Fortran 90 e implementando el método GMRES requirió una mayor cantidad de ciclos de Newton-Raphson, mientras que el simulador migrado en Python 3.8 e implementado el método de descomposición LU optimizado de SPSOLVE requirió una menor cantidad de ciclos. Esto es debido a que el método GMRES es de tipo iterativo, mientras que el método de descomposición LU optimizado de SPSOLVE es un método directo.

Fortran 90											
Num. de ciclos	**TIEMPO [Dias]**	**Pwf [Psi]**	**Num. de ciclos**	**TIEMPO [Dias]**	**Pwf [Psi]**	**Num. de ciclos**	**TIEMPO [Dias]**	**Pwf [Psi]**	**Num. de ciclos**	**TIEMPO [Dias]**	**Pwf [Psi]**
15	0.001445	4679.667	315	281.13549	4134.9651	615	719.257983	3761.5636	915	1680.027452	2962.7686
30	0.007482	4553.4532	330	317.976381	4101.071	630	729.442662	3753.9702	930	1785.027452	2870.932
45	0.032698	4462.202	345	339.415514	4081.672	645	763.747938	3726.6178	945	1890.027452	2775.9459
60	0.138035	4447.4472	360	355.018395	4067.0079	660	779.063484	3714.4781	960	1995.027452	2677.1516
75	0.57805	4443.4323	375	368.30754	4055.7815	675	801.161952	3697.2139	975	2100.027452	2573.4183
90	2.416103	4433.0822	390	386.85386	4039.4509	690	822.068825	3680.7859	990	2205.027452	2456.1907
105	10.094109	4409.1469	405	451.206897	3983.448	705	843.123538	3663.5547	1005	2310.027452	2326.4138
120	33.033523	4377.8632	420	480.688448	3958.1895	720	870.152148	3642.4263	1020	2415.027452	2189.7024
135	49.331543	4360.6091	435	487.935889	3952.0187	735	902.861004	3615.8202	1035	2520.027452	2045.1409
150	69.631015	4340.1192	450	501.220596	3941.0494	750	931.159239	3592.7665	1050	2623.843414	1885.0159
165	77.384755	4332.378	465	521.461669	3923.4054	765	964.859307	3564.3918	1065	2713.282534	1732.2954
180	98.446606	4311.4285	480	539.896352	3907.6411	780	993.098908	3540.0678	1080	2798.379167	1580.6342
195	118.208707	4291.7926	495	557.873514	3893.0227	795	1026.796889	3511.0033	1095	2889.149431	1500
210	143.825915	4266.563	510	585.063121	3870.9362	810	1069.705696	3474.0563	1110	2986.902442	1500
225	166.206913	4244.4263	525	619.085793	3843.0052	825	1119.052884	3432.0051	1125	3091.902442	1500
240	188.590013	4222.8188	540	630.375808	3833.6872	840	1177.735563	3382.4238	1140	3196.902442	1500
255	207.127402	4204.5269	555	647.55333	3819.3391	855	1260.027452	3313.1341	1155	3301.902442	1500
270	227.770552	4184.5666	570	664.884294	3805.8263	870	1365.027452	3226.5957	1170	3406.902442	1500
285	245.781521	4167.867	585	680.127849	3793.7403	885	1470.027452	3139.8074	1185	3511.902442	1500
300	261.227271	4154.0067	600	696.771778	3779.5569	900	1575.027452	3052.1599	1200	3616.902442	1500

Tabla 9. "Resultados de presión de fondo fluyendo $\left(P_{wf}\right)$ del simulador original en Fortran 90".

Python 3.8								
Num. de ciclos	**TIEMPO [Dias]**	**Pwf [Psi]**	**Num. de ciclos**	**TIEMPO [Dias]**	**Pwf [Psi]**	**Num. de ciclos**	**TIEMPO [Dias]**	**Pwf [Psi]**
15	0.001445129	4679.667042	315	1303.813162	3275.194921	615	3345.420133	1500
30	0.007481794	4553.453184	330	1408.813162	3188.511732	630	3450.420133	1500
45	0.032698439	4462.202024	345	1513.813162	3101.506643	645	3555.420133	1500
60	0.138034623	4447.447198	360	1618.813162	3013.255061			
75	0.578050005	4443.43231	375	1723.813162	2922.940696			
90	2.416103455	4433.082193	390	1828.813162	2829.881278			
105	10.09410886	4409.146919	405	1933.813162	2733.414927			
120	42.16704291	4368.083896	420	2038.813162	2632.782051			
135	95.25983761	4314.56	435	2143.813162	2525.072483			
150	159.0984757	4251.448268	450	2248.813162	2400.794644			
165	253.813162	4160.245167	465	2353.813162	2268.156839			
180	358.813162	4063.935448	480	2458.813162	2128.298406			
195	463.813162	3972.173538	495	2563.813162	1978.964149			
210	568.813162	3883.723936	510	2661.749244	1818.718008			
225	673.813162	3797.734102	525	2749.49715	1666.054636			
240	778.813162	3713.600036	540	2833.046174	1514.333463			
255	883.813162	3630.191399	555	2929.63852	1500			
270	988.813162	3542.037644	570	3030.420133	1500			
285	1093.813162	3451.514161	585	3135.420133	1500			
300	1198.813162	3362.628141	600	3240.420133	1500			

Tabla 10. "Resultados de presión de fondo fluyendo $\left(P_{wf}\right)$ del simulador migrado a Python 3.8".

Realizando una interpolación lineal de los valores de P_{wf} obtenidos del simulador migrado en los mismos puntos de tiempo que presenta el simulador original, se pudieron determinar los valores de P_{wf} del simulador migrado en los mismos puntos de tiempo que el simulador original, y con estos determinar el porcentaje de error de los resultados del simulador migrado respecto al simulador original.

Interpolación lineal:

$$y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$$

Porcentaje de error:

$$\% \; Error = \left| \frac{P_{wf \; simulador \; original} - P_{wf \; simulador \; migrado}}{P_{wf \; simulador \; original}} \right| x100$$

TIEMPO [Dias]	Pwf [Psi] Fortran	Pwf [Psi] Python interpolada	% Error	TIEMPO [Dias]	Pwf [Psi] Fortran	Pwf [Psi] Python interpolada	% Error
0.001445	4679.667	4679.667042	8.90659E-07	281.13549	4134.9651	4135.18416	0.005297754
0.007482	4553.4532	4553.452438	1.67323E-05	317.976381	4101.071	4101.39239	0.007836735
0.032698	4462.202	4462.203611	3.61124E-05	339.415514	4081.672	4081.727658	0.001363603
0.138035	4447.4472	4447.447195	1.19475E-07	355.018395	4067.0079	4067.416143	0.010037912
0.57805	4443.4323	4443.43231	2.20851E-07	368.30754	4055.7815	4055.638093	0.003535863
2.416103	4433.0822	4433.082196	9.52355E-08	386.85386	4039.4509	4039.430038	0.000516444
10.094109	4409.1469	4409.146919	4.37227E-07	451.206897	3983.448	3983.190443	0.006465688
33.033523	4377.8632	4379.777555	0.043728065	480.688448	3958.1895	3957.958183	0.005844013
49.331543	4360.6091	4360.861222	0.005781808	487.935889	3952.0187	3951.853104	0.004190159
69.631015	4340.1192	4340.396923	0.006398969	501.220596	3941.0494	3940.66237	0.009820472
77.384755	4332.378	4332.580224	0.004667742	521.461669	3923.4054	3923.611753	0.005259531
98.446606	4311.4285	4311.409518	0.000440271	539.896352	3907.6411	3908.082797	0.011303412
118.208707	4291.7926	4291.87244	0.001860306	557.873514	3893.0227	3892.939246	0.00214368
143.825915	4266.563	4266.546926	0.00037675	585.063121	3870.9362	3870.416019	0.013438107
166.206913	4244.4263	4244.603379	0.00417203	619.085793	3843.0052	3842.553125	0.011763583
188.590013	4222.8188	4223.050141	0.005478363	630.375808	3833.6872	3833.307158	0.009913223
207.127402	4204.5269	4205.200034	0.016009736	647.55333	3819.3391	3819.239613	0.002604833
227.770552	4184.5666	4185.322237	0.01805771	664.884294	3805.8263	3805.046406	0.020492115
245.781521	4167.867	4167.979032	0.002687993	680.127849	3793.7403	3792.67429	0.028099189
261.227271	4154.0067	4153.444684	0.013529493	696.771778	3779.5569	3779.337895	0.005794457

TIEMPO [Dias]	Pwf [Psi] Fortran	Pwf [Psi] Python interpolada	% Error	TIEMPO [Dias]	Pwf [Psi] Fortran	Pwf [Psi] Python interpolada	% Error
719.257983	3761.5636	3761.32022	0.006470174	1680.027452	2962.7686	2960.602397	0.073114151
729.442662	3753.9702	3753.159473	0.021596521	1785.027452	2870.932	2868.687685	0.078173746
763.747938	3726.6178	3725.67145	0.025394327	1890.027452	2775.9459	2773.642048	0.08299342
779.063484	3714.4781	3713.401188	0.028992286	1995.027452	2677.1516	2674.746641	0.089832772
801.161952	3697.2139	3695.846873	0.036974523	2100.027452	2573.4183	2569.988101	0.133293492
822.068825	3680.7859	3679.239123	0.042023019	2205.027452	2456.1907	2452.619343	0.145402269
843.123538	3663.5547	3662.513933	0.028408663	2310.027452	2326.4138	2323.467701	0.126636942
870.152148	3642.4263	3641.043271	0.037969989	2415.027452	2189.7024	2186.620318	0.140753449
902.861004	3615.8202	3614.199601	0.044819676	2520.027452	2045.1409	2041.237544	0.190860006
931.159239	3592.7665	3590.441547	0.064712057	2623.843414	1885.0159	1880.740736	0.226797235
964.859307	3564.3918	3562.148332	0.062941115	2713.282534	1732.2954	1729.060646	0.186732267
993.098908	3540.0678	3538.34278	0.048728435	2798.379167	1580.6342	1577.287148	0.211753708
1026.796889	3511.0033	3509.290794	0.04877541	2889.149431	1500	1506.008228	0.400548549
1069.705696	3474.0563	3472.297892	0.050615404	2986.902442	1500	1500	0
1119.052884	3432.0051	3430.14789	0.054114424	3091.902442	1500	1500	0
1177.735563	3382.4238	3380.471035	0.057732717	3196.902442	1500	1500	0
1260.027452	3313.1341	3311.655165	0.044638561	3301.902442	1500	1500	0
1365.027452	3226.5957	3224.659208	0.06001657	3406.902442	1500	1500	0
1470.027452	3139.8074	3137.788354	0.06430478	3511.902442	1500	1500	0
1575.027452	3052.1599	3050.056567	0.068912923	3616.902442	1500	1500	0

Tabla 11. “Porcentajes de error del simulador migrado respecto al original”.

El promedio de todos los porcentajes de error de los resultados obtenidos del simulador migrado respecto al simulador original fue del **0.0409 %**, con lo cual, se puede afirmar que los resultados y la migración del código haya sido realizada de manera correcta.

5.2. Tiempos de cómputo

Al comprobarse que el porcentaje de error de resultados del simulador migrado respecto al simulador original fuese bastante pequeño, se decidió llevar a cabo un comparativo de tiempos de cómputo, empleando ahora (además del método de descomposición LU optimizado de SPSOLVE) el método GMRES, igual de la librería de Scipy para Python (Scipy, 2022).

Propiedades de la computadora utilizada	
Procesador:	Intel(R) Core(TM) i5-8300H CPU @ 2.30 GHZ
RAM:	8GB

Tabla 12. "Propiedades de la computadora utilizada".

Intel (R) Fortran Compiler for One API		
GMRES (num. de corridas)	**Valor**	**Unidades**
1	26.2677	[minutos]
2	26.7742	[minutos]
3	25.4781	[minutos]
4	25.4966	[minutos]

Tabla 13. "Tiempos de cómputo del simulador original corrido con Intel (R) Fortran Compiler for One API". (Intel (R), 2022).

Python 3.8 Spyder		
SPSOLVE (num. de corridas)	**Valor**	**Unidades**
1	148.8366	[minutos]
2	148.2862	[minutos]
3	148.7866	[minutos]
4	150.4158	[minutos]
GMRES (num. de corridas)	**Valor**	**Unidades**
1	912.2423	[minutos]
2	911.2022	[minutos]
3	912.451	[minutos]
4	911.672	[minutos]

Tabla 14. "Tiempos de cómputo del código original corrido con Python 3.8 de Spyder". (Spyder, 2022).

De las tablas anteriores se puede observar que el tiempo de cómputo empleando el método GMRES para la solución del sistema de ecuaciones lineales del código migrado resulta realmente extenso (de aproximadamente 911 minutos), en contraste con el método SPSOLVE (descomposición LU optimizado) con un tiempo aproximado de 148 minutos. Mientras que el código original tarda aproximadamente 25 minutos en compilar.

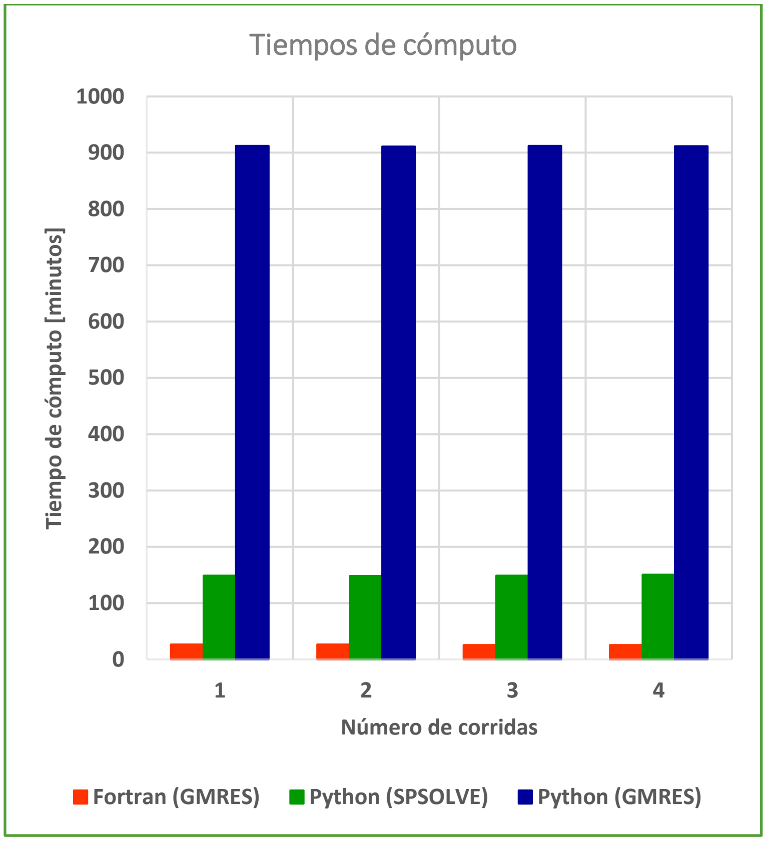

Fig. 46. "Comparaciones de tiempos de cómputo (Python vs Fortran)".

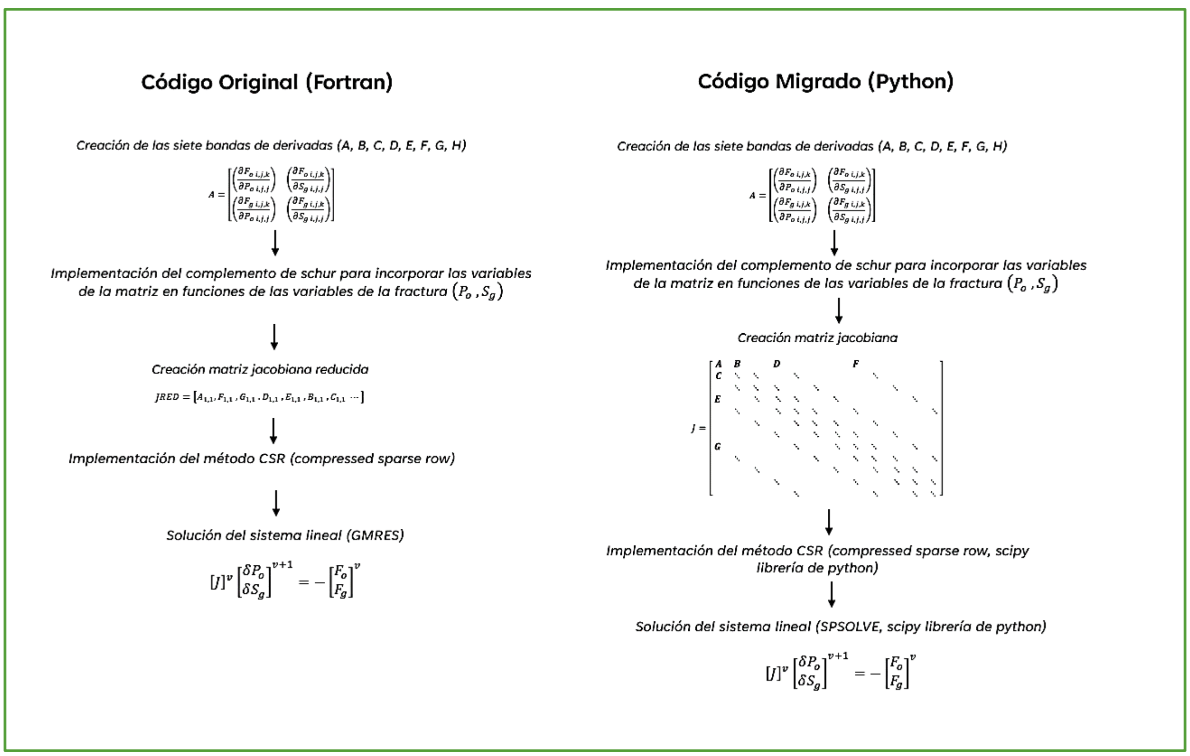

Fig. 47. "Esquema de solución entre ambos simuladores".

El esquema de solución que se presenta en la Fig. 47 muestra una de las principales diferencias entre ambos códigos. El código migrado, realiza las siete bandas de derivadas (A, B, C, D, E, F y G), y posteriormente lleva a cabo la creación de la matriz jacobiana (J), es decir, una matriz 2D que contara con las siete bandas requeridas por el jacobiano, y los demás valores que la componen serán cero. En cambio, el código original, también calcula las siete bandas de derivadas, sin embargo, al realizar la matriz jacobiana, lo que hace es guardar los resultados de las siete bandas en un vector de 1D, por así decirlo, en "una matriz jacobiana reducida" (J_{RED}), con lo cual, considerando esto, y que, además, Fortran es un lenguaje diseñado para la realización de cálculos numéricos, el código original puede ser bastante óptimo en tiempos de cómputo comparado con el código migrado.

5.3. Ventajas de implementar Python

Si bien, el tiempo de cómputo del simulador en Python 3.8 resultó ser más elevado que el original en Fortran 90, debido principalmente a que Fortran fue diseñado especialmente para llevar a cabo cálculos numéricos, a diferencia de Python, que además de poder utilizarse en la solución de problemas numéricos, tiene una amplia gama de oportunidades, por ejemplo, algunas ventajas que podemos encontrar son:

a) Es un lenguaje de programación de alto nivel, es decir, más amigable e intuitivo con el usuario, que logra acortar los tiempos de adaptación al lenguaje, desarrollo e implementación en la codificación.

b) Los proyectos desarrollados en este tipo de lenguaje pueden ser más fáciles de extender, al contar con un gran número de paquetes de código abierto que pueden ser usados.

c) Cuenta con librerías enfocadas en procurar mayor versatilidad, así como en la optimización de cómputo y procesamiento inmediato (como puede ser la librería Numba), las cuales están en constante actualización, tratando de reducir en la mayor medida de lo posible los tiempos de cómputo.

d) Tiene disponible una gran cantidad de herramientas para el post procesado de resultados de algún proyecto desarrollado en este lenguaje, por ejemplo, la extensa cantidad de opciones de visualización de resultados y generación de gráficos 2D y 3D (como puede ser la implementación de la librería Matplotlib o Plotly).

A continuación, se puede observar el comportamiento de la presión de aceite (P_o), presión de gas (P_g), saturación de aceite ($S_{at\ o}$) y saturación de gas ($S_{at\ g}$) de la fractura natural a través del modelo 3D realizado en Python, como ejemplo de una de las ventajas de la migración del simulador.

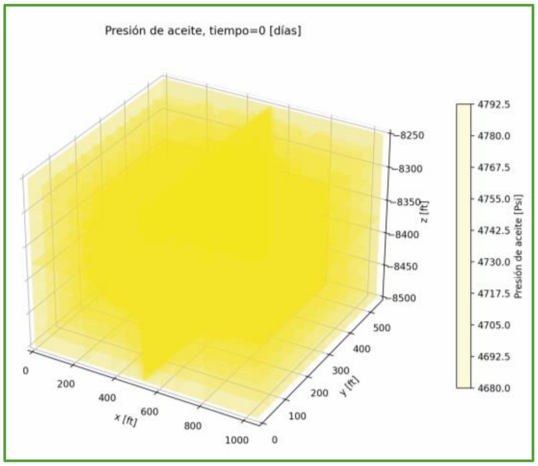

Fig. 48. **“Comportamiento de la presión de aceite de la fractura natural en el yacimiento”.**

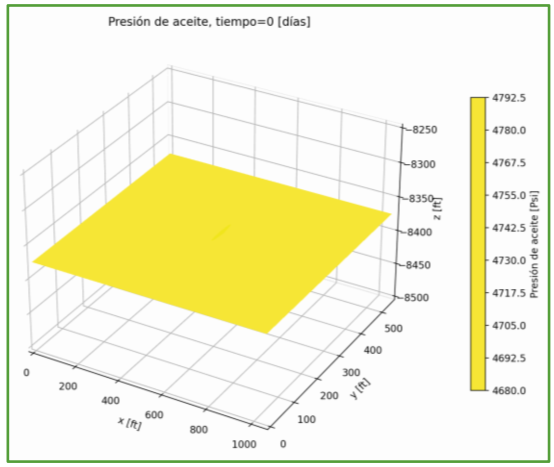

Fig. 49. “Comportamiento de la presión de aceite de la fractura natural en la sección del pozo (nx=11, ny=6, nz=5)”.

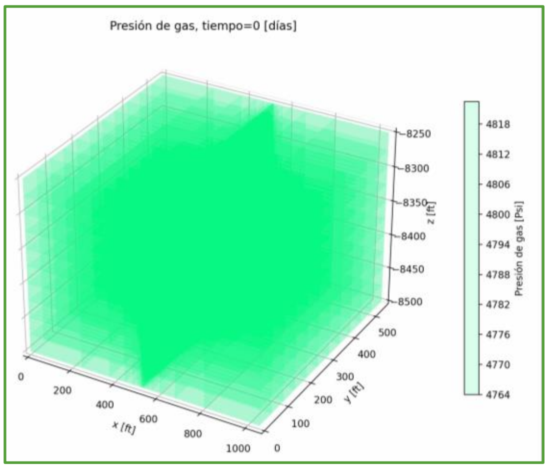

Fig. 50. “Comportamiento de la presión de gas de la fractura natural en el yacimiento”.

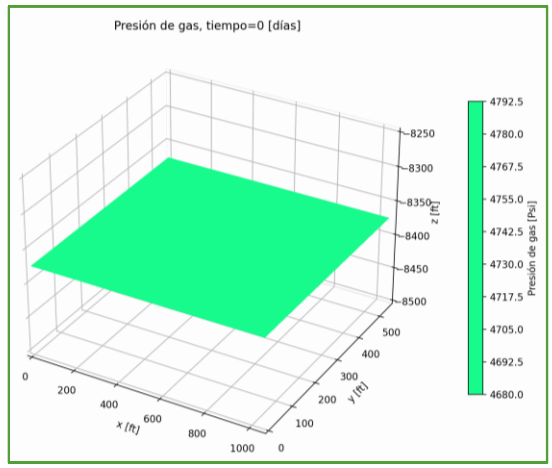

Fig. 51. “Comportamiento de la presión de gas de la fractura natural en la sección del pozo (nx=11, ny=6, nz=5)”.

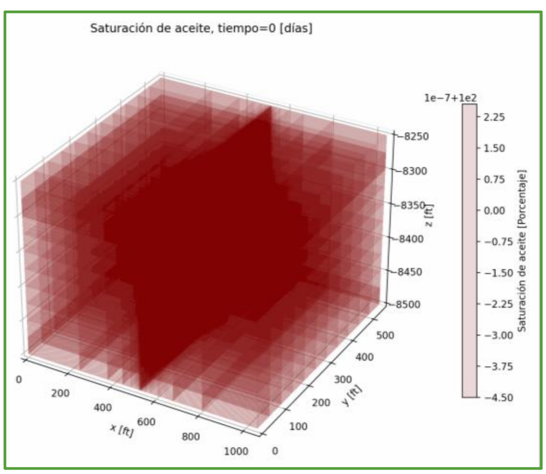

Fig. 52. “Comportamiento de la saturación de aceite de la fractura natural en el yacimiento”.

