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A Frailty Model for the Mexican Economy: A Copula Approach.

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This Thesis is dedicated to my Mother, Martha, my brother Ricardo, my nieces Michelle and Ximena, my nephew Emiliano, Cookie and all my friends, Thank You!

In total Gratitude to Dr. Armando Sánchez Vargas, for your unconditional friendship and endless mentoring lessons.

To the magnificent institution that gave me everything I need to make my dreams come true, Universidad Nacional Autónoma de México.

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Introduction.

Conventional econometric analysis usually assumes a distribution function underlying the process to be modeled. Frequently, models assume either a Normal distribution or a Student´s T distribution with very little room for other possibilities. Moreover, when trying to model a multivariate phenomenon, each variable may have a different distribution. Therefore, the issue is how to model this phenomenon's dependence-structure with a single function that accounts for the dissimilar distributions each variable may possess?

Copula functions are a way to address this question for they are functions that possesses a big amount of flexibility. This is a very important characteristic since they allow more realism to be incorporated in our models and hence, instead of assuming that data have a pre-determined distribution, we can study a phenomenon trying to preserve its original structure as much as possible.

This work's main goal is to explore how fragile the Mexican economy is given the high dependence that the *price-stability* and *export-led-growth* policies have with variables that policymakers do not control on its entirety. Moreover, the goal is to evaluate how easy it is for policymakers to implement countercyclical policies given that these stability variables' dependence structure may be affected when a shock hits the economy. Actually, if these variables become less dependent after a shock, then policymakers would have a very hard time trying to implement countercyclical policies since they would need to put the more effort to achieve the same result as before the crisis.

What is more, this work is concerned to study crisis-contagion from the U.S. to Mexico. If contagion indeed takes place, the goal is to study how the dependence structure changes with respect to key variables in which the stability model is placed upon. We asses contagion via copulas by evaluating the dependence change before and after the crisis.

In addition, this work's goal is to propose a method to assess structural break by means of copulas. The way we do this is by judging how different copula dependence parameters are before the crisis than after. We implement a 6-variable copula estimation utilizing four different distributions, Gumbel, Clayton, Rotated Gumbel and Frank. This copula selection tries to account for the possibility of upper tail dependence (Gumbel), lower tail dependence (Clayton) and symmetry (Frank). Estimations were made by rolling windows and by fixing a starting period and then adding one period at a time. We found structural break starting in 2008 as tested by the change in the theta parameter.

Once structural break was tested, we set our goals to assess if contagion took place among the variable associated with the *stability model*. We found contagion in every variable related to *stability*.

Finally, we made an actuarial Frailty Model to study which variables are more important to determine the hazard of having negative GDP growth. Moreover, via Canonical Correlation we found high association between two sets of variables, one containing Real Variables and the other containing Financial Variables.

This work is divided as follows: The first chapter presents a brief copula review containing the basic structures and characterizations that we are going to utilize in our empirical application. The second chapter presents an economic overview on the stability and export led growth model that Mexico is currently using, it presents the mainstream vision and a critique to it, pointing out the frailty factors that arise by following these policies. Finally, the third chapter is an implementation of copula methods to determine the following:

- Whether exists structural change as evaluated by rolling windows copula estimation and by fixing the starting date and then adding one at a time.
- Asses there was contagion or not by assessing copula change in the dependence structure.
- Determine the variables that influence the most to the hazard of having a negative GDP growth rate. Provide a relationship between two sets of variables, one real and the other financial, to get a measure of how related is the financial sector with the real sector. This later section was implemented with canonical correlations.

Chapter I

A Copula Review.

Introduction.

Copulas are functions that link two or more marginal distributions regardless of their initial distribution. This is a powerful characteristic since it allows the possibility of obtaining the dependence structure of several random variables as nature describes it, that is, without the need to apply transformations that forces data into a predetermined distribution. Copulas provide flexibility to the researcher and allow more realism to the probability model with which a phenomenon is captured.

The theory of copulas is relatively a new field of research, being Abe Sklar who set most of its foundations in his seminal work ”Distribution Functions for N-Dimensions and their Margins”[1]. Nevertheless, his work would seem to be forgotten for at least 20 years; apparently, nobody acknowledged the huge problem that this new theory was solving. By late 1980`s, researchers realized the immense potential of application to fields that ranged from Finance and Economics to Biology and Chemistry. The theory was reborn, and researchers all over the world have expanded this theory´s scope and applicability.

One of the problems that was addressed by copula researchers is the following: when trying to model dependence of a couple of random variables, it is often used the Pearson`s correlation coefficient. For two random variables X and Y, this coefficient is defined as:

$$\rho = \frac{cov(X,Y)}{\sqrt{var(x) * var(y)}}$$

Nevertheless, this coefficient can quantify dependence only when the variables exhibit a linear relation, that is, only when the relation among them can be expressed as $Y = a + bX$. For any non-linear relation this coefficient cannot convey an appropriate degree of dependence. In addition, this coefficient is valid only when both distributions are normally distributed, therefore, it provides representative results for elliptical phenomena and so, the range of applications of this dependence measure is somewhat limited. To solve this problem, researchers found a way to calculate dependence re-expressing Kendall´s Tau and Spearman`s Rho in terms of copula parameters, and therefore avoiding the normality problem.

[1] Sklar, Abe. “Fonctions de Répartition à n Dimensions et Leurs Margins”. *Publ. Inst. Statist. Univ.* Paris 8, (1959): 229-231.

Another issue that researchers often face in practical applications is that, when modeling a phenomenon it is not necessarily true that its distributions behavior have a symmetric dependence structure, that is, the variables of interest may have different degrees of dependence when moving in one direction as when moving in the opposite direction. Copula theory is able to consider this information by means of upper and a lower tail dependence coefficients. Finally, since Copulas are functions that allow a variety of densities to be fitted, it opens the possibility to study the problem of heavy tails, that is, extreme outcomes of a phenomenon are captured and modeled by the distribution instead of being considered as outliers.

In this chapter, we will briefly present the general concepts that address the problem of modeling dependence with copulas. Extensive approaches can be found in (Nelsen, 2006), (Manner, 2007) and (Matteis, 2001).

1.1. Preliminaries.

Let X and Y be two random variables and x and y their corresponding realizations. Let $\bar{\mathfrak{R}}$ denote the extended real line, that is, the conventional real numbers with the addition of the $-\infty, \infty$ signs. This is done in order to make the study set a compact set, which means that it can be characterized as closed and bounded. Let $\bar{\mathfrak{R}}^2 = \bar{\mathfrak{R}} \times \bar{\mathfrak{R}}$ be the conventional plane with the addition of the infinitum signs.

Definition 1: Let S_1, S_2 be nonempty subsets of $\bar{\mathfrak{R}}$, and let H be a two place real function such that Dom H = $S_1 \times S_2$. Let B = $[x_1, x_2] \times [y_1, y_2]$ be a rectangle whose vertices are in the Dom H. Then the *H-Volume* of B is given by:

$$V_H(B) = H(x_2, y_2) - H(x_2, y_1) - H(x_1, y_2) + H(x_1, y_1) \ldots..(1)$$

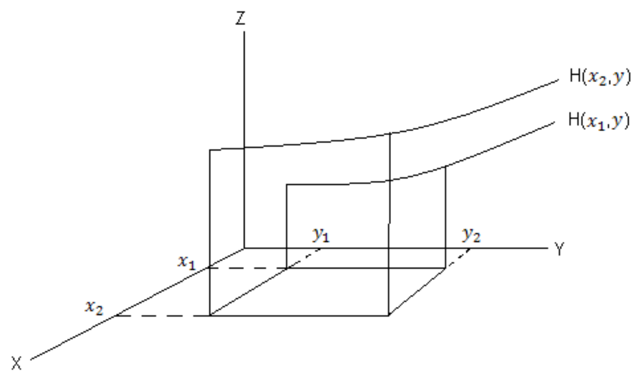
which can be restated as:

$$V_H(B) = H(x_2, y_2) - H(x_2, y_1) - (H(x_1, y_2) - H(x_1, y_1)) \ldots..(2)$$

The definition refers to the variation of a function $f: \mathfrak{R}^2 \rightarrow \mathfrak{R}$ in a particullar way. Let us take a look at the first part of the H-volume definition (1), $H(x_2, y_2) - H(x_2, y_1)$, this difference is similar to a transversal cut of the $f: \mathfrak{R}^2 \rightarrow \mathfrak{R}$ function at the point x_2 as shown in Figure 1. If this difference is greater or equal than zero it means that the function is monotone increasing parallel to the y axis and at the point x_2. The term monotone increasing refers to the possibility that a function continually increases or that it may be constant, but it won´t decrease any time on its interval. The second part is also a transversal cut at x_1 and it will be increasing if the difference $H(x_1, y_2) - H(x_1, y_1)$ is greater or equal than zero.

Figure 1

Transversal section of a function at $\mathbf{x_2}$ **and** $\mathbf{x_1}$.



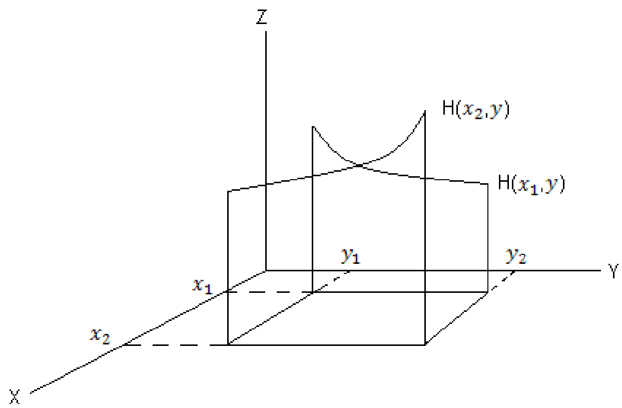
Definition 2: We say that a 2-place real function H is *2-increasing* if $V_H(B) \geq 0$ for all rectangles B whose vertices lie in Dom H.

This definition is clear when considering only one section of the function $H(x, y)$ since it is the typical $\Re^2$ definition of an increasing-monotone function, but what happen when we put these two statements together as they are at (2)? First of all, intuitively, if we expect a function to be strictly increasing in both sections, then by this definition we would have that $H(x_2, y_2) - H(x_2, y_1) > 0$ and that $H(x_1, y_2) - H(x_1, y_1) > 0$, but this last difference must be shortest in magnitude than that of the section at x_2, do not let yourself be blinded by intuition this time since this *2-increasing* concept is a little more elaborated.

Note that (2) may be positive if the difference $H(x_2, y_2) - H(x_2, y_1)$ is positive and greater in magnitude than that of $H(x_1, y_2) - H(x_1, y_1)$ being this last difference also positive. But if $H(x_1, y_2) - H(x_1, y_1) < 0$ then $-(H(x_1, y_2) - H(x_1, y_1)) > 0$, so if $H(x_2, y_2) - H(x_2, y_1) > 0$ meaning that the transversal section at x_2 is strictly increasing, the other section may even be decreasing and still we would have that the $V_H(B) \geq 0$ as shown in Figure 2.

Figure 2

Transversal sections of a 2-increasing function.



The other case is the opposite, when at x_2 the section is decreasing and at x_1 is decreasing, as long as the decrease in the section at x_1 is greater than that of x_2, then $V_H(B) \geq 0$ would still be positive. Therefore, if we would not consider the "for all rectangle" the 2-increasing definition would not imply that the function increases for every direction that could be taken over.

Definition 3: We say that that a function $H : S_1 \times S_2 \rightarrow \Re$ is grounded if

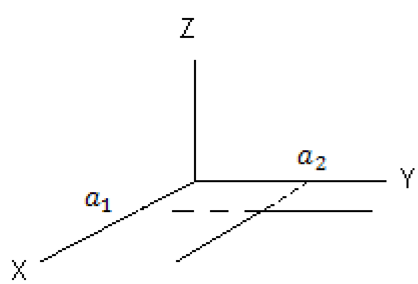
$$H(a_1, y) = 0 = H(x, a_2) \text{ for all } (x, y) \text{ in } S_1 \times S_2.$$

This concept refers to the set of points where the function touches the x-y plane at the constants a_1 in the x axis and a_2 in the y axis just as shown in the Figure 3.

Figure 3

Constants $\boldsymbol{a_1}$, $\boldsymbol{a_2}$ denote the places where the function

H touches the x-y plane.



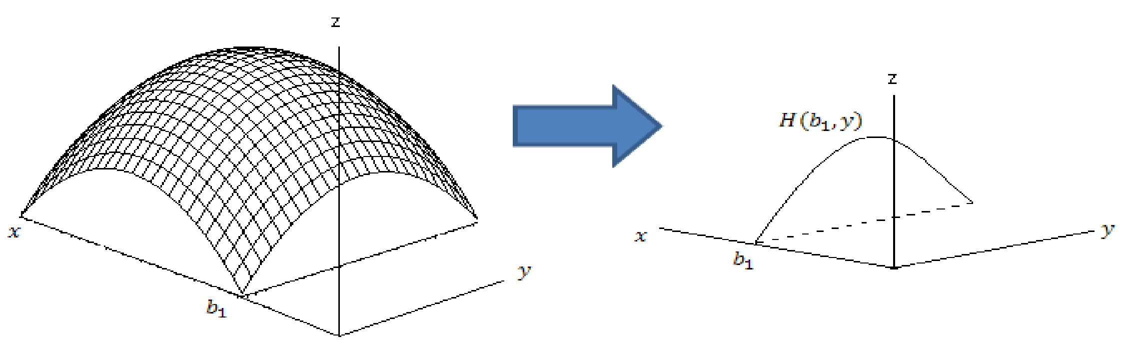
Definition 4: We say that a function $H : S_1 \times S_2 \rightarrow \Re$ has margins if there exists a greatest element b_1 in S_1 and a greatest element b_2 in S_2, and these are given by the following functions F and G.

- $F(x) = H(x, b_2)$ for all x in S_1
- $G(x) = H(b_1, y\,)$ for all y in S_2

The margins resemble the marginals concept in probability, nevertheless, here X and Y are not random variables so it is not exactly the same concept. On the other hand, the margin does represent a transversal cut of a function at the constant $y = b_2$ for F and $x = b_1$ for G as shown in Figure 4. Notice that what the figure is trying to depict, is that margins are primarily traversal cuts. Yet, this figure is not representing that these cuts are made at the domain´s maximum or supremum. The reason is that the current figure´s perspective (describing the domain minimums) is better to show the intended idea; otherwise, we would have to place the cut behind the function, and this is more difficult to spot.

Figure 4

A margin of a function at a point $\mathbf{b_1}$.



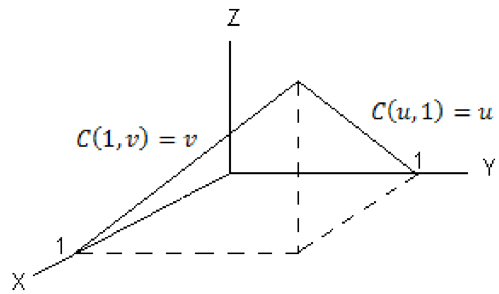
Definition 5: Let I be the interval $[0,1]$, let $I^2 = [0,1]\text{x}[0,1]$, a *2-dimensional copula* is a $C\colon I^2 \rightarrow I$ such that:

- C is grounded 2 increasing.
- Have margins $C(u, 1) = u$ and $C(1, v) = v$

A copula is a function that is defined in a *restricted* domain, and has the characteristic that at the constants $x = 1, y = 1$ the margins are the identity function.

Figure 5

Margins of a copula at x = 1 and y = 1.



Lemma 1. Let S_1 and S_2 nonempty sets of $\overline{\mathfrak{R}}$, and let *H* be a *2-increasing* function with domain $S_1 \times S_2$ .Let $x_1, x_2 \in S_1$ with $x_1 \le x_2$ and let $y_1, y_2 \in S_2$ with $y_1 \le y_2$. Then, the following functions are non-decreasing on S_1 and S_2 respectively.

$$t \mapsto H(t, y_2) - H(t, y_1)$$

$$t \mapsto H(x_2, t) - H(x_1, t)$$

So far, we have not guaranteed that a function may actually be increasing in every direction one could think of. Nevertheless, we have the necessary conditions to accomplish this goal and this is what we are going to do next.

Lemma 2. Let S_1 and S_2 nonempty sets of $\overline{\mathfrak{R}}$, and let *H* be a grounded *2-increasing* function with domain $S_1 \times S_2$. Then h is non-decreasing in each argument.

Proof:

Let k be a fixed quantity in S_2 and let a_2 be the infimum of S_2. Let $x_1, x_2 \in S_1$ with $x_1 \le x_2$. Knowing that *H* is non-decreasing we have,

$$H(x_1, a_2) - H(x_2, a_2) - H(x_1, t) + H(x_2, t) \ge 0$$

Given that *H* is grounded we know $H(x_1, a_2) = 0$ and $H(x_2, a_2) = 0$, therefore substituting in the previous inequality, $H(x_2, t) \ge H(x_1, t)$. Since this function now depends only on x, then it fulfills the single variable increasing definition; hence, it is increasing in each argument.■

An important characterization of copulas is the so-called Fréchet-Hoeffding bound theorem. It refers to functions that encapsulate every possible copula C, meaning that these functions constitute a floor and a ceiling for every copula.

Theorem 1 (Fréchet-Hoeffding Bounds): Let C be a subcopula, for every (u, v) in I^2 and for every copula,

$$Max(u + v - 1,0) \leq C(u, v) \leq Min(u, v)$$

Proof:

Let $(u, v) \epsilon Dom(C)$. We know that C is $2 - inxcreasing$, consequently we have that:

$$C(u, v) \leq C(u, 1) = u \text{ \& } C(u, v) \leq C(1, v) = v.$$

Therefore, any subcopula is below the minimum of any margin, $C(u, v) \leq \min(u, v)$. The other part of the inequality runs as follows. Consider the H-volume of a copula on $[u, 1] \times [v, 1]$, that is:

$$V_c([u, 1] \times [v, 1]) = C(1,1) - C(1, v) - C(u, 1) + C(u, v)$$

Knowing that a copula is limited by an upper bound which is 1 at the point $(1,1)$, provided it is two-increasing, consequently $C(1,1) = 1$. Furthermore, we know that

$$C(1, v) = v \quad \& \quad C(u, 1) = u$$

Consequently, joining all the pieces of information we have:

$$V_C([u, 1] \times [v, 1]) = 1 - v - u + C(u, v) \geq 0$$

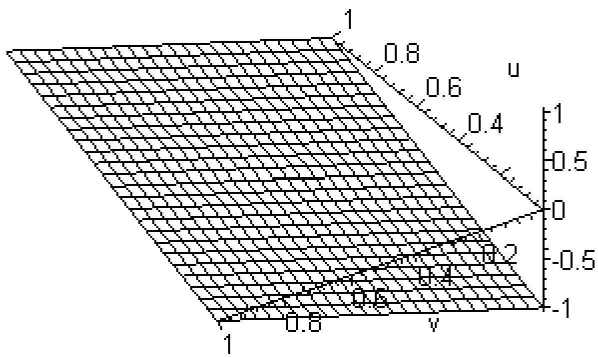
Solving for $C(u, v)$ we have, $-1 + v + u \leq C'(u, v)$. Finally, since any copula is greater or equal than zero we get to:

$$\max(u + v - 1,0) \leq C`(u, v). \blacksquare$$

Let´s take a look to the first part of the inequality, $Max(u + v - 1, 0) \leq C(u, v)$. First we should ask ourselves, what set could it represent? if the maximum is $u + v - 1$ then we would have $z = u + v - 1$ in a (u, v, z) space. This equation represents a plane displaced one unit from the origin to -1 and with positive identity slopes on u and v as is shown in Figure 6.

Figure 6

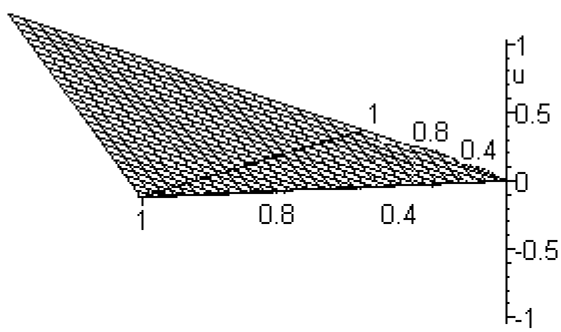
Plane $\mathbf{z = u + v - 1}$



But then again, we should ask ourselves for which values of (u, v), the set $z = u + v - 1$ could be the maximum when compared with $z = 0$? Note that all graph below the plane $z = 0$ is actually less than 0, when it is calculated the maximum between 0 and the $set\ z = u + v - 1$ the maximum is 0. Therefore, all this section do not correspond to the graph. In all, the interest set can be identified within the diagonal from $u = 1$, $v = 1$ and $u < 1$ and $v < 1$ and all the points at $z = 0$. This function of sets is the lower bound as shown in the Figure 7.

Figure 7

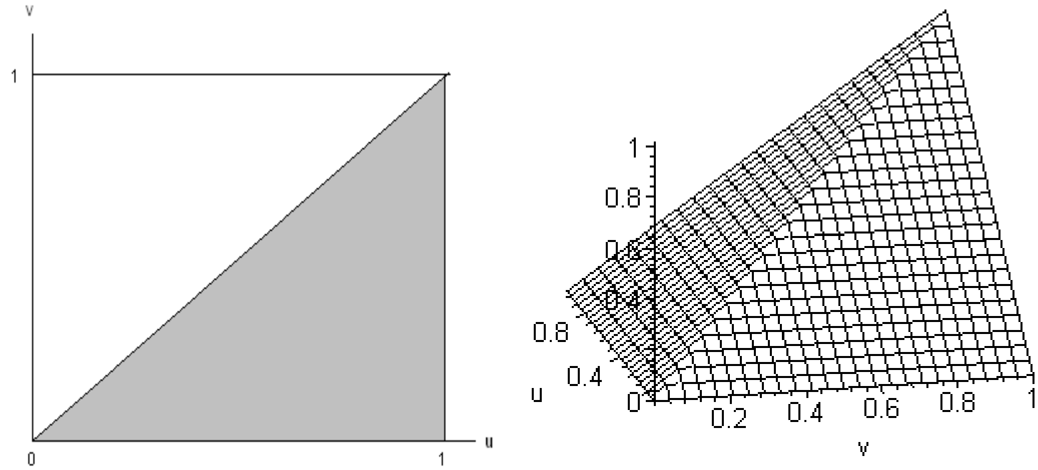
The lower bound provided by $\mathbf{Max(u + v - 1, 0)}$



The next set of interest is the upper bound given by $min(u, v)$. Let us first analyze what happens in the u-v plane by drawing the $u = v$ line. In the shaded region of Figure 8 panel a) any u component is larger than that of v, and in the non-shaded region, any v component is larger than that of u. In the first case, the minimum among u and v is v, so above the shaded region the set to be graphed is $z = v$ which is a *diagonal* plane. The same reasoning applies for the points in the shaded region, the set to be graphed is a *diagonal* plane $z = u$, combining this information for both, when $u < v$ and when $v < u$ we have the figure shown in panel b).

Figure 8

The upper Fréchet- Hoeffding Bound



Panel a) Panel b)

Theorem 2: Let S_1 and S_2 be non-empty subsets, and let H be a grounded-2increasing function with margins, whose domain, is $S_1 \times S_2$. Let $(x_1, y_1), (x_2, y_2)$ lie in $S_1 \times S_2$

$$|H(x_2, y_2) - H(x_1, y_1)| \leq |F(x_2) - F(x_1)| + |G(x_2) - G(x_1)|$$

Proof,

Using the triangle inequality we have,

$$|H(x_2, y_2) - H(x_1, y_1)| \leq |H(x_2, y_2) - H(x_1, y_2)| + |H(x_1, y_2) - H(x_1, y_1)|$$

Knowing that h is 2 increasing and also, increasing in each argument it can be inferred that,

$$0 \leq H(x_2, y_2) - H(x_1, y_2)$$

Given that, (x_2, y_2) and (x_1, y_2) are not always at the boundaries of $S_1 \times S_2$ then we can infer that as points get closer to the margins we get to,

$$0 \leq H(x_2, y_2) - H(x_1, y_2) \leq F(x_2) - F(x_1)$$

Similarly, we get the same when $x_2 \leq x_1$, as a consequence, for any $x_1, x_2 \in S_1$

$$|H(x_2, y_2) - H(x_1, y_2)| \leq |F(x_2) - F(x_1)|$$

The other inequality follows immediately. ■

Theorem 3: Let $C(u, v)$ be a copula, then for every $(u_1, u_2), (v_1, v_2)$ lie in the copula domain, then,

$$|C(u_2, v_2) - C(u_1, v_1)| \leq |u_2 - u_1| + |v_2 - v_1|$$

Proof, it follows from the preceding theorem.

This means that the copula is uniformly continuous on its domain. The next theorem characterizes the slope of a copula. It is defined in terms of measure theory, according to which, a property holds *almost everywhere* if the set where it does not hold, have Zero *Lebesgue measure*.

