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Introducción

La incorporación de la teoría inflacionaria a la cosmología permite un mejor entendimiento de algunos aspectos enigmáticos del Universo temprano y proporciona las condiciones físicas que permiten sembrar la estructura a grán escala del Universo. Es un concepto útil para el Universo primordial pero el escenario para su desarrollo no está aún bien establecido. Para construir un modelo inflacionario completamente satisfactorio se necesita cumplir con dos tipos de requisitos: lograr una solución efectiva a los problemas cosmológicos que la teoría inflacionaria tiene el potencial de resolver y ser consistente con una teoría realista de partículas elementales. Satisfacer simultáneamente estos requisitos no ha resultado ser una tarea fácil.

Los modelos inflacionarios propuestos hasta la fecha se dividen en dos clases: los *modelos inflacionarios estándar*, que aportan modificaciones al sector de materia introduciendo un campo escalar con un potencial efectivo particular, y los que introducen modificaciones también en el sector gravitacional. Este es el caso del recientemente propuesto modelo de *inflación extendida*, basado en la teoría de la gravedad de Jordan–Brans–Dicke.

Este trabajo vierte sobre los modelos inflacionarios que se desarrollan en el contexto de teorías de la gravedad del tipo escalar-tensorial. Atacamos el problema partiendo de los dos aspectos que abarca: modelos inflacionarios y teorías gravitacionales. Por un lado, estudiamos las teorías gravitacionales del tipo escalar–tensorial aplicándolas a la historia actual del Universo. Exponemos y delimitamos las variaciones al sector gravitacional permitidas por la confrontación con las observaciones. En particular, exploramos la posibilidad de introducir acoplamientos directos entre el campo escalar del sector gravitacional (campo de Jordan-Brans-Dicke) y algún tipo de materia oscura. El modelo consistente desarrollado se aplica luego al Universo temprano, tratando de construir un escenario inflacionario exitoso en cada uno de sus pasos.

El primer capítulo presenta un repaso de la teoría inflacionaria, sus ventajas y sus inconvenientes, y contiene una muy breve presentación de los modelos inflacionarios estándar. El segundo capítulo es un repaso de la teoría de la gravedad de Jordan–Brans–Dicke (JBD, de ahora en adelante) y del modelo de inflación extendida, recientemente propuesto. En el capítulo III se sistematizan y comparan los diferentes modelos propuestos en la literatura en el contexto de teorías escalar–tensoriales y su relación con teorías de más dimensiones.

El trabajo original está contenido principalmente en el capítulo IV, donde desarrollamos el escenario generalizado con acoplamientos directos, generales, del campo de JBD con el sector invisible, y en el capítulo VI, donde establecemos cotas observacionales actuales sobre los acoplamientos generalizados. El capítulo V está dedicado al Universo resultante de estas teorías, en particular al problema de sembrar la estructura del Universo. Esta parte también contiene cierta cantidad de trabajo original. Se investiga que tán sensibles son los resultados obtenidos para las perturbaciones de densidad a los detalles de la estructura del modelo. Se discute el significado y las implicaciones del procedimiento empleado en los trabajos que abordan el problema de formación de estructura y se exploran ulteriormente algunas extensiones del modelo original de inflación extendida y sus consecuencias para el Universo post–inflacionario.

Descripción

A continuación presentamos un breve resumen de la versión en inglés. Incluirá sólo los puntos y conceptos principales. Será particularmente breve en los capítulos de revisión y se extenderá un poco más en las partes originales. Las ecuaciones y definiciones de las cantidades involucradas, así como las referencias, se citarán de la versión íntegra en inglés.

Existe un modelo estándar para la cosmología, basado en la teoría de Einstein y las observaciones de Hubble, que describe correctamente la historia del Universo por lo menos desde la época de la síntesis de los elementos ligeros. Ha superado hasta ahora todas las pruebas observacionales pero encuentra dificultades al enfrentarse con la historia temprana del Universo: deben imponerse condiciones iniciales muy particulares para obtener un Universo como el que observamos actualmente. Las condiciones particulares que caracterizan al Universo temprano pueden resumirse de esta manera: existe un balance inicial preciso entre la densidad de energía y la tasa de expansión (Universo plano), con un inicio bien sincronizado en regiones aparentemente causalmente desconectadas del Universo (problema del horizonte). En este Universo uniforme a muy grán escala existen estructuras que requieren de perturbaciones de densidad que, en una época temprana, actuen como semillas para el crecimiento gravitacional. Estas fluctuaciones deben ser suficientemente grandes como para desarrollarse en las estructuras a grán escala que observamos (galaxias, cúmulos de galaxias y grandes regiones aparentemente vacías) y bastante pequeñas para caber en una región causalmente conectada al momento de su formación. La amplitud de las perturbaciones de densidad está además severamente restringida por observaciones: una inhomogeneidad en la densidad de materia presente en el momento del desacoplamiento radiación–materia deja huella en la radiación de fondo de microondas. Sin embargo, la radiación de fondo ha resultado ser altamente homogenea e isótropa: la detección de sus anisotropías por COBE ("Cosmic Background Explorer") entregó el siguiente resultado:

$\Delta T/T \approx 6 \times 10^{-6}$ [3].

Los desarrollos teóricos que se extienden más allá del modelo estándar son esencialmente un intento de comprender las bases físicas para condiciones iniciales tán específicas. Una de las posibilidades que han sido exploradas es la inclusión, en una época temprana, de una historia térmica que se desvía del modelo estándar, dejando inalterada la evolución posterior del Universo. La aplicación a la cosmología de las teorías de transición de fase en física de partículas nos ofrece esta posibilidad: un campo escalar que interactúa consigo mismo se puede comportar a altas energías como un fluido no–clásico y actuar como fuente para una expansión acelerada –*inflación*– del Universo. Este particular comportamiento termina al concluirse la transición de fase.

La idea básica de la teoría inflacionaria es que hubo una época en la que el Universo sufrió una expansión exponencial como resultado de la dominación de una densidad de energía constante (energía del vacío en Universo de de Sitter). Esta dominación debe ser temporal y la energía de vacío se tiene que transformar en energía de partículas, termalizada y uniformemente distribuida, para recobrar el Universo de Friedmann que observamos. En este sentido es útil trabajar con una configuración metaestable de un campo escalar que llena el Universo en expansión, y no con una verdadera constante cosmológica que refleja la propiedad del vacío: un campo escalar homogeneo, clásico, puede jugar el papel de un estado de vacío inestable y su decaimiento puede recalentar el Universo. En teorías de grán unificación se requieren potenciales de rompimiento de simetría para que la simetría existente a altas energías entre las diferentes componentes de una misma fuerza no se manifieste a bajas energías. El campo escalar responsable por esta transición de fase puede dominar el comportamiento del Universo mientras que se encuentra en el mínimo local de su potencial a altas energías. La figura 1 del capítulo I muestra un ejemplo de potencial con rompimiento espontáneo de simetría. (I.2.1) da la densidad Lagrangiana de un campo escalar real σ y (I.2.2) su densidad de energía potencial para que presente una transición de fase.

Se propusieron inicialmente tres modelos (que llamamos modelos estándar de inflación):

–Vieja inflación. Guth [8] construyó el primer modelo inflacionario, basándose en numerosos trabajos previos (vease [7] y Linde [5] para una breve historia del desarrollo de las ídeas inflacionarias y su bibliografía), con un potencial de transición de fase de primer orden. Una barrera de potencial alrededor del mínimo local (fig. 3 del cap. I) mantiene atrapado al campo durante un período de tiempo en una configuración de falso vacío con una densidad de energía constante. La transición de fase se lleva a cabo mediante tuneleo cuántico y se forman burbujas de verdadero vacío en un medio de falso vacío. La termalización de la energía (*recalentamiento* del Universo) se llevaría a cabo mediante choques de las paredes de las burbujas. Sin embargo, el escenario presentó un problema serio: las burbujas de verdadero vacío (que se expanden, al máximo, a la velocidad de la luz) no logran percolar puesto que el medio que las separa sigue expandiéndose exponencialmente.

–Nueva inflación [13]. La inflación ocurre durante el proceso de caída lenta del inflatón hacia el mínimo del potencial (σ_0). La transición de fase es suave (de segundo orden) y una sola región, conectada, abarca nuestro Universo. Para asegurar un comportamiento inflacionario, el potencial efectivo tiene que ser plano cerca de $\sigma = 0$ y, en una segunda fase, tiene que ser más escarpado para que la inflación termine y las oscilaciones del campo escalar alrededor del mínimo permitan volver a poblar el Universo con partículas producto de su decaimiento.

–Inflación caótica. Este modelo, propuesto por Linde [14], se basa en la suposición que la distribución inicial del campo escalar es caótica, i.e. toma diferentes valores en diferentes regiones del Universo. Esto se debería a que a altas energías (época de Planck) las fluctuaciones son tan grandes que el campo "no sabe" donde está el mínimo del potencial. En este caso, el potencial no es de rompimiento de simetría y tiene mínimo en $\sigma = 0$: $V(\sigma) = (\lambda/n)\sigma^n$, con constantes de acoplamiento, λ, pequeñas para que la evolución del campo sea lenta y lograr así una expansión quasi–exponencial.

La inflación resuelve el problema del horizonte puesto que lo aleja hasta distancias aún no observadas y predice un Universo plano puesto que el crecimiento exponencial del radio de curvatura del Universo en esa época asegura que sea hoy en día mucho más grande que el radio de Hubble. Resuelve además otro problema que se genera al introducir transiciones de fase: diluye los defectos topológicos. Su mayor logro es proporcionar un mecanismo de creación y amplificación de fluctuaciones de densidad. El hecho que la inflación estire exponencialmente las dimensiones espaciales hace posible que las fluctuaciones de densidad que crean la estructura a grán escala provengan de las fluctuaciones microscópicas de punto cero de los campos cuantizados. Las fluctuaciones del campo escalar se convierten, al final de la inflación, en perturbaciones de densidad de las partículas creadas en el decaimiento del inflatón. La teoría nos permite definir las fluctuaciones cuánticas en el momento en que cruzan el horizonte, es decir, cuando su longitud de onda es del tamaño de una región causalmente conectada. La amplitud de las fluctuaciones es una cantidad dependiente del modelo escogido pero la forma del espectro es una predicción genérica de la teoría inflacionaria: un Universo de de Sitter nos proporciona un espectro invariante de escala (i.e. amplitudes independientes de la longitud de onda al cruce del horizonte). Esta predicción coincide razonablemente con las características de la estructura a grán escala del Universo. Desafortunadamente, la amplitud de las fluctuaciones resulta demasiado grande (por varios órdenes de magnitud) comparada con las fluctuaciones de la radiación de fondo. Para obtener perturbaciones de densidad observacionalmente aceptables, es necesario ajustar las constantes de autoacoplamiento que intervienen en el potencial a valores muy pequeños ($\lambda \lesssim 10^{-12}$). Esto es lo que llamaremos el problema del ajuste fino. Puesto que los modelos inflacionarios estándar presentan, en mayor o menor medida, un problema de ajuste fino de alguna constante de la teoría, es necesario más trabajo para encontrar una representación concreta para el concepto de inflación.

La y Steinhardt [15] desarrollaron en 1989 un modelo inflacionario basado en la teoría de Jordan–Brans–Dicke de la gravedad [16] en el que la extraordinaria expansión que caracteriza a la inflación sigue ahora una ley de potencia. Modelos de inflación de ley de potencia habían sido obtenidos anteriormente con un potencial exponencial para el inflatón [18]. En este nuevo modelo, llamado inflación extendida, el mismo comportamiento es el resultado de la dinámica de dos campos: un inflatón, con un potencial del tipo de vieja inflación, y un campo escalar que juega el papel de un acoplamiento gravitacional variable en el tiempo ($\Phi = G^{-1}$). La expansión más lenta puede explicarse por el hecho que la densidad de energía del vacío es ahora compartida entre la expansión del Universo y la evolución del campo JBD. Al proponer su teoría, Brans y Dicke [16] introducen en el Lagrangiano para el sector gravitacional y la materia un campo escalar Φ acoplado con el escalar de curvatura, como se ve en la densidad Lagrangiana (II.1.2). El parámetro ω que interviene en el término cinético es una medida de la influencia adquirida por el campo escalar sobre el campo gravitacional con respecto al efecto de la curvatura del espacio-tiempo. De acuerdo con el principio de Mach, la fuente del campo Φ es la distribución espacial de la materia. La ecuación de onda para Φ y las ecuaciones de Einstein están dadas por (II.1.5) y (II.1.4) respectivamente. Para la inflación extendida se resuelven estas ecuaciones tomando como fuente dominante la densidad de energía del inflatón (con métrica de Friedmann-Robertson-Walker (FRW) y campos espacialmente homogéneos $\Phi = \Phi(t)$, $\sigma = \sigma(t)$). Se obtienen las soluciones (II.3.3) y (II.3.4) para la evolución del campo JBD y del factor de escala del Universo.

Los dos esquemas de inflación de ley de potencia están relacionados por una transformación conforme (vease e.g. [19]): un sector gravitacional de JBD más un inflatón con energía potencial constante (esquema de Jordan) se convierte, mediante la transformación (III.1.2) de la métrica, en un modelo con acoplamiento gravitacional constante más un campo con potencial inflacionario exponencial (esquema de Einstein). En este trabajo seguiremos la evolución del sistema en los dos esquemas: el de Jordan, que consideraremos como el sistema físico, y el de Einstein donde disponemos de una serie de resultados conocidos que podremos utilizar, lo cual nos será particularmente útil para el estudio de las fluctuaciones de densidad.

En inflación extendida, como en el modelo de vieja inflación, la transición de fase es de primer orden pero la expansión más lenta facilita la percolación de las burbujas de verdadero vacío y por lo tanto la formación de una región homogenea suficientemente grande como para contener nuestro Universo. Sin embargo la termalización y homogeinización de la energía contenida en las paredes de las burbujas más grandes siguen siendo una fuente de problemas. Para que estos procesos se lleven a cabo en buena medida antes de la época de la recombinación, se requiere de un valor bajo del parámetro de Brans–Dicke [20], [21]: $\omega \lesssim 25$, el cual está en conflicto con la cota observacional actual [22]: $\omega < 500$. Esta cota sobre ω se obtiene de medidas del retardo de señales por la gravedad solar, comparándolas con el valor predicho por la relatividad general.

Debido a estas discrepancias (que llamaremos el problema de ω), tenemos que renunciar al modelo más simple de inflación extendida. Diferentes variantes han sido propuestas: introducir un potencial para el campo JBD [23] que mantenga anclado el campo a algún valor de manera que el límite de baja energía de la teoría JBD coincida con la gravedad de Einstein, introducir un campo JBD en modelos de inflación caótica [24], [25] o nueva [25], dejar variar en el tiempo el parámetro de Brans–Dicke [26], [27], o permitir acoplamientos no–estándar del campo JBD con el inflatón. Esta última posibilidad surge de un trabajo de Damour, Gibbons y Gundlach [28] quienes consideran una teoría escalar–tensorial generalizada con acoplamientos directos del campo de JBD con el sector de materia oscura. Los efectos de un campo escalar del tipo JBD mezclado con la interacción tensorial usual están severamente restringidos por los experimentos. Sin embargo, puesto que las observaciones se llevan a cabo con materia visible, es posible construir una teoría en la cual el campo escalar está más fuertemente acoplado a una componente de materia que no esté involucrado en tests observacionales del principio de equivalencia. La restricción observacional se aplicaría entonces unicamente al acoplamiento con la materia visible. Damour, Gibbons y Gundlach encuentran un valor máximo para la variabilidad actual de la "constante" gravitacional compatible con –y no muy lejos de– las cotas observacionales.

Holman *et al.* [40] aplican la misma técnica al campo inflatón del modelo extendido, considerándolo como una componente de materia invisible con acoplamientos no–estándar con el campo de JBD. En este modelo generalizado de inflación extendida, la intervención de dos parámetros permite satisfacer simultáneamente los requerimientos actuales (ω observacional) y primordiales (condiciones para una inflación exitosa).

El modelo escalar–tensorial generalizado está descrito por el Lagrangiano (IV.1.1), con acoplamientos generalizados m y n arbitrarios. Derivamos las ecuaciones de Einstein (IV.1.3) y las ecuaciones de los campos (IV.1.4) y (IV.1.5). Trabajando con el campo redefinido de JBD $\varphi = \varphi_0 \ln(2\Phi)$ que tiene un término cinético convencional, con una métrica FRW, espacio plano ($k = 0$) y campos espacialmente homogéneos, llegamos al sistema de ecuaciones (IV.1.8), (IV.1.9) y (IV.1.10). De aquí en adelante trabajamos basándonos en dos suposiciones que permiten simplificar el problema y que consideramos describen correctamente nuestro Universo durante las épocas que nos interesan: la componente invisible se comporta como un fluido perfecto y representa la fuente dominante de energía para la expansión del Universo. Como fluido perfecto, se puede describir por su densidad de energía y de presión definidas a través de (IV.1.13) y cumple con la ecuación de estado $p_I = (\gamma_I - 1)\rho_I$. Al ser la componente dominante se puede despreciar la contribución de la materia visible, lo cual es una condición razonable tanto en la época inflacionaria, como en la época actual si suponemos un Universo plano con $\Omega_{barionica} \approx 0.1$ y $\Omega_{materia\ oscura} \approx 0.9$.

Nuestro problema estará entonces descrito por el sistema de ecuaciones (IV.1.17), (IV.1.18) y (IV.1.19), en función de las variables H y $y = \dot{\varphi}/\varphi$. El estudio de su espacio fase muestra que el sistema tiene tres líneas invariantes que corresponden a soluciones de ley de potencia: una solución atractor dada por (IV.3.1) y dos soluciones repulsor (IV.3.1a). El comportamiento del sistema dinámico puede verse en los diagramas de fase en las figuras (IV.1) y (IV.2). El punto crítico del sistema está en el origen del plano H–y. La flecha externa indica la solución atractor (IV.3.1) y las flechas pequeñas sobre las trayectorias señalan la dirección del tiempo. En los diagramas se puede ver que una región importante del espacio fase tiende asintóticamente a la línea atractor pero el punto crítico no es un atractor universal del sistema. Regiones separadas de la solución atractor por una línea repulsora no tienden a la solución (IV.3.1), es decir no todas las soluciones del sistema tienen un comportamiento de ley de potencia. La evolución del campo de JBD φ y del factor de escala a a lo largo de la solución atractor está dada por (IV.3.2).

Se puede encontrar el potencial para un campo escalar que actúa como un fluido perfecto dominante, resulta de la forma $V(\sigma) \propto \sigma^{-\alpha}$ (ec. (IV.3.13)). En la época inflacionaria, tenemos un potencial de tipo inflación caótica. La inflación de ley de potencia más general se obtiene pidiendo una potencia mayor que uno en la ley de evolución del factor de escala (expansión superlumínica): $f(n, m, \gamma_I) > 1$ en (IV.3.2). Si el potencial no se va a cero al final de la inflación, da lugar a un modelo con "constante" cosmológica que decae. En este contexto, el mismo campo que dominó durante la inflación puede volver a dominar en la época actual si su densidad de energía sufre un corrimiento al rojo menor que la densidad de energía de la radiación y la materia visibles. De esta manera, si existe una constante cosmológica no nula actualmente, este modelo permite relacionarla con la densidad de energía que sostuvo la inflación.

Podemos derivar la evolución del acoplamiento gravitacional y del parámetro de Hubble a lo largo de la solución atractor y compararlos con las cotas observacionales. La constante gravitacional medida en experimentos de retardo del tiempo es $G = [(2\omega + 4)/(2\omega + 3)](1/16\pi\Phi)$. La variabilidad de G en función de la evolución de Φ está dada por (IV.3.3). Un resultado interesante es que el acoplamiento gravitacional calculado a lo largo de la solución atractor varía con el tiempo aún cuando el tensor de energía-momento de la componente invisible tiene traza cero, i.e. cuando el Universo está dominado por la radiación de la componente invisible ($\gamma_I = 4/3$), como lo indica la expresión (IV.3.4). No es así en el modelo original de JBD, ni en las subsecuentes generalizaciones.

Considerando en cada era sólo el efecto de la componente dominante, se investigó, en el último capítulo, la posibilidad de establecer cotas sobre los acoplamientos generalizados del campo JBD. Suponiendo que el Universo está dominado hoy en día por la componente de materia oscura con ecuación de estado $p_I = 0$ ($\gamma_I = 1$), la variabilidad actual de G está dada por (IV.3.5) y el parámetro de Hubble por (IV.3.6). Para restringir los acoplamientos del dilatón con la materia oscura consideramos límites post–Newtonianos sobre el parámetro de Brans–Dicke ω, cotas observacionales sobre la edad del Universo, el parámetro de Hubble y la variabilidad de la "constante" de Newton hoy en día, así como sobre su valor durante la nucleosíntesis primordial. Queda un amplio intervalo para los parámetros m y n: $-7.7 \lesssim n - m \lesssim 4.5$. El límite superior resulta de incorporar consideraciones de la nucleosíntesis directamente en los modelos generalizados y el inferior proviene de la comparación de la variación de G predicha por nuestro modelo con el valor permitido observacionalmente.

Si nos remontamos a la época de la inflación, los límites más estrictos provienen de la radiación de fondo, traduciéndose en restricciones sobre las burbujas y sobre la amplitud y el espectro de las perturbaciones de densidad creadas por las fluctuaciones cuánticas de los campos escalares. El capítulo V está dedicado al estudio de estas cantidades: se exponen y comparan los resultados obtenidos en artículos recientes para los modelos de inflación extendida y se exploran ulteriormente los modelos generalizados y el escenario con un término de masa para el campo de JBD, durante y después de la inflación.

Al trabajar con modelos inflacionarios del tipo escalar–tensorial, disponemos de los siguientes ingredientes para sembrar la estructura del Universo: las fluctuaciones cuánticas de dos campos escalares –el inflatón y el campo de JBD– y tres tipos de perturbaciones de densidad: perturbaciones adiabáticas producidas por el campo con energía dominante, perturbaciones isotérmicas asociadas con las fluctuaciones del campo subdominante, e inhomogeneidades formadas por la estructura de burbujas del Universo emergente de la transición de fase. Surgen entonces diferentes posibilidades para la subsecuente evolución del Universo, dependiendo de cuál campo tiene las fluctuaciones dominantes y cuál campo es el principal responsable de recalentar el Universo. Como ya se sabía [18], la inflación de ley de potencia crea un espectro de fluctuaciones que se desvía ligeramente de la invariancia de escala, introduciendo más potencia a grandes escalas. Este es un resultado interesante puesto que, según algunos observadores (vease e.g. el estudio A.P.M. de Maddox *et al.* [64]), un espectro invariante de escala carece de suficiente potencia a grandes escalas para ser consistente con la estructura observada (movimientos de corrientes a grandes escalas, correlaciones cúmulo–cúmulo, regiones vacías). Por otro lado, la presencia de dos campos abre otra posibilidad en el diseño de espectros de fluctuaciones: se puede asociar un intervalo de escalas de las estructuras cósmicas a las fluctuaciones de un campo y otra parte del espectro puede corresponder a las fluctuaciones del otro campo.

El problema de las fluctuaciones puede ser atacado, en principio, en cualquiera de los dos sistemas conformes, pero el camino más directo es estudiarlo en el esquema de Einstein, donde el campo de JBD está minimamente acoplado, tiene un término cinético estándar y, bajo ciertas condiciones, juega el papel de un inflatón con potencial de caída lenta. El potencial inflacionario para el campo de JBD es el potencial (constante) del inflatón original multiplicado por un término exponencial en el campo de JBD. resultado de la transformación conforme. Este será el campo responsable de la estructura. En el esquema de Jordan tenemos la presencia de dos campos pero, mientras que el inflatón esté atrapado en el mínimo local de su potencial efectivo, el campo de JBD tendrá automaticamente las fluctuaciones dominantes [73], [78]. Si uno quisiera dejar evolucionar también el inflatón, ya sea con un tuneleo a través de la barrera del potencial (del tipo vieja inflación) o de caída lenta (inflación extendida nueva o caótica), tendría que tomar en cuenta las fluctuaciones de ambos campos y el acoplamiento del dilatón con el inflatón. Los resultados obtenidos serán facilmente transformados al marco original a tiempos grandes, donde ambos esquemas practicamente coinciden puesto que el factor conforme $2\Phi \rightarrow 1$ cuando $G \rightarrow G_N$. La densidad de energía y sus fluctuaciones se transforman mediante $\rho = (2\Phi)^2\tilde{\rho}$ y $\delta\rho/\rho = \delta\tilde{\rho}/\tilde{\rho} + 2\delta\Phi/\Phi$, respectivamente. Puesto que el campo de JBD varía muy lentamente en un régimen no–inflacionario, se puede considerar que al terminar la inflación las fluctuaciones ya coinciden en ambos esquemas.

La aplicación a una inflación de ley de potencia (de hecho, a una métrica inflacionaria general) de la fórmula estándar [66] $\delta\phi = H/2\pi$ para la fluctuación r.m.s de un campo escalar al cruzar el horizonte, derivada originalmente para el espacio de de Sitter, fue comprobada por Abbott y Wise [31]. Usando este resultado, Lucchin, Matarrese and Pollock [74] verificaron la validez de la relación $\delta\rho/\rho = \alpha H^2/\dot{\phi}$ para la inflación de ley de potencia y encontraron que la constante de proporcionalidad es $\alpha \approx 2 \times 10^{-2}$. Puesto que la amplitud de las perturbaciones de densidad es proporcional a H^2 y $H \sim t^{-1}$ en inflación de ley de potencia, las perturbaciones que salen del horizonte a tiempos anteriores son más grandes. Metiendo en (V.1.25) la dependencia en el tiempo de H, la evolución del campo clásico ϕ que se encuentra resolviendo la ecuación de onda (V.1.4) con un potencial exponencial $V(\varphi) = M^4 e^{-2\varphi/\varphi_0}$, y considerando que en este caso la longitud de onda crece con una ley de potencia, se encuentra que la amplitud de la fluctuación crece con la escala ([70], [72], [73], [19]), como se ve en las expresiones (V.2.1) y (V.2.2), donde p' es la potencia de la expansión en el esquema de Einstein.

De la expresión (V.2.2) se puede ver que la amplitud de las fluctuaciones está caracterizada por el cociente de la escala de unificación (M) sobre la escala de Planck (m_{Pl}) a una potencia α, donde $\alpha \approx 2$ para $\omega \gtrsim 10$, y está solo ligeramente en exceso de la cota observacional. La amplitud diverge en el límite $\omega \to \infty$ dado que el potencial efectivo se va a una constante y $\dot{\varphi} = 0$, pero disminuye si aumentamos ω en un intervalo intermedio de valores. Por otro lado, una ligera disminución de M puede resolver el problema puesto que la amplitud de las perturbaciones es muy sensible a variaciones en M: $\delta\rho/\rho \propto M^2$ para ω grande. No tenemos aquí la posibilidad de ajustar la constante de auto acoplamiento puesto que el potencial para σ se toma como estrictamente constante ($= M^4$); el ajuste fino recae sobre el parámetro de Brans–Dicke. Como habíamos mencionado, una de las posibilidades para evitar el problema de ω es introducir un potencial para el campo de JBD. La presencia de los dos campos escalares puede entonces llevar a un escenario inflacionario de dos rounds, con fluctuaciones importantes de ambos campos. Se exploraron las consecuencias, durante y después de la inflación, de incluir un término de potencial para el campo de JBD.

En el caso más simple el potencial es un término de masa. La ecuación de movimiento para el campo de JBD en el marco de Jordan es entonces una ecuación de Klein–Gordon modificada por la expansión (ec. (V.2.5)) y en las ecuaciones de Einstein aparece un término de constante cosmológica, como se ve en la ec. (V.2.6). Puesto que después de la inflación esta constante cosmológica puede llegar a dominar, tenemos que imponer que el campo de JBD decaiga. Por otro lado, para que la masa del campo de JBD no estorbe la evolución del Universo inflacionario, es necesario, en ambas ecuaciones: $m_\Phi^2 < \rho_V/\Phi$, con $\rho_V = M^4$. Puesto que durante la inflación $\Phi \leq m_{Pl}^2$, un límite seguro es: $m_\Phi < M^2/m_{Pl}$, que para $M \sim 10^{14} GeV$, implica: $m_\Phi \leq 10^9 GeV$. Este límite puede verse claramente en las figuras (V.1), (V.2), para H y Φ en la época inflacionaria. El comportamiento cambia drasticamente para $m_\Phi \geq 10^9 GeV$: H va rapidamente a una constante y Φ se establece en el mínimo de su potencial. Esto significa que se ha recuperado un régimen de inflación estándar, donde el término de masa dominante actúa como una constante cosmológica. Si consideramos un modelo de gravedad inducida el potencial para JBD es de rompimiento de simetría: $V(\Phi) = \lambda(\Phi - \Phi_0)^2$ con $\Phi_0 = m_{Pl}^2/16\pi$, y se puede ver que tenemos nuevamente la misma restricción sobre m_Φ. Este límite implica que se necesita una constante de auto acoplamiento pequeña ($\lambda \sim m_\Phi/m_{Pl}^2$), así que, aún en el contexto de una inflación de dos campos, es necesario un ajuste fino.

La posibilidad de que las grandes regiones casi vacías que se observan hoy en día sean residuos de las burbujas de la inflación extendida ha sido descartada porque la presencia de estas inhomogeneidades en la época de la recombinación provocaría una distorsión inaceptable sobre el fondo de microondas. Como ya se discutió, este argumento impuso restricciones severas sobre el parámetro de Brans–Dicke ω [21], [34], [82], [83]. Sin embargo, ha sido publicado recientemente un trabajo en el que se propone que el tiempo necesario para llenar una burbuja se reduce sustancialmente si se incorporan en el cálculo efectos relativistas [84]. Durante la época dominada por la radiación, las burbujas se contraen, a la velocidad de la luz para un vacío relativista, debido a la grán fuerza de presión que actúa sobre las paredes y este proceso de llenado puede llevarse a cabo en un tiempo corto, para un observador fuera de la burbuja, debido a la dilatación del tiempo causada por el potencial grande y negativo de la región vacía. Tomando en cuenta este efecto, se podría descartar el problema de las burbujas grandes y relajar los límites sobre ω aunque este problema está más directamente relacionado con el proceso de termalización que con el proceso de llenado. La termalización es una cuestión más complicada que no está del todo resuelta y que depende en cierta medida del tipo de materia que domina el comportamiento del Universo.

Los modelos generalizados no introducen nuevos ingredientes en este tema puesto que la estructura no proviene de las fluctuaciones del inflatón que es el sector que lleva los acoplamientos generalizados. Hay que imponer sin embargo algunas reestricciones sobre los parámetros del modelo para preservar las condiciones inflacionarias. En particular, para poder utilizar el formalismo estándar para las perturbaciones de densidad tenemos que asegurarnos de que, en el marco de Einstein, el potencial sea de caída lenta. Al cumplirse esta condición ($2\omega + 3 \gg (2-m)^2$), la fluctuación de densidad al cruzar el horizonte estará dada por (V.2.7).