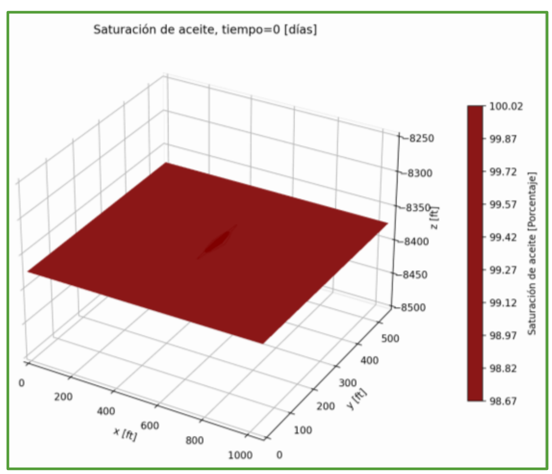

Fig. 53. “Comportamiento de la saturación de aceite de la fractura natural en la sección del pozo (nx=11, ny=6, nz=5)”.

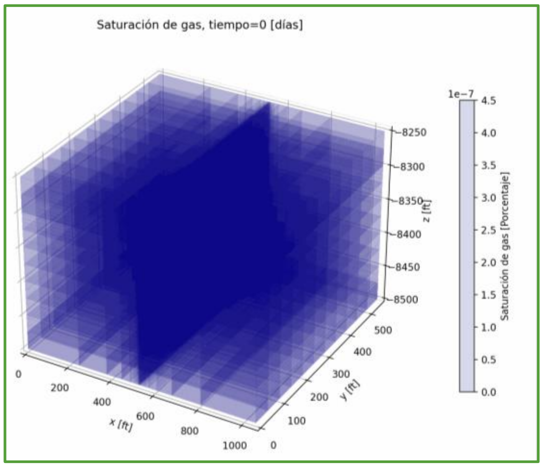

Fig. 54. "Comportamiento de la saturación de gas de la fractura natural en el yacimiento".

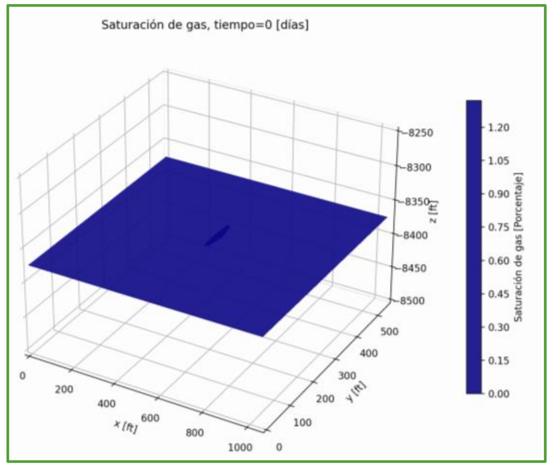

Fig. 55. "Comportamiento de la saturación de gas de la fractura natural en la sección del pozo (nx=11, ny=6, nz=5)".

El comportamiento de la presión de aceite (P_o) y gas (P_g) tanto en el modelo 3D como en las secciones cercanas al pozo disminuye conforme el yacimiento se va desarrollando, recordando que la presión de gas se determina a través de la presión capilar.

$$P_{cap\,go} = P_g - P_o$$

$$P_{g_f} = P_{cap_f\,go} + P_{o_f}$$

Respecto a la saturación de aceite ($S_{at\,o}$) al inicio de la simulación es de un 100%, al considerar el contacto gas-aceite en la cima del yacimiento, sin embargo, conforme se comienza a producir aceite en la superficie, y con ello a disminuir la presión, la saturación de aceite disminuye con el paso del tiempo. Recordando que la saturación de aceite se determina una vez obtenido el valor de la saturación del gas (variable primaria) de la simulación y a través de la definición de saturación para flujo bifásico.

$$S_{at\,o_f} = 1 - S_{at\,g_f}$$

Para el caso de la saturación de gas de la fractura natural en el yacimiento, de manera inversa a la saturación de aceite, aumenta conforme se desarrolla el yacimiento Fig. 52 y Fig. 54

6. Conclusiones y recomendaciones

En este capítulo se presentan las conclusiones y recomendaciones relacionadas al trabajo presentado en este escrito.

6.1. Conclusiones

El modelo de aceite negro modificado (o beta modificado) para el tratamiento de yacimientos de tipo gas y condensado o aceite volátil, realmente logró implementarse para la simulación del comportamiento del flujo de fluidos en yacimientos naturalmente fracturados.

Se logró modelar un caso de estudio correspondiente a un modelo numérico tridimensional, isotérmico, bifásico (aceite y gas), naturalmente fracturado, y con efectos de fracturas hidráulicas, para un yacimiento de tipo gas y condensado, y que contempla un pozo horizontal a lo largo de la dirección "x", presentado por (López Jiménez, 2017).

La migración presentada del código correspondiente al caso de estudio, y su optimización a través de la librería de Numba para Python (Numba, 2022) presentó resultados satisfactorios, al tener un buen empalme entre los resultados del simulador original codificado en Fortran 90 y el migrado a Python 3.8 (lenguaje de alto nivel), con un error promedio del 0.0409 % en los resultados del simulador migrado respecto al original.

El método SPSOLVE (descomposición LU optimizado) de la librería de Scipy para Python (Scipy, 2022) resultó ser el mejor método para la solución del sistema de ecuaciones lineales del caso de estudio, ya que empleando el método GMRES (residuo mínimo generalizado), igual de la librería de Scipy para Python (Scipy, 2022) se obtuvieron tiempos de cómputo mucho más largos para la solución.

Si bien el simulador migrado presentó tiempos de solución mayores al simulador original, se tuvieron ventajas en el post procesado de los resultados, al poder generar los modelos 2D y 3D del caso de estudio, así como la visualización del comportamiento de las presiones y saturaciones en cada paso de tiempo, además de tener una mayor facilidad de poder extenderse en un futuro e implementar demás métodos de optimización (como podría ser cómputo en paralelo), ya que se puede contar con un gran número de paquetes de código abierto que pudiesen ser utilizados.

6.2. Recomendaciones

El modelo presentado podría extenderse para la simulación de yacimientos con triple porosidad (Cortes Rubio, 2008).

Para reducir los tiempos de cómputo del simulador, valdría la pena implementar métodos que fuesen más rápidos para la solución del sistema de ecuaciones lineales, por ejemplo, la implementación del método de multimalla y con precondicionadores (Orozco Aguilar, 2013).

De igual forma, podría explorarse la posibilidad de emplear cómputo en paralelo en el simulador, se infiere que podría reducir de manera significativa los tiempos de cómputo (Alcántara Garcés, 2006).

Nomenclatura

$B_o\ \left[\frac{Vol_o\ c.y.}{Vol_o\ c.s.}\right]$ Factor de volumen de aceite

$B_g\ \left[\frac{Vol_g\ c.y.}{Vol_g\ c.s.}\right]$ Factor de volumen de gas

$B_w\ \left[\frac{Vol_w\ c.y.}{Vol_w\ c.s.}\right]$ Factor de volumen de agua

$b_o\ \left[\frac{Vol_o\ c.s.}{Vol_o\ c.y.}\right]$ Inverso del factor de volumen de aceite

$b_g\ \left[\frac{Vol_g\ c.s.}{Vol_g\ c.y.}\right]$ Inverso del factor de volumen de gas

$b_w\ \left[\frac{Vol_w\ c.s.}{Vol_w\ c.y.}\right]$ Inverso del factor de volumen de agua

C_{oo} Fracción de los pseudocomponentes de aceite presentes en la fase de aceite

C_{og} Fracción de los pseudocomponentes de aceite presentes en la fase de gas

C_{ow} Fracción de los pseudocomponentes de aceite presentes en la fase de agua

C_{go} Fracción de los pseudocomponentes de gas presentes en la fase de aceite

C_{gg} Fracción de los pseudocomponentes de gas presentes en la fase de gas

C_{gw} Fracción de los pseudocomponentes de gas presentes en la fase de agua

C_{wo} Fracción de los pseudocomponentes de agua presentes en la fase de aceite

C_{wg} Fracción de los pseudocomponentes de agua presentes en la fase de gas

C_{ww} Fracción de los pseudocomponentes de agua presentes en la fase de agua

$C_r\ \left[\frac{1}{psi}\right]$ Compresibilidad de la roca

F_o Función de residuos de la fase de aceite

F_g Función de residuos de la fase de gas

F_w Función de residuos de la fase de agua

$h_{x,y,z}$ Tamaño de los bloques de matriz (usado en el cálculo del de forma para la interacción matriz-fractura)

$k\ [mD]$ Permeabilidad absoluta

$k_{ro}[Fracción]$ Permeabilidad relativa del aceite

$k_{rg}[Fracción]$ Permeabilidad relativa del gas

$k_{rw}[Fracción]$ Permeabilidad relativa del agua

$P_{wf}[Psi]$ Presión de fondo fluyendo del pozo

$P_o[Psi]$ Presión de aceite

$P_g[Psi]$ Presión de gas

$P_w[Psi]$ Presión de agua

$P_{cap\ go}[Psi]$ Presión capilar gas-aceite

$P_{cap\ wo}[Psi]$ Presión capilar agua-aceite

q_o^* Producción volumétrica de aceite por unidad de volumen de roca

q_g^* Producción volumétrica de gas por unidad de volumen de roca

q_w^* Producción volumétrica de agua por unidad de volumen de roca

q_o Gasto volumétrico de la fase de aceite

q_g Gasto volumétrico de la fase de gas

q_w Gasto volumétrico de la fase de agua

R_s Relación de solubilidad de la fase de aceite en la fase de gas

r_s Relación de solubilidad de la fase de gas en la fase de aceite

r_e Radio equivalente del bloque donde se encuentra representado el pozo

r_w Radio del del pozo

S Daño (también conocido como skin factor)

$S_{at\ o}[Fracción]$ Saturación de aceite

$S_{at\ g}[Fracción]$ Saturación de gas

$S_{at\ w}[Fracción]$ Saturación de agua

V_o Velocidad de la fase de aceite (ley de Darcy)

V_g Velocidad de la fase de gas (ley de Darcy)

V_w Velocidad de la fase de agua (ley de Darcy)

$V_r\ [ft^3]$ Volumen de roca

$V_p\ [ft^3]$ Volumen poroso

WI Índice de productividad

$z, D\ [ft]$ Profundidad

$z_{goc}[ft]$ Profundidad del contacto gas-aceite

$z_{woc}[ft]$ Profundidad del contacto agua-aceite

$\tau^*_{o_{mf}}$ Función de transferencia matriz-fractura para la fase de aceite

$\tau^*_{g_{mf}}$ Función de transferencia matriz-fractura para la fase de gas

$\tau^*_{w_{mf}}$ Función de transferencia matriz-fractura para la fase de agua

σ Factor de forma para la interacción matriz-fractura (también conocido como shape factor)

$\emptyset\ [Fracción]$ Porosidad

γ_o Densidad relativa del aceite (o también conocido como peso específico)

$\rho_o \left[\frac{lbm}{ft^3}\right]$ Densidad del aceite

$\rho_g \left[\frac{lbm}{ft^3}\right]$ Densidad del gas

$\rho_w \left[\frac{lbm}{ft^3}\right]$ Densidad del agua

$\mu_o[cP]$ Viscosidad del aceite

$\mu_g[cP]$ Viscosidad del gas

$\mu_w[cP]$ Viscosidad del agua

λ_o Movilidad de la fase de aceite

λ_g Movilidad de la fase de gas

λ_w Movilidad de la fase de agua

<u>Subíndices y superíndices</u>

f Fracturas

m Matriz

mf Matriz-fractura

o Fase de aceite

g Fase de gas

w Fase de agua

x Dirección x

y Dirección y

z Dirección z

v Nivel de iteración

n Nivel de tiempo

ref Referencia

$i \pm \frac{1}{2}$ Fronteras de la celda i (i hace referencia al eje x)

$j \pm \frac{1}{2}$ Fronteras de la celda j (j hace referencia al eje y)

$k \pm \frac{1}{2}$ Fronteras de la celda k (k hace referencia al eje z)

Anexo. Definición de derivadas de las funciones de residuos

El método de Newton-Raphson, empleado para la linealización de las ecuaciones diferenciales ya discretizadas (que describen el flujo de fluidos en un medio poroso) requieren el uso de las derivadas de las funciones de residuos para las tres fases (aceite, gas y agua) considerando una simulación trifásica, con lo cual, a continuación se definen estas derivadas considerando una formulación totalmente implícita.

Para este caso, las derivadas de las funciones de residuos para las tres fases se desarrollan en la dirección "k", ya que el procedimiento para las otras dos direcciones en "i" y "j" es el mismo sin considerar los efectos gravitacionales. (Orozco Aguilar, 2013).

a) Si la función de residuos de la fase de aceite en la fractura natural $\left(F_{o_f}\right)$ es:

$$F_{o_{f_{i,j,k}}}^{n+1} = \left[\begin{array}{c}
T_{o_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i+\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i-\frac{1}{2},j,k}}}^{n+1} \, r_{s_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j+\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j-\frac{1}{2},k}}}^{n+1} \, r_{s_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)_f^{n+1} \\
+ \; T_{o_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - \left(\gamma_o \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{o_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - \left(\gamma_o \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \; T_{g_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)_f^{n+1} \\
- \; T_{g_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \, r_{s_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)_f^{n+1} \\
+ \left(b_{o_f} q_o\right)_{i,j,k}^{n+1} \\
+ \left(b_{g_f} r_{s_f} q_g\right)_{i,j,k}^{n+1} \\
+ \; T_{o_{mf_{i,j,k}}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_{i,j,k}^{n+1} \\
+ \; T_{g_{mf_{i,j,k}}}^{n+1} \, r_{s_{m_{i,j,k}}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}}\right)^{n+1} \\
- \dfrac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_{i,j,k} \end{array} \right]_f
\end{array} \right] = 0$$

Ahora, truncando únicamente en la dirección "k":

$$
F_{o_{f_k}}^{n+1} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \\
-\ T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\ \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \\
+\ T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_{k+1} - \left(P_{cap\ go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \\
-\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_k - \left(P_{cap\ go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \\
+\ \left(b_{o_f} q_o\right)_k^{n+1} \\
+\ \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1} \\
+\ T_{om f_k}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\
+\ T_{gm f_k}^{n+1}\ r_{s_{m_k}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \\
-\ \frac{Vp_k}{\Delta t} \left[\begin{array}{c}
\left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\ g} - S_{at\ w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\ g} - S_{at\ w}\right)\right]^n\right)_k \\
+\left(\ [1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\ g}\right]^{n+1} - \left[b_g r_s S_{at\ g}\right]^n\right)_k
\end{array}\right]_f
\end{array}\right]
$$

Si definimos los siguientes términos, de manera similar a (Cortes Rubio, 2008) y (Orozco Aguilar, 2013):

$$
\text{Term\_flujo\_aceite} = \left[\begin{array}{c}
\text{Trnsmsc\_aceite\_Z2}\ \ \text{Pot\_aceite\_Z2} \\
-\ \text{Trnsmsc\_aceite\_Z1}\ \ \text{Pot\_aceite\_Z1} \\
+\ \text{Trnsmsc\_gas\_Z2}\ \ rs\_Z2\ \ \text{Pot\_gas\_Z2} \\
-\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ \ \text{Pot\_gas\_Z1}
\end{array}\right]
$$

$$\text{Trnsmsc\_aceite\_Z2} = T_{o_{f_{k+\frac{1}{2}}}}^{n+1}$$

$$\text{Trnsmsc\_aceite\_Z1} = T_{o_{f_{k-\frac{1}{2}}}}^{n+1}$$

$$\text{Pot\_aceite\_Z2} = \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}$$

$$\text{Pot\_aceite\_Z1} = \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}$$

$$\text{Trnsmsc\_gas\_Z2} = T_{g_{f_{k+\frac{1}{2}}}}^{n+1}$$

$$\text{Trnsmsc\_gas\_Z1} = T_{g_{f_{k-\frac{1}{2}}}}^{n+1}$$

$$rs\_Z2 = r_{s_{f_{k+\frac{1}{2}}}}^{n+1}$$

$$rs\_Z1 = r_{s_{f_{k-\frac{1}{2}}}}^{n+1}$$

$$\text{Pot\_gas\_Z2} = \left(\left(P_{cap\ go} + P_o\right)_{k+1} - \left(P_{cap\ go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1}$$

$$\text{Pot\_gas\_Z1} = \left(\left(P_{cap\ go} + P_o\right)_k - \left(P_{cap\ go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1}$$

$$
\text{Term\_fuente} = \left[\begin{array}{c}
\left(b_{o_f} q_o\right)_k^{n+1} \\
+\ \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1}
\end{array}\right]
$$

$$\text{Term\_transf\_MF} = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \; \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + \; T_{g_{mf_k}}^{n+1} \; r_{s_{m_k}}^{n+1} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k} \right)^{n+1} \end{array} \right]$$

$$\text{Term\_acum} = \left[\; \frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\ + \left(\; [1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_k \end{array} \right]_f \; \right]$$

Entonces la función de residuos de la fase de aceite en la fractura natural truncada en la dirección “k” puede escribirse como:

$$F_{o_{f_k}}^{n+1} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \\ - \; \text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \\ + \; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \\ - \; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \\ + \; \text{Term\_fuente} \\ + \; \text{Term\_transf\_MF} \\ - \; \text{Term\_acum} \end{array} \right]$$

a.a) Derivando la función de residuos de aceite $\left(F_{o_f}\right)$ truncada en la dirección “k” respecto a la presión de aceite en la fractura natural $\left(P_{o_f}\right)$ en “k”:

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \\ - \; \text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \\ + \; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \\ - \; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \\ + \; \text{Term\_fuente} \\ + \; \text{Term\_transf\_MF} \\ - \; \text{Term\_acum} \end{array} \right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \;] \\ - \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \;] \\ + \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \;] \\ - \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \;] \\ + \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Term\_fuente} \;] \\ + \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Term\_transf\_MF} \;] \\ - \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; \text{Term\_acum} \;] \end{array} \right]$$

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
\text{Trnsmsc\_aceite\_Z2} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_aceite\_Z2}\ ] + \text{Pot\_aceite\_Z2} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_aceite\_Z2}\ ] \\
- \left(\text{Trnsmsc\_aceite\_Z1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_aceite\_Z1}\ ] + \text{Pot\_aceite\_Z1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_aceite\_Z1}\ ] \right) \\
+ \left(\text{Trnsmsc\_gas\_Z2} \ rs\_Z2 \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_gas\_Z2}\ ] + \text{Pot\_gas\_Z2} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_gas\_Z2} \ rs\_Z2\ ] \right) \\
- \left(\text{Trnsmsc\_gas\_Z1} \ rs\_Z1 \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_gas\_Z1}\ ] + \text{Pot\_gas\_Z1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_gas\_Z1} \ rs\_Z1\ ] \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_fuente}\ ] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_transf\_MF}\ ] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_acum}\ ]
\end{array} \right]
$$

Si sustituimos términos:

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
- \left(\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} q_o \right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g \right)_k^{n+1} \end{array} \right] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \ \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\ + \ T_{g_{mf_k}}^{n+1} \ r_{s_{m_k}}^{n+1} \ \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array} \right] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\frac{V p_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r (P_o^{n+1} - P_o^n)] [b_o (1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o (1 - S_{at\,g} - S_{at\,w})]^n \right)_k \\ + \left([1 + C_r (P_o^{n+1} - P_o^n)] [b_g r_s S_{at\,g}]^{n+1} - [b_g r_s S_{at\,g}]^n \right)_k \end{array} \right]_f \right]
\end{array} \right]
$$

Realizando las derivadas que se presentan dentro del corchete:

1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_o}{\mu_o}\right)_{f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{\mu_{o_{k-\frac{1}{2}}}\frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - b_{o_{k-\frac{1}{2}}}\frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{o_{k-\frac{1}{2}}}^2}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_o}{\mu_o}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{b_{o_{k-\frac{1}{2}}}}\frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}}\frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\right] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\left(\frac{1}{b_{o_{k-\frac{1}{2}}}}\frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}}\frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_o}{\mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{\mu_{o_{k+\frac{1}{2}}}\frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - b_{o_{k+\frac{1}{2}}}\frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{o_{k+\frac{1}{2}}}^2}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \left(\frac{A_z k k_{ro}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_o}{\mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{1}{b_{o_{k+\frac{1}{2}}}}\frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}}\frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\right] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\left(\frac{1}{b_{o_{k+\frac{1}{2}}}}\frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}}\frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_g}{\mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{\mu_{g_{k-\frac{1}{2}}}\dfrac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - b_{g_{k-\frac{1}{2}}}\dfrac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{g_{k-\frac{1}{2}}}^2}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_g}{\mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \right] = T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \text{Trnsmsc\_gas\_Z1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ rs\_Z1\ ] + rs\_Z1\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right] + r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\frac{\partial r_{s_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\left(T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\right)$$

4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_g}{\mu_g}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\mu_{g_{k+\frac{1}{2}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - b_{g_{k+\frac{1}{2}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{g_{k+\frac{1}{2}}}^2}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \left(\frac{A_z k k_{rg}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_g}{\mu_g}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}}\frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}}\frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\right] = T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}}\frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}}\frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \text{Trnsmsc\_gas\_Z2} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; rs\_Z2 \;] + rs\_Z2 \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right] + r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial r_{s_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}}\frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}}\frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\right)$$

<u>5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):</u>

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_o \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1}) \left(\frac{\gamma_{o_{f_k}} + \gamma_{o_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = 1 - (D_k - D_{k-1}) \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{o_{f_k}}^{n+1} + \gamma_{o_{f_{k-1}}}^{n+1}}{2}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\gamma_{o_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_o \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\left(\frac{\gamma_{o_{f_{k+1}}} + \gamma_{o_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k)\left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2}\right)\right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = -1 - (D_{k+1} - D_k)\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{o_{f_{k+1}}}^{n+1} + \gamma_{o_{f_k}}^{n+1}}{2}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = -1 - \frac{(D_{k+1} - D_k)}{2}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \gamma_{o_{f_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right] = -1 - \frac{(D_{k+1} - D_k)}{2}\frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

7) Derivada del potencial de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_g \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1})\left(\frac{\gamma_{g_{f_k}} + \gamma_{g_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ (P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (D_k - D_{k-1})\left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2}\right)\right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = 1 - (D_k - D_{k-1})\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{g_{f_k}}^{n+1} + \gamma_{g_{f_{k-1}}}^{n+1}}{2}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = 1 - \frac{(D_k - D_{k-1})}{2}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \gamma_{g_{f_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\ \right] = 1 - \frac{(D_k - D_{k-1})}{2}\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

8) Derivada del potencial de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_g \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\left(\frac{\gamma_{g_{f_{k+1}}} + \gamma_{g_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ (P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (D_{k+1} - D_k)\left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2}\right)\right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = -1 - (D_{k+1} - D_k)\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{g_{f_{k+1}}}^{n+1} + \gamma_{g_{f_k}}^{n+1}}{2}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = -1 - \frac{(D_{k+1} - D_k)}{2}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \gamma_{g_{f_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] = -1 - \frac{(D_{k+1} - D_k)}{2}\ \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

9) Derivada del término fuente en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1} \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \right] \right) \end{bmatrix}$$