Theorem 4: Let C be a copula, for any v in I, $\frac{\partial C(u,v)}{\partial u}$ exists for almost every u and $\frac{\partial C(u,v)}{\partial v}$ exists for almost every v and for such v and u we have,

$$0 \leq \frac{\partial C(u,v)}{\partial u} \leq 1 \text{ and } 0 \leq \frac{\partial C(u,v)}{\partial u} \leq 1$$

The functions, $u \mapsto \partial C(u, v)/\partial v$ & $u \mapsto \partial C(u, v)/\partial v$ are defined and are non-decreasing almost everywhere on [0,1]

Proof:

The copula partial derivatives exist given that monotone functions are differentiable almost everywhere (in the sense of measure theory). We now that,

$$|C(u_2, v_2) - C(u_1, v_1)| \leq |u_2 - u_1| + |v_2 - v_1|$$

setting $v_1 = v_2$ we have,

$$|C(u_2, v_1) - C(u_1, v_1)| \leq |u_2 - u_1|$$

Therefore,

$$\frac{|C(u_2, v_1) - C(u_1, v_1)|}{|u_2 - u_1|} \leq 1$$

$$\lim_{u_2 \to u_1} \frac{|C(u_2, v_1) - C(u_1, v_1)|}{|u_2 - u_1|} \leq \lim_{u_2 \to u_1} 1$$

Hence, $$\frac{\partial C(u,v)}{\partial u} \leq 1$$

Given that copulas are always greater or equal then zero, the lower bound of the above inequality follows immediately. Moreover, the other case is analogous. Furthermore, functions $u \mapsto \frac{\partial C(u,v)}{\partial v}$ and $v \mapsto \frac{\partial C(u,v)}{\partial v}$ are well defined and are non-decreasing almost everywhere in the interval $[0,1]$ ■ This theorem implies that copulas have relatively smooth slopes and are constant or increasing on the section where the partial derivate is calculated.

The next definitions, although they will resemble those of probability theory, they won`t actually involve any form of a random variables.

Definition 6. We define a distribution function as a function F with domain in $\bar{\mathfrak{R}}$ such that:

- F is non-decreasing
- $F(-\infty) = 0 \; and \; F(\infty) = 1$

Particular attention is needed to the concept of non-decreasing function. A function $f: \mathfrak{R} \to \mathfrak{R}$ is said to be increasing if given $x_1 < x_2$, then $f(x_1) < f(x_2)$, that is, the function is constantly increasing. A non-decreasing function does not necessarily increases continually, it can remain constant or at certain intervals being constant and others being increasing. This condition is expressed as $f(x_1) \leq f(x_2)$ allowing the possibility of equality.

Definition 7. Let H be a function with domain in $\bar{\mathfrak{R}}^2$, a joint distribution H is a function such that:

- H is 2-increasing,
- $H(x, -\infty) = H(-\infty, y) = 0$ and $H(-\infty, \infty) = 1$

The concept of a 2-increasing function has been already addressed. The second part of the definition may seem a little awkward. When the function H is evaluated at $y = -\infty$ leaving x as a free variable, we are calculating a copula section, i.e. we are taking $-\infty$ not as a concept but as a symbol. $H(x, -\infty) = 0$ means that at $y = -\infty$, the function H is equal to 0, so this could be thought as of a grounded function. In addition, this definition resembles that of an accumulated probability distribution function, which tends to 0 as x tends to minus infinity and have a value of 1 when tending to infinity. The margins of the function H in this case are given by $F(x) = H(x, \infty)$ and $G(y) = H(\infty, y)$.We now turn to the most important characterizations of copula theory, the Sklar´s theorem.

Theorem 5, Sklar (1959). Let H be a joint distribution function with margins F and G, then there exists a copula C such that for all x,y in $\bar{\mathfrak{R}}$,

$$H(x, y) = C(F(x), G(y))$$

If F and G are continuous, then C is unique; otherwise, C is uniquely determined on $Ran(F)xRan(G)$. Conversely, if C is a copula and F and G are distribution functions, then the function $H(x, y) = C\big(F(x), G(y)\big)$ is a joint distribution function with margins F and G.

Proof: See Nelsen (2008)

The first part of the theorem states that if F and G are margins of H, the initial joint distribution function could have been written in terms of F and G as inputs to the copula. The second part of the theorem is quite useful and relates the theory of copulas with real life problems. If the researcher has only the margins of a distribution F and G, and wishes to glue them into a single distribution function, it was complicated to get it when F and G had the different distributions; with copulas, it does not matter the margins' initial distribution for copulas can *mix* both margins in a joint function. Although Sklar´s theorem guarantees uniqueness upon continuity, in practice we actually will see that the best idea is to fit a series of copulas to data and then to choose the one with the best goodness-of-fit coefficient.

Sklar's theorem states that a copula function exists, but so far, it doesn't say how to find it. To find the copula it is first necessary to have the joint function H and the margins F and G, by inverting the margins and substituting these inverses at H is the way to get the copula. To accomplish this idea, a concept of inverses is needed.

Definition 8: Let F be a distribution function, it is said that $F^{(-1)}(t)$ with domain I is a quasi-inverse function of F if:

- t is in $RanF$, then $F^{-1}(t)$ is any number x in $\bar{\mathfrak{R}}$ such that $F(x) = t$ i.e. $\forall t \in RanF$,

$$F\left(F^{(-1)}(t)\right) = t;$$

- if t is not in $RanF$, then

$$F^{(-1)}(t) = \inf\{x: F(x) \geq t\} = \sup\{x: F(x) \leq t\}$$

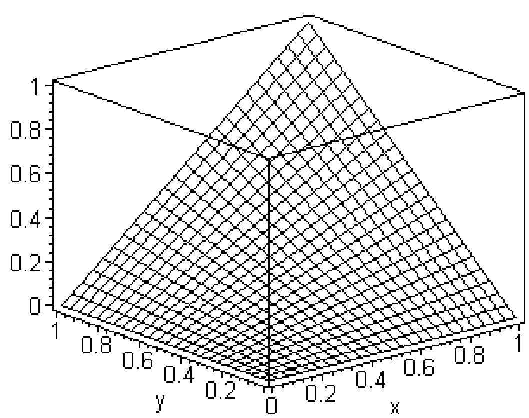
If F is increasing it only has one quasi inverse, the ordinary inverse F^{-1}. If we consider t as the significance in a probability distribution denoted by α, then the function F^{-1} becomes a quantile function.

Independence in copula terms it is also defined and is very important to our further empirical study. Its definition also resembles that of probability theory.

Definition 9: Let X and Y be continuous random variables. Then X and Y are independent if and only if $C_{XY} = \Pi(x,y) = xy$ it`s graph is plotted at Figure 9.

Figure 9

The product copula.



(Matteis, 2001) state one important characteristic about copulas which is that it can be expressed in terms of uniforms distributions. This proposition is useful since it helps us to transform a random variable into a uniform (statement two) and if we have a function in terms of the uniform valuated at the quasi-inverse, then we can recover the original function.

Proposition 1: Let X be a random variable with distribution F and let F^{-1}be a quantile function of F, then:

1. For any standard uniformly distributed $U \sim U(0,1)$ we have that $F^{-1}(U) \sim F$.

2. If F is continuous then the random variable $F(X) \sim U(0,1)$.

Proof:

Let´s calculate the U variable moment generating function:

$$m_x(t) = E(e^{tU}) = E\left(e^{tF(x)}\right) = \int_{-\infty}^{\infty} e^{tF(x)} f(x)dx$$

$$e^{tF(x)} = \int_{-\infty}^{x} \frac{d\left(e^{tF(x)}\right)}{dx} dx = t\int_{-\infty}^{x} e^{tF(x)} f(x)dx$$

$$m_u(t) = \lim_{x \to \infty} \int_{-\infty}^{x} e^{tF(x)} f(x)dx = \frac{e^{tF(x)}}{t} \Big|_{-\infty}^{\infty} = \frac{e^t - 1}{t}$$

Which is the uniform moment generating function along $[0,1]$. ■

Corollary 1 (Sklar, 1959): Let H be a joint distribution function with margins F and G all of them defined at I, let $F^{(-1)}$ and $G^{(-1)}$ be quasi-inverses of F and G. then for any (u,v) in $I \times I$ we have:

$$C(u,v) = H(F^{(-1)}(u), G^{(-1)}(v))$$

Proof: (Nelsen, 2006)

This result is absolutely relevant since it provides a mechanism for constructing copulas knowing the margins and the joint distribution function. For example if a researcher have the distribution of a random variable like the interest rate, and also have the distribution of another random variable like the exchange rate, and wishes to study the dependence structure among them, the researcher has now a way to link these distributions.

Although there has been no mention of random variables, every result established so far applies evenly for them. An interesting characterization regards the application of a transformation to copulas when these are difficult to deal or does not fit the application of interest. It is possible to modify a copula applying a strictly increasing transformation and still conserve the basic structure; this is stated in the following theorem.

Theorem 6: Let X and Y be continuous random variables with copula C_{xy}. If α and β are strictly increasing on $Ran(X)$ and $Ran(Y)$ respectively, then $C_{\alpha(X)\beta(Y)} = C_{XY}$ is invariant under strictly increasing transformations of X and Y.

Proof: See (Nelsen, 2006)

1.2. The study of dependence.

Random variables' dependence structure, refers to the degree of association between them and the way they are associated. Initially, it is needed a way of measuring that dependence such that the measure is scale invariant. Let us remember that Pearson correlation coefficient accomplishes this task by measuring the direction and the strength of the dependence in an invariant scale way.

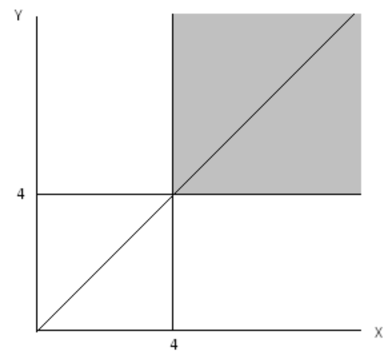
The first concept that is needed to achieve the goal of getting a scale invariant dependent measure is the so-called concordance. It is said that given X and Y random variables, they are concordant if large values of one are associated with large values of the other and vice versa. The idea of concordance tells us that the corresponding outcomes of the two random variables cannot stay away from the identity line x $= y$. Let us introduce the formal definition.

Definition 10: Let X and Y be two random variables and consider the vector (X, Y), let (x_i, y_i) and (x_j, y_j) realizations of those random variables, we say that these realizations are concordant if $x_i < x_j$ and $y_i < y_j$. Similarly these realizations are discordant if $x_i < x_j$ and $y_i > y_j$.

The concordance definition refers to concordance among two pairs so, if one of them is fixed at for example (4,4), then all the shaded region in Figure 10 contains points for which the x component is larger than 4 and for which the y component is larger than 5, that is, it contains all the points concordant with (4,4). Nevertheless, this region doesn´t exactly encompasses only values close to the identity line. Under this definition, one possible value that is concordant with (4,4) is (5,2000) which is far above.

Figure 10

Concordance region with respect to (4,4)



An alternate formulation of concordance is the following:

Definition 11: Let X and Y be two random variables, (x_i, y_i) and (x_j, y_j) are said to be concordant if and only if $(x_i - y_i)(x_j - y_j) > 0$ and discordant if $(x_i - y_i)(x_j - y_j) < 0$.

Popular dependence measures are the Kendall`s tau and Spearman`s rho. Nevertheless they are not the only ones that are going to be of our interest.

1.2.1 Kendall`s Tau

The sample version is defined in terms of concordance. Let $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ be a random sample of n observations, let c denote the number of concordant pairs and d the number of discordant pair among all possible combinations, then the Kendall`s Tau for the sample is defined as follows:

$$t = \frac{c - d}{c + d}$$

The population version is defined as:

$$\tau = P\big((X_1 - X_2)(Y_1 - Y_2) > 0\big) - P\big((X_1 - X_2)(Y_1 - Y_2) < 0\big)$$

Observe that if the probability of concordance is equal to one and the probability of discordance is zero, then Kendall`s tau will have a value of 1. If the opposite occurs, then the Kendall`s tau will have a value of -1. Therefore, the range of values which this coefficient can take is $[-1, 1]$, implying that when it`s equal to 1 the concordance is high and vice versa.

Kendall`s Tau measures the strength of the relationship and also measures its direction. In this specific sense, it is quite alike the Pearson´s correlation coefficient. Another dependence measure is the Spearman´s Rho, it is also based on the concordance concept.

1.2.2 Spearman`s Rho

Letting(X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) be independent random vectors with a common joint distribution function H with margins F, G and copula C. The population version of Spearman´s Rho is the probability of concordance minus the probability of discordance weighted by a constant. Note that a difference between the Kendall´s tau and Spearman`s rho is that the Kendall`s assume that observations comes from two independent and identically distributed random vectors whilst the Spearman`s rho assumes that observations come from three independent random vectors. Kendall`s Tau measure the concordance among vectors while the Spearman`s rho will attempt to measure the concordance among a combination between these three vectors. The population version is given by:

$$\rho_{X,Y} = 3(P[(X_1 - X_2)(Y_1 - Y_3) > 0] - P[(X_1 - X_2)(Y_1 - Y_3) < 0])$$

Without any normalization, this coefficient can range between 3 and -3 being 3 the value denoting high-perfect concordance and -3 for complete discordance. The properties of the Kendall`s tau and Spearman`s Rho are summarized in the following theorem as stated by (Matteis, 2001).

Theorem 7. Let X and Y be random variables with continuous distributions F_1 and F_2, joint distribution F and copula C, then:

1. $\rho_S(X,Y) = \rho_S(Y,X)$ and $\rho_t(X,Y) = \rho_t(Y,X)$
2. If X and Y are independent then $\rho_S(X,Y) = \rho_t(X,Y) = 0$
3. $-1 \le \rho_S(X,Y), \rho_S(X,Y) \le 1$
4. For $T: \mathfrak{R} \rightarrow \mathfrak{R}$, strictly monotonic on the range of X, both measures satisfy:

$$\rho(T(X),Y) = \begin{matrix} \rho(X,Y) \; if \; T \; is \; increasing \\ -\rho(X,Y) \; if \; T \; is \; decreasing \end{matrix}$$

1.2.3 Upper and Lower Tail Dependence

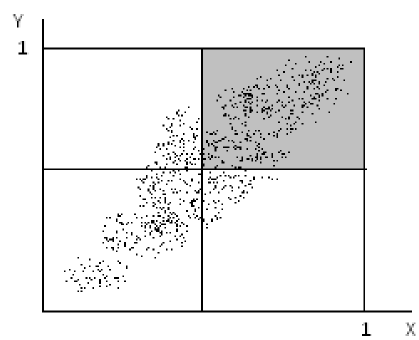
Another dependence measure regards the idea of measuring the degree of dependence for two random variables whenever large values of each random variable occur more often than not i.e. these coefficients will try to measure the degree of dependence among random variables' tails; notice that tail dependence is a relevant concept when the joint distribution is not symmetric. For a random vector with marginal functions F and G, the Upper tail dependence coefficient is defined as follows.

$$\lim_{u \to 1} P(Y > G^{-1}(u) \setminus X > \ F^{-1}(u)) \quad = \lambda_L$$

If the limit does exists then intuitively the researcher would be looking for the probability that Y is greater than the quasi-inverse *evaluated at one*, given that the quasi inverse function of F is evaluated at 1. Remembering that these functions are each constrained to the interval $[0,1]$ then the definition could be rephrased as, the researcher is looking for large values of Y given that there exists *large* values of X. When this probability is close to one or one, it would confirm the existence of large values of both random variables. Then again as (Nelsen, 2006) states, since these functions are restricted to the unit square, then an upper tail dependence imply that the values of each random variable lies in the shaded quadrant of Figure 11.

Figure 11

Region where upper tail dependence value lies.



The random variables exhibit upper tail dependence if the following limit exists

$$\lim_{u \to 0} P(Y \leq G^{-1}(u) \setminus X \leq \ F^{-1}(u)) \quad = \lambda_L$$

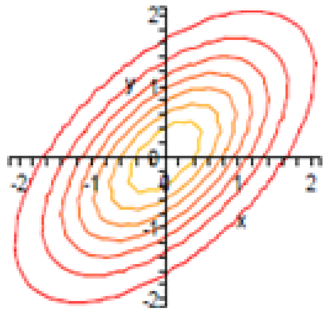
At this point, having the instruments to measure dependence, it would be appropriate to present some of the most useful families of copulas.

1.3. Copula Families.

Copula families can be classified as Elliptic and Archimedean. The former refers to symmetric copulas, such as the one in Figure 12.

Figure 12

An Elliptic Copula.



1.3.1 Gaussian Copula.

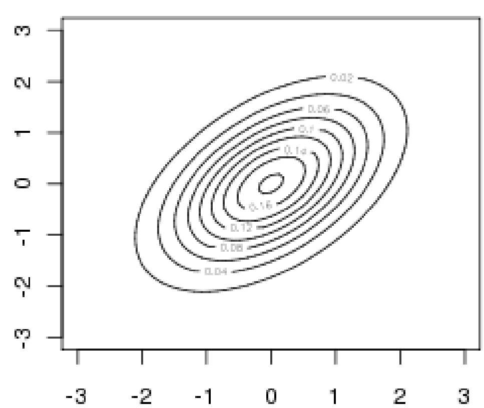
This is a one parameter family of pairs of random variables (X_1, X_2) is given by:

$$C_\rho^{G_\alpha}(x_1, x_2) = \int_{-\infty}^{\Phi^{-1}(x_1)} \int_{-\infty}^{\Phi^{-1}(x_2)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \exp\left(\frac{-(s^2 - 2\rho st + t^2)}{2(1-\rho^2)}\right) dsdt$$

ρ represents the usual Pearson correlation coefficient between random variables X_1 and X_2 and Φ is the typical single variable normal distribution, Matteis (2001). The contour levels for a gaussian copula with correlation equal to ½ is shown at Figure 13.

Figure 13

Level curves of a Gaussian copula.



From the figure, it can easily been noticed that its values distribute in a way such that, extreme values of both random variables are reached simultaneously. Nevertheless it can be said that this type of events correspond to phenomena for which extremes movements are not atypical.

The second family of copulas is the Archimedean family. Copulas belonging to this family are not constructed from the Sklar theorem; they are in general, non-symmetric and are derived from a Laplace transformation. Since they are not symmetric, its construction allow the researcher to model asymmetric dependence, i.e. to take into account the possibility that a pair of random variables depend differently when move together in one direction as when they move in the opposite direction. The most common copulas are the following:

1.3.2 Clayton Copula.

Its generator function is given by:

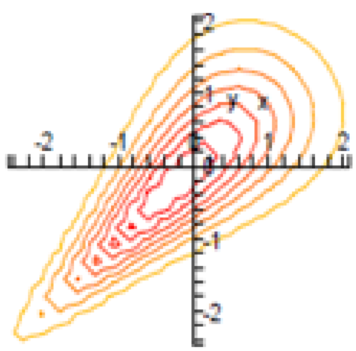
$$\varphi(t) = \frac{(t^{-\theta} - 1)}{\theta}, such\ that\ \theta \epsilon [-1, \infty] \backslash 0$$

Its density is given by:

$$C(u, v) = \max[(u^{-\theta} + u^{-\theta} - 1), 0]$$

Figure 14

Level curves of a Clayton Copula.



The most relevant aspect of Figure 14 is that it is clear that both random variables do not depend in the same way when they both increase as when they both decrease.

1.3.3 Gumbel Copula.

Its generator function is given by:

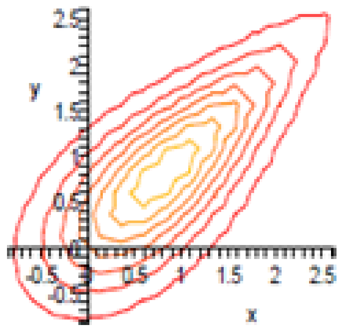
$$\varphi(t) = (-\ln((t)^{\theta})) \text{ with } \theta \geq 1$$

Its density is given by:

$$C = \exp(-[(-\ln(u))^{\theta} + (-\ln(u))^{\theta}]^{1/\theta})$$

Figure 15

Level curves of a Gumbel Copula.



Asymmetric dependence is also a characteristic of this copula.

1.3.4 Frank Copula.

Its generator function is given by:

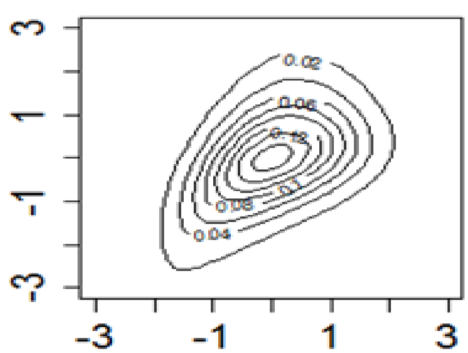
$$\varphi = -\ln\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right) \text{ con } \theta \neq 1$$

Its density is given by:

$$C = -\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$$

Figure 16

Level curves of a Frank Copula.



This type of copula is in general non-symmetric although an appropriate parameter specification can lead to symmetry.

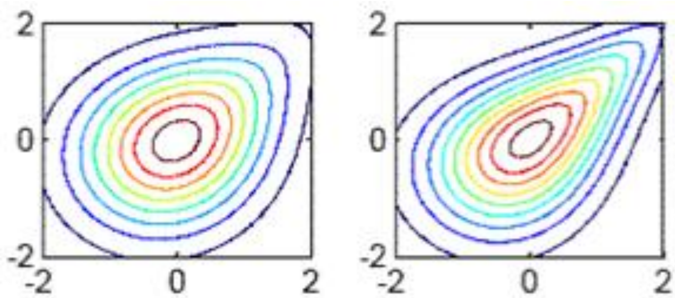
1.3.5 Joe-Clayton Copula.

Its generator function is given by:

$$C = 1 - (1 - \left(\left(1 - (1-u)^k\right)^{-\gamma} + \left(1 - (1-v)^k\right)^{-\gamma} - 1\right)^{-\frac{1}{\gamma}})^{\frac{1}{k}}$$

Figure 17

Level curves of a Joe-Clayton Copula.



The same as before, this type of copula is in general non symmetric although an appropriate parameter specification cal lead to a symmetric one

1.3.6 Characterizations for Archimedean Copulas.

According to (Matteis, 2001) there are two important theorems that distinguish Archimedean Copulas that are worthy of mentioning.

Theorem 8 (*Ling*): The Archimedean copulas are exactly the copulas $C(u,v)$ that satisfy the following properties:

1. $C(u,v)$ is associative in the copula sense, thus for all u, v, w $\in [0,1]$ we have $C(C(u,v),w) = C(u,C(v,w))$.
2. $C(u,u) < u$ for all $u \in (0,1)$.

Proof see (Ling, 1965).

Theorem 9 (*Abel´s Criterion)*: A copula C is an Archimedean Copula if it is twice differentiable and if it there exists an integrable function $f: (0,1) \to (0, \infty)$ such that:

$$f(v) \frac{\partial}{\partial u} C(u, v) = f(u) \frac{\partial}{\partial v} C(u, v) \; \forall \; 0 < u, v < 1$$

For a formal proof see (Matteis, 2001).

1.4. Estimation.

The process of estimating copula parameters can be performed by a one step and a two steps approach and both of them can be divided into a parametric and a non-parametric method. A parametric approach relies on the use of estimation parameters via the maximum likelihood method. In contrast, a non-parametric approach relies on the use of a procedure described at (Genest & Rivest, 2003). According to Manner (2007), there exist mainly five methods to perform copula estimation via the parametric approach. A glance to these methods is presented next.

1.4.1 Exact Maximum Likelihood Method (EML).

This is a one-step method estimates all the parameters at the same time. Letting $\theta \in \Theta$ be a parameter vector to be estimated which can be splitted as follows:

$$\theta = [\varphi', \gamma', \delta']'$$

Where $\varphi \in \phi$ denotes the parameters of $f(x)$, $\gamma \in \Gamma$ denotes the parameters of $g(y)$ and $\delta \in \Delta$ denotes the parameters of $c(F(x), G(y))$. Assuming that it is to be observed a sample x_t and y_t for $t = 1, \ldots, T$. The following representation is called the copula decomposition:

$$h(x_t, y_t\,; \theta) = f(x_t; \varphi) g(y_t; \gamma) c(F(x_t; \varphi), G(y_t); \delta)$$

The likelihood function is given by the next equation:

$$L_{X,Y} = \prod_{t=1}^{T} h(x_t, y_t\,; \theta) = \prod_{t=1}^{T} f(x_t; \varphi) g(y_t; \gamma) c(F(x_t; \varphi), G(y_t); \delta)$$

The product operator can distribute in the following way:

$$L_{X,Y} = \prod_{t=1}^{T} h(x_t, y_t\,; \theta) = \prod_{t=1}^{T} f(x_t; \varphi) \prod_{t=1}^{T} g(y_t; \gamma) \prod_{t=1}^{T} c(F(x_t; \varphi), G(y_t); \delta)$$

now applying natural logarithm:

$$\mathcal{L}_{\mathrm{X,Y}} = \ln(L_{X,Y}) = \sum_{t=1}^{T} h(x_t, y_t\,; \theta) = \sum_{t=1}^{T} f(x_t; \varphi) \sum_{t=1}^{T} g(y_t; \gamma) \sum_{t=1}^{T} c(F(x_t; \varphi), G(y_t); \delta)$$

which can be restated as:

$$\mathcal{L}_{\mathrm{X,Y}} = \mathcal{L}_{\mathrm{X}}(\varphi) + \mathcal{L}_{\mathrm{Y}}(\gamma) + \mathcal{L}_{\mathrm{C}}(\varphi, \gamma, \delta)$$

This is only the likelihood function which has to be optimized in order to get the maximum likelihood estimate of the parameter. When obtained, the estimator is given by:

$$\hat{\theta} = \arg\max \mathcal{L}_{XY}$$

This estimator is fully efficient.