Wang [45] analizó en detalle las restricciones impuestas sobre los modelos generalizados por los requisitos inflacionarios, obteniendo el siguiente intervalo para el parámetro m: $-11 \lesssim m \lesssim -8$. La necesidad de suprimir burbujas grandes, que no alcanzarían a llenarse y termalizar, fija la cota superior (usando $\omega > 500$) mientras que el límite inferior proviene de la cota impuesta por la isotropía de la rediación de fondo sobre la amplitud de las perturbaciones adiabáticas de densidad (tomando $(\delta T/T)_{\theta \sim 100^\circ} \lesssim 10^{-5}$). Las restricciones recaen basicamente sobre el parámetro m puesto que durante la inflación el término cinético es despreciable. Una restricción menos severa que las anteriores pero de cierta importancia es que para tener una expansión de ley de potencia con un campo de JBD creciente se necesita $m < 1$, es decir valores pequeños o negativos para m.

Otra cota severa proviene de las restricciones impuestas por COBE sobre el espectro de las fluctuaciones. Para comparar el espectro teórico con datos observacionales (disponibles a partir de una época muy posterior) tenemos que tomar en cuenta la evolución del espectro. La teoría inflacionaria nos permite determinar la amplitud de las perturbaciones en el momento que cruzan el horizonte, lo cual significa que la amplitud de modos diferentes está especificada a tiempos diferentes. La comparación con las observaciones requiere del espectro a un tiempo fijo, en particular en la época del desacoplamiento radiación–materia. Para tomar en cuenta la evolución de las perturbaciones fuera del horizonte se tiene que multiplicar el espectro (VI.2.1) por un factor de evolución k^4 (fuera del horizonte las perturbaciones son cantidades que no están bien definidas en el sentido que dependen de la norma seleccionada; trabajando en la norma síncrona, las perturbaciones crecen como λ^{-2}, donde λ es la longitud de onda que caracteriza a la fluctuación, vease, e.g. [6]). El espectro de fluctuaciones a la época de la recombinación es entonces $|\delta_k|^2 = k^{n_s} = k^{1-2/(p'-1)}$. Usando los resultados del primer año de COBE [3] $n_s = 1.1 \pm 0.5$,* se puede poner una cota sobre p' y restringir el acoplamiento m: $-11 \lesssim m \lesssim 15$. Es el mismo intervalo que el que resulta de las restricciones sobre la amplitud de las fluctuaciones.

Finalmente, si el mismo campo actúa como campo inflacionario en el Universo temprano y como la componente de materia oscura actual, tendrá que obedecer a las condi-

* Los resultados del segundo año de COBE DMR han sido publicados recientemente: "Cosmic Temperature Fluctuations from Two Years of COBE DMR Observations", C.L. Bennett *et al.*, enviado a *The Astrophysical Journal*. El valor más probable para el índice espectral reportado en este segundo análisis es $n = 1.59^{+0.49}_{-0.55}$ (68% CL). Si se comprueba este resultado, los modelos inflacionarios "normales" deberán ser descartados en su versión más simple puesto que no se pueden obtener índices espectrales mayores que uno, ni con una expansión de ley de potencia ni con una exponencial. Vease [87] para modelos inflacionarios con espectros azules.

ciones de ambas épocas. Combinando los dos grupos de restricciones se obtiene: $-11 \lesssim m \lesssim -8$ y $-18.7 \lesssim n \lesssim -3.5$.

Conclusiones

Hemos visto que un escenario inflacionario puede resultar de un amplio intervalo de teorías de partículas y de la gravitación, y que algunas de las dificultades encontradas pueden ser superadas introduciendo modificaciones en el término de potencial o en los términos de acoplamiento de los campos involucrados. En particular, las características problemáticas de una inflación de primer orden dejan entrever una solución: un proceso inflacionario más suave. Surge entonces la ***inflación extendida***: un modelo inflacionario que se desarrolla en el contexto de una teoría escalar–tensorial de la gravedad, en el cual el campo de Jordan–Brans–Dicke utiliza parte de la energía de vacío para su evolución, sustrayéndola a la expansión del Universo. El resultado de esta distribución de energía es una inflación de ley de potencia en lugar de una inflación exponencial.

Consideramos y comparamos las variaciones a la inflación extendida que han sido propuestas en la literatura y exploramos a fondo la posibilidad de introducir acoplamientos generalizados, más fuertes, del campo JBD con el sector invisible. Investigamos también las consecuencias de incluir un término de masa para el campo JBD.

Obtuvimos las ecuaciones de campo y sus soluciones atractor para la evolución del factor de escala y los campos escalares, considerando la componente de materia oscura como un fluido perfecto que constituye la fuente dominante para la expansión del Universo. Encontramos que el término potencial correspondiente a un campo escalar que se comporta como un fluido perfecto dominante decae con una ley de potencia del campo. Un aspecto particular de este modelo es que el campo JBD, y por lo tanto el acoplamiento gravitacional, varía con el tiempo aún cuando el universo está dominado por una componente oscura radiativa, i.e. por un fluido cuyo tensor de energía–momento tiene traza cero.

La dominación de un sector invisible es particularmente adecuada para describir dos épocas de la historia del Universo: la época actual, con una supuesta materia oscura no–bariónica que cierra el Universo, y el período inflacionario temprano. Obtenemos un modelo compatible con la evidencia observacional actual y cuyos parámetros libres (m y n) pueden ser acotados. Usando las cotas observacionales sobre el acoplamiento del campo JBD con la materia visible, cotas provenientes de cálculos de la nucleosíntesis primordial y límites sobre la edad del Universo y el parámetro de Hubble, pusimos restricciones sobre las constantes de acoplamiento de este modelo aplicado a una componente de materia oscura actualmente dominante. Un valor mayor de H_0t_0 reduce el intervalo permitido para $m-n$ pero la cota más severa proviene de la variabilidad de G permitida por consideraciones de nucleosíntesis. Con los valores observacionales actuales, queda un amplio intervalo para $m-n$. Los acoplamientos generalizados del campo JBD con la componente invisible generan contribuciones extra a la densidad de energía del Universo y a las posibilidades de variación de G. El intervalo permitido para $\dot{G}$ es así más grande que el que se obtiene en la teoría JBD estándar.

Yendo hacia atrás en el tiempo, aplicamos este modelo a la época inflacionaria. La isotropía de la radiación de fondo restringe los parámetros de la teoría cuando la componente invisible representa al campo inflatón en el Universo temprano: impone condiciones sobre la distribución y la evolución de las burbujas, sobre la amplitud de las perturbaciones de densidad que resultan de las fluctuaciones cuánticas del campo JBD y sobre el índice del espectro de potencia de las fluctuaciones. Consideradas en conjunto, estas condiciones restringen notablemente el intervalo permitido. En este caso las restricciones caen sobre el parámetro m, imponiéndole valores negativos y grandes ($-11 \lesssim m \lesssim -8$), puesto que el término cinético, que incluye el parámetro n, es despreciable durante la época inflacionaria.

En el contexto de una "constante" cosmológica que decae, el mismo campo que dominaba durante la inflación puede volver a ser dominante en la época actual si su densidad de energía sufre un menor corrimiento al rojo que la densidad de energía de la materia y de la radiación. En este caso, m y n tienen que cumplir simultáneamente con las condiciones primordiales y con los requerimientos actuales.

Un efecto importante de la inclusión de un campo JBD en escenarios inflacionarios es la obtebción de un espectro de perturbaciones de densidad con más potencia a grandes escalas. Esta es una característica que puede resultar de cierta utilidad al construir modelos que reproduzcan la estructura observada del Universo. Es un resultado general de los escenarios de inflación de ley de potencia. En los modelos de inflación extendida, la estructura resulta principalmente de las fluctuaciones del campo JBD y no de las del inflatón, como sucede en los escenarios inflacionarios estándar. Las amplitudes estimadas de las fluctuaciones están caracterizadas por la razón al cuadrado de la escala de unificación sobre la escala de Planck (para valores no muy pequeños de ω) y son solo ligeramente más grandes que la cota observacional. Esta es otra ventaja de estos modelos sobre los modelos de inflación exponencial los cuales producen fluctuaciones con amplitudes que exceden las observadas por varios órdenes de magnitud, a menos que se impongan restricciones severas y poco naturales sobre la constante de acoplamiento contenida en el término de potencial. Por otro lado, los posibles efectos observables de las fluctuaciones en el campo JBD, y por lo tanto en el acoplamiento gravitacional, merecen ulterior investigación.

Los acoplamientos generalizados no introducen ingredientes nuevos en este tema, salvo algunas limitaciones al parámetro m con el fin de respetar condiciones generales inflacionarias. Por otro lado, la inclusión de un término potencial para el campo JBD, útil para resolver el problema de ω, aumenta la amplitud de las fluctuaciones de densidad y vuelve a introducir un problema de ajuste fino de las constantes de acoplamiento de la teoría. Despues de la inflación, este término de masa tiende a volverse dominante, por lo que tenemos que imponer la condición de que el campo JBD decae.

Por lo que se refiere a la cuestión de ubicar los modelos inflacionarios del tipo escalar-tensorial en el contexto de una teoría fundamental, las teorías de cuerdas ofrecen una posibilidad atractiva. Al reducir estas teorías a cuatro dimensiones, tenemos un campo dilatón acoplado al escalar de curvatura de la métrica en cuatro dimensiones y directamente acoplado a los sectores no-gravitacionales. Desafortunadamente, un resultado general de las teorías de supercuerdas, y de otras teorías de más dimensiones, es que no logramos obtener bastante inflación. Sin embargo, uno de los grupos que trabaja sobre este punto reportó algunos resultados positivos al introducir dos campos del sector gravitacional (el dilatón y un campo modular). La obtención, a bajas energías, de una teoría JBD también encuentra algunos problemas, en particular cuando sus predicciones son comparadas con observaciones actuales.

Introduction

Inflation provides cosmology with a better understanding of some enigmatic features of the early Universe and provides the physical conditions for seeding the Universe large scale structure. It is a useful concept for the primeval Universe but it is still a developing scenario. The building of a successful inflationary model requires two things: that it provides an effective solution to the cosmological problems it has the potential to solve and that it is consistent with a realistic elementary particle theory. The simultaneous satisfaction of both requirements has not been an easy task.

The inflationary models proposed up to now may be devided in two classes: those which modify the matter sector by including a scalar field with some particular effective potential, which we call *standard inflationary models*, and those that also introduce variations in the gravitational sector, as compared to general relativity. This is the case of the recently proposed *extended inflation model* that works with a Jordan–Brans–Dicke gravity theory.

In this work we focus our attention on inflationary models embedded in scalar–tensor gravity theories. We approach the problem from the two aspects it embraces: inflationary scenarios and gravitational theories. On one hand, we study scalar–tensor gravitational theories applying them to the present Universe history. We outline and delimit the variations to the gravitational sector that are consistent with observations. In particular, we explore the possibility of direct couplings between the scalar field of the gravitational sector (Jordan–Brans–Dicke field) and some kind of invisible matter. We then apply the consistent developed model to the early Universe history, trying to implement a successful inflationary scenario in all its steps.

Chapter I presents a review of the inflationary theory, its advantages and its drawbacks, and contains a very brief description of the standard inflationary models. Chapter II is a review of the Jordan–Brans–Dicke theory of gravity and a presentation of the recently proposed extended inflation model. Chapter III is a systematization and comparison of the different models proposed in the literature in the context of scalar–tensor gravity theories and of their relation with a higher–dimensional theory.

The original work is mainly contained in chapter IV, where we develop the generalized scenario with direct, general, couplings of the Jordan–Brans–Dicke field with the invisible sector and in chapter VI, where we establish present experimental bounds and primordial constraints on the generalized couplings. Chapter V is dedicated to the Universe emerging from these theories, in particular to the problem of seeding the structure of the Universe. Some original work is also contained in this part, where we investigate how sensible are the results obtained for the density perturbation formalism, to the details of the structure of the model. We discuss the sense and the implications of the method employed in previous analysis and further explore some extensions of the original extended inflation model and their consequences for the post–inflationary Universe.

Chapter I

Inflationary Scenario

Introduction

The standard cosmological model, developed from Einstein's theory and Hubble observations, has so far withstood observational tests and successfuly accounts for the history of the Universe from at least the light elements synthesis epoch. It encounters nonetheless conceptual difficulties when dealing with the earliest history: very unnatural initial conditions have to be imposed in order to end up with the Universe as it appears today. Theoretical developments beyond the standard model consist mainly in trying to understand the physical grounds for these specific initial conditions. One of the possibilities that have been explored is to introduce, at some early epoch, a thermal history deviating from the standard model, while leaving unchanged the Universe evolution at later stages. Theories of phase transition in particle physics, applied to cosmology, offer this possibility: a self-interacting scalar field may behave, at high energies, as a non-classical fluid and act as a source of accelerated expansion –*inflation*– for the Universe. This particular behaviour ends with the phase transition.

This first chapter is dedicated to a review of the basic ideas of the inflationary model: what do we need it for, how to implement it and the issues that remain unsettled.

I.1) The Standard model

The standard cosmological model gives the following scenario for the early Universe: after the initial big bang, the Universe is filled with a hot gas of elementary particles in thermal equilibrium, adiabatically expanding, homogeneous and isotropic. The effects of gravitation are described by Einstein's general relativity (hereafter GR) and the fundamental laws of physics do not change with time. The Universe is supposed to have been homogeneous from the start and remained homogeneous as it evolved and changes in the state of matter and radiation are supposed to be smooth, with negligible effect on the thermodynamical history of the Universe.

An homogeneous and isotropic Universe is described by the Friedmann–Robertson–Walker metric (FRW):

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \qquad (I.1.1)$$

where $a(t)$ is the scale factor of the Universe and $k = +1, 0, -1$, for a closed, flat or open universe respectively. The evolution of the scale factor is governed by the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu} \qquad (I.1.2)$$

where $R_{\mu\nu}$ and R are the Ricci tensor and the Ricci scalar respectively and $T_{\mu\nu}$ is the stress–energy tensor for all the fields present (matter, radiation, etc...). In equation $(I.1.2)$ we have used units such that $8\pi G = 1$ and taken the cosmological constant $\Lambda = 0$ (such a term appears in the most general form of Einstein's equations consistent with general covariance). Greek indices run from 0 to 3 while Latin indices will run from 1 to 3.

The non–zero components of the Ricci tensor for the FRW metric are

$$\begin{aligned} R_{00} &= -3\frac{\ddot{a}}{a} \\ R_{ij} &= -\left[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + \frac{2k}{a^2}\right]\delta_{ij}, \end{aligned} \qquad (I.1.3)$$

and the Ricci scalar is

$$R = -6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right]. \qquad (I.1.4)$$

The energy–momentum tensor of the Universe must be diagonal in order to respect the symmetries of the metric and its spatial components must be equal, reflecting the isotropy of space; in such a way, it takes the same form as the one for a perfect fluid: $T^{\mu}_{\nu} =$

$diag(\rho, -p, -p, -p)$. The 00 component of the Einstein equations gives the Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{1}{3}\rho, \qquad (I.1.5)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, that determines the expansion rate of the Universe, and ρ is the energy density; the ii component gives

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}^2 + \frac{k}{a^2} = -p. \qquad (I.1.6)$$

Combining this two equations we get a useful expression for the acceleration

$$\ddot{a} = -\frac{1}{6}(\rho + 3p)a, \qquad (I.1.7)$$

which shows that, if $(\rho + 3p)$ is positive, as it is always the case in the standard model, $\ddot{a}$ is negative and the Universe expansion is slower and slower; it just reflects the deceleration due to matter gravitational attraction. On the other hand, it suggests the way to inflation: if we wanted the Universe to have an accelerating expansion phase, we would need a negative $\rho + 3p$ term. Equations (I.1.5) and (I.1.7) together lead to the continuity equation

$$\dot{\rho} = -3H(\rho + p), \qquad (I.1.8)$$

equivalent to the more familiar form

$$d(\rho a^3) = -pd(a^3), \qquad (I.1.9)$$

which is the $\mu = 0$ component of the conservation of the energy–momentum tensor $T^{\mu\nu}{}_{;\nu} = 0$.

To obtain the evolution with time of the Universe, we need the equation of state of its content; for a hot ultrarelativistic gas of non–interacting particles

$$p = \frac{1}{3}\rho \Rightarrow \rho \sim a^{-4}, \qquad (I.1.10)$$

and for non–relativistic cold matter

$$p = 0 \Rightarrow \rho \sim a^{-3}. \qquad (I.1.11)$$

Introducing these expressions in the Friedmann equation, we see that, for small a, in both cases, the quantity $\rho/3$ is much greater than k/a, therefore the scale factor goes as

$$a \sim t^{1/2} \qquad (I.1.12)$$

and

$$a \sim t^{2/3} \qquad (I.1.13)$$

respectively.

The Standard big bang (BB) model successfully accounts for the development of the Universe since, at least, the epoch of synthesis of nuclei and provides an explanation to many important aspects: the redshift of galaxies, the cosmic background radiation, and light elements abundance. There is however an important question left: what is the origin of the structures we see? This problem is enhanced by the lack of knowledge of the precise value of the parameters of the theory and by some enigmatic features of this cosmology. Some observations, some fundamental cosmological features, have no natural explanation in the context of the theory and this suggests that an important piece of the picture may be missing. We briefly go through these controversial points.

a) Isotropy

Homogeneity is a very useful principle in cosmology: a theory of the Universe would be intractable without a simplifying principle; but it also turns out to be excessively true, in the sense that the scale of homogeneity is so large that we do not find (in the standard model) an explanation for it.

The cosmic background radiation coming from different parts of the sky is (almost) exactly the same. This relic redshifted radiation, released when the thermal equilibrium between matter and radiation broke down, practically does not interact with the matter today: the universe became transparent to it after hydrogen recombination, so its homogeneity traces back to this epoch. Of course, what we observe does not guarantee that the entire Universe is smooth, it just gives information about the present observable region. By comparing radiation background intensisties we see that the temperature and expansion rate are precisely synchronized across the visible Universe. The problem appears if an explanation is sought for this, because distant regions observed in well separated parts of the sky are so far apart that there is not enough time since the BB for a signal to have traveled from one to the other. Assuming that causal relations require the transport of information at a velocity not exceeding that of light, we must conclude that microphysical processes (such as Compton scattering) could not operate to smooth temperature fluctuations and single out a unique temperature through such separated regions. In the past, these parts of the Universe were much closer together but this does not eliminate the problem since the zone of influence about an object, going backwards in time, decreases even faster. A light signal propagates along a geodesic $ds^2 = 0$, with $d\theta = d\phi = 0$, so the equation describing the ray propagation is $dr = -dt/a(t)$. The particle horizon is the distance light can travel from the beginning (of the particular cosmological epoch) to the time t we are considering:

$$d_H(t) = a(t) \int_{t_i}^{t} \frac{dt'}{a(t')} \qquad (I.1.14)$$

(in this expression we have placed ourselves at the origin $r = 0$ of coordinates, a mere convention according to isotropy and homogeneity). This quantity represents the constraint imposed by causality on dynamical evolution; if $d_H(t)$ is finite, our past light cone is limited by a particle horizon, otherwise all the Universe will be in causal contact. Taking $t_i = 0$ at the big bang, and a power-law expansion $a(t) \sim t^\alpha$ with $\alpha < 1$ for the Universe (according to both cases (I.1.12) and (I.1.13)) the particle horizon is

$$d_H(t) = \frac{t}{1-\alpha} \qquad (I.1.15)$$

so the distance of maximal causal connection goes like t and is finite. In spite of the fact that all the physical distances approach zero as $a \to 0$, the expansion of the universe precludes all but a very small part of the universe from being in causal contact. Then, why are all the disconnected regions so synchronised?

b) flatness

The energy density of the Universe is the quantity that determines in what type of Universe (open, flat or closed) we live. We call critical density ρ_c the value for which the Universe is flat

$$\rho_c = 3H^2 \tag{I.1.16}$$

and we define the density parameter

$$\Omega \equiv \frac{\rho}{\rho_c}, \tag{I.1.17}$$

where Ω and ρ_c change as the Universe expands. The correspondence between the value of Ω and the sign of k is

$$\begin{aligned} k &= +1 \Longrightarrow \Omega > 1 \quad \text{Closed} \\ k &= 0 \Longrightarrow \Omega = 1 \quad \text{Flat} \\ k &= -1 \Longrightarrow \Omega < 1 \quad \text{Open.} \end{aligned} \tag{I.1.18}$$

The observational value for the present density parameter Ω_0 is uncertain but is not far from 1. Dynamical estimations of Ω_0 on scales of $15 - 20 Mpc$ yield a lower limit of $\Omega_0 \approx 0.25$, while analysis of large scale flows, comparing the peculiar velocity field with the density distribution (with the assumption that IRAS galaxies trace the mass on large scales) seem to indicate $\Omega_0 \approx 1$, with a bias factor $b \approx 1 - 2$ [1].

This approximate balance between the effective kinetic energy of expansion (H^2) and the gravitational potential energy ($\rho/3$) becomes extremely accurate as $t \to 0$: the condition $\Omega = 1$ is unstable. This can clearly be seen if we write the Friedmann equation as an evolution equation for the density parameter

$$\Omega - 1 = \frac{k}{\dot{a}^2}. \tag{I.1.19}$$

Since in the standard model $\dot{a}$ decreases with time, Ω always deviates from 1. Eq. (I.1.19) can be written

$$\Omega(t) - 1 = \frac{\dot{a}_i^2}{\dot{a}^2(t)}(\Omega_i - 1), \tag{I.1.20}$$

where i indicates an "initial" time (that can be taken at the Planck time t_{Pl}). Since at the present time, $\Omega(t_0) \approx 1$, the value of $\Omega_{Pl} - 1$ must have been really small to balance the very big factor $\dot{a}^2_{Pl}/\dot{a}^2(t)$. If Ω were ever exactly equal to 1, it would remain exactly equal to 1 forever but if Ω differed slightly from 1 after the BB the deviation from 1 would rapidly grow with time and the universe would either soon ($few \times t_{Pl}$) recollapse, if it is a closed Universe, or expand so fast that matter condensations could never form, if it is open. A universe can survive $10^{10} yr$ only by extreme fine tuning of the initial values of ρ and H.

c) Smoothness and structure

Discrete structures are supposed to evolve from primordial seeds and, in this theoretical context, matter perturbations are found to effectively grow, under favorable circumstances, and reach values comparable to their mean values. Then, when this non–linear regime is attained, the overdense region decouples from the expansion of the universe and collapses to form a condensation. These theories of structure formation, based on the gravitational growth of small primordial perturbations, predict that some imprint of this event remains on the cosmic microwave background radiation (CMBR) in the form of small fluctuations: photons are gravitationally redshifted by inhomogeneities of the gravitational potential on the last scattering surface (Sachs–Wolfe effect, see e.g. [6]). But the Universe has shown to be very smooth. Till recently, no such fluctuations had been detected and we just had observable upper bounds on the anisotropy of the cosmic background: $\Delta T/T \lesssim few \times 10^{-5}$ (see e.g. [2]), on intermediate to large angular separations. The analysis of the first year of data from the Differential Microwave Radiometers of COBE (Cosmic Background Explorer) [3] has just detected structure with characteristic anisotropy of $\Delta T/T \approx 6 \times 10^{-6}$ (and whose measured parameters are consistent with a scale–invariant spectrum of perturbations). Their preliminary separation of galactic and cosmic microwave emission [4] suggests that this anisotropy signal is intrinsic to the cosmic background radiation. When we translate these bounds on temperature fluctuations to bounds on density perturbations at the time of decoupling, we obtain severe constraints for theories of structure formation. There are also other difficulties in this approach: on one hand there is no natural seed for these perturbations in the conventional BB model and, on the other. it is difficult to understand how any physical process could have contributed to it in the very early universe. This is because most of the astrophysically relevant scales (clusters, groups. galaxies) were much bigger than the causal horizon for early epochs.

So, this is the picture we have: a precise initial balance of density and expansion rate, with a well-synchronized start, which must apply to each separate part of the Universe, on one hand. On the other, the local balance of expansion and gravity in the limit $t \rightarrow 0$ must be supposed extremely accurate but not exact, in order to account for the observed large-scale clumping of matter. We have to reconcile the existence of galaxies, groups, clusters, with the overall homogeneity of the Universe. And we also wonder about some fundamental questions on the origin of the universe: the uniqueness of our Universe and the initial singularity. Why is nature just the way it is and not otherwise? and, was there anything *before* $t = 0$?

Facing these specific initial conditions (should we consider them just plausible enough?) some proposals have been put forward. A pulsating universe –successively expanding and contracting– could provide an escape from some of the conceptual puzzles, even if this would only shift some of the problems many cycles back (unless there were infinitely many cycles). Including a cosmological constant, the Friedmann regime $a \sim t^{\alpha}$ can be made to change into de Sitter $a(t) \sim e^{Ht}$ as $t \rightarrow 0$, so the scale factor bounces near the singularity: it comes to be very small but non–zero and all physical quantities remain finite. Anyway, what undoubtedly has to be done near the singularity, is turn to a quantum theory of gravitation. When density diverges it is essential to take into account quantum effects on the cosmic scale, in such a way, the relativistic singularity at the BB (and in a collapsing star) could be a problem of the theory, not of the universe, and flatness and horizon problems could arise just as a consequence of our application of classical physics beyond its domain of validity.

An important attempt to solve some of these problems is the theory of inflation. We will develop it in the rest of this chapter, starting with a brief presentation of phase transitions in the context of their cosmological application.

I.2) Unification theories

The basic idea of a unifying theory is that what we perceive to be independent forces are actually part of a single unified force, with an underlying symmetry relating each component of the force to the other. Since experimentally the forces are very different in strength and character, the theory is constructed so that the symmetry is spontaneously broken in the present Universe: the underlying symmetries are not manifest in the structure of the vacuum but are restored at high temperature. This means that the Universe must undergo a phase transition from a disordered phase, characterized by certain symmetries, to an ordered phase with a lower degree of symmetry, with the raising of an order parameter –a macroscopic quantity which was zero in the high temperature phase.

The idea of spontaneous symmetry breaking in unified theories can be built and understood exploiting the analogy with phenomena such as ferromagnetism, superfluidity or superconductivity. In a ferromagnetic substance, for instance, the order parameter that appears at $T \lesssim T_C$ is a non–zero magnetization whose direction breaks the rotational symmetry present in the Hamiltonean. The symmetry breaking may be induced, as a consequence of an external influence (an electromagnetic field), at any T, or spontaneously, when it depends on a gradual change of the system's parameters. The magnetization can, in principle, take any direction, but small fluctuations select one of the possible (degenerated) solutions. In general, in the case of a non–invariant vacuum state and a non–invariant Lagrangian we can speak of an explicit symmetry breaking, while a symmetry of the Lagrangian not respected by the vacuum is said to be spontaneously broken. In this case, we are in the presence of different vacuum states and the choice of one of them will define the "Universe". Dealing with unification theories, the order parameter is a scalar field σ whose ground state does not present the symmetry of the Lagrangian. To this respect, it is interesting to note that some kind of scalar field has been recurrently introduced in cosmology, with a variety of motivations: to lead inflation, to incorporate, as we will see in next chapter, Mach's principle in general relativity, as a candidate for cold dark matter, etc...

A real scalar field σ is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma) \qquad (I.2.1)$$

and, in order to present a phase transition, the potential energy density is chosen to have the form

$$V(\sigma) = -\frac{\mu^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 \qquad (I.2.2)$$

where μ is the mass of the scalar field and λ is its coupling constant (the only interaction we are considering for the σ field is the self interaction: the $\lambda\sigma^4$ term).

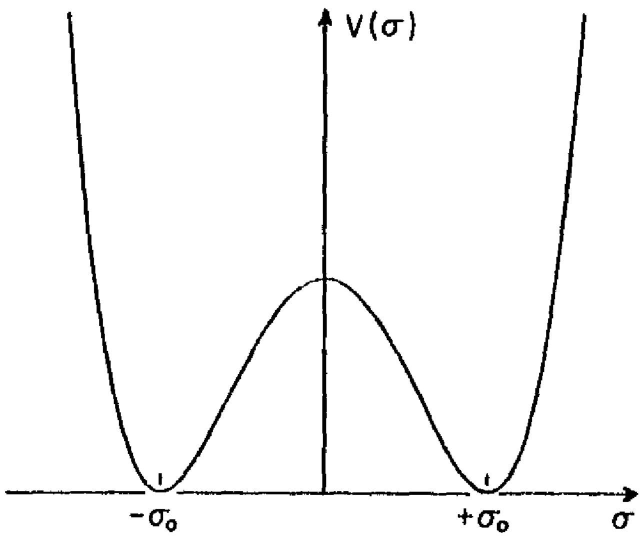

Fig. 1: An example of the potential for the model with spontaneous symmetry breaking.

The potential has a degenerate minimum at $\sigma_0 = \pm\mu/\sqrt{\lambda}$. The shape of the potential $V(\sigma)$ is shown in figure. 1. We have added a constant term $\mu^4/4\lambda$ whose sole effect will be to cancel the vacuum energy at points of minimum potential. To explain the symmetry restoration at some temperature T_c we must take into account the effect of the background gas in the calculation of higher–order quantum corrections to the classical potential. With this contribution, the equilibrium value of the scalar field at finite temperature ($T \neq 0$) is governed by the location of the minimum of the free energy density $V(\sigma, T)$, which reduces to the potential energy density $V(\sigma)$ at $T = 0$. Omitting terms that do not depend on σ, the complete expression for the finite temperature effective potential can be written in the form (see e.g. [5])

$$V(\sigma, T) = \frac{-\mu^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\lambda T^2}{8}\sigma^2 + \dots \qquad (I.2.4)$$

The temperature dependence of $V(\sigma, T)$ is shown in figure 2.

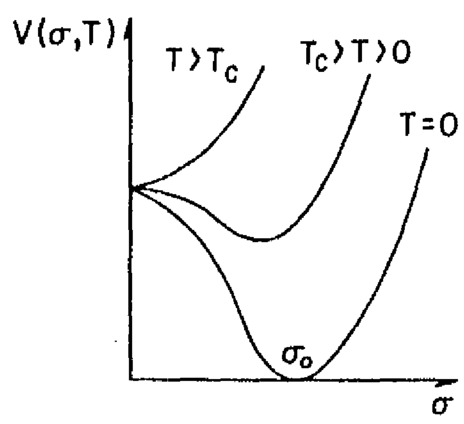

Fig. 2: The temperature dependence of $V(\sigma, T)$.

Considering the effect of the thermal bath of σ particles, we can define an effective mass of the scalar field about the classical solution $<\sigma>= 0$ including a temperature dependent term: $m^2 \equiv -\mu^2 + \frac{1}{4}\lambda T^2$. At $T > T_c$ the effective mass is real and $<\sigma>= 0$ is a stable classical minimum; at $T < T_c$, the effective mass squared is negative, the symmetry breaks and the field σ leaves the unstable point $<\sigma>= 0$, growing until it finds one of the true ground states, σ_0. From then onwards, the field σ will oscillate about the point σ_0 and it is about this point that the effective mass has to be defined. Making the change of variable $\sigma \to \sigma + \sigma_0$ in the Lagrangian density, it can be seen that the effective mass squared of the σ field has the correct sign at the minimum of the potential.

With an effective potential like the one in fig. 2, the change in the vacuum expectation value of the scalar field is continuous and the transition to the broken phase occurs smoothly, it is second order. There is also the possibility of a first order phase transition with a discontinuous change of the order parameter as the result of the presence, at $T = T_c$, of two local minima in the potential with a barrier separating the stable from the unstable state. The transition is then considerably delayed: even below T_c, the Universe stays in the symmetric phase, although the ground-state energy is lower in the broken phase, and supercools. After a period of cooling below T_c, quantum tunneling (at zero temperature) can induce the phase transition, releasing the *latent heat.* The symmetry breaking proceeds through the formation and subsequent expansion of bubbles of the stable phase within the unstable one. If the phase transition is second or weakly first-order, thermal fluctuations (at finite temperature) may drive the transition. The shape of the effective potential for a first order phase transition is shown in figure. 3.