Si definimos:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ r_{s_{f_k}}^{n+1}\ \right] + r_{s_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right]$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1} \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k} \left(b_{g_{f_k}}^{n+1} \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1} \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k} \left(b_{g_{f_k}}^{n+1} \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \end{bmatrix}$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} T_{o_{mf_k}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k}\right)^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_m} - P_{o_f}\right)_k^{n+1} \right] + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1} \right] \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k} \right)^{n+1} \right] \\ + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k} \right)^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1} \right] \end{array} \right) \end{array} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ (-1\ ) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ (-1\ ) \\ + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k} \right)^{n+1} \\ \left(T_{g_{mf_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[r_{s_{m_k}}^{n+1} \right] + r_{s_{m_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1} \right] \right) \end{array} \right) \end{array} \right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) \geq \mathbf{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{m\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\ i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{m_{i,j,k}}}^{n+1}$$

$$\boxed{\frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) < \mathbf{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\ i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[Vr_k \left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\ k}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_o}\ \right)_{f_k}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_o}{\mu_o} \right)_{f_k}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_o}\ \right)_{f_k}^{n+1} \left(\frac{\mu_{o_k} \frac{\partial b_{o_k}}{\partial P_{o_k}} - b_{o_k} \frac{\partial \mu_{o_k}}{\partial P_{o_k}}}{\mu_{o_k}^2} \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_o}\ \right)_{f_k}^{n+1} \left(\frac{b_o}{\mu_o} \right)_{f_k}^{n+1} \left(\frac{1}{b_{o_k}} \frac{\partial b_{o_k}}{\partial P_{o_k}} - \frac{1}{\mu_{o_k}} \frac{\partial \mu_{o_k}}{\partial P_{o_k}} \right)_f^{n+1}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right] = T_{o_{mf_k}}^{n+1} \left(\frac{1}{b_{o_k}} \frac{\partial b_{o_k}}{\partial P_{o_k}} - \frac{1}{\mu_{o_k}} \frac{\partial \mu_{o_k}}{\partial P_{o_k}} \right)_f^{n+1}}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) + \left(\boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{f}_{i,j,k}}}\right) \geq \boldsymbol{0}$$

$$T_{gmf_{i,j,k}}^{n+1} = Vr_{i,j,k}\left(\sigma k \lambda_g\right)_{m\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{m_{i,j,k}}}^{n+1}$$

$$\frac{\partial T_{gmf_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0$$

$$\frac{\partial r_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) + \left(\boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{f}_{i,j,k}}}\right) < \boldsymbol{0}$$

$$T_{gmf_{i,j,k}}^{n+1} = Vr_{i,j,k}\left(\sigma k \lambda_g\right)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ Vr_k\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\ k}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_g}\ \right)_{f_k}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ \frac{b_g}{\mu_g}\ \right)_{f_k}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_g}\ \right)_{f_k}^{n+1} \left(\frac{\mu_{g_k}\dfrac{\partial b_{g_k}}{\partial P_{o_k}} - b_{g_k}\dfrac{\partial \mu_{g_k}}{\partial P_{o_k}}}{\mu_{g_k}^2}\right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] = \left(\ Vr\ \sigma\ k\ k_{r_g}\ \right)_{f_k}^{n+1} \left(\ \frac{b_g}{\mu_g}\ \right)_{f_k}^{n+1} \left(\ \frac{1}{b_{g_k}}\frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}}\frac{\partial \mu_{g_k}}{\partial P_{o_k}}\ \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] = T_{gmf_k}^{n+1}\left(\ \frac{1}{b_{g_k}}\frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}}\frac{\partial \mu_{g_k}}{\partial P_{o_k}}\ \right)_f^{n+1}$$

$$\frac{\partial r_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \frac{\partial r_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Term\_transf\_MF}\ \right] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ +\ T_{gmf_k}^{n+1}\ r_{s_{m_k}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Term\_transf\_MF}\ \right] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ (-1\ ) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ +\left(\begin{array}{c} T_{gmf_k}^{n+1}\ r_{s_{m_k}}^{n+1}\ (-1\ ) \\ +\ \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \\ \left(\ T_{gmf_k}^{n+1}\ \dfrac{\partial r_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{m_k}}^{n+1}\ \dfrac{\partial T_{gmf_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \right) \end{array}\right) \end{array}\right]$$

11) Derivada del término de acumulación en la fractura natural en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\left[\begin{matrix}\left([1+C_r(P_o^{n+1}-P_o^n)][b_o(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_o(1-S_{at\,g}-S_{at\,w})]^n\right)_k \\ +\left([1+C_r(P_o^{n+1}-P_o^n)][b_g r_s S_{at\,g}]^{n+1}-[b_g r_s S_{at\,g}]^n\right)_k\end{matrix}\right]_f\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{matrix}\left([1+C_r(P_o^{n+1}-P_o^n)]\,[b_o(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_o(1-S_{at\,g}-S_{at\,w})]^n\right)_{f_k} \\ +\left([1+C_r(P_o^{n+1}-P_o^n)]\,[b_g r_s S_{at\,g}]^{n+1}-[b_g r_s S_{at\,g}]^n\right)_{f_k}\end{matrix}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{matrix}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] \\ +\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\left(\begin{matrix}b_{o_{f_k}}^n\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[(1-S_{at\,g}-S_{at\,w})_{f_k}^n\right] \\ +(1-S_{at\,g}-S_{at\,w})_{f_k}^n\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^n\right]\end{matrix}\right) \\ +\left(\begin{matrix}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\right] \\ +\left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g r_s S_{at\,g}\right)_{f_k}^n\right]\end{matrix}\right)\end{matrix}\right]$$

Si definimos:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1-S_{at\,g}-S_{at\,w}\right)_{f_k}^{n+1}\right] + \left(1-S_{at\,g}-S_{at\,w}\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = 0 + \left(1-S_{at\,g}-S_{at\,w}\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}\right]$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = \left(1-S_{at\,g}-S_{at\,w}\right)_{f_k}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] = C_{r_k}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] = C_{r_k}}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\right] = b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[S_{at\,g}{}_{f_k}^{n+1}\right] + S_{at\,g}{}_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\right] = 0 + S_{at\,g}{}_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\right] = S_{at\,g}{}_{f_k}^{n+1}\left(b_{g_{f_k}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[r_{s_{f_k}}^{n+1}\right] + r_{s_{f_k}}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = S_{at\,g}{}_{f_k}^{n+1}\left(\ b_{g_{f_k}}^{n+1}\ \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n}\ \right] = 0$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ \right] = 0$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = b_{g_{f_k}}^{n}\ r_{s_{f_k}}^{n}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ S_{at\,g}{}_{f_k}^{n}\ \right] + S_{at\,g}{}_{f_k}^{n}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n}\ r_{s_{f_k}}^{n}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = 0 + 0$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = 0$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} \left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ \left(\ 1 - S_{at\,g} - S_{at\,w}\ \right)_{f_k}^{n+1}\ \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ + \left(\ b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\ \left(\ C_{r_k}\ \right) \\ - \left(\begin{array}{c} b_{o_{f_k}}^{n}\ (\ 0\ ) \\ + \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n}\ (\ 0\ ) \end{array}\right) \\ + \left(\begin{array}{c} \left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ S_{at\,g}{}_{f_k}^{n+1}\left(\ b_{g_{f_k}}^{n+1}\ \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ + \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \left(\ C_{r_k}\ \right) \\ - (\ 0\ ) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} \left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ \left(\ 1 - S_{at\,g} - S_{at\,w}\ \right)_{f_k}^{n+1}\ \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ + \left(\ b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\ \left(\ C_{r_k}\ \right) \\ - (\ 0\ ) \\ + \left(\begin{array}{c} \left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ S_{at\,g}{}_{f_k}^{n+1}\left(\ b_{g_{f_k}}^{n+1}\ \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ + \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \left(\ C_{r_k}\ \right) \\ - (\ 0\ ) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1 + C_r(P_o^{n+1} - P_o^n))_{f_k}\left(\left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ + \left(b_o(1 - S_{at\,g} - S_{at\,w})\right)_{f_k}^{n+1} \left(C_{r_k}\right) \\ + \left(\begin{array}{c} (1 + C_r(P_o^{n+1} - P_o^n))_{f_k}\left(S_{at\,g}{}_{f_k}^{n+1}\left(b_{g_{f_k}}^{n+1}\frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ + \left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\left(C_{r_k}\right) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1 + C_r(P_o^{n+1} - P_o^n))_{f_k}\left(\begin{array}{c} \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\ + S_{at\,g}{}_{f_k}^{n+1}\left(b_{g_{f_k}}^{n+1}\frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \end{array}\right) \\ + \left(C_{r_k}\right)\left(\left(b_o(1 - S_{at\,g} - S_{at\,w})\right)_{f_k}^{n+1} + \left(b_g r_s S_{at\,g}\right)_{f_k}^{n+1}\right) \end{array}\right]$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] + \text{Pot\_aceite\_Z2}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] \\ - \left(\text{Trnsmsc\_aceite\_Z1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] + \text{Pot\_aceite\_Z1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }]\right) \\ + \left(\text{Trnsmsc\_gas\_Z2}\ rs\_Z2\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] + \text{Pot\_gas\_Z2}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2}\ rs\_Z2\ ]\right) \\ - \left(\text{Trnsmsc\_gas\_Z1}\ rs\_Z1\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] + \text{Pot\_gas\_Z1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1}\ rs\_Z1\ ]\right) \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] \end{array}\right]$$

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
- \left(\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} q_o \right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g \right)_k^{n+1} \end{array} \right] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1} \; r_{s_{m_k}}^{n+1} \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array} \right] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)] \left[b_o \left(1 - S_{at\,g} - S_{at\,w} \right) \right]^{n+1} - \left[b_o \left(1 - S_{at\,g} - S_{at\,w} \right) \right]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)] \left[b_g r_s S_{at\,g} \right]^{n+1} - \left[b_g r_s S_{at\,g} \right]^n \right)_k \end{array} \right]_f \right]
\end{array} \right]
$$

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \left(-1 - \frac{(D_{k+1}-D_k)}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1}-D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k+\frac{1}{2}}}} \frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}} \frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \\
- \left(\begin{array}{c}
T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \left(1 - \frac{(D_k-D_{k-1})}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(P_{o_k} - P_{o_{k-1}} - (D_k-D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k-\frac{1}{2}}}} \frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}} \frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)
\end{array} \right) \\
+ \left(\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \, r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(-1 - \frac{(D_{k+1}-D_k)}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - (D_{k+1}-D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \\
\left(T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial r_{s_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \right)
\end{array} \right) \\
- \left(\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \, r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(1 - \frac{(D_k-D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - (D_k-D_{k-1}) \left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \\
\left(T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial r_{s_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \right)
\end{array} \right) \\
+ \left(\begin{array}{c}
b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\
+ \left(b_{g_{f_k}}^{n+1} \, r_{s_{f_k}}^{n+1} \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k} \left(b_{g_{f_k}}^{n+1} \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right)
\end{array} \right) \\
+ \left(\begin{array}{c}
T_{o_{mf_k}}^{n+1} \left(-1 \right) + \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\
+ \left(\begin{array}{c}
T_{g_{mf_k}}^{n+1} \, r_{s_{m_k}}^{n+1} \left(-1 \right) \\
+ \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k} \right)^{n+1} \\
\left(T_{g_{mf_k}}^{n+1} \frac{\partial r_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{m_k}}^{n+1} \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)
\end{array} \right)
\end{array} \right) \\
- \frac{Vp_k}{\Delta t} \left[\begin{array}{c}
\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k} \left(\begin{array}{c}
\left(1 - S_{at\,g} - S_{at\,w} \right)_{f_k}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \\
+ S_{at\,g_{f_k}}^{n+1} \left(b_{g_{f_k}}^{n+1} \frac{\partial r_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)
\end{array} \right) \\
+ \left(C_{r_k} \right) \left(\left(b_o \left(1 - S_{at\,g} - S_{at\,w} \right) \right)_{f_k}^{n+1} + \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \right)
\end{array} \right]
\end{array}
\right]
$$

a.b) Derivando la función de residuos de aceite $\left(F_{o_f}\right)$ truncada en la dirección "k" respecto a la saturación de gas en la fractura natural $\left(S_{at\,g_f}\right)$ en "k":

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \\ - \; \text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \\ + \; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \\ - \; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \\ + \; \text{Term\_fuente} \\ + \; \text{Term\_transf\_MF} \\ - \; \text{Term\_acum} \end{array} \right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \;] \\ - \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \;] \\ + \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \;] \\ - \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \;] \\ + \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_fuente} \;] \\ + \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_transf\_MF} \;] \\ - \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_acum} \;] \end{array} \right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Pot\_aceite\_Z2} \;] \; + \; \text{Pot\_aceite\_Z2} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z2} \;] \\ - \left(\text{Trnsmsc\_aceite\_Z1} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Pot\_aceite\_Z1} \;] \; + \; \text{Pot\_aceite\_Z1} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z1} \;] \right) \\ + \left(\text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Pot\_gas\_Z2} \;] \; + \; \text{Pot\_gas\_Z2} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;] \right) \\ - \left(\text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Pot\_gas\_Z1} \;] \; + \; \text{Pot\_gas\_Z1} \;\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;] \right) \\ + \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_fuente} \;] \\ + \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_transf\_MF} \;] \\ - \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\; \text{Term\_acum} \;] \end{array} \right]$$

Si sustituimos términos:

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(
\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]
\end{array}
\right) \\
- \left(
\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array}
\right) \\
+ \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[
\begin{array}{c}
\left(b_{o_f} q_o\right)_k^{n+1} \\
+ \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1}
\end{array}
\right] \\
+ \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[
\begin{array}{c}
T_{o_{mf_k}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\
+ T_{g_{mf_k}}^{n+1} r_{s_{m_k}}^{n+1} \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1}
\end{array}
\right] \\
- \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\dfrac{Vp_k}{\Delta t} \left[
\begin{array}{c}
\left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o(1 - S_{at\,g} - S_{at\,w})\right]^{n+1} - \left[b_o(1 - S_{at\,g} - S_{at\,w})\right]^n \right)_k \\
+ \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_k
\end{array}
\right]_f \right]
\end{array}
\right]
$$

Realizando las derivadas que se presentan dentro del corchete:

1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_o k_{ro}}{\mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[k_{ro_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k + \frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_o k_{ro}}{\mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[k_{ro_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

<u>3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g k_{rg}}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] =$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[k_{rg_{f_{k-\frac{1}{2}}}}^{n+1} \right] + k_{rg_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right] \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rg_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + k_{rg_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{\mu_{g_{k-\frac{1}{2}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} - b_{g_{k-\frac{1}{2}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}}}{\mu_{g_{k-\frac{1}{2}}}^2} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rg_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + k_{rg_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rg_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + k_{rg_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} = \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] =$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_g}{\mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}}\right)_f^{n+1}\right)$$

$$= \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}\right)$$

$$= \left(\frac{A_z k k_{rg} b_g}{\Delta z \mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + \left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}\right)$$

$$= T_{g f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + \left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}\right)$$

$$= \frac{T_{g f_{k-\frac{1}{2}}}^{n+1}}{k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + T_{g f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

Si recordamos la derivada de la transmisibilidad del gas en la fractura natural en "$k-\frac{1}{2}$" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P_{o f_k}^{n+1}}\left[T_{g f_{k-\frac{1}{2}}}^{n+1}\right] = T_{g f_{k-\frac{1}{2}}}^{n+1}\left(\frac{1}{b_{g_{k-\frac{1}{2}}}}\frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}}\frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}}\right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{T_{g f_{k-\frac{1}{2}}}^{n+1}}{k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k-\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k-\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g f_k}^{n+1}}\left[T_{g f_{k-\frac{1}{2}}}^{n+1}\right] = \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\frac{\partial k_{rg\,f_{k-\frac{1}{2}}}^{\;n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k-\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}}\frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = \frac{\partial}{\partial S_{at\,g f_k}^{n+1}}\left[T_{g f_{k-\frac{1}{2}}}^{n+1}\; r_{s f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = \text{Trnsmsc\_gas\_Z1}\;\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\; rs\_Z1 \;] + rs\_Z1\;\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }]$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = T_{g f_{k-\frac{1}{2}}}^{n+1}\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}\left[r_{s f_{k-\frac{1}{2}}}^{n+1}\right] + r_{s f_{k-\frac{1}{2}}}^{n+1}\frac{\partial}{\partial S_{at\,g f_k}^{n+1}}\left[T_{g f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = 0 + r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 } \; rs\_Z1 \;] = r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_g}{\Delta z \mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{rg}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_g k_{rg}}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g k_{rg}}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] =$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[{k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \right] + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \right] \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\mu_{g_{k+\frac{1}{2}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} - b_{g_{k+\frac{1}{2}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}}}{\mu_{g_{k-\frac{1}{2}}}^2} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial S_{at\,g_k}} \right)_f^{n+1} = \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] =$$

$$= \left(\frac{A_z k}{\Delta z}\right)^{n+1}_{f_{k+\frac{1}{2}}} \left(\frac{b_g}{\mu_g}\right)^{n+1}_{f_{k+\frac{1}{2}}} \left(\frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g k}} \right)^{n+1}_f \right)$$

$$= \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)^{n+1}_{f_{k+\frac{1}{2}}} \left(\frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_f \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} \right)$$

$$= \left(\frac{A_z k k_{rg} b_g}{\Delta z \mu_g}\right)^{n+1}_{f_{k+\frac{1}{2}}} \left(\frac{1}{{k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_f \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} \right)$$

$$= T_{g f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{{k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_f \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} \right)$$

$$= \frac{T_{g f_{k+\frac{1}{2}}}^{n+1}}{{k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + T_{g f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_f \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

Si recordamos la derivada de la transmisibilidad del gas en la fractura natural en "$k+\frac{1}{2}$" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P_{o f_k}^{n+1}} \left[T_{g f_{k+\frac{1}{2}}}^{n+1} \right] = T_{g f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_f$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 }] = \frac{T_{g f_{k+\frac{1}{2}}}^{n+1}}{{k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k+\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}} \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 }] = \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k+\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}} \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} \left[T_{g f_{k+\frac{1}{2}}}^{n+1} \right] = \left(\frac{A_z k b_g}{\Delta z \mu_g}\right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g f_k}^{n+1}} + \frac{\partial T_{g f_{k+\frac{1}{2}}}^{n+1}}{\partial P_{o f_k}^{n+1}} \frac{\partial P_{cap\,go f_k}^{n+1}}{\partial S_{at\,g f_k}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} \left[T_{g f_{k+\frac{1}{2}}}^{n+1} \; r_{s f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \text{Trnsmsc\_gas\_Z2} \; \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\; rs\_Z2 \;] + rs\_Z2 \; \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 }]$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = T_{g f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} \left[r_{s f_{k+\frac{1}{2}}}^{n+1} \right] + r_{s f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} \left[T_{g f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g f_k}^{n+1}} [\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = 0 + r_{s f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,g f_k}^{n+1}} \left[T_{g f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \;\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } \; rs\_Z2 \;] = r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\left(\left(\frac{A_z k b_g}{\Delta z \mu_g}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{rg_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)$$

5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_o \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1})\left(\frac{\gamma_{o_{f_k}} + \gamma_{o_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1})\left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_o \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\left(\frac{\gamma_{o_{f_{k+1}}} + \gamma_{o_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k)\left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

7) Derivada del potencial de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_g \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1}) \left(\frac{\gamma_{g_{f_k}} + \gamma_{g_{f_{k-1}}}}{2} \right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (D_k - D_{k-1})\left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - (D_k - D_{k-1}) \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{\gamma_{g_{f_k}}^{n+1} + \gamma_{g_{f_{k-1}}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - \frac{(D_k - D_{k-1})}{2} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\gamma_{g_{f_k}}^{n+1} \right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\gamma_{g_{f_k}}^{n+1} \right] = \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \left(\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \left(1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \left(\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \left(1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)$$

<u>8) Derivada del potencial de gas en la fractura natural en $\left(k + \frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right]$$

$$(\gamma_g \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k) \left(\frac{\gamma_{g_{f_{k+1}}} + \gamma_{g_{f_k}}}{2} \right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = -\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - (D_{k+1} - D_k) \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{\gamma_{g_{f_{k+1}}}^{n+1} + \gamma_{g_{f_k}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = -\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - \frac{(D_{k+1} - D_k)}{2} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\gamma_{g_{f_k}}^{n+1} \right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\gamma_{g_{f_k}}^{n+1} \right] = \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = -\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} - \frac{(D_{k+1}-D_k)}{2}\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \left(-\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\left(1+\frac{(D_{k+1}-D_k)}{2}\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left((P_{cap\,go}+P_o)_{k+1}-(P_{cap\,go}+P_o)_k-(\gamma_g\Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \left(-\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\left(1+\frac{(D_{k+1}-D_k)}{2}\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

<u>9) Derivada del término fuente en (k) respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f}q_o\right)_k^{n+1} \\ +\left(b_{g_f}r_{s_f}q_g\right)_k^{n+1}\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{o_k}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ +\left(b_{g_{f_k}}^{n+1}r_{s_{f_k}}^{n+1}\frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\right]\right)\end{bmatrix}$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\right] = b_{g_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[r_{s_{f_k}}^{n+1}\right] + r_{s_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{o_k}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ +\left(b_{g_{f_k}}^{n+1}r_{s_{f_k}}^{n+1}\frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k}\left(b_{g_{f_k}}^{n+1}\frac{\partial r_{s_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right)\end{bmatrix}$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right] = \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + (\ 0\ ) \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k} \left(b_{g_{f_k}}^{n+1}\ \frac{\partial r_{s_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k} \left(b_{g_{f_k}}^{n+1}\ (\ 0\ ) + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1} \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c} b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k} \left(r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \end{array} \right]$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(P_{o_m} - P_{o_f}\right)_k^{n+1} \right] + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{mf_k}}^{n+1} \right] \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \right] \end{array} \right) \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ (\ 0\ ) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \frac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \left(- \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \\ + \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \\ \left(T_{g_{mf_k}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[r_{s_{mk}}^{n+1} \right] + r_{s_{mk}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{mf_k}}^{n+1} \right] \right) \end{array} \right) \end{array} \right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_o)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{m_{i,j,k}}}^{n+1}$$

$$\boxed{\frac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = 0}$$

Si: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) < \boldsymbol{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_o)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{o_{mf_k}}^{n+1}\,\right] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,Vr_k\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\,k}^{n+1}\,\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{o_{mf_k}}^{n+1}\,\right] = \left(\frac{Vr\,\sigma\,k\,b_o}{\mu_o}\right)_{f\,k}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,k_{ro_{f_k}}^{\;n+1}\,\right]$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{o_{mf_k}}^{n+1}\,\right] = \left(\frac{Vr\,\sigma\,k\,b_o}{\mu_o}\right)_{f\,k}^{n+1} \frac{\partial k_{ro_{f_k}}^{\;n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}}$$

Si: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) + \left(\boldsymbol{P_{cap\,go_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,go_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_g)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\,i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{m_{i,j,k}}}^{n+1}$$

$$\boxed{\frac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = 0}$$

Si: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) + \left(\boldsymbol{P_{cap\,go_{m_{i,j,k}}}} - \boldsymbol{P_{cap\,go_{f_{i,j,k}}}}\right) < \boldsymbol{0}$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k\lambda_g)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\,i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{g_{mf_k}}^{n+1}\,\right] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,Vr_k\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\,k}^{n+1}\,\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{g_{mf_k}}^{n+1}\,\right] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,(\,Vr\,\sigma\,k\,)_{f_k}^{n+1}\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_k}^{n+1}\,\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{g_{mf_k}}^{n+1}\,\right] = (\,Vr\,\sigma\,k\,)_{f_k}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_k}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,T_{g_{mf_k}}^{n+1}\,\right] =$$

$$= (\,Vr\,\sigma\,k\,)_{f_k}^{n+1}\left(\left(\frac{b_g}{\mu_g}\right)_{f_k}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\,k_{rg_{f_k}}^{\;n+1}\,\right] + k_{rg_{f_k}}^{\;n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{b_g}{\mu_g}\right)_{f_k}^{n+1}\right]\right)$$

$$= \left(Vr\,\sigma\,k \right)_{f_k}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_k}^{n+1} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_k}^{n+1} \left(\frac{\mu_{g_k} \frac{\partial b_{g_k}}{\partial S_{at\,g_k}} - b_{g_k} \frac{\partial \mu_{g_k}}{\partial S_{at\,g_k}}}{\mu_{g_k}^2} \right)_f^{n+1} \right)$$

$$= \left(Vr\,\sigma\,k \right)_{f_k}^{n+1} \left(\left(\frac{b_g}{\mu_g} \right)_{f_k}^{n+1} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_k}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

$$= \left(Vr\,\sigma\,k \right)_{f_k}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_k}^{n+1} \left(\frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial S_{at\,g_k}} \right)_f^{n+1} = \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{gmf_k}^{n+1} \right] =$$

$$= \left(Vr\,\sigma\,k \right)_{f_k}^{n+1} \left(\frac{b_g}{\mu_g} \right)_{f_k}^{n+1} \left(\frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,go_k}}{\partial S_{at\,g_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{Vr\,\sigma\,k\,b_g}{\mu_g} \right)_{f_k}^{n+1} \left(\frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + {k_{rg}}_{f_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

$$= \left(\frac{Vr\,\sigma\,k\,k_{rg}\,b_g}{\mu_g} \right)_{f_k}^{n+1} \left(\frac{1}{{k_{rg}}_{f_k}^{n+1}} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

$$= T_{gmf_k}^{n+1} \left(\frac{1}{{k_{rg}}_{f_k}^{n+1}} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

$$= \frac{T_{gmf_k}^{n+1}}{{k_{rg}}_{f_k}^{n+1}} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + T_{gmf_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Si recordamos la derivada de la transmisibilidad del gas entre la matriz y la fractura natural en "k" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{gmf_k}^{n+1} \right] = T_{gmf_k}^{n+1} \left(\frac{1}{b_{g_k}} \frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}} \frac{\partial \mu_{g_k}}{\partial P_{o_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{gmf_k}^{n+1} \right] = \frac{T_{gmf_k}^{n+1}}{{k_{rg}}_{f_k}^{n+1}} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{gmf_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{gmf_k}^{n+1} \right] = \left(\frac{Vr\,\sigma\,k\,b_g}{\mu_g} \right)_{f_k}^{n+1} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{gmf_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right] = \left(\frac{Vr\,\sigma\,k\,b_g}{\mu_g}\right)_{f_k}^{n+1} \frac{\partial {k_{rg}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1}\ \right] = T_{g_{mf_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ r_{s_{mk}}^{n+1}\ \right] + r_{s_{mk}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1}\ \right] = T_{g_{mf_k}}^{n+1}\ (\ 0\ ) + r_{s_{mk}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1}\ \right] = r_{s_{mk}}^{n+1}\ \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo en la derivada principal:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ (\ 0\ ) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \left(-\dfrac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right) \\ + \left(\left(P_{cap\,go} + P_o\right)_{mk} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \\ \left(T_{g_{mf_k}}^{n+1}\ (\ 0\ ) + r_{s_{mk}}^{n+1}\ \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right]\right) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \left(-\dfrac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right) \\ + \left(\left(P_{cap\,go} + P_o\right)_{mk} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \\ \left(r_{s_{mk}}^{n+1}\ \dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right]\right) \end{array}\right) \end{array}\right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1}\ \left(\left(P_{cap\,go} + P_o\right)_{mk} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{mk}}^{n+1} \left(-\dfrac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right) \\ + \left(\left(P_{cap\,go} + P_o\right)_{mk} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \left(r_{s_{mk}}^{n+1}\ \dfrac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right) \end{array}\right) \end{array}\right]$$

11) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\left[\begin{array}{c}\left([1+C_r(P_o^{n+1}-P_o^n)][b_o(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_o(1-S_{at\,g}-S_{at\,w})]^n\right)_k \\ +\left([1+C_r(P_o^{n+1}-P_o^n)][b_g r_s S_{at\,g}]^{n+1}-[b_g r_s S_{at\,g}]^n\right)_k\end{array}\right]_f\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{array}{c}\left([1+C_r(P_o^{n+1}-P_o^n)]\,[b_o(1-S_{at\,g}-S_{at\,w})]^{n+1} - [b_o(1-S_{at\,g}-S_{at\,w})]^n\right)_{f_k} \\ +\left([1+C_r(P_o^{n+1}-P_o^n)]\,[b_g r_s S_{at\,g}]^{n+1} - [b_g r_s S_{at\,g}]^n\right)_{f_k}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] \\ +\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\left(\begin{array}{c}b_{o_{f_k}}^n\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1-S_{at\,g}-S_{at\,w})_{f_k}^n\right] \\ +(1-S_{at\,g}-S_{at\,w})_{f_k}^n\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{o_{f_k}}^n\right]\end{array}\right) \\ +\left(\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g r_s S_{at\,g})_{f_k}^{n+1}\right] \\ +(b_g r_s S_{at\,g})_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g r_s S_{at\,g})_{f_k}^n\right]\end{array}\right)\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\right] + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}(-1) + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}(0)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_o(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}(-1)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g r_s S_{at\,g})_{f_k}^{n+1}\right] = b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[S_{at\,g_{f_k}}^{n+1}\right] + S_{at\,g_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g r_s S_{at\,g})_{f_k}^{n+1}\right] = b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}(1) + S_{at\,g_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g r_s S_{at\,g})_{f_k}^{n+1}\right] = b_{g_{f_k}}^{n+1}\,r_{s_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{n+1}\left(b_{g_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[r_{s_{f_k}}^{n+1}\right] + r_{s_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]\right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right] = \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ +\ S_{at\,g}{}_{f_k}^{n+1}\left(\ b_{g_{f_k}}^{n+1}\ (\ 0\ )\ +\ r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ +\ S_{at\,g}{}_{f_k}^{n+1}\left(\ (\ 0\ )\ +\ r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ +\ S_{at\,g}{}_{f_k}^{n+1}\left(\ r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)}$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n}\ \right] = 0}$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = b_{g_{f_k}}^{n}\ r_{s_{f_k}}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ S_{at\,g}{}_{f_k}^{n}\ \right]\ +\ S_{at\,g}{}_{f_k}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n}\ r_{s_{f_k}}^{n}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = 0\ +\ 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n}\ \right] = 0}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ \text{Term\_acum}\ \right] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c}\left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ b_{o_{f_k}}^{n+1}\ (-1\ )\ \right)\\ +\ \left(\ b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\ (\ 0\ )\\ -\left(\begin{array}{c} b_{o_{f_k}}^{n}\ (\ 0\ )\\ +\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n}\ (\ 0\ )\end{array}\right)\\ +\left(\begin{array}{c}\left(\ 1 + C_r(P_o^{n+1} - P_o^{n})\ \right)_{f_k}\left(\ b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ +\ S_{at\,g}{}_{f_k}^{n+1}\left(\ r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right)\\ +\ \left(\ b_g r_s S_{at\,g}\ \right)_{f_k}^{n+1}\ (\ 0\ )\\ -\ (\ 0\ )\end{array}\right)\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{o_{f_k}}^{n+1}(-1)\right) \\ +(0) \\ -(0) \\ +\left(\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{g_{f_k}}^{n+1} r_{s_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{n+1}\left(r_{s_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right) \\ +(0) \\ -(0)\end{array}\right)\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{o_{f_k}}^{n+1}(-1)\right) \\ +(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{g_{f_k}}^{n+1}\; r_{s_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{n+1}\left(r_{s_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right)\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(\begin{array}{c}\left(b_{o_{f_k}}^{n+1}(-1)\right) \\ + r_{s_{f_k}}^{n+1}\left(S_{at\,g_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + b_{g_{f_k}}^{n+1}\right)\end{array}\right)\right]$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] + \text{Pot\_aceite\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] \\ -\left(\text{Trnsmsc\_aceite\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] + \text{Pot\_aceite\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }]\right) \\ +\left(\text{Trnsmsc\_gas\_Z2}\;\; rs\_Z2\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] + \text{Pot\_gas\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2}\;\; rs\_Z2\text{ }]\right) \\ -\left(\text{Trnsmsc\_gas\_Z1}\;\; rs\_Z1\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] + \text{Pot\_gas\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1}\;\; rs\_Z1\text{ }]\right) \\ +\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ +\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ -\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }]\end{array}\right]$$

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] \; + \; \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] \; + \; \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(
\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \right] \\
+ \; \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]
\end{array}
\right) \\
- \left(
\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \right] \\
+ \; \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array}
\right) \\
+ \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[
\begin{array}{c}
\left(b_{o_f} q_o\right)_k^{n+1} \\
+ \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1}
\end{array}
\right] \\
+ \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[
\begin{array}{c}
T_{o_{mf_k}}^{n+1} \; \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\
+ \; T_{g_{mf_k}}^{n+1} \; r_{s_{m_k}}^{n+1} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1}
\end{array}
\right] \\
- \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; \frac{Vp_k}{\Delta t} \left[
\begin{array}{c}
\left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\
+ \left(\; [1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_k
\end{array}
\right]_f \; \right]
\end{array}
\right]
$$

$$
\begin{aligned}
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \Bigg[\; & T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \, (\,0\,) + \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \\
& - \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \, (\,0\,) + \left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \\
& + \Bigg(T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \, r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(- \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \left(1 + \frac{(D_{k+1} - D_k)}{2} \, \frac{\partial {\gamma_g}_{f_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\
& \quad + \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \\
& \quad \left(r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_g}{\Delta z \mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{rg}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \, \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \Bigg) \\
& - \Bigg(T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \, r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \left(1 - \frac{(D_k - D_{k-1})}{2} \, \frac{\partial {\gamma_g}_{f_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\
& \quad + \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \\
& \quad \left(r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_g}{\Delta z \mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{rg}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \, \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \Bigg) \\
& + \left(b_{o_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + \left(b_{g_{f_k}}^{n+1} \, r_{s_{f_k}}^{n+1} \, \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k} \left(r_{s_{f_k}}^{n+1} \, \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \, \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \right) \\
& + \Bigg(\left(P_{o_m} - P_{o_f}\right)_k^{n+1} \, \frac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \Bigg(T_{g_{mf_k}}^{n+1} \, r_{s_{m_k}}^{n+1} \left(- \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \\
& \quad + \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k} \right)^{n+1} \left(r_{s_{m_k}}^{n+1} \, \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \Bigg) \Bigg) \\
& - \frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P_o^{n+1} - P_o^{n}) \right)_{f_k} \left(\left(b_{o_{f_k}}^{n+1} \, (-1) \right) + r_{s_{f_k}}^{n+1} \left({S_{at\,g}}_{f_k}^{n+1} \, \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \, \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + b_{g_{f_k}}^{n+1} \right) \right) \right] \Bigg]
\end{aligned}
$$

a.c) Derivando la función de residuos de aceite $\left(F_{o_f}\right)$ truncada en la dirección "k" respecto a la saturación de agua en la fractura natural $\left(S_{at\,w_f}\right)$ en "k":

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2} \\ - \;\text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1} \\ + \;\text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2} \\ - \;\text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1} \\ + \;\text{Term\_fuente} \\ + \;\text{Term\_transf\_MF} \\ - \;\text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z2} \;\; \text{Pot\_aceite\_Z2}\;] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z1} \;\; \text{Pot\_aceite\_Z1}\;] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \;\; \text{Pot\_gas\_Z2}\;] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \;\; \text{Pot\_gas\_Z1}\;] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_fuente}\;] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_transf\_MF}\;] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_acum}\;] \end{bmatrix}$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_aceite\_Z2}\;] + \text{Pot\_aceite\_Z2} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z2}\;] \\ - \left(\text{Trnsmsc\_aceite\_Z1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_aceite\_Z1}\;] + \text{Pot\_aceite\_Z1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z1}\;] \right) \\ + \left(\text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2 \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_gas\_Z2}\;] + \text{Pot\_gas\_Z2} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z2} \;\; rs\_Z2\;] \right) \\ - \left(\text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1 \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_gas\_Z1}\;] + \text{Pot\_gas\_Z1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z1} \;\; rs\_Z1\;] \right) \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_fuente}\;] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_transf\_MF}\;] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_acum}\;] \end{bmatrix}$$

Si sustituimos términos:

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\ + \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ - \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \ r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g\right)_k^{n+1} \end{array} \right] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + \ T_{g_{mf_k}}^{n+1} \ r_{s_{m_k}}^{n+1} \ \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k} \right)^{n+1} \end{array} \right] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g r_s S_{at\,g}\right]^{n+1} - \left[b_g r_s S_{at\,g}\right]^n \right)_k \end{array} \right]_f \right] \end{array} \right]$$

Realizando las derivadas que se presentan dentro del corchete:

<u>1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_o k_{ro}}{\mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ ] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[k_{ro}{}_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro}{}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

<u>2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k + \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_o k_{ro}}{\mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ] = \left(\frac{A_z k b_o}{\Delta z \mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\ \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o}\right)_{f_{k+\frac{1}{2}}}^{n+1}\frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

<u>3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k-\frac{1}{2}}}^{n+1}\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_{k-\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = 0$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \text{Trnsmsc\_gas\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ rs\_Z1\ ] + rs\_Z1\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right] + r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = 0 + 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ r_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ ] = 0$$

<u>4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_g k_{rg}}{\mu_g}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = 0$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = \text{Trnsmsc\_gas\_Z2 } \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\, rs\_Z2 \,] + rs\_Z2 \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right] + r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = 0 + 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 } rs\_Z2 \text{ }] = 0$$

5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_o \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1})\left(\frac{\gamma_{o_{f_k}} + \gamma_{o_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1})\left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_o \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\ \left(\frac{\gamma_{o_{f_{k+1}}} + \gamma_{o_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k)\ \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2}\right)\ \right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right] = 0$$

7) Derivada del potencial de gas en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left((P_{cap\ go} + P_o)_k - (P_{cap\ go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_g \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1})\ \left(\frac{\gamma_{g_{f_k}} + \gamma_{g_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ (P_{cap\ go} + P_o)_k - (P_{cap\ go} + P_o)_{k-1} - (D_k - D_{k-1})\left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2}\right)\ \right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left((P_{cap\ go} + P_o)_k - (P_{cap\ go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] = 0$$

8) Derivada del potencial de gas en la fractura natural en $\left(k + \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left((P_{cap\ go} + P_o)_{k+1} - (P_{cap\ go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$(\gamma_g \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\ \left(\frac{\gamma_{g_{f_{k+1}}} + \gamma_{g_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ (P_{cap\ go} + P_o)_{k+1} - (P_{cap\ go} + P_o)_k - (D_{k+1} - D_k)\ \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2}\right)\ \right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left((P_{cap\ go} + P_o)_{k+1} - (P_{cap\ go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] = 0$$

9) Derivada del término fuente en (k) respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f}q_o\right)_k^{n+1} \\ +\left(b_{g_f}r_{s_f}q_g\right)_k^{n+1}\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{o_k}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \\ +\left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{g_k}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\right]\right)\end{bmatrix}$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\right] = b_{g_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[r_{s_{f_k}}^{n+1}\right] + r_{s_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{o_k}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \\ +\left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{g_k}\left(b_{g_{f_k}}^{n+1}\ \frac{\partial r_{s_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + r_{s_{f_k}}^{n+1}\ \frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)\right)\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\ (\ 0\ ) + q_{o_k}\ (\ 0\ ) \\ +\left(b_{g_{f_k}}^{n+1}\ r_{s_{f_k}}^{n+1}\ (\ 0\ ) + q_{g_k}\left(b_{g_{f_k}}^{n+1}\ (\ 0\ ) + r_{s_{f_k}}^{n+1}\ (\ 0\ )\right)\right)\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = [\ 0\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f}q_o\right)_k^{n+1} \\ +\left(b_{g_f}r_{s_f}q_g\right)_k^{n+1}\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] = [\ 0\ ]$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\begin{bmatrix} T_{o_{mf_k}}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1} \\ +\ T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ \left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\end{bmatrix}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \begin{bmatrix} T_{o_{mf_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_m}-P_{o_f}\right)_k^{n+1}\right] + \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\right] \\ +\left(\begin{matrix} T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\right] \\ +\left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\right]\end{matrix}\right)\end{bmatrix}$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ \right]\ =\ 0$$
$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{o_{mf_k}}^{n+1}\ \right]\ =\ 0$$
$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \right]\ =\ 0$$
$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ \right]\ =\ 0$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ]\ =\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ (\ 0\ )\ +\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ (\ 0\ ) \\ +\left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ (\ 0\ ) \\ +\ \left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ (\ 0\ )\end{array}\right)\end{array}\right]$$
$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ]\ =\ [\ 0\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ]\ =\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1} \\ +\ T_{g_{mf_k}}^{n+1}\ r_{s_{m_k}}^{n+1}\ \left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\end{array}\right]$$
$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ]\ =\ [\ 0\ ]$$

<u>11) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ]\ =\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \frac{Vp_k}{\Delta t}\left[\begin{array}{c}\left([1+C_r(P_o^{n+1}-P_o^n)][b_o(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_o(1-S_{at\,g}-S_{at\,w})]^n\right)_k \\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)][b_g r_s S_{at\,g}]^{n+1}-[b_g r_s S_{at\,g}]^n\right)_k\end{array}\right]_f\ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ]\ =\ \frac{Vp_k}{\Delta t}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c}\left(\ [1+C_r(P_o^{n+1}-P_o^n)]\ [b_o(1-S_{at\,g}-S_{at\,w})]^{n+1}\ -\ [b_o(1-S_{at\,g}-S_{at\,w})]^n\ \right)_{f_k} \\ +\left(\ [1+C_r(P_o^{n+1}-P_o^n)]\ [b_g r_s S_{at\,g}]^{n+1}\ -\ [b_g r_s S_{at\,g}]^n\right)_{f_k}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ]\ =\ \frac{Vp_k}{\Delta t}\left[\begin{array}{c}(\ 1+C_r(P_o^{n+1}-P_o^n)\ )_{f_k}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ b_o(1-S_{at\,g}-S_{at\,w})\ )_{f_k}^{n+1}\ \right] \\ +\ (\ b_o(1-S_{at\,g}-S_{at\,w})\ )_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ 1+C_r(P_o^{n+1}-P_o^n)\ )_{f_k}\ \right] \\ -\left(\begin{array}{c} b_{o_{f_k}}^n\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (1-S_{at\,g}-S_{at\,w})_{f_k}^n\ \right] \\ +\ (1-S_{at\,g}-S_{at\,w})_{f_k}^n\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^n\ \right]\end{array}\right) \\ +\left(\begin{array}{c}(\ 1+C_r(P_o^{n+1}-P_o^n)\ )_{f_k}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ b_g r_s S_{at\,g}\ )_{f_k}^{n+1}\ \right] \\ +\ (\ b_g r_s S_{at\,g}\ )_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ 1+C_r(P_o^{n+1}-P_o^n)\ )_{f_k}\ \right] \\ -\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ b_g r_s S_{at\,g}\ )_{f_k}^n\ \right]\end{array}\right)\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_o\left(1-S_{at\,g}-S_{at\,w}\right) \right)_{f_k}^{n+1} \ \right] = b_{o_{f_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(1-S_{at\,g}-S_{at\,w} \right)_{f_k}^{n+1} \ \right] + \left(1-S_{at\,g}-S_{at\,w} \right)_{f_k}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n+1} \ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_o\left(1-S_{at\,g}-S_{at\,w}\right) \right)_{f_k}^{n+1} \ \right] = b_{o_{f_k}}^{n+1} \ (-1) + \left(1-S_{at\,g}-S_{at\,w} \right)_{f_k}^{n+1} \ (0)$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_o\left(1-S_{at\,g}-S_{at\,w}\right) \right)_{f_k}^{n+1} \ \right] = b_{o_{f_k}}^{n+1} \ (-1)}$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(1+C_r(P_o^{n+1}-P_o^{n}) \right)_{f_k} \ \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = b_{g_{f_k}}^{n+1} \ r_{s_{f_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ S_{at\,g_{f_k}}^{\ \ n+1} \ \right] + S_{at\,g_{f_k}}^{\ \ n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1} \ r_{s_{f_k}}^{n+1} \ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = b_{g_{f_k}}^{n+1} \ r_{s_{f_k}}^{n+1} \ (0) + S_{at\,g_{f_k}}^{\ \ n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1} \ r_{s_{f_k}}^{n+1} \ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = (0) + S_{at\,g_{f_k}}^{\ \ n+1} \left(b_{g_{f_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ r_{s_{f_k}}^{n+1} \ \right] + r_{s_{f_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1} \ \right] \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = (0) + S_{at\,g_{f_k}}^{\ \ n+1} \left(b_{g_{f_k}}^{n+1} \ (0) + r_{s_{f_k}}^{n+1} \ (0) \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = (0)$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n+1} \ \right] = (0)}$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(1-S_{at\,g}-S_{at\,w}\right)_{f_k}^{n} \ \right] = 0}$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n} \ \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n} \ \right] = b_{g_{f_k}}^{n} \ r_{s_{f_k}}^{n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ S_{at\,g_{f_k}}^{\ \ n} \ \right] + S_{at\,g_{f_k}}^{\ \ n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n} \ r_{s_{f_k}}^{n} \ \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n} \ \right] = 0 + 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(b_g r_s S_{at\,g} \right)_{f_k}^{n} \ \right] = 0}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\,1 + C_r(P_o^{n+1} - P_o^n)\,)_{f_k}\left(\,b_{o_{f_k}}^{n+1}\,(-1\,)\right) \\ +\,(\,b_o(1 - S_{at\,g} - S_{at\,w})\,)_{f_k}^{n+1}\,(\,0\,) \\ -\left(\begin{array}{c} b_{o_{f_k}}^n\,(\,0\,) \\ +\,(1 - S_{at\,g} - S_{at\,w})_{f_k}^n\,(\,0\,) \end{array}\right) \\ +\left(\begin{array}{c} (\,1 + C_r(P_o^{n+1} - P_o^n)\,)_{f_k}\,(\,0\,) \\ +\,(\,b_g r_s S_{at\,g}\,)_{f_k}^{n+1}\,(\,0\,) \\ -\,(\,0\,) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\,1 + C_r(P_o^{n+1} - P_o^n)\,)_{f_k}\left(\,b_{o_{f_k}}^{n+1}\,(-1\,)\right) \\ +\,(\,0\,) \\ -\,(\,0\,) \\ +\,(\,0\,) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\,(\,1 + C_r(P_o^{n+1} - P_o^n)\,)_{f_k}\left(\,b_{o_{f_k}}^{n+1}\,(-1\,)\right)\right]$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\,(\,1 + C_r(P_o^{n+1} - P_o^n)\,)_{f_k}\left(\,b_{o_{f_k}}^{n+1}\,(-1\,)\right)\right]}$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] + \text{Pot\_aceite\_Z2}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] \\ -\left(\text{Trnsmsc\_aceite\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] + \text{Pot\_aceite\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }]\right) \\ +\left(\text{Trnsmsc\_gas\_Z2}\ \ rs\_Z2\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] + \text{Pot\_gas\_Z2}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2}\ \ rs\_Z2\text{ }]\right) \\ -\left(\text{Trnsmsc\_gas\_Z1}\ \ rs\_Z1\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] + \text{Pot\_gas\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1}\ \ rs\_Z1\text{ }]\right) \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ -\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] \end{array}\right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\ + \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(\left(P_{cap\,go} + P_o \right)_{k+1} - \left(P_{cap\,go} + P_o \right)_k - \left(\gamma_g \Delta D \right)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o \right)_{k+1} - \left(P_{cap\,go} + P_o \right)_k - \left(\gamma_g \Delta D \right)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ - \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(\left(P_{cap\,go} + P_o \right)_k - \left(P_{cap\,go} + P_o \right)_{k-1} - \left(\gamma_g \Delta D \right)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o \right)_k - \left(P_{cap\,go} + P_o \right)_{k-1} - \left(\gamma_g \Delta D \right)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} q_o \right)_k^{n+1} \\ + \left(b_{g_f} r_{s_f} q_g \right)_k^{n+1} \end{array} \right] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1} r_{s_{m_k}}^{n+1} \left(\left(P_{cap\,go} + P_o \right)_{m_k} - \left(P_{cap\,go} + P_o \right)_{f_k} \right)^{n+1} \end{array} \right] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left(\left[1 + C_r (P_o^{n+1} - P_o^n) \right] \left[b_o \left(1 - S_{at\,g} - S_{at\,w} \right) \right]^{n+1} - \left[b_o \left(1 - S_{at\,g} - S_{at\,w} \right) \right]^n \right)_k \\ + \left(\left[1 + C_r (P_o^{n+1} - P_o^n) \right] \left[b_g r_s S_{at\,g} \right]^{n+1} - \left[b_g r_s S_{at\,g} \right]^n \right)_k \end{array} \right]_f \right] \end{array} \right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\ - \left(\begin{array}{c} T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \end{array} \right) \\ + \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k+\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \\ (\;0\;) \end{array} \right) \\ - \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; r_{s_{f_{k-\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \\ (\;0\;) \end{array} \right) \\ + \;(\;0\;) \\ + \;(\;0\;) \\ - \frac{Vp_k}{\Delta t} \left[\;(\;1 + C_r(P_o^{n+1} - P_o^n)\;)_{f_k} \left(b_{o_{f_k}}^{n+1} \;(-1\;) \right) \right] \end{array} \right]$$

$$\frac{\partial F_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\ - \left(\left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right) \\ - \frac{Vp_k}{\Delta t} \left[\;(\;1 + C_r(P_o^{n+1} - P_o^n)\;)_{f_k} \left(b_{o_{f_k}}^{n+1} \;(-1\;) \right) \right] \end{array} \right]$$

b) Si la función de residuos de la fase de gas en la fractura natural $\left(F_{g_f}\right)$ es:

$$F^{n+1}_{g_{f_{i,j,k}}} = \left[\begin{array}{c} T^{n+1}_{o_{f_{i+\frac{1}{2},j,k}}} \; R^{n+1}_{s_{f_{i+\frac{1}{2},j,k}}} \; \left(P_{o_{i+1,j,k}} - P_{o_{i,j,k}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{o_{f_{i-\frac{1}{2},j,k}}} \; R^{n+1}_{s_{f_{i-\frac{1}{2},j,k}}} \; \left(P_{o_{i,j,k}} - P_{o_{i-1,j,k}}\right)^{n+1}_{f} \\ + \; T^{n+1}_{g_{f_{i+\frac{1}{2},j,k}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i+1,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)^{n+1}_{f} \\ - \; T^{n+1}_{g_{f_{i-\frac{1}{2},j,k}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i-1,j,k}\right)^{n+1}_{f} \\ + \; T^{n+1}_{o_{f_{i,j+\frac{1}{2},k}}} \; R^{n+1}_{s_{f_{i,j+\frac{1}{2},k}}} \; \left(P_{o_{i,j+1,k}} - P_{o_{i,j,k}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{o_{f_{i,j-\frac{1}{2},k}}} \; R^{n+1}_{s_{f_{i,j-\frac{1}{2},k}}} \; \left(P_{o_{i,j,k}} - P_{o_{i,j-1,k}}\right)^{n+1}_{f} \\ + \; T^{n+1}_{g_{f_{i,j+\frac{1}{2},k}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i,j+1,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k}\right)^{n+1}_{f} \\ - \; T^{n+1}_{g_{f_{i,j-\frac{1}{2},k}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j-1,k}\right)^{n+1}_{f} \\ + \; T^{n+1}_{o_{f_{i,j,k+\frac{1}{2}}}} \; R^{n+1}_{s_{f_{i,j,k+\frac{1}{2}}}} \; \left(P_{o_{i,j,k+1}} - P_{o_{i,j,k}} - (\gamma_o \Delta D)_{i,j,k+\frac{1}{2}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{o_{f_{i,j,k-\frac{1}{2}}}} \; R^{n+1}_{s_{f_{i,j,k-\frac{1}{2}}}} \; \left(P_{o_{i,j,k}} - P_{o_{i,j,k-1}} - (\gamma_o \Delta D)_{i,j,k-\frac{1}{2}}\right)^{n+1}_{f} \\ + \; T^{n+1}_{g_{f_{i,j,k+\frac{1}{2}}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i,j,k+1} - \left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(\gamma_g \Delta D\right)_{i,j,k+\frac{1}{2}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{g_{f_{i,j,k-\frac{1}{2}}}} \; \left(\left(P_{cap\,go} + P_o\right)_{i,j,k} - \left(P_{cap\,go} + P_o\right)_{i,j,k-1} - \left(\gamma_g \Delta D\right)_{i,j,k-\frac{1}{2}}\right)^{n+1}_{f} \\ + \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_{i,j,k} \\ + \left(b_{g_f} q_g\right)^{n+1}_{i,j,k} \\ + \; T^{n+1}_{o_{mf_{i,j,k}}} \; R^{n+1}_{s_{m_{i,j,k}}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_{i,j,k} \\ + \; T^{n+1}_{g_{mf_{i,j,k}}} \; \left(\left(P_{cap\,go} + P_o\right)_{m_{i,j,k}} - \left(P_{cap\,go} + P_o\right)_{f_{i,j,k}}\right)^{n+1} \\ - \dfrac{Vp_{i,j,k}}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^{n})]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n} \right)_{i,j,k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^{n})]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^{n} \right)_{i,j,k} \end{array} \right]_f \end{array} \right] = 0$$