1.4.2 Parametric two-step estimator (IFM).

Using a joint distribution function, in the first step the parameters of φ and γ of the marginal densities are estimated by maximum likelihood estimation to obtain:

$$\hat{\varphi} = argmax(\mathcal{L}_x)$$
$$\hat{\gamma} = argmax(\mathcal{L}_y)$$

With this estimates, the marginals are transformed into a functions distributed as uniform(0,1). At this point it is possible to estimate the copula parameters δ by maximizing the copula density:

$$\hat{\delta} = \mathrm{argmax}(\mathcal{L}_c(\hat{\varphi}, \hat{\gamma}, \delta))$$

This estimator is consistent, asymptotically normal and is easier to estimate, allows the two series x_t and y_t to have different lengths.

1.4.3 The semi-parametric two-step estimator (CML).

This is probably one of the most unrestrictive methods since it can be applied when the margins are unknown. At the beginning, the series of interest are transformed into uniform variables using the so-called probability integral transform. This transformation can transform data into uniform, regardless of the distribution of the random. The empirical distribution is defined as:

$$\hat{F}(\cdot) = \frac{1}{T}\sum_{t\,=\,1}^{T}\mathbf{1}_{\{x_{t\leq}\cdot\}}$$

Where $\mathbf{1}_{\{x_{t\leq}\cdot\}}$ is the indicator function. Let us remember that the indicator function for this case is defined as 1 if $x_t \leq a$ where a is a defined parameter, and 0 otherwise. Since the function depends on t, the whole function is multiplied by a constant $\frac{1}{T}$ so the indicator function is transformed as $\frac{1}{T}$ if $x_t \leq a$ and 0 otherwise which resembles the representation of a uniform function.

The copula parameter δ can be estimated maximizing the loglikelihood function of the copula density given by:

$$\mathcal{L}_C(\delta) = \sum_{t\,=\,1}^{T}\ln\left(c\left(\hat{F}(x_t), \hat{G}(y_t)\right)\right) = \sum_{t=1}^{T}\ln(c(\hat{u}_t, \hat{v}_t))$$

The function $\hat{G}(y_t)$ is built the same way as $\hat{F}(x_t)$, as an indicator function. The semi parametric estimator is:

$$\delta = \arg\max \mathcal{L}_C(\delta)$$

which is consistent and normally asymptotic and converges to the pseudo true parameter in the case of a wrongly specified copula.

1.4.4 The nonparametric estimator.

This method is based in the idea of concordance. The empirical copula is given by:

$$C_n\left(\frac{i}{T},\frac{j}{T}\right) = \frac{\text{númber of pairs } (x_t, y_t) \text{ in the sample such that } x_t \leq x_i \;\&\; y_t \leq y_j}{T}$$

If it is needed the nonparametric estimator given by Genest and Rivest we have that, as in the semi parametric two step method, the marginal distributions need not to be specified. It is defined in terms of the concordant (c) and discordant (d) pairs to calculate the Kendall`s tau:

$$\hat{\tau} = \frac{c - d}{c + d}$$

If there would not exist a closed form to the copula, it is possible to estimate the copula parameter using the general form of Kendall`s tau for Archimedean copulas which takes the form:

$$\tau_C = 1 + 4\int_0^1 \frac{\varphi(t)}{\varphi'(t)}dt$$

1.4.5 Model Selection.

Having fitted a series of copulas to data, the problem is to how to discriminate among competing models. A way to know which discriminate coefficient is the best to choose a model is by assessing their efficiency by using simulation. This is s suitable technique since each discrimination coefficient responds different under different circumstances. (Manner, 2007) performed simulation studies to assess the goodness of fit under a wide range of circumstances.

The study was divided in two parts: In the first, the properties of estimation techniques where tested when marginals are well specified and when they aren´t. In the second, he compares methods for selecting competing models. The tested models where IFC, EML, CML and MM standing for the non-parametric estimator based on Kendal`s tau abbreviating *Method of Moments*. Data where simulated from a Clayton Copula. Theoretically, the most efficient estimators ranked in ascending order are as follows.

1. EML
2. IFM

3. CML
4. MM

Manner´s results show that EML is the best, but for weak dependence structures the IFM is better. For correctly specified models, the MM is the worst. The CML is not as efficient as the fully parametric methods. When the marginals are misspecified the IFM estimator is superior to EML but worse to the CML. When margins are misspecified it is recommended to use CML.

Until this point, we have presented the theory that we will use on the empirical application, we now turn our attention to the limitations and criticisms that this theory have received, the main accusations of this theory refers to the use of Gaussian copulas to price on of the most controversial financial instruments that detonates the 2008 crisis, the CDO.

1.5. Time Varying Copulas.

For some economic variables, it is true that their variability may be quite unstable with a dependence structure which may change over time. This is the problem that addresses this section.

The first theoretic piece of information that we require refers to conditioning copula model. The most important advancement was given by Patton (2004), accordingly given X and Y random variables, these will be assumed to be conditioned to a variable W. The joint distribution of (X,Y,W) is H^*, and the conditional distribution of (X,Y) constraint to W as H. If conditional marginal distributions of X/W and Y/W are denoted by *F* and *G* then:

$$F(x/w) = H(\,x, \infty/w)$$
$$G(y/w) = H(\,\infty, y/w)$$

F, G and H are continuous and differentiable. W may be a vector of random variables but for the time being, as in Patton (2004) W will be a single random variable.

The conditional bivariate distribution of $(X,Y)/W$ is:

$$H(x,y/w) = \; f_w(w)^{-1} \cdot \frac{\partial H^*(x,y,w)}{\partial w} \text{ for } w\epsilon\mathcal{W}$$

Where f_w is the unconditional density of W and $\mathcal{W}$ is the closure of W.

Definition 12: Let X, Y and W be random Variables, the conditional Copula of $(X,Y)/W$, where $X/W \sim F$ and $Y/W \sim G$, is the conditional function of $U \equiv F(X/W)$ and $V \equiv G(Y/W)$ given W where U and V are the probability integral transforms necessary to induce a random variable into a $U(0,1)$. Some interesting properties for the two dimensional cases are the following:

- The copula function has a domain of the form (u, v, w) in which the first and second components are a value between 0 and 1 and the third component takes a value in the closure of the set $\mathcal{W}$, the image of the function continues to be in the [0,1] interval.

$$C: [0,1] \times [0,1] \times \mathcal{W} \to [0,1]$$

- The grounding property remains as in regular copulas, that is, all along the *axes*, the copula function worth 0 as in Figure 1.3 resembling the original copulas.

$$C(u, 0/w) = C(0, v/w) = 0, for\ every\ u\ and\ v\ in\ [0,1] and\ each\ w\ in\ \mathcal{W}$$

- Cutting with a perpendicular plane to the v axis at $v = 1$ we have that the cut represents the identity function as in Figure 5 and the same is true for a cut through the u axis, formally:

$$C(u, 1/w) = u\ and\ C(1, v/w) = v, for\ every\ u\ and\ v\ in\ [0,1]\ and\ each\ w \in \mathcal{W}$$

- There is a concept of similar to that of H-volume only that in this case, the function is constrained to $\mathcal{W}$.

$$V_C([u_1, u_2] \times [v_1, v_2]/w) \equiv$$
$$C(u_2, v_2/w) - C(u_1, v_2/w) - C(u_2, v_1/w) + C(u_1, v_1/w) \geq 0$$

$$for\ all\ u_1, u_2, v_1, v_2 \epsilon [0,1], such\ that\ u_1 \leq u_2, v_1 \leq v_2\ and\ each\ w \in \mathcal{W}$$

Now, the most important theorem in the copula theory, the Sklar´s theorem in the conditional version.

Theorem 10 (Sklar): Let F be the conditional distribution of X/W, G be the conditional distribution of Y/W, H be the joint conditional distribution of $(X,Y)/W$ and $\mathcal{W}$ the closure of W. Assume that F and G are continuous in x and y. Then there exists a unique conditional copula C such that

$$H(x, y/w) = C(F(x/w), G(y/w)/w)$$

$\forall\ (x, y) \in \bar{\mathfrak{R}} \times \bar{\mathfrak{R}}\ and\ each\ w \in \mathcal{W}$

1.6. Pitfalls of Copulas.

To model dependence, copula theory is perhaps the most unrestrictive and realistic approach that we currently have. Nevertheless, it is necessary to know its limitations. According to (Fermanian & Scaillet, 2004), in the process of obtaining the maximum likelihood estimators of a parameter that belongs to a parametric family that is asymptotically normally-distributed, if it is estimated via fully parametric method, the result depends on the right specification of all margins. If it is estimated via a semi-parametric method, it may not be efficient.

Additionally, in either case the researcher cannot be sure that the chosen copula is the right one. Simulations where conducted to determine the mean squared errors and the estimator bias; results indicate that a potential misspecification may lead to a severe estimator bias, which implies a possible dependence overestimation. In addition, this misspecification often leads to an increase in the mean squared error.

With respect to the criticisms related to the way that copula theory has been applied (not with respect to its technical limitations), a number of newspaper articles emerged tending to blame the formula to price CDO´s, that was largely based on the use of copulas, for the 2008 collapse of the financial markets. I consider this “pitfall” as a methodological one and not as a statistical one for the following reasons.

To begin with, in the most popular article “Recipe for Disaster: The Formula That Killed Wall Street”[2], the author assumes that the whole financial crisis occurred because the majority of market operators used an asset valuation method based on a Gaussian Copula. The author explicitly asks, “How can one formula pack such a devastating punch?” as far as I`m concerned it didn`t:

Each particular problem defines the type of copula to be used and not in the other way around, that is, each specific market and perhaps even each specific financial instrument would require its own model or copula that does not necessarily has to be Gaussian. That is, each time they need to make a CDO market valuation, they need to estimate which copula fits better.

[2] Felix Salomon. “Recipe for Disaster: The Formula That Killed Wall Street”. *Wired Magazine.*

Moreover, once a model has been chosen to price a financial instrument, it may be necessary to make parameter adjustments or even change the model as circumstances change. If financiers used the same formula to price a variety of financial instruments regardless of the specific characteristics of the financial instrument and irrespective of the deteriorating economic conditions that began on 2007, as they actually did, then the mistake was made by those who applied the formula, not the formula itself.

On the other hand, it seems to me quite simplistic to blame a single formula for the meltdown when many events caused it. Among those events we have the deregulation process that began in the 1980`s, the lowering of the interest rate in 2001 with a sudden hike that began at 2004 and the oversupply of government backed loans for subprime mortgages.

In addition, the bankers´ greed played a preponderant role which lead to many risky gambles, to over-leveraged transactions and financial deception in the form of "securitization". These events were only few of the ones that conforms the context of the crisis. Therefore, in the complete context is what it seems simplistic to me blaming only a mathematical formula although I do agree that a misuse of it was one of the causes.

Relying only in one model to make decisions leaving fundamental and technical analysis and even experience out of the game is a mistake not attributable to a model but to human error. Moreover and as the author points out, far beyond a copula model or a macroeconomic forecast, the main driver of the crisis was greed and as far as I´m concerned, there was a lot of moral hazard involved. Whether bankers used only the Gaussian model or not, what it is now certain is that they made very risky bets on a gigantic bubble assuming that some Federal Agency would come out and rescue them if things didn´t come out as planned.

Finally, although it is true that it was very difficult to anticipate the financial meltdown, if different copula models would have been used and not only the Gaussian, it is most likely that they would have captured more information about heavy tails and so, there would have been more likelihood of inferring the occurrence of the crisis. I can assert this since actually, copulas are designed to allow for margin-flexibility and therein, they can capture far more information than a normal multivariate distribution or an elliptical copula: they are designed to capture the effects of extreme asymmetric movements and dependencies.

Having completed the copulas presentation, the next chapter presents the economic framework in which this theory is applied.

Chapter II

Mexican Economic Policies.

The main purpose of this chapter is to provide an insight into the economic model used by Mexico, aiming to visualize the frailty factors upon which this model is placed. In addition, our aim is to show how these frailty factors are more relevant in the context of the current economic crisis.

During the 1980`s, Mexico suffered a period of heavy instability and economic stagnation; because of this, it was thought that the best way to promote economic growth was thorough stable fundamentals. This period coincides with the rise of the globalization process, which needs an open economy on goods and capital investments. This reform formally began in 1987 with the so-called “Pact for Stability and Economic Growth”, later modified in 1989. In essence, it aimed to stop the rising inflation and interest rates, as well as trying to control the exchange rate depreciation.

The new economic model`s perdurability required certain policies to be implemented. These reforms involved a reduction on government spending to get to nearby fiscal discipline. Likewise, it was of capital importance to avoid monetizing expenditure, this was achieved by providing independence to the central bank, *The Bank of Mexico* (BANXICO), and to enact as its mandate to provide stability. Later on, monetary policy changed from the so-called *short* to inflation targeting.

The Mexican liberalization process, economic and financial, was established with the *North America Free Trade Agreement* (NAFTA), which began operations in 1994. Moreover, a modification of several other laws took place. These modifications included the law regulating the Mexican Stock Exchange that now allowed foreigners to own a majority of a company`s shares. Thereafter, the economic pattern of stability and liberalization was going to be so prevalent that even the management change from one president to the next or even the change of political parties’ ideology would leave unaffected the stability-export-led-growth model.

2.1. The stability model.

The *stability model* comprises a set of monetary and fiscal policies aiming to achieve stability in certain economic variables. These are the exchange rate, inflation and the interest rate. We shall begin by presenting the monetary part of the strategy. It is important to acknowledge that this work does not support the idea of price stability as the best policy to achieve economic growth in Mexico. Nevertheless, since it is so prevalent it is more than necessary to present its ideas to better assess its glitches.

2.1.1 Price Stability Framework.

Price stability refers to a series of policies aiming to reduce the levels and variability of inflation to an appropriate point to pin-down inflation expectations. The final goal is to set the path for economic growth. There is no consensus regarding what price stability means, or about what exact level of prices they mean by price stability, it depends on the specific country and on the specific timeframe. (Fisher, 1996) acknowledges price stability as a range between one to three percent, that is, he acknowledges a numeric range for it. (Wynne, 2008) states that an appropriate definition is not a numeric one, and ideally, it should be made on the price index that better captures the welfare for society. This definition comes within the Greenspan´s definition of price stability: “Price stability exists when inflation is not a consideration in household and business decisions”, (Greenspan, 1994).

In this matter, the Federal Reserve Bank of the United States is almost unique in that it targets inflation in a qualitative way with no numerical inflation target. On the other hand, the European Central Bank defines it in the following way: “Price stability shall be defined as a year-on-year increase in the Harmonized Index of Consumer Prices for the Euro-Area of below two percent” (Castelnuovo *et al*., 2003); and finally, the Bank of Mexico defines it as a numerical target with a one percent error margin. The numeric value changes over time, but in 2012 for example, the target was three percent. In all, according to this monetary point of view, price stability is a strategy that should be applied in advance, since within the Milton Friedman`s framework, by targeting stability in advance, expectations about price increases may be contained and anchored (McDonough, 2006).

2.1.2 Where does the price stability idea comes from?

The motivation for policymakers to pursuit price stability come at least from the 1960`s, when around the world, governments decided to monetize expenditure in order to stimulate economic growth. An undesirable consequence of this strategy was inflation. In particular, the inflation hike in the U.S. during the 1970`s motivated mainly by the oil embargo and that of 1980`s related to the savings and loans societies crisis, brought about uncertainty and lack of stability within system.

During the 1970`s and the 1980`s, several other countries experienced huge inflation hikes that severely affected people’s wellbeing. Among those countries we can find Mexico, Israel, Brazil, Chile, Iceland, Turkey and Hungary. Other countries that experienced high inflation in a lesser degree are New Zealand, United Kingdom, Sweden and Canada.

Given that in some cases, the rate of inflation was very high and therefore, its consequences were harsh, a group of policymakers got to the *consensus* that, the best way in which economic policy can contribute to economic growth is by procuring price stability. Other countries soon followed that *consensus* and switched their policies towards that goal. Moreover, the *consensus* claims that price stability is justified provided the inverse relationship between inflation and economic growth (Bancomer, 2002).

The idea of achieving price stability within the *consensus’* framework comprises the acknowledgment that one of its causes is its expectations. The allowance of any level of inflation without the commitment to bring the level down may encourage expectations to “provide an opening for a demand-driven burst of inflation” (McDonought, 1997), therefore, beyond controlling any economic variable to keep inflation low, it is important to control the mere idea of it being high.

According to (Bernanke, 2006), by pursuing price stability, economic growth is an immediate consequence, therefore, this *consensus* establishes that price stability is an end and a mean itself which lead to the conclusion that price stability and growth are complements, the argument is threefold:

- Since price stability preserves the purchasing power integrity, money can be stored safely for current and future transactions. Consequently, long term planning and decision-making are possible with more confidence, luring consumption and investment.
- Additionally, since prices work as a mean to convey information with respect to supply and demand, the absence of inflation avoids distorting this information and therefore, agents have clearer signals. The final consequence of not distorting price information and allowing long term planning is that investment and consumption will bring economic growth.
- When inflation is anchored, there is no need for bankers to request additional compensation to hedge from it; the immediate consequence is that the interest rate will be driven down.

In sum, according to the price-stability point of view, the bottom-line of is that when it is achieved, it supposedly promotes economic growth, whenever it preserves the integrity of purchasing power and more importantly, when inflation expectations are kept in check. As a result, there is no tradeoff between growth and price stability.

Among many of the price stability defenders, there is not a clear consensus about what causes inflation. Even those in the very same Federal Reserve System (FED) have different views about it. Within the FED (Wynne, 1993) acknowledges only the growth of money stock as the source of inflation, others like (Bernanke, 2006) and (McDonough, 1997) leave the possibility open for other causes, like supply shocks.

On the other hand, although the argument of price stability is that stability implies economic growth, many central banks do not take that relationship for granted and enact as a Central Bank's objective, to procure economic growth. The best example of this case is the Federal Reserve System. On the other hand, the Bank of Mexico has a unique mandate, to procure stability alone. This is because it assumes that economic growth is an immediate consequence, just as it is stated by the most fundamentalist wing of the of price stability theory.

As part of justifying the existence of the stability polices, let us now look at the undesirable consequences of inflation.

2.1.3 Inflation consequences.

As conceived by this theory, the undesirable consequences of inflation damages society so much that, as stated in (Bancomer, 2002), stability policies represent an investment whose benefits compensate in excess any short term cost, provided that they are accompanied by other policies like fiscal discipline. Specifically, the consequences are:

- Inflation distorts the functioning of prices as a signaling mechanism, conveying ambiguous information to the market. This causes confusion among economic agents, for instance, there is no distinction for producers between an inflationary price increase and an increase that is consequence of an increase in demand.
- Inflation acts as a tax over capital causing a bias toward consumption in detriment of investment and savings.
- Inflation harms severely the group of people with low fixed income. Part of it is because the difference in education regarding investments among wealthy and low income classes.

- When high inflation exists, “people tend to change their investment habits and refrain from making long term commitments, upon which the organized economy depends”[3].
- Inflation has negative effects in terms of business’ decision making. This is because, when high inflation is foreseen, it has to be discounted from future investments’ profitability. Therefore, profitability that may have existed in the absence of inflation, is now reduced or absent. Therefore, when inflation is expected, it is more likely that investments are going to be stalled, which in turn affects economic growth.
- Inflation increases imports and reduces exports whenever local products are more expensive, thus this effect may have a negative effect on the current account.
- Inflation creates incentives to engage in non-productive activities such as speculation in financial markets, diverting resources from real to non-productive activities.

2.1.4 How can stability be reached?

According to (Mishkin, 1998), an economy can get to a price-stability state essentially in three ways. Each of which targets a variable in order to anchor inflation expectations, that is, a central bank uses this variable to communicate its intentions of lowering inflation and the expected path to do so. These strategies are the following:

Exchange-Rate Targeting: This strategy uses the exchange rate as a nominal anchor. It fixes the value of the domestic currency relative to that of a low inflation country. Another way to anchor expectations is by a crawling-peg with a low rate of depreciation and a preannounced rate of currency depreciation. In doing so, the central bank is obliged to limit its creation of money to a level similar to that of the anchor country; this can lower inflation expectations to a level that is similar to that of the anchor country.

The drawback of this way to achieve price stability is that monetary policy is completely constrained to target the exchange rate. The country can´t respond to domestic or foreign shocks provided a free capital mobility system (McDonough, 1996). This characteristic makes this strategy very prone to speculative attacks and if this target is maintained during a long attack, the central bank may ran out of reserves and may have to allow the currency to float freely. If the exchange rate targeting is used, the country must be sufficiently cautious as to peg its currency to an economy closely linked to it. Yet, another disadvantage is that a shock to the

[3] Wallis, Allen. “Price Stability and Economic Growth”, *Increasing Understanding of Public Problems and Policies*, Issue. 1960.

anchor country could directly transmit that tremor to the pegging country since the interest rate variations in the anchor country directly affect those of the pegging country (Banisadr, 2009).

Monetary Targeting: As simple as target a monetary aggregate which is assumed to have a relationship between that and inflation. Nowadays, this strategy is not common.

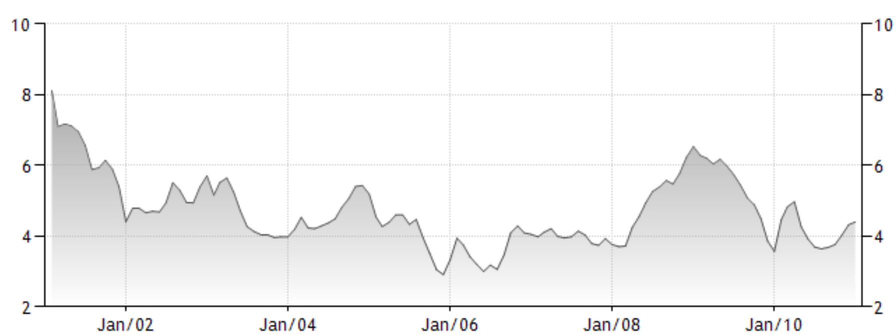
Inflation Targeting: This approach is the one that has been widely used among the most important economies in the world, including Mexico. As described by (Bernanke & Mishkin, 1997), its purpose is to anchor inflation expectations by committing to a target inflation rate. It seeks a nominal anchor for inflation while the policy commits for discipline in order to gain credibility with economic agents.

Its hallmark is the announcement and commitment that they will hold the inflation at a predefined level. It explicitly acknowledges price stability (and a reduced level of inflation) as the main objective of monetary policy. Additionally, it assumes that monetary policy can affect real quantities like output and employment only in the short run. Price Stability is usually associated with the central bank's independence. Inflation targeting strategy assumes that there is no long-run tradeoff between output and inflation, and it definitely assumes that price stability lures long-term growth.

The approach was first used in New Zealand, starting in 1990 it succeeded in lowering inflation from around 12% in 1982 to 2% in 1992. Mexico fully adopted it in 2001 and in terms of inflation reduction, it succeeded lowering it from around 8% in 2001 to less than 5% by the end of 2010 as shown in Figure 18.

Figure 18

Annual change on Mexico´s Consumer Price Index o (Annual Inflation Rate)



Source: Trading Economics.

Within the Inflation targeting framework, the benefits are that it provides a country with the flexibility to focus on domestic considerations to respond to them in the short term. Inflation targeting is not a rule not a discretionary policy, it is a “linear combination” of them, but more importantly, it is a strategy that is easily understood by the public. A drawback is that since inflation is not necessarily controllable by monetary authorities, the pursuit of an inflation target only takes effect with a lag. This fact combined with the ever-changing economic environment makes inflation targeting an elusive quest.

Finally, critics point out that the policy’s target should not be the inflation rate but economic growth. Yet, under its own assumptions, growth is an immediate consequence of price stability. This has been a brief description of what price stability comprises, we will now turn to how this objective is understood and pursued by the monetary authorities.

2.1.5 Empirical evidence.

According to (Bancomer, 2002), who surveyed a series of applied studies that investigate the relationship between inflation and growth for a myriad of countries (most of them OECD countries), found evidence supporting that countries who adopted the stability framework do have some positive growth rates.

Additionally, (Fisher, 1993) place strong attention to long-run inflation, he emphasizes the potential “inflationary consequences of monetary expansion under short term countercyclical purposes”. Even when he found a negative and significant correlation between growth and inflation between 1965 and 1994, for the countries he studied, he didn´t find a significant correlation coefficient for the entire period, implying non-conclusive evidence about the relationship between growth and inflation.

Furthermore, (Barro, 1995) found the same inverse relation, nevertheless, a significant inverse relationship is only found when included high inflation datasets. It is worthy of mentioning that (Bancomer, 2002), refers to these studies as if they would have found definitive evidence of this inverse relationship among inflation and growth. Nevertheless, when each material was assessed, we found that most references actually admit they do not have conclusive evidence.