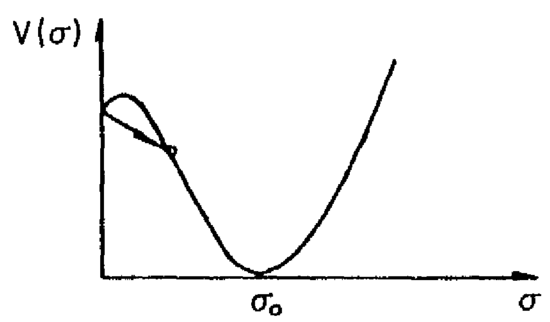

Fig. 3: An example of the potential for
a first order phase transition.

Grand unification theories make 3 cosmological predictions:

1) Baryogenesis

Two fundamental observations about the content of matter in the Universe are: the number of baryons is much smaller than the number of relic photons, with a baryon to photon ratio of $n_b/n_\gamma \simeq (4-7) \times 10^{-10}$, and there is no evidence for the presence of antimatter (see e.g. [6]).

Although we have no antimatter in the Universe today, at one time in the early universe, quarks and antiquarks were in thermal equilibrium with photons and hence they were present with similar abundances: $n_q \sim n_{\bar{q}} \sim n_\gamma$. This suggests the idea that the quantity of baryons present today corresponds to a small excess of baryons over antibaryons at early times, that consequently survived annihilation. Applying theories of Grand Unification (GUT) to an expanding Universe gave the possibility of developing this asymmetry in a Universe that was initially baryon symmetric. GUT brought the main ingredient: the new interactions that embrace strong and electroweak forces, combining quarks and leptons in multiplets of the unifying gauge group, violate baryon (B) and lepton number (L). Besides the requirement of baryon number violation, there are two other necessary conditions for generating a baryon asymmetry. First, violation of C (charge conjugation) and CP (charge conjugation combined with parity) invariance is needed in order to break the symmetry between particles and antiparticles, avoiding B-nonconserving reactions to produce baryon and antibaryon excesses at the same rate. Second, temporary loss of thermal equilibrium is required so that the annihilation rate for baryons and antibaryons cannot keep pace with their production rate. These ingredients are also available: C is violated in weak interactions; CP violation -although quite small- is observed in the interactions of K^0 and $\overline{K}^0$, finally, the necessary non-equilibrium condition is provided by the expansion of the Universe: a heavy particle which decays slowly will always go out of thermal equilibrium when the temperature of the Universe falls below its mass.

2)Monopoles

Another prediction of GUT, not so pleasant as the previous one, is the appearence of superheavy magnetic monopoles ($M_m \sim 10^{16} GeV$) that would contribute with an energy density far in excess of the critical one. Since they are stable objects, they cannot be destroyed, and we need a mechanism to suppress them.

The production mechanism of these topological defects (false vacuum remnants) is tied to the fact that during a cosmological phase transition any correlation length is limited by the particle horizon. Since in different correlation lengths the scalar field can take different vacuum expectation values (e.g. $+\sigma_0$ and $-\sigma_0$ in fig. 1) and the transition from one value to the other must be smooth, there must be a point where the scalar field takes the value $<\sigma>= 0$, i.e. a point of false vacuum.

3)Cosmological constant

Combining modern particle theories with gravity gives rise to another question. The discovery of the expansion of the Universe has made unnecessary the introduction of the cosmological constant (vacuum energy of the Universe) into the gravity equations, as first done by Einstein in order to compensate the matter attraction and obtain a stationary cosmological model. Astronomical observations indicate in fact that, if non-zero, it is very small. The upper limit is of the same order of magnitude as the critical energy density. This does not represent a very strong cosmological bound but from the point of view of elementary particles physics it is an extremely small quantity. Quantum field theory predicts a very large value (infinite) for the vacuum energy but one usually adjusts all scalar potentials so that $V(\sigma_0) = 0$ since the origin of vacuum energy is purely conventional in the absence of gravity. However, in general relativity vacuum energy affects the properties of spacetime. If the present value of $V(\sigma)$ is attained as a result of a series of symmetry breaking phase transitions, the vacuum energy is decreased by order M^4 (M = energy scale of SSB) in each transition and after all these enormous drops it turns out to be equal to zero with a great accuracy. It seems unlikely that the complete (or almost complete) cancellation of the vacuum energy should occur without some deep physical reason but we do not know any symmetry that forbids a cosmological term nor a cancellation mechanism for it by that degree, neither is there a reason why this vacuum should be transparent to gravity.

I.3) Inflation

Theories of Unification and their phase transitions opened the way to inflation. Just as the baryogenesis process has introduced the possibility of explaining the small ratio of n_B/n_γ instead of accepting it as an initial condition, the inflationary model proposes dealing with other features of the Universe connected with the initial conditions. The basic ideas for an inflationary scenario were developed by many authors –see [7] and Linde [5] for some history on the development of inflation and a detailed bibliography of early work– but the definite step corresponds to Guth [8] who suggested using the exponential expansion of the Universe during a phase transition with supercooled vacuum state to solve the horizon and flatness problems and the monopole problem raised with unification theories.

The basic idea of inflation is that there has been an epoch in which the Universe expanded exponentially as the result of the domination of the vacuum energy component on its energy density. This domination should be temporary and the vacuum energy should transform into energy of particles. In this sense, it is useful to work with a metastable configuration of a scalar field which fills the expanding Universe (instead of a real cosmological constant reflecting the vacuum property): an homogeneous, classical, scalar field can play the role of an unstable vacuum state, and its decay can heat up the Universe. Such a field, displaced from the minimum of its potential, causes a change in the vacuum energy density described by the quantity $V(\sigma)$ that enters into the Einstein equations, affecting the properties of spacetime. After the phase transition, the appearance over all space of $\sigma_0 \neq 0$ (true ground state displaced from zero) simply represents a restructuring of the vacuum state but also changes the masses of those particles with which it interacts: sign "correction" for the mass squared of the field σ, as we saw, but also masses of both gauge bosons and fermions arise as the result of its non-zero vacuum expectation value (Higgs mechanism).

As we saw, unification theories contain some scalar fields, displaced from the minimum of their potential. In a first order phase transition, during supercooling, the energy density of relativistic particles $\propto T^4$ becomes negligible and the presence of the barrier between the true and the false vacuum keeps the Universe "hung up" in the metastable false vacuum with a constant energy density $V(0)(\equiv V(\sigma = 0))$ during the supercooling. With a constant energy density $\rho \approx \rho_0$ in the Friedmann equation (I.1.5), neglecting the k/a^2 term,

$$H^2 \approx \frac{\rho_0}{3} \qquad (I.3.1)$$

the Hubble parameter is constant and the expansion of the Universe turns to be exponential

$$a(t) \sim e^{Ht}, \qquad (I.3.2)$$

the Universe asymptotically approaches a de Sitter Universe.

In spite of the exponential expansion of the Universe, the energy density remains constant: during inflation the amount of matter in the Universe grows exponentially as its volume.

Due to its Lorentz invariance, the energy-momentum tensor of the (false) vacuum state is of the form

$$T_{\mu\nu} = \Lambda g_{\mu\nu} \qquad (I.3.3).$$

with constant Λ. In such a way, the energy density ρ and the pressure p are constant, equal, and with opposite sign

$$p = -\rho. \qquad (I.3.4)$$

The negative pressure allows for the conservation of energy: $p = -\rho \Rightarrow \rho da^3 = -p da^3$ allows for $d(\rho a^3) = -p da^3$ with $\rho = cte$, and it is the driving force behind the exponential expansion since $\rho + 3p < 0$ in $\ddot{a} = -\frac{1}{6}G(\rho + 3p)a \Rightarrow \ddot{a} > 0$ and the Universe expansion becomes accelerated. And that is what uniquely characterizes inflation: $\dot{a}$ must increase with t. Indeed, as we will see in next chapter, inflation can also be described by a power-law expansion.

The negative pressure (≡ "tension") is a characteristic property of quantum vacuum in field theory (this is similar to the case of the Casimir Energy). According to general relativity, the pressure also contributes to the attraction, so, the notion of negative pressure would lead to the effect of a gravitational force that is effectively repulsive.

I.4) Inflationary models

Inflation involves, as we have seen, a scalar field whose expectation value is, for some reason, displaced from the true ground state of its potential; it will not necessarily be tied to a phase transition. Models of inflation will usually involve very flat scalar potentials and hence a scalar field weakly interacting with any other field in order to have $V(\sigma) \approx const.$ and $\dot{\sigma} = 0$. If we impose this conditions in the expressions for the energy and pressure density of a homogeneous scalar ($\vec{\nabla}\sigma = 0$) field

$$\rho_\sigma = \langle T_{\mu\nu} \rangle u^\mu u^\nu = \frac{1}{2}\langle \dot{\sigma}^2 \rangle + \langle V(\sigma) \rangle \qquad (I.4.1)$$

$$p_\sigma = -\frac{1}{3}h^{\mu\nu}\langle T_{\mu\nu} \rangle = \frac{1}{2}\langle \dot{\sigma}^2 \rangle - \langle V(\sigma) \rangle \qquad (I.4.2)$$

where $h^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, we see that the equation of state becomes $p = -\rho$. If $\dot{\sigma}$ is non-zero it has at least to satisfy: $\dot{\sigma} \ll V(\sigma = 0)$, and this will lead to a slow-rollover regime for inflation.

a) Old Inflation

The old inflation model is tied to a phase transition that occurs, for some values of the parameters, very slowly compared with the cooling rate. Its success depends on the possibility of a smooth completion of the phase transition accompanied by a large scale thermalization of the latent heat, in order that the inflationary period ends, followed by a Universe described by standard cosmology. When he was proposing inflation, Guth [8] already realized that this was a serious problem of his model: the volume inside bubbles expands with a power–law (speed of light) whereas the region between bubbles still undergoes a de Sitter expansion; in such a way, a very inhomogeneous Universe would appear since the nucleated bubbles could not keep pace with the cosmic expansion and could never reach each other.

Phase Transition

In the classical theory, the false vacuum would be completely stable; in the quantum version the tunneling through the energy barrier can achieve the transition, resulting in bubbles of the new phase growing at a speed that rapidly approaches the speed of light. Coleman and Callan [9], generalizing the tunneling of a particle in quantum mechanics, develop a Euclidean approach to the theory of the decay of a metastable vacuum state. The tunneling rate or the probability of forming a bubble of the broken phase can be written as

$$\Gamma = Ae^{-B} \qquad (I.4.3)$$

where B is the bounce action corresponding to the solution of the classical equation of motion for the field σ in Euclidean space and the prefactor $A \propto M^4$, with M some mass scale associated with the potential (e.g. the height of the barrier). A large action corresponds to a strong first order transition with a considerable supercooling. Coleman and De Luccia [10] have worked to generalize the Euclidean formalism to curved space. The progress in calculating the decay probability of the false vacuum in realistic theories is complicated by the fact that the solution to the σ equation of motion and the associated value of the Euclidean action must often be computed numerically and the prefactor can only be calculated in certain special cases, working in general with just a rough estimate.

Guth and Weinberg [11] and [12] investigate cosmological phase transitions driven by slow nucleation of bubbles via the zero temperature quantum tunneling process. They consider that bubble nucleation starts at some time t_0 and afterwards occurs at a constant rate per unit (physical) volume; bubbles are assumed to form with zero radius and then expand at the speed of light. They first consider whether the bubbles percolate and thermalize on a large scale. The fraction of space remaining in the old phase is given by

$$p(t) = \exp\left[-\int_{t_0}^{t} dt_1 \Gamma(t_1) a^3(t_1) V(t_1, t)\right] \qquad (I.4.4)$$

where

$$V(t_1, t) = \frac{4\pi}{3} \int_{t_1}^{t} \left[\frac{dt'}{R(t')}\right]^3 \qquad (I.4.5)$$

is the coordinate volume at time t of a bubble formed at time t_1. This volume is an increasing function of time but it tends to an asymptotic value ($= (4\pi/3H^3)e^{-3Ht_0}$) and hence there is always a fraction of space that remains in the old phase, no matter how long we wait. The finite limit of the volume occupied by bubbles represents an event horizon: two bubbles which born simultaneously, separated by a distance greater than the Hubble radius ($2H^{-1}$) will never collide. This means there will be no large scale percolation. We can define a measure of the possibility of percolation as the nucleation rate relative to the expansion rate of the Universe

$$\epsilon = \frac{\Gamma}{H^4}. \qquad (I.4.6)$$

In [11] it was found, on one hand, that there is some critical value

$$10^{-6} \lesssim \epsilon_{cr} \lesssim 0.24 \qquad (I.4.7)$$

at which percolation sets in, and, on the other hand,

$$\epsilon \lesssim 4 \times 10^{-3} \qquad (I.4.8)$$

is needed in order to have enough inflation to solve horizon and flatness problems (provided, of course, there is enough thermalization of the latent heat of transition). An estimation of the tunneling action in an $SU(5)$ phase transition [12] shows that values of ϵ of order 10^{-1000} are quite plausible. But a small percolation parameter obviously means poor thermalization. As for the possibility that our Universe is the product of the collision of a small number of bubbles, they prove that concentrations of bubbles form clusters which are each finally dominated by one bubble (the largest) and argue that, colliding with bubbles much smaller than itself, the energy in the walls of the largest bubble has no chance to be thermalized. The possibility that the presently observed Universe developed from a single bubble is ruled out because the region would fail to satisfy simultaneously the requirements of containing enough entropy to encompass our Universe and reheat to a high enough temperature for nucleosynthesis.

These considerations leave a really small window for the percolation parameter value: it has to be low enough to let inflation set in and high enough to restore a standard Universe. In such a way, the possibility of large scale thermalization is rejected: even a universe originating from the collisions of a small number of bubbles, as a consequence of the spread in the bubbles size, is unlikely to be a homogeneous and isotropic region containing sufficient entropy. They suggest that the solution would be to find a triggering mechanism for the phase transition to keep low the nucleation rate at the beginning and then become suddenly large, producing many bubbles of comparable size.

In the attempt to improve the situation, it was realized that inflation could be implemented in another type of phase transition [13].

b) New Inflation

Inflation occurs during the process of slowly growth of the inflaton field to its equilibrium value σ_0 and the phase transition is smooth (weakly first order or second order). We require a very flat potential barrier, that disappears for $T = 0$, in order to have a jump of the expectation value of the scalar field from $\sigma = 0$ to some initial value σ_i by quantum tunneling or through thermal fluctuations, and a slow evolution to the minimum σ_0 from there on. If, during this slow–rollover, inflation is large enough, the whole observable Universe evolves out of a single fluctuation region ("bubble"). The walls of this region will be far apart and will not engender any inhomogeneities in the observable part of the Universe. To ensure an inflationary behaviour, the effective potential must be flat near $\sigma = 0$, requiring very small self-coupling constants. The recovery of the standard Universe is not through the collisions of the bubble walls but rather through the decay of the σ field to other, lighter fields to which it couples. The inflationary potential has to become steep as it approaches the stable minimum σ_0. As the inflaton field begins to oscillate about σ_0, the vacuum energy is in the form of spatially coherent oscillations of the σ field, corresponding to zero-momentum σ particles. Their decay to other fields, coupled to the inflaton field, damp these oscillations and populate the Universe with matter and radiation. As the decay products thermalize, the Universe is reheated.

c) Chaotic Inflation

This model, suggested by Linde [14], is based on the assumption that the initial distribution of a scalar field is chaotic, i.e. it takes different values in different regions of the Universe. The reason for this would be that energy density fluctuations at the epoch of quantum cosmology are so big that the field does not "know" where the potential minimum is. The variation of the scalar field should be slow enough to ensure a quasi-exponential expansion requiring therefore very small coupling constants in the potential that is of the simple form

$$V(\sigma) = \frac{\lambda}{n}\sigma^n \qquad (I.4.9)$$

with n an even number. The minimum of this potential is at $\sigma = 0$. A chaotic distribution of the scalar field avoids making the assumption that the initial value of the scalar field corresponds to the minimum of its potential energy. And, indeed, with a small self-interaction coupling λ there is no reason to expect that at $t \sim t_{Pl}$ the field σ is equal to zero everywhere. In a Universe with a scalar field chaotic initial distribution, domains with a high enough initial value σ_i to ensure a sufficiently large inflation inevitably exist and give rise to mini-universes larger than the size of our observable part of the Universe. So, as in new inflation, we live in one single inflated "bubble" evolved from a small fluctuation region within one causal distance and there is no need for a bubble percolation process.

The chaotic inflation scenario differs from the other versions of inflationary Universe in that it is not based on the theory of high-temperature phase transitions in the early Universe. The scalar field is not attached to a unified theory and its only purpose is to implement inflation.

I.5) Inflationary predictions and solutions to cosmological problems

a) With inflation the horizon problem disappears: the horizon will just be moved to distances which have not been observed yet. Before inflation begins the region is much smaller than the horizon distance and it has time to homogenize and reach thermal equilibrium. And this small region is then inflated to become large enough to encompass our Universe.

Guth [8] has estimated that if the scale factor increases more than $\sim 10^{28}$ times during inflation, the horizon problem is solved.

b) During inflation the energy density of the Universe remains constant (or decreases very slowly) while the curvature term k/a^2 falls off exponentially, thereby explaining the flatness of the Universe. The global topological properties of the Universe will certainly remain unchanged but, for a sufficiently long inflationary period (that is approximately equal to the one required by the horizon problem [8]), the exponential growth of the radius of curvature of the Universe at that epoch ensures that it is still much greater than the Hubble radius today.

c) Inflation dilutes the monopole abundance: topological defects are created at the intersection of exponentially large bubbles and therefore have exponentially small density. To this end, GUT spontaneous symmetry breaking should of course occur before or during inflation.

d) An attractive feature of de Sitter expansion is that because of its rapidity, the universe loses all information on initial conditions.

e) The main success of the inflationary theory is the possibility of generating the seed perturbations that can grow to form the large scale structures. In the new and chaotic inflation models, where a single inflating "bubble" encompasses our present observable Universe, small quantum fluctuations of the scalar field in the homogeneously inflating region may, because of the exponential stretching of spatial dimensions, be at the origin of galaxies and clusters of galaxies. The inflationary theory provides the early Universe with a scale–invariant spectrum of pertubations (i.e. amplitudes almost independent of the wavelength). The amplitude of the spectrum is model dependent, but the form is a generic prediction. We will treat this topic extensively in chapter V. This great success unfortunately presents a drawback: the fluctuation amplitudes are much too large as compared with the observable fluctuations imprinted on the CMBR. Arranging for acceptable density perturbations results in a very restrictive constraint on inflationary potentials: one needs an extremely flat potential. For instance, with a potential of the form $\lambda\sigma^4$, the self–coupling constant λ must satisfy $\lambda < 10^{-12}$. We then have a fine–tuning problem: not only such a small value for the self–coupling costant sounds unnatural, but it seems hard to be preserved because of radiative corrections from interactions with other fields. The inflaton field must then be very weakly coupled to all fields so that one–loop corrections to the scalar potential do not interfere with the extremely flatness required. This fact has unpleasant implications for reheating (see below) and also for the new inflation potential: with an extremely weakly interacting field the high temperature corrections to the effective potential $V(\sigma, T)$ are negligibly small and this has the consequence that $\sigma = 0$ is no longer metastable.

f) An important question to be solved in all inflationary models is the thermalization mechanism of the vacuum energy density. An extremely weakly interacting field essentially decouples from any kind of particles leaving therefore no possibility of reheating, i.e. no transformation of vacuum into radiation.

g) The inflationary Universe requires a Λ term, but it arises and is supposed to last only during the transient stage in which the GUT phase transition is taking place. While inflation has the potential to solve all of the purely cosmological controversial issues of the standard cosmology, it does not address the puzzle of the cosmological constant (which is a problem of particle physics too).

h) Inflation explains the great degree of homogeneity of our observed Universe but one must assume however that in the early universe at least some regions were uniform and hot compared with the critical T of the phase transition. The scalar field must be smooth in a large enough region so that the energy density and pressure associated with spatial gradients in σ are smaller than the potential energy since if $(\nabla\sigma)^2$ dominates inflation will not occur.

i) There are some questions (certainly related) that raise now: what is σ? Why is it so weakly coupled? And are the initial conditions necessary for the realization of the inflationary regime sufficiently natural? The identity of the "inflaton" is not known. Presently, it is taken to be either some yet unidentified scalar particle, or an effective action term due to various interactions present. More or less successful models have been proposed where the inflaton field is related to the GUT phase transition, to supersymmetry spontaneous symmetry breaking, to higher-dimensional theories (where it is related to the radius of compactification of extra spatial dimensions), to a higher-derivative theory of gravity (where it is associated to the curvature scalar), where it is a non-minimally coupled scalar field or just a random scalar field as in chaotic inflation.

Conclusions

The inclusion of an inflationary period in the early history of the Universe is certainly useful, in particular to avoid the assumption of narrow initial conditions and to produce seeds for structure formation. The vagueness about the cause of inflation implies that it is a general concept rather than a specific physical theory. The important and difficult issue is to ensure that the inflaton field fits in the framework of a realistic elementary particle theory. Since we have encountered a, more or less pronounced fine--tuning problem in all the models we have discussed, the search for inflationary models continues.

Chapter II

Extended Inflation

Introduction

In the previous chapter I have depicted the "standard" inflationary models: all of them bring a modification to the matter sector (by including a scalar field with some particular effective potential). Other models introduce a modification in the gravity sector too, we will turn to them now.

La and Steinhardt [15] have recently developed a scenario for inflation based on the Jordan–Brans–Dicke gravity theory [16] in which the extraordinary expansion which characterizes inflation now follows a power law instead of an exponential one. Power–law inflation models had been previously proposed using an exponential potential for the inflaton [18]. In this new model, called extended inflation, the same behavior is the result of the dynamics of two fields: an inflaton, with an old–inflation type potential, and a scalar field, the Jordan–Brans–Dicke (hereafter JBD) field, which plays the role of a time–varying gravitational coupling. The slower expansion can be explained by the fact that the vacuum energy density is now shared between the Universe expansion and the evolution of the JBD (gravitational) scalar field. Actually the two schemes are related by a conformal transformation (see e.g. [19]): to the JBD frame with an inflaton field corresponds in the Einstein frame – where the gravitational coupling is constant – a model with a field having an exponential inflationary potential. In extended inflation, as in the old inflation model, the phase transition is of first order, but the slower expansion now allows bubbles of true vacuum to percolate and form a big enough region to contain our Universe. However bubbles are still a source of problems since the recovery at the end of inflation of such a region, homogeneous and isotropic, demands a low value of the Brans–Dicke parameter

[20], [21], which is in conflict with the lower observational bounds [22]. Taking this into account, we have to abandon the simplest models of extended inflation, e.g. introducing a potential for the JBD field [23]. Several other variations have been proposed: introducing a JBD field in chaotic [24], [25] or new [25] inflation, allowing the BD parameter to vary with time [26], [27], introducing non-standard couplings of the JBD field to matter [28].

In this chapter we will briefly discuss the JBD theory and review the work that has been done on power-law inflation, leaving the model with non-standard couplings for the next chapter.

II.1) Jordan–Brans–Dicke theory

In 1961 Brans and Dicke [16] developed a modified relativistic theory of gravitation in which the gravitational effects are in part geometrical and in part due to a scalar interaction. (In 1959 Jordan [17] had developed a theory formally similar to the Brans–Dicke one but with different physical interpretation). They were interested in a gravitational theory compatible with Mach's principle, in the sense that locally observed inertial reactions should depend upon the mass distribution of the Universe about the point of observation and consequently the physical "constants" should be position dependent. Being possible to reduce the variation of physical "constants" to that of a single parameter, they introduced a scalar field Φ whose primary function is the determination of the local value of the gravitational coupling. The ideas they present are incompatible with the strong equivalence principle.

Starting from the usual variational principle:

$$0 = \delta \int \left[R + (\frac{16\pi G}{c^4})\mathcal{L} \right] (-g)^{1/2}, \qquad (II.1.1)$$

they get the required generalization dividing by G – substituting it by Φ^{-1} – and introducing a Lagrangian density of a scalar field,

$$0 = \delta \int \left[\Phi R + (\frac{16\pi}{c^4})\mathcal{L} - \omega \frac{\partial_\alpha \Phi \partial^\alpha \Phi}{\Phi} \right] (-g)^{1/2} d^4x \qquad (II.1.2)$$

where the scalar field in the denominator has been introduced to let the constant ω be dimensionless. The parameter ω is a measure of the influence on the gravitational field acquired by the scalar field, with respect to the space–time curvature. The smaller ω, the more important is the effect of the scalar field. When $\omega \to \infty$ the JBD theory coincides with Einstein gravity.

In order not to interfere with the successes of the equivalence principle, the Lagrangian density of matter is identical in both equations and the equations of motion of matter are the same as in general relativity and the difference between the two theories lies in the gravitational field equations which determine $g_{\mu\nu}$.

The wave equation for Φ is:

$$\Box\Phi - \frac{1}{2\Phi}\partial_\mu \Phi \partial^\mu \Phi + \frac{R\Phi}{2\omega} = 0 \qquad (II.1.3)$$

with $\Box \equiv g^{\mu\nu}\nabla_\mu \nabla_\nu = (-g)^{-1/2}\partial_\mu[(-g)^{-1/2} g^{\mu\nu}\partial_\nu]$.

The field equations for the metric field are obtained from (II.1.2) by varying the components of the metric tensor and their first derivatives

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi}{c^4\Phi}T_{\mu\nu} + \frac{\omega}{\Phi^2}(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha \Phi \partial^\alpha \Phi) + \frac{1}{\Phi}(\nabla_\nu \partial_\mu \Phi - g_{\mu\nu}\Box\Phi) \quad (II.1.4)$$

The first term on the right is the usual source term of general relativity ($T_{\mu\nu}$ is the matter energy–momentum tensor corresponding to $\mathcal{L}$) with gravitational coupling Φ^{-1} and the second one is the energy–momentum tensor of the scalar field. The third term, results from the presence of second derivatives of the metric tensor in the variation of (II.1.2). These second derivatives are eliminated by integration by parts to give a divergence and the extra terms. The role of these extra terms is essential for a vanishing covariant divergence of the energy–momentum tensor of matter to be consistent with the equations of motion.

From (II.1.3) we see that the terms ΦR and the Φ–Lagrangian density serve as the source term for the generation of Φ waves. But it is interesting to see that, obtaining R from (II.1.4), this equation can be written:

$$\Box\Phi = \frac{8\pi}{(3+2\omega)c^4}T \qquad (II.1.5)$$

in such a way the source term appears as the trace of the energy–momentum tensor of matter alone (T), thus meeting the requirement of Mach's principle that Φ has as its source the matter distribution in space.

They also discuss the consequences of their theory on observable quantities, i.e. they check the classic tests of Einstein's theory. The gravitational redshift is computed in the weak field approximation. It is determined by g_{00} which contains, as compared to general relativity, the extra factor $[(4+2\omega)/(3+2\omega)]$. But this factor is absorbed into the definition of the gravitational constant: $G = \Phi^{-1}[(4+2\omega)/(3+2\omega)]$ and there will be no anomaly in the red shift. All metric theories of gravity predict the same gravitational redshift.

There is an anomaly in the deflection of light. This quantity differs from the GR value by the factor $[(3+2\omega)/(4+2\omega)]$, but at that time the accuracy of the light deflection observations was too poor to set any useful limit on ω. *

On the other hand, the accuracy in the observation of the perihelion rotation of the orbit of Mercury allowed for this limit. The precession is cumulative, so it can be observed over several years. The relativistic rotation rate of the perihelion of a planetary orbit is a factor $[(4+3\omega)/(6+3\omega)]$ times the GR value. There are other causes for precession (sun's oblateness, sun's rotation,...) but for comparison between measured and theoretical precessions it is usually taken: $\Delta\varphi_{GR} = \Delta\varphi_{obs} - \Delta\varphi_{Newt}$, with $\Delta\varphi$ precession angle in

* The first experiments were done with visible light, during an eclipse. Later, developments in radio astronomy made possible a far greater accuracy than was possible with optical astronomy. Along a decade, a number of radio–wave deflection measurements with groups of quasars that pass very close to the sun yielded different determinations of the metric parameter γ with encreasing accuracy, the last of which, in 1975, gave an interval centered in the general relativity value, see, e.g. Will [35], p. 172.

each revolution, which are the ones large enough to be measured. Taking into account a $\Delta\varphi$ caused by the sun oblateness (measurement from Dicke et al., in disagreement with other measurements, see e.g. Weinberg [29]) would enhance the discrepancy between the measured and the theoretical GR values. With an accuracy of 8% of agreement between the observed and the GR computed results, the bound they find is: $\omega \geq 6$.

Since then, the accuracy of the observations has notably improved. The previous tests dealt only with the shape of the trajectories of light or planets but it has then been possible to follow their time history. As predicted by general relativity, the round-trip times of light signals traveling to the inner planets or to artificial satellites and reflected back to the earth are increased by the direct effect of solar gravity on spacetime. Upon comparison between the theoretical value and measured echo delays we can put limits on the metric parameter. The estimation of the theoretical value of each corrected echo delay needs a metric theory and a model of the solar system. For this model we need distances, radii, masses, planet rotations, and we do not know them with a great accuracy; then a large set of unknown parameters are determined by fitting observed times with theoretical formulas. The comparison of metric theories with each other and with observations, at least for solar system tests, can be made in the post-Newtonian limit (i.e. weak field, slow motion limit). The *parametrized post-Newtonian (PPN) formalism* embraces most metric theories and contains a set of parameters (PPN parameters) whose values fix the particular metric theory we are dealing with. One of this parameters is γ, which is a measure of how much space curvature (g_{ik}) is produced by unit rest mass, the general relativity value being $\gamma = 1$ (see e.g. [30]). For a scalar-tensor theory, the parameter γ is related to the coupling parameter ω through: $\gamma = (\omega + 1)/(\omega + 2)$.

The most accurate value, obtained from radio ranging to the Viking spacecraft is [22] $\gamma = 1.000 \pm 0.002$, where the uncertainty given, about twice the formal standard deviation, is based on the spread obtained in the estimates of γ from the many measured times and on a judgement of the reliability of all the procedures used in the collection and analysis of the solar system data. From the uncertainty on γ, the restriction on ω is:

$$\omega > 500.$$

II.2) Power–law inflation

The idea of inflation was continuously developed and it was realized that the cosmological problems which led to the proposal of inflation could also be solved by any accelerated phase of expansion, even if not exponential. In this way, a family of general inflationary models characterized by a scale factor which grows like $a \sim t^p$, with p a constant greater than 1, was investigated by Abbott and Wise [31] and Lucchin and Matarrese [18]. The first interesting result [18] is that the potential which leads to this power–law inflation is exponential:

$$V(\Phi) \sim e^{-(\Phi-\Phi_i)/\sigma} \qquad (II.2.1)$$

where Φ is the inflaton and the Φ–solution increasing with time is considered, i refers to an initial time t_i where $\Phi_i \neq 0$ and $\sigma = (p/4\pi)^{1/2} m_{pl}$; $V(\Phi)$ is assumed to depend on t only through Φ. Since the authors did not consider any physics underlying the model, this potential should be considered as a way to mimic the source for power–law inflation during a time interval. Then, considering the conditions that the combination of the observed isotropy of the cosmic background radiation and the requirement of forming galaxy protostructures imposes on the amplitude of quantum fluctuations of the scalar field it emerges that $p = 1.9$ is the lowest permitted value but in general, low values of p, e.g. $p = 2$, give a reheating temperature only marginally compatible with the usual baryosynthesis constraint and a perturbation spectrum not completely satisfactory on large scales. Abbott and Wise get the same result imposing observational bounds on the amplitude of gravitational waves generated from quantum fluctuations during the inflationary period. They relate this quantity to the maximum reheating temperature and find that for power–law inflation it is an increasing function of p.