Ahora, truncando únicamente en la dirección “k”:

$$F^{n+1}_{g_{f_k}} = \left[\begin{array}{c} T^{n+1}_{o_{f_{k+\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k+\frac{1}{2}}}} \; \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{o_{f_{k-\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k-\frac{1}{2}}}} \; \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)^{n+1}_{f} \\ + \; T^{n+1}_{g_{f_{k+\frac{1}{2}}}} \; \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_{k} - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)^{n+1}_{f} \\ - \; T^{n+1}_{g_{f_{k-\frac{1}{2}}}} \; \left(\left(P_{cap\,go} + P_o\right)_{k} - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)^{n+1}_{f} \\ + \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_{k} \\ + \left(b_{g_f} q_g\right)^{n+1}_{k} \\ + \; T^{n+1}_{o_{mf_k}} \; R^{n+1}_{s_{m_k}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_{k} \\ + \; T^{n+1}_{g_{mf_k}} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \\ - \dfrac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^{n})]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n} \right)_{k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^{n})]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^{n} \right)_{k} \end{array} \right]_f \end{array} \right]$$

Si definimos los siguientes términos, de manera similar a (Cortes Rubio, 2008) y (Orozco Aguilar, 2013):

$$\text{Term\_flujo\_gas} = \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \\ -\; \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \\ +\; \text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \\ -\; \text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \end{bmatrix}$$

$$\text{Trnsmsc\_aceite\_Z2} = T^{n+1}_{o_{f_{k+\frac{1}{2}}}}$$

$$\text{Trnsmsc\_aceite\_Z1} = T^{n+1}_{o_{f_{k-\frac{1}{2}}}}$$

$$Rs\_Z2 = R^{n+1}_{s_{f_{k+\frac{1}{2}}}}$$

$$Rs\_Z1 = R^{n+1}_{s_{f_{k-\frac{1}{2}}}}$$

$$\text{Pot\_aceite\_Z2} = \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)^{n+1}_f$$

$$\text{Pot\_aceite\_Z1} = \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)^{n+1}_f$$

$$\text{Trnsmsc\_gas\_Z2} = T^{n+1}_{g_{f_{k+\frac{1}{2}}}}$$

$$\text{Trnsmsc\_gas\_Z1} = T^{n+1}_{g_{f_{k-\frac{1}{2}}}}$$

$$\text{Pot\_gas\_Z2} = \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)^{n+1}_f$$

$$\text{Pot\_gas\_Z1} = \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)^{n+1}_f$$

$$\text{Term\_fuente} = \begin{bmatrix} \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_k \\ +\left(b_{g_f} q_g\right)^{n+1}_k \end{bmatrix}$$

$$\text{Term\_transf\_MF} = \begin{bmatrix} T^{n+1}_{o_{mf_k}} \; R^{n+1}_{s_{m_k}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_k \\ +\; T^{n+1}_{g_{mf_k}} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{bmatrix}$$

$$\text{Term\_acum} = \left[\frac{Vp_k}{\Delta t} \begin{bmatrix} \left([1 + C_r(P^{n+1}_o - P^n_o)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\ + \left([1 + C_r(P^{n+1}_o - P^n_o)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_k \end{bmatrix}_f \right]$$

Entonces la función de residuos de la fase de gas en la fractura natural truncada en la dirección "k" puede escribirse como:

$$F_{g_{f_k}}^{n+1} = \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \\ - \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \\ + \text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \\ - \text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \\ + \text{Term\_fuente} \\ + \text{Term\_transf\_MF} \\ - \text{Term\_acum} \end{bmatrix}$$

b.a) Derivando la función de residuos de gas $\left(F_{g_f}\right)$ truncada en la dirección "k" respecto a la presión de aceite en la fractura natural $\left(P_{o_f}\right)$ en "k":

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \\ - \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \\ + \text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \\ - \text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \\ + \text{Term\_fuente} \\ + \text{Term\_transf\_MF} \\ - \text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \;] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \;] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \;] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \;] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Term\_fuente} \;] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Term\_transf\_MF} \;] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\; \text{Term\_acum} \;] \end{bmatrix}$$

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
\text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Pot\_aceite\_Z2}\ \right] + \text{Pot\_aceite\_Z2} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2\ \right] \\
- \left(\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Pot\_aceite\_Z1}\ \right] + \text{Pot\_aceite\_Z1} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1\ \right] \right) \\
+ \left(\text{Trnsmsc\_gas\_Z2} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Pot\_gas\_Z2}\ \right] + \text{Pot\_gas\_Z2} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Trnsmsc\_gas\_Z2}\ \right] \right) \\
- \left(\text{Trnsmsc\_gas\_Z1} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Pot\_gas\_Z1}\ \right] + \text{Pot\_gas\_Z1} \;\; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Trnsmsc\_gas\_Z1}\ \right] \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Term\_fuente}\ \right] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Term\_transf\_MF}\ \right] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \text{Term\_acum}\ \right]
\end{array}\right]
$$

Si sustituimos términos:

$$
\frac{\partial F_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
- \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} q_g\right)_k^{n+1} \end{array} \right] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \; R_{s_{mk}}^{n+1} \; \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + \; T_{g_{mf_k}}^{n+1} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{array} \right] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_k \end{array} \right]_f \right]
\end{array}\right]
$$

Realizando las derivadas que se presentan dentro del corchete:

1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k-\frac{1}{2}}}} \frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}} \frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = \text{Trnsmsc\_aceite\_Z1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; Rs\_Z1 \;] + Rs\_Z1 \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 }]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial R_{s_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k-\frac{1}{2}}}} \frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}} \frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)$$

2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k+\frac{1}{2}}}} \frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}} \frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \text{Trnsmsc\_aceite\_Z2} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\; Rs\_Z2 \;] + Rs\_Z2 \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial R_{s_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k+\frac{1}{2}}}} \frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}} \frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)$$

3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] = T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] = T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] = -1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

7) Derivada del potencial de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1} \right] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

8) Derivada del potencial de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1} \right] = -1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

9) Derivada del término fuente en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f}R_{s_f}q_o\right)_k^{n+1} \\ +\left(b_{g_f}q_g\right)_k^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \begin{bmatrix} b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ ] \\ +\ b_{g_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \end{bmatrix}$$

Si definimos:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ ] = b_{o_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ R_{s_{f_k}}^{n+1}\ ] + R_{s_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ b_{o_{f_k}}^{n+1}\ ]$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \begin{bmatrix} \left(b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k}\left(b_{o_{f_k}}^{n+1}\ \frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\ \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\ +\ b_{g_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} \left(b_{o_f}R_{s_f}q_o\right)_k^{n+1} \\ +\left(b_{g_f}q_g\right)_k^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \begin{bmatrix} \left(b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k}\left(b_{o_{f_k}}^{n+1}\ \frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\ \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\ +\ b_{g_{f_k}}^{n+1}\ \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \end{bmatrix}$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\begin{bmatrix} T_{omf_k}^{n+1}\ R_{sm_k}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1} \\ +\ T_{gmf_k}^{n+1}\ \left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1} \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \begin{bmatrix} T_{omf_k}^{n+1}\ R_{sm_k}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ \right] + \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{omf_k}^{n+1}\ R_{sm_k}^{n+1}\ \right] \\ +\left(\begin{matrix} T_{gmf_k}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ \left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \right] \\ +\left(\left(P_{cap\,go}+P_o\right)_{m_k}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{gmf_k}^{n+1}\ \right] \end{matrix} \right) \end{bmatrix}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \; R_{s_{m_k}}^{n+1} \; (-1) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \left(T_{o_{mf_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[R_{s_{m_k}}^{n+1} \right] + R_{s_{m_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1} \right] \right) \\ + \left(T_{g_{mf_k}}^{n+1} \; (-1) + \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \end{array} \right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{m_{i,j,k}}}^{n+1}$$

$$\frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0$$

$$\frac{\partial R_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) < \boldsymbol{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1} \right] = T_{o_{mf_k}}^{n+1} \left(\frac{1}{b_{o_k}}\frac{\partial b_{o_k}}{\partial P_{o_k}} - \frac{1}{\mu_{o_k}}\frac{\partial \mu_{o_k}}{\partial P_{o_k}} \right)_f^{n+1}$$

$$\frac{\partial R_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \frac{\partial R_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}} - P_{o_{f_{i,j,k}}}}\right) + \left(\boldsymbol{P_{cap\,go_{m_{i,j,k}}} - P_{cap\,go_{f_{i,j,k}}}}\right) \geq \boldsymbol{0}$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_g)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\,i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{m_{i,j,k}}}^{n+1}$$

$$\frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = 0$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) + \left(\boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{f}_{i,j,k}}}\right) < \boldsymbol{0}$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_g)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right] = T_{g_{mf_k}}^{n+1}\left(\frac{1}{b_{g_k}}\frac{\partial b_{g_k}}{\partial P_{o_k}} - \frac{1}{\mu_{g_k}}\frac{\partial \mu_{g_k}}{\partial P_{o_k}}\right)_f^{n+1}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{mk}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ +\ T_{g_{mf_k}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{mk}}^{n+1}\ (-1) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1}\left(T_{o_{mf_k}}^{n+1}\frac{\partial R_{s_{mk}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{mk}}^{n+1}\frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +\left(T_{g_{mf_k}}^{n+1}\ (-1) + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1}\frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \end{array}\right]$$

<u>11) Derivada del término de acumulación en la fractura natural en (k) respecto a la presión de aceite en la fractura natural en (k):</u>

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\left[\begin{array}{c}\left([1 + C_r(P_o^{n+1} - P_o^n)][b_o R_s(1 - S_{at\ g} - S_{at\ w})]^{n+1} - [b_o R_s(1 - S_{at\ g} - S_{at\ w})]^n\right)_k \\ +\left([1 + C_r(P_o^{n+1} - P_o^n)][b_g S_{at\ g}]^{n+1} - [b_g S_{at\ g}]^n\right)_k\end{array}\right]_f\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\begin{array}{c}\left([1 + C_r(P_o^{n+1} - P_o^n)]\ [b_o R_s(1 - S_{at\ g} - S_{at\ w})]^{n+1} - [b_o R_s(1 - S_{at\ g} - S_{at\ w})]^n\right)_{f_k} \\ +\left([1 + C_r(P_o^{n+1} - P_o^n)]\ [b_g S_{at\ g}]^{n+1} - [b_g S_{at\ g}]^n\right)_{f_k}\end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c}\left(1 + C_r(P_o^{n+1} - P_o^n)\right)_{f_k}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o R_s(1 - S_{at\ g} - S_{at\ w})\right)_{f_k}^{n+1}\right] \\ +\left(b_o R_s(1 - S_{at\ g} - S_{at\ w})\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 + C_r(P_o^{n+1} - P_o^n)\right)_{f_k}\right] \\ -\left(\begin{array}{c} b_{o_{f_k}}^n\ R_{s_{f_k}}^n\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 - S_{at\ g} - S_{at\ w}\right)_{f_k}^n\right] \\ +\left(1 - S_{at\ g} - S_{at\ w}\right)_{f_k}^n\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^n\ R_{s_{f_k}}^n\right]\end{array}\right) \\ +\left(\begin{array}{c}\left(1 + C_r(P_o^{n+1} - P_o^n)\right)_{f_k}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g\ S_{at\ g}\right)_{f_k}^{n+1}\right] \\ +\left(b_g\ S_{at\ g}\right)_{f_k}^{n+1}\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 + C_r(P_o^{n+1} - P_o^n)\right)_{f_k}\right] \\ -\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g\ S_{at\ g}\right)_{f_k}^n\right]\end{array}\right)\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o R_s (1 - S_{at\,g} - S_{at\,w})\right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1}\right] + \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o R_s (1 - S_{at\,g} - S_{at\,w})\right)_{f_k}^{n+1}\right] = 0 + \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[R_{s_{f_k}}^{n+1}\right] + R_{s_{f_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}\right]\right)$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_o R_s (1 - S_{at\,g} - S_{at\,w})\right)_{f_k}^{n+1}\right] = \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1} \frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 + C_r (P_o^{n+1} - P_o^n)\right)_{f_k}\right] = C_{r_k}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 + C_r (P_o^{n+1} - P_o^n)\right)_{f_k}\right] = C_{r_k}}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n+1}\right] = b_{g_{f_k}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[S_{at\,g_{f_k}}^{\;n+1}\right] + S_{at\,g_{f_k}}^{\;n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n+1}\right] = 0 + S_{at\,g_{f_k}}^{\;n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n+1}\right] = S_{at\,g_{f_k}}^{\;n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n+1}\right] = S_{at\,g_{f_k}}^{\;n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n}\right] = 0}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n}\right] = b_{o_{f_k}}^{n} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[R_{s_{f_k}}^{n}\right] + R_{s_{f_k}}^{n} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n}\right] = 0$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n}\right] = 0}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n}\right] = b_{g_{f_k}}^{n} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[S_{at\,g_{f_k}}^{\;n}\right] + S_{at\,g_{f_k}}^{\;n} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n}\right] = 0 + 0$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g}\right)_{f_k}^{n}\right] = 0}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left((1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ +\left(b_o R_s(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}(C_{r_k}) \\ -\left(\begin{array}{c} b_{o_{f_k}}^n R_{s_{f_k}}^n\ (0) \\ +(1-S_{at\,g}-S_{at\,w})_{f_k}^n\ (0)\end{array}\right) \\ +\left(\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(S_{at\,g_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +(b_g S_{at\,g})_{f_k}^{n+1}(C_{r_k}) \\ -(0)\end{array}\right)\end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left((1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ +\left(b_o R_s(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}(C_{r_k}) \\ -(0) \\ +\left(\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(S_{at\,g_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +(b_g S_{at\,g})_{f_k}^{n+1}(C_{r_k}) \\ -(0)\end{array}\right)\end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left((1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)\right) \\ +\left(b_o R_s(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}(C_{r_k}) \\ +\left(\begin{array}{c}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(S_{at\,g_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +(b_g S_{at\,g})_{f_k}^{n+1}(C_{r_k})\end{array}\right)\end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(\begin{array}{c}(1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +S_{at\,g_{f_k}}^{n+1}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\end{array}\right) \\ +(C_{r_k})\left(\left(b_o R_s(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1} + (b_g S_{at\,g})_{f_k}^{n+1}\right)\end{array}\right]$$

Es así que nuestra derivada final queda de la siguiente forma:

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
\text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] + \text{Pot\_aceite\_Z2}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] \\
- \left(\text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z1}\ ] + \text{Pot\_aceite\_Z1}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ ] \right) \\
+ \left(\text{Trnsmsc\_gas\_Z2}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] + \text{Pot\_gas\_Z2}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2}\ ] \right) \\
- \left(\text{Trnsmsc\_gas\_Z1}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] + \text{Pot\_gas\_Z1}\ \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] \\
- \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ]
\end{array}
\right]
$$

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\ R_{s_{f_{k+\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1}\ R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\ R_{s_{f_{k-\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \right] + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1}\ R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \right) \\
+ \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
- \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} q_g\right)_k^{n+1} \end{array} \right] \\
+ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1}\ \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array} \right] \\
- \frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s(1 - S_{at\,g} - S_{at\,w})\right]^{n+1} - \left[b_o R_s(1 - S_{at\,g} - S_{at\,w})\right]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_k \end{array} \right]_f
\end{array}
\right]
$$

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} =
\left[
\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(-1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \\
\left(T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial R_{s_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k+\frac{1}{2}}}} \frac{\partial b_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k+\frac{1}{2}}}} \frac{\partial \mu_{o_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \right) \\
\\
- \left(
\begin{array}{c}
T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \\
\left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial R_{s_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{o_{k-\frac{1}{2}}}} \frac{\partial b_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{o_{k-\frac{1}{2}}}} \frac{\partial \mu_{o_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \right)
\end{array}
\right) \\
\\
+ \left(
\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(-1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \\
\left(T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k+\frac{1}{2}}}} \frac{\partial b_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k+\frac{1}{2}}}} \frac{\partial \mu_{g_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)
\end{array}
\right) \\
\\
- \left(
\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \left(1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \\
\left(T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{g_{k-\frac{1}{2}}}} \frac{\partial b_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{g_{k-\frac{1}{2}}}} \frac{\partial \mu_{g_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)
\end{array}
\right) \\
\\
+ \left(
\begin{array}{c}
\left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} \frac{\partial q_{o_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{o_k} \left(b_{o_{f_k}}^{n+1} \frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\
+ b_{g_{f_k}}^{n+1} \frac{\partial q_{g_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{g_k} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}
\end{array}
\right) \\
\\
+ \left(
\begin{array}{c}
T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} (-1) + \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \left(T_{o_{mf_k}}^{n+1} \frac{\partial R_{s_{m_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{m_k}}^{n+1} \frac{\partial T_{o_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(T_{g_{mf_k}}^{n+1} (-1) + \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)
\end{array}
\right) \\
\\
- \frac{Vp_k}{\Delta t} \left[
\begin{array}{c}
\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k}
\left(
\begin{array}{c}
\left(1 - S_{at\,g} - S_{at\,w} \right)_{f_k}^{n+1} \left(b_{o_{f_k}}^{n+1} \frac{\partial R_{s_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1} \frac{\partial b_{o_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ S_{at\,g_{f_k}}^{n+1} \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}
\end{array}
\right) \\
+ \left(C_{r_k} \right) \left(\left(b_o R_s \left(1 - S_{at\,g} - S_{at\,w} \right) \right)_{f_k}^{n+1} + \left(b_g S_{at\,g} \right)_{f_k}^{n+1} \right)
\end{array}
\right]
\end{array}
\right]
$$

b.b) Derivando la función de residuos de gas $\left(F_{g_f}\right)$ truncada en la dirección "k" respecto a la saturación de gas en la fractura natural $\left(S_{at\ g_f}\right)$ en "k":

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \\ - \;\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \\ + \;\text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \\ - \;\text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \\ + \;\text{Term\_fuente} \\ + \;\text{Term\_transf\_MF} \\ - \;\text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2}\ ] \\ - \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1}\ ] \\ + \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2}\ ] \\ - \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1}\ ] \\ + \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] \\ + \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] \\ - \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] \end{bmatrix}$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}} = \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z2}\ ] \; + \; \text{Pot\_aceite\_Z2} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2\ ] \\ - \left(\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Pot\_aceite\_Z1}\ ] \; + \; \text{Pot\_aceite\_Z1} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1\ ] \right) \\ + \left(\text{Trnsmsc\_gas\_Z2} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z2}\ ] \; + \; \text{Pot\_gas\_Z2} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z2}\ ] \right) \\ - \left(\text{Trnsmsc\_gas\_Z1} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Pot\_gas\_Z1}\ ] \; + \; \text{Pot\_gas\_Z1} \;\; \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] \right) \\ + \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] \\ + \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] \\ - \;\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] \end{bmatrix}$$

Si sustituimos términos:

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(\begin{array}{c} T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ - \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o \right)_k^{n+1} \\ + \left(b_{g_f} q_g \right)_k^{n+1} \end{array} \right] \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1} \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array} \right] \\ - \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)][b_o R_s(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o R_s(1 - S_{at\,g} - S_{at\,w})]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)][b_g S_{at\,g}]^{n+1} - [b_g S_{at\,g}]^n \right)_k \end{array} \right]_f \right] \end{array} \right]$$

Realizando las derivadas que se presentan dentro del corchete:

<u>1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = \text{Trnsmsc\_aceite\_Z1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\, Rs\_Z1 \,] + Rs\_Z1 \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 }]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = 0 + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \;] = R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \text{Trnsmsc\_aceite\_Z2} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\; Rs\_Z2 \;] + Rs\_Z2 \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 }]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = 0 + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2 } \; Rs\_Z2 \;] = R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right)$$

3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_g}{\Delta z \mu_g} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rg_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_g}{\Delta z \mu_g} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{rg_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + \frac{\partial T_{g_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

7) Derivada del potencial de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \left(\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\left(1 - \frac{(D_k - D_{k-1})}{2}\,\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

8) Derivada del potencial de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \left(-\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\left(1 + \frac{(D_{k+1} - D_k)}{2}\,\frac{\partial \gamma_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