Other sources like (Escrivá, 2010), also state that “a great deal of macroeconomic stability” has been achieved in Latin-America, conquering inflation and implying a sustained growth rate. Nevertheless, as the case of Mexico best exemplifies, although price stability was achieved, this strategy only has gotten a sluggish GDP growth rate.

Other cases such as those mentioned in (Cavallo, 1996), (Frenkel, 1996), (Brash, 1996) and (Tosovsky, 1996) show that stability brought a GDP increase in the cases of Argentina, Israel, New Zealand and the Czech Republic. Moreover, most of them state that inflation reduction strategies like inflation targeting must also be accompanied by a series of policies such as:

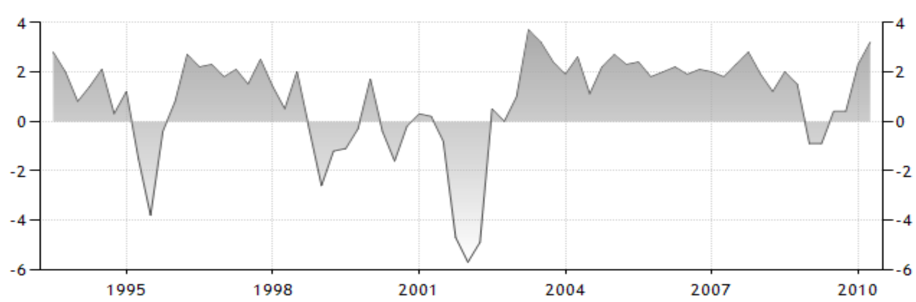
- Liberalization of trade and capital movements.
- Privatization of public companies and deregulation to achieve competition and discipline.
- An increase on foreign reserves.
- Monetary reforms tending to provide autonomy of the central bank.
- A tendency to reduce the interest rate and to stabilize the exchange rate.
- A reduction of fiscal deficits to nearby surplus levels.
- Elimination of subsidies and other incentives.
- Increase flexibility in the labor market.

Although it cannot be denied that these countries achieved stability, their growth rate is not necessarily substantial. The average growth has been less than three percent in the best of cases. As it can be seen, starting from 1996 and for a relevant part of the study period, economic growth is lethargic and in most cases, the growth rate rarely was above three percent per year.

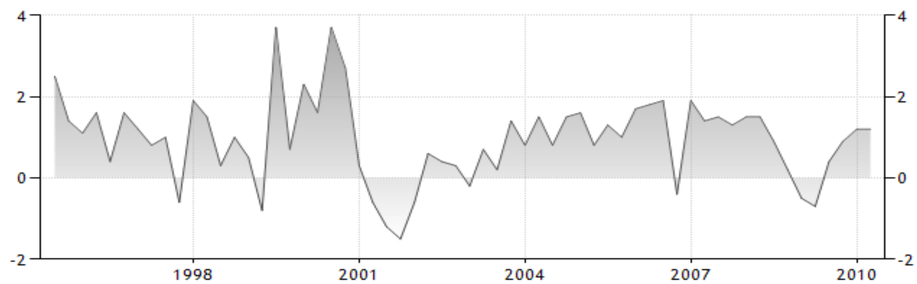
Figure 19

GDP Growth Rates

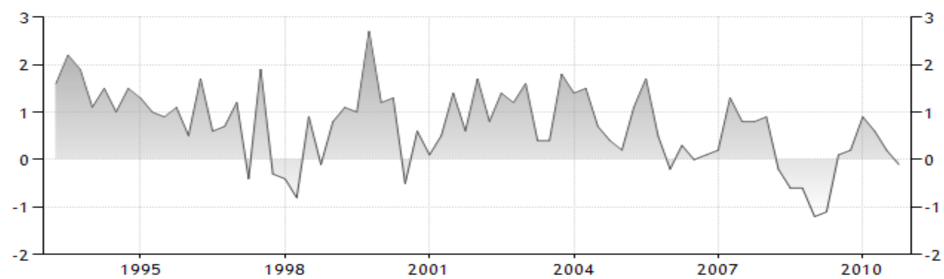
Argentina.



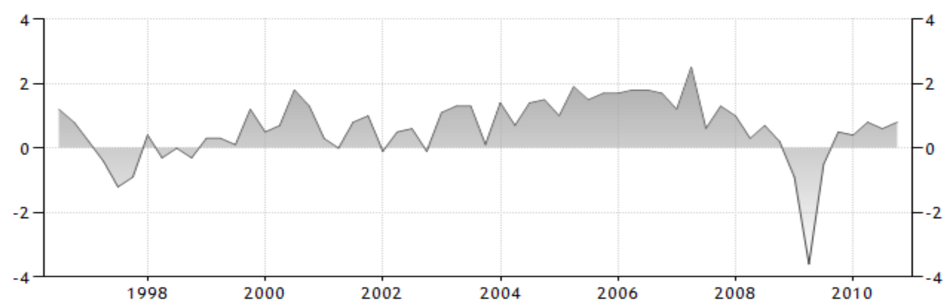
Israel



New Zealand



Czech Republic



Source: Trading Economics.

We will now turn our attention to show how this stability framework was implemented in Mexico.

2.1.6 Price stability as conceived by the Bank of Mexico.

In the view of the Mexican Central Bank, Inflation is ultimately a monetary phenomenon so it corresponds to the central bank to head expectations and price formation, (Sanchez, 2010). Inflation is a "chronic malady which must be addressed energetically"[4]; it has many

[4] Sánchez, 2010, Bank of México.

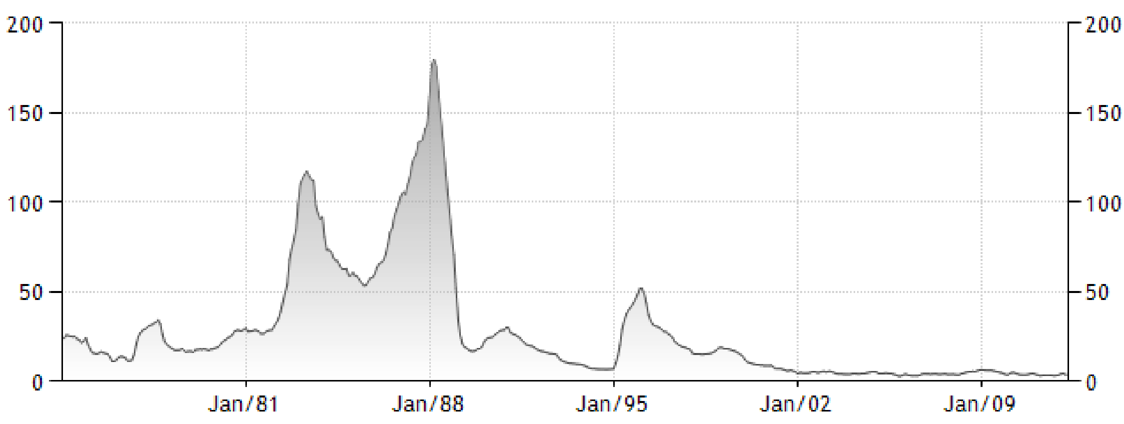
unfortunate consequences that can be avoided provided that the economic policy to address this issue is available.

According to (Sanchez, 2002), huge amounts of effort must be devoted to avoid inflation since, if it is anticipated, economic agents will look for a money substitute and companies would change prices frequently wasting a considerable amount of resources. If it is not anticipated, uncertainty generates additional costs and instability; this is because, wages and other prices are rigid and are not able to adjust to the rising prices.

Mexico has had a turbulent inflationary history, therefore, standards of living has been damaged as salaries do not cope with these constant and pronounced inflation hikes. As it can be seen in Figure 20, a relative price stability period occurred in early periods until 1973. According to (Sanchez, 2011(a)) it was during this early stage that policymakers committed to procure stability by means of combining both monetary and fiscal policies. Monetary policy comprised low inflation and the accumulation of currency reserves, their main weapon was the so-called ”encaje legal” (a mandatory amount or of bank reserves or legal reserves, it is the deposits proportion that banks cannot lend) which was accompanied by a reduction on the fiscal deficit.

Figure 20

Inflation, CPI Annual Change.



Source: Trading Economics.

According to (Sanchez, 2002), in Mexico there exists an inverse relation among public spending and inflation. He calculated that if average inflation had been three percent during the past 50 years, *per capita* income would have been 49 percent higher. Although this statement sounds alarming, we would add that if price stability policies had been in place earlier, economic growth would have been far smaller than it reality was. Consequently, although inflation would have been lower, per-capita income would not have been as high as Sanchez state given that income would have been smaller.

In addition, Sanchez found a negative correlation between inflation and economic growth (a coefficient of -0.48). Nevertheless, within the same timeframe, Mexican policymakers implemented a huge state intervention program with an additional industrial policy to achieve development and growth. Many of the reasons why these policies did not work have to do more with mismanagement and corruption than with excessive spending.

Starting at 1973, external shocks hit the public finances and policymakers decided to pursuit economic growth by expansionary policies. Given the decline in the oil price, public expenditure was monetized until 1982, when deficit spending could no longer be sustained. Mexico then ran into an ever-increasing external debt spiral. At the same time, the U.S. raised their interest rates making it impossible for Mexico to service its debt; the consequence was a balance of payment crisis.

In order to put an end to these unstable stages policymakers decided that a commitment to stability was the best course of action. During 1987, authorities signed the “Pact of Stability and Economic Growth”. One of the basic premises was that: “It is clear that monetary policy cannot stimulate directly and systematically economic activity and employment”.

To avoid monetizing expenditure from ever happening again, governments around the world opted to provide independence to their central banks. The Bank of Mexico gain its autonomy during the early 90`s. The new objective enacted on the Bank´s mission article 28th was: “The state will have an autonomous Central Bank in the execution of its functions and its management. Its main objective will be to procure the stability of the purchasing power of national money…no authority may order the central bank concede financing”.

Later on, because of the 1995 crisis it was decided to adopt a flexible exchange rate. Fiscal policy was gradually headed to fiscal discipline and monetary policy was tightened. As inflation was driven down, monetary policy changed from the “corto” to inflation targeting.

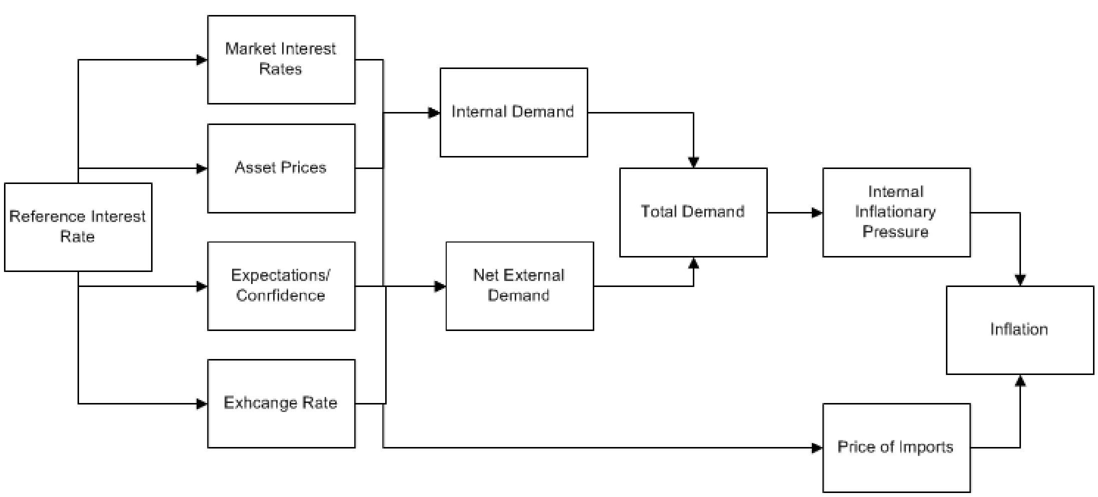
To deal with the inflation *pass-through* effect from the exchange rate to inflation, the Bank of Mexico needed one anchor that could be controlled by the Bank. It was required that the monetary base do not expand more than the GDP’s potential long term growth. Given that variations in the exchange rate yield a change in prices, the nominal demand for money would vary as well, therefore, the variation on the exchange rate alter inflation and money. This was an important task, since from the central bank's point of view, the exchange rate variations are the main cause of inflation in Mexico. The inflation persistence is a consequence of the following causes as conceived by Bank of Mexico's Deputy Governor, Manuel Sanchez:

- Over the long run, Inflation is always a monetary phenomenon; with a correlation coefficient of 0.997 between inflation and the monetary base/GDP ratio.
- A constant and ever increasing deficit spending.
- Inflation has been highly linked to the variations in the exchange rate.

From the monetary policy point of view, the variables that the Bank of Mexico could try to control to administer the inflation rate (operational objectives), are the short-run interest rate and the private bank´s current accounts balances. Having the Central Bank to pick one of these, during the first part of the decade, it picked the current account balances by means of the so-called "corto", and from 2008 it shifted to the short term interest rate. With this strategy, the Bank of Mexico incentivizes the interbank lending´s interest rate to that of the target rate, which is the one-day interbank funding rate (Banxico, 2007). With this newly adopted operational objective mechanism the central bank knows that, when inflation or its expectations are high, they can raise the interest rate by means of the transmission mechanism shown in Figure 21

Figure 21

Monetary policy´s transmission mechanism.



Source: (Bancomer, 2002)

When the Bank of Mexico moves the interest rate, it affects demand and the exchange rate. When it reduces demand, it reduces economic activity to cope with current supply and therefore, it stabilizes prices. Nevertheless, since this economic policy reduces demand, it also stalls economic growth.

On the other hand, inflation could be contained by having an appreciated exchange rate. This is because, Mexican consumption is heavily based on imports; therefore, an appreciated exchange rate lowers imports prices and helps to contain inflation.

In all, Inflation was driven down from around 10% in 2000 to less than 5% by the end of the study period. It is worthy of mentioning that although inflation was driven down, just a meager amount of economic growth took place. Actually, growth was so low that this recent phase of Mexican stability could even be conceived as a stagnation spell.

According to (BANXICO, 2008), price stability's main achievements are:

- Real and nominal interest rate reduction.
- A reduction in the transmission of shocks from the exchange rate to inflation and to its expectations.
- A reduction on the persistence of the inflationary process.
- A stimulation of the efficiency, flexibility and profoundness of the financial system.
- An increase on the levels and a reduction of the variability of relevant variables such as consumption, investment and product.

Finally, (Banxico, 2008) claims that all these measures have resulted in clear benefits such as:

- Greater credit availability for mortgages and consumption.
- Greater resources and financing alternatives for long-term investment projects.
- Grater increase in the economic activity, employment and income.
- A better planning context for individuals and companies and hence, a better resource allocation.

In sum, monetary policy in Mexico is conducted by raising the interest rate that in one hand, reduces the overall demand reducing inflationary pressures, and in the other, it appreciates the exchange rate making imports cheaper and thus reducing inflation.

2.1.7 Fiscal Policy and Price Stability.

Monetary policy on its own is not able to achieve stability, it needs for demand pressures to be contained. Given that monetary policy is already committed to price stability, Fiscal policy does not really have much room to implement a policy different to that. This is because, if fiscal and monetary policies are not aligned, an expansionary fiscal policy could end up in the bottom of the ocean if its monetary counterpart does not follow it.

The essence of fiscal policy in terms of stability is to maintain a near balanced budget in order to keep demand pressures at bay. What is more, fiscal discipline, as viewed by these policymakers, avoids the crowding out effect which theoretically happens when the government takes resources that could have been allocated in productive investment. Moreover, fiscal discipline is now an enacted mandate for policymakers in Mexico. The Mexican Internal Revenue Service's *Fiscal Responsibility Act* now forbids the Federal Government to spend much beyond its revenue.

2.2. A second thought on Price Stability.

Monetary authorities are seeking for a reduction in the levels and in the variability of the described variables. Stability is always a desirable property. Nevertheless, the methods to achieve stability may cost more to the economy than the benefits it supposedly gets. In reality, what occurred is that the downsizing of the Mexican government, who was the driver of economic growth, has not been replaced by the private sector (Moreno-Brid *et al.*, 2009). Actually, it is worth of taking into consideration that Ben Bernanke, former chairman of the Federal Reserve System (FED), admits that research evidence is not conclusive with respect to the hypothesis that price stability boosts economic growth.

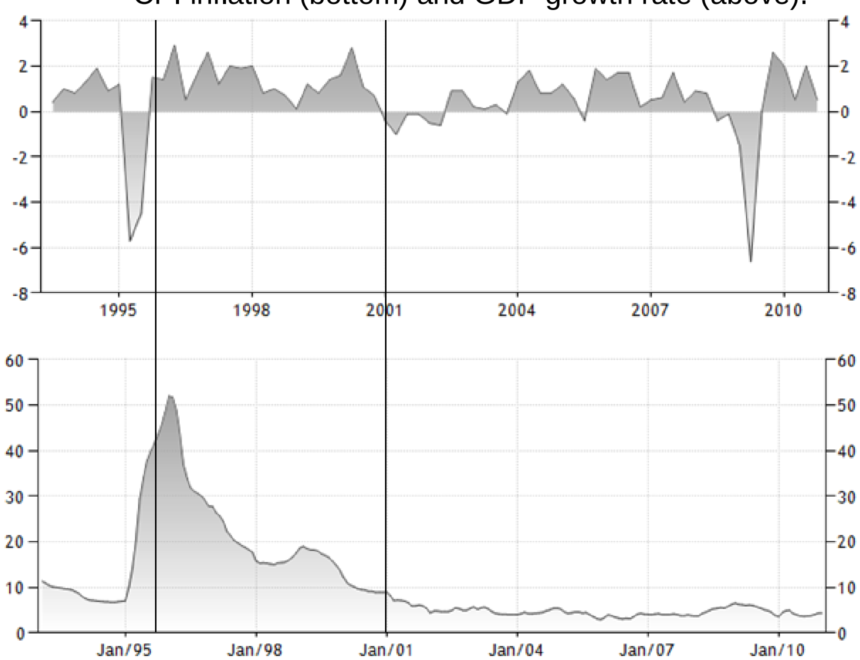
The strategy that is used to achieve price stability constraints the central bank's ability to exert monetary policy. According to (Sanchez, 2002), if a policy leads the country to the benefits of price stability, then it may easily sacrifice the "absence of an independent monetary policy to face external shocks". We definitely rebate this statement provided the recent Mexican experience during the crisis: Mexican performance was the worst of any Latin countries, only comparable to that of Venezuela, this makes Sanchez's assertion ridiculous. When committing to stability, policymakers lose the ability to create money and so, the government depends on debt if it would require additional resources. Moreover, the policymakers´ ability to exert countercyclical policies is compromised as well.

Furthermore, (Sanchez, 2002) claims that having the ability of conducting monetary policy to deal against external shocks is a disadvantage, provided that in the past, Mexican policymakers dealt very poorly against turmoil. Although he is right with respect to the policy makers´ past performance in the face of an external shock, this doesn´t necessarily means that Mexico needs to get rid of its monetary policy whatsoever. Rephrasing Sánchez´ assertion, we would have something like the following: *since in the past money caused me problems, I need to give away all my money to avoid any further complication, instead of correcting what caused problem in the first place*. This conclusion underestimates the policymaker's ability to learn from the past and apply countercyclical policy correctly in the phase of an external shock; it assumes that every policymaker that sits on the central bank´s chair is always going to underachieve. Again, having no monetary policy to fight the current crisis made the Mexican economy to underperform compared when compared gainst the majority of Latin American countries.

Let us look at Figure 22, what it is clear from it is that in the period where inflation was brought down, economic growth was sluggish at best. This fact will naturally lead us to question the stability-policy´s effectiveness. Furthermore, having a constantly stagnant GDP growth rate, as the one Mexico had during the 2000's decade, could be even worse than having a recession.

Figure 22

CPI inflation (bottom) and GDP growth rate (above).



Source: Trading Economics

Let us turn out attention to the interest rate, which is another key variable for the stability policy.

2.2.1 Interest Rate.

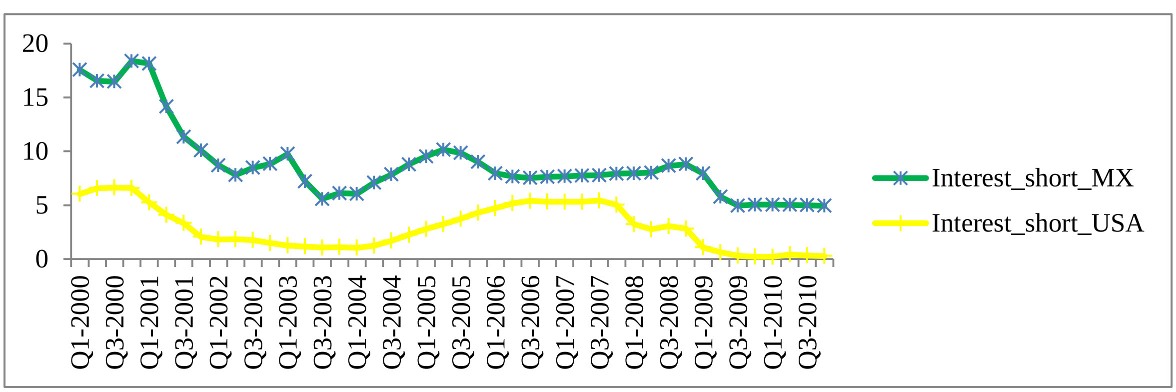
Any government needs to have the ability to manipulate the interest rate at will. In particular, if inflation is high, a high interest rate is needed to keep inflation at bay. Nevertheless, according to (Frenkel, 1996) in an open market economy with free capital movements, a high interest rate means that there will be an inflow of capitals that is going to appreciate the exchange rate, export-competitiveness is therefore compromised.

Countries that fight inflation this way find that after a period of appreciation, it is necessary a central bank intervention in the foreign exchange market to sterilize these flows. In all, this process cannot be sustained for long time. Yet, Mexico's stability policy is heavily supported on having a high interest rate to control inflation and its expectations. If this rate is not able to stay at the necessary high levels, it could compromise the model's credibility. Likewise, a higher interest rate constrains demand and constrains credit, having the overall consequence of stalling economic activity.

Monetary authorities' claim that the Mexican interest rate was driven down. The catch is that, the interest rate is indeed high when compared with Mexico´s main trading partner, the U.S. In fact, we see from Figure 23 that the difference between these interest rates is large and relatively constant; this is what motivates capitals to access the Mexican economy and in the process, compensate for the current account deficit. In the end, it is not necessary to increase further the interest rate to attract capitals as long as there is a relevant difference.

Figure 23

Short-term interest rates for the USA and Mexico.



Source: OECD.

The other negative effect is that, an elevated interest rate compared with that of countries with similar speculative risk, promotes capital inflows that appreciate the exchange rate. This will end up affecting the export and the domestic sectors. This is because, on one hand, it is clear that an appreciated exchange rate makes Mexican exports more expensive and, on the other hand, imports are now cheaper in the Mexican market and are competing with advantages over Mexico´s local production.

2.2.2 Fiscal Discipline.

According to (Gayer & Rosen, 2008), the so-called neoclassical view regarding government spending states that when a government performs any project with funds in excess of its revenue, it is made in exchange of private resources. This process is supposed to happen weather this project´s funds are obtained through taxes or bond issuance. In particular, when they issue bonds, the claim is that this issuance competes against private debt. Consequently, public debt is placed in the markets in exchange of productive investment. This is called the *crowding-out* effect. It is worthy of mentioning that crowding-out occurs only under the full employment assumption.

Acknowledging this crowding out effect, the Mexican government seeks fiscal discipline to avoid it. Nevertheless, even if crowding-out occurs as the theory states, it is generally admitted that public investment increases the total investment, diminishing or eliminating the crowding out negative consequences (Ros, 2009).

Fiscal discipline is supposed to generate trust in the commitment to maintain a stable exchange rate. This in turn, makes it difficult to lower interest to increase real investment. Another consequence of fiscal discipline is that it affects the fiscal balance negatively. When economic activity is reduced, tax revenue is reduced as well. Additionally, government spending is always thought to be an excellent booster for economic activity. Even republican-neoclassic members in the U.S. government admitted this, by changing from a free-market scheme to a government interventionist state during the 2008 crisis.

Additionally, this crowding out hypothesis does not take into consideration that currently, there is many times more money in the financial markets than in the real economy. This implies that there is enough money into the economy to pay for both, the government newly issued debt and the bonds and stocks issued by private companies (as long as each makes a profitable proposition) therefore, there would not be any crowding out effect.

2.2.3 Exchange rate.

The Central Bank claims to have a flexible exchange rate. This is true to the extent that the Central Bank fixes a range within which the exchange rate can move. Yet, since they fix upper and lower bounds, then we cannot say that this scheme is actually *flexible,* provided that the exchange will not move outside those bands. Furthermore, an exchange rate can also be influenced by the interest rate. Mexico uses the interest rate as an anchor for the exchange rate; it attracts capitals looking for carry-trades and capital gains from Mexican financial assets.