A possible disease of this scenario is that the mechanism whereby inflation ends is unclear.

II.3) Extended inflation

In 1984, Mathiazhagan and Johri [32] reanalyse inflationary scenarios under the framework of JBD theory. Their model contains two scalar fields: the JBD field and another scalar which mediates the transition. The resulting expansion is power–law. With a Coleman–Weinberg potential for the inflaton (in the standard SU(5) model), they estimate the time required after the tunnelling event for the inflaton to roll down to the global minimum and find that the amount of inflation is sufficient to solve the cosmological problems.

In 1989, La and Steinhardt [15] and [19] develop this scenario with an old inflation type potential, giving special emphasis on the percolation of bubbles of true vacuum and the thermalization during collisions, of the energy contained in the bubble walls.

Equations (II.1.4) and (II.1.5) with a FRW line element with scale factor $a(t)$ and a spatially homogeneous JBD field $\Phi = \Phi(t)$ are:

$$H^2 = \frac{8\pi\rho}{3\Phi} - \frac{k}{a^2} + \frac{\omega}{6}[\frac{\dot{\Phi}}{\Phi}]^2 - H[\frac{\dot{\Phi}}{\Phi}] \qquad (II.3.1)$$

and

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{8\pi T}{3+2\omega} \qquad (II.3.2)$$

where ρ is the Universe energy density, H is the Hubble parameter $H = \dot{a}/a$, a dot denotes differentiation with respect to t and $k = 0, 1, -1$ corresponds to a flat, closed or open Universe.

The solution to the JBD equations when the Universe is essentially vacuum energy density dominated is:

$$\Phi(t) = m_p^2(1 + \frac{\chi t}{\alpha})^2 \qquad (II.3.3)$$

$$a(t) = (1 + \frac{\chi t}{\alpha})^{\omega+1/2} \qquad (II.3.4)$$

where $\chi^2 = 8\pi\rho_v/3m_p^2$ (Hubble constant in the Einstein theory), ρ_v is the vacuum energy density, m_p is an arbitrary integration constant corresponding to the effective Planck mass at the beginning of inflation, and $\alpha^2 = (3+2\omega)(5+6\omega)/12$.

For short times, $\chi t < \alpha$, the JBD solution has an exponential regime (for large ω) $a(t) \approx \exp(\chi t)$, with Φ nearly constant. Then, when $\chi t > \alpha$, this solutions can be written:

$$\Phi \approx m_p^2(\frac{\chi}{\alpha}t)^2 = \frac{32\pi\rho_v}{(3+2\omega)(5+6\omega)}t^2 \qquad (II.3.5)$$

$$a(t) \approx (\frac{\chi}{\alpha}t)^{\omega+1/2} \qquad (II.3.6)$$

It has to be noted that the solution for the JBD field used in extended inflation is a special case of a more general class of solutions: it meets the boundary condition $\Phi(0) = 0$. This ensures that the JBD solution is dominated by the vacuum energy and not by the JBD scalar field as $t \to 0$. The same initial condition is also usually imposed when solving for matter domination. After the inflationary epoch, the evolution of the JBD field is much slower. During the radiation dominated epoch, the Universe expansion is the same as in general relativity

$$a(t) \sim t^{2(\omega+2)/(3\omega+4)},$$

$$\Phi = const,$$

since the source for the JBD field evolution is the trace of the energy-momentum tensor of matter which is zero for an ultrarelativistic fluid. In the matter-dominated era, the presence of the JBD field has again some effect on the Universe behaviour; with $k = 0$ we have

$$a(t) \propto t^{(2\omega+2)/(3\omega+4)},$$

$$\Phi(t) \propto t^{2/(3\omega+4)}.$$

This solution corresponds to the simple case of zero integration constant in the field equation for Φ, i.e. it is obtained imposing the constraint $\dot{\Phi}a^3 \to 0$ on the initial singularity ($a = 0$). While this solution is obtained with special values for the initial conditions that may not be the appropriate ones, all other solutions approach this one at large time, see e.g. [29].

There have also been attempts to implement inflation by relating directly the inflaton field with the gravity sector. In the *induced gravity* model [33], gravity arises as a symmetry breaking phenomenon; it is a consequence of vacuum fluctuations of quantum fields in curved backgrounds. Thus, the theory deviates from general relativity only at high energies. In this context, gravity is a phenomenological theory instead of a fundamental one. With, for instance, a Ginzburg–Landau symmetry breaking, the transition can be inflationary. Since the new feature, as compared to conventional scenarios, is the substitution in the Lagrangian of the term $(16\pi G_N)R$ by a term $1/2\epsilon\sigma^2 R$, one would expect, as long as the slow–rolling approximation is valid, little change in the course of inflation. This turns out to be true and we consequently have the same fine–tuning problems as in standard inflation. That is why we turn to inflationary models with two fields. Inserting extended inflation in the context of induced gravity, a more complicated model is obtained, where both the inflaton and the JBD field have inflationary potentials [23]. The result is a multiple–episodes inflation in which at low energies the gravitational coupling would be driven to its present constant value by the JBD field potential. Furthermore, the fluctuations generated in the initial inflationary phase could establish the large–scale structure, while those of the second phase would be responsible for the perturbation spectrum at small scales.

La and Steinhardt calculate (in the usual way defined by Guth and Weinberg [11]) the probability $p(t)$ for a point to remain in the false vacuum phase (eq (1.3.8)), substituting here a scale factor which grows as a power–law of time. With this dependence on time, $p(t)$ decreases faster than the Universe volume $a^3(t)$ increases and as the physical volume occupied by the false vacuum is $p(t)a^3(t)$, the Universe exits from the false vacuum phase. Another approach is considering the number of bubbles created in a Hubble volume in one Hubble time $\epsilon \equiv \Gamma/H^4$, where the nucleation rate per unit time per unit volume (Γ) is again considered as a constant. In standard inflation H was also a constant and so was ϵ In the power-law expansion regime $H \equiv \dot{a}/a = H_0(1 + H_0 t/\omega)^{-1}$, resulting in $\epsilon \propto t^4$. In this way, ϵ can be very small at the beginning, allowing for inflation to occur, and then grow above the critical value where percolation is achieved. Here, the filling of space with true vacuum still occurs exponentially in time but now the expansion is only power law. The time variation of ϵ also alters the bubble size distribution, this being an important quantity for the estimation of the time necessary for thermalization. Since the energy is concentrated in the bubble walls, the spread of energy through the bubble interior, after collision, is faster in small bubbles than in big ones. On the other hand, for the same reason, large bubbles could lead to voids in the subsequent matter distribution.

We can then hope that a small but non-negligible number of big bubbles exists. It could influence the large-scale structure and be at the origin of voids and at the same time it would be so small not to imprint inadmissible distortions to the background radiation.

Additionally, with this thermalisation process we have a mechanism for non adiabatic fluctuations: the radiation pressure makes radiation separate from matter in their diffusion through the true vacuum region. Isothermal fluctuations may be useful for structure formation.

When $\omega \to \infty$ the model recovers the old inflationary scheme that fails precisely in the percolation and thermalisation processes. We can therefore expect an upper limit on the ω parameter in order to get successful inflation. If the change from exponential to power-law expansion is enough to ensure percolation, it is also necessary to verify that a satisfactory reheating process is achieved [20], [34]. Considering astrophysical and cosmological restrictions on the model, the most stringent bound that has been obtained comes from the isotropy of the cosmic background radiation (CBR). To limit the distortions in the CBR due to the bubbles energy, a heuristic constraint is imposed: that only a small fraction of space (at most $10^{-4} - 10^{-3}$) is still in the thermalisation process at the recombination epoch. The upper limit obtained for ω at the end of inflation is:

$$\omega \lesssim 25$$

Although this value is not very precise, it is definitely far from the observed present bound [22] $\omega > 500$. So, the quantity of big bubbles turns out to be too high. However, since the conflict originates from constraints applied at very different times, it is possible to think to an "evolutionary" solution to the problem.

The general features for a successful extended theory of inflation are outlined: an epoch of sufficient inflation with a high initial value of the Hubble parameter –consequently a small initial ϵ– decreasing slow, in order to prolong the inflationary period, to a value H_{crit} –ϵ_{crit}– at which percolation is achieved. At the end the decrease in H should be faster than in purely Brans–Dicke, to obtain an adequate reheating. To meet this requirement of a steeper variation of the bubble nucleation parameter in order to suppress big bubbles and have, towards the end of inflation, a boom of small bubbles, all of them nearly of the same dimensions, Steinhardt and Accetta [26] and García–Bellido and Quirós [27] propose to generalize the model, introducing a variable BD parameter $\omega(\Phi)$. Thus, a dynamical mechanism to keep low the ω value during inflation and let it grow to the present value during the subsequent epoch is available. (This more general scalar–tensor theory of gravity had already been proposed in 1968–1970 [35]) In their *Hyperextended model*, Steinhardt and Accetta introduce a non–minimal coupling for a field ϕ:

$$\mathcal{L} = -f(\phi)R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 16\pi\mathcal{L}_{matt}, \qquad (II.3.7)$$

where $\mathcal{L}_{matt}$ does not include ϕ, and then recast this Lagrangian in a form reminiscent of JBD theory writing $f(\phi) = \Phi$ and absorbing in ω the extra factors in the second term

$$\mathcal{L} = -\Phi R + \frac{\omega(\Phi)}{\Phi}\partial_\mu\Phi\partial^\mu\Phi + 16\pi\mathcal{L}_{matt} \qquad (II.3.8)$$

with $\omega = f/2[f']$ where $f' \equiv df/d\phi$.

The equations for a variable ω are

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3\Phi} + \frac{\omega}{6}(\frac{\dot{\Phi}}{\Phi})^2 - H\frac{\dot{\Phi}}{\Phi} \qquad (II.3.9)$$

which is the same as in extended inflation, and

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{d\omega/d\Phi}{2\omega+3}\dot{\Phi}^2 = \frac{8\pi}{2\omega+3}(\rho - 3p) \qquad (II.3.10)$$

where the extra term takes into account the variation of the parameter.

The advantage of the extended model over the old inflation model was that the nucleation parameter ϵ changed from a constant to a quantity growing in time –as the fourth power of time. But this growth rate has still shown to be inadequate. The hyperextended model obtains a nucleation parameter that grows exponentially with time.

To support the departure from standard gravity in order to get a successful inflation, Accetta and Steinhardt [36] present the role that inflation can play in altering the gravitational force. They consider a non-minimal coupling for the JBD field, that takes the form $f(\Phi)R$ where $f(\Phi) \approx M_0^2 + \xi\Phi^2 + \xi'\Phi^4/M_0^2 + ...$ for $\Phi << M_0$. Typically, the initial value of Φ and the coupling constants $\xi, \xi', ...$ are small and all but the first term are taken to be negligible. Inflation can amplify the effects of the non-minimal coupling since the false-vacuum energy pushes Φ to high values and thus the higher order terms, usually ignored, become important.

In the context of a time-dependent (Φ-dependent) ω, an interesting alternative arises for the mode in which inflation ends [37]. Since the exponent of the power-law expansion depends on ω, below a certain value of ω ($\omega = 1/2$, in standard extended inflation) the expansion of the Universe becomes subluminal. Therefore, for an ω decreasing with Φ, inflation ends while the Universe is still trapped in the false vacuum and most of the false vacuum is converted to true vacuum by bubbles nucleated after the end of inflation. In this case, most of the bubbles do not inflate and the big bubble problem is avoided.

It is worth noticing, however, that there could be no big bubble problem. A generalization of the thin wall formalism (Coleman [9]) to extended inflation [39] seems to indicate that gravity can lead to the recollapse of bubbles at the beginning of the inflationary period, thus altering the bubbles distribution. This happens in general relativity too –in curved spacetime, a newly nucleated bubble cannot, at first, keep pace with the expansion of the Universe– but it is more important in JBD theory since gravity is stronger during the early stages. Only after the gravitational constant has decreased sufficiently can the bubbles be treated as if they expand immediately after nucleation with the speed of light. The source of energy for the expansion of bubbles is the energy stored in the false vacuum. In flat space it is just the potential energy of the inflaton while in curved space it has an additional contribution: the gravitational energy of the inflaton, which represents a considerable fraction of the energy of early bubbles. In the context of a decreasing–G theory, the conversion of gravitational energy into kinetic energy of the JBD field is responsible for the depletion of energy of these early bubbles which will consequently start to shrink and recollapse. For this mechanism to be useful to our purposes, it must last long enough (i.e. gravity must inhibit the growth of bubbles during most of the inflationary epoch) and this imposes restrictions on the model. For instance, Goldwirth and Zaglauer [39] find that for a double–well inflaton potential with non–degenerate minima, the self–coupling constant must satisfy $\lambda \approx M/M_{Pl}$, where M represents the typical mass scale of the phase transition. It is a moderate fine tuning for λ and it is consistent with the thin–wall approximation requirement ($\lambda \ll 1$). The implications of these values on the inflationary model (reheating, amplitude of density fluctuations) should, of course, be checked.

Another solution for the discrepancy between the value of ω implied by present observations with the ω value required by a successful inflation is to resort to the induced extended inflation model. The key difference is the existence for a potential for the JBD field: if there is some potential keeping Φ anchored at some value, then the low–energy limit of JBD will resemble Einstein gravity.

Among all these attempts to construct an appealing, "natural", version of inflation, the idea of extending the other inflation models to the JBD frame is quite immediate. Linde suggests a chaotic extended inflation scenario [24], introducing in the matter sector of the JBD action a chaotic type potential for the inflaton: $V(\sigma) = \lambda\frac{\sigma^{2n}}{2n}$. He just shows that it is a possible model and puts special emphasis on the feature of self-regeneration of inflationary domains leading to an eternal process of inflation -as in the original chaotic inflation model- and on the possibility of relating the present large value of the Planck mass to other coupling constants in the theory or else to anthropic arguments. The model has further been investigated by McDonald [24] obtaining the following constraints on the BD parameter. To obtain sufficient inflation $\omega \gtrsim 0.25$ but the model does not conflict with the observed isotropy of the Universe provided that $\omega \gtrsim 16$. On the other hand, only for $\omega < 250$ does the model differ significantly from standard chaotic inflation. The interesting interval would then be: $16 \lesssim \omega \lesssim 250$. but in this range the bound imposed by the CBR on the density fluctuations requires a much smaller self-coupling of the inflaton than in conventional chaotic inflation. So it seems that although chaotic inflation in JBD theory is possible it is not of great benefit.

A chaotic version of the hyperextended scenario have also been proposed by Lidsey [37], with an exponential potential $V(\sigma) = V_0 \exp(-\lambda\kappa\sigma)$ for the inflaton.

More work has been done about chaotic extended inflation but with further sophistications; we include it in the next section.

Conclusions

The JBD gravitational theory introduces interesting possibilities for the inflationary scenario: a softer inflation that avoids some of the problems of the original model, while still linked to a (first order) phase transition. Unfortunately, once again, the model does not go successfuly through the whole inflationary process and variations of the extended model have to be considered. Facing the discrepancies between present observations and inflationary requirements -in particular the pressing issue of successful recovery of a Friedmann Universe- the simplest possibility seems to be the inclusion of a potential term for the JBD field, that allows for different evolutions of the gravitational sector at high and low energies. We will investigate this possibility more extensively when considering the Universe evolution resulting from the inflationary period (chapter V).

An important aspect is that inflation seems to be a very general concept –an accelerated expansion epoch for the Universe– that may be implemented in the context of a wide range of particle and gravitational theories, introducing modifications in the potential terms or in the coupling terms in order to overcome the problematic steps of the process.

Chapter III

Generalized Scalar–Tensor Theories of Gravity

Introduction

The effects of the presence of a JBD type scalar mixed to the usual tensor interaction are severely constrained by experiments but, since observations only refer to visible matter, it is possible to construct a theory where a scalar field is coupled more strongly to a possible invisible or dark matter component than to visible matter. This is the key idea in a work by Damour, Gibbons and Gundlach [28] (hereafter DGG) who consider a generalized (or "amplified") JBD theory, where a dilaton field couples with different strengths to visible and dark matter. The observational bound would then apply only to the visible matter coupling. They find a maximum value for the present rate of change of Newton's constant compatible with – and not far from – observational bounds.

The same technique was applied to the inflaton field driving extended inflation, considering it as an invisible matter component having non-standard coupling to the JBD field [40]. In this amplified extended inflation model, the combination of the restrictions imposed by the observational limits on the BD parameter ω and by the requirements of a successful inflation, leads to a considerable region in the parameter space of the theory where all constraints are satisfied. It is the presence of two parameters –instead of one– that gives enough freedom to keep $\omega > 500$ and at the same time satisfy the isotropy of the CBR requirement.

Holman *et al.* [41] go beyond this model, now allowing non-standard kinetic and potential terms for the inflaton in the JBD frame, i.e. more general couplings (that is the model we will call "generalized" from now on) between the JBD field and the inflaton.

Generalized chaotic and new extended inflation have also been analysed by Berkin and Maeda [52] and several works have been done trying to constrain these generalized couplings from observational bounds [42], [43] and [44] and from requirements of successful inflation [45].

In this chapter we review the DGG work and the subsequent generalized extended inflation models. Constraints on these models are treated in more detail in chapter VI.

III.1) Dark Matter, non–standard dilaton couplings

As we have seen, extended inflation models are not easily implemented: they are subject to a stringent restriction that leads to introduce sophistications to the simplest model. The fact is that the JBD theory itself is severely constrained by present experiments and it is in this sector that we have to impose further modifications. The scalar field ϕ that is introduced in JBD is coupled to matter only through the metric: it appears in the gravitational field equations, determining the metric, and then intervenes in the matter equations of motion only through it. We may look for the possibility of a more general JBD theory where the scalar field couples with different stengths to different matter sectors. This leads to a violation of the weak equivalence principle and is severely restricted by experiments. If we, however, postulate some kind of "dark" matter, since experiments usually work with visible matter, the equivalence principle would be violated in a matter sector that eludes experimental tests. This would also reconcile the theoretically preferred spatially flat universe with the presently observed matter density.

Modified theories of gravity may result from higher-dimensional theories upon compactification to 4 dimensions: supergravity or superstring models produce a decaying exponential potential coupled to other scalar fields and generalized Einstein theories –by generalized Einstein theories we mean JBD, induced gravity, any theory with non–minimal coupling, R^2 theory and effective four–dimensional theories arising from compactification from higher-dimensions. So we may search for the more general couplings we need in the dimensional-reduced Lagrangians. In superstring theories (see e.g. [46]), the couplings of the effective four-dimensional theory (i.e. the measured experimental values) are determined by the vacuum expectation values of the fields; the coupling "constants" are in fact not constant. One of these fields, coming from the gravitational multiplet, is the dilaton, a neutral scalar field coupled to R (the Ricci scalar) in a way reminiscent of a JBD theory.

DGG work with a dilaton field coupled with different strengths to visible and invisible matter, their action functional in the Jordan (physical) frame reads

$$S = \int d^4x\sqrt{-g}\Big[-\Phi R + \frac{\omega}{\Phi} g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\Big] + S_V[\Psi_V, g_{\mu\nu}] + S_I[\Psi_I, (2\Phi)^{1-\beta_I/\beta_V} g_{\mu\nu}], \tag{III.1.1}$$

where S_V and S_I denote the action functionals for the visible matter fields, Ψ_V, and invisible ones, Ψ_I, respectively. We have again adopted units such that $8\pi G_0 = 1$, with G_0 the value of Newton's constant; this implies that the present value of the JBD field is $\Phi(t_0) = 1/2$. The visible sector couples only to the metric and not to the JBD field. The standard JBD theory corresponds to the simple case $\beta_V = \beta_I$. DGG actually define their model in the Einstein conformal frame, which is defined as that frame where the gravitational action takes the standard Einstein–Hilbert form and is obtained via the conformal (Weyl) transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = 2\Phi g_{\mu\nu}. \tag{III.1.2}$$

The rescaled action is

$$S = \int d^4x\sqrt{-\tilde{g}}\Big[-\frac{\tilde{R}}{2} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\Big] + S_V[\Psi_V, e^{-2\beta_V\varphi}\tilde{g}_{\mu\nu}] + S_I[\Psi_I, e^{-2\beta_I\varphi}\tilde{g}_{\mu\nu}] \tag{III.1.3}$$

where we also defined the new JBD field variable

$$\varphi = \varphi_0 \ln(2\Phi) \quad \text{with } \varphi_0 \equiv \sqrt{\omega + 3/2}. \tag{III.1.4}$$

In the Einstein frame both material systems couple to the dilaton field, but with different metric: $e^{2\beta_v \phi} g_{\mu\nu}$ and $e^{2\beta_i \phi} g_{\mu\nu}$ for visible and dark matter respectively.

Through a Weyl rescaling and redefinitions of the scalar field, every scalar–tensor theory can exhibit standard gravitational and kinetic terms (the two first terms of eq. (III.1.3)) and then the difference resides in the couplings to matter and in the self–interaction terms. In the transformation from a theory with a variable G to a theory with a constant G we have shifted the time dependence from the gravitational coupling to the inflaton vacuum energy density (and to couplings to the other material systems). The fact that extended inflation models can be transformed to a frame where the gravitational action assumes the standard form suggests that the physical justification could come from the modification of the particle physics scalar sector instead of the modification of the gravitational interaction. The solution to the percolation problem is power–law inflation, not some theory of non–minimal gravity; the latter is just a scenario for the achievement of power–law inflation. In this respect, it is interesting to keep in mind that in all these models there can be different versions for the physical idea that underpins the non–standard couplings proposed to overcome the difficulties encountered.

By working in this frame and assuming that the universe is dynamically dominated by an invisible fluid. DGG find a dynamical system which, transformed back to the Jordan frame adapted to visible matter, reads

$$\frac{dy}{dt} = -3r\beta_I H^2 + (6r\beta_I\beta_V - 3)Hy + (2\beta_V - 3r\beta_I\beta_V^2 + \frac{1}{2}r\beta_I)y^2, \qquad (III.1.5)$$

$$\begin{aligned}\frac{dH}{dt} &= (-\frac{3}{2}\gamma - 3r\beta_I\beta_V)H^2 + (3\gamma\beta_V - 4\beta_V + 6r\beta_I\beta_V^2)Hy \\ &\quad - (\frac{1}{4}\gamma - \frac{1}{2} - \frac{3}{2}\gamma\beta_V^2 + 3\beta_V^2 - 3r\beta_I\beta_V^3 + \frac{1}{2}\beta_I\beta_V)y^2,\end{aligned} \qquad (III.1.6)$$

with $y = d\varphi/dt$, $r = 4 - 3\gamma$ and $p_I = (\gamma - 1)\rho_I$. They find the attractor solution

$$H = \frac{2 - \gamma - 2r\beta_I\beta_V}{3\gamma - \frac{3}{2}\gamma^2 + r^2\beta_I^2 - 2r\beta_I\beta_V} t^{-1}, \qquad (III.1.7)$$

$$y = \frac{r\beta_I}{r\beta_I\beta_V - 1 + \frac{1}{2}\gamma} H. \qquad (III.1.8)$$

The variation of the scalar field gives the variation of the gravitational "constant", which can be compared with observational values. Using the observational constraints on the JBD coupling to visible matter and considering lower limits on the Hubble constant and the age of the Universe, the coupling constant β_I of the dilaton to invisible matter is constrained, nearly independent of β_V, to $|\beta_I| \lesssim 1$. They find a maximum value for the present rate of change of Newton's constant, $| d \ln G/dt |_0 \lesssim 6.6\times10^{-12} yr^{-1}$, consistent with the bounds based on the Viking–lander ranging data [47], $| d \ln G/dt |_0 \lesssim 10^{-11} yr^{-1}$, or with the less restrictive bounds based on binary pulsar measurements [48]. An improvement in the precision of $\dot{G}$ experiments can be a possible test for their model and indeed their theoretical value is slightly beyond the more recent bound [49] obtained incorporating new measurements of the neutron half–life and reaction uncertainties in nucleosynthesis calculations.

For radiation domination ($\gamma = 4/3$) the model gives $dG/dt = 0$ since the scalar field φ couples to the trace of the energy–momentum tensor only, as in the simple JBD theory.

III.2) Generalized extended inflation

Inspired by this possibility, Holman. Kolb and Wang [40], propose to solve the problem of thermalizing the energy in the bubble walls by the necessary epoch associating the inflationary field to the invisible matter having a different coupling to the JBD field; the identity of the inflaton being unknown allows such an assumption. The process of thermalization of the bubbles energy involves both types of matter and, consequently, the limit deriving from this requirement is now imposed on a combination of the parameters of the theory. On the other hand, timing measurements, as signal delay or orbital period change of a binary pulsar, refer only to visible couplings –with the assumption of a smooth distribution of dark matter over the solar system or the binary pulsar scales– thus the bound on ω is unaltered. Combining the restrictions imposed by the observational limit on ω and by requirements of a successful inflation, they obtain a considerable region in the ω–β space ($\beta \equiv \beta_I/\beta_V$) where all the constraints are satisfied: occurrence of an inflationary regime, sufficient inflation, percolation and thermalization, recovery of a common Robertson–Walker frame in all the bubbles that form our Universe. As in the simplest model of extended inflation, the truly restrictive bound comes from the requirement that a small fraction of space should be contained in big bubbles and leads to the relation $\omega/\beta^2 \lesssim 25$, instead of the standard limit $\omega \lesssim 25$. With $\beta \gtrsim 5$ one recovers $\omega \gtrsim 500$, in agreement with the observational limit.

Considering the action (III.1.1) in the inflationary regime (the invisible component being now associated to the inflaton field) we have $\Psi_I = \Psi_0$ with $V(\Psi_0) = \rho_v = const.$ (in extended inflation as in the old model), and the energy density of the false vacuum ρ_v dominates the total energy density (visible matter will play no role during inflation). With $k = 0$ the equations of motion for $a(t)$ and $\Phi(t)$ are

$$H^2 = \frac{\rho_v}{3}(2\Phi)^{1-2\beta} + \frac{\omega}{6}(\frac{\dot\Phi}{\Phi})^2 - H\frac{\dot\Phi}{\Phi}, \qquad (III.2.1)$$

$$\frac{\ddot\Phi}{\Phi} + 3H\frac{\dot\Phi}{\Phi} = \frac{4\beta\rho_v}{2\omega+3}(2\Phi)^{1-2\beta}. \qquad (III.2.2)$$

This system of equations admits power–law solutions just as the original extended inflation scenario did

$$\begin{aligned} a(t) &= a(0)(1+Bt)^p, \ p = (\omega - \beta + 3/2)/(2\beta - 1)\beta, \\ \Phi(t) &= \Phi(0)(1+Bt)^q, \ q = 2/(2\beta - 1), \end{aligned} \qquad (III.2.3)$$

where $t = 0$ means the beginning of inflation and

$$B^2 = \frac{4\beta^2(2\beta-1)^2\rho_v[2\Phi(0)]^{1-2\beta}}{(2\omega+3)(6\omega+9-4\beta^2)}.$$

With $\beta \equiv \beta_I/\beta_V = 1$, these results reduce to those for extended inflation (II.3.3) and (II.3.4).

Holman *et al.* [41] further generalize the model considering non–standard kinetic and potential terms for the inflaton σ in the Jordan frame

$$S = \int d^4x\sqrt{-g}\Big[-\Phi R + \frac{\omega}{\Phi}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + (2\Phi)^n\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - (2\Phi)^m V(\sigma)\Big] \qquad (III.2.4)$$

Comparison of the actions in eq.(III.1.1) and eq.(III.2.4) with $\Phi_I = \sigma$, shows that the DGG model corresponds to the particular case $1 - \beta_I/\beta_V = n = m/2$. They argue that in dimensionally reduced theories the generic action has $m, n \neq 0$. Starting from a higher dimensional (4+D) gravity model coupled to a scalar field whose potential allows for a phase transition to occur via bubble nucleation, they recast the Kaluza–Klein action into a "generalized JBD form", containing Φ–σ cross terms with $m = n = 1$. The Φ-field is defined, during the process of dimensional reduction, as a function of the scale factor $b(t)$ of the internal dimensions:

$$\Phi(t) \equiv \frac{1}{2}\left(\frac{b(t)}{b_0}\right)^D, \qquad (III.2.5)$$

and the BD parameter is defined as $\omega \equiv 1 - 1/D$ (which is of order unity for any D and hence far from the observational value). There are however two more differences with a JBD action: the effective JBD field has a kinetic term with "wrong" sign and a non-trivial self-interaction term. However their result is that their equations admit no power–law solution and the exponential expansion solution leads to an insufficient amount of inflation. Further attempts to implement generalized extended inflation from higher–dimensional theories will be analysed in the next section.

Holman *et al.* examine the bubble nucleation and percolation processes [50],[40]. The time evolution of the JBD field and its non–trivial couplings cause a time–varying false vacuum energy during (the Euclidian bounce, corresponding to) the tunneling process. In order to compute the bubble nucleation rate, the mechanism of false vacuum tunneling under these circumstances should be understood (see [51] for some work in this direction). Nevertheless, to ease the task, Holman *et al.* [50] establish an approximate expression that systematically "freezes out" gravitational effects taking $G_0 \to 0$. In this limit they also have to neglect the JBD field kinetic term, since it has the same G_0 dependence as the gravitational term: $\Phi \to \Phi_0 \propto G_0^{-1/2}$ as $G_0 \to 0$. On the other hand, the JBD field is of course not considered a constant during this treatment –its variation is the main feature of the process under description. The way out is to argue that the imaginary time bounce configuration, which is the one used to compute the tunnelling action. is a different situation from the real time configuration, where the JBD field can remain time-dependent, governing the evolution of the universe (after all. something similar happens in the original calculations [9] where the metric is frozen out from the bounce solution although the real time Universe was still expanding). They expect that their approximation ignoring terms of order G_0 will be reliable when the effective Planck mass induced by the JBD is much greater than the mass scales associated with the inflaton. If Φ increases with time, the approximation works better at late times. They work in the Einstein frame, to have standard gravitational couplings, with the truncated Euclidean action for the inflaton

$$S_E = \int d^4x[f(\Phi)\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + g(\Phi)V(\sigma)] \qquad (III.2.6)$$

where the JBD field time evolution is accounted for in the non-trivial couplings of the inflaton; and then they follow the same procedure as in the calculation for old inflation. They find, in the general situation, a time-dependent bubble nucleation rate per unit volume. If $2n - m \neq 0$ (i.e. the most general couplings) the time dependence of the nucleation probability (Γ) can be exponentially strong through the time dependence of the JBD field:

$$\Gamma = A_0(2\Phi)^{2n}e^{-B_0(2\Phi)^{2n-m}}$$

where A_0 and B_0 are Φ independent (they depend only on the inflaton potential). It has to be stressed out that Accetta and Romanelli [51], applying the formalism of Coleman and De Luccia [10] to the false vacuum decay in a scalar-tensor theory of gravity (where confident results are only obtained in the limit of small values of the non-minimal coupling parameter), find that Γ exponentially decreases during inflation, with a cutoff at Γ_0 (the constant decay rate calculated in flat space). This situation would not help in the search for an increasing Γ at the end of inflation, as suggested by the difficulty in thermalizing the energy in the walls of large bubbles. In the original extended inflation model one would also expect a time dependent nucleation rate due to the time dependence of the JBD field in the false vacuum but, applying the same technique to this case [50], it can be seen that, at late times (large JBD field), the rate becomes approximately constant in the Jordan frame (although time dependent in the Einstein frame). The different time dependence of the nucleation rate in the two frames may look uncomfortable but it is the result of making use of a time–dependent rescaling of space–time and is eliminated if one asks physical questions such as whether and when the true vacuum percolates.