9) Derivada del término fuente en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{array}{c}\left(b_{o_f} R_{s_f} q_o\right)_k^{n+1} \\ + \left(b_{g_f} q_g\right)_k^{n+1}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \left[\begin{array}{c} b_{o_{f_k}}^{n+1}\, R_{s_{f_k}}^{n+1}\,\frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{o_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}\, R_{s_{f_k}}^{n+1}\right] \\ + b_{g_{f_k}}^{n+1}\,\frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{g_k}\,\frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; b_{o_{f_k}}^{n+1}\; R_{s_{f_k}}^{n+1} \;\right] \;=\; b_{o_{f_k}}^{n+1}\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; R_{s_{f_k}}^{n+1} \;\right] \;+\; R_{s_{f_k}}^{n+1}\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; b_{o_{f_k}}^{n+1} \;\right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \;=\; \left[\begin{array}{c} \left(b_{o_{f_k}}^{n+1}\; R_{s_{f_k}}^{n+1}\; \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{o_k} \left(b_{o_{f_k}}^{n+1}\; \frac{\partial R_{s_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; R_{s_{f_k}}^{n+1}\; \frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \right) \right) \\ +\; b_{g_{f_k}}^{n+1}\; \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{g_k}\; \frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \end{array}\right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; b_{g_{f_k}}^{n+1} \;\right] \;=\; \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\; \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \;=\; \left[\begin{array}{c} b_{o_{f_k}}^{n+1}\; R_{s_{f_k}}^{n+1}\; \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{o_k} \left(b_{o_{f_k}}^{n+1}\;(\,0\,) \;+\; R_{s_{f_k}}^{n+1}\;(\,0\,) \right) \\ +\; b_{g_{f_k}}^{n+1}\; \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{g_k}\; \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\; \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \;=\; \left[\begin{array}{c} b_{o_{f_k}}^{n+1}\; R_{s_{f_k}}^{n+1}\; \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{o_k}\,(\,0\,) \\ +\; b_{g_{f_k}}^{n+1}\; \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{g_k}\; \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\; \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \;=\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o\right)_k^{n+1} \\ +\left(b_{g_f} q_g\right)_k^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \;=\; \left[\begin{array}{c} b_{o_{f_k}}^{n+1}\; R_{s_{f_k}}^{n+1}\; \frac{\partial q_{o_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \\ +\; b_{g_{f_k}}^{n+1}\; \frac{\partial q_{g_k}}{\partial S_{at\,g_{f_k}}^{n+1}} \;+\; q_{g_k}\; \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\; \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} \end{array}\right]$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \;=\; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\; R_{s_{m_k}}^{n+1}\; \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ +\; T_{g_{mf_k}}^{n+1}\; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(P_{o_m}-P_{o_f}\right)_k^{n+1}\right] + \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\right] \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\right] \\ + \left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1}\right] \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\ (\ 0\ ) + \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\left(T_{o_{mf_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[R_{s_{m_k}}^{n+1}\right] + R_{s_{m_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\right]\right) \\ + \left(T_{g_{mf_k}}^{n+1}\left(-\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right) + \left(\left(P_{cap\,go}+P_o\right)_{m_k} - \left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1}\right]\right) \end{array}\right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) \geq \mathbf{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{m\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{m_{i,j,k}}}^{n+1}$$

$$\boxed{\frac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = 0}$$

$\boldsymbol{Si}$: $\left(\boldsymbol{P_{o_{m_{i,j,k}}}} - \boldsymbol{P_{o_{f_{i,j,k}}}}\right) < \mathbf{0}$

$$T_{o_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_o)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_o k_{r_o}}{\mu_o}\right)_{f\,i,j,k}^{n+1} \quad ; \quad R_{s_{m_{i,j,k}}}^{n+1} = R_{s_{f_{i,j,k}}}^{n+1}$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\right] = \left(\frac{Vr\ \sigma\ k\ b_o}{\mu_o}\right)_{f_k}^{n+1} \frac{\partial {k_{ro}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\right] = T_{o_{mf_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[R_{s_{m_k}}^{n+1}\right] + R_{s_{m_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\right] = T_{o_{mf_k}}^{n+1}\ (\ 0\ ) + R_{s_{m_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\right]$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\right] = R_{s_{m_k}}^{n+1}\left(\left(\frac{Vr\ \sigma\ k\ b_o}{\mu_o}\right)_{f_k}^{n+1} \frac{\partial {k_{ro}}_{f_k}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) + \left(\boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{f}_{i,j,k}}}\right) \geq \boldsymbol{0}$$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}\left(\sigma k \lambda_g\right)_{m\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{m\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{m_{i,j,k}}}^{n+1}$$

$$\frac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}} = 0$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) + \left(\boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\ go}_{\boldsymbol{f}_{i,j,k}}}\right) < \boldsymbol{0}$$

$$T_{g_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}\left(\sigma k \lambda_g\right)_{f\ i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_g k_{r_g}}{\mu_g}\right)_{f\ i,j,k}^{n+1} \quad ; \quad r_{s_{m_{i,j,k}}}^{n+1} = r_{s_{f_{i,j,k}}}^{n+1}$$

$$\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}\left[\ T_{g_{mf_k}}^{n+1}\ \right] = \left(\frac{Vr\ \sigma\ k\ b_g}{\mu_g}\right)_{f_k}^{n+1} \frac{\partial k_{rg_{f_k}}^{\ n+1}}{\partial S_{at\ g_{f_k}}^{n+1}} + \frac{\partial T_{g_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\ go_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}$$

Sustituyendo en la derivada principal:

$$\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\ (\ 0\ ) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1}\left(\ T_{o_{mf_k}}^{n+1}\ (\ 0\ ) + R_{s_{m_k}}^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \\ + \left(\ T_{g_{mf_k}}^{n+1}\left(-\dfrac{\partial P_{cap\ go_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \dfrac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} \left(P_{o_m} - P_{o_f}\right)_k^{n+1}\left(\ R_{s_{m_k}}^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \\ + \left(\ T_{g_{mf_k}}^{n+1}\left(-\dfrac{\partial P_{cap\ go_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \dfrac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \end{array}\right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1}\ R_{s_{m_k}}^{n+1}\ \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + \ T_{g_{mf_k}}^{n+1}\ \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\ g_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{c} \left(P_{o_m} - P_{o_f}\right)_k^{n+1}\left(\ R_{s_{m_k}}^{n+1} \dfrac{\partial T_{o_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \\ + \left(\ T_{g_{mf_k}}^{n+1}\left(-\dfrac{\partial P_{cap\ go_{f_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) + \left(\left(P_{cap\ go} + P_o\right)_{m_k} - \left(P_{cap\ go} + P_o\right)_{f_k}\right)^{n+1} \dfrac{\partial T_{g_{mf_k}}^{n+1}}{\partial S_{at\ g_{f_k}}^{n+1}}\right) \end{array}\right]$$

11) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\left[\begin{matrix}\left([1+C_r(P_o^{n+1}-P_o^n)][b_oR_s(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_oR_s(1-S_{at\,g}-S_{at\,w})]^n\right)_k \\ +\left([1+C_r(P_o^{n+1}-P_o^n)][b_gS_{at\,g}]^{n+1}-[b_gS_{at\,g}]^n\right)_k\end{matrix}\right]_f\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\begin{matrix}\left([1+C_r(P_o^{n+1}-P_o^n)]\ [b_oR_s(1-S_{at\,g}-S_{at\,w})]^{n+1}-[b_oR_s(1-S_{at\,g}-S_{at\,w})]^n\right)_{f_k} \\ +\left([1+C_r(P_o^{n+1}-P_o^n)]\ [b_gS_{at\,g}]^{n+1}-[b_gS_{at\,g}]^n\right)_{f_k}\end{matrix}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{matrix}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_oR_s(1-S_{at\,g}-S_{at\,w}))_{f_k}^{n+1}\right] \\ +(b_oR_s(1-S_{at\,g}-S_{at\,w}))_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\left(\begin{matrix}b_{o_{f_k}}^n R_{s_{f_k}}^n\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1-S_{at\,g}-S_{at\,w})_{f_k}^n\right] \\ +(1-S_{at\,g}-S_{at\,w})_{f_k}^n\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{o_{f_k}}^n R_{s_{f_k}}^n\right]\end{matrix}\right) \\ +\left(\begin{matrix}(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g S_{at\,g})_{f_k}^{n+1}\right] \\ +(b_g S_{at\,g})_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] \\ -\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_g S_{at\,g})_{f_k}^n\right]\end{matrix}\right)\end{matrix}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_oR_s(1-S_{at\,g}-S_{at\,w}))_{f_k}^{n+1}\right] =$$

$$= b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\right] + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}\right]$$

$$= b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}(-1) + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)$$

$$= b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}(-1) + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}(0) + R_{s_{f_k}}^{n+1}(0)\right)$$

$$= b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}(-1) + (1-S_{at\,g}-S_{at\,w})_{f_k}^{n+1}(0)$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(b_oR_s(1-S_{at\,g}-S_{at\,w}))_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}R_{s_{f_k}}^{n+1}(-1)}$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\right] = 0}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ S_{at\,g}{}_{f_k}^{n+1}\ \right] + S_{at\,g}{}_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ (\ 1\ ) + S_{at\,g}{}_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right] = \frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1} + S_{at\,g}{}_{f_k}^{n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1} + S_{at\,g}{}_{f_k}^{n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)}$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (1 - S_{at\,g} - S_{at\,w})_{f_k}^{n}\ \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ R_{s_{f_k}}^{n}\ \right] = b_{o_{f_k}}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[R_{s_{f_k}}^{n}\ \right] + R_{s_{f_k}}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ R_{s_{f_k}}^{n}\ \right] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n}\ R_{s_{f_k}}^{n}\ \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n}\ \right] = b_{g_{f_k}}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ S_{at\,g}{}_{f_k}^{n}\ \right] + S_{at\,g}{}_{f_k}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n}\ \right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n}\ \right] = 0 + 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\ (\ b_g\, S_{at\,g}\ )_{f_k}^{n}\ \right] = 0}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{matrix} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1}\;(-1)\right) \\ +\left(b_o R_s(1-S_{at\,g}-S_{at\,w})\right)_{f_k}^{n+1}\;(0) \\ -\begin{pmatrix} b_{o_{f_k}}^{n} R_{s_{f_k}}^{n}\;(0) \\ +(1-S_{at\,g}-S_{at\,w})_{f_k}^{n}\;(0)\end{pmatrix} \\ +\begin{pmatrix} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{g_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{\;n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right) \\ +\left(b_g S_{at\,g}\right)_{f_k}^{n+1}\;(0) \\ -(0)\end{pmatrix}\end{matrix}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{matrix} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1}\;(-1)\right) \\ +(0) \\ -(0) \\ +\begin{pmatrix} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{g_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{\;n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right) \\ +(0) \\ -(0)\end{pmatrix}\end{matrix}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{matrix} (1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1}\;(-1)\right) \\ +(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\left(b_{g_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{\;n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\right)\end{matrix}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[(1+C_r(P_o^{n+1}-P_o^n))_{f_k}\begin{pmatrix}\left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1}\;(-1)\right) \\ + b_{g_{f_k}}^{n+1} + S_{at\,g_{f_k}}^{\;n+1}\left(\frac{\partial b_{g_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,go_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right)\end{pmatrix}\right]$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{matrix} \text{Trnsmsc\_aceite\_Z2}\;\; Rs\_Z2\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] + \text{Pot\_aceite\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z2}\;\; Rs\_Z2\;] \\ -\left(\text{Trnsmsc\_aceite\_Z1}\;\; Rs\_Z1\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] + \text{Pot\_aceite\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_aceite\_Z1}\;\; Rs\_Z1\;]\right) \\ +\left(\text{Trnsmsc\_gas\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] + \text{Pot\_gas\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }]\right) \\ -\left(\text{Trnsmsc\_gas\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] + \text{Pot\_gas\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z1 }]\right) \\ +\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ +\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ -\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }]\end{matrix}\right]$$

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(\begin{array}{c}
T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
+ \left(\begin{array}{c}
T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
- \left(\begin{array}{c}
T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right]
\end{array} \right) \\
+ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c}
\left(b_{o_f} R_{s_f} q_o \right)_k^{n+1} \\
+ \left(b_{g_f} q_g \right)_k^{n+1}
\end{array} \right] \\
+ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c}
T_{o_{mf_k}}^{n+1} \; R_{s_{m_k}}^{n+1} \; \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\
+ \; T_{g_{mf_k}}^{n+1} \; \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1}
\end{array} \right] \\
- \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c}
\left([1 + C_r(P_o^{n+1} - P_o^n)] \left[b_o R_s (1 - S_{at\,g} - S_{at\,w}) \right]^{n+1} - \left[b_o R_s (1 - S_{at\,g} - S_{at\,w}) \right]^n \right)_k \\
+ \left([1 + C_r(P_o^{n+1} - P_o^n)] \left[b_g S_{at\,g} \right]^{n+1} - \left[b_g S_{at\,g} \right]^n \right)_k
\end{array} \right]_f \right]
\end{array} \right]
$$

$$
\frac{\partial F^{n+1}_{g_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} = \left[
\begin{array}{l}
+\left(\begin{array}{c} T^{n+1}_{o_{f_{k+\frac{1}{2}}}}\ R^{n+1}_{s_{f_{k+\frac{1}{2}}}}\ (\ 0\ ) \\ +\ \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k)\ \left(\frac{\gamma_{o\,k+1} + \gamma_{o\,k}}{2} \right) \right)^{n+1}_{f} \left(R^{n+1}_{s_{f_{k+\frac{1}{2}}}} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial {k_{ro}}^{n+1}_{f_{k+\frac{1}{2}}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \right) \end{array} \right) \\
-\left(\begin{array}{c} T^{n+1}_{o_{f_{k-\frac{1}{2}}}}\ R^{n+1}_{s_{f_{k-\frac{1}{2}}}}\ (\ 0\ ) \\ +\ \left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1})\ \left(\frac{\gamma_{o\,k} + \gamma_{o\,k-1}}{2} \right) \right)^{n+1}_{f} \left(R^{n+1}_{s_{f_{k-\frac{1}{2}}}} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)^{n+1}_{f_{k-\frac{1}{2}}} \frac{\partial {k_{ro}}^{n+1}_{f_{k-\frac{1}{2}}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \right) \end{array} \right) \\
+\left(\begin{array}{c} T^{n+1}_{g_{f_{k+\frac{1}{2}}}} \left(\left(-\frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \left(1 + \frac{(D_{k+1} - D_k)}{2}\ \frac{\partial {\gamma_g}^{n+1}_{f_k}}{\partial P^{n+1}_{o_{f_k}}} \right) \right) \\ +\ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (D_{k+1} - D_k)\ \left(\frac{\gamma_{g\,k+1} + \gamma_{g\,k}}{2} \right) \right)^{n+1}_{f} \\ \left(\left(\frac{A_z k b_g}{\Delta z \mu_g} \right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial {k_{rg}}^{n+1}_{f_{k+\frac{1}{2}}}}{\partial S^{n+1}_{at\,g_{f_k}}} + \frac{\partial T^{n+1}_{g_{f_{k+\frac{1}{2}}}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \end{array} \right) \\
-\left(\begin{array}{c} T^{n+1}_{g_{f_{k-\frac{1}{2}}}} \left(\left(\frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \left(1 - \frac{(D_k - D_{k-1})}{2}\ \frac{\partial {\gamma_g}^{n+1}_{f_k}}{\partial P^{n+1}_{o_{f_k}}} \right) \right) \\ +\ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{g\,k} + \gamma_{g\,k-1}}{2} \right) \right)^{n+1}_{f} \\ \left(\left(\frac{A_z k b_g}{\Delta z \mu_g} \right)^{n+1}_{f_{k-\frac{1}{2}}} \frac{\partial {k_{rg}}^{n+1}_{f_{k-\frac{1}{2}}}}{\partial S^{n+1}_{at\,g_{f_k}}} + \frac{\partial T^{n+1}_{g_{f_{k-\frac{1}{2}}}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \end{array} \right) \\
+\left(\begin{array}{c} b^{n+1}_{o_{f_k}}\ R^{n+1}_{s_{f_k}}\ \frac{\partial q_{o_k}}{\partial S^{n+1}_{at\,g_{f_k}}} \\ +\ b^{n+1}_{g_{f_k}}\ \frac{\partial q_{g_k}}{\partial S^{n+1}_{at\,g_{f_k}}} + q_{g_k} \frac{\partial b^{n+1}_{g_{f_k}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \end{array} \right) \\
+\left(\begin{array}{c} \left(P_{o_m} - P_{o_f} \right)^{n+1}_{k} \left(R^{n+1}_{s_{mk}}\ \frac{\partial T^{n+1}_{o_{mf_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \\ +\ \left(T^{n+1}_{g_{mf_k}} \left(-\frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) + \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \left(\frac{\partial T^{n+1}_{g_{mf_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \right) \end{array} \right) \\
-\ \frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P^{n+1}_o - P^n_o) \right)_{f_k} \left(\begin{array}{c} \left(b^{n+1}_{o_{f_k}}\ R^{n+1}_{s_{f_k}}\ (-1) \right) \\ +\ b^{n+1}_{g_{f_k}} + {S_{at\,g}}^{n+1}_{f_k} \left(\frac{\partial b^{n+1}_{g_{f_k}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,go_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} \right) \end{array} \right) \right]
\end{array}
\right]
$$

b.c) Derivando la función de residuos de gas $\left(F_{g_f}\right)$ truncada en la dirección "k" respecto a la saturación de agua en la fractura natural $\left(S_{at\,w_f}\right)$ en "k":

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2} \\ - \;\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1} \\ + \;\text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2} \\ - \;\text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1} \\ + \;\text{Term\_fuente} \\ + \;\text{Term\_transf\_MF} \\ - \;\text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \text{Pot\_aceite\_Z2}\;] \\ - \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \text{Pot\_aceite\_Z1}\;] \\ + \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z2} \;\; \text{Pot\_gas\_Z2}\;] \\ - \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z1} \;\; \text{Pot\_gas\_Z1}\;] \\ + \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_fuente}\;] \\ + \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_transf\_MF}\;] \\ - \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_acum}\;] \end{bmatrix}$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_aceite\_Z2}\;] \; + \; \text{Pot\_aceite\_Z2} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2\;] \\ - \left(\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_aceite\_Z1}\;] \; + \; \text{Pot\_aceite\_Z1} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1\;] \right) \\ + \left(\text{Trnsmsc\_gas\_Z2} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_gas\_Z2}\;] \; + \; \text{Pot\_gas\_Z2} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z2}\;] \right) \\ - \left(\text{Trnsmsc\_gas\_Z1} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Pot\_gas\_Z1}\;] \; + \; \text{Pot\_gas\_Z1} \;\; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Trnsmsc\_gas\_Z1}\;] \right) \\ + \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_fuente}\;] \\ + \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_transf\_MF}\;] \\ - \;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;\text{Term\_acum}\;] \end{bmatrix}$$

Si sustituimos términos:

$$
\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\
- \left(\begin{array}{c} T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
+ \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (\gamma_g \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
- \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\
+ \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (\gamma_g \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\
+ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o \right)_k^{n+1} \\ + \left(b_{g_f} q_g \right)_k^{n+1} \end{array} \right] \\
+ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} \; R_{s_{m_k}}^{n+1} \left(P_{o_m} - P_{o_f} \right)_k^{n+1} \\ + \; T_{g_{mf_k}}^{n+1} \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array} \right] \\
- \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)][b_o R_s(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o R_s(1 - S_{at\,g} - S_{at\,w})]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)][b_g S_{at\,g}]^{n+1} - [b_g S_{at\,g}]^n \right)_k \end{array} \right]_f \right]
\end{array} \right]
$$

Realizando las derivadas que se presentan dentro del corchete:

<u>1) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}
$$

Entonces:

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = \text{Trnsmsc\_aceite\_Z1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\, Rs\_Z1 \,] + Rs\_Z1 \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 }]
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_aceite\_Z1 } \; Rs\_Z1 \,] = T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \; \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} \right]
$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ ] = 0 + R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial T_{o_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ ] = R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z1}\ \ Rs\_Z1\ ] = R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

<u>2) Derivada de la transmisibilidad de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = \text{Trnsmsc\_aceite\_Z2} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ Rs\_Z2\ ] + Rs\_Z2 \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right] + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = 0 + R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial T_{o_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_aceite\_Z2}\ \ Rs\_Z2\ ] = R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{ro}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

<u>3) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Trnsmsc\_gas\_Z1}\ ] = 0$$

4) Derivada de la transmisibilidad de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_gas\_Z2 }] = 0$$

5) Derivada del potencial de aceite en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_k}-P_{o_{k-1}}-(\gamma_o\Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

6) Derivada del potencial de aceite en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_aceite\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_{k+1}}-P_{o_k}-(\gamma_o\Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] = 0$$

7) Derivada del potencial de gas en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go}+P_o\right)_k-\left(P_{cap\,go}+P_o\right)_{k-1}-\left(\gamma_g\Delta D\right)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z1 }] = 0$$

8) Derivada del potencial de gas en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_{cap\,go}+P_o\right)_{k+1}-\left(P_{cap\,go}+P_o\right)_k-\left(\gamma_g\Delta D\right)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_gas\_Z2 }] = 0$$

9) Derivada del término fuente en (k) respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c}\left(b_{o_f}R_{s_f}q_o\right)_k^{n+1}\\ +\left(b_{g_f}q_g\right)_k^{n+1}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c} b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{o_k}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ \right]\\ + b_{g_{f_k}}^{n+1}\frac{\partial q_{g_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{g_k}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ \right] = b_{o_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ R_{s_{f_k}}^{n+1}\ \right] + R_{s_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n+1}\ \right]$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c}\left(b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\frac{\partial q_{o_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{o_k}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)\right)\\ + b_{g_{f_k}}^{n+1}\frac{\partial q_{g_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{g_k}\frac{\partial b_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[\begin{array}{c}\left(b_{o_{f_k}}^{n+1}\ R_{s_{f_k}}^{n+1}\ (\ 0\ ) + q_{o_k}\left(b_{o_{f_k}}^{n+1}\ (\ 0\ ) + R_{s_{f_k}}^{n+1}\ (\ 0\ )\right)\right)\\ + b_{g_{f_k}}^{n+1}\ (\ 0\ ) + q_{g_k}\ (\ 0\ )\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = [\ 0\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c}\left(b_{o_f}R_{s_f}q_o\right)_k^{n+1}\\ +\left(b_{g_f}q_g\right)_k^{n+1}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = [\ 0\ ]$$

10) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c} T_{omf_k}^{n+1}\ R_{s_{mk}}^{n+1}\ \left(P_{o_m}-P_{o_f}\right)_k^{n+1}\\ + T_{gmf_k}^{n+1}\ \left(\left(P_{cap\,go}+P_o\right)_{mk}-\left(P_{cap\,go}+P_o\right)_{f_k}\right)^{n+1}\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_m} - P_{o_f}\right)_k^{n+1} \right] + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} \right] \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \right] \\ + \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1} \right] \end{array}\right) \end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(P_{o_m} - P_{o_f}\right)_k^{n+1} \right] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} \right] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \right] = 0$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{g_{mf_k}}^{n+1} \right] = 0$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} (0) + \left(P_{o_m} - P_{o_f}\right)_k^{n+1} (0) \\ + \left(\begin{array}{c} T_{g_{mf_k}}^{n+1} (0) \\ + \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} (0) \end{array}\right) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = [\ 0\ ]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c} T_{o_{mf_k}}^{n+1} R_{s_{m_k}}^{n+1} \left(P_{o_m} - P_{o_f}\right)_k^{n+1} \\ + T_{g_{mf_k}}^{n+1} \left((P_{cap\,go} + P_o)_{m_k} - (P_{cap\,go} + P_o)_{f_k} \right)^{n+1} \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = [\ 0\ ]$$

<u>11) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)][b_o R_s(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o R_s(1 - S_{at\,g} - S_{at\,w})]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)][b_g S_{at\,g}]^{n+1} - [b_g S_{at\,g}]^n \right)_k \end{array}\right]_f \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\ [b_o R_s(1 - S_{at\,g} - S_{at\,w})]^{n+1} - [b_o R_s(1 - S_{at\,g} - S_{at\,w})]^n \right)_{f_k} \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\ [b_g S_{at\,g}]^{n+1} - [b_g S_{at\,g}]^n \right)_{f_k} \end{array}\right]$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c}
(\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\ \right] \\
+\ \left(\ b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \right] \\
-\left(\begin{array}{c}
b_{o_{f_k}}^n\ R_{s_{f_k}}^n\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^n\ \right] \\
+\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^n\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^n\ R_{s_{f_k}}^n\ \right]
\end{array}\right) \\
+\left(\begin{array}{c}
(\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] \\
+\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \right] \\
-\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^n\ \right]
\end{array}\right)
\end{array}\right]
$$

Si definimos:

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\ b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\right] =
$$

$$
= b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1}\right] + \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ \right]
$$

$$
= b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ (-1\ ) + \left(\ 1 - S_{at\,g} - S_{at\,w}\ \right)_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\frac{\partial R_{s_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + R_{s_{f_k}}^{n+1}\frac{\partial b_{o_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)
$$

$$
= b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ (-1\ ) + \left(\ 1 - S_{at\,g} - S_{at\,w}\ \right)_{f_k}^{n+1}\left(b_{o_{f_k}}^{n+1}\ (\ 0\ ) + R_{s_{f_k}}^{n+1}\ (\ 0\ )\right)
$$

$$
= b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ (-1\ ) + \left(\ 1 - S_{at\,g} - S_{at\,w}\ \right)_{f_k}^{n+1}\ (\ 0\ )
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\ b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\ \right)_{f_k}^{n+1}\right] = b_{o_{f_k}}^{n+1}\,R_{s_{f_k}}^{n+1}\ (-1\ )
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \right] = 0
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ {S_{at\,g}}_{f_k}^{n+1}\ \right] + {S_{at\,g}}_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ b_{g_{f_k}}^{n+1}\ \right]
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = b_{g_{f_k}}^{n+1}\ (\ 0\ ) + {S_{at\,g}}_{f_k}^{n+1}\ (\ 0\ )
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = (\ 0\ )
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(\ b_g\,S_{at\,g}\ \right)_{f_k}^{n+1}\ \right] = (\ 0\ )
$$

$$
\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^n\ \right] = 0
$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n} \right] = b_{o_{f_k}}^{n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[R_{s_{f_k}}^{n} \right] + R_{s_{f_k}}^{n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n} \right] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{o_{f_k}}^{n} R_{s_{f_k}}^{n} \right] = 0}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g} \right)_{f_k}^{n} \right] = b_{g_{f_k}}^{n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[S_{at\,g}{}_{f_k}^{n} \right] + S_{at\,g}{}_{f_k}^{n} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{g_{f_k}}^{n} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g} \right)_{f_k}^{n} \right] = 0 + 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_g S_{at\,g} \right)_{f_k}^{n} \right] = 0}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left(1 + C_r(P_o^{n+1} - P_o^{n}) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \\ + \left(b_o R_s \left(1 - S_{at\,g} - S_{at\,w}\right) \right)_{f_k}^{n+1} (0) \\ - \left(\begin{array}{c} b_{o_{f_k}}^{n} R_{s_{f_k}}^{n} (0) \\ + \left(1 - S_{at\,g} - S_{at\,w}\right)_{f_k}^{n} (0) \end{array} \right) \\ + \left(\begin{array}{c} \left(1 + C_r(P_o^{n+1} - P_o^{n}) \right)_{f_k} (0) \\ + \left(b_g S_{at\,g} \right)_{f_k}^{n+1} (0) \\ - (0) \end{array} \right) \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left(1 + C_r(P_o^{n+1} - P_o^{n}) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \\ + (0) \\ - (0) \\ + \left(\begin{array}{c} (0) \\ + (0) \\ - (0) \end{array} \right) \end{array} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t} \left[\left(1 + C_r(P_o^{n+1} - P_o^{n}) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \right]$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t} \left[\left(1 + C_r(P_o^{n+1} - P_o^{n}) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \right]}$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2 \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Pot\_aceite\_Z2}\,] \; + \; \text{Pot\_aceite\_Z2} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Trnsmsc\_aceite\_Z2} \;\; Rs\_Z2\,] \\ - \left(\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1 \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Pot\_aceite\_Z1}\,] \; + \; \text{Pot\_aceite\_Z1} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Trnsmsc\_aceite\_Z1} \;\; Rs\_Z1\,] \right) \\ + \left(\text{Trnsmsc\_gas\_Z2} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Pot\_gas\_Z2}\,] \; + \; \text{Pot\_gas\_Z2} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Trnsmsc\_gas\_Z2}\,] \right) \\ - \left(\text{Trnsmsc\_gas\_Z1} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Pot\_gas\_Z1}\,] \; + \; \text{Pot\_gas\_Z1} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Trnsmsc\_gas\_Z1}\,] \right) \\ + \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Term\_fuente}\,] \\ + \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Term\_transf\_MF}\,] \\ - \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}[\,\text{Term\_acum}\,] \end{array}\right]$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T^{n+1}_{o_{f_{k+\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k+\frac{1}{2}}}} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)^{n+1}_f \right] \\ + \left(P_{o_{k+1}} - P_{o_k} - (\gamma_o \Delta D)_{k+\frac{1}{2}}\right)^{n+1}_f \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[T^{n+1}_{o_{f_{k+\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k+\frac{1}{2}}}} \right] \\ - \left(\begin{array}{c} T^{n+1}_{o_{f_{k-\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k-\frac{1}{2}}}} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)^{n+1}_f \right] \\ + \left(P_{o_k} - P_{o_{k-1}} - (\gamma_o \Delta D)_{k-\frac{1}{2}}\right)^{n+1}_f \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[T^{n+1}_{o_{f_{k-\frac{1}{2}}}} \; R^{n+1}_{s_{f_{k-\frac{1}{2}}}} \right] \end{array} \right) \\ + \left(\begin{array}{c} T^{n+1}_{g_{f_{k+\frac{1}{2}}}} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)^{n+1}_f \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_{k+1} - \left(P_{cap\,go} + P_o\right)_k - \left(\gamma_g \Delta D\right)_{k+\frac{1}{2}}\right)^{n+1}_f \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[T^{n+1}_{g_{f_{k+\frac{1}{2}}}} \right] \end{array} \right) \\ - \left(\begin{array}{c} T^{n+1}_{g_{f_{k-\frac{1}{2}}}} \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)^{n+1}_f \right] \\ + \left(\left(P_{cap\,go} + P_o\right)_k - \left(P_{cap\,go} + P_o\right)_{k-1} - \left(\gamma_g \Delta D\right)_{k-\frac{1}{2}}\right)^{n+1}_f \;\; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[T^{n+1}_{g_{f_{k-\frac{1}{2}}}} \right] \end{array} \right) \\ + \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\begin{array}{c} \left(b_{o_f} R_{s_f} q_o\right)^{n+1}_k \\ + \left(b_{g_f} q_g\right)^{n+1}_k \end{array} \right] \\ + \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\begin{array}{c} T^{n+1}_{o_{mf_k}} \; R^{n+1}_{s_{m_k}} \; \left(P_{o_m} - P_{o_f}\right)^{n+1}_k \\ + \; T^{n+1}_{g_{mf_k}} \; \left(\left(P_{cap\,go} + P_o\right)_{m_k} - \left(P_{cap\,go} + P_o\right)_{f_k}\right)^{n+1} \end{array} \right] \\ - \; \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}}\left[\frac{Vp_k}{\Delta t} \left[\begin{array}{c} \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^{n+1} - \left[b_o R_s\left(1 - S_{at\,g} - S_{at\,w}\right)\right]^n \right)_k \\ + \left([1 + C_r(P_o^{n+1} - P_o^n)]\left[b_g S_{at\,g}\right]^{n+1} - \left[b_g S_{at\,g}\right]^n \right)_k \end{array} \right]_f \right] \end{array}\right]$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{o_{f_{k+\frac{1}{2}}}}^{n+1} R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \ (0) \\ + \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right) \\ - \left(\begin{array}{c} T_{o_{f_{k-\frac{1}{2}}}}^{n+1} R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \ (0) \\ + \left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right) \end{array} \right) \\ + \left(\begin{array}{c} T_{g_{f_{k+\frac{1}{2}}}}^{n+1} \ (0) \\ + \left((P_{cap\,go} + P_o)_{k+1} - (P_{cap\,go} + P_o)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{g_{k+1}} + \gamma_{g_k}}{2} \right) \right)_f^{n+1} \\ (0) \end{array} \right) \\ - \left(\begin{array}{c} T_{g_{f_{k-\frac{1}{2}}}}^{n+1} \ (0) \\ + \left((P_{cap\,go} + P_o)_k - (P_{cap\,go} + P_o)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{g_k} + \gamma_{g_{k-1}}}{2} \right) \right)_f^{n+1} \\ (0) \end{array} \right) \\ + (0) \\ + (0) \\ - \frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \right] \end{array} \right]$$

$$\frac{\partial F_{g_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} \left(P_{o_{k+1}} - P_{o_k} - (D_{k+1} - D_k) \left(\frac{\gamma_{o_{k+1}} + \gamma_{o_k}}{2} \right) \right)_f^{n+1} \left(R_{s_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right) \\ - \left(\left(P_{o_k} - P_{o_{k-1}} - (D_k - D_{k-1}) \left(\frac{\gamma_{o_k} + \gamma_{o_{k-1}}}{2} \right) \right)_f^{n+1} \left(R_{s_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(\frac{A_z k b_o}{\Delta z \mu_o} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{ro_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right) \right) \\ - \frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k} \left(b_{o_{f_k}}^{n+1} R_{s_{f_k}}^{n+1} (-1) \right) \right] \end{array} \right]$$

c) Si la función de residuos de la fase de agua en la fractura natural $\left(F_{w_f}\right)$ es:

$$F_{w_{f_{i,j,k}}}^{n+1} = \begin{bmatrix} T_{w_{f_{i+\frac{1}{2},j,k}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i+1,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k} \right)_f^{n+1} \\ - \; T_{w_{f_{i-\frac{1}{2},j,k}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i-1,j,k} \right)_f^{n+1} \\ + \; T_{w_{f_{i,j+\frac{1}{2},k}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j+1,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k} \right)_f^{n+1} \\ - \; T_{w_{f_{i,j-\frac{1}{2},k}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j-1,k} \right)_f^{n+1} \\ + \; T_{w_{f_{i,j,k+\frac{1}{2}}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k+1} - \left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(\gamma_w \Delta D\right)_{i,j,k+\frac{1}{2}} \right)_f^{n+1} \\ - \; T_{w_{f_{i,j,k-\frac{1}{2}}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{i,j,k} - \left(P_o - P_{cap\,wo}\right)_{i,j,k-1} - \left(\gamma_w \Delta D\right)_{i,j,k-\frac{1}{2}} \right)_f^{n+1} \\ + \left(b_{w_f} q_w\right)_{i,j,k}^{n+1} \\ + \; T_{w_{mf_{i,j,k}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_{i,j,k}} - \left(P_o - P_{cap\,wo}\right)_{f_{i,j,k}} \right)^{n+1} \\ - \dfrac{Vp_{i,j,k}}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_{i,j,k} \right]_f \end{bmatrix} = 0$$

Ahora, truncando únicamente en la dirección "k":

$$F_{w_{f_k}}^{n+1} = \begin{bmatrix} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - \left(\gamma_w \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1} \\ - \; T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - \left(\gamma_w \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1} \\ + \left(b_{w_f} q_w \right)_k^{n+1} \\ + \; T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \\ - \dfrac{Vp_k}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \end{bmatrix}$$

Si definimos los siguientes términos, de manera similar a (Cortes Rubio, 2008) y (Orozco Aguilar, 2013):

$$\text{Term\_flujo\_agua} = \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \;\; \text{Pot\_agua\_Z2} \\ - \; \text{Trnsmsc\_agua\_Z1} \;\; \text{Pot\_agua\_Z1} \end{bmatrix}$$

$$\text{Trnsmsc\_agua\_Z2} = T_{w_{f_{k+\frac{1}{2}}}}^{n+1}$$

$$\text{Trnsmsc\_agua\_Z1} = T_{w_{f_{k-\frac{1}{2}}}}^{n+1}$$

$$\text{Pot\_agua\_Z2} = \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - \left(\gamma_w \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1}$$

$$\text{Pot\_agua\_Z1} = \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - \left(\gamma_w \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1}$$

$$\text{Term\_fuente} = \left[\left(b_{w_f} q_w \right)_k^{n+1} \right]$$

$$\text{Term\_transf\_MF} = \left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right]$$

$$\text{Term\_acum} = \left[\frac{Vp_k}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right]$$

Entonces la función de residuos de la fase de agua en la fractura natural truncada en la dirección "k" puede escribirse como:

$$F_{w_{f_k}}^{n+1} = \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \;\; \text{Pot\_agua\_Z2} \\ - \;\text{Trnsmsc\_agua\_Z1} \;\; \text{Pot\_agua\_Z1} \\ + \;\text{Term\_fuente} \\ + \;\text{Term\_transf\_MF} \\ - \;\text{Term\_acum} \end{bmatrix}$$

c.a) Derivando la función de residuos de agua $\left(F_{w_f}\right)$ truncada en la dirección "k" respecto a la presión de aceite en la fractura natural $\left(P_{o_f}\right)$ en "k":

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \;\; \text{Pot\_agua\_Z2} \\ - \;\text{Trnsmsc\_agua\_Z1} \;\; \text{Pot\_agua\_Z1} \\ + \;\text{Term\_fuente} \\ + \;\text{Term\_transf\_MF} \\ - \;\text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2} \;\; \text{Pot\_agua\_Z2 }] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1} \;\; \text{Pot\_agua\_Z1 }] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] \end{bmatrix}$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] + \text{Pot\_agua\_Z2} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] \\ - \left(\text{Trnsmsc\_agua\_Z1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] + \text{Pot\_agua\_Z1} \; \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }] \right) \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_acum }] \end{bmatrix}$$

Si sustituimos términos:

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - \left(\gamma_w \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - \left(\gamma_w \Delta D\right)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - \left(\gamma_w \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - \left(\gamma_w \Delta D\right)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(b_{w_f} q_w \right)_k^{n+1} \right] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right] \end{array} \right]$$

Realizando las derivadas que se presentan dentro del corchete:

1) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_w k_{rw}}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \left(\frac{A_z k k_{rw}}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \left(\frac{A_z k k_{rw}}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{\mu_{w_{k-\frac{1}{2}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - b_{w_{k-\frac{1}{2}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{w_{k-\frac{1}{2}}}^2} \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \left(\frac{A_z k k_{rw}}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] = T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

2) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_w k_{rw}}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \left(\frac{A_z k k_{rw}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \left(\frac{A_z k k_{rw}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\mu_{w_{k+\frac{1}{2}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - b_{w_{k+\frac{1}{2}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}}}{\mu_{w_{k+\frac{1}{2}}}^2} \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \left(\frac{A_z k k_{rw}}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right] = T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

3) Derivada del potencial de agua en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right]$$

$$(\gamma_w \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1}) \left(\frac{\gamma_{w_{f_k}} + \gamma_{w_{f_{k-1}}}}{2} \right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{w_k} + \gamma_{w_{k-1}}}{2} \right) \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = 1 - (D_k - D_{k-1}) \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{w_{f_k}}^{n+1} + \gamma_{w_{f_{k-1}}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\gamma_{w_{f_k}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = 1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

4) Derivada del potencial de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right]$$

$$(\gamma_w \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k) \left(\frac{\gamma_{w\,f_{k+1}} + \gamma_{w\,f_k}}{2} \right)$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{w\,k+1} + \gamma_{w\,k}}{2} \right) \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = -1 - (D_{k+1} - D_k) \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{\gamma_{w_{f_{k+1}}}^{n+1} + \gamma_{w_{f_k}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = -1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\gamma_{w_{f_k}}^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = -1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

5) Derivada del término fuente en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w \right)_k^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \left[b_{w_{f_k}}^{n+1} \frac{\partial q_{w_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{w_k} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w \right)_k^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \left[b_{w_{f_k}}^{n+1} \frac{\partial q_{w_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{w_k} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right]$$

6) Derivada del término transferencia matriz-fractura en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{wmf_k}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{wmf_k}^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{wmf_k}^{n+1} \right] \end{array} \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{wmf_k}^{n+1} \; (-1) \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[T_{wmf_k}^{n+1} \right] \end{array} \right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$$\boldsymbol{Si}\text{: } \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) - \left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{f}_{i,j,k}}}\right) \geq \boldsymbol{0}$$

$$T^{n+1}_{w_{mf_{i,j,k}}} = Vr_{i,j,k}(\sigma k \lambda_w)^{n+1}_{m\,i,j,k} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)^{n+1}_{m\,i,j,k}$$

$$\frac{\partial T^{n+1}_{w_{mf_k}}}{\partial P^{n+1}_{o_{f_k}}} = 0$$

$$\boldsymbol{Si}\text{: } \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) - \left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{f}_{i,j,k}}}\right) < \boldsymbol{0}$$

$$T^{n+1}_{w_{mf_{i,j,k}}} = Vr_{i,j,k}(\sigma k \lambda_w)^{n+1}_{f\,i,j,k} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)^{n+1}_{f\,i,j,k}$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = \frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right]$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = \frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, Vr_k\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)^{n+1}_{f\,k} \,\right]$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = (\, Vr\,\sigma\,k\,k_{rw}\,)^{n+1}_{f_k}\ \frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\left(\frac{b_w}{\mu_w}\right)^{n+1}_{f_k}\right]$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = (\, Vr\,\sigma\,k\,k_{rw}\,)^{n+1}_{f_k}\left(\frac{\mu_{w_k}\dfrac{\partial b_{w_k}}{\partial P_{o_k}} - b_{w_k}\dfrac{\partial \mu_{w_k}}{\partial P_{o_k}}}{\mu^2_{w_k}}\right)^{n+1}_{f}$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = (\, Vr\,\sigma\,k\,k_{rw}\,)^{n+1}_{f_k}\left(\frac{b_w}{\mu_w}\right)^{n+1}_{f_k}\left(\frac{1}{b_{w_k}}\frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}}\frac{\partial \mu_{w_k}}{\partial P_{o_k}}\right)^{n+1}_{f}$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}} \,\right] = T^{n+1}_{w_{mf_k}}\left(\frac{1}{b_{w_k}}\frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}}\frac{\partial \mu_{w_k}}{\partial P_{o_k}}\right)^{n+1}_{f}$$

Entonces:

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\,\text{Term\_transf\_MF}\,\right] = \frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\, T^{n+1}_{w_{mf_k}}\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right]$$

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}}\left[\,\text{Term\_transf\_MF}\,\right] = \left[\begin{array}{c} T^{n+1}_{w_{mf_k}}\ (-1\,) \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1} \dfrac{\partial T^{n+1}_{w_{mf_k}}}{\partial P^{n+1}_{o_{f_k}}} \end{array}\right]$$

7) Derivada del término de acumulación en la fractura natural en (k) respecto a la presión de aceite en la fractura natural en (k):

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\ [\ ([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n)_k\ ]_f \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ ([1 + C_r(P_o^{n+1} - P_o^n)]\ [b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n\ )_{f_k}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ ] \\ +\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] \\ -\left(\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n}\ ]\right) \end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ ] = b_{w_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ S_{at\,w_{f_k}}^{n+1}\ \right] + S_{at\,w_{f_k}}^{n+1}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{w_{f_k}}^{n+1}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ ] = 0\ +\ S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ ] = S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] = C_{r_k}$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] = C_{r_k}}$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n}\ ] = b_{w_{f_k}}^{n}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ S_{at\,w_{f_k}}^{\ n}\ \right] + S_{at\,w_{f_k}}^{\ n}\ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}}\left[\ b_{w_{f_k}}^{n}\ \right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n}\ ] = 0\ +\ 0$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n}\ ] = 0}$$

Entonces:

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\left(S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) \\ +\ (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ (\ C_{r_k}\ ) \\ -\ (\ 0\ ) \end{array}\right]$$

$$\boxed{\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\left(S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right) + (\ b_w\ S_{at\,w}\ )_{f_k}^{n+1}\ (\ C_{r_k}\ )\ \right]}$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_agua\_Z2} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_agua\_Z2}\ ] + \text{Pot\_agua\_Z2} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z2}\ ] \\ - \left(\text{Trnsmsc\_agua\_Z1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Pot\_agua\_Z1}\ ] + \text{Pot\_agua\_Z1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z1}\ ] \right) \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_fuente}\ ] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_transf\_MF}\ ] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} [\ \text{Term\_acum}\ ] \end{array} \right]$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \ \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\left(b_{w_f} q_w \right)_k^{n+1} \right] \\ + \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ - \frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[\frac{Vp_k}{\Delta t} \ \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right] \end{array} \right]$$

$$
\frac{\partial F_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \left(-1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{w_{k+1}} + \gamma_{w_k}}{2} \right) \right)_f^{n+1} \\
\left(T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right) \\
- \left(\begin{array}{c}
T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(1 - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{w_k} + \gamma_{w_{k-1}}}{2} \right) \right)_f^{n+1} \\
\left(T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \right)
\end{array} \right) \\
+ \left(b_{w_{f_k}}^{n+1} \frac{\partial q_{w_k}}{\partial P_{o_{f_k}}^{n+1}} + q_{w_k} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \\
+ \left(\begin{array}{c}
T_{w_{mf_k}}^{n+1} \left(-1 \right) \\
+ \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \frac{\partial T_{w_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}
\end{array} \right) \\
- \left(\frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k} \left(S_{at\,w_{f_k}}^{n+1} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) + \left(b_w\, S_{at\,w} \right)_{f_k}^{n+1} \left(C_{r_k} \right) \right] \right)
\end{array} \right]
$$

c.b) Derivando la función de residuos de agua $\left(F_{w_f}\right)$ truncada en la dirección "k" respecto a la saturación de gas en la fractura natural $\left(S_{at\,g_f}\right)$ en "k":

$$
\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} \left[\begin{array}{c}
\text{Trnsmsc\_agua\_Z2 Pot\_agua\_Z2} \\
- \text{Trnsmsc\_agua\_Z1 Pot\_agua\_Z1} \\
+ \text{Term\_fuente} \\
+ \text{Term\_transf\_MF} \\
- \text{Term\_acum}
\end{array} \right]
$$

$$
\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c}
\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z2 Pot\_agua\_Z2 }] \\
- \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 Pot\_agua\_Z1 }] \\
+ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Term\_fuente }] \\
+ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Term\_transf\_MF }] \\
- \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}} [\text{ Term\_acum }]
\end{array} \right]
$$

$$\frac{\partial F^{n+1}_{w_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} = \left[\begin{array}{c} \text{Trnsmsc\_agua\_Z2} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Pot\_agua\_Z2}\ ] + \text{Pot\_agua\_Z2} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z2}\ ] \\ - \left(\text{Trnsmsc\_agua\_Z1} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Pot\_agua\_Z1}\ ] + \text{Pot\_agua\_Z1} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z1}\ ] \right) \\ + \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Term\_fuente}\ ] \\ + \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Term\_transf\_MF}\ ] \\ - \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Term\_acum}\ ] \end{array} \right]$$

Si sustituimos términos:

$$\frac{\partial F^{n+1}_{w_{f_k}}}{\partial S^{n+1}_{at\,g_{f_k}}} = \left[\begin{array}{c} T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)^{n+1}_f \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)^{n+1}_f \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \right] \\ - \left(\begin{array}{c} T^{n+1}_{w_{f_{k-\frac{1}{2}}}} \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k} - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)^{n+1}_f \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k} - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)^{n+1}_f \ \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[T^{n+1}_{w_{f_{k-\frac{1}{2}}}} \right] \end{array} \right) \\ + \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[\left(b_{w_f} q_w \right)^{n+1}_k \right] \\ + \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[T^{n+1}_{w_{mf_k}} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ - \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[\frac{Vp_k}{\Delta t} \ \left[\left([1 + C_r(P^{n+1}_o - P^n_o)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right] \end{array} \right]$$

Realizando las derivadas que se presentan dentro del corchete:

<u>1) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z1}\ ] = \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[T^{n+1}_{w_{f_{k-\frac{1}{2}}}} \right]$$

$$\frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z1}\ ] = \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[\left(\frac{A_z k}{\Delta z} \right)^{n+1}_{f_{k-\frac{1}{2}}} \left(\frac{b_w k_{rw}}{\mu_w} \right)^{n+1}_{f_{k-\frac{1}{2}}} \right]$$

$$\frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z1}\ ] = 0$$

$$\frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}[\ \text{Trnsmsc\_agua\_Z1}\ ] = \frac{\partial}{\partial S^{n+1}_{at\,g_{f_k}}}\left[T^{n+1}_{w_{f_{k-\frac{1}{2}}}} \right] = 0$$

2) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \;\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1}\left(\frac{b_w k_{rw}}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\; T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \;\right] = 0}$$

3) Derivada del potencial de agua en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_w \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1})\left(\frac{\gamma_{w_{f_k}} + \gamma_{w_{f_{k-1}}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (D_k - D_{k-1})\left(\frac{\gamma_{w_{f_k}}^{n+1} + \gamma_{w_{f_{k-1}}}^{n+1}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] = 0}$$

4) Derivada del potencial de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$(\gamma_w \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k)\left(\frac{\gamma_{w_{f_{k+1}}} + \gamma_{w_{f_k}}}{2}\right)$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (D_{k+1} - D_k)\left(\frac{\gamma_{w_{f_{k+1}}}^{n+1} + \gamma_{w_{f_k}}^{n+1}}{2}\right)\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] = 0}$$

5) Derivada del término fuente en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_{w_f}q_w\right)_k^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \left[b_{w_{f_k}}^{n+1}\frac{\partial q_{w_k}}{\partial S_{at\,g_{f_k}}^{n+1}} + q_{w_k}\frac{\partial b_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \left[b_{w_{f_k}}^{n+1}(\,0\,) + q_{w_k}(\,0\,)\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_{w_f}q_w\right)_k^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_fuente }] = [\,0\,]$$

6) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{w_{mf_k}}^{n+1}\dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\dfrac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right]\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{w_{mf_k}}^{n+1}(\,0\,) \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}(\,0\,)\end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = [\,0\,]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = [\,0\,]$$

7) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de gas en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n\right)_k\right]_f\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left([1 + C_r(P_o^{n+1} - P_o^n)]\,[b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n\right)_{f_k}\right]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ ] \\ +\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] \\ -\left(\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n}\ ]\right) \end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ ] = b_{w_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ S_{at\,w_{f_k}}^{n+1}\ ] + S_{at\,w_{f_k}}^{n+1}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ b_{w_{f_k}}^{n+1}\ ]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ ] = 0\ +0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ ] = 0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n}\ ] = b_{w_{f_k}}^{n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ S_{at\,w_{f_k}}^{\ \ n}\ ] + S_{at\,w_{f_k}}^{\ \ n}\ \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ b_{w_{f_k}}^{n}\ ]$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n}\ ] = 0\ +0$$

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n}\ ] = 0$$

Entonces:

$$\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\ (\ 0\ ) \\ +\ (\ b_w\, S_{at\,w}\ )_{f_k}^{n+1}\ (\ 0\ ) \\ -\ (\ 0\ ) \end{array}\right]$$