Moreover, as we previously said, fiscal discipline appreciates the exchange rate provided that it reduces demand for foreign goods and therefore, imports tend to be contained. The interest rate is reduced to the extent that its difference with that of the U.S. is relevant to attract capitals. Given these assumptions, (Sanchez, 2005) claims that the exchange rate regime in Mexico is flexible, and because of that, it has proven successful in discouraging speculative attacks. On one hand, Mexico's exchange rate is not completely flexible as he himself admits when claiming that monetary authorities may fall in the so-called “fear of floating” implying that the central bank is not so willing to leave the exchange rate to float.

Moreover, the abrupt changes in the USD/MXN exchange rate during 2008 and 2011 proved that this variable is subject to harsh speculative attacks, even when authorities were so secure that speculative attacks were a distant possibility, given their flexible exchange rate. Since price stability is quite dependent on exchange rate movements, we can say that, price stability is therefore dependent on capital flows that determine the exchange rate movements. Hence, the exchange rate stability is placed upon quite volatile and unpredictable factors, which in turn imply that price stability is placed upon frail factors as well.

Finally, (Sanchez, 2002) claims that “exchange rate variations even if abrupt, should not be interpreted as a regime failure, but rather as a reflection of the speed with which information flows into the markets”. He argues that some authors may claim that given the degree of openness of Mexico's balance of payments, it makes the peso an ideal candidate for speculation, Sanchez states that this conception is misunderstood since many other countries exhibit the same volatility as the peso, even with a lower inflation. He doesn´t takes into account the frailty factors that are valid for the Mexican stability scheme, which are not necessarily the same in other countries, and which are of interest to speculators precisely because of its frailty.

In sum, inflation has indeed been reduced significantly and has being stabilized in the study period, as it was intended to be made by the Bank of Mexico. Nevertheless, this reduction has not resulted in lower interest rates as compared with Mexico's main trading partner, nor in a significant GDP growth. What is more, these policies caused a severe damage to the export sector and to the production sector that targets national consumption, given the cheap imports they promote (Huerta, 2010). The high interest rate and fiscal discipline has managed to constrain demand and consequently, these policies stabilize the economy but they also stall growth.

Once monetary and fiscal policies are confined to stability, and once they do so by means of restrictive policies, they compromise endogenous growth (Huerta, 2010). Contrary to what BANXICO states, restrictive policies constrain the internal market despite the fact that stability may have being achieved. This is why low inflation, fiscal discipline and an appreciated exchange rate does not necessarily implies a sustained and noteworthy economic growth.

2.2.4 Summarizing stability pitfalls.

Economic growth and full employment are no longer economic policy objectives. The objective is to preserve stability. Policymakers prefer inflation reduction without evaluating its consequences. Although some stability has been achieved, the means to get it place the system into a state of frailty. According to (López *et al*. 2011), Mexico´s output performance is fairly tied to that of the US. Therefore, in the context of the current crisis and in the context of the lackluster recovery of the US, it is imperative to rethink the Mexican growth strategy to try to decouple it from foreign variables.

As long as the government keeps trying to maintain fiscal discipline and an appreciated exchange rate, it will gain stability conditioned to volatile factors such as exports, foreign exchange and foreign investment (most likely to end up in stock investments, which are even more volatile). Moreover, foreign investment whether it is speculative or real, depends to a big extent on how confidence in Mexico is perceived. The downside of this policy is that perceptions can change as quickly, as a blow of the wind, and they are completely out of control from authorities.

In fact, not only foreign investment but also the entire Mexican economic scheme depends on variables that Mexican policymakers cannot control on its entirety. Because of this, stability is not fully under policymaker's control, and so, long term planning could be compromised as well despite the existence of a “foreseeable future”. Likewise, no successful economy has achieved development without substantial state intervention, nor does the U.S. nor does the Asian tigers nor does China. The path that Mexican policymakers are carrying could be inadequate (Marquez, 2009).

Mexico’s economic growth is based on stability and a hybrid of an export-led-growth model, which is the one that we are going to address next.

2.3. Export Led Growth Model.

According to (Dreger & Herzer, 2011), the Export-Led-Growth (ELG) hypothesis states on its demand side, that sustained economic growth cannot be maintained in domestic markets because they are limited by the size of the local economy. Consequently, by placing economic growth on external markets, expansion opportunities are practically unlimited acknowledging that they do not involve restrictions on local demand. Furthermore, the hypothesis claims that growth comes not only from an increased volume of exports but also, from an improvement in productivity that stems from knowledge of foreign technology that promotes cross-border-knowledge spillovers. Also, the ELG hypothesis implies that investment is going to be focused in the most efficient sectors, generating economies of scale and generating externalities on the non-export sector.

Additionally, ELG`s core theory posits that whenever countries become increasingly open and enjoy a comparative advantage, benefits of supporting economic growth on exports constitute best practices in terms of production and sales, promoting product development and exposing firms to competition. Therefore, exports are the main driver of economic growth by means of an increase in factor productivity, (Lorde, 2011).

In accordance to (Palley, 2011), the rise of this model can be conceived in three phases. In the first, the ELG model was adopted by Japan and Germany. This phase comprises the period 1945-1970. Both economies had their own indigenous industrial base and so, economic growth by means of trade was achieved with an undervalued exchange rate. The second phase refers to the Asian tigers experience; it runs from 1970 to 1985. Of capital importance for these countries were the acquisition of foreign technology and an undervalued exchange rate, as well as having an undervalued labor-force.

The third phase as epitomized by Mexico, is a phase in which countries become an export platform for transnational corporations since they do not create their own indigenous capacity. The difference with the other phases is that these countries do not own technology nor do they own these exporting corporations. Consequently, production is labor-intensive and these exports´ added-value is marginal. These conditions severely affect wages and living standards.

Of particular importance is the incorporation of these economies to global trade by multilateral trade agreements, the objective is not to promote trade *per se,* but to create production zones. In the case of Mexico, this country was set as a production zone mainly for the US and in a lesser degree to Canada throughout the North America Free Trade Agreement.

The fourth phase is an extension of the third but in a magnified scale. It is best exemplified by China, and differs from that of Mexico in that China imposes greater tariffs on imports, it has an undervalued exchange rate with respect to its main trading partners and it attempts to build its own indigenous technological base. In addition, China owns the major dollar reserves in the world that allows it to influence the currency market. Let us now take a look into the consequences that the ELG model has brought to the Mexican economy.

2.3.1 A second thought on the Export-Led Growth strategy.

First, there is no conclusive evidence that the ELG hypothesis applies for every country. (Yang, 2008) found that it is not necessarily true that exports causes growth, but that growth comes upon the increase in the non-tradable sector, that is, when this sector grows it promotes exports in a way that causality runs the other way around. Although this doesn´t seem to be the case for Mexico, authors have found evidence to support that the ELG model does bring economic growth in the short run. Nevertheless, in the long run the relationship is inverse (Dreger & Herzer, 2011) and (Lorde, 2011), (López *et al*., 2011).

On the other hand, in the case of Mexico, even if the ELG hypothesis is true to the extent that it fosters growth, since countries adopting the ELG model on the same way that Mexico did (without creating an owning productive capital), growth and development cannot stem from it. This is because the scarce spillovers and the low aggregate value that comes from that production type (Palley, 2001). Additionally, if the ELG model is applied on a global scale, there exists the risk of falling into the *fallacy of composition*, according to which, all countries will grow relying on foreign demand. Since trade is a sum zero game, the earnings of some are loses of others and so, the overall benefit is constant. Today, more than 70 countries follow this export-led strategy (Palley, 2002).

Since the adoption of the ELG model, Mexico has not recovered the growth pace that prevailed during the 1960-1980 period. Mexico haven´t had a substantial capital accumulation and its labor productivity have not increased, nor does its *Total Factor Productivity*, (Palley, 2011). It is worthy of mentioning that there is strong evidence to believe that the Export-Led Growth model for Mexico is exhausted to the extent that this country cannot continue to base its growth on in-bond exports trying to compete in productivity and wages against China and other Asian countries (Palley, 2002), (Palley, 2011), (Lorde, 2011) (Moreno-Brid *et al*, 2005).

Other export-led-growth countries, relentless to leave its share of production to China, have to compete against China´s substantially low wages. These low wages deteriorate China´s living conditions. In addition, the main market for these export countries was actually those countries that nowadays are suffering the current economic crisis and therefore, their demand for imports is stalled. Therefore, defenders of the export-led growth model have to ask themselves to whom are the export countries going to sell?

This stage of globalization is so competitive that every day there is a smaller fraction of the export market for emerging economies provided the Chinese expansion. As (Palley, 2011) posits, conditions have changed in a way that the only big suppliers like China are likely to benefit from the ELG model. This is a consequence of their scale and the fact that they did built their own indigenous technological base, the cheap workforce remuneration and the massive capacity to undervalue their exchange rate. Therefore, the demand-side assumption of the Export-Led Growth hypothesis regarding that, growth is constrained by the size of the local markets may no longer apply to most countries.

The Export-led growth model, when instituted to create a processing zone such as in the case of Mexico, often leads producers to acquire inputs and raw materials outside the country since in general, they are cheaper. In the case of Mexico, foreign inputs were at least 30% cheaper (Moreno-Brid *et al*,. 2005). Since the hosting country is only a plant, spillovers from this industry to the rest of the economy are often scarce. Moreover, although a substantial increase in exports is envisioned, an ever increase in imports is also to be expected.

Let us now take a look into how Mexico was involved in such a growth scheme as the ELG model.

2.3.2 The road to the ELG model in Mexico and its consequences.

During the 1940`s, Mexico`s economic model was that of import substitution. One of the objectives was to change from an agrarian scheme to industrialization. The Federal Government through huge interventionism and protectionism led this strategy. To achieve their goal, substantial changes had to be implemented: tax cuts, trade restrictions and a minimum participation of local producers in any industry were required, (Lorde, 2011). Of particular interest was the so-called *Maquiladora* or In-bond export program which aimed to establish a manufacture export platform: it had priority tax exceptions and their imported inputs were supplied within a free- tax scheme.

Among the main positive achievements of this period we have that, manufacturing industry increased its GDP share from 15% in 1940`s to 25% at 1970, a real average increase of 3% annually from 1940 to 1970. This growth strategy had its faults that eventually led to drop it. According to (Moreno-Brid *et al*, 2005) these mistakes where an uneven wealth distribution, a failure to implement a fiscal reform that improves public income and a lack of an appropriate export program for the non in-bond exports, as well as corruption and inefficiency.

During the 1970`s the economy lost momentum, the Federal Government rapidly implemented an expansionist program based in the ever increasing oil income. Moreover, the oil industry was poorly managed, (Lorde, 2011). By early 1980`s Mexico`s import substitution program came to an end with the collapse of the oil prices and the rise of the US interest rate. Some consequences were an exploding inflation and an exchange rate destabilization. This was the beginning of a reform that envisioned export-led-growth as the most suitable policy for México. In 1986, Mexico entered the General Agreement on Taxation and Trade and later in 1994, Mexico formally joined the North America Free Trade Agreement (NAFTA).

According to (Lustig, 1994), by joining the NAFTA Mexico would be able to double its GDP in a relatively short time and so, living conditions would improve substantially. According to (Burfisher *et al*., 2001) several quantitative studies were done within the frame of general equilibrium under a myriad of methodologies, all concluding that the benefits from NAFTA would be reduced for the US and very large for Mexico.

After decades of protectionism, it was expected that imports would flood the market for some limited time. Nevertheless, manufactures imports grew more than twice as quickly as exports from 1988 to 2003. Moreover, it doubled the long-term income elasticity of imports; this means that in order to maintain a well-adjusted trade-balance, Mexico would need to expand exports at an annual rate of 15%, whereas in reality, Mexican income grew at an annual average of 5% (Moreno-Brid *et al*, 2005). Moreover, the added-value stemming from in-bond exports was almost constant during the period 1983-2003. Moreover, spillovers were quite reduced provided the low wages pay to workers and the low usage of local inputs.

Additional to the lack of benefits from trade, there also exists the problem of the scarce rate of capital accumulation. (Ros, 2009) states that a major problem is the fall of the investment rate. This is mainly the consequence of a decline in public investment which declined approximately 6.6% from 2004-2007. Also, nor does trade activities nor does the actual public investment has many linkages with local Mexican sectors. This creates an almost null spillover from the highly dynamic export sector.

What is more, trade deficit in the manufacturing sector has been increasing provided the high import content on exports. Thus, the balance of payments is more dependent on capital inflows to finance this deficit. Finally, Mexico´s economic growth depends heavily on that of the US provided that around 88% of Mexico´s exports are to the US. Therefore, under the current economic crisis, where the US economy is not responding adequately, Mexico´s performance is condemned given that Mexico is unable to apply substantial counter-cyclical policies provided its insertion in the price-stability scheme.

Additionally, Mexico has to keep the exchange rate appreciated with respect to the USD and to relegate the non in-bond export sector. This is because, Mexico failed to develop a robust domestic consumption market, building growth on foreign demand. In addition, intermediate imports are quite sensitive to exchange rate variations (Palley, 2002). Likewise, this appreciation phenomenon is encouraged by the central bank since it alleviates inflationary pressures by appreciating the exchange rate. Finally, evidence was found to support the statement that Mexico´s import share is largely caused by the manufacture exports of which in-bond or *maquila* exports are dominant (Ibarra, 2011).

Perhaps, in an attempt to save the current economic scheme and to compete with the low labor-cost of other countries, Mexico`s authorities proposed a labor reform in which workers can be hired hourly and layoff restrictions are reduced. In all likelihood, these reforms would only deteriorate economic conditions provided the already low minimum wage in Mexico.

Empirical studies found that in the long-run, an increase in exports will bring a decrease in economic growth provided the low ties to local Mexican suppliers. The exports´ low and stagnant added-value and the high amount of imported parts that are required to manufacture export goods, mainly for the in-bond sector, makes the supposed spillovers to the rest of the economy practically null (Lorde, 2011), (Moreno-Brid *et al*. 2005), (Dreguer & Herzer, 2011), (Marquez, 2009).

Therefore, even if exports continue to increase, there will not be a substantial economic improvement in terms of job creation and better wages. In the short-run however, there is evidence that an increase in exports, does boosts economic growth quantified as Granger-causality, (Lorde, 2011). In terms of GDP growth, exports is its main driver as found by (Lorde, 2011), nevertheless and as it has been said, a GDP increase in this context does not implies spillovers nor an increase in the population wellbeing.

The negative consequences of the ELG model were shortly acknowledged after the institutionalization of NAFTA. During Ernesto Zedillo´s tenure (1994-2000) and during Vicente Fox´s tenure (2000-2006) it was recognized that this new trade scheme broke productive chains in the Mexican manufacturing sector and that it was crowding-out local producers. Nevertheless, none of them nor does the following president, Felipe Calderon (2006-2012), changed the policy towards an effective industrial policy to strength the non in-bond production.

Macroeconomic and trade reforms had a remarkable impact in Mexico´s economy. Given that the state diminished its presence as a growth agent, economic growth has diminished and also there has been a weakening on fiscal income. Moreover, the exchange rate has served as a sort of an anchor for inflation; its overvalued level as measured by PPP ended up hurting the non in-bond export sector.

An appreciated exchange rate makes exports more expansive and therefore, the Mexican non in-bond export sector is severely affected. Moreover, cheap imports displace Mexican products in the local market. In all, the exchange rate does not work as an instrument to promote trade policy to benefit Mexico. The only industry able to benefit in this situation is the export-transnational companies that buy inputs at low prices in the rest of the world, assemble their products in Mexico, with a very low added-value placed upon them, and then these products are exported to the target countries. Since these goods are sold in target country´s currency, they do not loose price competitiveness regardless of the exchange rate in Mexico (Huerta, 2010).

In all, Mexico´s experience shows that although a substantial increase in exports did occur, economic growth has been scarce, actually, Mexico`s GDP is growing slower than during the import substitution phase. Furthermore, Mexico´s growth is currently constrained by the balance of payments, and on an appreciated exchange rate. Mexico should be wary of any persistent appreciation on its exchange rate since it tends to increase external indebtedness levels and it tends to deteriorate the trade balance (Moreno-Brid *et al*, 2005)

2.4. Summarizing ELG and Stability polices.

If economic growth ever takes place in Mexico, it will come upon the growth of the rest of the world. This is why there are few chances to expect grow in Mexico. The best chance for Mexico is to promote endogenous growth. The difficulty is that the very same stability model constrains the internal market. Despite the achieved stability, the economy continues to be sluggish. Also, these policies are confined to the stability objective so there are no chances to implement expansionary counter-cyclical policies, and so, the likelihood of recovery are not envisioned at all, stability is more and more apparent and frailty is more real (Huerta, 2010).

Most of the other economies lower their interest rates to promote investment, Mexico couldn´t do that because that action would stop the capital inflow and the desired stability would be jeopardized (Huerta, 2010). The problem is that, whenever there exists an abrupt disruption on the accumulation pattern as that which occurred during the 2008 crisis, what is needed are counter-cyclical policies. They wouldn´t be that inflationary since the crisis caused slack capacity. Nevertheless, given the Central Banks independence, these policies are hard to coordinate and implement.

Having presented the stability model, we can inquire why if price stability is such an effective strategy, if there was so much credit available and if the future's predictability is that important for growth, why is it that Mexico grew slower than the average of Latin-American countries before and during the crisis. In particular, during the crisis, economic growth was mostly absent.

In contrast with what (Sanchez, 2005) states, the stabilization process has not generated substantial and significant economic growth; real salaries have not increased, nor does economic activity. In all, Mexico`s growth strategy has being placed upon exogenous factors. Nevertheless, if external factors would somehow stop working in Mexico's benefit as It occurred during the 2008 crisis, Mexican growth and its stability would be compromised. Consequently, given that Mexico's stability variables are dependent on many exogenous factors, the entire system possesses an inherent frailty.

Chapter III

Parameter Estimation and Data Analysis.

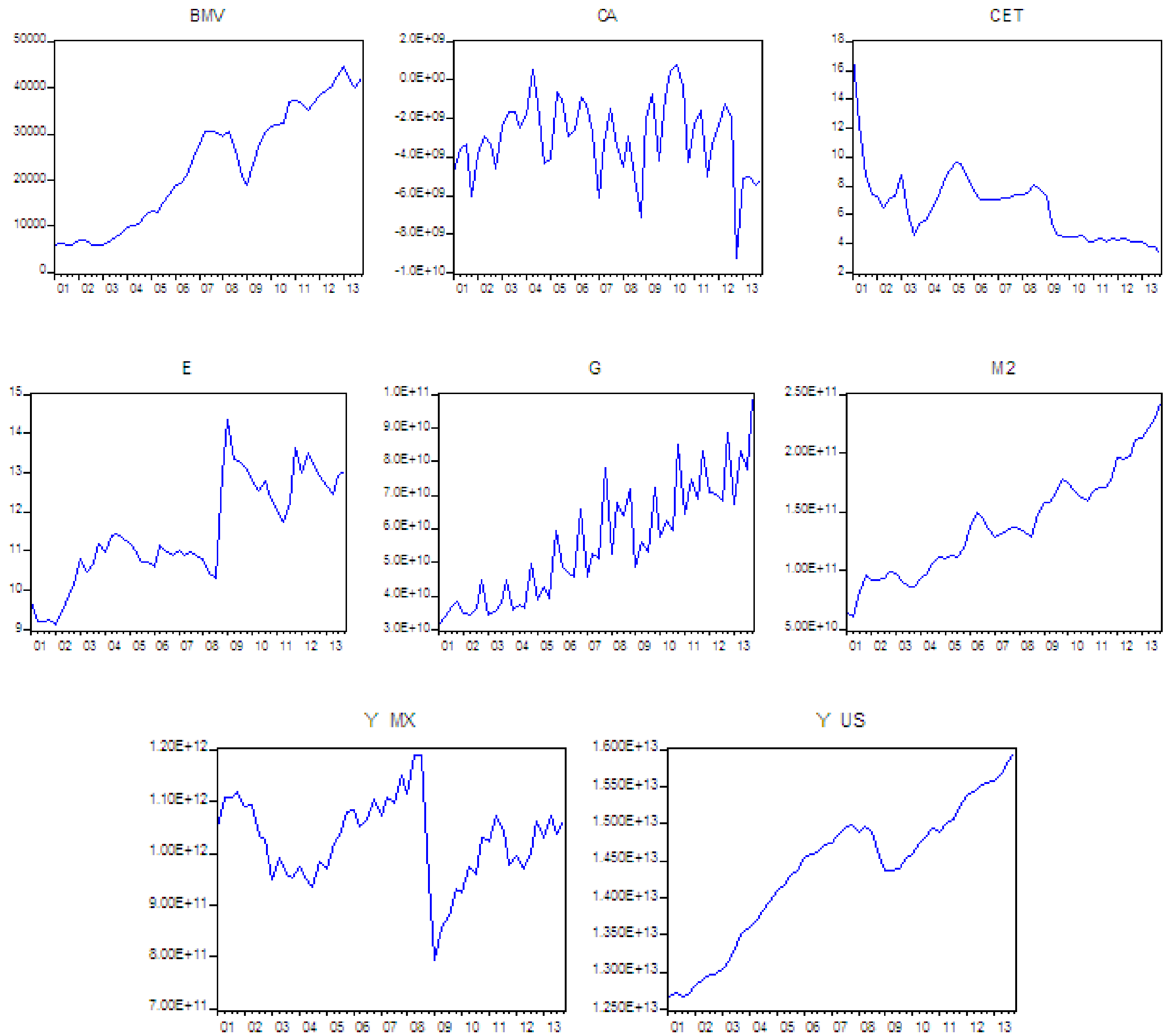
Given the stability policy and the export-led-growth model braced by Mexico, it is clear that frailty factors mainly arise from Mexico's dependence on the U.S. economy. Other frailty factors are those upon which, the stability model depends, these are: low government spending (near fiscal discipline), having an appreciated exchange rate and having a relatively high interest rate. If these conditions were compromised by turmoil, like during the 2008 crisis, then not only economic growth would be compromised, but also, the government's ability to respond to any shock. In part, this is a consequence of the central bank´s independence and its mandate to procure price stability alone. On the fiscal side, policy is also limited to contain shocks since the Mexican IRS also has an enacted mandate to perform with nearby fiscal discipline levels.

Hence, economic frailty for the Mexican economy comes in the form of contagion from the outside and from the potential contagion to the local key economic variables that lead the stability policy. Consequently, this chapter´s goal is to assess contagion, structural break and to determine the overall dependence of our study variables. Finally, an actuarial frailty model is implemented to determine the hazard of a negative GDP growth as dependent of economic covariates associated with the stability model.

Our economic variables are: US GDP (Y US), Mexican GDP (Y MX) Mexican Current Account (CA), the exchange rate MXN/USD (e), Mexican Government Spending (G), Mexican monetary aggregate M2 (M2), Mexican Stock Exchange index (BMV) and Cetes-28 interest rate corresponding to the rate of the leading treasury bond in Mexico (CETE). All variables are quarterly spaced. We divided our study timeframe in two sections, one before the 2008 crisis (2001:1-2008:3) and the other after it (2008:4-2013:4). Data is taken as first difference and logarithms. Figure 24 plots our study variables in levels, most of them will show at least a shock during the 2008 crisis.

Figure 24

Study variables in levels.



Part of our main concern in this work is contagion. There is not a unique contagion definition, although, one of the most widely adopted in finance is that *financial contagion* occurs when “there is a significant increase in dependence between price movements across markets during the crisis period’’ (Cheen & Poon, 2007). Alternatively, contagion can be understood as a “significant increase in cross-market correlation during a turmoil period” (Forbes & Rigobon, 2002).

These authors define contagion mainly as a dependence increase, whether is symmetric or not (referring to tail dependence). Nevertheless, contagion does not necessarily means more dependence, for instance, (Patton, 2009) defines contagion as “a phenomenon whereby crises, somehow defined, that occur in one market lead to problems in other markets beyond what would be expected on the basis of fundamental linkages between markets”. Consequently, it can also occur that given a shock, dependence among the study variables may actually decrease, what matters is that given a turmoil, dependence gets altered (leading to problems in other variables, as in Dr. Patton´s terminology) it doesn’t matter the direction, that is, it can either increase or decrease, this also is contagion. Therefore, we will not to take the financial definition of contagion as it is; rather, we will define contagion as (Patton, 2009) a substantial change in dependence whether it is one direction or the other.

3.1. Literature Review.

(Arakelian & Dellaportas, 2005) attempt to model contagion by the so-called threshold copulas, which are one of the increasing ways that allow the parameter theta to change in time. Other approaches are Dynamic Conditioning and Time-Varying Copulas. Based on simulation studies and an empirical application, these authors reached the conclusion that copula-flexibility allows for a wide range of data sets to be examined provided it takes into account a more dynamic parameter estimation. They do find contagion in their dataset. Furthermore, (Adam *et al*. 2013) presents a comprehensive copula review with a special emphasis on contagion. They find that not every financial asset within their studied geographical area, Poland, presents contagion despite the big shockwave the 2008 crisis sent all over the world.

(Gaiduchevici, Gabriel, 2013) uses time varying dependence to measure dependence change in European assets. The author’s interest is oriented to credit risk contagion between corporate and financial sectors. His way of specifying dependence is as a function of past values and standardized score of their copula log likelihood. This approach is able to capture contagion effect between real and financial sector. Gaiduchevici’s work is relevant to our study since we too want to study contagion to real variables.