So, we now have generalized extended inflation models where the time variation of ϵ may come not only from a varying H but also from a varying Γ.

The *Soft Inflation* model by Berkin, Maeda and Yokoyama [25] also involves non–standard couplings which can be seen to correspond to the case $m = n = 1$. They work with two coupled scalar fields: the inflaton driving new or chaotic inflation, and a field with exponentially decaying potential that slows the expansion rate. In a second paper, Berkin and Maeda [52] allow for arbitrary m and n as may result from fundamental theories or from conformal transformations on generalized Einstein theories. In the former case the kinetic term is standard ($n = 1$) [53] while in the latter one there may also appear an exponential coupling in the inflaton kinetic term. If the starting point is higher–dimensional theories, there are two possibilities for the inflaton: it can be defined in (4+D)–dimensions or introduced in the effective 4–dimensional theory.

They work in the inflationary regime, starting from the general action

$$S = \int d^4x\sqrt{-\tilde{g}}\Big[-\frac{\tilde{R}}{2} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}e^{-N\varphi}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - e^{-M\varphi}V(\sigma)\Big], \qquad (III.2.7)$$

which can be obtained from Eq.(III.2.4) via the conformal transformation leading to the Einstein frame (III.1.2) and the rescaling (III.1.4). The new parameters N and M are related to n and m through $N = (1-n)/\varphi_0$ and $M = (2-m)/\varphi_0$. They work with a new inflation type Coleman–Weinberg potential [38]

$$V(\sigma) = V_0 - \frac{\lambda}{4}\sigma^4, \qquad (III.2.8)$$

and with a chaotic type potential

$$V(\sigma) = \frac{1}{n}\lambda_n\sigma^n. \qquad (III.2.9)$$

and apply cosmological restrictions to impose limits on the parameters of the inflationary potential. The exponential potential multiplied by the coupling constant acts as an effective constantly decreasing coupling "constant", thus suppressing the amplitude of density fluctuations, softening the restrictions for standard values of the parameters. Reducing M corresponds to reducing the exponential potential importance thus making more and more severe the CMB constraint on the perturbations amplitude, till the standard regime is recovered when $M = 0$ ($m = 2$). Both with a new inflation potential and a chaotic inflation one the $N = 0$ ($n = 1$), arising from a fundamental theory [18], seems to give better results than the $N = M/2$ ($m = 2n$) case which is the case of almost all the generalized Einstein theories considered so far. With $N = 0$ they find a broad allowed region in the space of the new inflation potential parameters while a narrow one –containing anyhow the desirable value of the self–coupling constant near unity– for a chaotic inflationary potential. The $N = M/2$ case seems to be of little advantage compared to the standard inflationary models.

Wang [45] uses the conditions for successful inflation to put constraints on the exponents m and n of the couplings of the JBD field to the inflaton sector. The resulting allowed region in m–n is quite large. Casas, García–Bellido and Quiros [54] recently considered improved nucleosynthesis limits on the parameters β_I and β_V of the DGG model (see also [42]). We will report their procedures and results in chapter VI.

III.3) Generalized extended Inflation from higher–dimensional theories

The main problem encountered by Holman *et al.* in their higher–dimensional (Kaluza–Klein) model [41] is that it cannot be made inflate enough. The generic situation is that the scale factor $b(t)$ of the internal dimensions goes to its minimum without allowing enough time for sufficient inflation. It could perhaps be possible to put together all the conditions to make the model work (construct a potential stable at large b, that has a minimum at a nonzero value of b to avoid the internal dimensions from shrinking to zero, and flat enough to allow sufficient inflation) but the need for an adjustment of parameters makes it an unnatural construction.

Superstring theories present several different candidates for the inflaton field and are particularly suitable for some kind of extended inflation since the scalar fields of the gravitational multiplet (the dilaton and the moduli) are coupled to the curvature scalar of the four-dimensional metric in the same way as the JBD field and are also coupled to the non-gravitational sector.

Exploring the claimed connection between string theory and extended inflation, Campbell, Linde and Olive [53] conclude that the existence of a dilaton, by itself, does not provide a natural basis for a realization of extended inflation. This is the result of the dynamics of the dilaton alone in the presence of a source of vacuum energy, it does not represent the exhaustive exploration of the possibility of realization of an inflationary regime, which would require involving all the scalar fields present in every possible string ground-state construction. They start from the string effective action [55]

$$S_{string} = \int d^4x\sqrt{g}e^{-\sqrt{2}\kappa\phi}\Big[\frac{R}{2\kappa^2} - \partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu y\partial^\mu y - m^2y^2 - V_0\Big] \qquad (III.3.1)$$

where ϕ is the dilaton, y represents any other massive scalar and V_0 is the potential at string tree level (genus 0). Applying a conformal transformation $g_{\mu\nu} \rightarrow e^{-\sqrt{2}\kappa\phi}g_{\mu\nu}$, one obtains:

$$S_{string} = \int d^4x\sqrt{g}\Big[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu y\partial^\mu y - e^{-\sqrt{2}\kappa\phi}(m^2y^2 + V_0)\Big]. \qquad (III.3.2)$$

By comparison with the extended inflation action in the Einstein frame

$$S = \int d^4x\sqrt{g}\Big[\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}e^{-\kappa\varphi/\varphi_0}\partial_\mu y\partial^\mu y - e^{-2\kappa\varphi/\varphi_0}(m^2y^2 + V_0)\Big], \quad (III.3.3)$$

with $\varphi_0 \equiv \sqrt{\omega + 3/2}$ (which is the M = 2N case of the more general ($III.2.6$), exhibiting explicitly here the inflaton mass term), it can be seen that the full string theory effective action is not of the JBD type: there is no value of the parameter ω for which the matter (field y) kinetic and potential terms take the same form in both actions. The two theories agree only under some extreme assumptions. A JBD form with $\omega = 1/2$ is obtained considering a truncated action, retaining only the graviton and the dilaton kinetic terms and the potential $V_0^{(0)}$ (genus 0). There seems to be no way to recover a JBD theory for any value of ω. No inflation solutions are found, anyway. Including, and taking as dominant, a higher genus potential (ignoring corrections to kinetic terms), Campbell *et al.* [53] are able to find a power–law expansion with exponent < 1, which is not inflation. Their conclusion is that if extended inflation is to occur, it will not be due solely to dilaton evolution.

A better connection between effective string theories and scalar–tensor theories is obtained if what we want to get is generalized extended inflation. Indeed, the generalized action (III.2.2) is of the same form of the effective string action (III.3.2) if $N = 0$ and $M = \sqrt{2}$. So string theory seems to suggest "non–standard" couplings of the dilaton to kinetic and potential matter terms.

Casas, García–Bellido and Quirós [58] analyze the cosmological solutions in the radiation and matter dominated regimes from the gravitational sector of four–dimensional heterotic strings. Their starting point is the action

$$S = \frac{1}{\alpha'} \int d^4x \sqrt{-g} e^{-2\phi} (R + 4g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \alpha' \mathcal{L}_M) \qquad (III.3.4)$$

obtained by a truncation procedure developed by Witten [57], from the higher–dimensional action computed at tree–level in string loop perturbation theory and keeping only linear terms in the string tension α' and in the curvature. Here, ϕ is the dilaton and the matter Lagrangian $\mathcal{L}_M$ contains at least one scalar field whose effective potential precise form depends on the details of the compactification procedure. After conformal transformations and field redefinitions they obtain general scalar–tensor theories of gravity, with a value of the ω parameter which depends on the particular conformal transformation, corresponding to different supersymmetric and non–supersymmetric string scenarios. The result is that all of them are in conflict with bounds on the time variation of gauge couplings and/or the post Newtonian bounds of general relativity. Bounds from the electromagnetic coupling, when they hold, yield in fact much stronger constraints. They have assumed a flat direction involving the dilaton field and a constant vacuum expectation value for the moduli: a more complicated scalar–tensor theory of the same type, that could maybe lead to different conclusions, would result keeping the moduli fields as dynamical degrees of freedom and taking into account non–trivial potentials that can appear from non–perturbative effects.

García–Bellido and Quirós [56] study the possibility of extended inflation in string scenarios with spontaneously broken supersymmetry, involving another field of the gravitational sector: a modulus χ. They consider one direction in the (χ, ϕ) configuration space fixed to its vacuum expectation value and one runaway direction (if both fields were fixed to their vacuum expectation value there would be no extended inflation) and they suggest that the non–constant value of the moduli along the runaway direction will help to overcome the problems found by Campbell *et al.*. They are able, through conformal redefinitions, to put the effective string action under a generalized extended inflation form. They impose all the conditions for successful extended inflation and find that they can be satisfied in a region of the parameters space, wherefrom the constant moduli region is excluded, meeting the results of ref. [53].

Conclusions

In this chapter we have reviewed different extensions of the original extended inflation model. We were particularly interested in the possibility of introducing direct, stronger couplings of the JBD field with an invisible matter sector, which means any matter that is not "common" matter involved in observational tests of the equivalence principle. Today, this model can be constrained by, and is compatible with, observational bounds on the variability of the gravitational coupling. In the next chapter, we will amplify and further develop this possibility. If the invisible sector is taken to be the inflaton field, we have the advantage of another available parameter which gives more freedom for satisfying simultaneously present and primordial requirements. Bounds on generalized inflationary models will be presented in the last chapter.

We have also reported some attempts to relate extended inflation models with a fundamental theory. String theories are a good candidate for supply generalized scalar–tensor gravity theories: upon reduction to four dimensions we find a scalar field coupled to the curvature scalar of the four–dimensional metric and directly coupled to non–gravitational sectors. Nonetheless, inflation from strings is not very promising: a general result is that there is not enough inflation, nor standard, nor extended. Although some positive results are found when introducing two scalar fields from the gravitational sector. On the other hand, one may ask if a JBD theory is obtainable from higher–dimensional theories. In spite of the characteristics of the dilaton field in four dimensions, its couplings with the non–gravitational sector do not correspond to a Brans–Dicke field. Allowing for generalized couplings, we can avoid this problem, yet the predictions of the model are found to be in contradiction with present bounds on the time variation of gauge couplings and of the gravitational coupling.

Chapter IV

Generalized Dilaton Couplings to Dark Matter

Introduction

Inspired by these "generalized extended inflation" models, we work with the most general couplings between the JBD field, or dilaton field, and the conjectured dark matter component, i.e. we extend the DGG model allowing for arbitrary m and n in the dark matter kinetic and potential terms [59]. We work in two conformally related frames: the Jordan (physical) frame and the Einstein one. We obtain the field equations and their attractor solution, considering the dark matter component as an invisible perfect fluid which gives the dominant source to the universe expansion. We discuss how to implement this model in a Lagrangian formalism and argue that a natural choice of coupling constant would actually exclude the DGG model. Finally, the potential for the scalar field acting as a dominant perfect fluid, is presented. This could correspond to the inflationary epoch or to the present one and, in the context of a decaying cosmological constant, the same field that dominated during inflation may dominate again in the present epoch if the energy density associated with it redshifts slower than the energy density of matter and radiation.

IV.1) Model in the Jordan–Brans–Dicke frame

We start in the Jordan frame from the general action of eq.(III.2.4) adding to it the contribution of visible matter fields through a Lagrangian $\mathcal{L}_V$

$$S = \int d^4x\sqrt{-g}\Big[-\Phi R + \frac{\omega}{\Phi}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + (2\Phi)^n\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - (2\Phi)^m V(\sigma) + \mathcal{L}_V\Big] \quad (IV.1.1)$$

with n and m arbitrary parameters. Applying

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\mu\nu}} = 2\frac{\partial\mathcal{L}}{\partial g^{\mu\nu}} - g^{\mu\nu}\mathcal{L}$$

to the matter Lagrangian $\mathcal{L}_m = \mathcal{L} + \Phi R$, we obtain the energy-momentum tensor, containing contributions from the fields Φ, σ and from visible matter,

$$T_{\mu\nu} = \frac{2\omega}{\Phi}\partial_\mu\Phi\partial_\nu\Phi + (2\Phi)^n\partial_\mu\sigma\partial_\nu\sigma - \frac{\omega}{\Phi}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - \frac{1}{2}(2\Phi)^n g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma +$$

$$+(2\Phi)^m g_{\mu\nu}V(\sigma) + T_{V\mu\nu}. \qquad (IV.1.2)$$

From the complete Lagrangian $\mathcal{L}$ we get the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{T_{V\mu\nu}}{2\Phi} + \frac{\omega}{\Phi^2}(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi) + \frac{1}{\Phi}(\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\Box\Phi)$$

$$+\frac{1}{2\Phi}[(2\Phi)^n(\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\alpha\sigma\partial^\alpha\sigma) + (2\Phi)^m g_{\mu\nu}V(\sigma)] \qquad (IV.1.3)$$

and the field equations

$$\Box\Phi = \frac{1}{2(2\omega+3)}T_V + (n-1)\frac{(2\Phi)^n}{2(2\omega+3)}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma - (m-2)\frac{(2\Phi)^m}{2\omega+3}V(\sigma), \qquad (IV.1.4)$$

$$(2\Phi)^n\Box\sigma = -2n(2\Phi)^{n-1}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\sigma - (2\Phi)^m\frac{\partial V(\sigma)}{\partial\sigma}, \qquad (IV.1.5)$$

with $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$. With a (spatially flat) FRW line element with scale factor $a(t)$, the field equations are

$$\frac{d^2\Phi}{dt^2} + 3H\frac{d\Phi}{dt} - a^{-2}\Delta^{(3)}\Phi = \frac{\rho_V - 3p_V}{2(2\omega+3)} + \frac{n-1}{2(2\omega+3)}(2\Phi)^n\Big[(\frac{d\sigma}{dt})^2 - a^{-2}(\vec{\nabla}\sigma)^2\Big]$$

$$-\frac{m-2}{2\omega+3}(2\Phi)^m V(\sigma), \qquad (IV.1.6)$$

$$(2\Phi)^n\Big(\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} - a^{-2}\Delta^{(3)}\sigma\Big) = 2n(2\Phi)^{n-1}\Big(\frac{d\Phi}{dt}\frac{d\sigma}{dt} - a^{-2}\vec{\nabla}\Phi.\vec{\nabla}\sigma\Big)$$

$$+(2\Phi)^m\frac{\partial V(\sigma)}{\sigma}, \qquad (IV.1.7)$$

where $\Delta^{(3)} = \partial_i\partial_j$. Defining now the new JBD field variable $\varphi = \varphi_0\ln(2\Phi)$, in order to work with a conventional kinetic energy term, and considering spatially homogeneous fields $\varphi = \varphi(t)$, $\sigma = \sigma(t)$, the field equations reduce to

$$\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} + \frac{1}{\varphi_0}\left(\frac{d\varphi}{dt}\right)^2 = \frac{e^{-\varphi/\varphi_0}}{2\varphi_0}(\varrho_V - 3p_V) + \frac{(n-1)e^{(n-1)\varphi/\varphi_0}}{2\varphi_0}\left(\frac{d\sigma}{dt}\right)^2 - \frac{(m-2)e^{(m-1)\varphi/\varphi_0}}{\varphi_0}V(\sigma), \qquad (IV.1.8)$$

$$e^{n\varphi/\varphi_0}\left(\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + \frac{n}{\varphi_0}\frac{d\varphi}{dt}\frac{d\sigma}{dt}\right) = -e^{m\varphi/\varphi_0}\frac{\partial V(\sigma)}{\partial\sigma}, \qquad (IV.1.9)$$

where we assumed that the visible matter is described by a perfect fluid with energy density ϱ_V and isotropic pressure p_V. From Eq.(IV.1.3) the Friedmann equation follows

$$3H^2 = e^{-\varphi/\varphi_0}\varrho_V + \frac{1}{2}\left(1 - \frac{3}{2\varphi_0^2}\right)\left(\frac{d\varphi}{dt}\right)^2 - \frac{3H}{\varphi_0}\frac{d\varphi}{dt} + \frac{1}{2}e^{(n-1)\varphi/\varphi_0}\left(\frac{d\sigma}{dt}\right)^2 + e^{(m-1)\varphi/\varphi_0}V(\sigma). \qquad (IV.1.10)$$

Since in the Jordan frame the visible matter has no couplings with the JBD field, it satisfies the standard conservation law $\nabla_\lambda T_V^{\lambda\mu} = 0$; in our case only the $\mu = 0$ component matters,

$$\frac{d\varrho_V}{dt} = -3H(\varrho_V + p_V). \qquad (IV.1.11)$$

Considering the σ field as an invisible fluid, from its energy-momentum tensor,

$$T_{I\mu\nu} = e^{n\varphi/\varphi_0}(\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\rho\sigma\partial^\rho\sigma) + g_{\mu\nu}e^{m\varphi/\varphi_0}V(\sigma), \qquad (IV.1.12)$$

the energy density and isotropic pressure can be obtained

$$\varrho_I \equiv T_{I\mu\nu}u^\mu u^\nu = \frac{1}{2}e^{n\varphi/\varphi_0}\left(\frac{d\sigma}{dt}\right)^2 + e^{m\varphi/\varphi_0}V(\sigma),$$
$$p_I \equiv -\frac{1}{3}h^{\mu\nu}T_{I\mu\nu} = \frac{1}{2}e^{n\varphi/\varphi_0}\left(\frac{d\sigma}{dt}\right)^2 - e^{m\varphi/\varphi_0}V(\sigma). \qquad (IV.1.13)$$

We are implicitly assuming that both the visible and dark matter components are at rest in the FRW metric. We can then rewrite the σ field equation as a conservation law

$$\frac{d\varrho_I}{dt} = -3H(\varrho_I + p_I) + \frac{1}{2\varphi_0}\frac{d\varphi}{dt}[(m-n)\varrho_I - (m+n)p_I]. \qquad (IV.1.14)$$

We can also rewrite our Eq.(IV.1.8) and Eq.(IV.1.10) in terms of the invisible fluid

$$\frac{d^2\varphi}{dt^2}+\frac{1}{\varphi_0}\left(\frac{d\varphi}{dt}\right)^2+3H\frac{d\varphi}{dt}=\frac{e^{-\varphi/\varphi_0}}{2\varphi_0}[\varrho_V-3p_V+(n-m+1)\varrho_I+(n+m-3)p_I]. \quad (IV.1.15)$$

$$3H^2=\frac{1}{2}\Big(1-\frac{3}{2\varphi_0^2}\Big)\Big(\frac{d\varphi}{dt}\Big)^2-\frac{3H}{\varphi_0}\frac{d\varphi}{dt}+e^{-\varphi/\varphi_0}(\varrho_V+\varrho_I). \quad (IV.1.16)$$

Equation (IV.1.16) (Friedmann equation), Eq.(IV.1.15) (JBD field equation), Eq.(IV.1.11) (energy conservation for the visible matter) and Eq.(IV.1.14) (energy conservation for the dark matter component) completely describe our physical system. We will therefore *define* the generalized couplings of the JBD field to an invisible fluid with energy density ϱ_I and pressure p_I through these equations.

Let us now consider the universe dominated by the invisible fluid, assumed to obey the equation of state $p_I=(\gamma_I-1)\varrho_I$, and neglect ϱ_V and p_V. In terms of the variables $y\equiv d\varphi/dt$ and H, the system of Eq.(IV.1.16), Eq.(IV.1.15) and Eq.(IV.1.14) reduces to

$$\frac{dy}{dt}=3\mu H^2+3\Big(\frac{\mu}{\varphi_0}-1\Big)Hy-\Big[\frac{1}{\varphi_0}+\frac{\mu}{2}\Big(1-\frac{3}{2\varphi_0^2}\Big)\Big]y^2, \quad (IV.1.17)$$

$$\frac{dH}{dt}=-\frac{3}{2}\Big(\gamma_I+\frac{\mu}{\varphi_0}\Big)H^2+\frac{1}{2\varphi_0}\Big(4-3\gamma_I-3\frac{\mu}{\varphi_0}\Big)Hy$$
$$+\frac{1}{4}\Big(1-\frac{3}{2\varphi_0^2}\Big)\Big(\gamma_I-2+\frac{\mu}{\varphi_0}\Big)y^2, \quad (IV.1.18)$$

where $\mu\equiv[2(2-m)-\gamma_I(3-m-n)]/2\varphi_0$. The dark matter energy density is given by

$$\varrho_I=e^{\varphi/\varphi_0}\Big[3H^2+\frac{3}{\varphi_0}Hy-\frac{1}{2}\Big(1-\frac{3}{2\varphi_0^2}\Big)y^2\Big]. \quad (IV.1.19)$$

From now on, we work with this system of equations ((IV.1.17), (IV.1.18) and (IV.1.19) since we consider that it correctly describes our Universe during the two epochs we are interested in: inflation and the present epoch with a dark matter dominating component. We will solve it and compare its predictions with observational bounds.

IV.2) Model in the Einstein frame

The system of equations (IV.1.17) and (IV.1.18) can also be obtained, following DGG, by working in the Einstein frame and then transforming back to the Jordan one. The action in the Einstein frame can be obtained from that in the Jordan one by performing the conformal transformation (III.1.2) $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \exp(\varphi/\varphi_0) g_{\mu\nu}$ Of course, the visible matter Lagrangian is also affected by the transformation. The complete action is then that of Eq.(III.2.6) with the addition of the visible matter contribution through the Lagrangian $\tilde{\mathcal{L}}_V$

$$S = \int d^4x \sqrt{-g} \Big[-\frac{\tilde{R}}{2} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}e^{-N\varphi}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - e^{-M\varphi}V(\sigma) + \tilde{\mathcal{L}}_V \Big] \quad (IV.2.1)$$

(with $N = (1-n)/\varphi_0$, $M = (2-m)/\varphi_0$ and $\varphi_0 = \sqrt{\omega + 3/2}$). As in the Jordan frame we define a matter Lagrangian from which the energy–momentum tensor follows

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi + e^{-N\varphi}\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi - \frac{1}{2}e^{-N\varphi}g_{\mu\nu}\partial_\rho\sigma\partial^\rho\sigma$$
$$+ g_{\mu\nu}e^{-M\varphi}V(\sigma) + T_{V\mu\nu} \quad (IV.2.2)$$

Here and in the following we drop the tildes unless we compare quantities in the two frames.

The field equations read

$$\Box\varphi = -Ne^{-N\varphi}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + Me^{-M\varphi}V(\sigma) + \frac{1}{2\varphi_0}T_V, \quad (IV.2.3)$$

$$e^{-N\varphi}\Box\sigma = Ne^{-N\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\sigma - e^{-M\varphi}\frac{\partial V(\sigma)}{\partial\sigma}, \quad (IV.2.4)$$

while the Einstein equations take the standard form with the r.h.s. provided by the energy–momentum tensor of Eq.(IV.2.2). With spatially homogeneous fields and a FRW line element these equations reduce to

$$\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} = Me^{-M\varphi}V(\sigma) - N\frac{e^{-N\varphi}}{2}\Big(\frac{d\sigma}{dt}\Big)^2 + \frac{1}{2\varphi_0}(\varrho_V - 3p_V), \quad (IV.2.5)$$

$$e^{-N\varphi}\Big(\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} - N\frac{d\varphi}{dt}\frac{d\sigma}{dt}\Big) = -e^{-M\varphi}\frac{\partial V(\sigma)}{\partial\sigma}, \quad (IV.2.6)$$

$$3H^2 = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + \frac{1}{2}e^{-N\varphi}\left(\frac{d\sigma}{dt}\right)^2 + e^{-M\varphi}V(\sigma) + \varrho_V. \qquad (IV.2.7)$$

Under the conformal transformation, the proper time, the scale factor, the energy density and the pressure transform as $d\tilde{t} = \exp(\varphi/2\varphi_0)dt$; $\tilde{a} = \exp(\varphi/2\varphi_0)a$; $\tilde{\varrho} = \exp(-2\varphi/\varphi_0)\varrho$; $\tilde{p} = \exp(-2\varphi/\varphi_0)p$. We can therefore transform Eq.(IV.1.11) to the Einstein frame and obtain

$$\frac{d\varrho_V}{dt} = -3H(\varrho_V + p_V) - \frac{1}{2\varphi_0}\frac{d\varphi}{dt}(\varrho_V - 3p_V). \qquad (IV.2.8)$$

Taking the σ field as an invisible fluid, from its energy–momentum tensor, we obtain its energy density and isotropic pressure

$$\begin{aligned} \varrho_I &= \frac{1}{2}e^{-N\varphi}\left(\frac{d\sigma}{dt}\right)^2 + e^{-M\varphi}V(\sigma), \\ p_I &= \frac{1}{2}e^{-N\varphi}\left(\frac{d\sigma}{dt}\right)^2 - e^{-M\varphi}V(\sigma). \end{aligned} \qquad (IV.2.9)$$

The σ field equation, written as a conservation law, reads

$$\frac{d\varrho_I}{dt} = -3H(\varrho_I + p_I) - \frac{1}{2}\frac{d\varphi}{dt}[\varrho_I(M - N) - p_I(M + N)]. \qquad (IV.2.10)$$

and in terms of the invisible fluid, equations (IV.2.8) and (IV.2.5) take the form

$$3H^2 = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + \varrho_V + \varrho_I, \qquad (IV.2.11)$$

$$\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt} = \frac{1}{2\varphi_0}(\varrho_V - 3p_V) + \frac{1}{2}[\varrho_I(M - N) - p_I(M + N)]. \qquad (IV.2.12)$$

In a universe dominated by the invisible fluid, with equation of state $p_I = (\gamma_I - 1)\varrho_I$, the system of Eq.(IV.2.11), Eq.(IV.2.12) and Eq.(IV.2.10) reduces to

$$\frac{dH}{dt} = \frac{1}{2}\left(\frac{\gamma_I}{2} - 1\right)\left(\frac{d\varphi}{dt}\right)^2 - \frac{3\gamma_I}{2}H^2, \qquad (IV.2.13)$$

$$\frac{d^2\varphi}{dt^2} = -3H\frac{d\varphi}{dt} + \mu\left[3H^2 - \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2\right], \qquad (IV.2.14)$$

with $\mu = M - \gamma_I(M + N)/2$. The dark matter energy density reads $\varrho_I = 3H^2 - (1/2)(d\varphi/dt)^2$. The two equations (IV.2.13) and (IV.2.14), transformed back to the Jordan frame, reduce to Eq.(IV.1.17) and Eq.(IV.1.18), respectively.

IV.3) Dark matter as a dominant invisible fluid

a) Attractor solutions

A study of the phase space of the system of equations (IV.1.17) and (IV.1.18) shows that there are three invariant lines: an attractor solution:

$$
\begin{aligned}
H &= \frac{2-\gamma_I-\mu/\varphi_0}{\mu^2+3\gamma_I(1-\gamma_I/2)-\mu/\varphi_0}t^{-1}, \\
y &= \frac{2\mu}{\mu^2+3\gamma_I(1-\gamma_I/2)-\mu/\varphi_0}t^{-1}.
\end{aligned}
\qquad (IV.3.1)
$$

and two repeller solutions:

$$
\begin{aligned}
H &= -\frac{(2\varphi_0)^{-1}\pm(6)^{-1/2}}{(2\varphi_0)^{-1}\pm 3(6)^{-1/2}}t^{-1}, \\
y &= \frac{1}{(2\varphi_0)^{-1}\pm 3(6)^{-1/2}}t^{-1}.
\end{aligned}
\qquad (IV.3.1a)
$$

This behaviour can be seen in the phase portraits of the dynamical system, in figures (IV.1) and (IV.2), at the end of this chapter. The critical point of the system is located at the origin of the H-y plane. The external arrow indicates the attracting line solution (IV.3.1) and small arrows on the trajectories show the time direction. On the diagram it can be seen that points in an important region of phase space tend asymptotically to the attracting line but the critical point is not a universal attractor for the system. Regions separated from the attracting line by a repulsive line (IV.3.1a) do not tend asymptotically to solution (IV.3.1), meaning that not all solutions of the system have a power-law behaviour. Figure (IV.1) has been drawn for the dark dust ($\gamma_I = 1$) domination case and fig. (IV.2) corresponds to an inflaton field ($\gamma_I = 0$).

Note that Eq.(IV.1.16) and Eq.(IV.1.17) and their solutions Eq.(IV.3.1) reduce to the corresponding ones in DGG (*III*.1.5, *III*.1.6, *III*.1.7 and *III*.1.8) when $m = 2n$, with $\beta_V = 1/2\varphi_0$, $\beta_I = (1-n)/2\varphi_0$ (our φ field corresponds to DGG $(-\sigma)$ field). Moreover, our results can be obtained directly from theirs if the replacement $(4-3\gamma_I)\beta_I \rightarrow \mu$ is performed in their equations.

The time evolution of the JBD field and of the scale factor are easily obtained from (IV.3.1)

$$\varphi = g \ln \frac{t}{t_i} + \varphi_i$$
$$a = a_i (\frac{t}{t_i})^f \qquad (IV.3.2)$$

where t_i, φ_i and a_i refer to the initial time of invisible matter domination and $f = f(n, m, \gamma_I) = Ht$; $g = g(n, m, \gamma_I) = yt$.

The gravitational constant measured in time–delay experiments is $G = [(2\omega + 4)/(2\omega + 3)](1/16\pi\Phi)$, the time–variation of G is therefore related to the variation of Φ by: $\dot{G}/G = -\dot{\Phi}/\Phi$. Assuming that the universe is close to the attractor solution we can write

$$\frac{d \ln G}{d \ln t} = -\frac{1}{\varphi_0} \frac{2\mu}{\mu^2 + 3\gamma_I(1 - \gamma_I/2) - \mu/\varphi_0}. \qquad (IV.3.3)$$

Note that if $p_I = \varrho_I/3$ ($\gamma_I = 4/3$) the expression for the rate of change of the gravitational constant reduces to

$$\frac{d \ln G}{d \ln t} = -\frac{6(2n - m)}{12\varphi_0^2 + (2n - m)(2n - m - 3)}, \qquad (IV.3.4)$$

which vanishes only in the $m = 2n$ case: the gravitational coupling continues to change along the attractor solution even if the universe is dominated by an invisible radiation component, unlike the standard case, where the JBD field couples to other fields only through the trace of their energy–momentum tensor. In this respect we mention that a modification of the JBD theory has been proposed [86] that takes into account the gravitational effect of electromagnetic radiation from a Machian point of view. For this purpose, the source for the scalar field Φ has been taken as $\rho + 3p$ (like the source for the gravitational field in general relativity) instead of the energy–momentum trace $\rho - 3p$. Both theories coincide during dust domination.