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}}[\ \text{Term\_acum}\ ] = [\ 0\ ]$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_agua\_Z2} \;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Pot\_agua\_Z2}\;] + \text{Pot\_agua\_Z2}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Trnsmsc\_agua\_Z2}\;] \\ -\left(\text{Trnsmsc\_agua\_Z1} \;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Pot\_agua\_Z1}\;] + \text{Pot\_agua\_Z1}\;\frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Trnsmsc\_agua\_Z1}\;] \right) \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Term\_fuente}\;] \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Term\_transf\_MF}\;] \\ - \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}[\;\text{Term\_acum}\;] \end{array} \right]$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ -\left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w \right)_k^{n+1} \right] \\ + \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ - \frac{\partial}{\partial S_{at\,g_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right] \end{array} \right]$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (D_{k+1} - D_k)\left(\frac{\gamma_{w_{f_{k+1}}}^{n+1} + \gamma_{w_{f_k}}^{n+1}}{2} \right) \right)_f^{n+1} \;(\;0\;) \\ -\left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \;(\;0\;) \\ + \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (D_k - D_{k-1})\left(\frac{\gamma_{w_{f_k}}^{n+1} + \gamma_{w_{f_{k-1}}}^{n+1}}{2} \right) \right)_f^{n+1} \;(\;0\;) \end{array} \right) \\ + \;(\;0\;) \\ + \;(\;0\;) \\ - \;(\;0\;) \end{array} \right]$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,g_{f_k}}^{n+1}} = [\;0\;]$$

c.c) Derivando la función de residuos de agua $\left(F_{w_f}\right)$ truncada en la dirección "k" respecto a la saturación de agua en la fractura natural $\left(S_{at\,w_f}\right)$ en "k":

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \quad \text{Pot\_agua\_Z2} \\ - \ \text{Trnsmsc\_agua\_Z1} \quad \text{Pot\_agua\_Z1} \\ + \ \text{Term\_fuente} \\ + \ \text{Term\_transf\_MF} \\ - \ \text{Term\_acum} \end{bmatrix}$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z2} \quad \text{Pot\_agua\_Z2}\ ] \\ - \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z1} \quad \text{Pot\_agua\_Z1}\ ] \\ + \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_fuente}\ ] \\ + \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_transf\_MF}\ ] \\ - \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_acum}\ ] \end{bmatrix}$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \begin{bmatrix} \text{Trnsmsc\_agua\_Z2} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Pot\_agua\_Z2}\ ] + \text{Pot\_agua\_Z2} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z2}\ ] \\ - \left(\text{Trnsmsc\_agua\_Z1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Pot\_agua\_Z1}\ ] + \text{Pot\_agua\_Z1} \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Trnsmsc\_agua\_Z1}\ ] \right) \\ + \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_fuente}\ ] \\ + \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_transf\_MF}\ ] \\ - \ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\ \text{Term\_acum}\ ] \end{bmatrix}$$

Si sustituimos términos:

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right] \\ - \left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] \end{array} \right) \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w \right)_k^{n+1} \right] \\ + \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right] \\ - \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t} \left[\left([1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n \right)_k \right]_f \right] \end{array}\right]$$

Realizando las derivadas que se presentan dentro del corchete:

<u>1) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k - \frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_w k_{rw}}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }] = \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w k_{rw}}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }] =$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1} \right] + k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \right] \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1} \left(\frac{\mu_{w_{k-\frac{1}{2}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} - b_{w_{k-\frac{1}{2}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}}}{\mu_{w_{k-\frac{1}{2}}}^2} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1} \left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k-\frac{1}{2}}}}^{\,n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} = \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] =$$

$$= \left(\frac{A_z k}{\Delta z} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k b_w}{\Delta z \mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= \left(\frac{A_z k k_{rw} b_w}{\Delta z \mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \left(\frac{1}{k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= \frac{T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Si recordamos la derivada de la transmisibilidad del agua en la fractura natural en "$k - \frac{1}{2}$" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] = T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k-\frac{1}{2}}}} \frac{\partial b_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k-\frac{1}{2}}}} \frac{\partial \mu_{w_{k-\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \frac{T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \left(\frac{A_z k b_w}{\Delta z \mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Trnsmsc\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \right] = \left(\frac{A_z k b_w}{\Delta z \mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

2) Derivada de la transmisibilidad de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_w k_{rw}}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] = \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w k_{rw}}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] =$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \right] + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \right] \right)$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{\mu_{w_{k+\frac{1}{2}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} - b_{w_{k+\frac{1}{2}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}}}{\mu_{w_{k+\frac{1}{2}}}^2} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial S_{at\,w_k}} \right)_f^{n+1} = \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] =$$

$$= \left(\frac{A_z k}{\Delta z}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{b_w}{\mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{A_z k b_w}{\Delta z \mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_{k+\frac{1}{2}}}}^{n+1} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= \left(\frac{A_z k k_{rw} b_w}{\Delta z \mu_w}\right)_{f_{k+\frac{1}{2}}}^{n+1} \left(\frac{1}{k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \left(\frac{1}{k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S^{n+1}_{at\,w_{f_k}}} + \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_{f} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}} \right)$$

$$= \frac{T^{n+1}_{w_{f_{k+\frac{1}{2}}}}}{k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S^{n+1}_{at\,w_{f_k}}} + T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_{f} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}}$$

Si recordamos la derivada de la transmisibilidad del agua en la fractura natural en "$k+\frac{1}{2}$" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P^{n+1}_{o_{f_k}}} \left[T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \right] = T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \left(\frac{1}{b_{w_{k+\frac{1}{2}}}} \frac{\partial b_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} - \frac{1}{\mu_{w_{k+\frac{1}{2}}}} \frac{\partial \mu_{w_{k+\frac{1}{2}}}}{\partial P_{o_k}} \right)^{n+1}_{f}$$

Sustituyendo:

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Trnsmsc\_agua\_Z2 }] = \frac{T^{n+1}_{w_{f_{k+\frac{1}{2}}}}}{k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S^{n+1}_{at\,w_{f_k}}} + \frac{\partial T^{n+1}_{w_{f_{k+\frac{1}{2}}}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}}$$

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Trnsmsc\_agua\_Z2 }] = \left(\frac{A_z k b_w}{\Delta z \mu_w} \right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S^{n+1}_{at\,w_{f_k}}} + \frac{\partial T^{n+1}_{w_{f_{k+\frac{1}{2}}}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}}$$

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Trnsmsc\_agua\_Z2 }] = \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[T^{n+1}_{w_{f_{k+\frac{1}{2}}}} \right] = \left(\frac{A_z k b_w}{\Delta z \mu_w} \right)^{n+1}_{f_{k+\frac{1}{2}}} \frac{\partial k_{rw_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial S^{n+1}_{at\,w_{f_k}}} + \frac{\partial T^{n+1}_{w_{f_{k+\frac{1}{2}}}}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}}$$

<u>3) Derivada del potencial de agua en la fractura natural en $\left(k-\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[\left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)^{n+1}_{f} \right]$$

$$(\gamma_w \Delta D)_{k-\frac{1}{2}} = (D_k - D_{k-1}) \left(\frac{\gamma_{w_{f_k}} + \gamma_{w_{f_{k-1}}}}{2} \right)$$

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[\left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{w_{f_k}} + \gamma_{w_{f_{k-1}}}}{2} \right) \right)^{n+1}_{f} \right]$$

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Pot\_agua\_Z1 }] = -\frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}} - (D_k - D_{k-1}) \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[\frac{\gamma_{w_{f_k}}^{n+1} + \gamma_{w_{f_{k-1}}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} [\text{ Pot\_agua\_Z1 }] = -\frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}} - \frac{(D_k - D_{k-1})}{2} \frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[\gamma_{w_{f_k}}^{n+1} \right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S^{n+1}_{at\,w_{f_k}}} \left[\gamma_{w_{f_k}}^{n+1} \right] = \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P^{n+1}_{o_{f_k}}} \frac{\partial P^{n+1}_{cap\,wo_{f_k}}}{\partial S^{n+1}_{at\,w_{f_k}}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = -\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \left(-\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \left(1 + \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}} \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = -\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }] = \left(-\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \left(1 + \frac{(D_k - D_{k-1})}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)$$

<u>4) Derivada del potencial de agua en la fractura natural en $\left(k+\frac{1}{2}\right)$ respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}} \right)_f^{n+1} \right]$$

$$(\gamma_w \Delta D)_{k+\frac{1}{2}} = (D_{k+1} - D_k) \left(\frac{\gamma_{w_{f_{k+1}}} + \gamma_{w_{f_k}}}{2} \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (D_{k+1} - D_k) \left(\frac{\gamma_{w_{f_{k+1}}} + \gamma_{w_{f_k}}}{2} \right) \right)_f^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - (D_{k+1} - D_k) \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\frac{\gamma_{w_{f_{k+1}}}^{n+1} + \gamma_{w_{f_k}}^{n+1}}{2} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - \frac{(D_{k+1} - D_k)}{2} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\gamma_{w_{f_k}}^{n+1} \right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\gamma_{w_{f_k}}^{n+1} \right] = \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }] = \left(\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \left(1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right)$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_agua\_Z2}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_agua\_Z2}\ ] = \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} - \frac{(D_{k+1} - D_k)}{2}\frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Pot\_agua\_Z2}\ ] = \left(\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)\left(1 - \frac{(D_{k+1} - D_k)}{2}\frac{\partial \gamma_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\right)$$

<u>5) Derivada del término fuente en (k) respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w\right)_k^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[b_{w_{f_k}}^{n+1}\frac{\partial q_{w_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{w_k}\frac{\partial b_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right]$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[b_{w_{f_k}}^{n+1}\right] = \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[b_{w_{f_k}}^{n+1}\frac{\partial q_{w_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{w_k}\frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(b_{w_f} q_w\right)_k^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_fuente}\ ] = \left[b_{w_{f_k}}^{n+1}\frac{\partial q_{w_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{w_k}\frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right]$$

<u>6) Derivada del término transferencia matriz-fractura en (k) respecto a la saturación de agua en la fractura natural en (k):</u>

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\ \text{Term\_transf\_MF}\ ] = \left[\begin{array}{l} T_{w_{mf_k}}^{n+1}\dfrac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right] \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\dfrac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{w_{mf_k}}^{n+1}\left(\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right) \\ +\left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1} \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] \end{array}\right]$$

En esta parte es importante indicar que las derivadas de las transmisibilidades entre la matriz y la fractura existirán o no, dependiendo del concepto de corriente arriba (Rodríguez de la Garza & Galindo Nava, 2000) o "upstream weighting" (Peaceman D. W., 1977) y (Aziz, 1979), ya que estas transmisibilidades emplearán las propiedades de la matriz o la fractura, dependiendo del resultado de la evaluación del potencial:

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) - \left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{f}_{i,j,k}}}\right) \geq \boldsymbol{0}$$

$$T_{w_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_w)_{m\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)_{m\,i,j,k}^{n+1}$$

$$\boxed{\frac{\partial T_{w_{mf_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = 0}$$

$$\boldsymbol{Si}:\ \left(\boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{o}_{\boldsymbol{f}_{i,j,k}}}\right) - \left(\boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{m}_{i,j,k}}} - \boldsymbol{P}_{\boldsymbol{cap\,wo}_{\boldsymbol{f}_{i,j,k}}}\right) < \boldsymbol{0}$$

$$T_{w_{mf_{i,j,k}}}^{n+1} = Vr_{i,j,k}(\sigma k \lambda_w)_{f\,i,j,k}^{n+1} = Vr_{i,j,k}\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)_{f\,i,j,k}^{n+1}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[Vr_k\left(\frac{\sigma k b_w k_{r_w}}{\mu_w}\right)_{f\,k}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[(Vr\,\sigma\,k)_{f_k}^{n+1}\left(\frac{b_w\,k_{rw}}{\mu_w}\right)_{f_k}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] = (Vr\,\sigma\,k)_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w\,k_{rw}}{\mu_w}\right)_{f_k}^{n+1}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[T_{w_{mf_k}}^{n+1}\right] =$$

$$= (Vr\,\sigma\,k)_{f_k}^{n+1}\left(\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[k_{rw_{f_k}}^{n+1}\right] + k_{rw_{f_k}}^{n+1}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\right]\right)$$

$$= (Vr\,\sigma\,k)_{f_k}^{n+1}\left(\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_k}}^{n+1}\left(\frac{\mu_{w_k}\frac{\partial b_{w_k}}{\partial S_{at\,w_k}} - b_{w_k}\frac{\partial \mu_{w_k}}{\partial S_{at\,w_k}}}{\mu_{w_k}^2}\right)_f^{n+1}\right)$$

$$= (Vr\,\sigma\,k)_{f_k}^{n+1}\left(\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_k}}^{n+1}\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\left(\frac{1}{b_{w_k}}\frac{\partial b_{w_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_k}}\frac{\partial \mu_{w_k}}{\partial S_{at\,w_k}}\right)_f^{n+1}\right)$$

$$= (Vr\,\sigma\,k)_{f_k}^{n+1}\left(\frac{b_w}{\mu_w}\right)_{f_k}^{n+1}\left(\frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_k}}^{n+1}\left(\frac{1}{b_{w_k}}\frac{\partial b_{w_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_k}}\frac{\partial \mu_{w_k}}{\partial S_{at\,w_k}}\right)_f^{n+1}\right)$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial S_{at\,w_k}} \right)_f^{n+1} = \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \right] =$$

$$= \left(Vr\,\sigma\,k \right)_{f_k}^{n+1} \left(\frac{b_w}{\mu_w} \right)_{f_k}^{n+1} \left(\frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_k}}^{n+1} \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \frac{\partial P_{cap\,wo_k}}{\partial S_{at\,w_k}} \right)_f^{n+1} \right)$$

$$= \left(\frac{Vr\,\sigma\,k\,b_w}{\mu_w} \right)_{f_k}^{n+1} \left(\frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + k_{rw_{f_k}}^{n+1} \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= \left(\frac{Vr\,\sigma\,k\,k_{rw}\,b_w}{\mu_w} \right)_{f_k}^{n+1} \left(\frac{1}{k_{rw_{f_k}}^{n+1}} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= T_{w_{mf_k}}^{n+1} \left(\frac{1}{k_{rw_{f_k}}^{n+1}} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)$$

$$= \frac{T_{w_{mf_k}}^{n+1}}{k_{rw_{f_k}}^{n+1}} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + T_{w_{mf_k}}^{n+1} \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \right)_f^{n+1} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Si recordamos la derivada de la transmisibilidad del agua entre la matriz y la fractura natural en "k" respecto a la presión de aceite en la fractura natural en "k":

$$\frac{\partial}{\partial P_{o_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \right] = T_{w_{mf_k}}^{n+1} \left(\frac{1}{b_{w_k}} \frac{\partial b_{w_k}}{\partial P_{o_k}} - \frac{1}{\mu_{w_k}} \frac{\partial \mu_{w_k}}{\partial P_{o_k}} \right)_f^{n+1}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \right] = \frac{T_{w_{mf_k}}^{n+1}}{k_{rw_{f_k}}^{n+1}} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \right] = \left(\frac{Vr\,\sigma\,k\,b_w}{\mu_w} \right)_{f_k}^{n+1} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \right] = \left(\frac{Vr\,\sigma\,k\,b_w}{\mu_w} \right)_{f_k}^{n+1} \frac{\partial k_{rw_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{mf_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Term\_transf\_MF }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} \left[T_{w_{mf_k}}^{n+1} \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}} [\text{ Term\_transf\_MF }] = \left[\begin{array}{c} T_{w_{mf_k}}^{n+1} \left(\dfrac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\ + \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k} \right)^{n+1} \dfrac{\partial T_{w_{mf_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \end{array} \right]$$

7) Derivada del término de acumulación en la fractura natural en (k) respecto a la saturación de agua en la fractura natural en (k):

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\;[\;(\,[1+C_r(P_o^{n+1}-P_o^n)][b_w S_{at\,w}]^{n+1}-[b_w S_{at\,w}]^n\,)_k\;]_f\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;(\,[1+C_r(P_o^{n+1}-P_o^n)]\;[b_w S_{at\,w}]^{n+1}\;-\;[b_w S_{at\,w}]^n\;)_{f_k}\;\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c}(\;1+C_r(P_o^{n+1}-P_o^n)\;)_{f_k}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;]\\ +\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;1+C_r(P_o^{n+1}-P_o^n)\;)_{f_k}\;]\\ -\left(\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n}\;]\right)\end{array}\right]$$

Si definimos:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;] = b_{w_{f_k}}^{n+1}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;S_{at\,w_{f_k}}^{n+1}\;\right]\;+\;S_{at\,w_{f_k}}^{n+1}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;b_{w_{f_k}}^{n+1}\;\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;] = b_{w_{f_k}}^{n+1}\;(\;1\;)\;+\;S_{at\,w_{f_k}}^{n+1}\;\frac{\partial b_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Aplicando la regla de la cadena podemos hacer la siguiente igualación:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;b_{w_{f_k}}^{n+1}\;\right] = \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\;\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

Sustituyendo:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;] = b_{w_{f_k}}^{n+1}\;(\;1\;)\;+\;S_{at\,w_{f_k}}^{n+1}\;\frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\;\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n+1}\;] = b_{w_{f_k}}^{n+1}\;+\;S_{at\,w_{f_k}}^{n+1}\;\frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\;\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;1+C_r(P_o^{n+1}-P_o^n)\;)_{f_k}\;] = 0$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;1+C_r(P_o^{n+1}-P_o^n)\;)_{f_k}\;] = 0}$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n}\;] = b_{w_{f_k}}^{n}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;{S_{at\,w_{f_k}}}^{n}\;\right]\;+\;{S_{at\,w_{f_k}}}^{n}\;\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\;b_{w_{f_k}}^{n}\;\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n}\;] = 0\;+\;0$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\;(\;b_w\,S_{at\,w}\;)_{f_k}^{n}\;] = 0}$$

Entonces:

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\begin{array}{c} (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\left(\ b_{w_{f_k}}^{n+1}\ +\ S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right) \\ +\ (\ b_w\,S_{at\,w}\ )_{f_k}^{n+1}\ (\ 0\ ) \\ -\ (\ 0\ ) \end{array}\right]$$

$$\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\left(\ b_{w_{f_k}}^{n+1}\ +\ S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)\ \right]$$

$$\boxed{\frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] = \frac{Vp_k}{\Delta t}\left[\ (\ 1 + C_r(P_o^{n+1} - P_o^n)\ )_{f_k}\left(\ b_{w_{f_k}}^{n+1}\ +\ S_{at\,w_{f_k}}^{n+1}\ \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}}\ \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}\right)\ \right]}$$

Es así que nuestra derivada final queda de la siguiente forma:

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} \text{Trnsmsc\_agua\_Z2}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z2 }]\ +\ \text{Pot\_agua\_Z2}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z2 }] \\ -\left(\ \text{Trnsmsc\_agua\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Pot\_agua\_Z1 }]\ +\ \text{Pot\_agua\_Z1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Trnsmsc\_agua\_Z1 }]\right) \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_fuente }] \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_transf\_MF }] \\ -\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}[\text{ Term\_acum }] \end{array}\right]$$

$$\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c} T_{w_{f_{k+\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\right] \\ +\ \left(\left(P_o - P_{cap\,wo}\right)_{k+1} - \left(P_o - P_{cap\,wo}\right)_k - (\gamma_w \Delta D)_{k+\frac{1}{2}}\right)_f^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{w_{f_{k+\frac{1}{2}}}}^{n+1}\right] \\ -\left(\begin{array}{c} T_{w_{f_{k-\frac{1}{2}}}}^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\right] \\ +\ \left(\left(P_o - P_{cap\,wo}\right)_k - \left(P_o - P_{cap\,wo}\right)_{k-1} - (\gamma_w \Delta D)_{k-\frac{1}{2}}\right)_f^{n+1}\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{w_{f_{k-\frac{1}{2}}}}^{n+1}\right] \end{array}\right) \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\left(\ b_{w_f} q_w\ \right)_k^{n+1}\right] \\ +\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\ T_{w_{mf_k}}^{n+1}\ \left(\left(P_o - P_{cap\,wo}\right)_{m_k} - \left(P_o - P_{cap\,wo}\right)_{f_k}\right)^{n+1}\right] \\ -\ \frac{\partial}{\partial S_{at\,w_{f_k}}^{n+1}}\left[\frac{Vp_k}{\Delta t}\ [\ (\ [1 + C_r(P_o^{n+1} - P_o^n)][b_w S_{at\,w}]^{n+1} - [b_w S_{at\,w}]^n\ )_k\ ]_f\ \right] \end{array}\right]$$

$$
\frac{\partial F_{w_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} = \left[\begin{array}{c}
T_{w_{f_{k+\frac{1}{2}}}}^{n+1} \left(\left(\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \left(1 - \frac{(D_{k+1} - D_k)}{2} \frac{\partial {\gamma_{w}}_{f_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\
+ \left((P_o - P_{cap\,wo})_{k+1} - (P_o - P_{cap\,wo})_k - (D_{k+1} - D_k) \left(\frac{\gamma_{w_{f_{k+1}}} + \gamma_{w_{f_k}}}{2} \right) \right)_f^{n+1} \\
\left(\left(\frac{A_z k b_w}{\Delta z \mu_w} \right)_{f_{k+\frac{1}{2}}}^{n+1} \frac{\partial {k_{rw}}_{f_{k+\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{f_{k+\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\
- \left(\begin{array}{c}
T_{w_{f_{k-\frac{1}{2}}}}^{n+1} \left(\left(- \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \left(1 + \frac{(D_k - D_{k-1})}{2} \frac{\partial {\gamma_{w}}_{f_k}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \right) \right) \\
+ \left((P_o - P_{cap\,wo})_k - (P_o - P_{cap\,wo})_{k-1} - (D_k - D_{k-1}) \left(\frac{\gamma_{w_{f_k}} + \gamma_{w_{f_{k-1}}}}{2} \right) \right)_f^{n+1} \\
\left(\left(\frac{A_z k b_w}{\Delta z \mu_w} \right)_{f_{k-\frac{1}{2}}}^{n+1} \frac{\partial {k_{rw}}_{f_{k-\frac{1}{2}}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} + \frac{\partial T_{w_{f_{k-\frac{1}{2}}}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right)
\end{array} \right) \\
+ \left(b_{w_{f_k}}^{n+1} \frac{\partial q_{w_k}}{\partial S_{at\,w_{f_k}}^{n+1}} + q_{w_k} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\
+ \left(\begin{array}{c}
T_{w_{mf_k}}^{n+1} \left(\frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \\
+ \left((P_o - P_{cap\,wo})_{m_k} - (P_o - P_{cap\,wo})_{f_k} \right)^{n+1} \frac{\partial T_{w_{mf_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}}
\end{array} \right) \\
- \frac{Vp_k}{\Delta t} \left[\left(1 + C_r (P_o^{n+1} - P_o^n) \right)_{f_k} \left(b_{w_{f_k}}^{n+1} + S_{at\,w_{f_k}}^{n+1} \frac{\partial b_{w_{f_k}}^{n+1}}{\partial P_{o_{f_k}}^{n+1}} \frac{\partial P_{cap\,wo_{f_k}}^{n+1}}{\partial S_{at\,w_{f_k}}^{n+1}} \right) \right]
\end{array} \right]
$$

Lo que se realizó en este anexo, fue desarrollar las derivadas de las funciones de residuos de las tres fases correspondientes a la fractura natural $\left(F_o,\ F_g,\ F_w\right)_f$ respecto a las variables primarias de la fractura natural $\left(S_{at\,o},\ S_{at\,g},\ S_{at\,w}\right)_f$, sin embargo, recordemos que al inicio de este anexo, se indicó que estas derivadas se realizaron únicamente en la dirección "k", entonces, un paso siguientes sería determinar las derivadas en "k+1" y "k-1", aunque varios términos que ya se obtuvieron en este anexo se repetirían en estas direcciones faltantes, lo que haría más sencillo de determinarlas, y para las direcciones "i" y "j", el procedimiento sería el mismo sin considerar los efectos gravitacionales. (Orozco Aguilar, 2013).

Posteriormente sería necesario determinar:

a) Las derivadas de las funciones de residuos de las tres fases en la fractura natural $\left(F_o, F_g, F_w \right)_f$ respecto a las variables primarias de la matriz $\left(P_o,\ S_{at\,g},\ S_{at\,w} \right)_m$

b) Las derivadas de las funciones de residuos de las tres fases en la matriz $\left(F_o, F_g, F_w \right)_m$ respecto a las variables primarias de la fractura natural $\left(P_o,\ S_{at\,g},\ S_{at\,w} \right)_f$

c) Las derivadas de las funciones de residuos de las tres fases en la matriz $\left(F_o, F_g, F_w \right)_m$ respecto a las variables primarias de la matriz $\left(P_o,\ S_{at\ g},\ S_{at\ w} \right)_m$

El proceso para determinar estas derivadas sería el mismo que se describe en este anexo, siendo así, una posible guía al momento de determinar estas derivadas.

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