(Forbes & Rigobon, 2002) is a very influential work. They specify contagion as an increase in cross-market linkages occurring if two markets do not present high degree of co-movement during both stability and crisis. They further use this and other definitions to study the Mexican, American and East Asian financial markets. They find a dependence increase and more importantly, a bias towards the tails when turmoil hits markets.

Finally, (Chen & Poon, 2007) present a comprehensive study of contagion in financial markets utilizing Appetite Indexes. They use a Dummy T-copula and time varying copulas to conclude that contagion took place though, they also found that Latin American markets were resilient to the 1997 Asian crisis. What we can see from these works is that, although the 2008 crisis expanded all over the world, even in this globalized economy, a shockwave as big as that not necessarily spreads to every financial assets and markets; some do have a degree of resilience.

3.2. Data pre-processing.

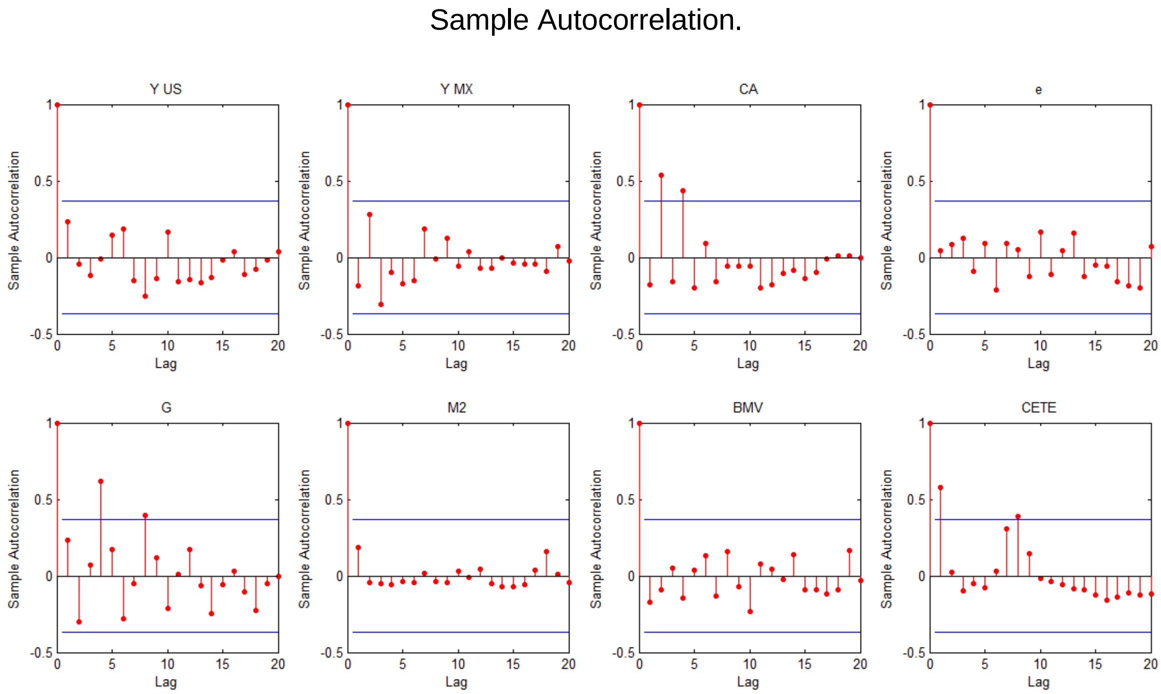
We will first assess normality in the entire timeframe by the Kolmogorov-Smirnov test, for that, we found that all variables reject normality except the Current Account. For the Jarque-Bera test, the variables that do not reject normality are the Current Account, Government Spending and the Mexican Stock Exchange index BMV. Tests information is summarized in Table 1 . In any case, since we are fitting are copulas, we do not need to worry by the lack of normality. Actually, copulas take into account information that the Normal distribution is unable to capture.

Table 1

Normality Tests							
Kolmogorov Smirnov							
Ho: Normality; Value =1 implies rejection of Null at 5%							
Y US	**Y MX**	**CA**	**e**	**G**	**M2**	**BMV**	**Cetes28**
1	1	0	1	1	1	1	1
Jarque Bera							
Ho: Normality; Value =1 implies rejection of Null at 5%							
1	1	0	1	0	1	0	1

Given that usually economic time series carry some kind of autocorrelation, our next step is to asses for its presence. As it can be seen on Figure 25 some lags lie outside the sample autocorrelation bands. What is more, we will need to determine if our data possess some kind of heteroscedastic effect and we do this by performing the ARCH test. We found that Mexican GDP and Cete's reject the homoscedasticity hypothesis, therefore we'll have to filter the series. For that matter, we applied a GARCH(1,1) to consider that information.

Figure 25



Filtering the series as an initial step is important since as (Cheen & Poon, 2007) posits, dependence measures such as the correlation coefficient will tend to be biased if heteroskedasticity is not accounted from the very beginning. After filtering our data, we applied the ARCH test again and in no case the homoscedasticity hypothesis is rejected. ARCH effects test information is summarized in Table 2.

Table 2

ARCH Tests							
ARCH Test, before filtering							
Ho: No conditional heteroscedasticity; Value = 1 imples rejection of Null at 5%							
Y US	Y MX	CA	e	G	M2	BMV	Cetes28
0	1	0	0	0	0	0	1
ARCH Test after filtering							
0	0	0	0	0	0	0	0

Next, any copula estimation deals only with data in the $[0,1]$ interval. Actually all of our data must be distributed as $U\sim(0,1)$. In part, this is because Copulas are objects that lie in the $[0,1]^3$ cube for the bivariate case or in the n-th dimensional hypercube with the $[0,1]$ interval as the limiting space for each variable, but the actual reason lies far beyond that. The natural next step is to transform our data to that scale by a kernel smoothing density function. A univariate kernel estimator is given by the followingexpression:

$$\hat{f}(x,h) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

where:

K = kernel

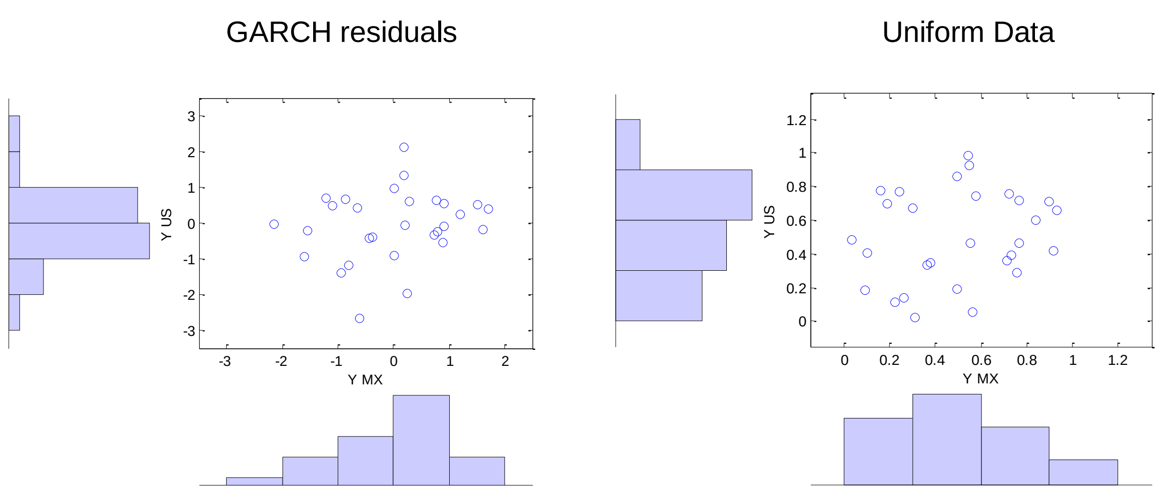
h = bandwidth

As it can be seen, this function works as a weighted standardizer. The general kernel function is given by:

$$K(x,p) = \frac{(1-x^2)^p}{2^{2p+1}\Gamma(\mathrm{p}+1)\Gamma(\mathrm{p}+1)/\Gamma(\mathrm{p}+1+\mathrm{p}+1)}$$

When $p = 0$ we get the uniform kernel. Our data then is going to be transformed from GARCH residuals to Uniform data this way. As an example, Figure 26 shows this transformation for two variables, the US GDP and Mexican GDP.

Figure 26



After this process is completed, we test whether each variable has successfully been transformed to $U\sim(0{,}1)$. We do this with the Kolmogorov-Smirnov test and in no case, the null hypothesis that vectors are from the same distribution is rejected; see Table 3 for the statistical summary (values in the diagonal are set to 0).

Table 3

Variables are from the same distribution								
Two sample Kolmogorov-Smirnov								
Ho: Data in vectors x1 and x2 are from the same distribution; Value = 1 implies rejection of Nell at 5%								
	Y US	Y MX	CA	e	G	M2	BMV	Cetes28
Y US	0	0	0	0	0	0	0	0
Y MX	0	0	0	0	0	0	0	0
CA	0	0	0	0	0	0	0	0
e	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0
M2	0	0	0	0	0	0	0	0
BMV	0	0	0	0	0	0	0	0
Cetes28	0	0	0	0	0	0	0	0

At this point, we have the dataset ready to serve as input for copulas, hence, we continue with the estimation process.

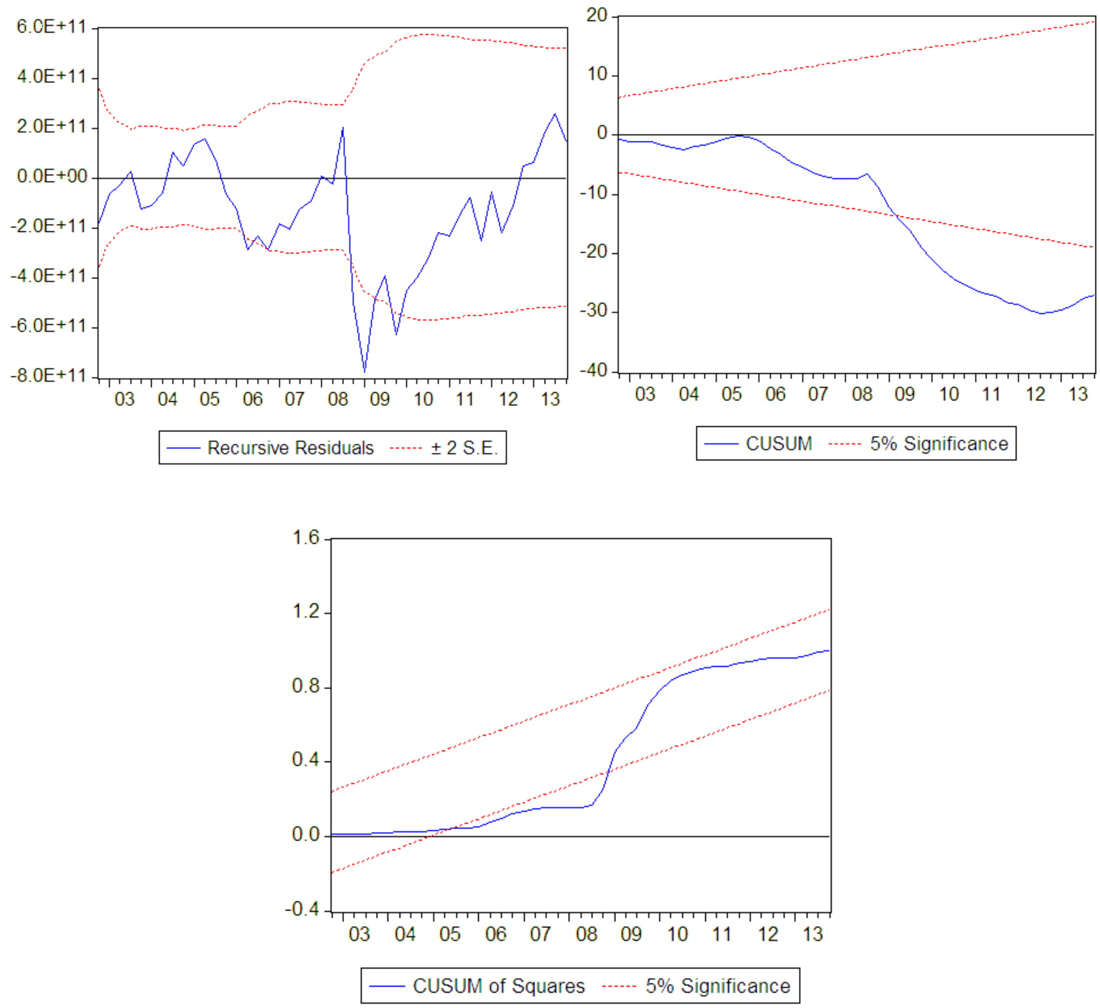
3.3. Structural Break and Multivariate Estimation.

The objective in this section is to estimate the dependence parameter for a six variable set and to propose an alternative way to detect structural break. In order to estimate this multivariate dependence, extensions were implemented to estimate the theta parameter for 6-variable copulas corresponding to three families: Gumbel, Clayton and Frank. We used these specific families because they preserve the idea that one accounts for upper tail dependence (Gumbel), other to for lower tail dependence (Clayton) and finally, a symmetric one which is the Frank copula. We also estimate the Rotated Gumbel copula.

Some of the most common ways to detect Structural Breaks are the so-called stability tests; these are the CHOW test, CUSUM test, Recursive Residuals tests etc. These tests can be performed upon implementing a regression since they are based on its residuals. We will briefly show the results by these methods and then we will continue with the method that we are proposing to show the copula parameters behavior. Since in this study we are trying to detect not only structural break but also contagion from the American economy, we will include the US output in our regression to perform stability tests.

Figure 27 illustrates the graphical output of three tests, CUSUM, CUSUM of squares and that of recursive residuals. Recursive Residuals shows a change outside the allowed two standard deviations bands; although this change is only temporal, it shows the impact's magnitude of the 2008 crisis. CUSUM shows a change that starts in 2009 and does not get back to stability again. Finally, CUSUM of squares test also shows a temporary break. We also performed the period-to-period CHOW test and found structural break evidence for 2008.

Figure 27



3.3.1 Copula Structural-Break with a fixed start.

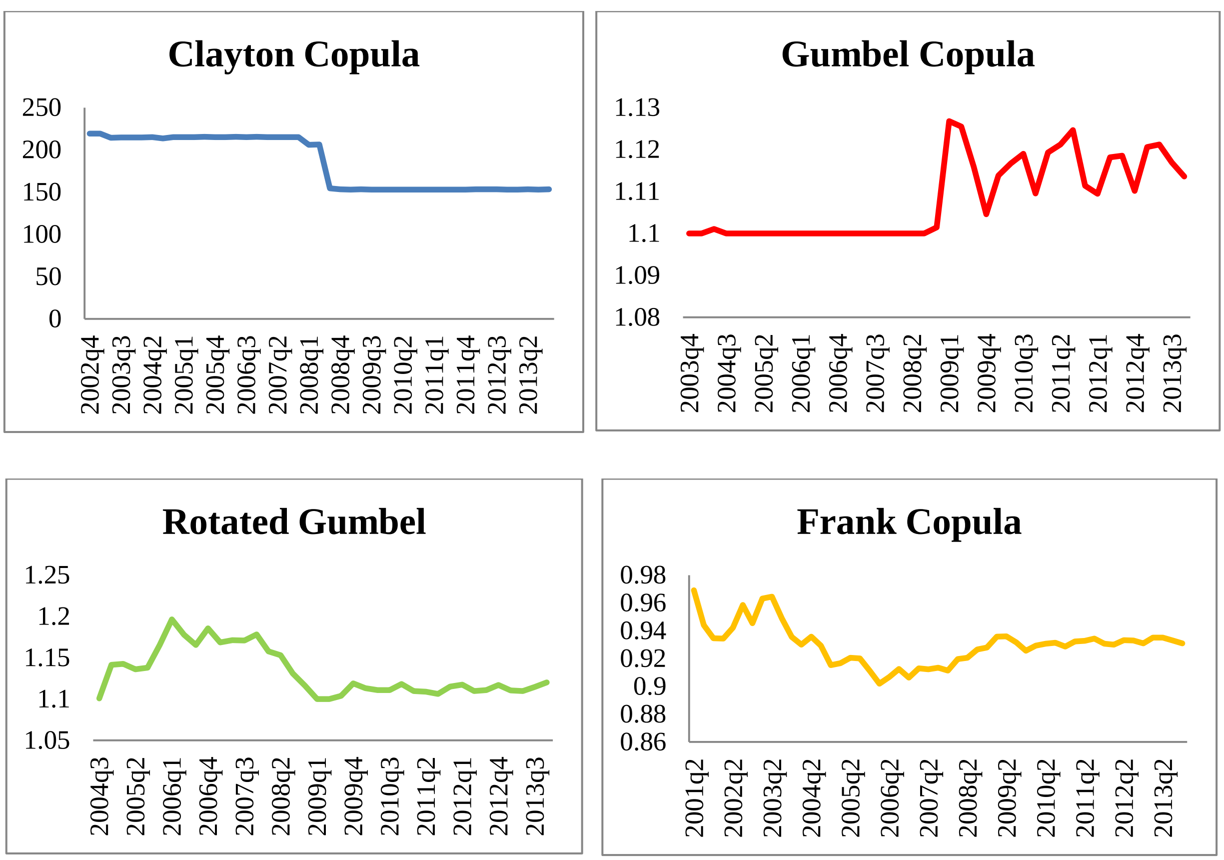
This work proposes a way to identify structural break through copulas. The way we do this is by estimating the parameter theta for a moving time-window which starts in 2001. Then, we add a quarter at a time and re-estimate the parameter. The dependence parameter theta, therefore, incorporates new information each step and the rationale is that, if a structural break takes place, then dependence among variables is going to be affected as well. If this is the case, we will have two sets of values before and after the break. If the difference between these values is statistically significant, then we have evidence supporting a structural break.

We estimate theta parameters for Gumbel, Rotated Gumbel, Clayton, and Frank Copulas. The estimation method was maximum likelihood although, the numerical method we used was Trust-Region Reflective Algorithm. In order to assess the best fit we will use the likelihood outcome as a goodness of fit indicator and we will take the smaller value. This is because the optimization algorithm minimizes the minus-likelihood. In addition, we will estimate the likelihood for the whole period, not in a rolling-window scheme.

We estimate the theta parameter for six-variable copula, which included Mexican GDP, U.S. GDP, real exchange rate MXN/USD, government spending, M2, and Cete's interest rate. Figure 28 shows theta's behavior through time. It has to be noticed that the series do not start at the same date since it takes few periods at the beginning to each copula to gather enough information to produce relevant outputs.

As it can be seen, there are different behaviors before and after the crisis. Most importantly, Clayton, Gumbel and Rotated Gumbel do show a graphical change which we could interpret as a potential structure break.

Figure 28



We then split data to have two periods, one after and other before the crisis. With these two sets, we perform a difference of means test for uneven variances to know if there exists statistical evidence supporting the hypothesis that data before the crisis is different from that after it. This information is shown in Table 4, which shows that for most copulas (Clayton, Gumbel and Rotated Gumbel) the null hypothesis of equal means is rejected at 5%. The only one not rejecting is the Frank Copula.

Table 4

Null: the two data vectors are from populations with equal means, without assuming equal variances.							
The returned value of h = 0 indicates that test does not reject Null at 5%.							
Clayton		Gumbel		Rotated Gumbel		Frank	
Decision	1	Decision	1	Decision	1	Decision	0
P-Value	1.86E-22	P-Value	4.03E-13	P-Value	3.11E-09	P-Value	0.273

To assess which statistical decision is more reliable, we have to know which copula best fits our data. We assess this by comparing the likelihood coefficient. Since the optimization was made by minimizing the minus-likelihood, we have to look for the smallest likelihood coefficient. Table 5 displays the likelihood for the four copulas and theta values as estimated in the complete period at once. We can see that Clayton copula is the one that fits best followed by the Rotated Gumbel, then the Gumbel and finally the Frank copula. Consequently, given that for copulas with higher fit the null hypothesis of equal means is rejected, we can assert that evidence suggest that structural break took place as measured by the statistically significant change in theta.

Table 5

Clayton		Gumbel		Rotated Gumbel		Frank	
theta	153.293439	theta	1.11357139	theta	1.12006896	theta	0.93096642
LogLikelihood	-12691.4926	LogLikelihood	-5.59313701	LogLikelihood	-6.23963673	LogLikelihood	199.115095

In order to interpret our theta estimations we have to remember what theta means for each copula in terms of dependence. For a Frank copula, a theta tending to 0 means independence; for a Clayton copula, a theta tending to 0 means independence; for a Gumbel copula, a theta tending to one means independence.

As we can see from **¡Error! No se encuentra el origen de la referencia.**, Clayton estimation tells us that our economic variables had an initial degree of dependence, and after the crisis, they became more independent. For the Frank Copula, although there is not a clear pattern, we can say that it trends downward, towards independence although it does so in a very smooth way.

Finally, both Gumbel copulas slither upside down, starting from independence before the crisis; then it draws from independence to dependence back and forth. This change in dependence has to be considered when elaborating countercyclical policies. If variables are less dependent, as it seems they are, then policies would be less effective provided that variables are less responsive; therefore, policymakers could be underestimating their strategies' impact.

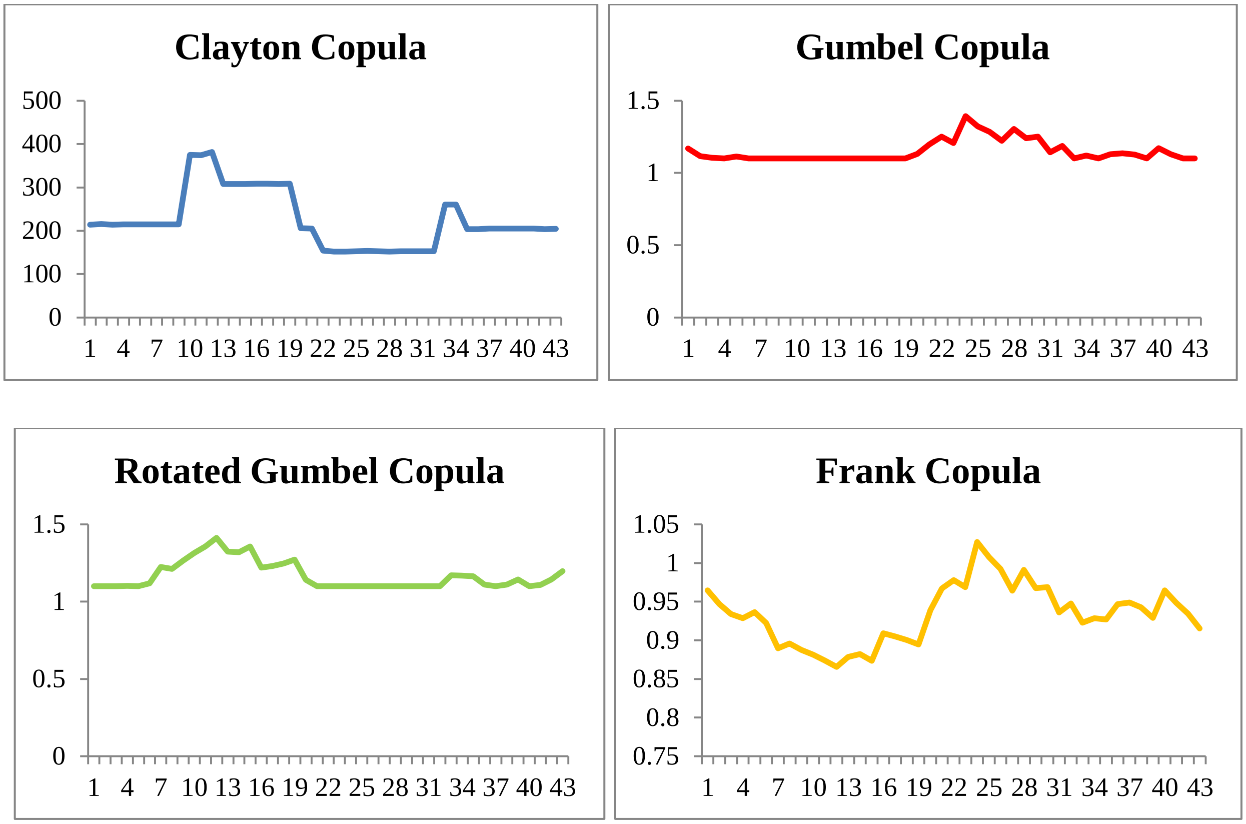
3.3.2 Copula Structural Break by Rolling Windows.

As confirmatory evidence for structural break, we performed an additional type of assessment. Instead of letting the initial estimation date fixed and then add a quarter at a time, we estimate theta by *rolling windows*. Estimating a suitable window-width is important, the reason is that if the width is very slim, theta estimation may be unreliable provided it won´t have enough information to calculate dependence between the variables; and if it is very broad, then we might question how well it is capturing the structural break information. Given that we have quarterly data, we fixed the width as two years.