Assuming that the Universe today is matter dominated, which implies $p_I = 0$ ($\gamma_I = 1$), we have that the variation of G at the present time is

$$\frac{d \ln G}{d \ln t} = -\frac{4(1 + n - m)}{6\varphi_0^2 + (1 + n - m)(n - m - 1)} \qquad (IV.3.5)$$

and the Hubble parameter

$$H = \frac{4\varphi_0^2 - 2(1+n-m)}{6\varphi_0^2 + (1+n-m)(n-m-1)} t^{-1}. \qquad (IV.3.6)$$

These quantities can be compared to the observational limits and it will be done in chapter VI. On the other hand, it is interesting to note that on the attractor solution, with a particular coupling ($m = n+1$) of the JBD field to dark matter, the presence of the scalar field seems to have no effect on the Universe behaviour: $d \ln G/d \ln t = 0$ and $Ht = 2/3$, as in the standard cosmological model. And if we ask for this particular coupling in the DGG case $m = 2n$ ($n = 1$ and $m = 2$), it comes out, in the Einstein frame, that $N = 0$ and $M = 0$, so that the ω parameter completely disappears from action (IV.2.1) in the dark matter sector. Going from the Jordan frame to the Einstein one, the coupling between the JBD field and dark matter seems to disappear.

b) **Dark matter Lagrangian**

So far we have defined our generalized dilaton–dark matter couplings only through the set of equations that governs their dynamics, eq.(IV.1.14), eq.(IV.1.15) and eq.(IV.1.16). One would like, instead, to write down a Lagrangian containing the explicit couplings between the invisible matter field and the JBD one [59] similarly to what is done for the DGG model through eq.(III.1.1). This is obvious if ψ_I is a neutral scalar field σ whose self–interactions are described by the potential $V(\sigma)$: in such a case, in fact, our recipe would just reduce to the action of eq.(III.2.4), with general parameters m and n. It is much less obvious if the dark matter component is represented by a perfect fluid, the problem being that. in the latter case the Lagrangian, which is just the isotropic pressure (see, e.g., ref.[60]), cannot be explicitly written in terms of the field variables and the metric tensor. However, it has been shown in Ref.[61] that, in the particular case of an irrotational, isentropic perfect fluid with equation of state $p/\varrho = \gamma - 1 =$ constant, one can write

$$\mathcal{L}_{fluid} = p[\psi, g_{\mu\nu}] = \frac{\gamma-1}{\gamma}(g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi)^{\gamma/2(\gamma-1)}. \qquad (IV.3.7)$$

where the real scalar field ψ is the velocity potential, defined by

$$u^{\mu} \equiv (g^{\sigma\tau}\partial_{\sigma}\psi\partial_{\tau}\psi)^{-1/2} g^{\mu\nu}\partial_{\nu}\psi. \qquad (IV.3.8)$$

The main limitation of this approach is that it is not suitable to describe a fluid of dust, since, in such a case, the Lagrangian would identically vanish. (Other approaches are possible, which overcome this difficulty, see, e.g., ref.[62]). Restricting our analysis to the $\gamma_I \neq 1$ case we can use the Lagrangian of Eq.(IV.3.7) for our purposes. Noting that the formal change $1 - \beta_I/\beta_V \to [2m - \gamma_I(m+n)]/(4 - 3\gamma_I)$ maps the DGG model into ours, and using Eq.(III.1.1) and Eq.(IV.3.7), we arrive to the following action functional for the invisible matter component

$$\begin{aligned} S_I &= \int d^4x \sqrt{-g}\; p[\psi_I, g_{\mu\nu}, \Phi] = \\ &= \int d^4x \sqrt{-g}\; \frac{\gamma_I - 1}{\gamma_I}\; (2\Phi)^{\gamma_I(m+n)-2m}\; (g^{\mu\nu}\partial_{\mu}\psi_I\partial_{\nu}\psi_I)^{\gamma_I/2(\gamma_I-1)}, \end{aligned} \qquad (IV.3.9)$$

which reduces to the DGG model for $m = 2n$ and to the standard perfect fluid one in the JBD case, $m = n = 0$.

This clearly shows that, at least in the constant p_I/ϱ_I case, there is a unique combination of the parameters m, n and γ_I, namely $\alpha_I \equiv \gamma_I(m+n) - 2m$ playing the role of dilaton–dark matter coupling constant. It seems rather unnatural, however, that the coupling constant depends upon the fluid equation of state; note that this is also true for the DGG model. The only case in which α_I has the desirable feature of being equation-of-state-independent is when $m = -n$, which would then exclude the DGG case.

c) Dark matter potential

From the Friedmann equation (IV.1.10) and the Jordan–Brans–Dicke field equation (IV.1.8), neglecting the visible matter contribution and substituting the attractor solutions (IV.3.1), we find the time variation of the potential for the "invisible" scalar field

$$V[\sigma(t)] = Zt^{\varsigma} \qquad (IV.3.10)$$

with

$$\zeta = \frac{2[-\varphi_0\mu^2 + (2-m)\mu - 3\gamma_I(1-\gamma_I/2)\varphi_0]}{\varphi_0\mu^2 - \mu + 3\gamma_I(1-\gamma_I/2)\varphi_0}$$

$$Z = (1-\frac{\gamma_I}{2})\frac{3(\gamma_I-2)^2 - 2\mu^2}{\mu^2 - \mu/\varphi_0 + 3\gamma_I(1-\gamma_I/2)}\times$$
$$\epsilon^{(1-m)\varphi_i/\varphi_0} t_i^{2(m-1)\mu[\varphi_0\mu^2-\mu+3\gamma_I(1-\gamma_I/2)\varphi_0]^{-1}}$$

(we remember that μ was defined as $\mu = [2(2-m)+\gamma_I(n+m-3)]/2\varphi_0)$). Finding the time dependence of σ from the σ–field equation (IV.1.9) allows to write the effective potential that makes the scalar field act as a dominant invisible fluid. Assuming a solution of the form

$$\sigma = Bt^{\beta}. \qquad (IV.3.11)$$

it comes out that β and B are:

$$\beta = \frac{(1-n)\mu}{\varphi_0\mu^2 - \mu + 3\gamma_I(1-\gamma_I/2)}$$

and

$$B = \pm\frac{\varphi_0}{\mu}\left\{\frac{(2-\gamma_I)[\mu^2\varphi_0 + (m-2)\mu + 3\gamma_I\varphi_0(1-\frac{\gamma_I}{2})][3(\gamma_I-2)^2 - 2\mu^2]}{[-\mu^2\varphi_0 + (n-1)\mu + 6\varphi_0(1-\gamma_I/2)^2]}\right\}^{\frac{1}{2}}$$
$$\epsilon^{(1-n)\varphi_i/2\varphi_0} t_i^{(n-1)\mu[\varphi_0\mu^2-\mu+3\gamma_I(1-\gamma_I/2)\varphi_0]^{-1}}.$$

Finally, the potential $V(\sigma)$ can be written as a power–law of σ, depending upon the parameters m and n and the equation of state (γ_I)

$$V(\sigma) = \frac{Z}{B^{2[(1-m)(1-n)^{-1}-\beta^{-1}]}}\sigma^{2(n-1)^{-1}[\varphi_0\mu+m-2+3\gamma_I(1-\gamma_I/2)\varphi_0/\mu]} \qquad (IV.3.12)$$

or, substituting μ and φ_0:

$$V(\sigma) \propto \sigma^{\gamma_I(n-1)^{-1}\{n+m-3+3(2-\gamma_I)(2\omega+3)[2(2-m)+\gamma_I(n+m-3)]^{-1}\}}. \qquad (IV.3.13)$$

Assuming that at the present time the invisible fluid dominates, with equation of state $p = 0$ ($\gamma_I = 1$), the σ–potential reduces to

$$V(\sigma) \propto \sigma^{(n-1)^{-1}[n+m-3+3(2\omega+3)(n-m+1)^{-1}]}. \qquad (IV.3.14)$$

It may be of some interest to exhibit explicitly the evolution for the σ-field and its potential in some special cases, in which the exponent takes a simpler form:

* with $m = 2n$, as in DGG original model, $\sigma \propto t^{2(1-n)^2/(n^2+6\varphi_0-1)}$ and $V(\sigma) \propto \sigma^{3[1-\frac{2\omega+3}{(1-n)^2}]}$

* for $m = n = 0$, i.e. the standard JBD model with dark, dominating matter. we have $\sigma \propto t^{2/(6\varphi_0-1)}$ and $V(\sigma) \propto \sigma^{-6(\omega+1)}$.

These potentials decrease as σ increases, for $\omega > 0$, and if they are non–zero at the end of inflation and apply up to the present epoch, they give rise to models with a decaying cosmological costant [63]. In such models, the scalar field that drove inflation has a potential with a power–law tail at large σ: $V \propto \sigma^{-\alpha}$, acting like a cosmological constant that decreases toward the "natural value" $\Lambda = 0$ less rapidly than the energy densities of matter and radiation. This could happen if the inflaton energy density had been converted only in part to entropy at the end of inflation, leaving a part decreasing much more slowly. If there is any non-zero cosmological constant at present, the model allows a relation between Λ and the energy density that drove inflation.

In order to avoid affecting the usual nucleosynthesis theory, the assumption $\rho_\sigma \ll \rho_{\text{ordinary matter}}$ at that epoch, has to be done, and, in this sense, a cosmological constant Λ or a rolling, homogeneous scalar field very weakly coupled to ordinary matter are good candidates as they resist gravitational collapse up to large scales and dominate the energy density only at low redshifts. This assumption is indeed present in all the models with a time–varying G.

The constraint placed by experiments –the Eötvös–Dicke experiment that probes the independence of the acceleration towards the sun upon the material– is that the scalar field can only be exceedingly weakly coupled to ordinary matter.

e) Inflationary epoch

Turning to the inflationary epoch, the potential (IV.3.13) can be considered of the chaotic type, in the sense that it is a rolling potential, with no local minima. The most general power–law inflation is obtained by requiring:

$$f(n,m,\gamma_I) \equiv \frac{2-\gamma_I-\mu/\varphi_0}{\mu^2+3\gamma_I(1-\gamma_I/2)-\mu\varphi_0} > 1$$

in $a \propto t^{f(n,m,\gamma_I)}$. On the other hand, imposing the standard requirement for first-order inflation $\gamma_I = 0$, we obtain a constant effective potential since its exponent (in IV.3.13) becomes zero

$$V(\sigma) = \frac{2(2-m)(6\varphi_0^2-1)}{1-m} e^{(1-m)\varphi/\varphi_0} t_i^{-2\varphi_0}.$$

The n parameter, i.e. the one that is present in the kinetic term, cancels out in the solution –as we should expect, since the kinetic term is usually taken to be negligible during inflation– and we recover the generalized extended solution considered so far in the literature [40], [45]: a power–law expansion with

$$f(\omega,n,m) = \frac{2\omega+m+1}{(2-m)(1-m)}.$$

Conclusions

We have considered a scalar–tensor theory of gravity, in which, in analogy with the so–called generalized extended inflation model, two parameters determine the couplings of the dilaton to the invisible matter sector. Assuming that the invisible sector is a dominant perfect fluid, we have found the attractor solution for the system, along which we have derived the evolution of the gravitational and the Hubble parameters. We have also been able to write the potential for the dominating invisible fluid and the dark matter action functional containing the explicit coupling to the dilaton field, from where a special coupling seems to emerge.

A new aspect of this model is that the JBD field, and therefore the gravitational constant, varies with time even if the universe is dominated by a dark radiative component, i.e. by a fluid whose energy–momentum tensor has vanishing trace. It is not so in the original JBD model and in almost all subsequent generalizations.

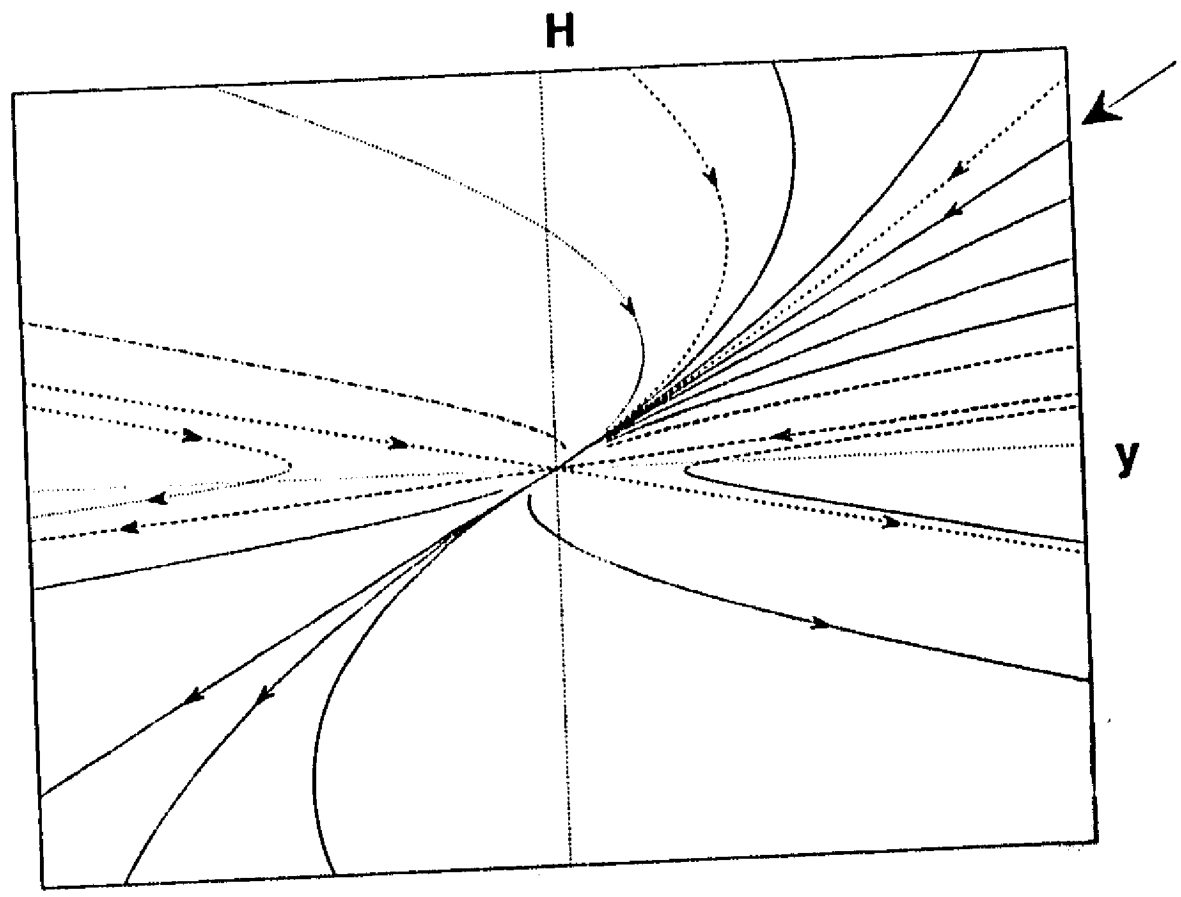

Figure IV.1

The H and y trajectories in phase space for the parameters: $\omega = 10$, $\gamma_I = 1$, $m = 1$, $n = 2$. The arrow indicates the attractor solution (IV.3.1).

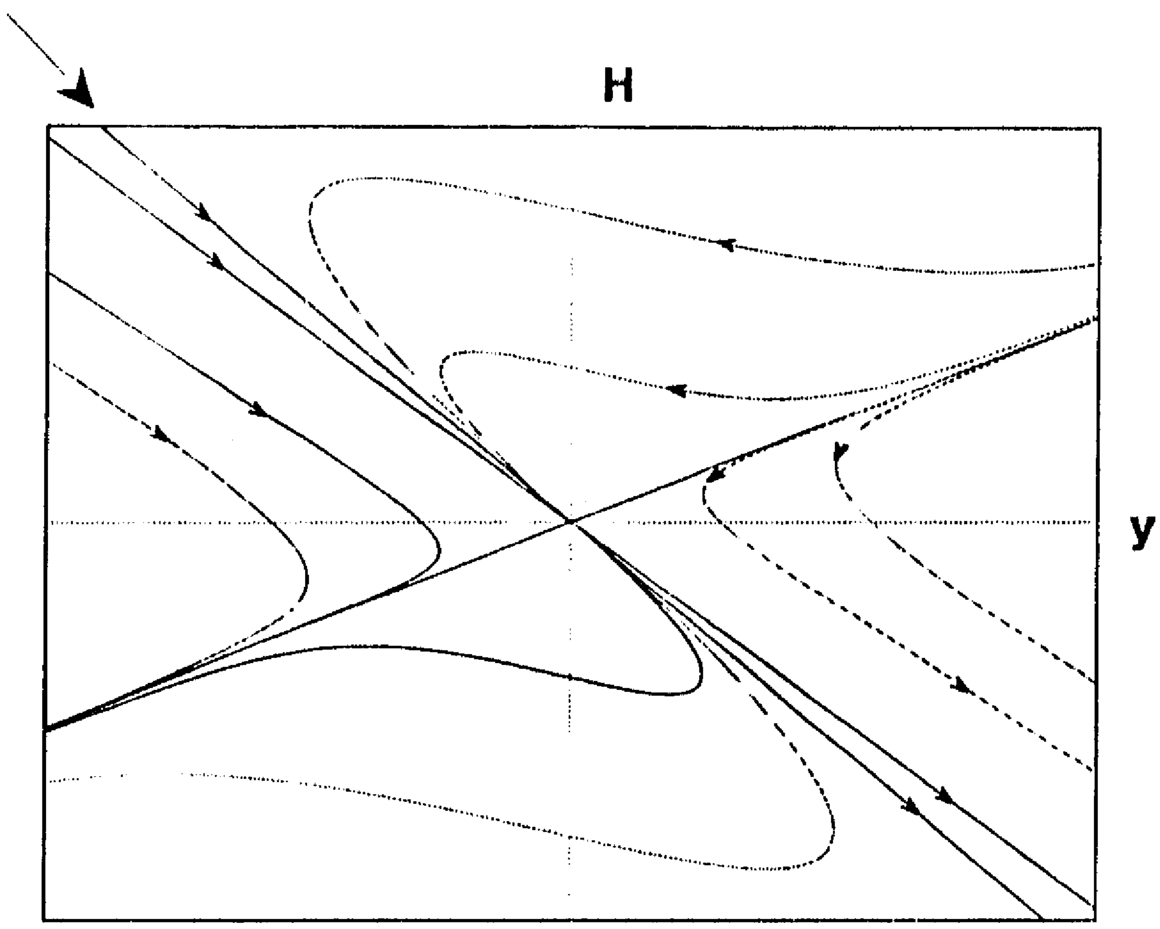

Figure IV.2

The H and y trajectories in phase space for the parameters: ω=10, γ_1=0, m=12, n=0. The external arrow indicates the attractor solution (IV.3.1).

Chapter V

Density Fluctuations in Extended Inflationary Models

Introduction

Dealing with inflationary models imbedded in scalar–tensor theories of gravity, we have the following ingredients for seeding the Universe structure: the quantum fluctuations of two scalar fields -the inflaton and the JBD field- and three types of density perturbations: adiabatic perturbations, produced by the field with dominating energy, isothermal perturbations, associated with the fluctuations of the subdominant field, and inhomogeneities resulting from the bubbly structure of the Universe emerging from the phase transition. Then, different possibilities arise for the subsequent evolution of the Universe, depending on which field has the dominant fluctuations and which field is mainly responsible for the reheating of the Universe.

As it is already known [18], power–law inflation leads to a fluctuation spectrum which slightly deviates from scale invariance, introducing more power on large scales. This is an interesting result since, according to some observers (see e.g. the A.P.M. survey by Maddox *et al.* [64]), a perturbation spectrum with more power on large scales than the scale–invariant one seems to be required to be consistent with large scale structure. Nevertheless, it has to be noticed that the COBE DMR results [65] do not seem to suggest such an excess power. On the other hand, the presence of two fields in extended inflation models, introduces another possibility for designing the fluctuation spectrum: a range of scales of cosmic stuctures can be associated to the fluctuations of one field and another part of the spectrum may correspond to the other's field fluctuations. This could happen in a double episode inflation, or in a scenario where a weakly interacting, initially subdominant field, lately dominates because its energy density redshifts less than matter and radiation.

The problem of fluctuations may, in principle, be addressed in either of the two conformal frames –Jordan's or Einstein's– but the most direct approach is in the Einstein frame. In this frame the JBD field is minimally coupled, has a standard kinetic term and, under some conditions, plays the role of a slow-rolling inflaton, for which there already exists a standard procedure for computing density fluctuations. The inflationary potential for the JBD field is supplied by the potential of the original inflaton field multiplied by an exponential term in the JBD field resulting from the conformal transformation. The results will be easily transformed back to the original frame at late times, when both frames practically coincide. In the Jordan frame, we have the presence of two fields but, as long as the inflaton is trapped in the local minimum of its effective potential, the JBD field will automatically have the dominant fluctuations.

In fact, the generalized models do not introduce any new ingredient in this topic since structure does not arise from fluctuations of the inflaton, which is the sector that carries the generalized couplings. Some care must be taken, anyway, in order to preserve the inflationary conditions: some constraints must be imposed on the parameters of these models [45].

This chapter begins with a review of the basic ideas about density perturbations from inflation. In section (V.2), I report and compare the results obtained for extended inflationary models of perturbations arising from scalar field quantum fluctuations and, briefly, from the bubble distribution. Finally, I further explore the generalized models and the scenario with a mass for the JBD field, during and after inflation.

V.1) Review of quantum fluctuations and density perturbations in theories of inflation

The fact that inflation exponentially stretches spatial dimensions, suggests that density fluctuations that are at the origin of large scale structure, may come from microscopic zero-point fluctuations of the quantized fields. The main idea of the theory of generation of density perturbations in inflationary cosmology would be that the stretched, long-wave fluctuations of the scalar field transform at a later stage to perturbations of the density of particles that were created during the decay of the inflaton field. In a rather direct way, the development of density perturbations can be viewed as:

$$\frac{\delta\rho}{\rho} \sim \frac{\delta V(\phi)}{V(\phi)} \sim \frac{(\partial V/\partial\phi)\delta\phi}{V(\phi)}, \tag{V.1.1}$$

that means that, a fluctuation in the inflaton field results in a fluctuation in the Universe energy density since the inflaton potential is the dominating energy during the inflationary stage.

a) Scalar field quantum fluctuations

When the Universe is expanding faster than the horizon growth, as it does in an inflationary regime, the wavelength of a fluctuation becomes greater than a causally connected region and causal microphysics do not operate anymore, so the fluctuation amplitude freezes at some nonzero value $\delta\phi(x)$ i.e. it remains almost unchanged for a long time, until it reenters the horizon. Such a frozen fluctuation is equivalent to the appearence of a classical field $\delta\phi(x)$ whose average over macroscopic intervals of space and time, do not vanish. The equation of motion for a scalar field in the background of a de Sitter metric is

$$\ddot{\phi} + 3H\dot{\phi} - e^{-2Ht}\partial_i^2\phi = -\frac{\partial V}{\partial\phi}. \tag{V.1.2}$$

If we want to consider the possibility that the scalar field undergoes small inhomogeneous quantum fluctuations, we may write

$$\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t). \tag{V.1.3}$$

where $\phi_0(t)$ is the classical homogeneous field which obeys to the equation

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 = -\frac{\partial V}{\partial\phi_0}, \tag{V.1.4}$$

and the quantum fluctuations $\delta\phi(\mathbf{x},t)$ imposed on the classical solution $\phi_0(t)$ satisfy the equation

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - e^{-2Ht}\partial_i^2\delta\phi = -\frac{\partial^2 V(\phi_0)}{\partial\phi^2}\delta\phi, \qquad (V.1.5)$$

that is obtained substituting (V.1.3) into (V.1.2), using (V.1.4), and retaining terms linear in $\delta\phi$. The amplitude of the field fluctuations can be estimated by quantizing the scalar field in de Sitter space ([66], or see e.g. 5]) decomposing the field fluctuations into their Fourier components

$$\delta\phi(\mathbf{x},t) = (2\pi)^{-3/2}\int d^3p\left[a_p^+\phi_p(t)e^{i\mathbf{p}.\mathbf{x}} + a_p^-\phi_p^*(t)e^{-i\mathbf{p}.\mathbf{x}}\right], \qquad (V.1.6)$$

where $\mathbf{p}$ is the, time-independent, conformal momentum and a_p^+, a_p^- are creation and annihilation operators. For a (nearly) massless field ($m \ll H$), with a flat potential, the equation for $\phi_p(t)$ reads:

$$\ddot{\phi}_p(t) + 3H\dot{\phi}_p(t) + \mathbf{p}^2 e^{-2Ht}\phi_p(t) = 0. \qquad (V.1.7)$$

Solving for $\phi_p(t)$:

$$\phi_p(t) = \frac{iH}{\sqrt{2}p^{3/2}}\left(1 + \frac{p}{iH}e^{-Ht}\right)\exp\left\{\frac{ip}{H}e^{-Ht}\right\}, \qquad (V.1.8)$$

we can estimate the field fluctuations

$$\langle(\delta\phi)^2\rangle = \frac{1}{(2\pi)^3}\int|\phi_p|^2 d^3p = \frac{1}{(2\pi)^3}\int\left(\frac{e^{-2Ht}}{2p} + \frac{H^2}{2p^3}\right)d^3p, \qquad (V.1.9)$$

whose interpretation becomes clearer in terms of the physical momentum $k = pe^{-Ht}$ (which decreases as the Universe expands)

$$\langle(\delta\phi)^2\rangle = \frac{1}{(2\pi)^3}\int\frac{d^3k}{k}\left(\frac{1}{2} + \frac{H^2}{2k^2}\right). \qquad (V.1.10)$$

The first term is the usual contribution from vacuum fluctuations in Minkowski space ($H = 0$) and can be eliminated by renormalization: the remaining term

$$\langle(\delta\phi)^2\rangle = \frac{1}{(2\pi)^3}\int\frac{d^3k}{k}\left(\frac{H^2}{2k^2}\right) \qquad (V.1.11)$$

is directly related to the inflationary regime. The main contribution to the average deviation of the field ϕ from its homogeneous value φ_0 comes from the long–wave fluctuations, which are, at the same time, the important ones for seeding structure formation. So, restricting our attention to wavenumbers from $k = O(H)$ to a $k_{min} = He^{-Ht}$, which corresponds to a long–wave cut–off due to the fact that inflation starts at a finite time t_i, this deviation grows as

$$\langle(\delta\varphi)^2\rangle = \frac{H^3 t}{4\pi^2}. \qquad (V.1.12)$$

The source of this growth is the contribution of newborn fluctuations, stretched by the expansion to wavelengths bigger than the horizon. Of more interest for the characterization of the perturbations spectrum is the field fluctuation power on each scale k defined as the contribution to $\langle(\delta\phi)^2\rangle$ in a given logarithmic interval in k:

$$\langle(\delta\phi)^2\rangle = \int d^3k \mid \delta\varphi_k \mid^2 = \int \frac{dk}{k}(\Delta\phi)_k^2. \qquad (V.1.13)$$

From a direct comparison of equations (V.1.13) and (V.1.11), we obtain

$$(\Delta\phi)_k^2 = k^3|\delta\phi_k|^2/2\pi^2 = (H/2\pi)^2, \qquad (V.1.14)$$

which is a scale–independent quantity, since $H \approx const.$ during inflation. The fluctuation can also be calculated taking into account the field mass, with the following result (see e.g. [67]):

$$(\Delta\phi)_k = \frac{H}{2\pi}\left(\frac{k}{H}\right)^{m^2/3H^2}, \qquad (V.1.15)$$

and this coincides with the previous expression (eq. (V.1.14)), provided that $m^2 \ll H^2$. So, any effectively massless scalar field will have fluctuations of order $H/2\pi$ imprinted upon it, on all scales, as they cross outside the horizon. By effectively massless we mean a field whose mass is smaller than the amplitude of its fluctuations $H/2\pi$; this will be generally true since inflationary potentials are very flat.

From the solution (V.1.8), it can be seen that a "freezing" of the amplitude of the field $\phi_p(t)$ occurs when the physical momentum k, for any mode, becomes smaller than H.

The leading effect of quantum fluctuations of the scalar field is to alter the time needed for $\phi_0(t)$ to reach the minimum of the potential, so that the transition from de Sitter to Friedmann expansion starts at different times in different regions of space. At large times ($t \gg H^{-1}$), the term with spatial derivatives in equation (V.1.5) becomes negligible and $\delta\phi(\mathbf{x},t)$ starts to satisfy the same equation as $\dot{\phi}_0$ (compare eq. (V.1.5) with the time derivative of eq. (V.1.4)). Therefore, since the solution to the equation is essentially unique,* the ratio between these two quantities approaches a constant. The proportionality constant may depend on $\mathbf{x}$ and has time dimensions, we may then write

$$\delta\phi(\mathbf{x},t) = -\delta t(\mathbf{x})\dot{\phi}_0(t). \qquad (V.1.16).$$

Thus, to linear order in $\delta\phi$, eq. (V.1.3) can be written as

$$\phi(\mathbf{x},t) = \phi_0(t) + \delta\phi(\mathbf{x},t) \approx \phi_0(t - \delta t(\mathbf{x})) \qquad (V.1.17)$$

and $\delta t(\mathbf{x})$ can be interpreted as a time-delay (position dependent) function for the evolution of $\phi_0(t)$. Since the oscillations of the scalar field in its potential minimum result in the production of radiation and particles, this inhomogeneous time delay results in inhomogeneities in the mass density of the Universe.

b) Density perturbations

The density contrast can be expressed in a Fourier expansion:

$$\delta(\mathbf{x},t) = \frac{\delta\rho(\mathbf{x},t)}{\rho} = \int \frac{d^3k}{(2\pi)^3}\delta_k(t)e^{-i\mathbf{k}.\mathbf{x}}, \qquad (V.1.18)$$

and its r.m.s value can be calculated averaging over all space

$$\frac{\delta\rho}{\rho} = < \delta(\mathbf{x},t)\delta(\mathbf{x},t) >^{1/2},$$

* This can be easily proved if one notices that, at large times ($t \gg H^{-1}$), the Wronskian vanishes $W(\dot{\phi}_0, \delta\phi) = W_0 e^{-3Ht} \approx 0$.

yielding

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \int \left(\frac{\delta\rho}{\rho}\right)_k^2 \frac{dk}{k}, \tag{V.1.19}$$

where we have defined the density fluctuation power per logarithmic interval $(\delta\rho/\rho)_k^2 = (2\pi^2)^{-1}k^3 \mid \delta_k \mid^2$.

In order to translate scalar field fluctuations at the inflationary stage into density perturbations that develop later, when the Universe is filled with matter and radiation, we have to relate quantities that correspond to different epochs. To this end, we work at the transition time between the inflationary and radiation dominated regimes, assuming that the transition takes place sharply at time t_0. A variety of methods have been followed to treat the problem, with agreement about the answer. I shall present here a rather simple reasoning that reproduces the correct order of magnitude result and more rigorous treatments can be found in [68].

At late times, after the phase transition from de Sitter to FRW regime, i.e. at $t > t_0$, the Hubble parameter is given by $H = 1/2t$. As we saw in the previous section, there is a fluctuation in t_0, and therefore in t, of order $\delta t(\mathbf{x})$ (eq. (V.1.16)). This time shift can be related to a perturbation in the expansion rate: $\delta H = -(1/2)t^{-2}\delta t = -2H^2\delta t$. Inhomogeneities in the expansion may in turn be related to fluctuations in the energy density through the Friedmann equation: $\delta\rho/\rho = 2\delta H/H$. This results in

$$\frac{\delta\rho(\mathbf{x})}{\rho} = -4H\delta t(\mathbf{x}). \tag{V.1.20}$$

Therefore, once we have defined the retardation time at each point through eq. (V.1.16), we can substitute it in eq. (V.1.20) to estimate the resulting fluctuation density:

$$\frac{\delta\rho}{\rho} = \frac{4H\delta\phi}{\dot{\phi}_0}. \tag{V.1.21}$$

Decomposing in Fourier components on both sides, this relation can be written for each mode as

$$\delta_k(t) = \frac{4H(t)}{\dot{\phi}_0(t)}\delta\phi_k(t) \tag{V.1.22}$$

and, from this equation, the contribution to $(\delta\rho/\rho)^2$ from a unit logarithmic interval in k is found to be related to the field fluctuation power by

$$\left(\frac{\delta\rho}{\rho}\right)^2_k = 16\frac{H^2}{\dot{\phi}_0^2}(\Delta\phi)^2_k. \qquad (V.1.23)$$

Introducing (V.1.14), we finally have

$$\left(\frac{\delta\rho}{\rho}\right)_k = \frac{2H^2(t)}{\pi\dot{\phi}_0(t)}. \qquad (V.1.24)$$

Expression (V.1.14) is accurate only for small times, when the potential term is negligible and the fluctuation is still not subject to unstable growth, at late times the behaviour of $\delta\phi(k,t)$ should had been determined using its complete equation of motion. On the other hand, the time–delay function is accurate only at late times, when the spatial derivatives become unimportant; however, both results should be reasonable estimates at the time of transition between the two regimes. So, matching two results at the interphase of their validity domains and assuming that the phase transition is instantanous, we can obtain a relation between the scalar field fluctuations and the density perturbations. The quantities on the right hand side of the previous equation correspond to the inflationary regime. when the scalar field fluctuation becomes larger then the horizon and is frozen in as a classical field (so that the field fluctuation is treated as a classical object), while the density contrast refers to the radiation dominated epoch, after the decay of the inflaton field into matter and radiation. A more rigorous calculation shows that the quantity in the left hand side should strictly correspond to the density contrast when the k–modes reenter the horizon. Therefore, the correct result at the horizon scale is

$$\left(\frac{\delta\rho}{\rho}\right)_{hor} = C\frac{H^2}{2\pi\dot{\phi}_0}\bigg|_{k\sim H}, \qquad (V.1.25)$$

where $C = 4$ or $C = 2/5$ if the mode k reenters the horizon when the Universe is radiation or matter dominated (see e.g. [5]). effectively given by (V.1.1).

During inflation, H and $\dot{\phi}$ vary slowly, yielding a very nearly scale invariant spectrum of perturbations. Let us consider a specific potential for illustrating the dependence on time, and hence on wavenumber, of the density contrast. With $V(\phi) = -(\lambda/4)\phi^4$, typical for many theories, and neglecting $\ddot{\phi}_0$ (i.e. with the slow-rollover condition) in the field equation (V.1.4), the solution for ϕ is

$$\phi_0 = \left(\frac{3H}{2\lambda}\frac{1}{t_e - t}\right)^{1/2},$$

where t_e is the time when inflation ends (actually, at large values of ϕ_0 the potential differs from the one we use here and ϕ_0 does not tend to infinity). We can then calculate $d\phi_0/dt$ and, taking $t = t_{hor}$, substitute it in eq. (V.1.25). Thus,

$$\left(\frac{\delta\rho}{\rho}\right)_{hor} \sim \lambda^{1/2}H^{3/2}(t_e - t_{hor})^{3/2}.$$

In the time interval $(t_e - t_{hor})$ a wavelength grows from H^{-1} to $k^{-1} = H^{-1}e^{H(t_e - t_{hor})}$, so, the density fluctuation at horizon scale has a weak k–dependence:

$$\left(\frac{\delta\rho}{\rho}\right)_{hor} \sim \lambda^{1/2}ln^{3/2}\left(\frac{H}{k}\right).$$

Nonetheless, it should be noted that if the inflating Universe were indistinguishable from de Sitter space and the Hubble parameter and the field ϕ were really constant, the result would be a far too inhomogeneous Universe. This is, in fact, the origin of the amplitude excess of the perturbations in inflationary models: to obtain a nearly constant energy density and sufficient inflation, $\dot{\phi}_0$ is kept small and this tends to increase $\delta\rho/\rho$.

Density fluctuations can be classified, at early times, when they are super–horizon sized, as adiabatic and isothermal. Adiabatic fluctuations arise from scalar fluctuations of the metric (different expansion rates in different spacetime points) in an initially homogeneous distribution of matter and are density fluctuations in which the ratio of matter density to radiation density (ρ_{mat}/ρ_{rad}) is not altered. They are possible when the matter radiation interaction is strong. On the other hand, fluctuations in the composition of matter at a constant total energy density are called isocurvature fluctuations. If matter density changes slightly from point to point and radiation is left homogeneous, there are no temperature fluctuations; in this way they are also called isothermal. They correspond to fluctuations in the local equation of state. So, the term isothermal fluctuation usually refers to the case where the ratio of baryonic matter concentration to radiation density changes, however, it can also be applied to fluctuations of a field (connected with the hidden sector of the theory) that interacts with usual matter only gravitationally and whose fluctuations do not consequently alter the plasma temperature.

The field fluctuation amplitude $H/2\pi$ is valid for the fluctuation of any effectively massless scalar field; if we are dealing with the field σ that drives inflation, the total energy density of matter being $\rho_{tot} \approx V(\sigma)$, then, the inhomogeneities of $V(\sigma)$, after its decay, give rise to adiabatic density perturbations of $\rho_{tot} \sim T^4$:

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \sim \frac{\delta\rho_{tot}}{\rho_{tot}} \sim \frac{\delta T}{T}.$$

Fluctuations in any other field, with energy $\rho_\varphi \ll \rho_{infl} \approx \rho_{tot}$, will not, initially, lead to considerable perturbations of the total energy, nor to the associated metric and temperature perturbations. Such isothermal perturbations may become important at later stages of the evolution of the Universe: if a field interacts weakly with other particles, it, or its decay products, can eventually give the main contribution to the total energy of the Universe. It will act as a decaying cosmological constant, as already suggested in chapter IV, for the remnant of the inflaton field (see anyhow [69] for constraints on this kind of models).

V.2) Scalar field fluctuations and density perturbations in extended inflation models

Inhomogeneities in the JBD field correspond to spatial variations of the gravitational constant. The consequences of these variations and the observational constraints that can be imposed remain to be dicussed in detail, but, dealing with density fluctuations, the variation of G will be of interest only as long as it affects the expansion rate H.

a) Perturbations in Standard Brans–Dicke Theory

As already mentioned, the expression for the fluctuation amplitude $\delta\varphi = H/2\pi$ at the epoch of horizon crossing, applies, in de Sitter space, to a minimally coupled scalar field with standard kinetic term. The JBD field Φ has not a standard kinetic term and has dimensions of mass squared but it can be written in terms of a field ϕ with usual dimensions (mass) and kinetic term: $\Phi = 2\pi\phi^2/\omega$. This is the field to which it seems more natural to ascribe this fluctuation, although it will be strictly correct only in the limit $\omega \gg 1$, where ϕ is minimally coupled. So, an even more natural approach is to analyze curvature fluctuations in the conformally rescaled Einstein frame, where the JBD field looks like a minimally coupled field with an exponential potential $V(\varphi) = M^4 \exp(-2\varphi/\varphi_0)$ and hence plays the role of the inflaton in the context of a slow–rollover inflation [70]. From then on, the usual formula for density fluctuations $\delta\rho/\rho = \alpha H^2/\dot{\phi}$, is directly applied. Lucchin, Matarrese and Pollock [74] have verified the application of this formula to power–law inflation and found the constant of proportionality is $\alpha \approx 2 \times 10^{-2}$. The fluctuations spectrum behaves as: $\delta\rho/\rho \propto \lambda^{1/p-1}$, where λ is, here and in the following, the wavelenght associated to a perturbation and p is the power of the expansion law $a \propto t^p$. The application of the standard formula for the r.m.s. fluctuation of a scalar field, derived for de Sitter space, to power–law inflation has been investigated by Abbott and Wise [31]. They have shown that, in a general inflationary metric, the amplitude of scalar field fluctuations for wavelengths well outside the horizon, can be written as: $\mid \Delta\varphi_k \mid^2 \propto k^2/a^2(t_{hc}) \propto H_{hc}^2$, where use have been made of the fact that the time of horizon crossing (sub–index hc) is defined by $k/a(t_{hc}) = H_{hc}$.

Since the Jordan frame is our physical frame, we will have to transform this expression back to it. Under the conformal transformation, the energy density and the density contrast transform as:

$$\rho = (2\Phi)^2\tilde{\rho}$$

and

$$\frac{\delta\rho}{\rho} = \frac{\delta\tilde{\rho}}{\tilde{\rho}} + 2\frac{\delta\Phi}{\Phi}.$$

At late times, the conformal factor $2\Phi \rightarrow 1$ since $\Phi = 1/2$ when G reaches its present value G_N (in units such that $8\pi G_N = 1$). The JBD field varies very slowly in a non-inflationary regime, so, after inflation, the two frames are already approximately the same and the density fluctuations in both frames coincide. In the Jordan frame, density perturbations generated from the JBD field fluctuations are, initially, isothermal fluctuations since the JBD field has prevailing fluctuations but makes an important contibution to the energy density of the Universe only after the inflaton phase transition. From here on, we work in the Einstein frame and omit tildes except when comparing quantities in the two frames.

The amplitude of density perturbations in extended inflationary models increases with scale [70], [72], [73], [19]:

$$\left(\frac{\delta\rho}{\rho}\right)_{hor} = f(\omega, M)\lambda^{1/(p'-1)} = f(\omega, M)\lambda^{4/(2\omega-1)}, \tag{V.2.1}$$

where $p' = (2\omega + 3)/4$ is the power in the expansion law, in the Einstein frame: $a \propto t^{p'}$, and, raising ω, the scale dependence becomes negligible, as expected. This stronger scale dependence as compared to the exponential inflation case can be found following the same procedure as in section (V.1): solving eq. (V.1.4) with an exponential potential $V(\varphi) = M^4 e^{-2\varphi/\varphi_0}$ and considering that in this case the wavelength grows as a power-law. In fact, as the density perturbation spectrum is proportional to H^2, one expects, in power-law inflation models, perturbations leaving the horizon at earlier times to be larger. This means the spectrum has more power on large scales. The function $f(\omega, M)$, where M indicates the scale of the phase transition, differs somewhat from one author to another. In ref. [70] it is

$$f(\omega, M) \approx 4\pi 10^{100/(2\omega-1)} \left[\frac{2\omega+3}{6}\right]^{1/2} \left[\frac{(6\omega+5)(2\omega+3)}{32\pi\omega^2}\right]^{-2/(2\omega-1)} \left[\frac{M}{m_{Pl}}\right]^{2(2\omega+1)/(2\omega-1)} \tag{V.2.2}$$

for λ in eq. (V.2.1) given in Mpc: $\lambda_{Mpc} \equiv \lambda/Mpc = \lambda\ 10^{-38} GeV$. From this last equation we see that, on one hand, the fluctuation amplitude diverges in the very large

ω limit because the effective potential goes to a constant and $\dot{\varphi} = 0$, but, on the other hand, it is lessened, if we increase ω in the range of intermediate values. This expression leads to interesting amplitudes $\delta\rho/\rho \sim 10^{-4}$ for energy scales of the phase transition $M \leq 10^{14} GeV$; while computing the associated temperaure fluctuations on large angular scales (corresponding to scales from 100 to 10 000Mpc), with this value for M and, e.g. $\omega = 10$, too large values are obtained to be consistent with current limits on quadrupole anisotropy ($\Delta T/T \approx 6 \times 10^{-6}$, see [3]). Increasing ω –while still in the range allowed by successful extended inflation– or decreasing M slightly –as $\delta\rho/\rho \propto M^2$ for large ω, it is very sensitive to variations in M– can remedy this problem. We do not have here the possibility of fine–tuning the self–coupling constant, since the σ potential is taken strictly constant $= M^4$; in (old) extended inflation models the fine–tuning falls on the Brans–Dicke parameter. The fact that the spectrum behaves as a positive power of the wavelength is a feature that might be useful in building models that account for the observed large–scale structure of the Universe. Depending on the value of ω, this can represent a very slight increase of power at large scales, or a more substantial one. For $\omega = 25$, $\delta_\lambda|_{hc} \propto \lambda^{0.08}$; with $\omega = 10$, $\delta_\lambda|_{hc} \propto \lambda^{0.2}$. (with $\omega = 500$ the spectrum is, as expected, practically scale invariant). Which, if any, of these deviations from the scale invariance could account for the very large structures observed remains to be worked out in detail.

Now, if we want to compare the theoretical spectrum with observational data (that are available from a much later epoch), we have to work out its evolution. From inflation, we calculated the perturbation amplitude at horizon crossing, but this means that the amplitude for different modes is specified at different times; for comparison with observations, we have to estimate the spectrum of perturbations at a fixed time, in particular, at the epoch of decoupling, when the background radiation last scattered. As we saw in section (V.I), the density contrast can be written for each mode k: $(\delta\rho/\rho)_k^2 = k^3 \mid \delta_k \mid^2$. Introducing the usual assumption that there is no preferred primordial scale, the fluctuation spectrum is a power–law $\mid \delta_k \mid^2 \propto k^n$, implying $n = -3$ for scale invariance. In order to take the evolution of the perturbation outside the horizon into account, the processed spectrum have to include an evolution factor k^4 (density perturbations outside the horizon are gauge-dependent and hence not well-defined quantities; working in the synchronous gauge, density fluctuations grow as λ^{-2}, keeping metric perturbations constant. see e.g. [6]). The redefined fluctuation spectrum, $|\delta_k|^2 = k^{n_s}$, will then have index $n_s = 1$. All the above mentioned exponents for λ are well inside the range allowed by COBE results for the index of the primordial fluctuation power-law spectrum $|\delta_k|^2 \propto k^{n_s}$: $n_s = 1.1 \pm 0.5$ [3].

It is worth noticing that power-law inflation models with small p values, exist: $a \sim t^2$ in wall-dominated inflation, in a model of broken symmetric theory of gravity, or a model of Kaluza-Klein cosmology during the initial compactification of the D extra dimensions, whose spectra can therefore be very far from the scale invariance [74].

Guth and Jain [72] have pointed out some subtleties of the procedure for the estimation of density fluctuations that could question previous results. The main doubt is, of course, in which frame the field should be quantized. Since the conformal transformation, transforms the JBD field to a new field non-linearly related to the original one, it is not clear if the results will be the same with both fields. But they do not attack this problem: they achieve the calculation employing the standard formula in the Einstein frame, albeit avoiding some of the usual simplifying assumptions. In this way, they find results that coincide with previous ones for not too small values of ω ($\gtrsim O(few)$). The points they work on more carefully relate to the evaluation time of the quantities envolved in the standard formula. Even in the context of standard inflation, the formula for $\delta\rho/\rho$ is only an approximation, matching quantities corresponding to different epochs, but here H depends on time more strongly and any answear that depends on H must specify precisely the time at which it should be evaluated. Instead of using the standard approximation of slow-rollover. it is possible, here, to evaluate $\dot{\varphi}(t)$ by differentiating the exact solution for the evolution of the scalar field. In fact, the assumption that $\ddot{\varphi}$ is negligible must be taken with care for small values of ω since:

$$\ddot{\varphi}(t) \sim \frac{H\dot{\varphi}(t)}{(2\omega+3)/4}.$$

Also, a standard convention for the time of horizon crossing (at which the right hand side of the density fluctuation formula has to be evaluated) is $\lambda_{phys} = H^{-1}$, but, with $a \propto t^n$, $H = nt^{-1}$ and $\lambda_{phys} = nH^{-1}$. This would produce, through n $(= 2\omega + 3/4)$, an ω dependent correction, enhancing the fluctuations amplitude. Incorporating this corrections, the resulting discrepancy factor is near unity for values of ω usually employed in extended inflation ($\gtrsim 10$).

The results obtained applying directly the standard procedure in the Jordan frame, can be compared with the result in the Einstein frame [72], [70]. The answer in the Jordan frame is smaller by a factor that is near unity and becomes large only for very small values of ω ($\sim 2-3$). Anyhow, we do not expect the standard procedure to be applicable in the Jordan frame, because of the non-minimal coupling.

Some work has been done [75] addressing the question of consistency between the two frames, but only with near classical states, i.e. decomposing each of the dynamical variables into the homogeneous classical background part and a small fluctuating quantum part: $\phi = \phi_c - \delta\phi$, where ϕ_c plays the role of G^{-1} and $\delta\phi$ is a dynamical field. Following a Hamiltonian formalism for constrained systems to extract the true dynamical degrees of freedom and canonically quantizing the resulting system, they find that the quantum part can be analyzed in either of the frames, since the conformal transformation will induce only a linear transformation in the dynamical variables, so the amplitude of density perturbations in the original frame coincides with the one in the Einstein frame. The problem with pure quantum fields has not been addressed.

Seshadri quantizes directly the JBD field in the Jordan frame [76] for a model in which ω varies with time (hyperextended inflation). It should be stressed, anyhow, that his results are not so promising: density perturbations are large in amplitude and have a scale invariant spectrum, implying further constraints on the model. Large amplitudes arise due to the large value of the parameter β contained in the expression for the ω evolution: $2\omega + 3 = 2\beta/(\phi_c - \phi)$, which in turn is required for the source of the Einstein equation to be dominated by the constant term. The value of β affects both the magnitude and the spectrum of density perturbations.

Lidsey [77] works out the fluctuation spectrum for Extended Chaotic Inflation and finds the same scale dependence as the spectrum of old extended inflation, from which it can be seen that, as long as the inflaton potential is taken to be approximately constant ($V(\sigma) \approx const$), it does not affect the scale dependence of the spectrum, but does determine the amplitude.

What about the σ field (the inflaton in the original frame) fluctuations? In [70], it is argued that the σ quantum fluctuations should be highly supressed because its effective mass ($\approx V_f^{1/4}$) is much larger than the Hawking-Gibbons temperature $H/2\pi$ (temperature associated to de Sitter space due to the periodicity of the S^4 sphere obtained in the Euclidean formulation, which gives the characteristic amplitude of fluctuations). This however may not be true at the beginning of inflation, as the time dependence of the gravitational constant ($G_{eff} = e^{-\varphi/\varphi_0} = (2\Phi)^{-1}$) leads to a Hubble parameter $H \approx (G_{eff}V_f)^{1/2}$ larger than its general relativity value $(G_N V_f)^{1/2}$. Comparing the mass m_σ of the inflaton, defined by: $m_\sigma^2 = \partial V(\sigma)/\partial\sigma|_0 \sim M^2$ with the fluctuation amplitude $H/2\pi$, using a roughly estimate for H:

$$H^2 \sim \frac{8\pi\rho_v}{3\phi} \sim \frac{8\pi M^4 G_{eff}}{3} \Longrightarrow H \sim 2(2\pi/3)^{1/2}(G_{eff})^{1/2}M^2,$$

we have

$$\frac{m_\sigma}{H/2\pi} \sim (\sqrt{G_{eff}}M)^{-1}. \qquad (V.2.3)$$

Since $(G_{eff})^{-1} < m_{Pl}^2$ during inflation and $M \ll m_{Pl}$ is required to ensure that the JBD field fluctuations are acceptably small (as can be seen from eq. (V.2.2)), then $m_\sigma > H/2\pi$. The strength of this argument of course depends on how small G_{eff}^{-1} can be at this epoch and could be wrong, as we said, at the beginning of the inflationary period. These estimations correspond to the Jordan frame, since it is in this frame where the σ field has canonical kinetic term, but the ratio between the σ-mass and the Hubble constant should of course hold in the Einstein frame, using the redifined field and the conformally transformed expansion rate $\tilde{H}$. The key argument for ignoring the inflaton perturbations is rather that this field is confined to a passive role and then its dynamics are irrelevant. Deruelle *et al.* [78] calculate explicitly the contribution of the perturbation of the inflaton and show that, to linear order, the perturbation of any field sitting in a local minimum of its potential, and hence behaving as an effective cosmological constant, decouples from the perturbations of the other fields, including the metric, and consequently the density. To first order, the density fluctuation will depend on $\delta\phi$ alone (more precisely, to its gauge invariant version) and the mixed terms in $\delta\phi$ and $\delta\sigma$ will vanish, precluding any implication of the coupling of the dilaton and the inflaton on the initial spectrum. So, this is the reason why, in old extended inflation, the σ-field fluctuations can be ignored, and not because they are suppressed. However, if one wants to allow for background inflaton dynamics, in the form of tunnelling (old Extended Inflation) or of slow rolling (new or chaotic Extended Inflation), inflaton fluctuations must be considered and the coupling of the dilaton to the inflaton taken into account. In old extended inflation, the fact that the JBD field is evolving while the inflaton is tunnelling and how to follow the production and evolution of fluctuations during the bubble coalescence process are questions that have not been adressed. Besides, the presence of two scalar fields could lead to a scenario of double-round inflation, with relevant fluctuations of both fields.

Two-field inflation models have already been considered [79], in which two scalar fields are coupled together, one field rolling and the other trapped in the false vacuum, with canonical gravity and kinetic terms. They suffer from the general problem of fine-tuning of the coupling constant to keep the potential of the rolling field flat. There are two effects that work in the direction of increasing the perturbations amplitude: first, H is determined by the strictly constant energy of the infalton field σ, so that it does not decrease at the end of the inflationary epoch; second, inflation ends when the σ field makes the transition to the true vacuum, hence there is no need for the denominator on the right-hand side of the density fluctuation formula ($\dot{\varphi}$) to be increasing at the end of inflation. In this approach, the features of the effective potential $V(\sigma, \varphi)$ are: a barrier in the σ direction (quantum evolution), and a smooth rolling down for φ (classical evolution); for large values of φ, the barrier can be smoothed out as an effect of the coupling between the two fields. In Extended Inflation models, if we introduce a potential for the JBD field and work in the Einstein frame, we have a similar behaviour. The introduction of this potential is suggested, as we saw, by observational requirements: if, at some time after inflation, a potential anchors the JBD field at some value, the low energy limit of the theory coincides with Einstein gravity. During inflation, this potential should be negligible compared with the energy density of the inflaton field and is expected not to affect the inflationary Universe evolution. The most simple potential is a mass term, and, in fact, the scalar field is expected to acquire mass due to quantum effects: a primarily massless field emitting and absorbing quanta of other fields will become massive, unless there is, in the theory, an invariance principle that forbids mass. The interaction mediated by this field then becomes short range and its influence at large distances becomes negligible. There is no difficulty from the experimental point of view for modifying gravity at small scales: astronomical tests would not detect it. If, introducing another ingredient, a symmetry breaking potential is chosen for the (redefined) JBD field: $V(\phi) = \lambda(\phi^2 - \phi_0^2)$ (equivalent to $V(\Phi) = \lambda\omega(\Phi - \Phi_0)^2$), we have an induced gravity scenario [23], [73]. Let us explore the consequences, during and after inflation, of including a potential for the JBD field, considering also the JBD field evolution after the σ-field phase transition.

If the JBD field settles to its general relativity value before the inflaton has tunnelled (and the barrier is not completely smoothed out), there is a phase of standard inflation, with its inherent problem of bubble percolation. So, the σ tunnelling must succeed first and Φ might evolve significantly after inflation, one should therefore keep in mind that the value of Φ at the end of inflation (Φ_e) will affect the results for the amplitude of the density fluctuations, enhancing it by a factor of $(\sqrt{\Phi_e}/m_{Pl})^{-(2\omega+1)/(\omega-1/2)}$ if $\Phi_e < m_{Pl}^2$, as calculated in [70]. After the inflaton transition, there are a number of possibilities. As in old inflation, the collision of bubbles should generate a hot fluid of radiation; if this energy density dominates, the standard FRW cosmology is recovered. This could, however, not be the case, since reheating due to bubble collisions is not expected to be very efficient (see below). If, on the other hand, the JBD field potential energy dominates over radiation produced by the inflaton decay, extended inflation reduces to some kind of chaotic inflation and reheating really happens when the JBD field oscillates before settling, as in new or chaotic inflation. In the case that reheating is efficient, with a symmetry breaking potential, it could also happen that the ϕ symmetry is restored, leading to a round of induced gravity inflation in the percolated region. We may then expect a second inflationary episode, after the σ phase transition, with an effective potential $V(\varphi) \equiv e^{-2\varphi/\varphi_0} V_{JBD}(e^{\varphi/\varphi_0})$. Depending on the form of V_{JBD}, $V(\varphi)$ may be an extended inflation type exponential potential (when $V_{JBD} \approx const.$), reduce to an effective cosmological constant, or even grow exponentially. Here, if $V(\varphi)$ dominates, constraints on density fluctuations once again imply a small value for the self-coupling constant λ_φ entering in $V(\varphi)$. When $V(\varphi)$ does not dominate, we are still left with constraints on ρ_v and $V(\varphi = 0)$. It is interesting to note that the fine–tuning problems of the inflationary models are hardly eluded. It has been proposed that, in a $\lambda\phi^4$ chaotic inflation model with non–minimal coupling $\xi R\phi^2$, the contraint on λ is substantially weakened if very strong non–minimal coupling is allowed. But one cannot really say the fine–tuning problem is resolved since an unnaturally small λ is merely replaced by an unnaturally large ξ (see e.g. [75], [80]).

b) Mass for the JBD field, picture in the Jordan frame.

The $(\Phi)^{-1}$ in the kinetic term was originally introduced by Brans and Dicke, to permit ω to be dimensionless since Φ has dimensions of mass squared: $[\Phi] = [G^{-1}] = [m^2]$. A mass term for this field should be introduced in an analogous way and will have the form

$m^2\phi$. Then, for a massive JBD field the Lagrangian reads:

$$\mathcal{L}_\Phi = -\Phi R + \frac{1}{\Phi}\left[\omega g_{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - f(\omega)m_\Phi^2\Phi^2\right]. \qquad (V.2.4)$$

The equation of motion for the JBD field is a Klein–Gordon equation modified by the expansion of the Universe (for $f(\omega) = 2\omega + 3$):

$$(\Box + m_\Phi^2)\Phi = \frac{1}{2(2\omega+3)}T_V - \frac{1}{2(2\omega+3)}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma + \frac{2}{2\omega+3}V(\sigma), \qquad (V.2.5)$$

and a cosmological constant term appears in the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}f(\omega)m_\Phi^2 g_{\mu\nu} = \frac{T_{V\mu\nu}}{2\Phi} + \frac{\omega}{\Phi^2}(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi) + \frac{1}{\Phi}(\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\Box\Phi)$$

$$+\frac{1}{2\Phi}[(\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\alpha\sigma\partial^\alpha\sigma) + g_{\mu\nu}V(\sigma)]. \qquad (V.2.6)$$

After inflation, this cosmological constant will eventually dominate and hence the JBD field must decay. During inflation, where $V(\sigma) = \rho_v$ dominates, with a FRW metric and $\Phi = \Phi(t)$, we have:

$$\ddot{\Phi} + 3H\dot{\Phi} + m_\Phi^2\Phi = \frac{2\rho_v}{3+2\omega},$$

and

$$H^2 = \frac{8\pi\rho_V}{3\Phi} + \frac{2\omega+3}{2}m_\Phi^2 + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - H\left(\frac{\dot{\Phi}}{\Phi}\right).$$

For m_Φ not to disturb the evolution of the inflationary Universe, we must have, in both equations, $m_\Phi^2 < \rho_V/\Phi$, with $\rho_V = M^4$. Since, during inflation, $\Phi \le m_{Pl}^2$, a safe limit for the JBD field mass is: $m_\Phi < M^2/m_{Pl}$, which, for $M \sim 10^{14}GeV$, implies: $m_\Phi \le 10^9 GeV$. This limit can clearly be seen in the plots (V.1) and (V.2) at the end of this chapter, for H and Φ during the inflationary epoch. The behaviour changes drastically for $m_\Phi \ge 10^9 GeV$: H quickly goes to a constant value and the Φ field settles to its potential minimum. This means that a standard inflation regime has been recovered, with the dominating mass term acting as a cosmological constant. In figures (V.1) and (V.2), τ and Ψ are dimensionless quantities: $\tau = 10^{-4}\left[2M^4/(2\omega+3)\right]^{1/4}\ t$ and $\Psi = 10^{-8}\left[2M^4/(2\omega+3)\right]^{-1/2}\ \Phi$, where $M = 10^{14}GeV$.

If we now want to introduce a symmetry breaking potential $V(\Phi) = \lambda(\Phi - \Phi_0)^2$ with $\Phi_0 = m_{Pl}^2/16\pi$, the equations are:

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{2}{3+2\omega}\left[\rho_v + V(\Phi) - \frac{1}{2}\Phi\frac{\partial V(\Phi)}{\partial \Phi}\right],$$

and

$$H^2 = \frac{8\pi\rho_v}{3\Phi} + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 - H\left(\frac{\dot{\Phi}}{\Phi}\right) + \frac{1}{2}\frac{V(\Phi)}{\Phi}.$$

The mass term is $m_\Phi^2 = \lambda\Phi_0 \sim \lambda m_{Pl}^2$ and we still have the same constraint on m_Φ. This limit for the value of m_Φ shows that a small self-coupling constant λ is required ($\lambda \sim m_\Phi/m_{Pl}^2$), so that, even in the context of two-fields inflation, a fine tuning is needed. The Φ^2 term cancels out in the Klein-Gordon equation but appears in Einstein equations as $m_\Phi^2 \Phi g_{\mu\nu}$; when $\Phi \simeq const.$, after inflation, it is again a cosmological constant term.

c) Perturbations from generalized models

Turning to the generalized models, the Lagrangian for σ in the Einstein frame is:

$$\mathcal{L}_\sigma = \frac{1}{2}\epsilon^{-N\varphi}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \epsilon^{-M\varphi}V(\sigma),$$

as we saw in chap. IV. The difference with standard Brans–Dicke, regarding fluctuation production, will only be the power of the exponential factor in the potential term:

$$e^{-2\varphi/\varphi_0} \rightarrow e^{-M\varphi/\varphi_0},$$

with $M = (2-m)/\varphi_0$. Then, we exclude the value $m = 2$, that leads us to exponential inflation in the Einstein frame. At the same time, care must be taken to avoid a potential that would be too steep. In order to have a slowly decreasing potential that allows us to apply the standard density perturbations formula to these models, we first have to check that generalized extended inflation transforms to slow-rollover inflation in the Einstein frame. Imposing the slow-rollover conditions $\mid \partial^2\varphi/\partial t^2 \mid \ll H(\partial\varphi/\partial t)$ and $(1/2)(\partial\varphi/\partial t)^2 \ll V(\varphi)$ in the equations of motion, we obtain constraints on the exponent of the power–law expansion: $p' \gg 1$ and $3p' - 1 \gg 1$ [45], where the value of p' is now $p' = (2\omega+3)/(2-m)^2$. This implies $2\omega + 3 \gg (2-m)^2$. We can then estimate the density fluctuations through:

$$\left.\frac{\delta\rho}{\rho}\right|_{hor} \simeq \frac{H^2}{d\varphi/dt} \simeq -\frac{3H^3}{dV/d\varphi},$$

where, for the last equality, use has been made of the slow roll-over conditions. Remembering that, restoring normal units, $H^2 = (8\pi/3m_{Pl}^2)V(\varphi)$, where $V(\varphi) = e^{-M\varphi}V(\sigma) = e^{-M\varphi}M^4$, and $e^{\varphi/\varphi_0} = 16\pi\Phi/m_{Pl}^2$, we get

$$\left.\frac{\delta\rho}{\rho}\right|_{hor} \simeq \left(\frac{2\omega+3}{6}\right)^{1/2}\left(\frac{M}{m_{Pl}}\right)^2 \frac{m_{Pl}^2}{(m-2)\Phi^{(2-m)/2}}. \qquad (V.2.7)$$

With $m = 0$, we have the expression for standard Brans–Dicke theory:

$$\left.\frac{\delta\rho}{\rho}\right|_{hor} \simeq \left(\frac{2\omega+3}{6}\right)^{1/2}\left(\frac{M}{m_{Pl}}\right)^2\left(\frac{m_{Pl}^2}{\Phi}\right)$$

which is the expression employed to get eqs. (V.2.1) and (V.2.2).