Figure 29 illustrates theta behavior. Notice that the x axis is not labeled as time, rather it represents the period that was taken, for instance, the third data on the Clayton Copula doesn't refers to the third quarter of 2001, instead it refers to the theta that was estimated as in the third window comprising third quarter of 2001 to the third quarter of 2003. What is more, the first quarter that is affected by the crisis is the 30^{th} quarter, therefore, in order to test for structural break we split our estimations as those before reaching that quarter and those after that quarter.

Figure 29 show that estimating theta by rolling windows produces a smother outcome as compared to the previous case. Yet, in both cases there is one pattern before the crisis and another one after it. As we said, we need to test whether this difference is significant and we do this with a difference of means test without assuming equal variances. This outcome is shown in Table 6.

Figure 29



As with the previous estimation, we have to know which copula fits best our data and we measure that by comparing the likelihood. As before, it was taken as a theta estimation for the whole period and hence, the outcome is the same as in the preceding case where the Clayton copula fits best then Rotated Gumbel, then Gumbel and then Frank. All tests reject the null hypothesis of parameter equality; consequently, as measured by rolling windows, we conclude that there was a structural break.

Table 6

Null hypothesis that the two vectors are from populations with equal means, unequal variances.							
h = 0 indicates that ttest does not reject the null hypothesis at the default 5%							
Clayton		Gumbel		Rotated Gumbel		Frank	
Decision	1	Decision	1	Decision	1	Decision	1
P-Value	0.000119	P-Value	0.00830	P-Value	0.00062083	P-Value	0.0001396

Given the supporting evidence we found to reject the hypothesis of null-structural-break in 2008, we now asses individually how dependence affect our study variables. We make this by a bivariate dependence assesment based on theta estimation and dependence measures such as rank correlations.

3.4. Bivariate Estimation.

The objective is to determine whether there exists economic contagion from the U.S. to the Mexican economy after the 2008 crisis. As we outlined at the outset, our study period is separated in two sections, one being 2001:1-2008:3, and the other 2008:4-2013:4. Dependence can be calculated in different ways. Theta parameter for the Archimedean copulas measure dependence in its own scale, for instance, having a Gumbel theta close to 1 implies independence whereas for a Clayton copula, independence is reached as theta tends to 0.

We calculate these copula-theta dependence parameters and we also calculated the Spearman's Rho, Kendall's Tau and the Pearson's correlation coefficient. Kendall's Tau and Spearman's Rho were calculated because they possess some advantages over linear correlation; one of these advantages is that Tau and Rho are relatively unbiased to outliers whereas Pearson's is not. In addition, Tau and Rho correlations are *distribution independent* in contrast to Pearson's correlation coefficient which assumes Normality.

Copulas can be depicted graphically; at this point, it is informative to show different cases in order to grasp how they can be interpreted. We will start with the Gumbel copula; Figure 30 (Left) shows a $\theta = 1$ Gumbel-copula corresponding to independence, and a dependent copula with $\theta = 213$ (right). It can be seen that the more dependent it is, the graph becomes a soft sheet. In contrast, when theta is big, it becomes a double sheet.

Figure 30

$\theta = 1$, Independence $\theta = 213$, Dependence

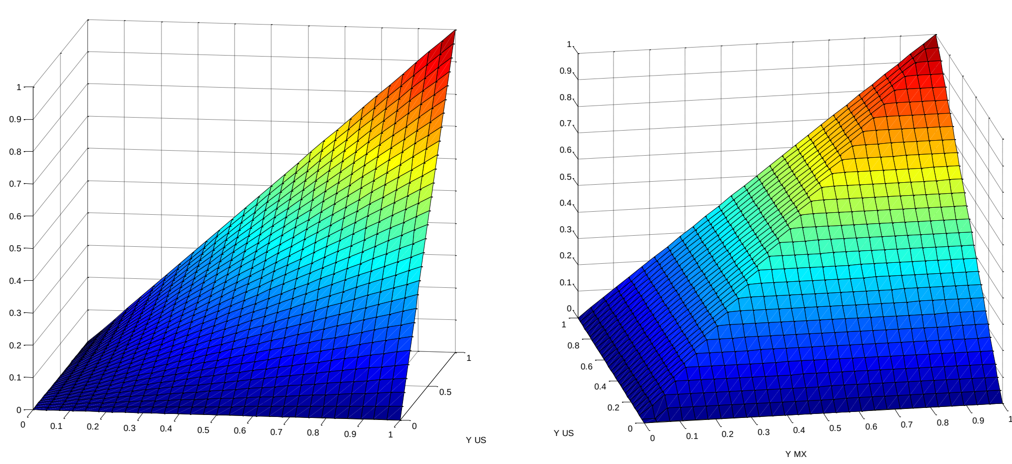
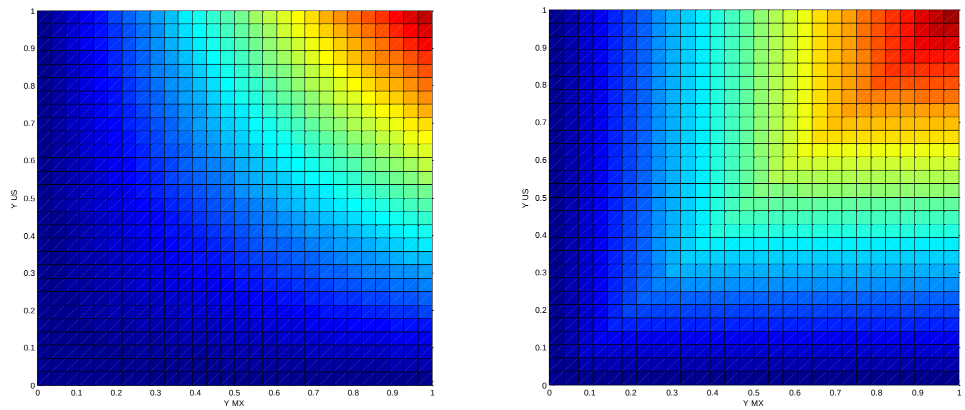


Figure 31 shows Clayton's copula level curves. The more independent the variables are, the more clustered they are near the upper left corner.

Figure 31

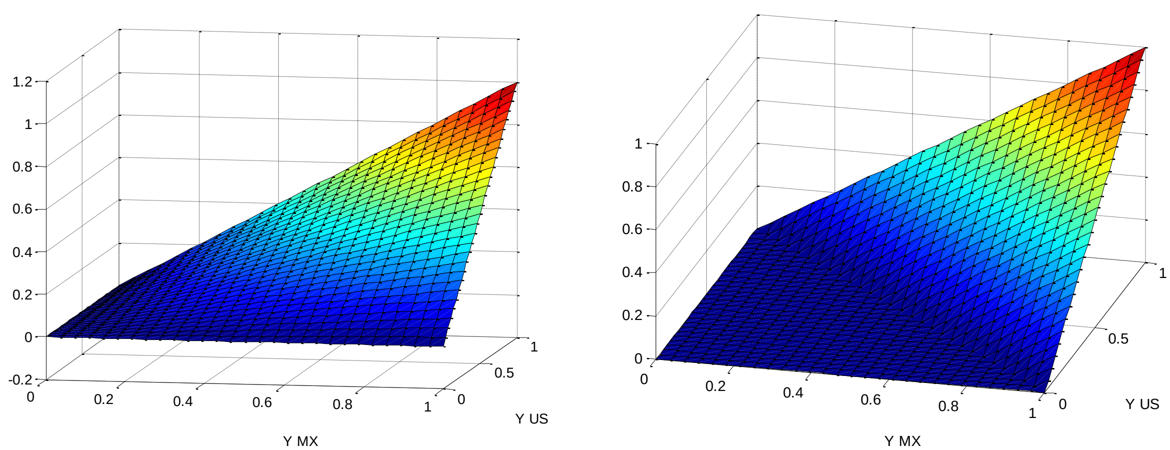
$\theta = 1$, Independence $\qquad$ $\theta = 213$, Dependence



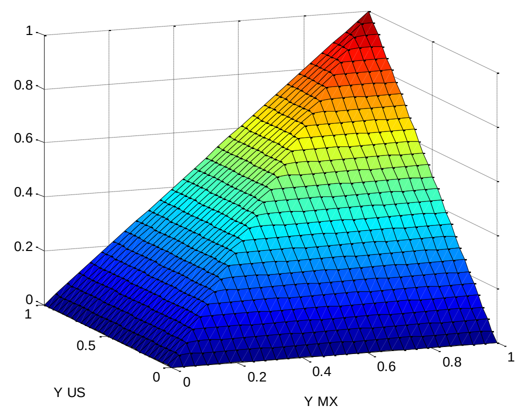
Frank copula allows for a negative dependence parameter, therefore, there would be a third case. Figure 32 plots them for $\theta = 0.0001,\ \theta = -300$ & $\theta = 300$ respectively.

Figure 32

$\theta = 0.0001$, Independence $\qquad$ $\theta = -300$, Negatively dependent



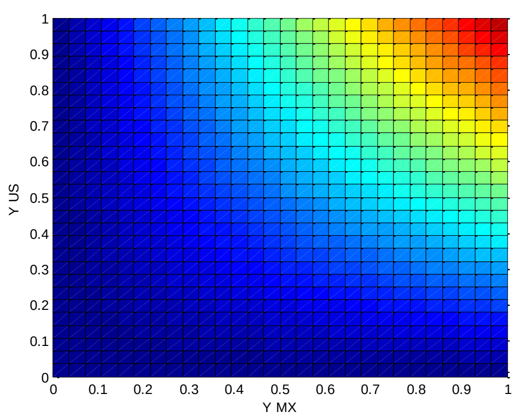
$\theta = 300$, Positively dependent



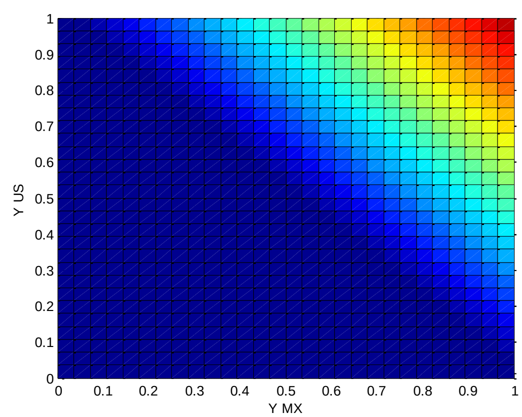
The corresponding level curves are the following.

Figure 33

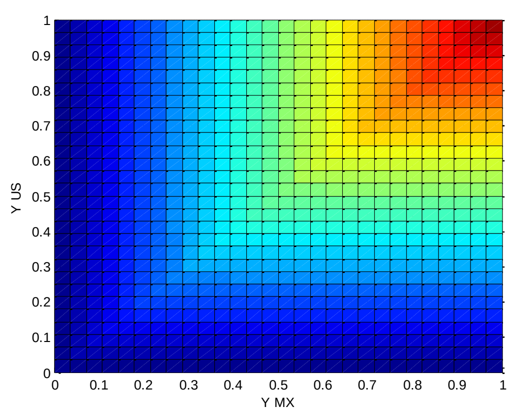
$\theta = 0.0001$, Independence



$\theta = -300$, Negatively dependent



$\theta = 300$, Positively dependent



Plotting copulas is a useful way of analyzing dependence. Nevertheless, since we have eight variables, we would have to show up to $\binom{8}{2} = 28$ combinations, which is an unfeasible task. This is why it is necessary to base our analysis in comparing dependence coefficients instead.

First, copula parameters are estimated directly, without being function of theta parameters (estimations labeled as COFIT). We have that the best fit among all was given by the Clayton Copula, followed by Frank and Gumbel copulas. Then we calculated theta parameters as function of rank correlation (estimations labeled as COPARAM). We also calculated Pearson and Rank correlation indexes as a secondary reference to asses if a dependence shift took place.

3.4.1 US GDP vs. Mexican GDP.

The relationship is direct, that is, the more the US GDP grows the more the Mexican GDP grows as well. In terms of dependency, Clayton's parameter changes from 0.2939 before the crisis to 0.334 after the crisis. That is, there was a dependence increase. Nevertheless, what is more interesting, is that it was the only indicator showing a dependency increase. Even the tau-dependent Clayton shows a dependence decrease. This could be a rare result provided the high interrelation between the Mexican and the American economy. Nevertheless, just after the crisis exploded, economic agents were not driven by rational decisions based on macroeconomics. They ultimately were driven by panic and uncertainty, which could have momentarily detached the behavior of these two economies. Table 7 shows this output.

Table 7

	Y US vs. YMX		
Index	**PRE**	**POST**	
Pearson	0.20484304	0.05549914	
Kendall	0.09359606	0.05714286	
Spearman	0.14285714	0.03246753	
Gumbel	1.10326087	1.06060606	COPARAM
Frank	0.84839739	0.51565064	COPARAM
Clayton	0.20652174	0.12121212	COPARAM
Gumbel	1.10585023	1.00000136	COFIT
Frank	1.24875096	0.49488238	COFIT
Clayton	0.293955983	0.334298953	COFIT

Additionally, although dependency decreased it didn’t decreased so much, it was barely few points that were reduced, meaning that the Mexican economy still is very dependent on the American economy, only that it detached temporarily given the big impact caused by the crisis. According to our definition, we can say that contagion took place from the American economy given there was a change in dependence. Let´s now see how this shock is transmitted to the rest of the Mexican economy.

3.4.2 Mexican GDP vs. Exchange rate MXN/USD (e).

The exchange rate is one of the most crucial variables for the so-called stability policy. As we previously said, this policy tends to appreciate the exchange rate to reduce inflation pressure. Moreover, nailing down inflation in this way could have the side effect of affecting negatively Mexico’s export sector. Let us see what happened. The relationship is inverse, hence, the more depreciated the peso is, the more the GDP increases. Once again, there is a tendency towards independence in most indicators, see Table 8. Consequently, Mexican policymakers cannot rely on maintaining an appreciated exchange rate to foster growth.

Table 8

	Y MX vs. e		
	PRE	POST	
Pearson	-0.8260828079	-0.6745194614	
Kendall	-0.6354679803	-0.5142857143	
Spearman	-0.8290640394	-0.6818181818	
Gumbel	2.7432432432	2.0588235294	COPARAM
Frank	-8.9600660417	-6.0021355736	
Clayton	3.4864864865	2.1176470588	
Gumbel	1.0000013575	1.0000013575	COFIT
Frank	-9.2807575472	-6.0022351874	
Clayton	0.0000014509	0.0000014509	

3.4.3 Mexican GDP vs. Government Expenditure.

Stability policies state that fiscal discipline, or near discipline levels are needed to foster growth. Policymaker’s rationale is that by avoiding overspending, the government does not *crowds-out* resources that could have been allocated to private investment. Moreover, by maintaining fiscal discipline, aggregate demand gets to be contained and, therefore, inflation is contained as well. If inflation is contained, the policy states, investment takes place provided that investors have a foreseeable future.

What we found is a direct relationship between GDP and government spending, implying that, the more the government spends, the more the GPD increases, no quite as the stability policy claims. Nevertheless, we see an acute loose of dependency between these variables, starting from Pearson's correlation through copula dependency parameters, see Table 9. Thus, even if the transmission mechanism would work as it is established by the stability policy, these variables tends to be less reactive.

Table 9

	Y MX vs. G		
	PRE	POST	
Pearson	0.375493802	0.096038217	
Kendall	0.24137931	-0.038095238	
Spearman	0.366502463	0.045454545	
Gumbel	1.318181818	1.03960396	COPARAM
Frank	2.281638309	-0.343260791	
Clayton	0.636363636	0.079207921	
Gumbel	1.465058369	1.228965182	COFIT
Frank	2.941467946	0.806648943	
Clayton	0.847612604	0.075430999	

3.4.4 Mexican GDP vs. BMV.

Given that the Mexican Stock Exchange index is an investment barometer for the economy, we found that there is a positive relationship between the GDP and BMV. We also found dependence lost between these variables. For the Clayton copula –which has the best fit - there was a dependence lost from 0.89 to 0.51 on the fitted parameters and a loss of about the same amount in the tau-dependent parameters, see Table 10. Somehow, this effect was to be expected since it is known that financial investors tend to detach their decision making from real variables during euphoria or panic stages.

Table 10

	Y MX vs. BMV		
	PRE	POST	
Pearson	0.421219033	0.311539746	
Kendall	0.266009852	0.219047619	
Spearman	0.374876847	0.288311688	
Gumbel	1.362416107	1.280487805	COPARAM
Frank	2.542355947	2.052125067	
Clayton	0.724832215	0.56097561	
Gumbel	1.635504337	1.3014213	COFIT
Frank	3.274078349	2.392172553	
Clayton	0.893283934	0.512119351	

3.4.5 Mexican GDP vs. Interest rate (Cete's).

The relationship is inverse, though, as always, there was a dependence decrease. In this case, dependence got down by about 0.2 as measured by Pearson and almost any other indicator. Again, the dependence reduction is substantial and is quite relevant for policymaking. Gumbel's parameter is very close to independence at 1.02, and the same is true for the Clayton's tau-dependent parameter, see Table 11. Since interest rate is one anchor for the stability policy, it is clear that this variable must be kept under control at all times. Yet, if the state would want to promote growth by lowering the interest rate, they will have to lower the rate more than in normal times to get the same output.

Table 11

	Y MX vs. CETES		
	PRE	POST	
Pearson	-0.3557266	-0.143016188	
Kendall	-0.256157635	-0.028571429	
Spearman	-0.349753695	-0.093506494	
Gumbel	1.344370861	1.029411765	COPARAM
Frank	-2.437043985	-0.257313033	
Clayton	0.688741722	0.058823529	
Gumbel	1.000001358	1.000001358	COFIT
Frank	-2.824138238	-1.12001444	
Clayton	0.000001451	0.000001451	

3.4.6 Interest rate (Cete's) vs. Exchange rate MXN/USD (e).

Interest rate is one of the main instruments to control the exchange rate without directly intervening in the Foreign Exchange market. This is because, when the rate is high (higher than other competing foreign investments) it attracts foreign capitals to Mexico, in the process, these capitals are converted from their local currency, most of which are dollars, to Mexican pesos. By doing so, the demand for pesos is decreased and its supply increases (alternatively, it can be said that the dollar's demand is decreased), hence the peso is appreciated against the U.S. dollar.

We got a positive relationship between these variables. The higher interest rate the lower levels of exchange rate (the more the MXN/USD gets appreciated). As with all the other variables, dependence shrinks to almost half of what it was before the crisis, which coincides with the fact that Mexico's central bank, BANXICO, had a challenging time trying to stabilize the exchange rate with the interest rate. Actually, at some point it had to intervene directly in the foreign exchange market with up to 15 billion. We can see from Table 12 that Clayton's theta is cut by less than half in both cases; the same is true with the Frank copula and the Gumbel copula approaches close to independence in the tau-dependent estimation.

Table 12

	Cetes28 vs e		
	PRE	POST	
Pearson	0.318867932	0.122838669	
Kendall	0.216748768	0.019047619	
Spearman	0.286206897	0.053246753	
Gumbel	1.27672956	1.019417476	COPARAM
Frank	2.028834544	0.17147897	
Clayton	0.553459119	0.038834951	
Gumbel	1.473919961	1.203867183	COFIT
Frank	2.488300311	0.937132633	
Clayton	0.790340018	0.307545170	

These are the relations we considered as the most important for our document. In every relation we studied, we found a dependence decrease, which means that at least momentarily, transmission mechanisms became clumsy, and therefore, not as effective as before. Moreover, the stability model used by Mexican policymakers who state that, economic growth is going to sprout from price-stability, is going to have more difficulties after the crisis given the new variables' lack of responsiveness. As the matter of fact, if before the crisis this stability policy was already having problems to stimulate growth, these problems are magnified now.

We must notice that this lack of responsiveness goes beyond Keynes' trap for liquidity, since in our case, it is not only that interest rate gets irresponsive to an increase in money supply, it is that all variables stop responding as we knew them. Therefore, in order to implement an expansionary policy, the government must now exaggerate their policies to move the economy out of danger.

3.5. The Frailty Model.

Frailty models lie within a field called survival analysis, which in general is interested on studying the *time to an event*. In clinical data, for instance, researchers are interested in quantifying the time in which a patient relapses of a certain illness or occasionally, to quantify the time from the starting event to death. Therefore, this formulation comprises two events, an initial and a final event, the latter is usually called *failure*.

Within this framework, T denotes time to failure; it is a variable with continuous distribution and finite expectation. Let $f(t), t \geq 0$ be the density function for T, therefore, we are going to be interested on the cumulative density function, which we are going to define as the survival function $S(t) = P(T > t) = \int_t^{\infty} f(x)dx = 1 - F(t), t \in [0, \infty)$, it represents the probability that an individual would survive more than t. Consequently, the probability that a failure takes place before time *t* is given by $F(t) = P(T \leq t) = 1 - S(t)$.

Also, a hazard function is defined as $\lambda(t) = f(t)/S(t) = -\partial S(t)/\partial t/S(t)$, and expresses the probability that a person have an event (failure) on the next period. The cumulative hazard function has a curious property. First, it is given by $\Lambda(t) = \int_0^t \lambda(x)dx$, and it has the property that $S(t) = \exp(-\Lambda(t))$.

Sometimes, it is of interest to study the relation between the survival time and certain variables. Usually, to study this relationship, researchers used the Cox-model. This model admits not only continuous but also categorical variables. The relative risk is given parametrically and the hazard function is not parametric. The hazard function is given by:

$$\lambda_i(t) = \lambda_0(t)\exp(\beta^T X_i)$$

Where:

λ_0 = baseline hazard function, the most common types of baseline functions are Exponential, Weibull, Gompertz, Lognormal and a Loglogistic.

$\exp(\beta^T X_i)$ = represents the relative risk of individual *i*,

X_i = Vector of covariates,for individual *i*.

We will turn out attention to the study of frailty models. These types of constructions are ideal for random effects models that take into consideration unobserved heterogeneity. We will center our attention to the multivariate frailty models for clustered data. One of the most interesting features of these types of models is that the type of data that they deal with, clustered, opens the possibility for dependence of individuals between clusters. The formulation for frailty models in terms of conditional hazards is the following:

$$\lambda_{ij}(t/u_i) = \lambda_0(t)u_i exp\left(X_{ij}^T\beta\right)$$

$\lambda_0(t)$ = baseline hazard function.

u_i = heterogeneity component (frailty term in group *i*).

X_{ij}^T = vector of covariates.

β = vector of regression coefficients.

Frailty, as given by the u_i term, is an unobservable realization of a non-negative random variable U with mean one (whenever it exists). We can choose between different types of frailty functions: Gamma frailty, Lognormal frailty, Positive Stable frailty and Inverse Gaussian frailty. The intuition of this model is that the variability between groups causes different risks for each group. A small value of the frailty parameter θ implies more heterogeneity among groups, and stronger dependence within groups.

In our setup, we are interested on studying the hazard of having a negative GDP growth. This hazard is going to be modeled as function of the Mexican GDP (mx_gdp_usd), M2 (m2_dollars), the Mexican stock market index (bmv) and government spending (gov_spend_usd). We picked a Gamma frailty distribution and a Weibull baseline hazard function, the numerical method used was Simulated Annealing.

The Gamma frailty has the following formulation:

$$U \sim Gamma(\theta); \qquad f(u) = \frac{\theta^{-\frac{1}{\theta}} u^{\frac{1}{\theta}-1} exp(-u/\theta)}{\Gamma(1/\theta)}$$

The baseline hazard function we choose is a Weibull, and it is given by the following formulation:

$$\lambda_0(t) = \lambda \rho t^{\rho-1}$$

Nevertheless, before showing the results we are going to describe how we built the database. The risk of recurrence needs independent variables, which are, time to recurrence, and a Status variable. In our case, the time to recurrence is the time between one quarter with a negative GDP growth and another. For the status variable, which signals if the event is present or not, we counted the number of quarters between one event and the other and then we signaled it as 1 present, 0 absent. Since variations are prevalent it doesn´t make sense to account for every negative variation, we took as a failure only those quarters in which the negative variation was bigger than 1.5 percent.

Every "patient" then, is going to be represented by every year from 2001 to 2013, and within that timeframe, we divided our timespan in four clusters, two before the crisis and two other after the crisis. More specifically, cluster "1" comprises the timeframe 2001-2003 inclusive, cluster "2" comprises 2004-2006, cluster "3" comprises 2007-2009 and cluster "4" comprises 2010-2013. This clustering information was stored in a summarizing variable called "cluster2". In order to perform a cox regression to estimate frailty, we had to do further processing in order to simplify computations, this simply was a variable re-scaling to make them more manageable.

Variables GDP, M2 and BMV had a significance of 0.01 and Government Spending had a significance of 0.05. The frailty estimation as given by the theta coefficient is 0.097, which is relatively low. This implies a greater degree of heterogeneity among groups and a stronger association within groups, that is, we have well distinguished clusters. We got a Kendall's tau of $\tau = \frac{1}{1+2\theta} = 0.046$. This value is also low, but confirms the result obtained by the θ parameter, this is because large θ values implies independence.

Now, it is time to show the hazard of GDP decrease as influenced by the covariates we choose. For the GDP, it redundantly influences on the hazard of GDP decrease, still, we report that we got a coefficient of -2.67. This implies an inverse relationship, the bigger the size of the GDP growth the less the hazard of recurring into a negative GDP growth.

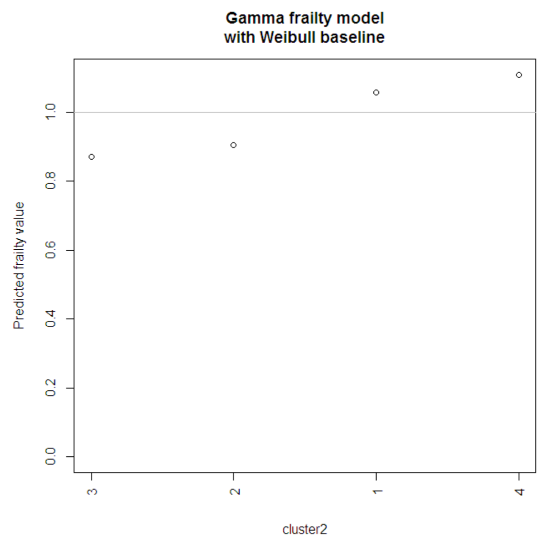
For M2, we got a coefficient of -10.71. Consequently, the more money there is in circulation the biggest the hazard of GDP decrease. Notice that as long as there is enough economic activity, there can be a big amount of money in circulation without negative consequences. The problem is that in Mexico, the economic activity is not buoyant and therefore, it is a risky trait to have so much money circulating.

For the Mexican stock exchange BMV we got a coefficient of 1.135. This implies that the more the stock market increases, the bigger the hazard of having a negative GDP growth. Finally, for government spending we got a coefficient of -40.31. This is by far the most important variable that determines the hazard of negative growth: the bigger the government spending the less likely the chances of relapsing into a negative growth.

It is also possible to plot the predictive values in frailty plot. We have the vales above the line are the ones that are most likely to present failure. Notice that on Figure 34 the label on the horizontal axis is called cluster2. As we explained at the outset, this does not mean that it is plotting the information for the second cluster alone. Instead, cluster2 is the summarizing variable that stores all four clusters. We can see that the clusters that are more prone to failure are “1” and “4” (those above the line). Cluster 1 corresponds to 2001-2003 and 4 to 2010-2013.

This information obviously makes sense. In the first cluster, from 2001-2003 the United States faced a small recession, the 9/11 attacks and a major military operation. On the fourth cluster, the world economy was facing the consequences of the crisis and the posterior instability that brought about the European Union. These events have a notable effect on the hazard of having a negative GDP growth in Mexico.

Figure 34



This concludes our frailty analysis, yet we have a final word to say with respect to the association between the set of financial variables and the set of real variables as accounted by canonical correlation.

3.6. Canonical Correlation.

In multiple regression, the idea is to relate or explain a single dependent variable with a set of other independent variables. In Canonical Correlation, the idea is to extend the concept to something more useful and more general, which is to investigate the relationships between several variables in a set A, and a second set of variables B.

The goal in Canonical Correlation is to find functions of one set of variables that maximally correlate with linear functions on the other set. It is important to notice that in contrast to other methods such as Principal Components, not standardizing the variables does not have an effect on the final output. In addition, the amount of variables in each set does not have to be the same.

According to (Everit, 2005) the idea is to take the association between A and B as the largest correlation between two single variables u_i & v_j derived from each set of variables as linear combinations. Each u_i is a linear combination of the variables in A, and the same is true for v_j and B. Each linear combination's coefficients are chosen such that the following conditions are fulfilled:

- $u_i's$ are mutually uncorrelated $\forall\ i \neq j$
- $v_i's$ are mutually uncorrelated $\forall\ i \neq j$
- The correlation between u_i & v_j is R_i for $i = 1, \ldots, s$ with $R_1 > R_2 > \cdots > R_s$
- $u_i's$ are uncorrelated with $v_j's$ except for v_i, that is, $cov(u_i, v_j) = 0\ \forall\ i \neq j$

The coefficients for the linear combinations are determined as the eigenvectors of the following matrices:

$$E_1 = R_{11}^{-1} R_{12} R_{22}^{-1} R_{21} \ \ \& \ E_2 = R_{22}^{-1} R_{21} R_{11}^{-1} R_{12}$$

Where:

R_{11} = Correlation between the vectors in A.

R_{22} = Correlation between the vectors in B.

$R_{12} = R_{21}$ = Matrix of correlations across the two sets.

The s canonical correlations express the association between A and B after removal of the within set correlation and are obtained as the square roots of the non-zero eigenvalues of either E_1 or E_2.

In our empirical study, we are interested on quantifying the relation between the financial and the real-economic variables. To that end, we allocate the real variables in set A: US GDP, Mexican GDP and Government Spending, and we allocate the financial variables in set B: exchange rate, monetary aggregate M2, Mexican Stock Exchange Index (BMV) and the Cete's interest rate. In this case, we have that $p = 3$ for set A and $p = 4$ in set B, consequently our system of equations has a structure similar to the following:

$$\begin{aligned}
\boldsymbol{A_1} &= \boldsymbol{\beta_{10}} + \boldsymbol{\beta_{11} B_1} + \boldsymbol{\beta_{12} B_2} + \boldsymbol{\beta_{13} B_3} + \boldsymbol{\beta_{14} B_4} + \boldsymbol{\varepsilon_1} \\
\boldsymbol{A_2} &= \boldsymbol{\beta_{20}} + \boldsymbol{\beta_{21} B_2} + \boldsymbol{\beta_{22} B_2} + \boldsymbol{\beta_{23} B_3} + \boldsymbol{\beta_{24} B_4} + \boldsymbol{\varepsilon_2} \\
\boldsymbol{A_3} &= \boldsymbol{\beta_{30}} + \boldsymbol{\beta_{31} B_3} + \boldsymbol{\beta_{32} B_2} + \boldsymbol{\beta_{33} B_3} + \boldsymbol{\beta_{34} B_4} + \boldsymbol{\varepsilon_3}
\end{aligned}$$

To start with, we must be sure that there is a relationship of some kind between sets A and B. We evaluate this by testing whether these coefficients are different from zero. This is similar to testing whether variables in A are independent from those in B. The test is based on Wilk's lambda with the following hypothesis: $H_o: \beta_{ij} = 0\ \forall\ i, j$. Our estimation gives a value of 0.06527 with a p-value of 0.0000, we therefore, reject the null hypothesis of no relationship, and conclude that statistical evidence supports the existence of dependence among A and B.

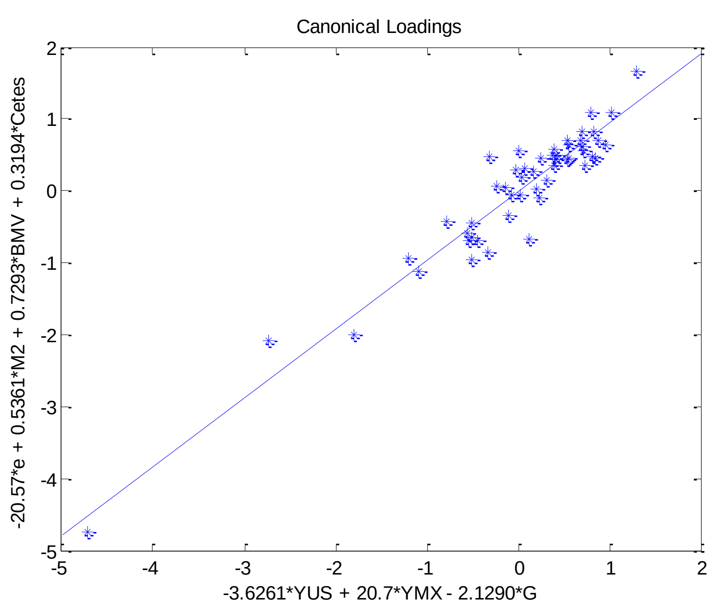
Knowing that it makes sense to apply Canonical Correlation to our data, we now proceed to coefficient estimation. This is a set of correlations that relate how strong dependence is between each linear relationship from set A to set B. As Table 13 shows, the only high correlation is given between the first pair of relationships.

Table 13

Relation label	Canonical Correlation
1	0.91472
2	0.21787
3	0.02148

The 0.91472 coefficient implies that about 91.5% of the variation of A_1 is explained by the variation of B_1 and so on. Correlation for the second and third relations is very low, which is why we only are going to take the first. We can see the high degree of association by plotting the canonical scores. As Figure 35 depicts, the association between these sets is remarkably high without having any outlier.

Figure 35



Summarizing the equation estimations we have:

$$A_1 = -3.6261 * YUS + 20.6815 * YMX - 2.129 * G$$

$$B_1 = -20.5702 * e + 0.5361 * M2 + 0.7293 * BMV + 0.3194 * Cetes$$

As in regression analysis, each coefficient gives the individual contribution of each variable to the canonical variable A_1 and B_1 respectively. Finally, we must estimate the correlations between each variable and the canonical variable $(A_i \& B_i)$. Table 14 shows that for variable A, the biggest correlation is given by Mexico's GDP and that of the U.S. that is, interpreting A variable as a canonical variable for the performance of real variables Y MX and Y US are the most important. For B variable, interpreted as a set of financial variables we see that the most important variable is the exchange rate followed by the Mexican Stock exchange, M2 and the interest rate.

Table 14

Correlations between COVARIATES and canonical variables			
A		B	
Y US	-0.48259	**e**	0.99798
Y MX	-0.91929	**M2**	0.23365
G	-0.07862	**BMV**	-0.57767
-	-	**Cetes**	0.18383

Now, wrapping everything up we state that the best explanatory variables for the real-economy, set A, are the exchange rate and the stock exchange. Moreover, this analysis does not refers to causality, meaning that we cannot state that if e depreciates then government spending will grow, what we can say is that, if initially there was an increase in government spending, then probably the exchange rate will depreciate and because of that, we have this direct relation between the exchange rate and A.

Going the other way around, the best explanatory variables for the financial variables, set B, is Mexico's GDP and the U.S. GDP. Secondly, the US GDP is also hugely important; the least important is government spending. Since the relationship is inverse, we can state that the more the GDP increases, the more appreciated the exchange rate becomes.

Canonical Correlation analysis is a very powerful almost unknown technique. Its importance is that it allows us to relate two sets of variables and also to calculate a correlation between these sets which is something that is rarely possible with other methods. In our case, this correlation is 0.91472 and connects the real economy and the financial sector with a high degree of association.

FINAL REMARKS

- We presented an alternative method based on copulas to test structural break. This method is different to the stability tests based on regression residuals.
    - Results based on Rolling Windows and adding a period at a time confirm a significant change in the dependence structure when the 2008 crisis hit the global economy.
    - Parameter estimations before the crisis were compared to those after it, showing a statistically significant change, which supports the structural break hypothesis.
    - In the cases with best fit, dependence patterns actually tends to independence, though in some cases, as with the Gumbel copula, dependence slithers upside down as uncertainty raises, this could be attributed to economic agent's irrationality arising from the previous tremor.
- Bivariate estimations show that there was a change in dependence from the US to the Mexican GDP, that is, we found Contagion from the American economy to the Mexican. In many cases, dependence decreased as uncertainty governed the agent's behavior after the shock.
- What bivariate results tell us is that the Mexican economy became less responsive, at least for a period of time. Consequently, if the stability model was having problems procuring growth before the crisis, stability is going to have even more problems after it. This is because, policymakers have to do more to achieve the same results as before.
- Frailty estimations yielded a greater amount of heterogeneity among groups and a stronger association within groups.
- Frailty-Cox regression yielded the following:
    - M2 has an inverse relation of -10.71with the hazard of having a negative GDP growth (relapse).
    - The stock exchange a positive relation of 1.35, the more the capital-speculative investments grow the more the hazard or relapse.
    - Government spending is by far the most important covariate, the bigger the spending the lesser the hazard of relapse. Currently, Mexican economic authorities are procuring the opposite, a decrease in spending.
- Frailty estimation yielded that clusters that are more prone to failure are one and four.
- Through Canonical Correlation we found a high degree of dependence between real and financial variables. Therefore, it does not come as a surprise that the 2008 crisis which

started as a financial phenomenon in the US, was transmitted very quickly to the rest of the Mexican economic variables.

- In addition, we found that the best predictor for the Mexican Financial variables set is the Mexican GDP, and in second place the US GDP. Conversely, the best predictor for the real variables set is the exchange rate.
- After the 2008 crisis stroke the Mexican economy, its negative effects diffused to the rest of the variables by reducing the dependence among them. This means that the Mexican economy in general, becomes less reactive to any kind of stimulus.
- During the crisis, we saw a sluggish reaction from the government. This was in part because the Stability Model constrains the government's ability to implement countercyclical policies.
    - In particular, Mexico's government loosed its ability to control monetary policy by providing the central bank with independence. Moreover, the central bank's enacted objective of pursuing price-stability alone obliged the bank to implement contractive policies to reduce demand in order to decrease inflation, this in turn, made it more difficult to fight back the 2008 crisis.
- As we have seen, given the Stability Export-Led-Growth model, Contagion took place from the American economy to the Mexican economy. More importantly, Mexico's ability to grow was compromised given Mexico's high dependence with the American economy.
- Summarizing, in terms of the Stability-Model that Mexico currently follows, the theoretic outcomes that we should be enjoying are, a stable inflation rate together with a steady interest rate that produce a foreseeable future which will end up in investment and in an increased output. Instead, what it really is happening is:
    - We have had a sluggish economic growth, despite the fact that *price-stability* has been reached. Moreover, the government´s ability to respond to the crisis was compromised and some instability and uncertainty took place despite the fact that Mexico utilized a model that was supposed to procure stability. Consequently, México's response after the crisis was appalling.
    - Since government spending and the GDP became less dependent after the crisis we have that, if policymakers want to stimulate the economy, they have to spend more than before to get the same outcome. This fact runs contrary to the stability main point of reducing government spending to reduce inflation.
    - Something similar happens with the interest rate. If policymakers want to stimulate the economy, they need to lower the interest rate more than before.

This is definitely a dangerous for it would stop foreign investment and it could stop the inflow of foreign capitals from appreciating the exchange rate that could compromise the inflation rate, which is an integral part of the stability policy.

REFERENCES.

- Bancomer. “Hacia la Estabilidad Permanente de los Precios”. *BBVA Research, Serie de Propuestas*, No. 24, (December 2002).
- BANXICO. “Instrumentación de la Política Monetaria a través de un Objetivo Operacional de Tasas de Interés”. In *Annex 3 of the Inflation Report*, Mexico City: The Bank of Mexico, (July-September, 2007).
- BANXICO. “La Banca Central y la Importancia de la Estabilidad Económica”. June 16th, 2008. Mexico City: The Bank of Mexico. Conference available at Bank of Mexico website: http://www.banxico.org.mx/politica-monetaria-e-inflacion/material-de-referencia/intermedio/politica-monetaria/%7B3C1A08B1-FD93-0931-44F8-96F5950FC926%7D.pdf. Accessed March 2011.
- BANXICO. “Inflation Targeting Regime”. Mexico City: The Bank of Mexico. Document available at Bank of Mexico website: http://www.banxico.org.mx/politica-monetaria-e-inflacion/material-de-referencia/intermedio/politica-monetaria/%7B0C315B3B-2E91-C5B3-4B46-EF566A29AF11%7D.pdf. Accessed March 2011.
- Barro, Robert. “Inflation and Economic Growth”. Working Paper Series, 5326, National Bureau of Economic Research, (October 1995).
- Burnam, Len. “Rising Economic Inequality and Tax Policy”. Power Point presentation to the Urban Institute Board, 1st May 2007. Tax Policy Center, Washington, DC.
- Bernanke, Ben. “The Benefits of Price Stability”. Speech at the Center for Economic Policy Studies. Princeton University, February 24, 2006.
- Bernanke, Ben. & Mishkin, Frederic. “Inflation Targeting: A New Framework for Monetary Policy”. *Journal of Economic Perspectives*, Vol. 11 (Spring, 1997), 97-116.
- Brash, Donald. ”Reducing Inflation in New Zealand: Some Practical Issues”. Proceedings, Federal Reserve Bank of Kansas City, (1996): 123-138.

  Carstens, Agustin. “La Economía Mexicana”. 6th of April 2011. Governor of the Bank of Mexico, BANXICO. Presentation to the Senate. Conference available on BANXICO website:http://www.banxico.org.mx/publicaciones-y-discursos/discursos-y-presentaciones/presentaciones/%7BCC1D3C87-74EB-936F-058B-C904E5B40B22%7D.pdf

  CEFP. “Evolución de “
- Castelnuovo, E. Nicoletti-Altimari, S. & Rodríguez-Palenzuela, D. “Definition of Price Stability, Range and Point Inflation Targets: The anchoring of Long-Term Inflation

Expectations". Working Paper Series, No. 273. European Central Bank: (September 2003).

- Cavallo, Domingo. "Lessons From the Stabilization Process in Argentina, 1990-1996". Proceedings, Federal Reserve Bank of Kansas City. (1996): Pages 169-178.
- Chavez, Fernando & Cuevas, Víctor. "Déficit, Deuda y Reforma Fiscal en México". Problemas del Desarrollo, Revista Latinoamericana de Economía. Vol. 38, No. 147 (2007). las Tasas de Interés Activas y Pasivas y su Entorno Internacional". Centro de Estudios de las Finanzas Públicas, Cámara de Diputados. 2009. Available at CEFP website:http://www.cefp.gob.mx/intr/edocumentos/pdf/cefp/2009/cefp0202009.pdf. Accessed June 2011.
- Curry, Timothy & Shibut, Lynn. "The Cost of the Savings and Loans Crisis: Truth and Consequences", Federal Deposit Insurance Corporation Banking Review, Vol 30, No.2, December 2000.
- Davidson, Adam. "How AIG Fell Apart." Reuters, September 18, 2008. http://www.reuters.com/article/2008/09/18/us-how-aig-fell-apart-idUSMAR85972720080918
- De Matteis, Robeto. "Fitting Copulas to Data". Diploma Thesis, Institute of Mathematics of the University of Zurich, 2001.
- Escrivá, José. "Latin America shines: Is it for real this time?". PowerPoint presentation, Nov 2010, BBVA Research.
- Fisher, Stanley. "Why are Central Banks Pursuing Long-Run Price Stability?". *Proceedings*, Federal Reserve Bank of Kansas City, (1996): 7-34.
- Fergusson, Nial. *The Ascent of* Money. New York: Penguin Press, 2008.
- Frenkel, Jacob. "Israel´s Experience with Inflation". Proceedings, Federal Reserve Bank of Kansas City, (1996): pages 139-146.
- Gayer, Ted & Rosen. Harvey. *Public Finance*. 8th Ed. McGraw-Hill/Irwin. 2008.
- Greenspan, Alan. "Testimony before the Subcommittee on Economic Growth and Credit Formation of the Committee on Banking, Finance and Urban Affairs". U.S. House of Representatives, (February 22, 1994).
- Huerta, Arturo. "La Liberalización Económica y la Estabilidad Macroeconómica: Modelo Fracasado de Desarrollo". *Economía UNAM*, Vol.6 No.018 (2009 a)
- Huerta, Arturo. *La Crisis en EUA y México: La dificultad de su salida*. Dirección General de Publicaciones y Fomento Editorial, Mexico City: Facultad de Economía UNAM. 2010.

- Hult, John. *Options Futures and Other Derivatives*. 7th Edition, New Jersey: Pearson-Prentice Hall, 2009.
- Issing, Otmar "The Role of Fiscal and Monetary Policies in the Stabilization of the Economic Cycle", Speech on 14 Nov 2004, Mexico City. The ECB conference Board. Available at ECB press section on its website http://www.ecb.int/press/key/date/2005/html/sp051114.en.html, Accessed January 2011.
- Labaton, Stephen. "S.E.C. Concedes Oversight Flaws Fueled Collapse", The *New York Times*, September 26, 2008.
- Ling, C.h. "Representation of Associative Functions". *Publicaciones Methematicae Debrecen*, Num. 12 (1965):189 – 212.
  http://www.nytimes.com/2008/09/27/business/27sec.html?em
- López, Diana. Cruz, Mario. Zuñiga, Karla and Galvan, Carlos. "Cronología de la Crisis". Economía Informa, UNAM. No. 357 (March-April 2009): 93-109.
- Lorde, Troy. "Export-led Growth: A Case Study of Mexico". *International Journal of Business, Humanities and Technology*. Vol. 1 No.1 (July 2011): 33-44.
- Lustig, Nora. "NAFTA: Doing Well by doing Good", *The Brookings Review*, in The World Watch, March 1994 p.47
- Manner, Hans. "Estimation and Model Selections of Copulas with an Application to Exchange Rates". *Research Memoranda 056*, Maastricht: METEOR, Maastricht Research School of Economics of Technology and Organizations, 2007.
- Marquez, Yanod. "Balanza de Pagos, Estabilidad y Crecimiento en México 1979-2005". *Revista de Economía Internacional*. Vol. 11, No. 21, Segundo semestre (2009): 179-203.
- McDonough, William. "A Framework for the Pursuit of Price Stability". Remarks made by Mr. McDonough for the speech at the Annual Financial Services Forum of the New York State Bankers Association, March 21st 1996 and the speech at the Economic Club of New York, October 2nd, 1996.
  Federal Reserve Bank of New York Economic Policy Review, August 1997
- Mishkin, Frederic. "Inflation Targeting". Prepared For Brian Vane and Howard Vine. *An Encyclopedia of Macroeconomics*. 2001. Document available at Columbia University website:
  http://www0.gsb.columbia.edu/faculty/fmishkin/PDFpapers/01ENCYC.pdf. Accessed January 2011

- ______________. "International Experiences with Different Monetary Policy Regimes". *Institute for International Economic Studies*, Stockholm. Seminar Paper No. 648, 1998.
- Matteis, Roberto. *Fitting Copulas to Data*. Diploma thesis. Institute of Mathematics, Swiss Federal Institue of Technology Zurich (ETH), University of Zurich. 2001.
- Moreno-Brid, Juan; Rivas, Juan; & Santamaria, Jesus. "Mexico: Economic Growth Exports and Industrial Performance after NAFTA". *Estudios y Perspectivas*. CEPAL, No.42 (December 2005).
- Nelsen, Roger. *An Introduction to Copulas*. Springer, USA: 2006.
- Palley, Thomas. "Export-Led Growth: Evidence of Developing Country Crowding Out". In *Economic Integration, Regionalism, and Globalization*. Arestis, Baddeley, & McCombie (eds), Cheltenham: Edward Elgar, 2003.
- ______________. "Financialization: What Is and Why It Matters", The Levy Institute of Bard College, Working Paper No. 535. 2007.
- ______________. "The Rise and Fall of the Export-led Growth". Levy Economics Institute of Bard College, Working Paper No. 675, 2011.
- Patton, Andrew. "Modelling Asymmetric Exchange Rate Dependence". Financial Econometrics Research Centre. Working Paper WP04-04. London School of Economics. (2004).
- Sánchez, Manuel. "Objective and Functioning of the Bank of Mexico". 1st July 2010. Sub-Governor of Mexican Central Bank, BANXICO. Closing speech on the Bank of Mexico´s Cathedra at UAM. Conference available on BANXICO website:http://www.banxico.org.mx/material-educativo/informacion-general/catedra-banco-de-mexico/universidad-autonoma-de-mexico-uam-mayo-julio-2/%7B86959E7A-2B91-9D43-6558-DF2515CA3817%7D.pdf. Accessed February 2011.
- __________. "La interminable Tarea de la Estabilidad de Precios". 26th January 2011(a). Sub-Governor of Mexican Central Bank, BANXICO. Financial Conference Meeting at ITESM CCM. Conference available on BANXICO website: http://www.banxico.org.mx/dyn/publicaciones-y-discursos/discursos-y-presentaciones/discursos/%7B43BCFB1D-52B1-9DC5-929C-8025A454F522%7D.pdf. Accessed February 2011.
- __________. "Contribución del Banco de México al Crecimiento". 7th September 2011(b). Member of the Government Board, BANXICO. National College of Economists. Conference available on BANXICO website: http://www.banxico.org.mx/publicaciones-y-

discursos/discursos-y-presentaciones/presentaciones/%7BE2FA01C2-38E6-1814-DE38-E2ADA39A2BF8%7D.pdf. Accessed February 2011.

- __________. "The Need for a Monetary Reform in Mexico". *CATO Journal.* Vol. 22, No. 2 (Fall 2002).
- Seidman, Willliam. "The World Financial System: Lessons Learned and Challenges Ahead". Proceedings of the 7th Special Semminar on International Finance; 1996 Conference Tokyo September 18. Lessons From the Eighties: What Does the Evidence Show?. Conference Paper, available on FDIC Web site, http://www.fdic.gov/bank/historical/history/vol2/panel3.pdf
- Tosovsky, Josef. "Disinflation in the Czech Republic: Looking both Backward and Forward". Proceedings, Federal Reserve Bank of Kansas City. (1996): 147-159.
- Wallis, Allen. "Price Stability and Economic Growth". *Increasing Understanding of Public Problems and Policies,* Issue. 1960
- Wynne, Mark. "Price Stability and Economic Growth", Federal Reserve Bank of Dallas. *The Southwest Economy*, issue May/June 1993.
- ___________. "How Should Central Banks Define Price Stability?".*Working Papers Series*, No. 8, Federal Reserve Bank of Dallas: (April 15 2008).