The analysis of which values of m are requested to get interesting fluctuations amplitudes and to observe the allowed deviation from the scale invariance of the spectrum will be included in Chap.VI.

d) Inhomogeneities from bubbles

Finally, we briefly address the second mechanism for creating density inhomogeneities. In all this work on the creation and evolution of perturbations, the phase transition is really ignored, i.e. it is supposed to leave the Universe homogeneous and isotropic on cosmic scales. Nevertheless, the percolation, collision and thermalization of bubbles generated during the tunnelling process produce inhomogeneities as well and it has been suggested [81] that nearly energy empty regions today (voids) could be remnants of extended inflation bubbles. But this possibility has been discarded by the fact that the distortion they would cause on the CMBR, were they present at the recombination epoch, would be inacceptably high (see e.g. [83]). This argument has been used to impose stringent bounds on the Brans–Dicke parameter ω [21], [34], [82], [83], as we already discussed in chapter II, and these constraints preclude individual bubbles from providing an interesting source of density perturbations. The estimation of the time needed for radiation to cross the bubble was based on the assumption that a superhorizon–sized void would either conformally expand with spacetime (and photons just leak into the void) [21], [82], or fill in with matter and radiation that stream into the void, with the approximation of a FRW metric in and inside the bubble wall with the same scale factor as the outer region [83]. Very recently, however, the possibility that the time required for filling the bubbles can be substantially reduced incorporating relativistic effects in the calculation, has been considered [84]. During the radiation dominated epoch, bubbles shrink, at the speed of light for a relativistic void, due to the large pressure force acting on the fluid in the wall, and this filling process can take place in a short time, as observed from outside, due to a time dilation caused by the large, negative potential of the void relative to the outer spacetime. In this work, care has been taken to distinguish between the inner and the outer scale factor and Hubble parameter. Taking this effect into account, the so called "big bubble problem" could be discarded and constraints on the Brans–Dicke parameter ω relaxed, even though this problem is directly related to the thermalization process, rather than to the filling process. The thermalization time is certainly greater than the time required for photons originally in the void walls to reach the center of the void, and this is a more complicated question to deal with, that depends to some extent on the type of matter dominating the Universe behaviour.

Conclusions

From inflationary models with Jordan–Brans–Dicke gravity theory we obtain tilted spectra (positive power of the wavelength) that could lead to a better agreement with the observed large–scale structure of the Universe. The estimated fluctuation amplitudes are only slightly in excess of the observational bound. There is no need to impose a very small value on any (dimensionless) parameter; the fluctuation amplitude is characterized by the ratio of the unification scale to the Planck scale. In the simplest extended inflation model (old inflation type potential with a massless JBD field), the only adjustable quantity is the Brans–Dicke parameter. To avoid any mismatching between the required value of ω and the one suggested by observations, a potential for the JBD field is usually invoked. But, if the inclusion of a potential is useful to solve the ω problem, it also increases the amplitude of density fluctuations and re-introduces the fine-tuning problems of standard inflation models. In addition, we have to impose the condition that the JBD field decays in order to preclude it from dominating. On the other hand, the presence of a potential term for the JBD field causes, in most cases, two rounds of inflation, with the possibility of two power spectra.

Regarding the Generalized Models, we must impose some limitations on the parameter m not to upset the inflationary behaviour of the inflaton field σ. m is limited by the slow roll-over constraint, that depends on the ω value, and $m \neq 2$ is required for a power-law inflation. We may expect that tighter constraints on m will come from bounds on the fluctuations amplitude and spectrum imposed by COBE but this will be included in the next chapter.

The possibility that inhomogeneities caused by bubbles created in a first order transition could provide an interesting source of density perturbations, which was one of the initial motivations for these kind of models, seems to be discarded. On the other hand, the constraint on the bubble distribution was the main limitation of the theory and now it is proposed that a careful approach could invalidate this limit.

More work has to be done in order to be able to follow the creation and development of inhomogeneities originating from the scalar fields fluctuations, allowing for the evolution of both fields, e.g. the σ field tunnelling while the JBD field is slowly evolving. The observable effects of fluctuations in the gravitational coupling and any resulting costraints remain to be analyzed.

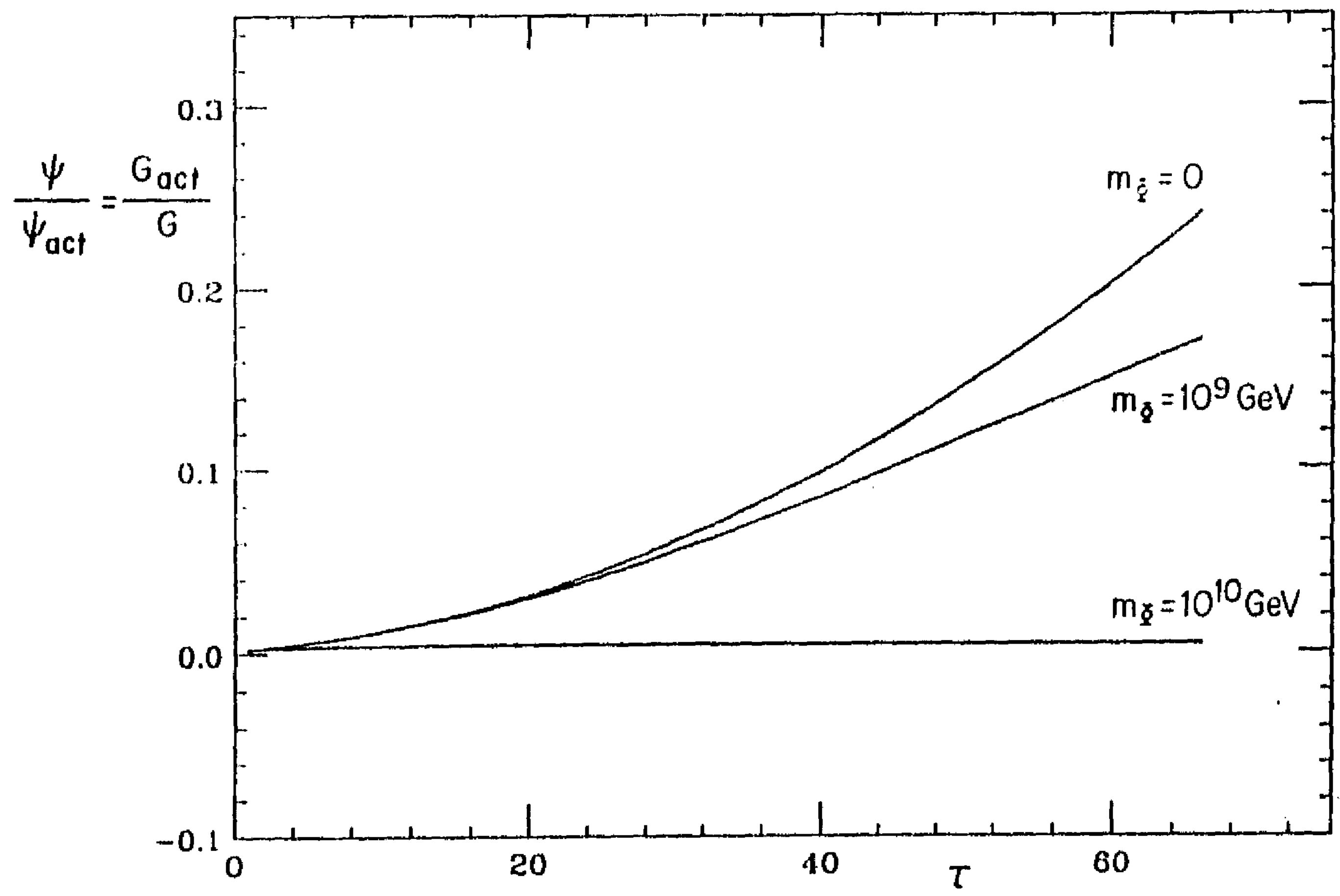

Figure V.1:

Evolution of Φ during the inflationary epoch with a mass term for the JBD field. τ and Ψ are dimensionless quantities and $\omega = 25$.

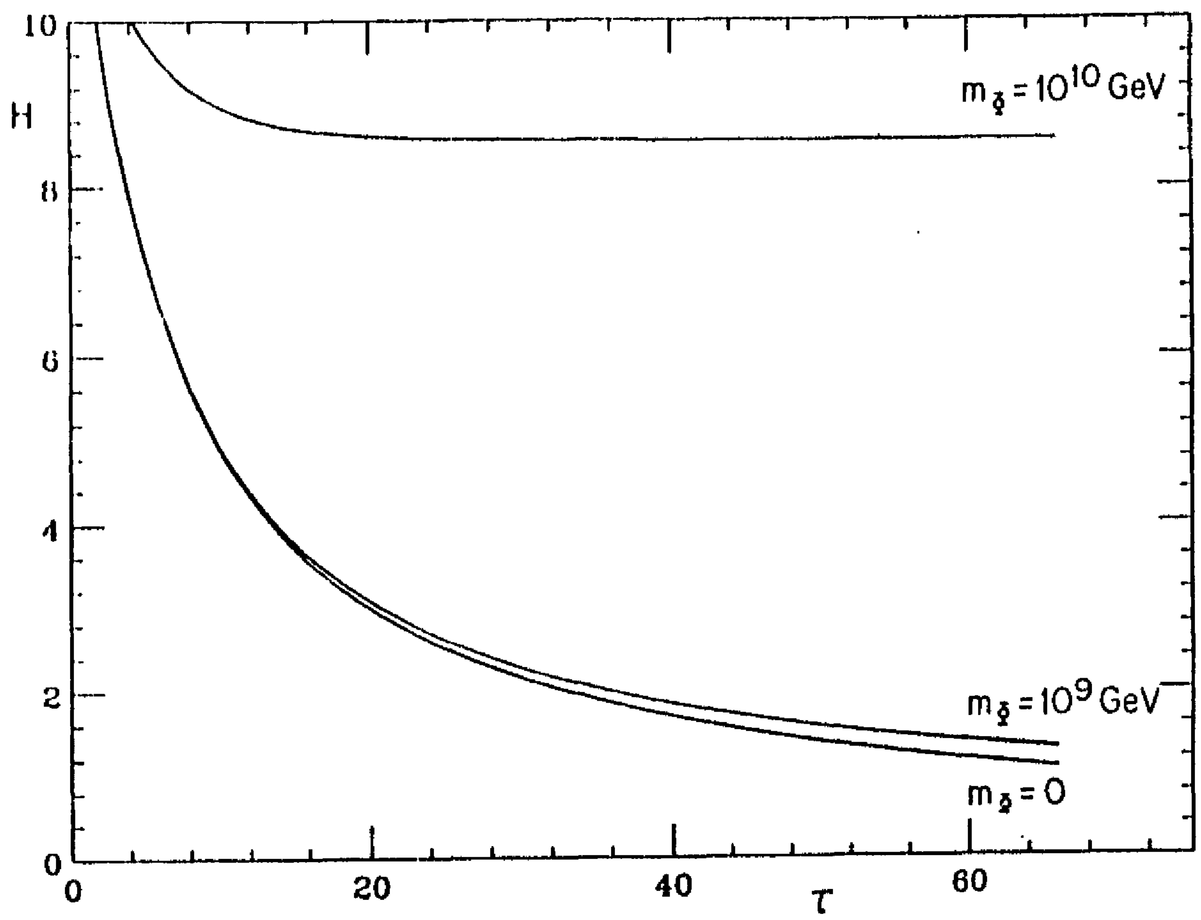

Figure V.2:

Evolution of H during the inflationary epoch with a mass term for the JBD field. τ is a dimensionless time and $\omega = 25$.

Chapter VI

Observational Limits on the Generalized JBD Field Couplings

Introduction

Considering in each era only the effect of the dominant component, we investigate here the possibility of establishing bounds on the general kinetic and potential couplings of the JBD field. So, assuming a dark matter dominated Universe today, we consider post–Newtonian bounds on the Brans–Dicke parameter ω, observational bounds on the age of the Universe, on the Hubble parameter and on the variability of the Newtonian constant today, as well as on its value during primordial nucleosynthesis, to constrain the dilaton couplings to dark matter.

Going back in time and applying the generalized model to the inflaton field, another set of conditions must be satisfied. We first report the constraints on the parameters of the theory imposed by inflationary requirements and then discuss COBE results for the primordial density fluctuations power spectrum index.

Finally, if we think on the present dark matter component as a remnant of the inflaton field, a combination of both sets of bounds should be considered.

VI.1) Present constraints on the generalized dilaton couplings to dark matter

a) Post–Newtonian bounds

We can constrain the generalized couplings today using the lower bound $\omega \gtrsim 500$, obtained from radar time–delay measurement [22] and the expressions for the present time variation of G (eq. IV.3.5) or the evolution of the Hubble parameter (eq. IV.3.6). There are two possibilities: one way is to constrain the parameters with bounds on $H_0 t_0$ and then check that these values are compatible with limits on the variation rate of G. The other possibility is to constrain m and n directly with bounds on the variability of G.

Following the first way, we consider a firm lower limit on the age of the universe ($t_0^{min} \simeq 7.8 Gyr$) and the present Hubble constant ($H_0^{min} \simeq 48 km\ s^{-1}\ Mpc^{-1}$) (see, e.g., Ref.[28] and references therein), corresponding to $H_0 t_0 \gtrsim 0.4$; we get a bound on the dark matter component couplings, $|1 + n - m| \lesssim 2\varphi_0 = 2\sqrt{\omega + 3/2}$. This represents, for $\omega \sim 500$, very relaxed bounds: $-44 \lesssim m - n \lesssim 46$.

Comparing this result with the one from DGG, we notice that:

–in the Einstein frame this bound reads $|M - N| \lesssim 2$, which would reduce to their result $\beta_l \lesssim 1$ with $M = 2N$, and

–introducing, as they do, these limits in eq. (IV.3.5), we find, in spite of the more general couplings, the same present rate of variation of G ($|d \ln G/dt|_0 \lesssim 5 \times 10^{-12} yr^{-1}$), only marginally consistent with recent limits based on primordial nucleosynthesis [49].

Investigating the possibility of improving this bound, we can use a higher value for $H_0 t_0$, that corresponds, in any case, to more accepted values for these quantities. Values such as $H_0 = 50 km\ s^{-1}\ Mpc^{-1}$ and $t_0 \approx 1.3 - 1.5 \times 10^{10} yr$ lead to $H_0 t_0 \sim 0.7$, but we find that this model, with positive values of ω, does not allow for such a high value. From eq. (IV.3.6) we see that the maximum value is $Ht \approx 2/3$, which is the standard FRW value (in a matter dominated, $k = 0$ Universe). So, if future observational tests point to high H values –as the one reported, e.g. [85], where the adopted value for the Hubble constant is $H_0 = 67 \pm 15 km s^{-1} Mpc^{-1}$ (other quoted values lying within 1 or 2 standard deviations of this value)– these models do not help.

Among all the observational limits on a varying gravitational coupling, those due to conditions for a successful (standard) nucleosynthesis appear to provide the most stringent bounds. However, it must be noticed that nucleosynthesis conditions are less reliable than astrophysical tests since they do not constrain the present value, but rather the average variability from nucleosynthesis until now, and depend on simplifying assumptions that enter the big bang model. We will turn to this in the next section, but let us first consider the necessary conditions for the switchover from visible to invisible matter domination. The invisible component energy redshift can be obtained following its evolution (eq. (IV.1.14)) on the attractor solution (IV.3.1), written as $d\varphi/dt = 2\mu H/(2-\gamma_I-\mu/\varphi_0)$. We obtain $\rho_I \propto a^\alpha$ with

$$\alpha = -3\gamma_I + \frac{(1+n-m)[m(2-\gamma_I)-n\gamma_I]}{2\varphi_0^2(2-\gamma_I)-(1+n-m)}. \qquad (VI.1.1)$$

In the case of invisible pressure–free matter domination, $\gamma_I = 1$, this redshift reads

$$\alpha = -3 + \frac{(1+n-m)(m-n)}{2\varphi_0^2-(1+n-m)}.$$

On the other hand, when the invisible pressure was $\rho_I/3$,

$$\alpha = -4 + \frac{2(1+n-m)(m-2n)}{4\varphi_0^2-3(1+n-m)},$$

that reduces to the usual radiation energy redshift $\rho \propto a^{-4}$ when $m = 2n$. During visible radiation/matter domination the evolution of ρ_I obeys to the usual scaling laws. From these expressions we see that the deviation from the standard energy redshift laws, i.e. the second term on the r.h.s., can be positive or negative, leading respectively to less or more redshift then for the visible component, but is, for values $\varphi_0^2 \sim O(500)$ and $|1+n-m| < 2\varphi_0$, in both cases small. So the most efficient way to have the domination switchover is to assume that invisible matter becomes pressure–free during visible radiation domination. In this case, we must require $\alpha = -3 + (1+n-m)(m-n)/[2\varphi_0^2-(1+n-m)] > -4$, which leads to $|1+n-m| < \sqrt{2}\varphi_0$, tightening the previous constraint. Then, if invisible matter dominates now, it must be pressure–free and this is the reason why we use $\gamma_I = 1$ in the expressions for $\dot{G}$ and Ht for comparison with their present values.

b) Nucleosynthesis bounds

On the other hand, we can get a more stringent limit on the parameters m and n using bounds on the present G rate of change. Accetta, Krauss and Romanelli [49] recently incorporated new measurements of the neutron half-life and reaction uncertainties in nucleosynthesis calculations and thus improved the constraint on the allowed range for G at this epoch. Then, assuming that G decreases in time as a power-law: $G(t)/G(t_0) \propto (t/t_0)^{-\alpha}$, from nucleosynthesis until now, it is possible to constrain its variability. They obtain: $|d \ln G/dt|_0 \lesssim 0.9 \times 10^{-12} yr^{-1}$. With the post-Newtonian bound $\varphi_0 \gtrsim 22.4$ ($\omega \gtrsim 500$) and $t_0 \simeq 1.5 \times 10^{10} yr$ we get $|1+n-m| \lesssim 0.3\varphi_0$.

An independent bound on our parameters can be obtained from the nucleosynthesis calculations of ref. [42] (see also refs.[43], [44], [54]), exploiting the formal analogy between the DGG model and ours when $\beta_I \rightarrow \mu$, where we remember that $\mu = (n-m+1)/(2\varphi_0)$ when $\gamma_I = 1$. They find bounds on $\dot{G}_0/(G_0 H_0) \simeq -4\beta_V \beta_I$: $-0.011 \leq \dot{G}_0/(G_0 H_0) \leq 0.039$ (compatible with and slightly beyond the purely phenomenological limit of [49]), from where we get $-0.011\varphi_0^2 \lesssim m-n-1 \lesssim 0.039\varphi_0^2$. These calculations are based on the requirement that nucleosynthesis, proceeding in the same scenario as in the standard model, yields abundances compatible with observations. This constrains the expansion rate at that epoch and can be translated, through eq. (IV.3.5), to limits on the range of the coupling constants. In the context of generalized scalar–tensor theories of gravity, the expansion rate of the Universe can be affected in 3 ways: G has a different value at nucleosynthesis than today, dark matter contributes to the total energy density, the dilaton field energy can also add its contribution if the dominating matter during nucleosynthesis is the invisible one. In all this work, only a conservative scenario is considered, in which the visible component dominates at the nucleosynthesis epoch (with $\gamma_V = 4/3$), so that the only effect to be taken into account is the different value of G, that will be constant until after nucleosynthesis. Since in the generalized models, with $m \neq 2n$, the gravitational coupling varies during invisible radiation domination, we should explore the implications of the variability of G during nucleosynthesis. However, from expressions (IV.3.4) and (IV.3.5), we see that the variation rate of G during domination of invisible radiation and of invisible matter are comparable. Then, even if G varies at this epoch, to the allowed variation between nucleosynthesis and now corresponds a small variation during the nucleosynthesis process. On the other hand, the contribution of the dilaton field to the total energy density could be more relevant, especially if the JBD field is not massless as we considered in last chapter. In this case, the evolution of the scalar field is no more coupled only to the trace of the matter energy-momentum tensor and consequently continues during radiation domination. In the present work we will not explore this possibility any further.

Altogether these limits imply

$$-7.7 \lesssim n - m \lesssim 4.5,$$

where the upper limit comes from incorporating nucleosynthesis considerations directly in the generalized model and the lower one from the previous result, obtained comparing our model with the presently allowed variability of G.

VI.2) Early Universe constraints on the generalized dilaton couplings to the inflaton field

a) Inflationary requirements

Wang [45] has analyzed in detail the constraints imposed on the generalized model by inflationary requirements. Working in the Einstein frame, first of all, $m < 1$ is needed for a power-law expansion of the Universe with a growing JBD field. Then, The more stingent constraints are, as usually, the ones originated from the high degree isotropy of the cosmic microwave background radiation (CMBR). This condition has to be imposed on the two mechanisms for generating density inhomogeneities: the incomplete filling process of bubbles and inhomogeneities derived from scalar field quantum fluctuations, and corresponds to the relevant lower bound on m in the latter case and to the upper one in the former. In order that the bubble nucleation and percolation processes accomplish successfuly, we have to ask that the percolation probabibility grows with time and, with the same heuristic procedure as in chapter II, require that no more than a fraction $\sim O(10^{-N})$ of the total volume, with $N \simeq 5$, be occupied by false vacuum at the recombination epoch. A constraint on a combination of the parameters n, m and ω is thus obtained, wherefrom, taking $\omega \approx 500$ and using the range of values of n that ensures that ϵ increases in time $(m-1 < n \leq m/2)$, a lower bound on m can be established: $m \lesssim -8$.

From considering density inhomogeneities created by scalar field quantum fluctuations, two conditions must be checked: first, as we said in chapter V, the slow rollover conditions are required in order to apply the standard procedure in calculating density fluctuations and lead to $2\omega + 3 \gg (2-m)^2$. Second, we must convert eq. (V.2.1), together with eq. (V.2.2), to an expression for temperature fluctuations and constrain it with observational bounds. Using the (conservative) bound $\delta T/T \leq 10^{-5}$, for $\theta \geq 1^o$, and making also use of the bubbles constraint, the isotropy from the microwave background reads $p' \gtrsim 6$, that translates in $\omega + 3/2 > 3(2-m)^2$. For $\omega \simeq 500$, the bounds on m are $-11 \lesssim m \lesssim 15$.

From all these considerations, the resulting allowed interval is:

$$-11 \lesssim m \lesssim -8,$$

where the bound from above comes from the requirement of suppression of large bubbles an the one from below is imposed by constraints (from the CMBR) on adiabatic density perturbations.

With a lower value of ω, e.g. $\omega = 25$, the allowed range is very narrow (albeit with the advantage that the power of the coupling term is near unity): $-1 \lesssim m \lesssim -0.2$ and also $4.2 \lesssim m \lesssim 5$. Although this is somehow meaningless since the main motivation for introducing stronger dilaton–inflaton couplings was to avoid the discrepancy of ω with its present observational value, i.e. allow a larger value of $\dot{G}$ than in the original JBD theory, with a small JBD coupling to visible matter (large ω).

b) Bounds on the fluctuation power spectrum from COBE

As we saw in chapter V (eq. (VI.2.1)), the density contrast resulting from power-law inflation models is $(\delta\rho/\rho)^2_{hor} \propto \lambda^{2/(p'-1)} = k^{-2/(p'-1)}$, and an evolution factor k^4 has to be included to obtain the density contrast at the recombination epoch: $(\delta\rho/\rho)^2_{rec} \equiv k^3|\delta_k|^2 = k^{4-2/(p'-1)}$. This leads to a fluctuation power spectrum

$$|\delta_k|^2 = k^{n_s} = k^{1-2/(p'-1)},$$

that can be constrained using COBE results [3] $n_s = 1.1 \pm 0.5$*:

$$1 - \frac{2}{(p'-1)} > 0.6 \Rightarrow p' > 6.$$

Since $p' = (2\omega+3)/(2-m)^2$, for $\omega = 500$, the allowed range for m is:

$$-11 \lesssim m \lesssim 15.$$

* Very recently, the results from the second year of COBE DMR observations have been published: "Cosmic Temperature Fluctuations from Two Years of COBE DMR Observations", C.L. Bennett *et al.*, submitted to *The Astrophysical Journal*. The most likely value for the spectral index resulting from this second analysis is $n = 1.59^{+0.49}_{-0.55}$ (68% CL). If this result comes to be true, "normal" inflation models will be in trouble since spectral indexes larger than unity cannot be obtained with a power-law expansion nor with an exponential one. See anyhow [87] and references therein for inflationary models with blue perturbation spectra.

VI.3) Constraints on the dark matter component as a remnant of the inflationary field

If the same field acts as the inflaton field in the early Universe and accounts at present for the dark matter component, it has to fulfill requirements from both epochs. Combining the two sets of constraints, we obtain:

$$-11 \lesssim m \lesssim -8,$$

$$-18.7 \lesssim n \lesssim -3.5.$$

Conclusions

Using the observational bounds on the JBD field coupling to visible matter, bounds coming from primordial nucleosynthesis calculations, the limits on the age of the universe and the Hubble parameter, we have restricted the coupling constants of this model applied to a today dominant dark matter component: $-7.7 \lesssim n - m \lesssim 4.5$. A higher $H_0 t_0$ value reduces the allowed interval for $(m - n)$ but the most stringent bound comes, at present, from the variability of G allowed by nucleosynthesis considerations. Forthcoming, more stringent observational limits on $\dot{G}$ can substantially reduce this interval.

CMBR isotropy constrains the parameters of the theory when the invisible component is thought of as an inflaton field in the early universe. It imposes conditions on the bubble distribution and evolution, on the amplitude of density fluctuations that result from the JBD field quantum fluctuations and on the fluctuations power spectrum index. Altogether, this conditios are quite stringent: $-11 \lesssim m \lesssim -8$. Here, the constraints fall mainly on the parameter m since the kinetic term (which carries the parameter n) is negligible during the inflationary epoch.

Thinking of the "invisible" field σ as being both the inflaton and the present dark matter component, m and n have to satisfy early conditions and present requirements simultaneously.

More work has to be done with non–conventional scenarios at nucleosynthesis in generalized JBD models. We have presented modifications to the standard JBD theory –direct. generalized JBD couplings to the invisible component and a mass term for the JBD field– that generate extra contibutions to the Universe energy density and to the variation possibilities of G. The allowed interval for $\dot{G}$ is then larger than in the purely phenomenological approach and in standard JBD theory. The implications of the eventual domination of the invisible component and the variability of G during the nucleosynthesis process should be explored.

Conclusions

We have seen that inflation may be implemented in a wide range of particle and gravitational theories and that some of the encountered difficulties can be overcome introducing modifications in the potential terms or in the coupling terms of the involucrated fields. In particular, the problematic feaures of a first order inflation foresee that a softer inflationary process may represent a solution. We then turn to ***extended inflation***: an inflationary model imbedded in a scalar–tensor gravity theory, where the Jordan–Brans–Dicke field extracts some vacuum energy for its evolution, stealing it from the Universe expansion. The result of this energy distribution is a power–law inflation instead of an exponential one.

We have considered and compared many variations to extended inflation proposed in the literature and we have explored in depth the possibility of stronger, generalized couplings of the JBD field with the invisible sector. We have also investigated the consequences of the inclusion of a mass term for the JBD field.

Considering the dark matter component as an invisible perfect fluid which gives the dominant source of the Universe expansion, we have obtained the field equations and their attractor solutions for the evolution of the scale factor and the scalar fields. We have found that the potential term that corresponds to a scalar field that behaves as a dominating perfect fluid decays as a power–law. A particular aspect of this model is that the JBD field, and therefore the gravitational constant, varies with time even if the universe is dominated by a dark radiative component, i.e. by a fluid whose energy–momentum tensor has a vanishing trace.

The domination of an invisible sector is particularly suitable for describing two epochs of the Universe history: today, with a conjectured non–baryonic dark matter that closes the

Universe, and in the early inflationary period. We obtain a model compatible with present observational evidence and whose free parameters (m and n) may be constrained. Using the observational bounds on the JBD field coupling to visible matter, bounds coming from primordial nucleosynthesis calculations, limits on the age of the universe and the Hubble parameter, we have restricted the coupling constants of this model applied to a today dominant dark matter component. A higher $H_0 t_0$ value reduces the allowed interval for $(m-n)$ but the most stringent bound comes, at present, from the variability of G allowed by nucleosynthesis considerations. With present observational values, the resulting interval is not very stringent. Generalized JBD couplings to the invisible component and the inclusion of a mass term for the JBD field generate extra contibutions to the Universe energy density and to the variation possibilities of G. The allowed interval for $\dot{G}$ is then larger than in the purely phenomenological approach and in standard JBD theory.

Going back in time, we apply this model to the inflationary epoch. The cosmic microwave background radiation isotropy constrains the parameters of the theory when the invisible component is thought of as an inflaton field in the early Universe. It imposes conditions on the bubble distribution and evolution, on the amplitude of density fluctuations that result from the JBD field quantum fluctuations and on the fluctuations power spectrum index. Altogether, these conditions are quite stringent. Here, the constraints fall mainly on the parameter m, imposing on it large and negative values ($-11 \lesssim m \lesssim -8$), since the kinetic term, which carries the parameter n, is negligible during the inflationary epoch.

In the context of a decaying cosmological constant, the same field that dominated during inflation may dominate again in the present epoch if the energy density associated with it redshifts slower than the energy density of matter and radiation. In this case, m and n have to satisfy early conditions and present requirements simultaneously.

An important effect of introducing a Jordan–Brans–Dicke field in inflationary scenarios is to obtain tilted models: more power on very large scales in the density perturbation spectrum. This is a feature that may be useful in building models that account for the observed large-scale structure of the Universe. It is a general result of power-law inflation scenarios. In extended inflationary models, structure arises from the JBD field fluctuations rather than from fluctuations of the inflaton field. The estimated fluctuation amplitudes are characterized by the squared ratio of the unification scale to the Planck scale (for not too small ω values) and are slightly in excess of the observational bound. This is another advantage of these models on the exponential inflation models that end up with fluctuation amplitudes of many order-of-magnitude in excess, unless the coupling constant in the potential term is fine-tuned. On the other hand, the observable effects of fluctuations in the JBD field, and hence in the gravitational coupling, is a topic that deserves further investigation. Generalized couplings do not introduce any new ingredients in this topic besides some limitations on the parameter m to observe general inflationary conditions. On the other hand, the inclusion of a potential term for the JBD field, useful to solve the ω problem, increases the amplitude of density fluctuations and re-introduces a fine-tuning problem. After inflation, this mass term tends to dominate and we have to impose the condition that it decays.

Regarding the question of finding a place for scalar-tensor inflationary models in the context of a fundamental theory, an appealing possibility is string theories. In these theories we have, upon reduction to four dimensions, a dilaton field coupled to the curvature scalar of the four-dimensional metric and directly coupled to non-gravitational sectors. Unfortunately, a general result of superstring theory and other higher-dimensional theories is that we can not get enough inflation. Nonetheless, one of the groups working on this point reported some positive results when introducing two scalar fields from the gravitational sector. The implementation of a JBD theory also has some problems, in particular when its predictions are confronted with present observational bounds.

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